String Theoretic Evidence for the Weak Gravity Conjecture

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CAU BSM Workshop 2023, CAU

21-02-2023

Disclaimer

What this talk is (not) for

- I was told to talk about the **current status** of the **Weak Gravity Conjecture (WGC)** — a prominent example of **swampland conjectures**.

- There have been so many developments on the WGC that a single talk cannot cover them all.

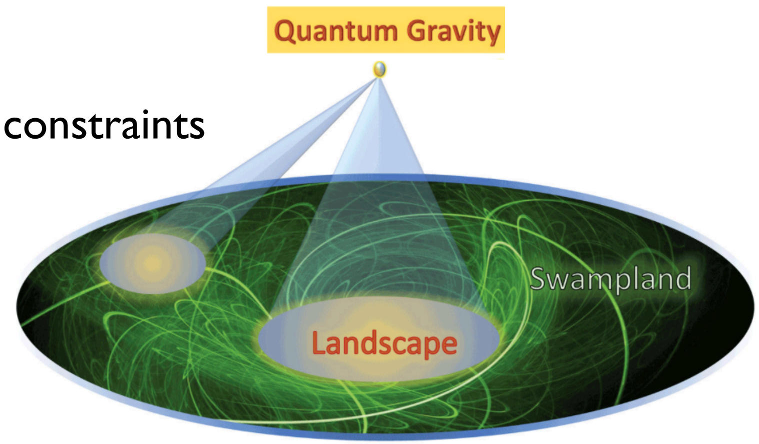
- I will focus on the **string theoretic aspects of the WGC** (mostly on the recent developments I myself was part of through [S.-J.L., Lerche, Weigand]'s).
I will also address **a few other swampland conjectures**.

- For other aspects, see e.g. the comprehensive reviews: [Palti '19] (on swampland conjectures); [Harlow, Heidenreich, Reece, Rudelius '22] (on the WGC). In particular, for the particle physics aspects, see [talk by Reece] just before mine.

Motivation

Quantum Gravity and String Theory

- **Swampland Conjectures**
  - Which effective field theories (EFTs) couple to Quantum Gravity?
    - **Swampland v.s. Landscape**
  - EFTs in the Landscape subject to **universal** consistency constraints
    - **Swampland Conjectures**
      - ⇢ general, useful, but **not** fully understood



picture from [Palti '19]

- **Stringy Realization**
  - Quantitative verification of the explicit conjectures
  - Manifestations in string geometry
  - Refinement

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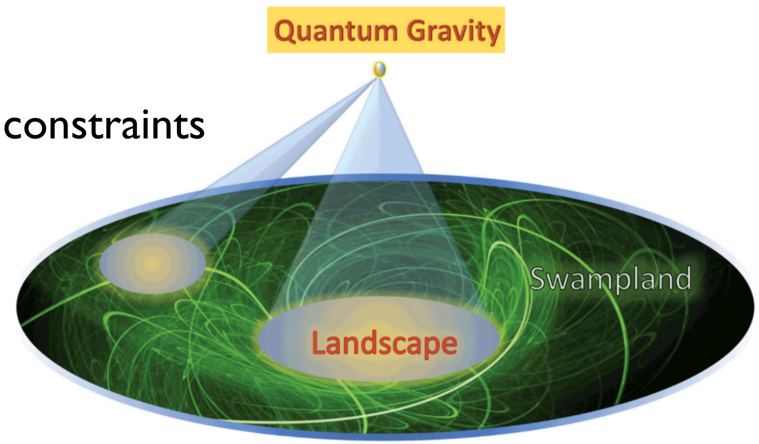

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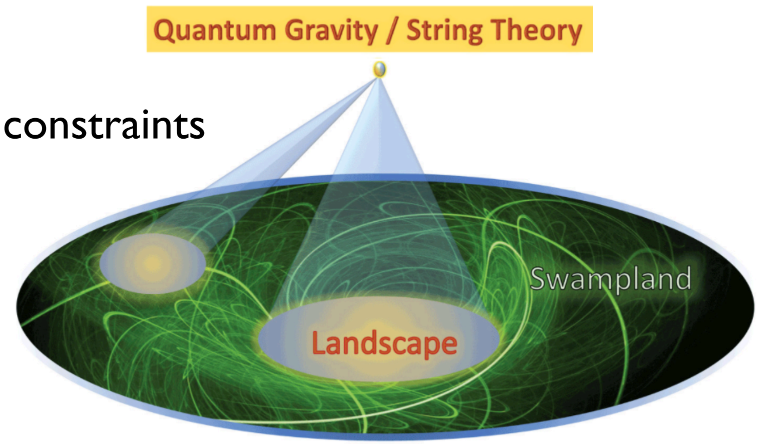

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Swampland Conjectures

Examples

Cobordism Conjecture
[McNamara, Vafa, '19]

AdS Distance Conjecture
[Lust, Palti, Vafa, '19]

No Global Symmetry
[Banks, Dixon, '88], [Harlow, Ooguri, '18]

Completeness
[Polchinski '03]

Distance Conjecture
[Ooguri, Vafa, '06]

Emergent String Conjecture
[S.-J.L., Lerche, Weigand '19]

Trans-Planckian Censorship
[Bedroya, (Brandenberger, Loverde,) Vafa, '19]

sub-Lattice WGC
[Heidenreich, Reece, Rudelius, '16]
[Montero, Shiu, Soler, '16]

Scalar WGC
[Palti, '17] [S.-J.L., Lerche, Weigand '18]
[Heidenreich, Reece, Rudelius,'19]

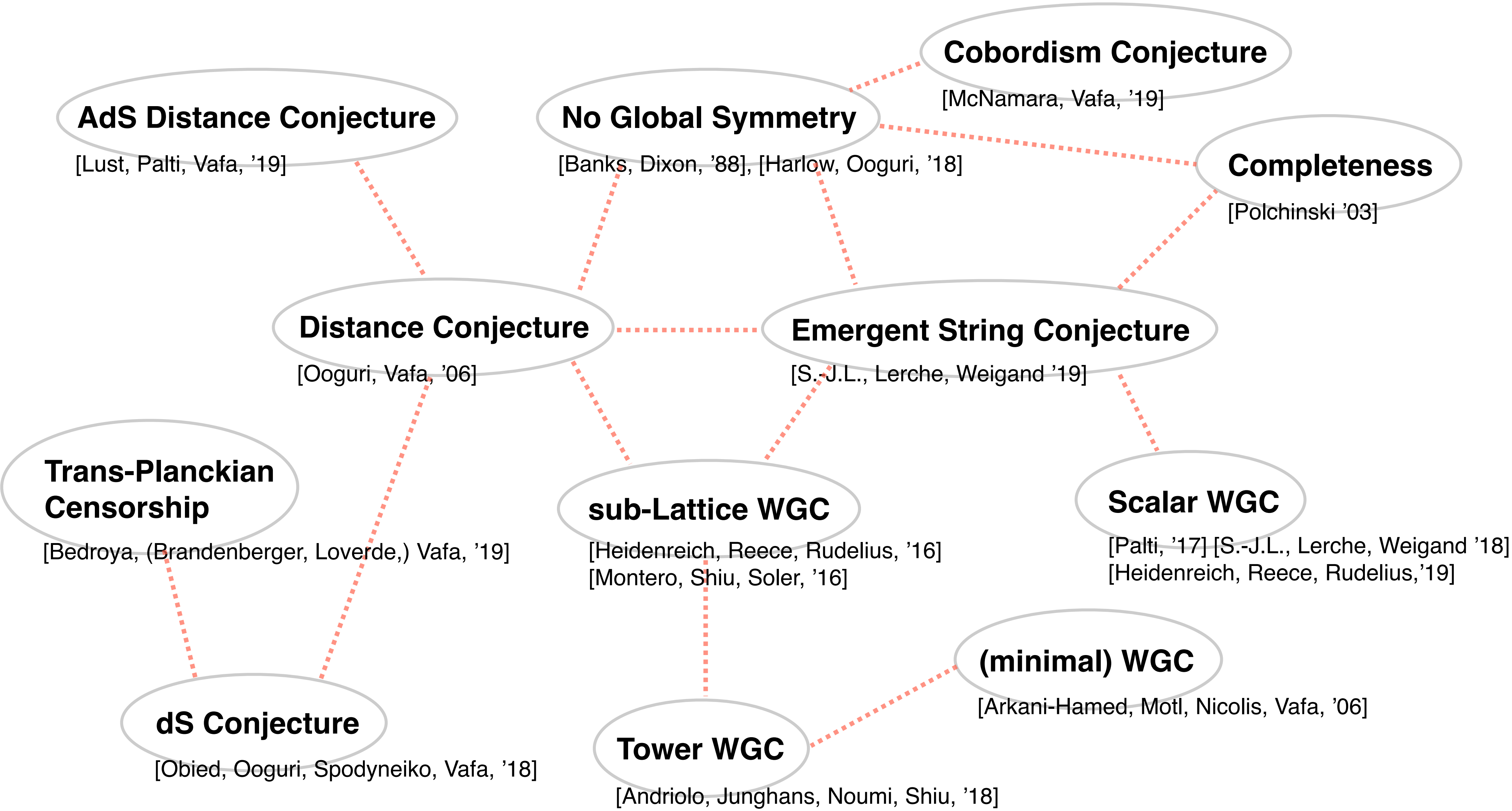
(minimal) WGC
[Arkani-Hamed, Motl, Nicolis, Vafa, '06]

dS Conjecture
[Obied, Ooguri, Spodyneiko, Vafa, '18]

Tower WGC
[Andriolo, Junghans, Noumi, Shiu, '18]

