Supernova Axion Emissivity with $\Delta(1232)$ Resonance in Heavy Baryon Chiral Perturbation Theory

Shu-Yu Ho

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In collaboration with J. Kim, P. Ko & J. Park (KIAS)

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The Strong CP Problem in QCD

💥The CP-violating term in QCD

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi} G^{c\mu\nu} \widetilde{G}^c_{\mu\nu}$$

strong CP phase

💥Experimental bound from neutron EDM : $|\theta| < 10^{-10}$

💥Theoretically, this problem even more puzzling

$$\theta = \theta_0 + \arg\det(M_u M_d)$$

theta vacuum　　chiral transformation

Why θ is so small is the strong CP problem.

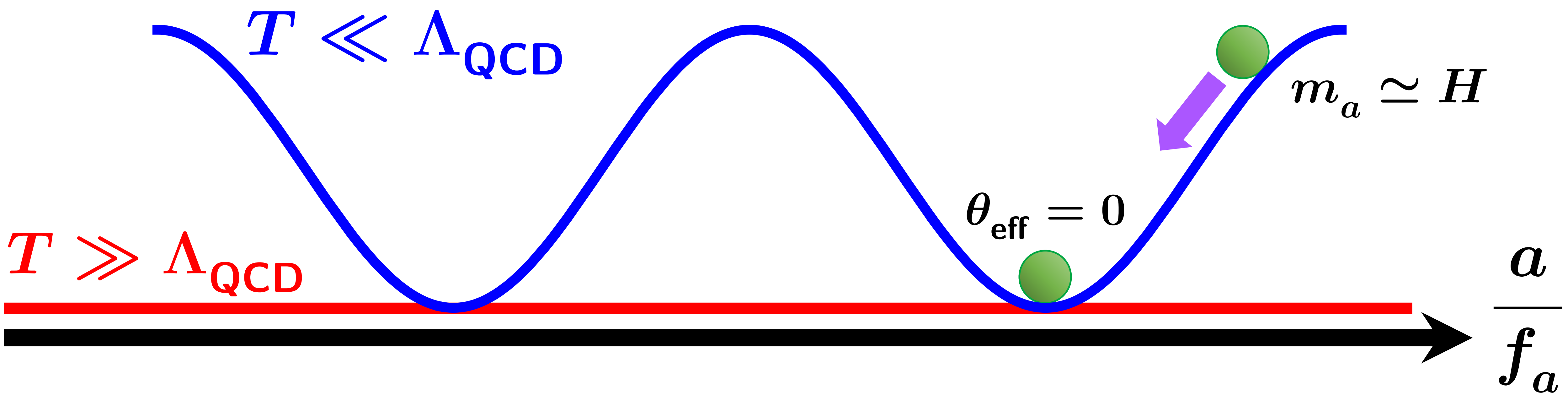
The QCD axion

💥Peccei-Quinn (PQ) mechanism : Strong CP phase is promoted to a dynamical variable :

Peccei, Quinn `77, Weinberg `78, Wilczek `78

$$\mathcal{L}_\theta = \underbrace{\left[\theta + \frac{a(x)}{f_a}\right]}_{\theta_{\rm eff}(x)} \frac{g_s^2}{32\pi} G^{c\mu\nu}\widetilde{G}^c_{\mu\nu}$$

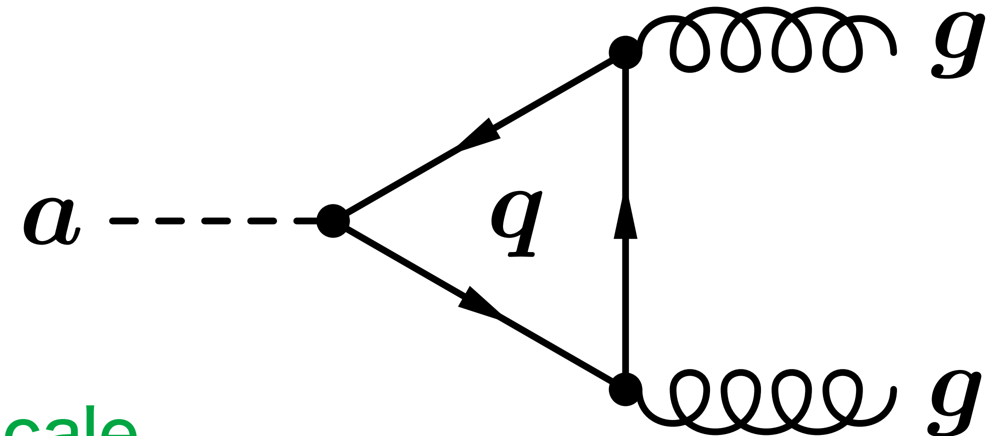
f_a : decay constant



Axion Interactions with SM particles

💥Axion-gluon interaction

$$\mathcal{L}_{agg} = \frac{g_s^2}{32\pi}\frac{a}{f_a}G^{c\mu\nu}\widetilde{G}^c_{\mu\nu}$$



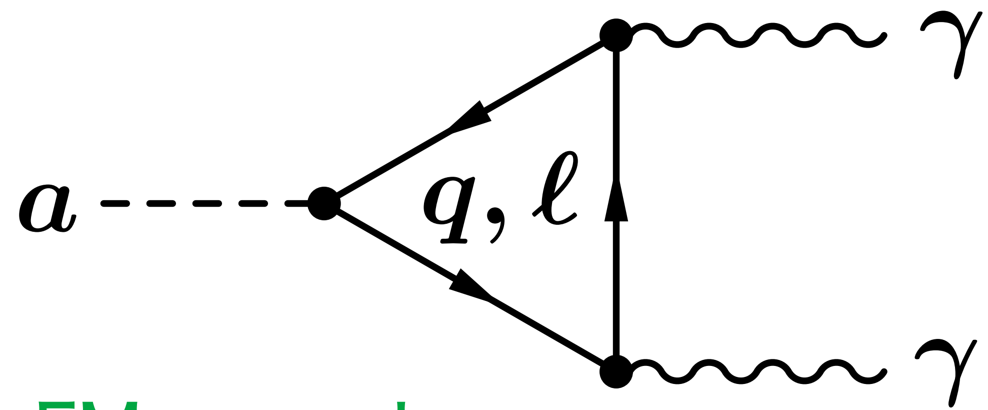
$$f_a = \frac{v_{\mathrm{PQ}}}{N_{\mathrm{DW}}}$$

→ PQ symmetry breaking scale

→ domain wall number

💥Axion-photon interaction

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma}}{4}aF^{\mu\nu}\widetilde{F}_{\mu\nu}$$



$$g_{a\gamma} = \frac{\alpha_{\mathrm{EM}}}{2\pi f_a}\left(-\frac{2}{3}\frac{m_u + 4m_d}{m_u + m_d} + \frac{\mathcal{E}}{\mathcal{N}}\right)$$

→ EM anomaly

→ color anomaly

Axion Interactions with SM particles

💥Axion-electron interaction

E.O.M. & I.P.

$$\mathcal{L}_{aee} = -iC_{ae}\frac{m_e}{f_a}a\overline{\psi_e}\gamma^5\psi_e = C_{ae}\frac{\partial_\mu a}{2f_a}\overline{\psi_e}\gamma^\mu\gamma^5\psi_e$$

C_{ae} : model-dependent coefficient

💥Axion-nucleons interaction

$$\mathcal{L}_{aNN} = \sum_{N=p,n} C_{aN}\frac{\partial_\mu a}{2f_a}\overline{\psi_N}\gamma^\mu\gamma^5\psi_N$$

