Generalized Global Symmetries in Particle Physics

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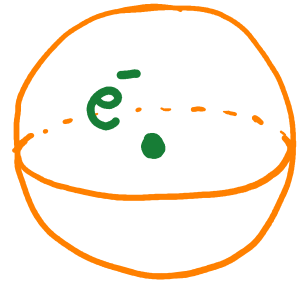
2023 Chung-Ang University
Beyond the Standard Model Workshop

Generalized Global Symmetries!

Most Symmetries in particle physics act on local operators

$$\psi(x) \rightarrow e^{i\alpha Q}\,\psi(x)$$



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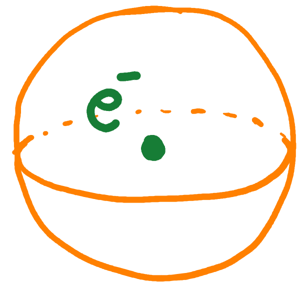


Recently, concept of symmetry has gone through explosive generalizations!

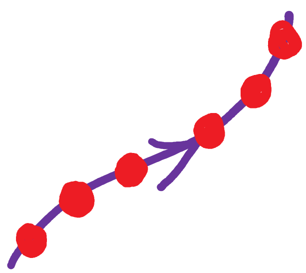
"Generalized Global Symmetries (GGS)"

Generalized Global Symmetries!

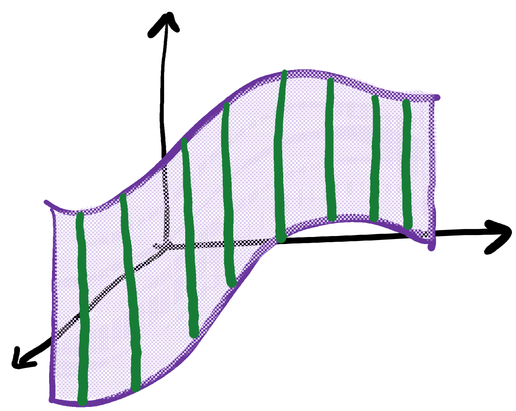
I. Higher-form symmetries

Various **extended objects** appear in broad class of theories.

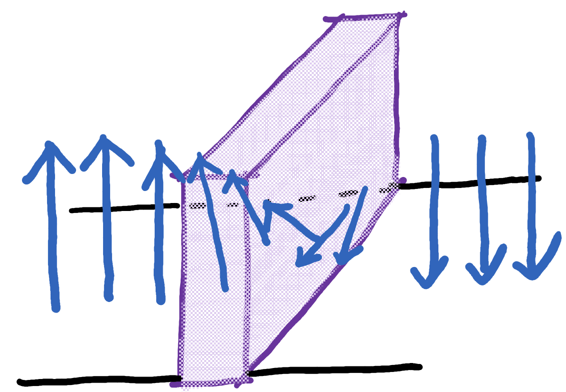
Local operator
e.g. particle
0-form symmetry



Line operator
e.g. Wilson loop
't Hooft loop
1-form symmetry



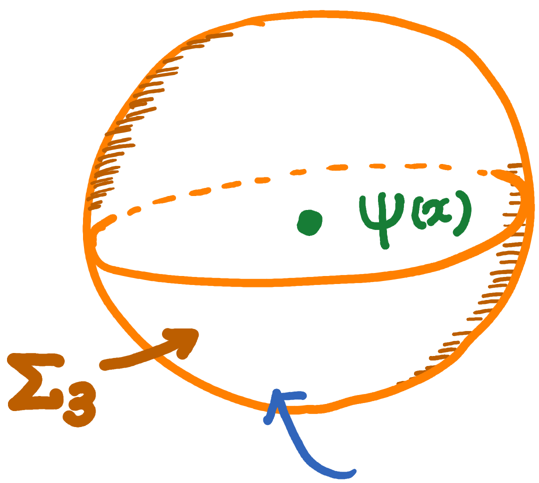
Surface operator
e.g. Cosmic string
2-form symmetry



Volume operator
e.g. Domain Wall
3-form symmetry

Generalized Global Symmetries!

II. Non-Invertible Symmetries



$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC$$

$$C \to C + \frac{1}{N}\epsilon_1, \int \frac{\epsilon_1}{2\pi} \in Z$$

$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right) \text{ with } \frac{p}{N} = \frac{N_f}{k}$$

Generalized Global Symmetries!