Swampland Conjectures

Charting the Conjectures



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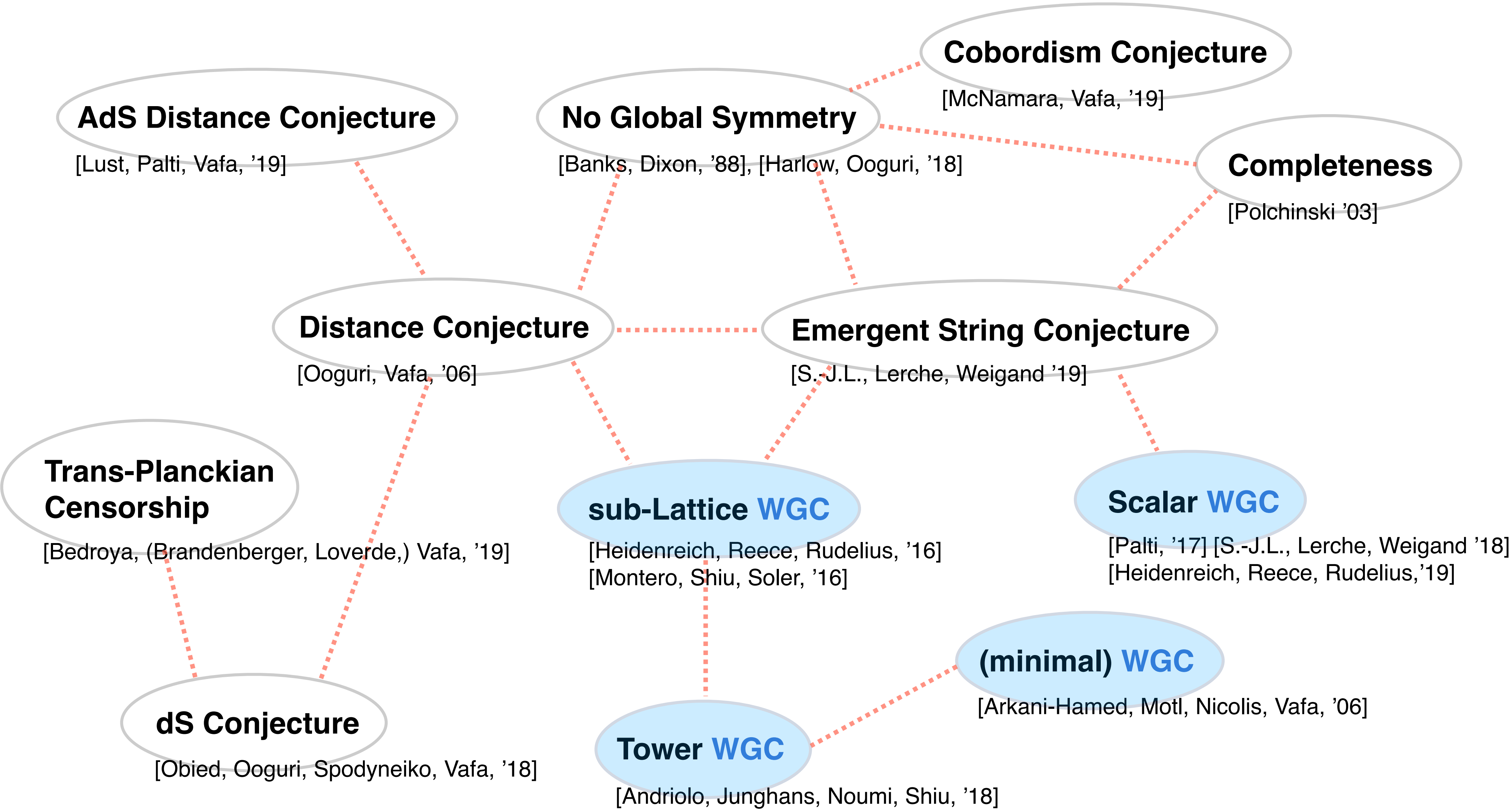

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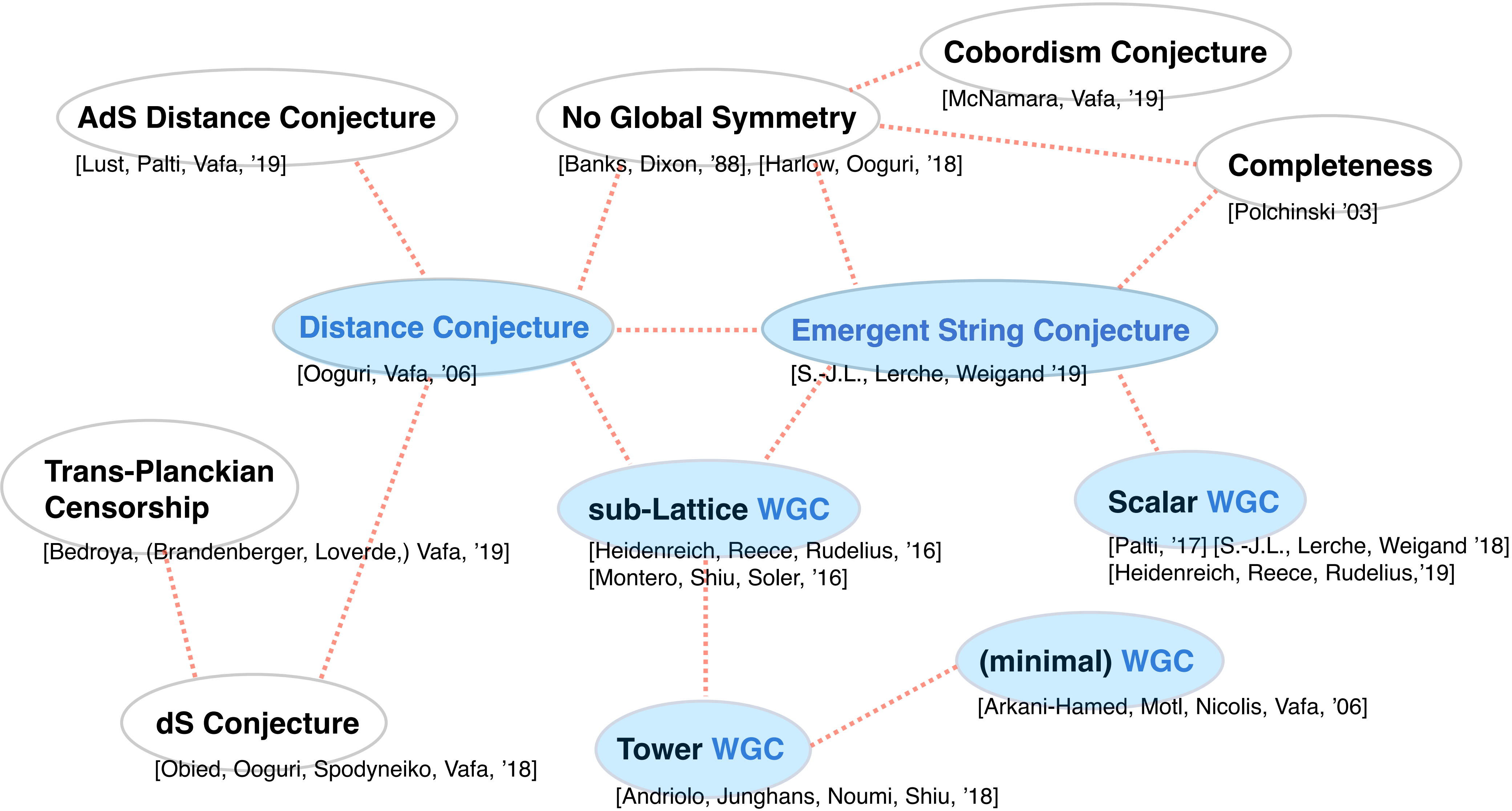

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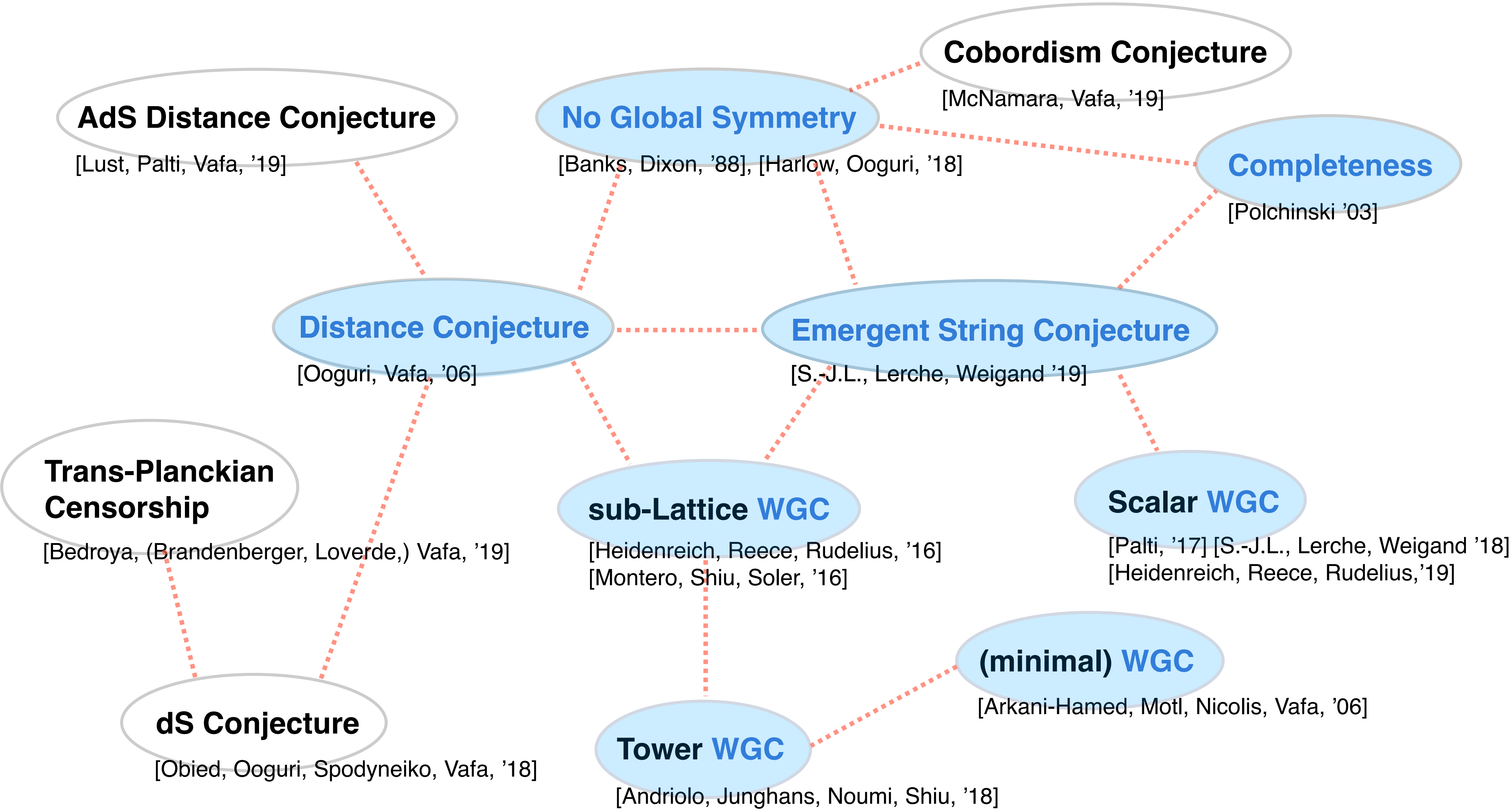