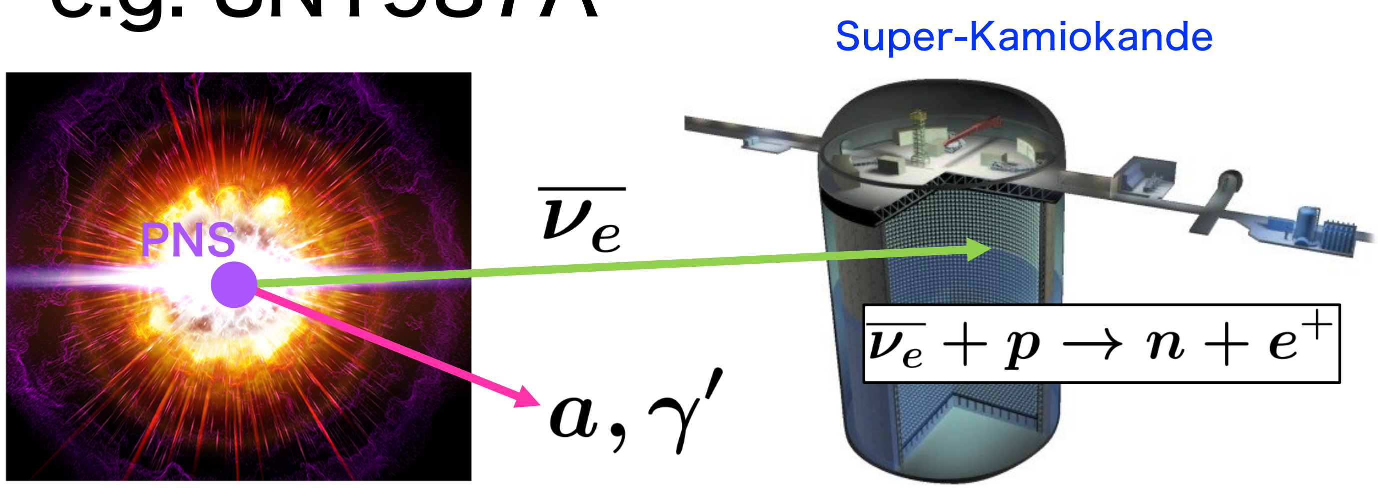
(related to our work)

💥The axion couples to the SM particles with strength inversely proportional to the decay constant. Hence, the axion feebly couples to the SM particles due to the large decay constant.

Axion emission from celestial bodies

💥The axions can be produced copiously from some and hot dense celestial objects such as supernovae (SNe), neutron stars, and white dwarfs.

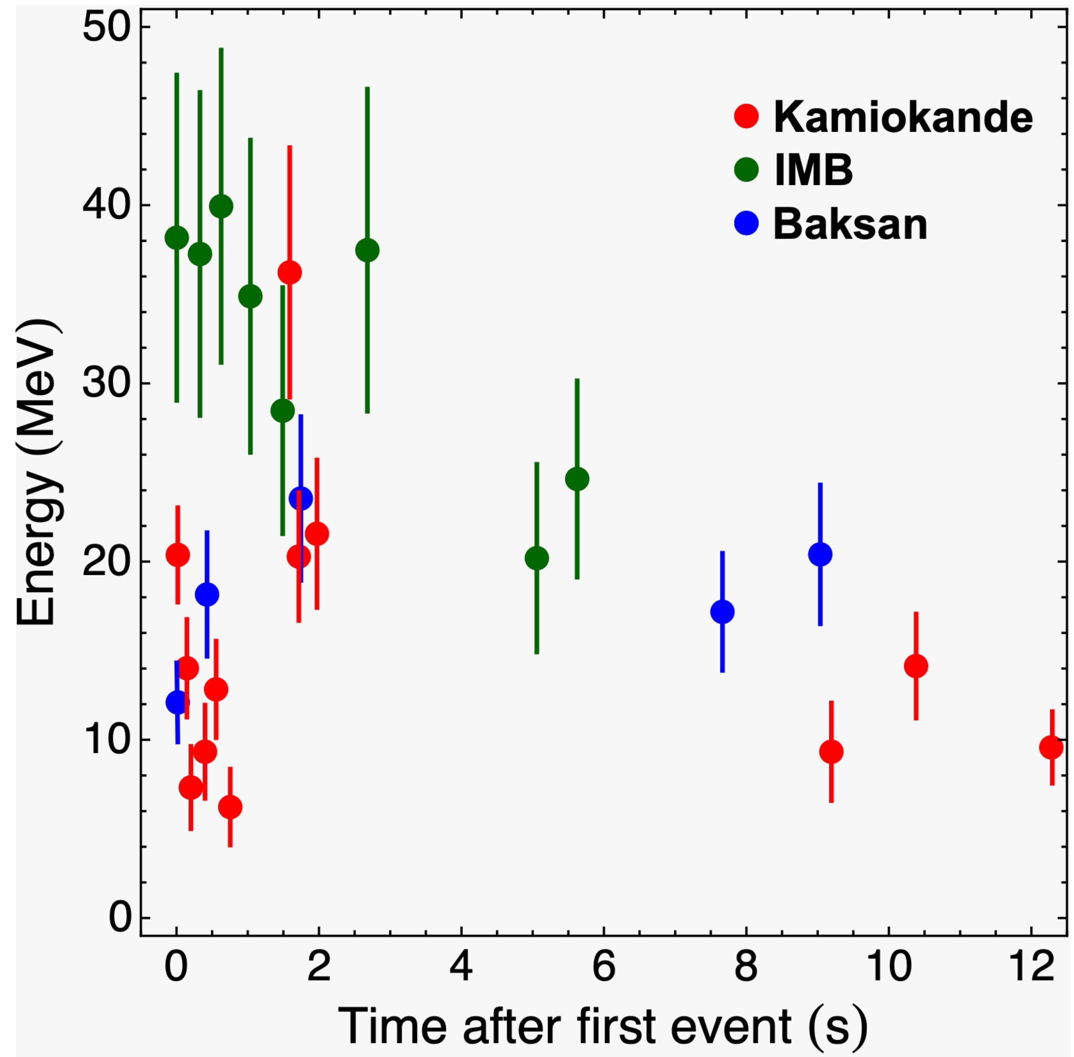
➤e.g. SN1987A



➤Raffelt's criteria

$$L_{\text{new particle}} < L_\nu \sim 3 \times 10^{52} \text{erg/s}$$

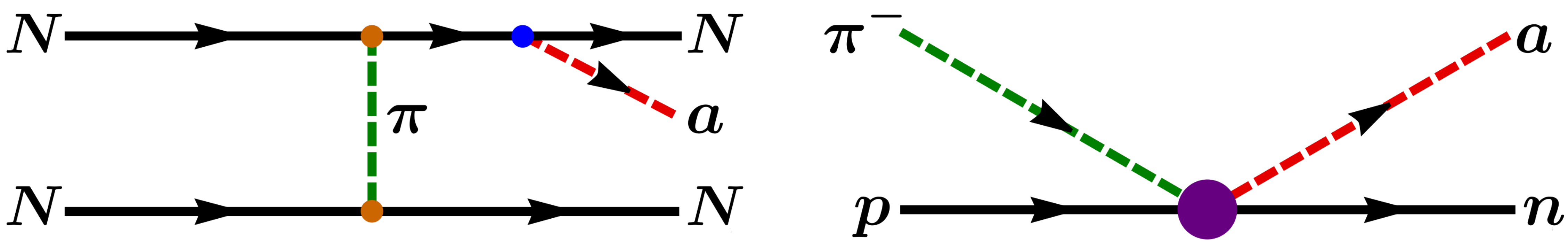
Raffelt `90



Axion emission from Supernovae

💥Two hadronic processes that can create axions inside SNe

- Nucleon-nucleon bremsstrahlung (NNB) : $NN \to NNa$
- Pion-induced Compton-like scattering (PCS) : $\pi^- p \to na$

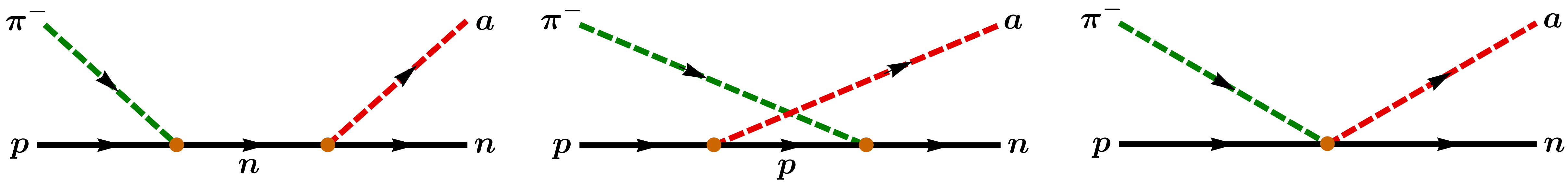


- It has been thought the NNB as the dominant axion emission for a while due to the underestimate of the n_π inside SNe.
- Recent studies have shown that the PCS dominates over the NNB to be the main source of the axion emission inside SNe.