- **Generalized Global Symmetries (GGS)** have shown to be extremely powerful in deepening our understanding of QFT

  - Aharony, Seiberg, Tachikawa '13
  - Kapustin, Seiberg '14
  - Gaiotto, Kapustin, Seiberg, Willett '14
  - Gaiotto, Kapustin, Komargodski, Seiberg '17
  - Anber, Poppitz '18
  - Cordova, Dumitrescu '18
  - Cordova, Dumitrescu, Intriligator '18
  - Benini, Cordova, Po-Shen-Hsin '18
  - Cordova, Ohmori '19
  - …. Anber, **Hong**, Son '21 ….
  - Kaidi, Ohmori, Zheng '21
  - Choi, Cordova, Po-Shen Hsin, Ho Tat Lam, Shu-Heng Shao '21
  - Many many more

Generalized Global Symmetries in Particle Physics?

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(Q1) Are there **generalized symmetries** in **(3+1)d QFTs** that relevant for **particle physics**?

(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

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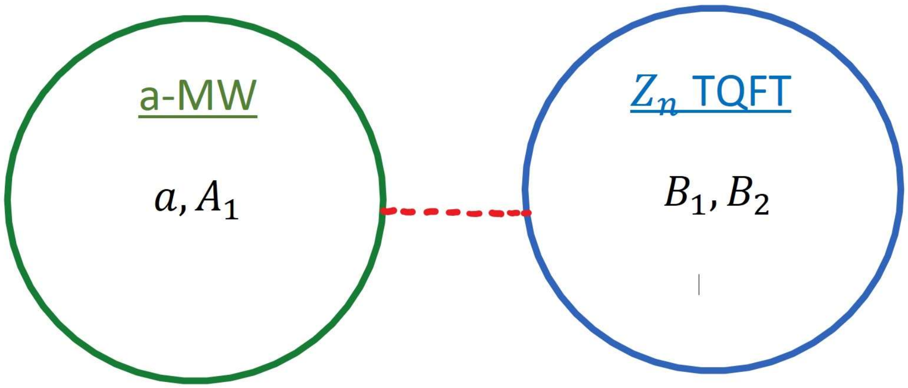
(Q2) Can there be **observable signals** (even in principle) associated with (due to) the presence of those **generalized symmetries**?

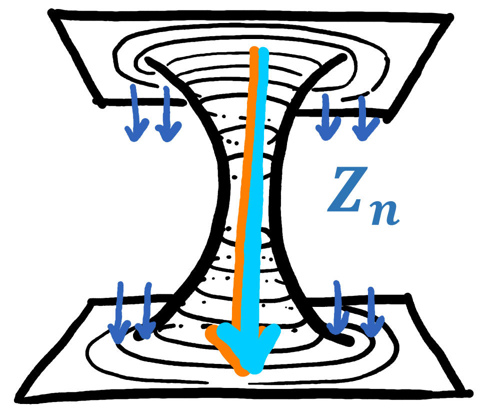
(Q3) Can **generalized symmetry** provide **novel or meaningful solutions** to problems in **particle physics**?

Outline

Coupling a Cosmic String to a TQFT

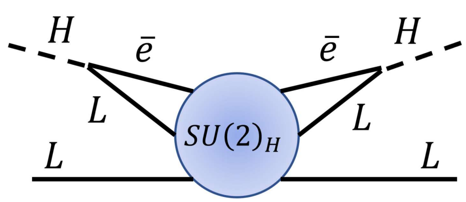
(with T. Daniel Brennan and Liantao Wang)





Neutrino Masses from Generalized Symmetry Breaking

(with Clay Córdova, Seth Koren, and Kantaro Ohmori)



$$D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right)$$

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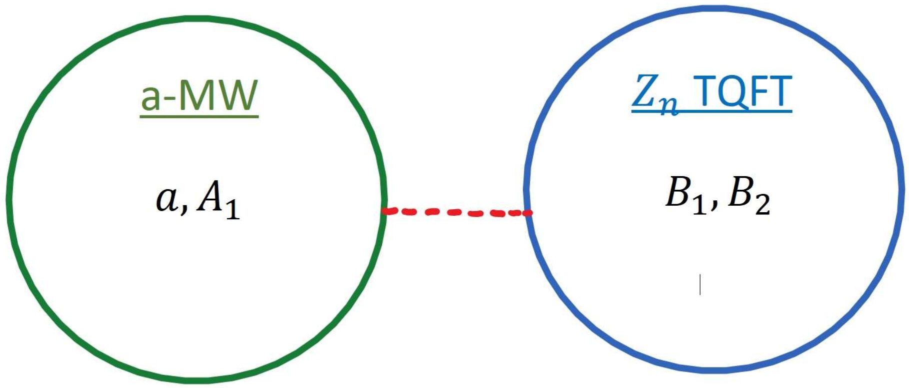