The Weak Gravity Conjectures

The Claim

- **The WGC, minimal ver.** [Arkani-Hamed, Motl, Nicolis, Vafa '06]

$$S = \int d^4x \sqrt{-G} \left(M_{\rm Pl}^2 R - \frac{1}{4g^2} F^2 + \cdots \right)$$

> **There must exist a particle with mass "smaller than" charge:** $\frac{g^2 q^2}{m^2} \geq \left. \frac{g^2 Q^2}{M^2} \right|_{\rm ExtBH} = \frac{\mu_{\rm ext}}{M_{\rm Pl}^2}$

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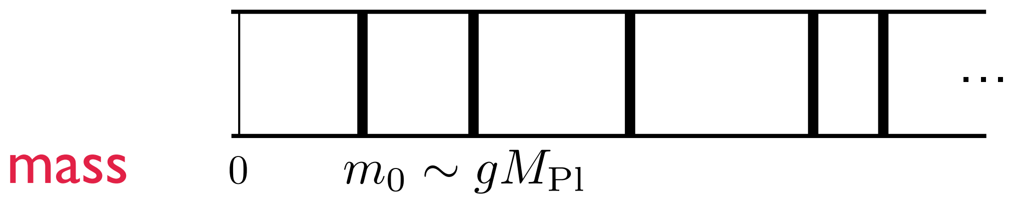
There must exist a tower / sublattice of such particles with mass scale $m_0 \sim g M_{\rm Pl}$



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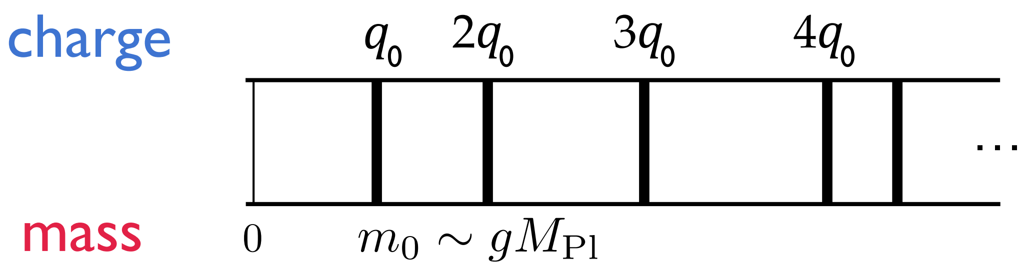

The Menu

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Swampland Conjectures & String Theory

The Weak Gravity Conjecture(s)

Second Course

Preliminary Evidence (16 Qs)

- Heterotic String on a Torus -

Main Course

Stronger Evidence (8 Qs)

- 6d F-theory Vacua -

Strongest Evidence (4 Qs)

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Preliminary Evidence

- Heterotic String Theory -

Preliminary Evidence: Upshot

Heterotic String on a Torus

- Heterotic string theory is a **closed-string** theory and reduces at very low energies to **10d SUGRA** coupled to **10d SYM** w/ **G=SO(32)** (or $E_8 \times E_8$).

- Such a 10d EFT alone does not serve as a testing ground for the WGC.

- The heterotic string also produces a tower of **massive** excitations; we can quantize the heterotic string on a flat space, such as $\mathbb{R}^{1,9}$ or $\mathbb{R}^{1,3} \times T^6$.

- The idea is to check if some of those excitations serve as "WG particles", i.e., if they obey $\frac{g^2q^2}{m^2} \geq \frac{g^2Q^2}{M^2}\Big|_{\mathrm{ExtBH}} = \frac{\mu_{\mathrm{ext}}}{M_{\mathrm{Pl}}^2}$; the WGC will be tested this way.

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Quantization of Heterotic String

10d Supergravity

- **Low-Energy EFT**
  - 10d N=1 SUGRA coupled to 10d SYM w/ G=SO(32) ($\supset$ $\mathbf{U(1)}^{16}$)

$$S=\int_{\mathbb{R}^{1,9}}\frac{M_{\rm Pl}^8}{2}(\sqrt{-G}R-\frac{1}{2}\mathrm{d}\phi\wedge*\mathrm{d}\phi)-\frac{1}{2g_0^2}e^{-\phi/2}\,\mathrm{Tr}\,F\wedge*F$$

  - The extremal charge-to-mass ratio of dilatonic RN BHs

$$\left.\frac{g^2Q^2}{M^2}\right|_{\rm ExtBH}=\frac{\mu_{\rm ext}}{M_{\rm Pl}^8}\quad \text{where}\quad \mu_{\rm ext}=\frac{d-3}{d-2}+\frac{\alpha^2}{2}=\frac{7}{8}+\frac{1}{8}=1$$

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String Excitations

- Mass spectrum:

$$m^2 = 8\pi(n - 1 + \frac{1}{2}|\vec{q}|^2)M_{\rm str}^2 \ , \quad n \in \mathbb{Z}_{\geq 0}$$

- Charge spectrum:

$$\vec{q} = (q_1 + \frac{c}{2}, \ldots, q_{16} + \frac{c}{2})$$

with $c = 0$ or 1 , $q_i \in \mathbb{Z}$, $\sum_i q_i \in 2\mathbb{Z}$

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WG Particles?

- Find states in the spectrum with

$$\frac{g^2 q^2}{m^2} \geq \frac{\mu_{\rm ext}}{M_{\rm Pl}^8} = \frac{1}{M_{\rm Pl}^8}$$

- Charge-to-mass ratio (with n=0):

$$\frac{g^2 q^2}{m^2} = \frac{1}{M_{\rm Pl}^8} \cdot \frac{|\vec{q}\,|^2}{|\vec{q}\,|^2 - 2} > \frac{1}{M_{\rm Pl}^8}\ , \quad \forall \vec{q}$$

Heterotic String on a Six-Torus

4d N=4 Supergravity (16 Q's)

- **Low-Energy EFT**
  - 10d N=1 SUGRA coupled to 10d SYM w/ G=SO(32)*

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  - Reduces to 4d N=4 SUGRA coupled to 16 U(1)s* (w/ Wilson lines)
  - The extremal charge-to-mass ratio is **stable** under the reduction [Heidenreich, Reece, Rudelius '15]

$$\left.\frac{g^2Q^2}{M^2}\right|_{\rm ExtBH} = \frac{\mu_{\rm ext}}{M_{\rm Pl}^{d-2}}, \quad \mu_{\rm ext} = \frac{d-3}{d-2} + \frac{\alpha^2}{2}$$

    - $M_{\rm Pl}$ and g acquire a common volume factor
    - Changes in d and α cancel in $\mu_{\rm ext}$!

- **WG Particles**
  - The 10d WG particles in the (sub)lattice w.r.t. the Cartan lead to 4d WG particles

*The same results follow for $E_8 \times E_8$ heterotic string.

*To be precise, additional U(1)s arise from the reduction but the sublattice WGC persists.

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Stronger Evidence

- 6d F-theory -

Stronger Evidence: Upshot

6d F-theory

- F-theory is another name for Type IIB string theory with 7-branes and hence, with a non-trivial dilation profile along the internal compact directions.

- E.g. $6d$ F-theory is the $10d$ IIB string theory put on a compact (curved) 4-manifold; it preserves 8 Q's (to be contrasted with the heterotic on a torus preserving 16 Q's).

- The 7-branes carry gauge algebras and charged particles arise from the open string ending on them with a fixed "charge factor". It may look impossible to fulfill the sublattice WGC.

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- A special **2-cycle** can be identified so that the resulting string is the (dual) heterotic string; its excitations are captured by the (index-ver.) partition function, which we can control though the internal space is not flat!