B. Fore and S. Reddy (2020), P. Carenza, et al. (2021), T. Fischer, et al. (2021)

What we did

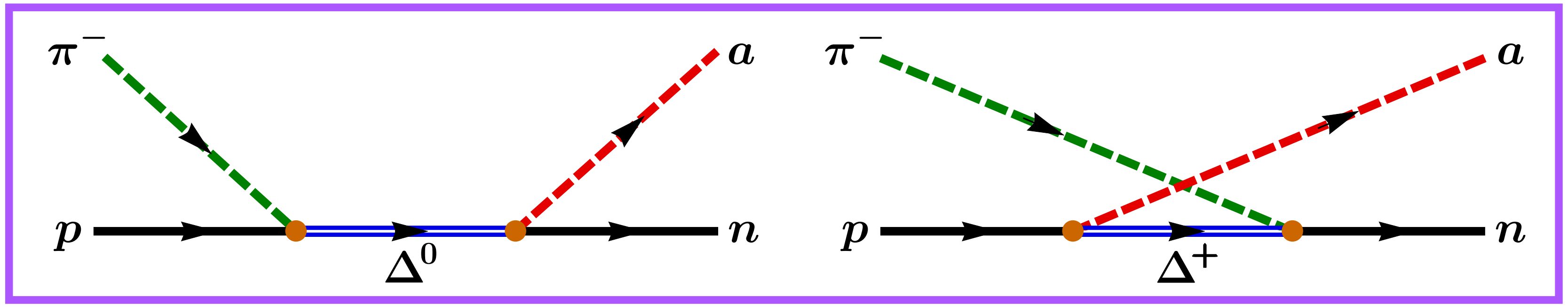
💥We evaluate the supernova axion emission rate including the Δ resonance in the heavy baryon chiral perturbation theory



P. Carenza, B. Fore, M. Giannotti, A. Mirizzi and S. Reddy (2021)

K. Choi, H. J. Kim, H. Seong & C. S. Shin (2022)

$|\boldsymbol{k}_\pi| \simeq m_\pi \ll m_p$



In our work

- ➢For $T_{\text{SN}} \sim 30$ MeV, $|k_\pi| \simeq \sqrt{3m_\pi T_{\text{SN}}} \simeq m_\pi$, $E_\pi \sim 180$ MeV
- ➢The $m_{\pi^- p}$ is somewhere in the middle of Δ and N masses.

Outline

- 💥Introduction
- 💥Heavy Baryon Chiral Perturbation Theory
- 💥Axion Couplings to Baryons and Mesons
- 💥Scattering Cross Section of $\pi^- + p \longrightarrow n + a$
- 💥Supernova Axion Emissivity with $\Delta(1232)$ Resonance
- 💥Summary

Heavy Baryon Formalism

Jenkins & Manohar `91

💥In this formalism, the nucleon is almost on-shell with a nearly unchanged velocity when it exchanges some tiny momentum with the pion

$$p_N^\mu = m_N v^\mu + \delta k_\pi^\mu \qquad v^2 = 1$$

$$m_N/\Lambda_\chi \sim 1$$

$$\delta k_\pi^\mu/\Lambda_\chi \ll 1$$

- Velocity-dependence baryon field

$\Lambda_\chi \sim 1\,\mathrm{GeV}$

$$\mathcal{B}_v(x) = e^{im_B v\cdot x}\mathcal{B}(x) \longrightarrow \overline{\mathcal{B}}(i\not\partial - m_B)\mathcal{B} \to \overline{\mathcal{B}_v} i\not\partial \mathcal{B}_v$$

- The power counting expansion of the effective field theory for pions and baryons can be systematic and well-behaved.
- The algebra of the spin operator formalism can be much simpler than that of the gamma matrix formalism.

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{ \mathcal{A}_\mu, \mathcal{B}_v \right\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\mathcal{A}_\mu, \mathcal{B}_v \right] \right\rangle + \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q \left(\mathbf{\Pi} + \mathbf{\Pi}^\dagger \right) \right\rangle + \cdots ,$$

$\langle \cdots \rangle = \mathrm{tr}(\cdots)$

$f_\pi \simeq 92.4\,\mathrm{MeV}$

➤ $$\mathcal{B}_v = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & \Sigma_v^+ & p_v \\ \Sigma_v^- & -\frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & n_v \\ \Xi_v^- & \Xi_v^0 & -\frac{2}{\sqrt{6}}\Lambda_v \end{pmatrix} , \quad \mathcal{D}_\mu \mathcal{B}_v = \partial_\mu \mathcal{B}_v + \left[\mathcal{V}_\mu, \mathcal{B}_v \right]$$

baryon octet

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) , \quad \mathcal{A}_\mu = \frac{i}{2}\left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) ,$$

$$\xi = \exp\left(\frac{i\boldsymbol{\pi}}{f_\pi} \right) , \quad \mathbf{\Pi} = \xi^2 , \quad \boldsymbol{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}_0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

meson octet

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \rangle + 2D\langle \overline{\mathcal{B}_v} S_v^\mu \{\mathcal{A}_\mu, \mathcal{B}_v\}\rangle + 2F\langle \overline{\mathcal{B}_v} S_v^\mu [\mathcal{A}_\mu, \mathcal{B}_v]\rangle + \frac{1}{4} f_\pi^2 \langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \rangle + b\langle \mathcal{M}_q (\mathbf{\Pi} + \mathbf{\Pi}^\dagger)\rangle + \cdots \ ,$$

- Spin operator : $S_v^\mu = \gamma^5 [\not{v}, \gamma^\mu]/4$ $\quad v \cdot S_v = 0$
- Quark mass matrix : $\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s)$ $\quad \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \Rightarrow \mathrm{SU}(3)_V$
- Under the $\mathbf{SU(3)}_L \otimes \mathbf{SU(3)}_R$ symmetry

$$\mathcal{B}_v \to \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger \ , \quad \mathcal{D}_\mu \mathcal{B}_v \to \mathcal{U}_H (\mathcal{D}_\mu \mathcal{B}_v) \mathcal{U}_H^\dagger \ , \quad \mathbf{\Pi} \to \mathcal{U}_L \mathbf{\Pi} \mathcal{U}_R^\dagger \ ,$$

$$\xi \to \mathcal{U}_L \xi \mathcal{U}_H^\dagger = \mathcal{U}_H \xi \mathcal{U}_R^\dagger \ , \quad \mathcal{V}_\mu \to \mathcal{U}_H \mathcal{V}_\mu \mathcal{U}_H^\dagger + \mathcal{U}_H \partial_\mu \mathcal{U}_H^\dagger \ , \quad \mathcal{A}_\mu \to \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$\mathcal{U}_{L,R} \in \mathrm{SU}(3)_{L,R} \qquad \mathcal{U}_H = \mathcal{U}_H(x) \in \mathrm{SU}(3)_H$$ (local)

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \{\mathcal{A}_\mu, \mathcal{B}_v\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu [\mathcal{A}_\mu, \mathcal{B}_v] \right\rangle + \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q (\mathbf{\Pi} + \mathbf{\Pi}^\dagger) \right\rangle + \cdots ,$$

➢To the first order in $\boldsymbol{\pi} / f_\pi$

$$\xi = \mathbb{I}_{3\times 3} + i\boldsymbol{\pi}/f_\pi \quad \mathcal{A}_\mu = \partial_\mu \boldsymbol{\pi}/f_\pi \quad \mathcal{V}_\mu = 0$$

$$\longrightarrow \mathcal{L}_{\pi B} \supset \frac{2(D+F)}{f_\pi}\left\langle \overline{\mathcal{B}_v} S_v^\mu (\partial_\mu \boldsymbol{\pi}) \mathcal{B}_v \right\rangle + \frac{2(D-F)}{f_\pi}\left\langle \overline{\mathcal{B}_v} S_v^\mu \mathcal{B}_v (\partial_\mu \boldsymbol{\pi}) \right\rangle$$

$$g_A = D + F \simeq 1.254$$

$$\mathcal{L}_{\pi N} = \frac{\sqrt{2} g_A}{f_\pi} \left(\overline{p_v} S_v^\mu n_v \partial^\mu \pi^+ + \overline{n_v} S_v^\mu p_v \partial^\mu \pi^- \right)$$

pion-nucleon interaction

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interactions of meson octet, baryon octet & baryon decuplet

$$\mathcal{L}_{\pi BT} = -i\,\overline{(\mathcal{T}_v^\mu)}_{ijk} v^\rho \mathcal{D}_\rho (\mathcal{T}_{v\mu})_{ijk} + \Delta m_{TB} \overline{(\mathcal{T}_v^\mu)}_{ijk} (\mathcal{T}_{v\mu})_{ijk}$$
$$+\mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)}_{i\ell m}(\mathcal{A}_\mu)_{\ell j}(\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)}_{km}(\mathcal{A}_\mu)_{j\ell}(\mathcal{T}_{v\mu})_{i\ell m}\right] + \cdots\,,$$