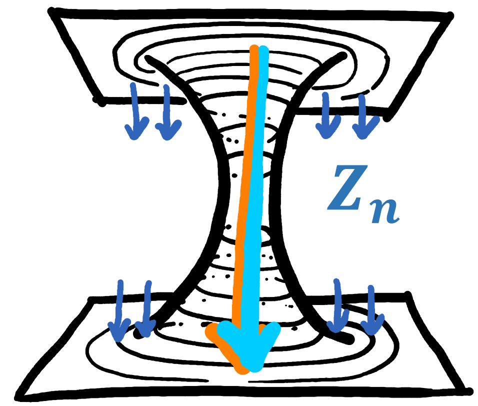

I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

II. TQFT-Coupling 2: Z_M Discrete Gauging

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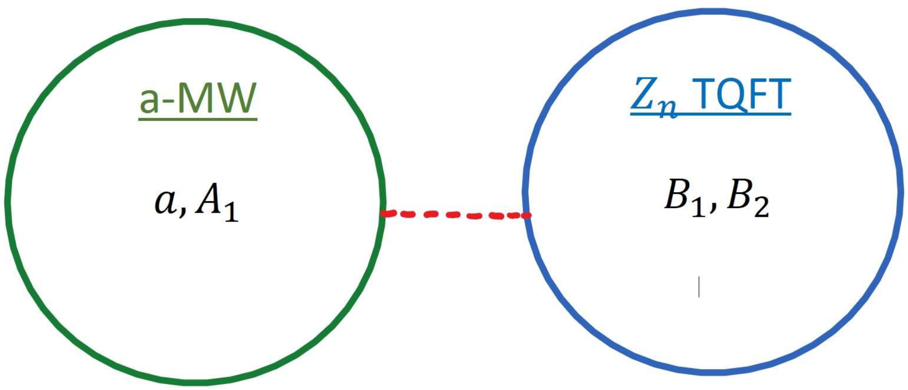

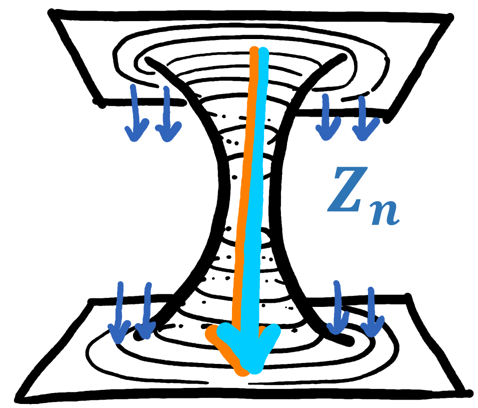

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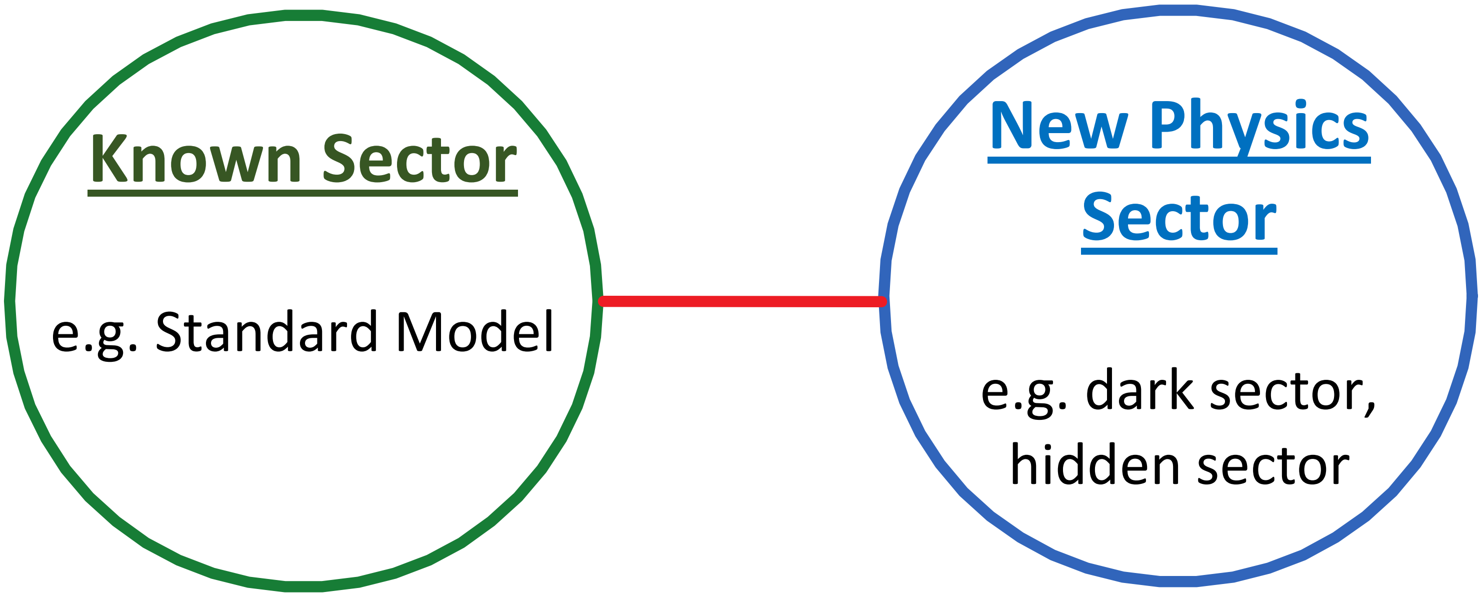
A General Setup in Particle Physics