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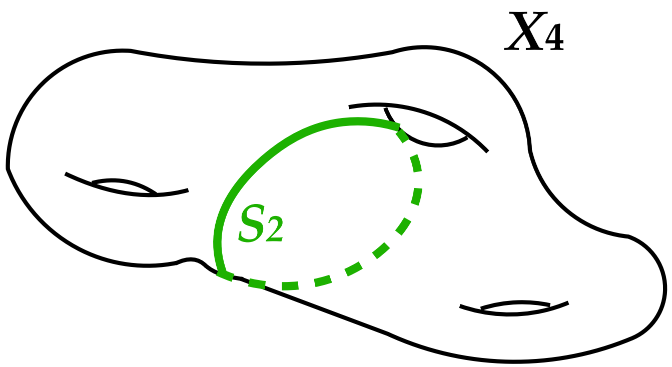
F-theory in 6 Dimensions

Couplings via Kahler Moduli

- **6d F-theory**
  - IIB string theory on a compact 4-manifold X_4 with 7-branes on an internal 2-dim'l surface S_2
  - external 6-dim'l gauge fields
  - a non-trivial dilaton profile



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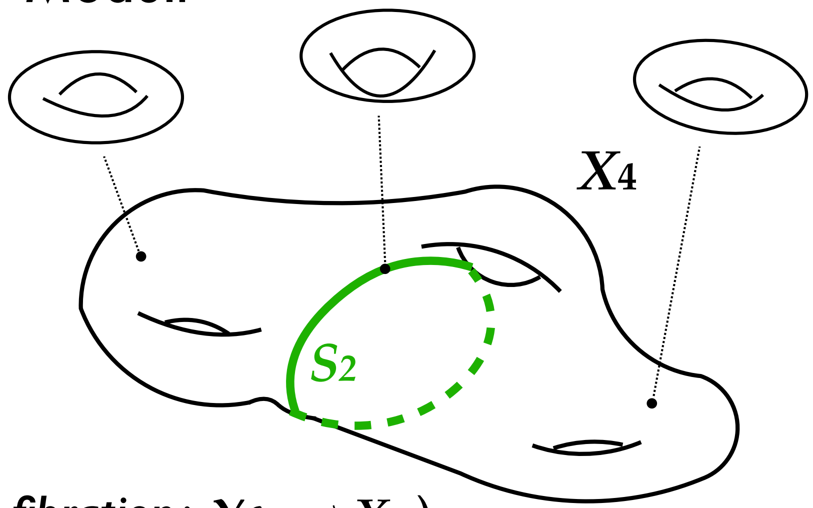
F-theory in 6 Dimensions

Couplings via Kahler Moduli



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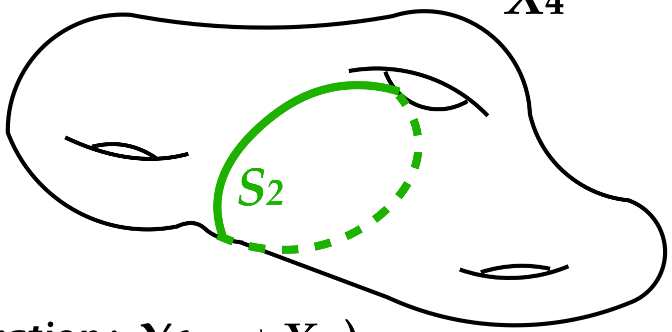

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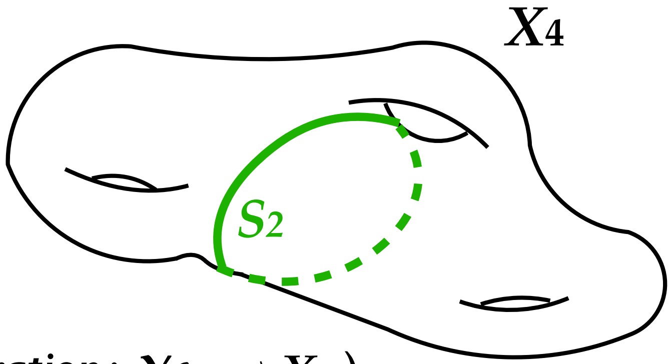

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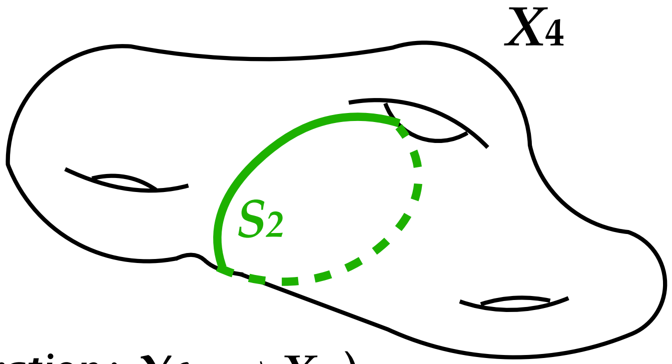

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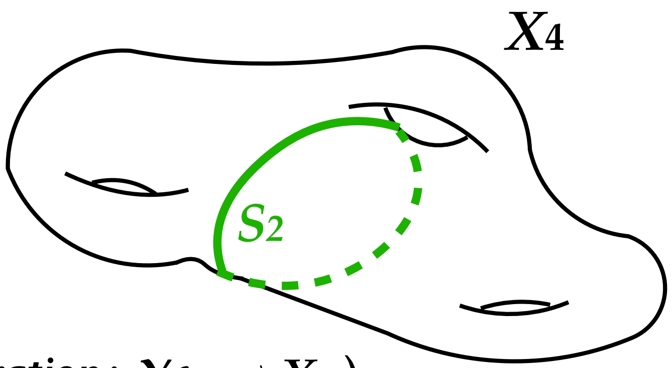

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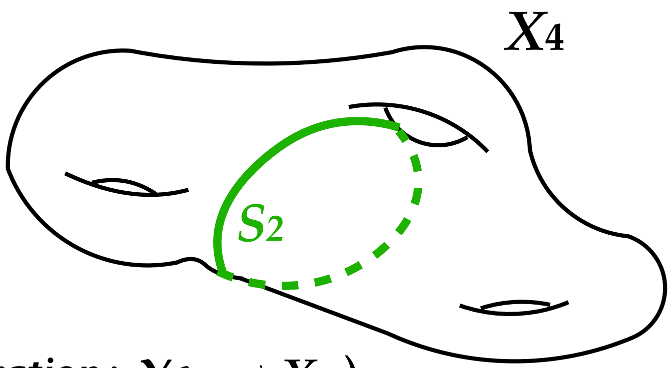

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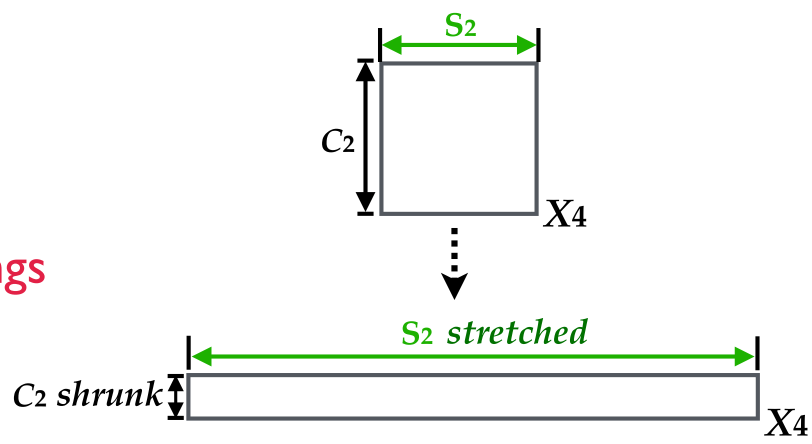

Emergent String at Weak Gauge Coupling

D3 Brane on Shrinking 2-cycle [S.-J.L., Lerche, Weigand, '18]

- **Geometry**

  - Dialing $g \to 0$ deforms X_4 such that a certain 2-cycle C_2 therein **shrinks**, where C_2 is

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  - D3-brane on C_2 leads to heterotic string with a *vanishing tension*

    $$\frac{M_{\rm str}^2}{M_{\rm Pl}^2} \sim \frac{\mathcal{V}_{C_2}}{\mathcal{V}_{X_4}^{1/2}} \to 0$$

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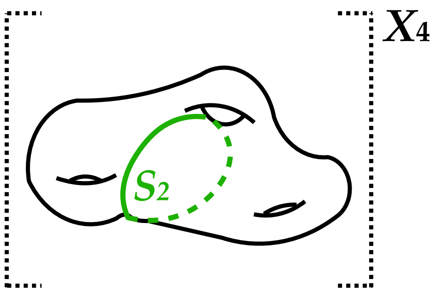

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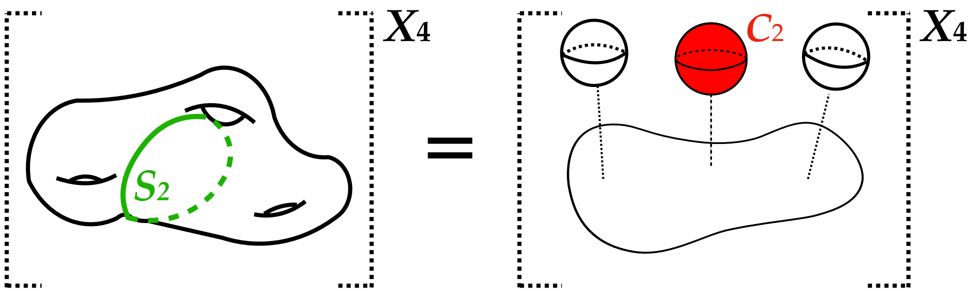