- Spin-3/2 Rarita-Schwinger field : $(\mathcal{T}_v^\mu)_{ijk}$

$$\Delta m_{TB} = m_T - m_B$$
$$\mathcal{C} \simeq 3g_A/2$$

- Under the $\mathbf{SU(3)}_L \otimes \mathbf{SU(3)}_R$ symmetry

$$(\mathcal{T}_v^\mu)_{ijk} \to (\mathcal{U}_H)_{i\ell}(\mathcal{U}_H)_{jm}(\mathcal{U}_H)_{kn}(\mathcal{T}_v^\mu)_{lmn}$$

- Rep. of the Delta baryon : $(\mathcal{T}_{v\mu})_{112} = \frac{1}{\sqrt{3}}\Delta^+_{v\mu}\,, \quad (\mathcal{T}_{v\mu})_{122} = \frac{1}{\sqrt{3}}\Delta^0_{v\mu}$

$$\longrightarrow \quad \mathcal{L}_{\pi N\Delta} = \frac{\mathcal{C}}{\sqrt{6}f_\pi}\left(\overline{n_v}\Delta^+_{v\mu}\partial^\mu\pi^- + \overline{\Delta^+_{v\mu}}n_v\partial^\mu\pi^+ - \overline{p_v}\Delta^0_{v\mu}\partial^\mu\pi^+ - \overline{\Delta^0_{v\mu}}p_v\partial^\mu\pi^-\right)$$

pion-nucleon-delta interaction

Hadronic Axial Vector Currents

💥The Lagrangian invariant under the local $\mathrm{SU(3)}_H$ symmetry

$$\mathcal{L}_{\pi B} \supset i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{\mathcal{A}_\mu, \mathcal{B}_v\right\}\right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\mathcal{A}_\mu, \mathcal{B}_v\right]\right\rangle$$

$$\mathcal{L}_{\pi BT} \supset \mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)_{i\ell m}}(\mathcal{A}_\mu)_{\ell j}(\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}}(\mathcal{A}_\mu)_{j\ell}(\mathcal{T}_{v\mu})_{i\ell m}\right]$$

$$\mathcal{B}_v \to \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger\ , \quad \mathcal{D}_\mu \mathcal{B}_v \to \mathcal{U}_H (\mathcal{D}_\mu \mathcal{B}_v) \mathcal{U}_H^\dagger\ , \quad \mathcal{A}_\mu \to \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$(\mathcal{T}_v^\mu)_{ijk} \to (\mathcal{U}_H)_{i\ell}(\mathcal{U}_H)_{jm}(\mathcal{U}_H)_{kn}(\mathcal{T}_v^\mu)_{lmn}$$

➤Noether's theorem : $\xi \to \mathcal{U}_H \xi \mathcal{U}_R^\dagger \to (1+i\epsilon^A t^A)\xi$ $\quad \epsilon^A \to 0$

$$\mathcal{J}_{\pi B}^{A\mu} = D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{\xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v\right\}\right\rangle + F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v\right]\right\rangle$$

$$+\frac{1}{2} v^\mu \left\langle \overline{\mathcal{B}_v}\left[\xi^\dagger t^A \xi - \xi t^A \xi^\dagger, \mathcal{B}_v\right]\right\rangle\ ,$$

$t^A\ (A = 1, 2, \cdots, 8)$

Gell-Mann matrices

$$\mathcal{J}_{\pi BT}^{A\mu} = \frac{\mathcal{C}}{2}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)_{i\ell m}}\left(\xi^\dagger t^A \xi + \xi t^A \xi^\dagger\right)_{\ell j}(\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}}\left(\xi^\dagger t^A \xi + \xi t^A \xi^\dagger\right)_{\ell j}(\mathcal{T}_{v\mu})_{i\ell m}\right]$$

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The QCD axion Lagrangian

💥 $v_{\mathrm{PQ,EW}} \gg T \gg \Lambda_{\mathrm{QCD}}$ & at leading order in a/f_a

$$\mathcal{L}_{aqg} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\widetilde{G}^{c\mu\nu} + \overline{q} i \gamma^\mu \partial_\mu q$$
$$- \left(\overline{q_L}\mathcal{M}_q q_R + \mathrm{h.c.}\right) + \frac{\partial_\mu a}{2 f_a}\overline{q}\gamma^\mu\gamma^5 \mathcal{X}_q q$$

axion derivative interactions

- Light quark fields : $q = (u, d, s)^{\mathsf{T}}$
- Axion coupling matrix : $\mathcal{X}_q = \mathrm{diag}(X_u, X_d, X_s)$
- Typically, one introduces an SM-singlet complex scalar field $\Phi \sim (1,1)_0$ with a PQ charge in UV models. After the ~~v_{PQ}~~, the phase of $\Phi \propto e^{ia/f_a}$ is then identified as the axion.

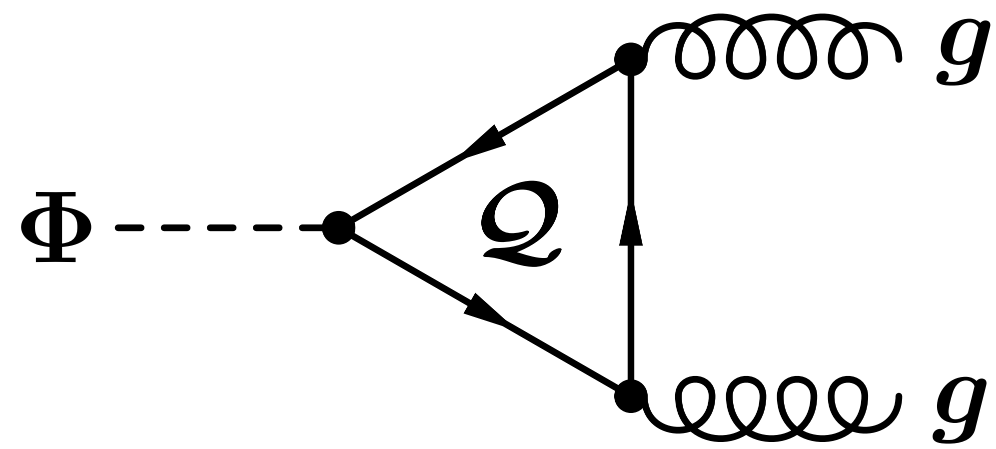
Axion models

💥KSVZ model

Kim `79,
Shifman, Vainshtein, Zakharov `80

- ➢The QCD anomaly is realized by introducing a heavy vector-like fermion.

$$\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R \sim (\mathbf{3}, \mathbf{1})_0$$



- ➢Interactions : $y_Q \Phi \overline{\mathcal{Q}_L} \mathcal{Q}_R + \mathrm{h.c.}$
- ➢Under PQ symmetry

$$\Phi \to e^{iq_{\mathsf{PQ}}} \Phi \qquad \mathcal{Q}_L \to e^{iq_{\mathsf{PQ}}/2} \mathcal{Q}_L \qquad \mathcal{Q}_R \to e^{-iq_{\mathsf{PQ}}/2} \mathcal{Q}_R$$

- ➢Only $\boldsymbol{\Phi}$ and $\boldsymbol{\mathcal{Q}}$ have PQ charges : $X_u = X_d = X_s = 0$

(at tree level)

Axion models

💥DFSZ model

Dine, Fischler, Srednicki `81
Zhitnitsky `80

- The QCD anomaly is induced by assuming 2HDM $\boldsymbol{H_u}$ & $\boldsymbol{H_d}$ couples to the SM quark fields.
- Interactions : $H_u^\dagger H_d (\Phi^*)^2$ $\quad \overline{Q_L}(\mathcal{Y}_u \widetilde{H}_u U_R + \mathcal{Y}_d H_d D_R) + \mathrm{h.c.}$
- Under PQ symmetry

$$\Phi \to e^{iq_{\mathsf{PQ}}}\Phi \quad H_u \to e^{-iq_{\mathsf{PQ}}} H_u \quad H_d \to e^{iq_{\mathsf{PQ}}} H_d$$

$$Q_L \to Q_L \quad U_R \to e^{-iq_{\mathsf{PQ}}} U_R \quad D_R \to e^{-iq_{\mathsf{PQ}}} D_R$$

- The axion as a linear combination of the CP-odd scalars can couple to the SM quarks : $X_u = \dfrac{\cos^2\beta}{3}\ , \quad X_{d,s} = \dfrac{\sin^2\beta}{3}$

$\tan\beta = \dfrac{\langle \boldsymbol{H_u} \rangle}{\langle \boldsymbol{H_d} \rangle}$