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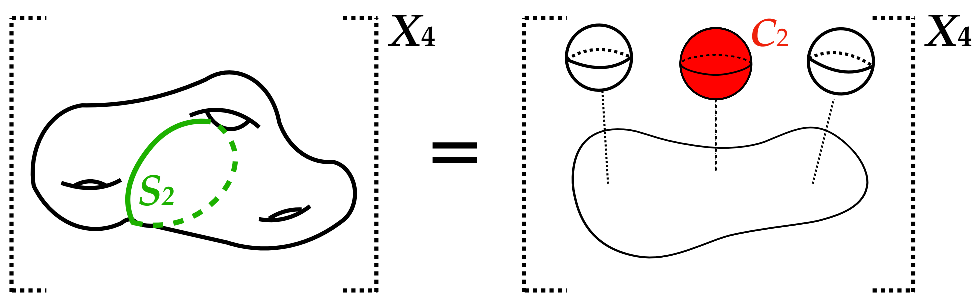

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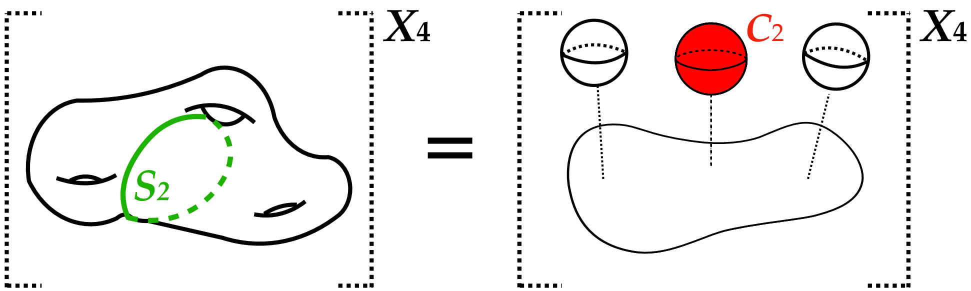

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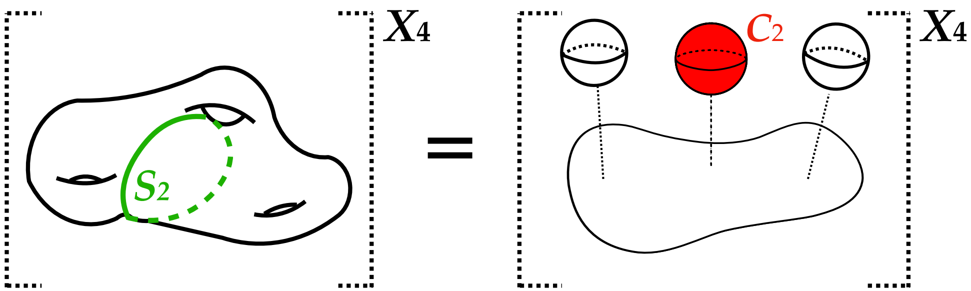

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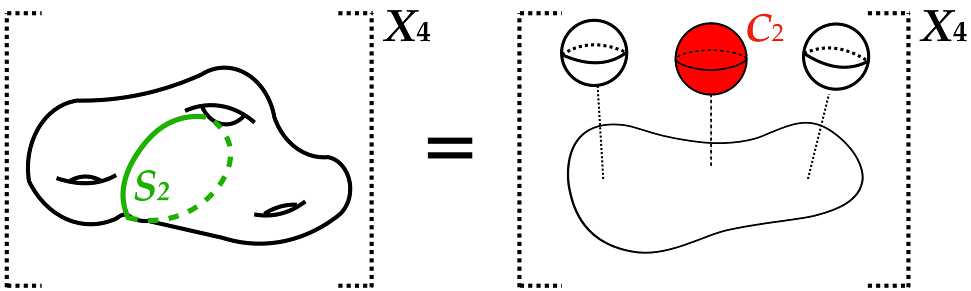

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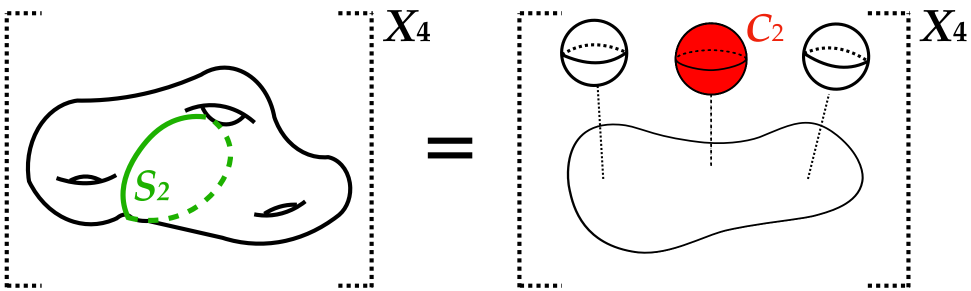

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Light tower of string excitations $\overset{?}{\supset}$ Sublattice of *WG particles*

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The WGC at Weak Gauge Coupling

Main Ideas

- **The 6d Effective Action** [S.-J.L., Lerche, Weigand, '18]
  - Asymptotically turns into a dilatonic Einstein-Maxwell theory w/ dil. coupling $\alpha = 1$

$$S = \int_{\mathbb{R}^{1,5}} \frac{M_{\rm Pl}^4}{2}(\sqrt{-G}R - {\rm d}x \wedge *{\rm d}x) - \frac{1}{2g_0^2} e^{\alpha x} F \wedge *F + \cdots$$

  - The extremal charge-to-mass ratio of dilatonic RN BHs

$$\frac{g^2Q^2}{M^2}\bigg|_{\rm ExtBH} = \frac{\mu_{\rm ext}}{M_{\rm Pl}^4} \quad \text{where} \quad \mu_{\rm ext} = \frac{d-3}{d-2} + \frac{\alpha^2}{4} = \frac{3}{4} + \frac{1}{4} = 1$$

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- Mass spectrum of the light tower:

$$m^2 = 8\pi(n-1)M_{\text{str}}^2$$

- Charge spectrum at each level n:

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WG Particles?

- Find $(n, q^{\rm max}(n))$ in the spectrum w/

$$\frac{g^2 q^{\rm max}(n)^2}{m(n)^2} \geq \frac{\mu_{\rm ext}}{M_{\rm Pl}^4} = \frac{1}{M_{\rm Pl}^4}$$

- Test if such $q^{\rm max}(n)$ fill a sublattice

A Distinguished Sublattice

Universal Feature of the Excitation Spectrum

- **The Spectrum**
  - Info needed of the maximal charge $q^{\max}$ at a given mass level n
  - Captured by the "partition function" (or its index version) of the 6d heterotic string:

$$\mathcal{Z}(\nu, \eta) = -\nu^{-1} \sum N(n,q)\, \nu^n\, \eta^q$$

where $N(n,q)$ = "Degeneracy at excitation mass level n and charge q"

- **A Distinguished Sublattice** [S.-J.L., Lerche, Weigand '18]
  - A general property of $\mathcal{Z}(\nu, \eta)$: "quasi-modularity"

  ⟶ $N(n,q)$ non-zero for charges up to $q_k^{\max} = 2rk \;\; (n_k = rk^2)$ for $k \in \mathbb{Z}$
  - This charge sublattice (of index $2r$) is filled by physical states
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Verification for 6d F-theory [S.-J.L., Lerche, Weigand '18]

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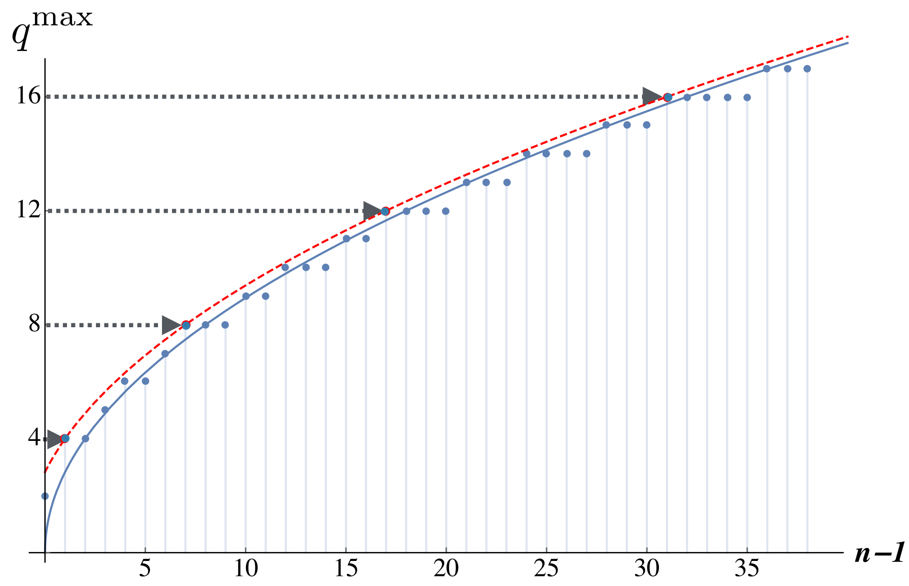
  - Quasi-Jacobi nature of the heterotic worldsheet partition function guarantees that

$q_k^{\max} = 2rk \ (n_k = rk^2)$ for $k \in \mathbb{Z}$ are filled by physical states, with the index $2r = C_2 \cdot \mathbf{S}_2$



maximal charge *vs.* mass level

(an example w/ sublattice index $2r=4$)

$$q_k^{\max} = \sqrt{4\,r\,n_k} \geq \sqrt{4\,r\,(n_k - 1)}$$

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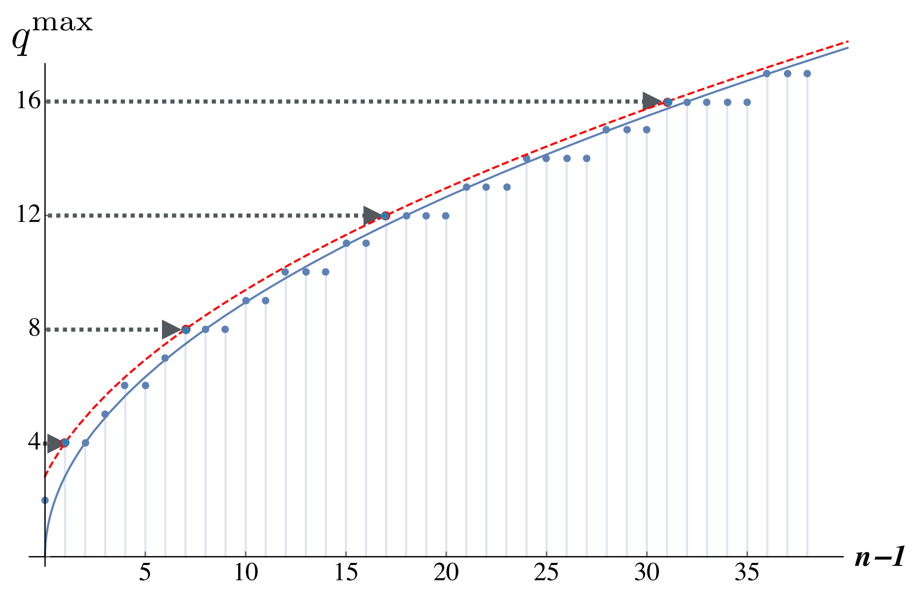
- **The Distinguished Sublattice**
  - Quasi-Jacobi nature of the heterotic worldsheet partition function guarantees that

$q_k^{\max} = 2rk \ (n_k = rk^2)$ for $k \in \mathbb{Z}$ are filled by physical states, with the index $2r = C_2 \cdot \mathbf{S}_2$



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(an example w/ sublattice index $2r=4$)

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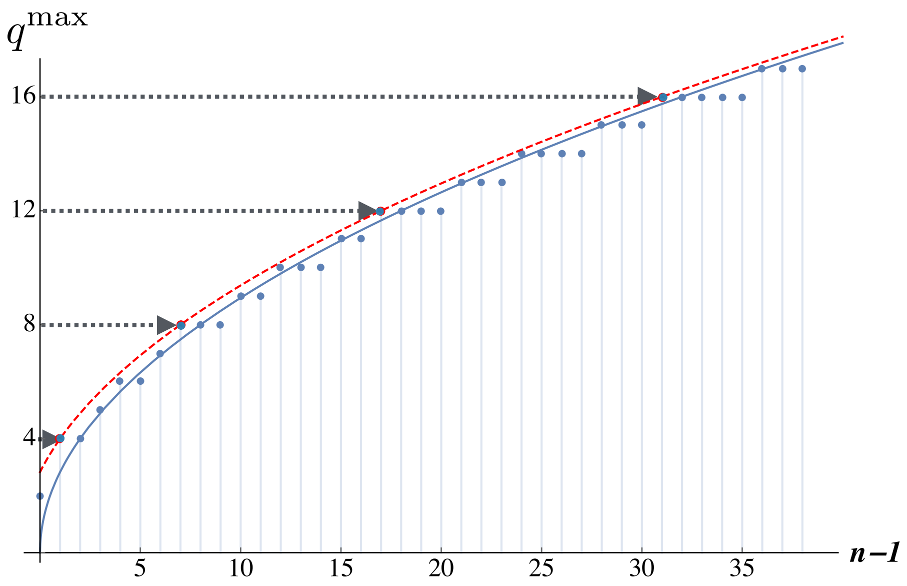

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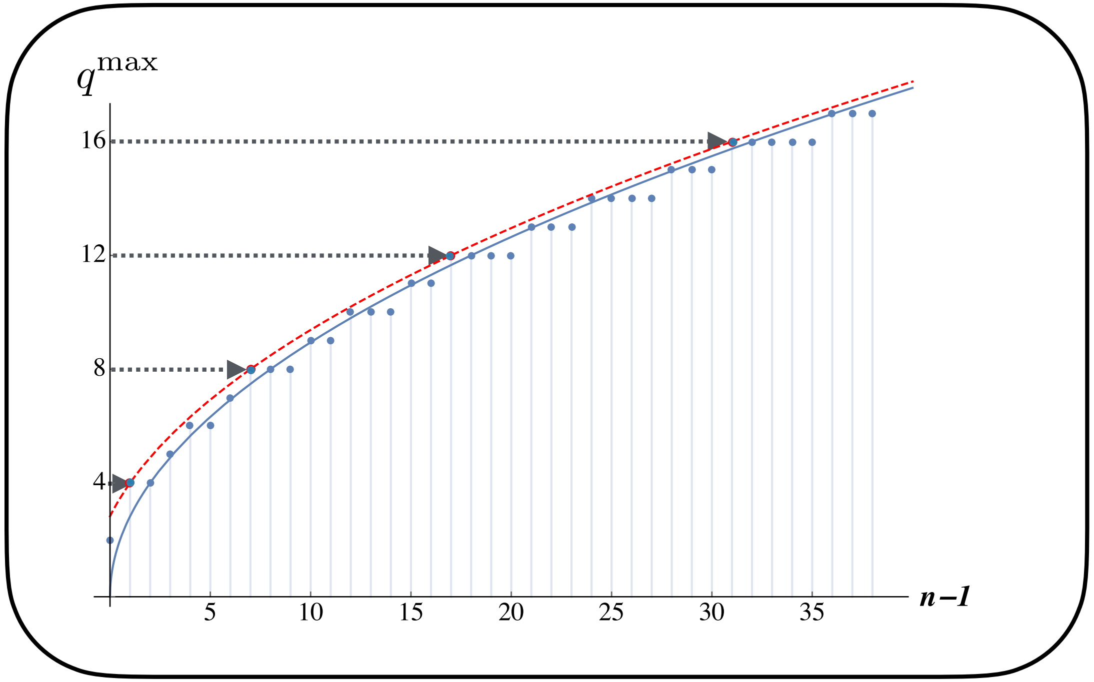

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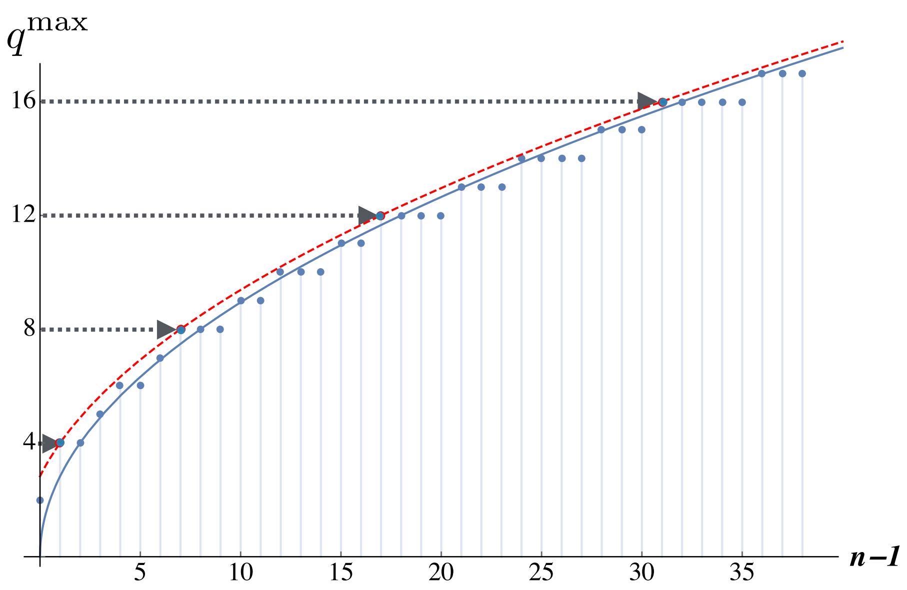

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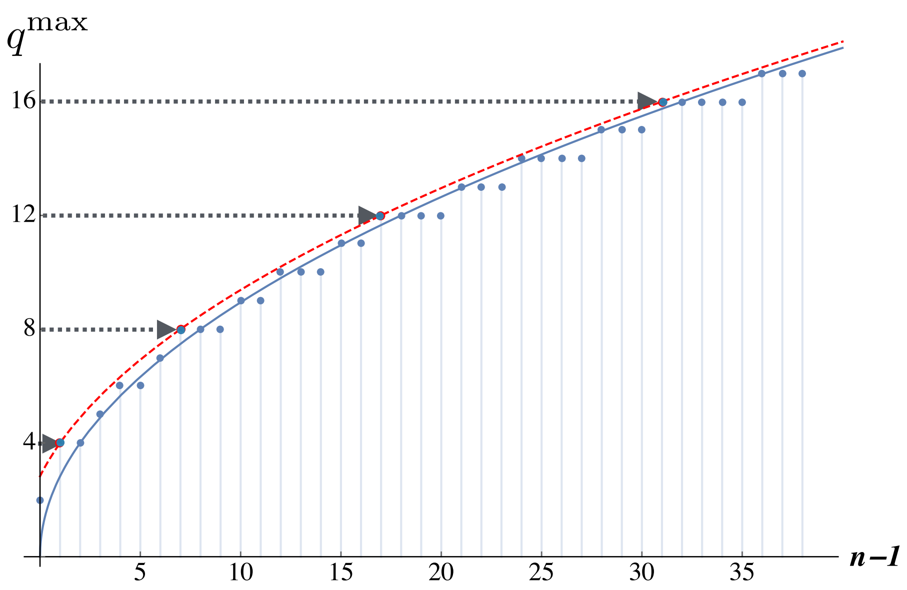