Axion couplings to hadrons

💥Below the QCD confinement scale, one can remove the axion-gluon coupling by the chiral trans. on the light quark fields

$$q \to \mathcal{R}_a q = \exp\left(-i\gamma^5 \frac{a}{2f_a}\mathcal{Q}_a\right) q \,, \quad \langle \mathcal{Q}_a \rangle = 1$$

act on the quark flavor space

$$\longrightarrow \int \mathcal{D}q\mathcal{D}\bar{q} \to \int \mathcal{D}q\mathcal{D}\bar{q} \exp\left[i \int \mathrm{d}^4x \left(-\frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\widetilde{G}^{c\mu\nu}\langle \mathcal{Q}_a \rangle \right) \right]$$

$\epsilon^{0123} = +1$

💥To avoid the axion-π^0 mass mixing, the customary choice is

$$\mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\mathrm{tr}\left(\mathcal{M}_q^{-1}\right)} = \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\,\mathrm{diag}\left(\frac{1}{m_u}, \frac{1}{m_d}, \frac{1}{m_s}\right)$$

Axion couplings to hadrons

💥Under the chiral trans., the quark kinetic term is shifted as

$$\overline{q} i\gamma^\mu \partial_\mu q \to \overline{q} i\gamma^\mu \partial_\mu q + \frac{\partial_\mu a}{2f_a} \overline{q}\gamma^\mu \gamma^5 \mathcal{Q}_a q + \mathcal{O}\left(\frac{a^2}{f_a^2}\right)$$

💥The light quark mass term becomes

$$\overline{q_L} \mathcal{M}_q q_R \to \overline{q_L} \mathcal{M}_a q_R \ , \quad \overline{q_R} \mathcal{M}_q q_L \to \overline{q_R} \mathcal{M}_a^\dagger q_L$$

$$\mathcal{M}_a \equiv \mathcal{R}_a \mathcal{M}_q \mathcal{R}_a$$

Up to the second order in the axion field

$$\mathcal{M}_a = \mathcal{M}_q - i\frac{a}{2f_a}\{\mathcal{M}_q, \mathcal{Q}_a\} - \frac{a^2}{8f_a^2}\{\{\mathcal{M}_q, \mathcal{Q}_a\}, \mathcal{Q}_a\} + \mathcal{O}\left(\frac{a^3}{f_a^3}\right)$$

Axion couplings to hadrons

💥The resulting Lagrangian with only the axion and quark fields

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{q} i\gamma^\mu \partial_\mu q - \left(\overline{q_L}\mathcal{M}_a q_R + \overline{q_R}\mathcal{M}_a^\dagger q_L\right) + \frac{\partial_\mu a}{2f_a}\overline{q}\gamma^\mu\gamma^5\left(\mathcal{X}_q + \mathcal{Q}_a\right)q$$

➢Use $M_{3\times3} = 2\langle M_{3\times3}\,\hat{t}^A\rangle\hat{t}^A$ for any 3x3 Hermitian matrix $M_{3\times3}$

$$\{\hat{t}^A\} = \{t^A\}\cup\{t^0\} \quad t^0 = \mathbb{I}_{3\times3}/\sqrt{6}$$

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{q} i\gamma^\mu \partial_\mu q + \left\langle \mathcal{M}_a q_R \overline{q_L} + \mathcal{M}_a^\dagger q_L \overline{q_R}\right\rangle + \frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A\right\rangle \mathcal{J}_q^{A\mu}$$

quark axial vector currents $\mathcal{J}_q^{A\mu} = \overline{q}\gamma^\mu\gamma^5\hat{t}^A q$

➢The next step is to replace the light quark fields with the corresponding hadron fields in the HBChPT.

Axion couplings to hadrons

💥Axion couplings to pions : $\mathcal{U}_L(q_L\overline{q_R})\mathcal{U}_R^\dagger \sim \mathcal{U}_L\mathbf{\Pi}\,\mathcal{U}_R^\dagger$

$$\langle \mathcal{M}_a q_R\overline{q_L} + \mathcal{M}_a^\dagger q_L\overline{q_R}\rangle \longrightarrow \mathcal{L}_{a\pi} = \frac{1}{2}f_\pi^2 B_0\langle \mathcal{M}_a\mathbf{\Pi}^\dagger + \mathcal{M}_a^\dagger\mathbf{\Pi}\rangle$$

➤ $\mathcal{M}_a = \mathcal{M}_q - i\frac{a}{2f_a}\{\mathcal{M}_q, \mathcal{Q}_a\} - \frac{a^2}{8f_a^2}\{\{\mathcal{M}_q, \mathcal{Q}_a\}, \mathcal{Q}_a\}$

pion mass — axion-π^0 mass mixing — axion mass

$$\mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\mathrm{tr}\left(\mathcal{M}_q^{-1}\right)}$$

➤ Axion mass : $m_a = \sqrt{\frac{z}{(1+z)(1+z+w)}}\frac{f_\pi m_\pi}{f_a} \simeq 6\,\mathrm{meV}\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)$

$$z \equiv m_u/m_d \simeq 0.485$$

$$w \equiv m_u/m_s \simeq 0.025$$

$$m_\pi = \sqrt{B_0(m_u+m_d)} \simeq 139.57\,\mathrm{MeV}$$

Axion couplings to hadrons

💥Axion couplings to pions and baryons : $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q+\mathcal{Q}_a\right)\hat{t}^A\right\rangle\mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi B} = \frac{\partial_\mu a}{f_a}\left[\left\langle\left(\mathcal{X}_q+\mathcal{Q}_a\right)t^A\right\rangle\mathcal{J}_{\pi B}^{A\mu}+\frac{1}{3}S\left\langle\mathcal{X}_q+\mathcal{Q}_a\right\rangle\mathcal{J}_{\pi B}^{0\mu}\right]$$

$$\mathcal{J}_{\pi B}^{0\mu} = \left\langle\overline{\mathcal{B}_v}S_v^\mu\mathcal{B}_v\right\rangle$$

➢ Axion couplings to pions and nucleons :

$$\mathcal{L}_{a\pi N} = \frac{\partial_\mu a}{f_a}\left[C_{ap}\overline{p_v}S_v^\mu p_v+C_{an}\overline{n_v}S_v^\mu n_v+\frac{i}{2f_\pi}C_{a\pi N}\left(\pi^+\overline{p_v}v^\mu n_v-\pi^-\overline{n_v}v^\mu p_v\right)\right]$$

$$C_{ap} = X_u\Delta u+X_d\Delta d+X_s\Delta s+\frac{\Delta u+z\Delta d+w\Delta s}{1+z+w}$$

$$C_{an} = X_d\Delta u+X_u\Delta d+X_s\Delta s+\frac{z\Delta u+\Delta d+w\Delta s}{1+z+w}$$

$$C_{a\pi N} = \frac{1}{\sqrt{2}}\left(X_u-X_d+\frac{1-z}{1+z+w}\right) = \frac{C_{ap}-C_{an}}{\sqrt{2}g_A}$$

nucleon matrix element

$$\langle p|\overline{q}S_v^\mu q|p\rangle = s^\mu\Delta q/2$$

$$\Delta u = 0.847$$

$$\Delta d = -0.407$$

$$\Delta s = -0.035$$

Axion couplings to hadrons

💥Axion couplings to pions and baryons : $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A\right\rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi BT} = \frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q + \mathcal{Q}_a\right)t^A\right\rangle \mathcal{J}_{\pi BT}^{A\mu}$$

$$\epsilon_{ijk}\overline{(\mathcal{T}_v^\mu)_{ijm}}(\mathcal{B}_v)_{mk} = 0$$

➢Axion couplings to pions, nucleons and Delta baryons :

$$\mathcal{L}_{aN\Delta} = \frac{\partial_\mu a}{2f_a}\left[C_{ap\Delta}\left(\overline{p_v}\Delta_\mu^+ + \overline{\Delta_\mu^+}p_v\right) + C_{an\Delta}\left(\overline{n_v}\Delta_\mu^0 + \overline{\Delta_\mu^0}n_v\right)\right]$$

$$C_{ap\Delta} = C_{an\Delta} \equiv C_{aN\Delta} = -\frac{\mathcal{C}}{\sqrt{3}}\left(X_u - X_d + \frac{1-z}{1+z+w}\right) = -\frac{\sqrt{3}}{2}\left(C_{ap} - C_{an}\right)$$

➢Note that $C_{a\pi N}$ and $C_{aN\Delta}$ are not independent parameters since they can be expressed in terms of $C_{ap} - C_{an}$.