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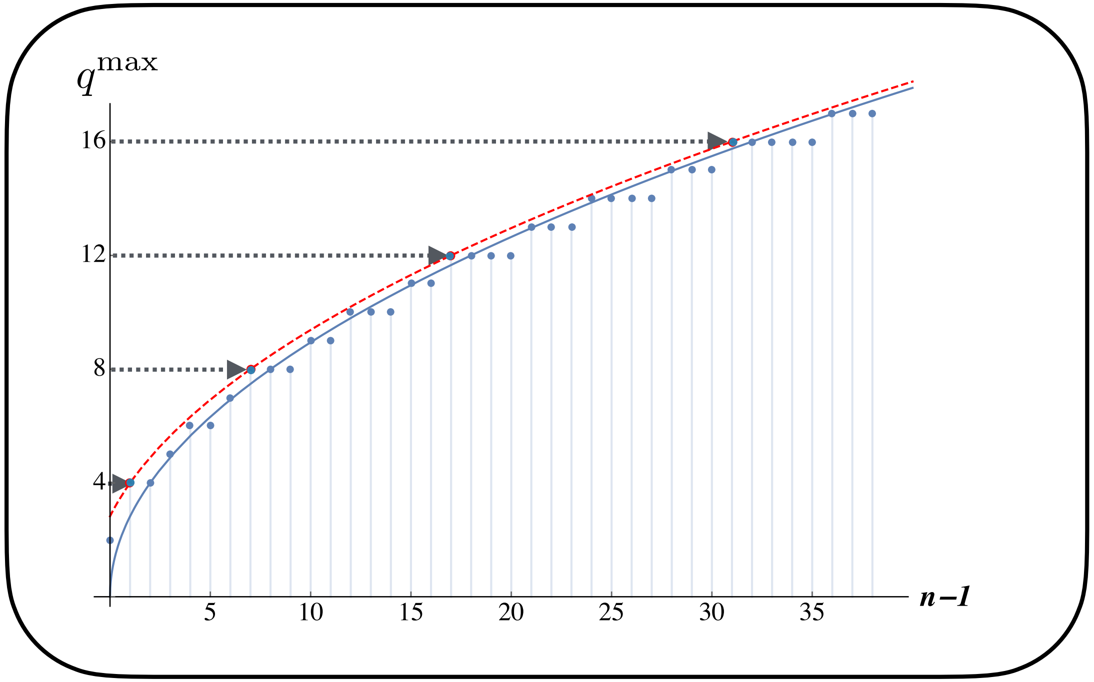

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Strongest Evidence

- 4d F-theory -

Strongest Evidence: Upshot

4d F-theory

- 4d F-theory is the 10d IIB string theory put on a compact (curved) 6-manifold, it preserves 4 Q's (to be contrasted with the 6d F-theory, which preserves 8 Q's).

- Once again, a special internal 2-cycle can be identified so that the 3-brane wrapping it produces an effective heterotic string in the 4d EFT.

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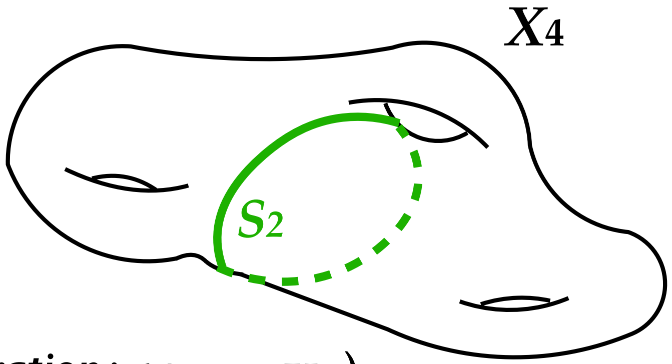
F-theory in 6 Dimensions

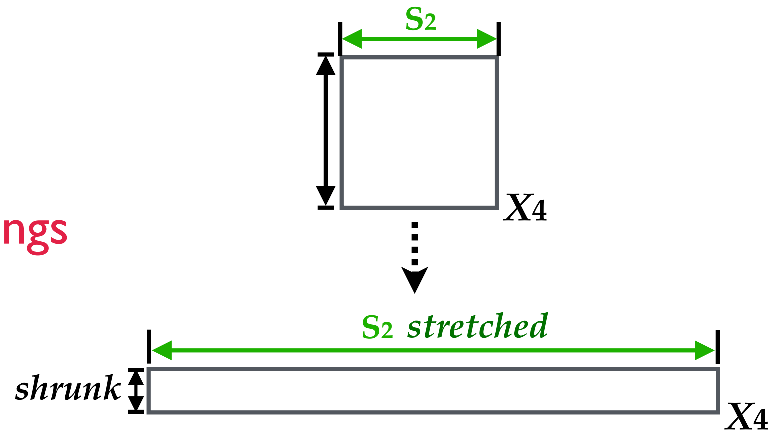
Couplings via Kahler Moduli

- **6d F-theory**



  - IIB string theory on a compact 4-manifold X_4 with 7-branes on an internal 2-dim'l cycle S_2
    - external 6-dim'l gauge fields
    - a non-trivial dilaton profile (described by an *elliptic fibration*: $Y_6 \longrightarrow X_4$)

- **Physics via Geometry**



  - We will look into the "Kahler moduli" of X_4
  - Govern the cycle volumes and in turn, the couplings
    - gravity: $(M_{\mathrm{Pl}}/M_{\mathrm{IIB}})^4 = 4\pi\mathcal{V}_{X_4}$
    - gauge: $(1/g^2)/M_{\mathrm{IIB}}^2 = (2\pi)^{-1}\mathcal{V}_{S_2}$
  - Weak gauge coupling (w/ gravity fixed): $\mathcal{V}_{S_2} \to \infty \quad (\mathcal{V}_{X_4} \sim 1) \quad \Longrightarrow$ exists C_2 with $\mathcal{V}_{C_2} \to 0$

F-theory in 4 Dimensions

Couplings via Kahler Moduli

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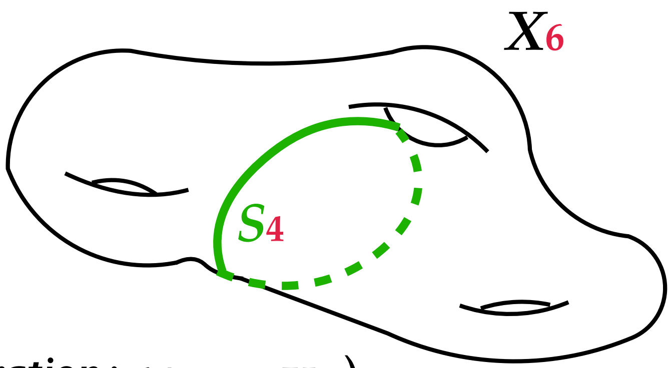

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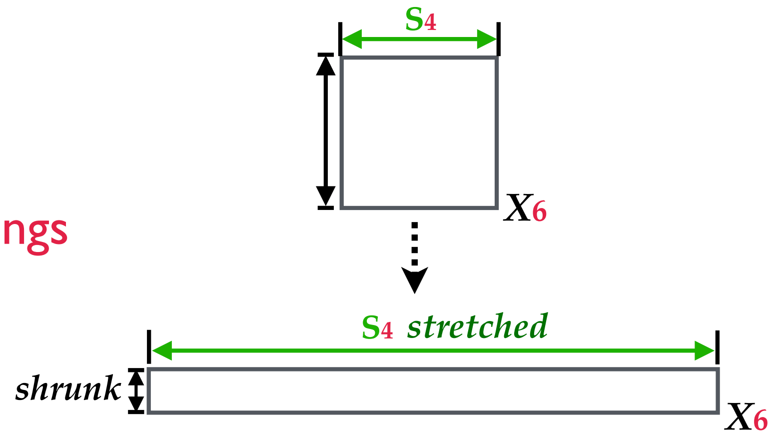

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The WGC at Weak Gauge Coupling

Main Ideas: Same as in 6 Dimensions

- **The 4d Effective Action** [S.-J.L., Lerche, Weigand '19]
  - Asymptotically turns into a dilatonic Einstein-Maxwell theory w/ dil. coupling $\alpha = \sqrt{2}$

$$S = \int_{\mathbb{R}^{1,3}} \frac{M_{\rm Pl}^2}{2}(\sqrt{-G}R - {\rm d}x \wedge *{\rm d}x) - \frac{1}{2g_0^2} e^{\alpha x} F \wedge *F + \cdots$$

  - The extremal charge-to-mass ratio of dilatonic RN BHs

$$\left.\frac{g^2Q^2}{M^2}\right|_{\rm ExtBH} = \frac{\mu_{\rm ext}}{M_{\rm Pl}^2} \quad \text{where} \quad \mu_{\rm ext} = \frac{d-3}{d-2} + \frac{\alpha^2}{4} = \frac{1}{2} + \frac{1}{2} = 1$$

String Excitations

- Mass spectrum of the light tower:

$$m^2 = 8\pi(n-1)M_{\rm str}^2$$

- Charge spectrum at each level n:

$$q \in [0, q^{\max}(n)]$$

→

WG Particles?

- Find $(n, q^{\max}(n))$ in the spectrum w/

$$\frac{g^2 q^{\max}(n)^2}{m(n)^2} \geq \frac{\mu_{\rm ext}}{M_{\rm Pl}^2} = \frac{1}{M_{\rm Pl}^2}$$

- Test if such $q^{\max}(n)$ fill a sublattice

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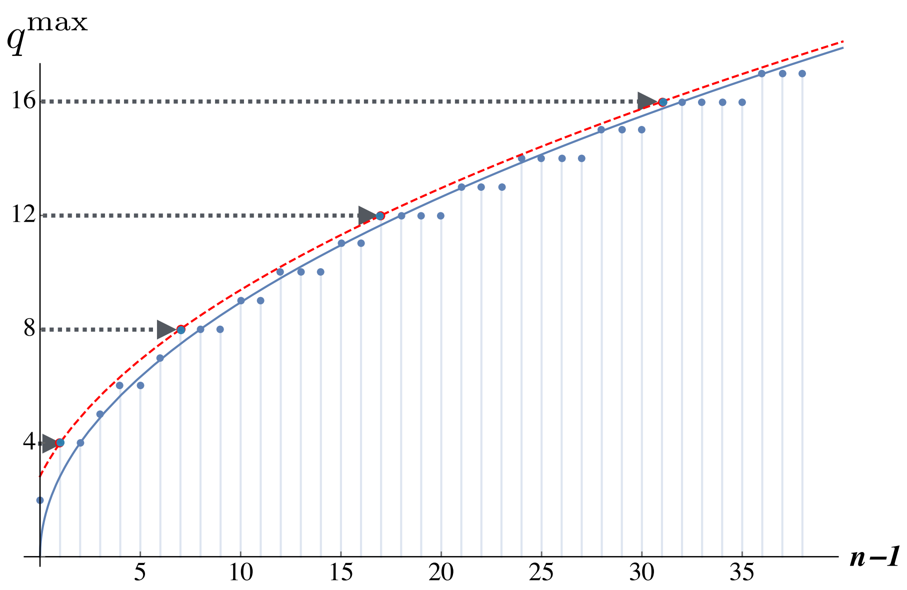

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Less Protection with Less SUSY [Klawer, S.-J.L., Weigand, Wiesner '20]

- **Origin of Corrections**
  - Shrinking cycles in principle lead to corrections to the classical geometric calculations
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  - Classical $\mu_{\rm str}^{\rm cl} = 1$

    in other words: $\mathcal{V}_{\rm S_4}^{\rm cl}\,\mathcal{V}_{C_2}^{\rm cl} \overset{!}{=} (C_2 \cdot {\rm S}_4)\,\mathcal{V}_{X_6}^{\rm cl}$
  - Quantum $\mu_{\rm str} = 1 - \Delta$

    leading corrections to the volumes cancel in the ratio, i.e., $\Delta \to 0$ *at strict weak gauge coupling*
  - Subleading correction to the WG criterion

$$\frac{g^2 q^2}{m^2} \overset{?}{\geq} \frac{1-\Delta}{M_{\rm Pl}^2}$$

Mass renormalization ansatz: $\frac{m^2}{M_{\rm Pl}^2} = 8\pi(n-1)\frac{M_{\rm str}^2}{M_{\rm Pl}^2}(1+\rho)$

If the sublattice still provides WG particles: $\rho = -\frac{1}{2}\Delta$

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The Menu

First Course

Swampland Conjectures & String Theory

The Weak Gravity Conjecture(s)

Second Course

Preliminary Evidence (16 Qs)

- Heterotic String on a Torus -

Main Course

Stronger Evidence (8 Qs)

- 6d F-theory Vacua -

Strongest Evidence (4 Qs)

- 4d F-theory Vacua -

Dessert

Evidence for Other Conjectures

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a light tower of *either* *Kaluza-Klein excitations* *or* *string excitations*

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"emergence of **unique** critical tensionless string!"

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Evidence for Other Conjectures

The Emergent String Conjecture [S.-J.L., Lerche, Weigand, '19]

At infinite distance in moduli space a quantum gravity theory *either* *decompactifies* *or* reduces to a *weakly coupled string theory*

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Completeness [Polchinski, '03]

▪ The excitations of our emergent heterotic string provide massive particles with an **arbitrary charge**.

▪ The number theoretic (*quasi-modular* and *quasi-Jacobi*, resp., in 6d and 4d) property of the heterotic partition function allows no gaps in the charge spectrum.*

* For a non-generic EFT w/ a sufficiently tuned gauge flux, charge gaps may appear at least in the heterotic index.

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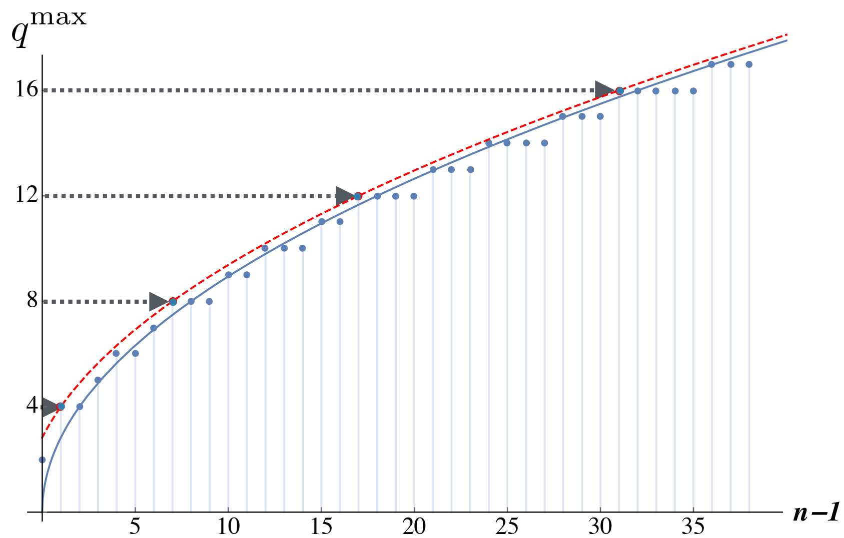

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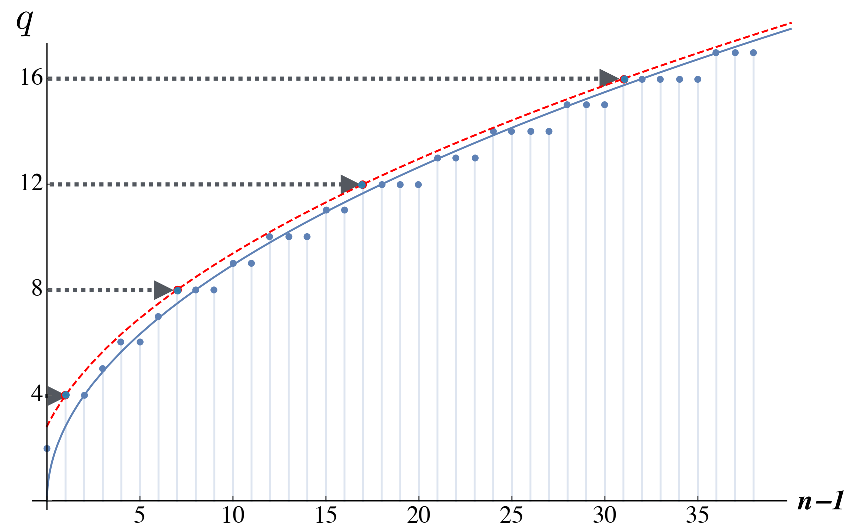

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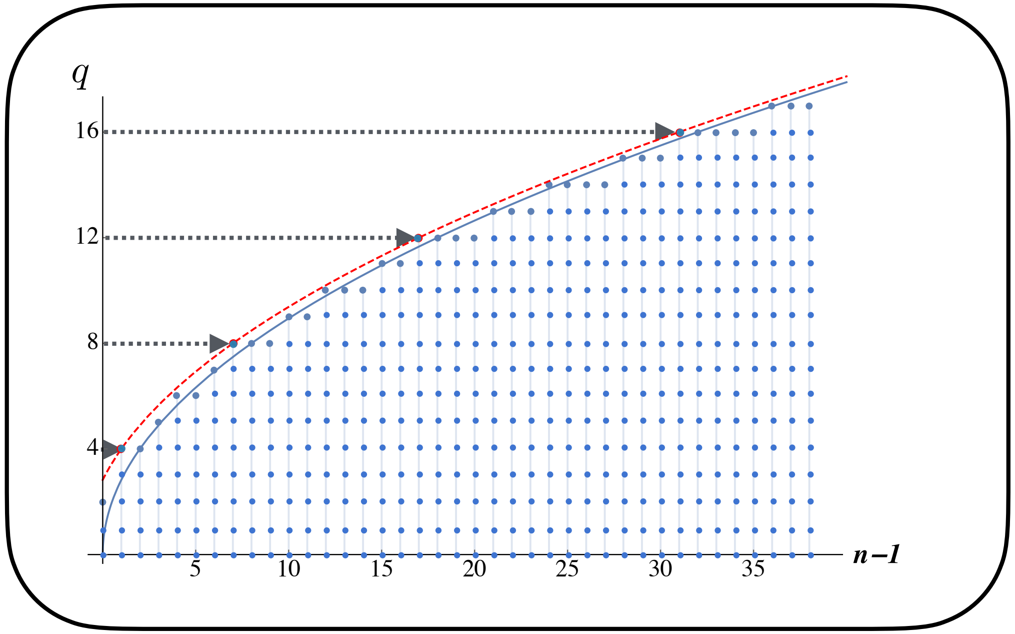

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Evidence for Other Conjectures

No Global Symmetry [Banks, Dixon, '88]

- At weak gauge coupling, we have observed an infinite tower of particles (the heterotic excitations) become light.

- This indicates that the effective description of the physics breaks down and in particular that the strict weak gauge coupling limit is not part of the moduli space.

- In other words, the $\mathrm{U}(1)$ gauge vector **cannot** lead to a strict **global** $\mathrm{U}(1)$ symmetry; this supports the No Global Symmetry Conjecture.

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- The WGC claims **weakness of gravity** as a general **feature of quantum gravity**.

- The **minimal WGC** predicts a WG particle (w/ mass smaller than charge), while the **sublattice version** conjectures a charge **sublattice** to be filled by WG particles.

- We have discussed strong evidence for the WGC from string theory. Microscopically, the sublattice of WG particles arise from the **excitations of an (emergent) critical string**.

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- Much stronger evidence is found in the weak gauge coupling limits of the **F-theoretic EFTs** (w/ 8 or 4 Q's), where a light critical string necessarily **emerges** to realize the sublattice WGC!

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