Axion couplings to hadrons

💥Numerically, we obtain

$$C_{ap} = \begin{cases} +0.430 & \text{KSVZ model} \\ +0.712 - 0.430\sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{an} = \begin{cases} +0.002 & \text{KSVZ model} \\ -0.134 + 0.406\sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{a\pi N} = \begin{cases} +0.241 & \text{KSVZ model} \\ +0.477 - 0.471\sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{aN\Delta} = \begin{cases} -0.370 & \text{KSVZ model} \\ -0.732 + 0.724\sin^2\beta & \text{DFSZ model} \end{cases}$$

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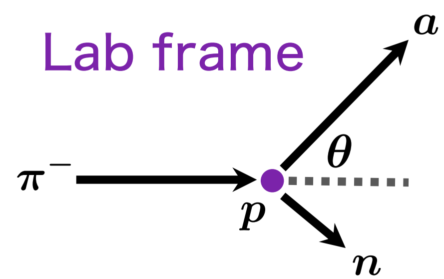
Squared matrix element

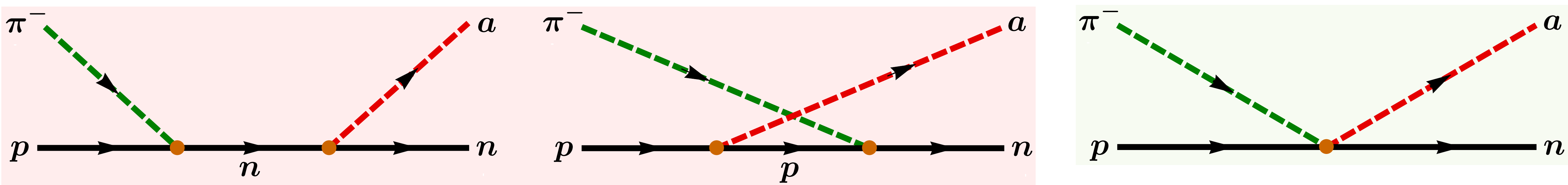
$$\overline{\left|\mathcal{M}_{\pi^- p\to na}\right|^2} = \frac{2m_N^2}{f_\pi^2 f_a^2}\left\langle P_+\Omega^\dagger P_+\Omega\right\rangle$$

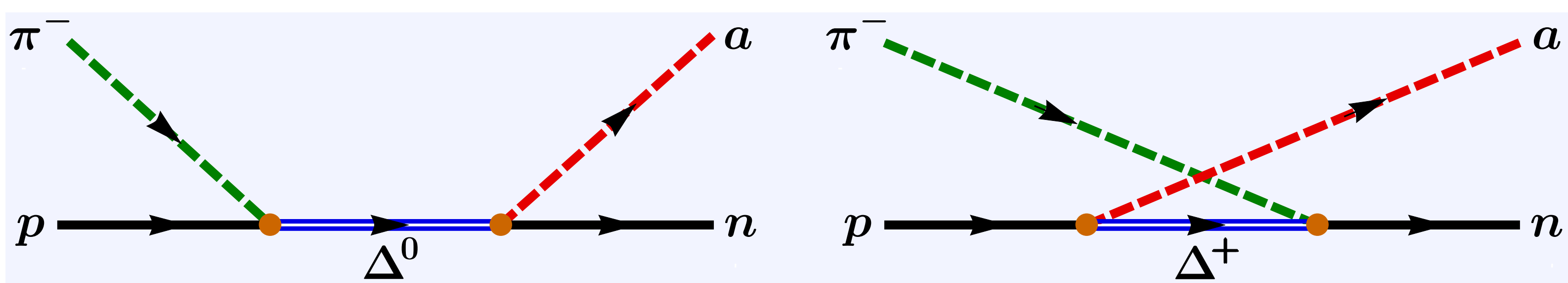
$$P_+ = \mathrm{diag}(1,1,0,0)$$
$$\Theta = \mathrm{diag}\left(e^{+i\theta}, e^{-i\theta}, e^{+i\theta}, e^{-i\theta}\right)$$



$$\Omega = \frac{\sqrt{2}g_A|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{4E_\pi}\left(C_{ap}\Theta - C_{an}\Theta^\dagger\right) + \frac{C_{a\pi N}|\boldsymbol{k}_a|}{2}\mathbb{I}_{4\times 4}$$
$$+\frac{\mathcal{C}|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{6\sqrt{6}}\left[\frac{C_{an\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4} - \Theta^\dagger\right)}{E_\pi - \Delta m + i\Gamma_\Delta/2} + \frac{C_{ap\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4} - \Theta\right)}{E_\pi + \Delta m - i\Gamma_\Delta/2}\right]$$

$$\Delta m = m_\Delta - m_N \simeq 293\,\mathrm{MeV}$$
$$\Gamma_\Delta \simeq 117\,\mathrm{MeV}$$





Scattering cross section

💥Cross section formula

$$\sigma_{\pi^- p\to na} = \int \frac{\mathrm{d}^3\boldsymbol{k}_a}{(2\pi)^3 2E_a}\frac{\mathrm{d}^3\boldsymbol{k}_n}{(2\pi)^3 2E_n}(2\pi)^4\delta^{(4)}(k_\pi + k_p - k_a - k_n)\frac{\overline{\left|\mathcal{M}_{\pi^- p\to na}\right|^2}}{4\left[(k_\pi\cdot k_p)^2-(m_\pi m_N)^2\right]^{1/2}}$$

➤ $$\sigma_{\pi^- p\to na} = \frac{E_\pi m_N^2}{16\pi f_\pi^2 f_a^2|\boldsymbol{k}_\pi|}\mathcal{G}_a(|\boldsymbol{k}_\pi|)$$

$$C_\pm \equiv (C_{ap}\pm C_{an})/2 \qquad \bar{\Gamma}_\Delta = \Gamma_\Delta/2$$

$$\mathcal{G}_a(|\boldsymbol{k}_\pi|) = \frac{2g_A^2(2C_+^2+C_-^2)}{3}\left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 + C_{a\pi N}^2\left(\frac{E_\pi}{m_N}\right)^2 + \frac{8\sqrt{2}g_A C_{a\pi N}C_-}{3}\left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2\left(\frac{E_\pi}{m_N}\right)$$
$$+\frac{4C_{aN\Delta}^2\mathcal{C}^2}{81}\frac{E_\pi^2(\Delta m^2+2E_\pi^2+\bar{\Gamma}_\Delta^2)}{\left[(\Delta m - E_\pi)^2+\bar{\Gamma}_\Delta^2\right]\left[(\Delta m + E_\pi)^2+\bar{\Gamma}_\Delta^2\right]}\left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2$$

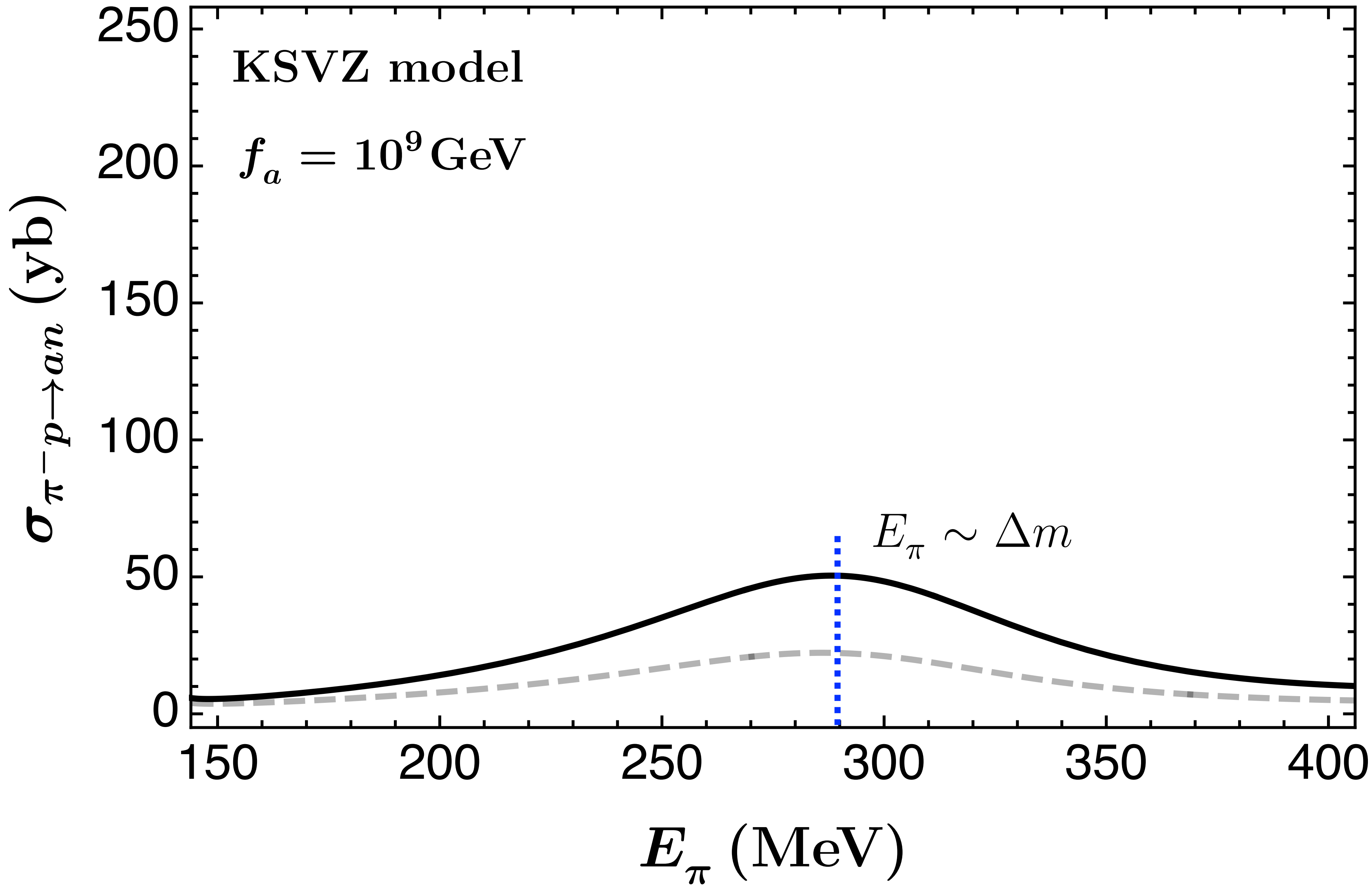
Delta resonance

$$-\frac{8\sqrt{3}g_A C_{aN\Delta}\mathcal{C}}{27}\frac{E_\pi\left[(\Delta m^2-E_\pi^2)(C_+\Delta m + C_-E_\pi)+\bar{\Gamma}_\Delta^2(C_+\Delta m - C_-E_\pi)\right]}{\left[(\Delta m - E_\pi)^2+\bar{\Gamma}_\Delta^2\right]\left[(\Delta m + E_\pi)^2+\bar{\Gamma}_\Delta^2\right]}\left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2$$
$$-\frac{16\sqrt{6}C_{a\pi N}C_{aN\Delta}\mathcal{C}}{27}\frac{E_\pi^2(\Delta m^2-E_\pi^2-\bar{\Gamma}_\Delta^2)}{\left[(\Delta m - E_\pi)^2+\bar{\Gamma}_\Delta^2\right]\left[(\Delta m + E_\pi)^2+\bar{\Gamma}_\Delta^2\right]}\left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2\left(\frac{E_\pi}{m_N}\right)$$

$\boldsymbol{m_N \gg |k_\pi|, E_\pi}$

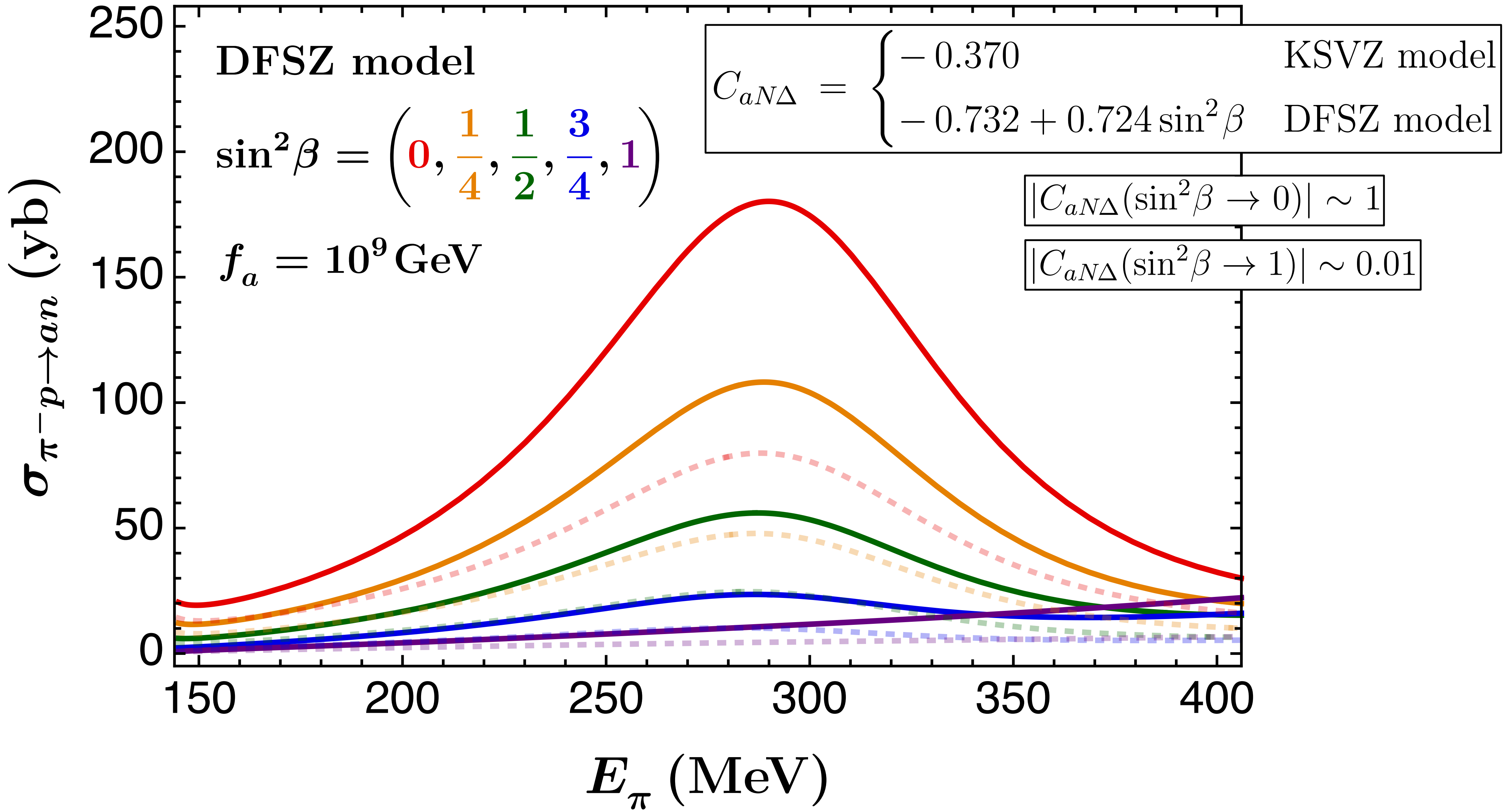
Scattering cross section v.s. E_π

💥KSVZ model



Scattering cross section v.s. E_π

💥DFSZ model



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Supernova Axion Emissivity

💥Emission rate (the energy loss per unit volume and time)

$$\dot{\mathcal{E}}_a = \int \frac{\mathrm{d}^3\boldsymbol{k}_\pi}{(2\pi)^3 2E_\pi} \frac{\mathrm{d}^3\boldsymbol{k}_p}{(2\pi)^3 2E_p} \frac{\mathrm{d}^3\boldsymbol{k}_a}{(2\pi)^3 2E_a} \frac{\mathrm{d}^3\boldsymbol{k}_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}\left(k_\pi + k_p - k_a - k_n\right)$$
$$\times f_\pi(|\boldsymbol{k}_\pi|) f_p(|\boldsymbol{k}_p|) \left[1 - f_n(|\boldsymbol{k}_n|)\right] \left|\mathcal{M}_{\pi^- p \to na}\right|^2 E_a ,$$

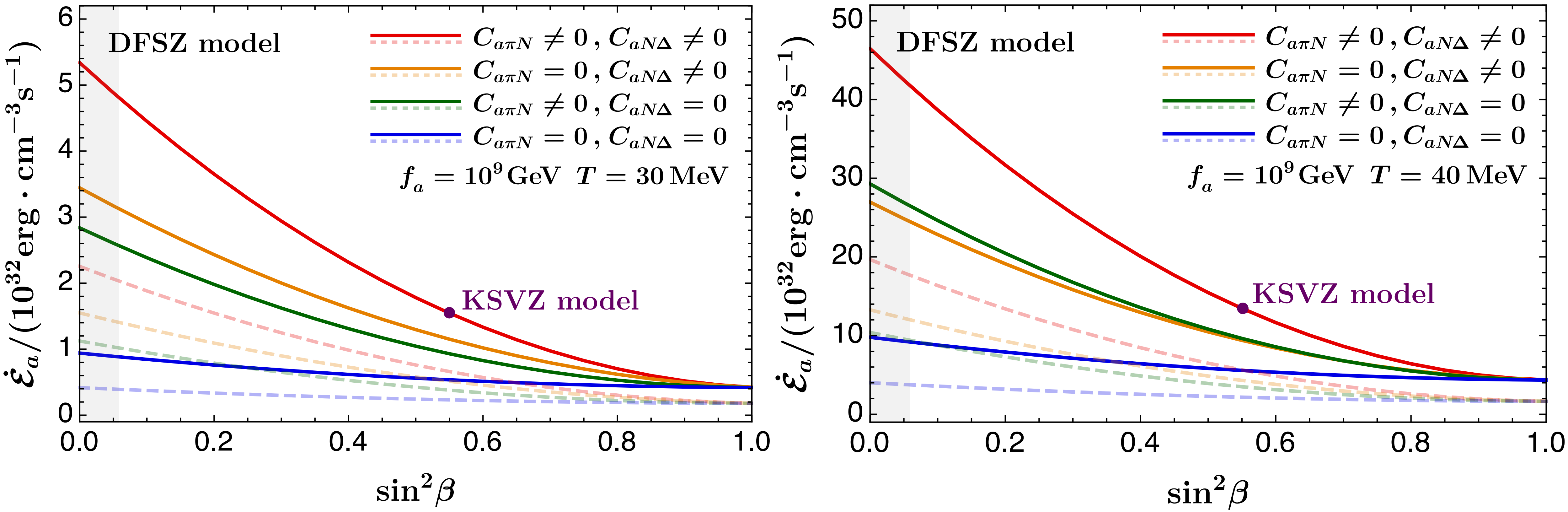
$$f_j(|\boldsymbol{k}_j|) = 1/\left[e^{(E_j-\mu_j)/T} \pm 1\right]$$

➤ $$\dot{\mathcal{E}}_a = \frac{z_\pi z_p}{f_\pi^2 f_a^2} \sqrt{\frac{m_N^7 T^{11}}{128\pi^{10}}} \int_0^\infty \mathrm{d}x_p \frac{x_p^2 e^{x_p^2}}{\left(e^{x_p^2} + z_n\right)\left(e^{x_p^2} + z_p\right)} \int_0^\infty \mathrm{d}x_\pi \frac{x_\pi^2 \epsilon_\pi \left[\mathcal{G}_a(x_\pi) + \Delta\mathcal{G}_a(x_\pi)\right]}{e^{\epsilon_\pi - y_\pi} - z_\pi}$$

$$x_p = \frac{|\boldsymbol{k}_p|}{\sqrt{2m_N T}}, \quad x_\pi = \frac{|\boldsymbol{k}_\pi|}{T}, \quad \epsilon_\pi = \frac{E_\pi}{T}, \quad y_\pi = \frac{m_\pi}{T} \qquad z_j = e^{(\mu_j - m_j)/T}$$

$$\Delta\mathcal{G}_a(x_\pi) = \frac{\sqrt{2} g_A C_{a\pi N} C_-}{3} \frac{E_\pi^4 - 3\left(\Delta m^2 - \bar{\Gamma}_\Delta^2\right)E_\pi^2 + 2\left(\Delta m^2 + \bar{\Gamma}_\Delta^2\right)^2}{\left[(\Delta m - E_\pi)^2 + \bar{\Gamma}_\Delta^2\right]\left[(\Delta m + E_\pi)^2 + \bar{\Gamma}_\Delta^2\right]} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right)$$

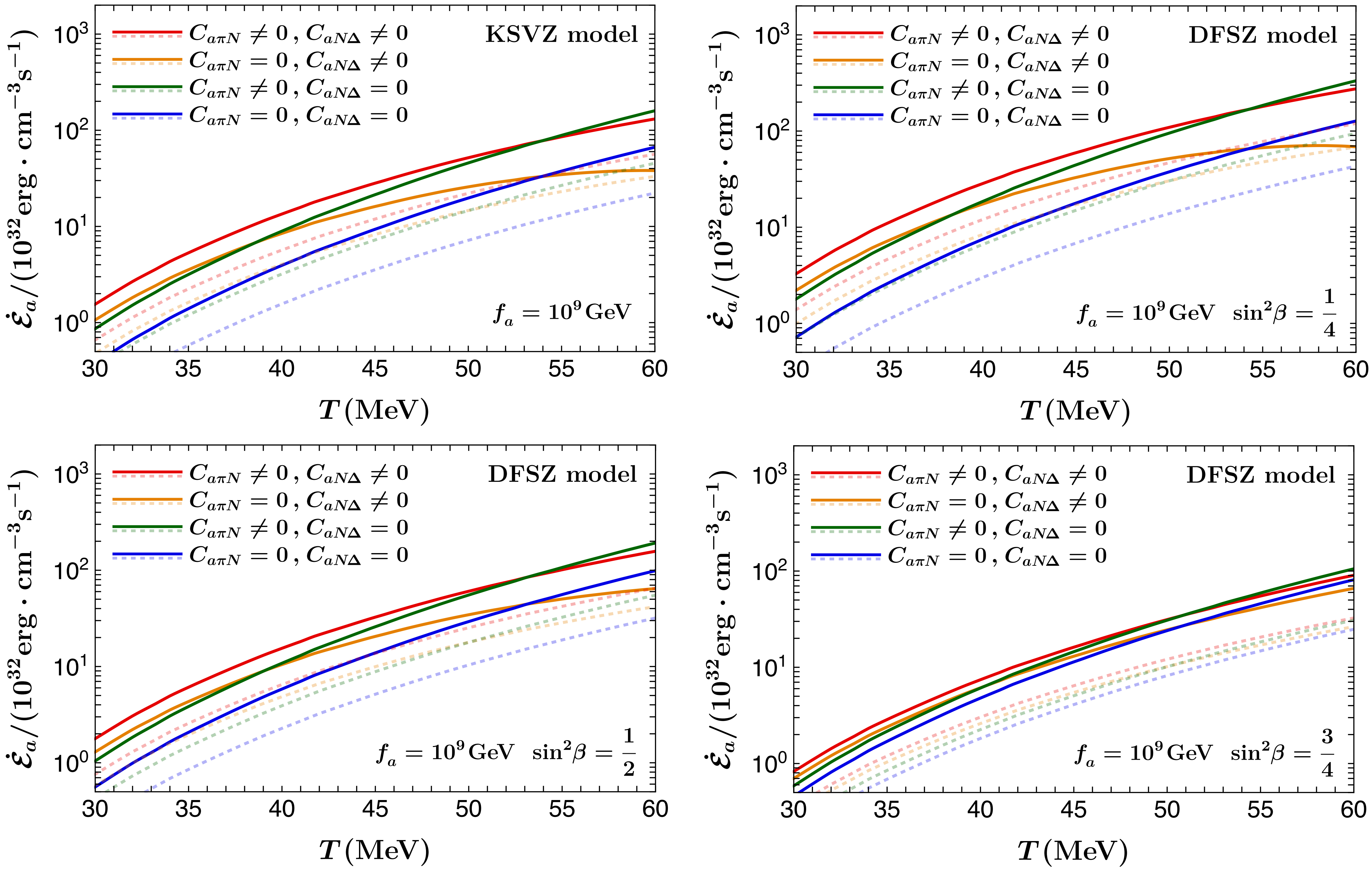
Supernova Axion Emissivity v.s. $\sin^2\beta$



💥The gray band is excluded by tree-level unitarity of fermion scattering : $0.25 \lesssim \tan\beta \lesssim 170$ Luzio, et al. 2021

💥Supernova axion emissivity can be enhanced at most by a factor of ~5 for $\beta \to 0$ compared to the earlier studies.

Supernova Axion Emissivity v.s. T



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Summary

- 💥 We have estimated the supernova axion emissivity with the Δ(1232) resonance in the HBChPT.
- 💥 We have noticed that the supernova axion emissivity was overestimated by $m_N \to \infty$ in DFSZ and KSVZ models.
- 💥 We have shown that the supernova axion emissivity can be enhanced by a factor of $\sim$4 in the KSVZ model and up to a factor of $\sim$5 in the DFSZ model with $\tan\beta \to 0$ compared to the case without the $C_{a\pi N}$ and $C_{aN\Delta}$.
- 💥 We have found that the Δ resonance can give a destructive contribution to the supernova axion emissivity at high $T_{\rm SN}$.