E.g. Dark (matter) sector,

SUSY breaking sector and SUSY breaking mediation,

Composite-Elementary sector, ...

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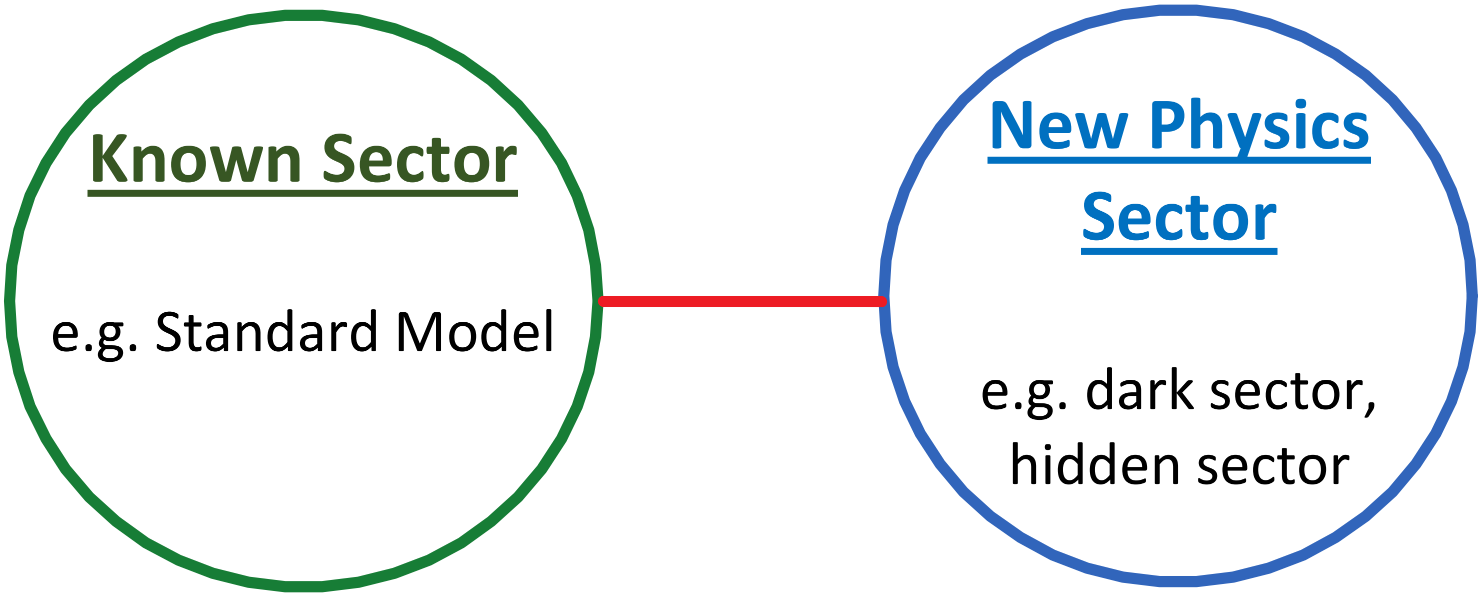
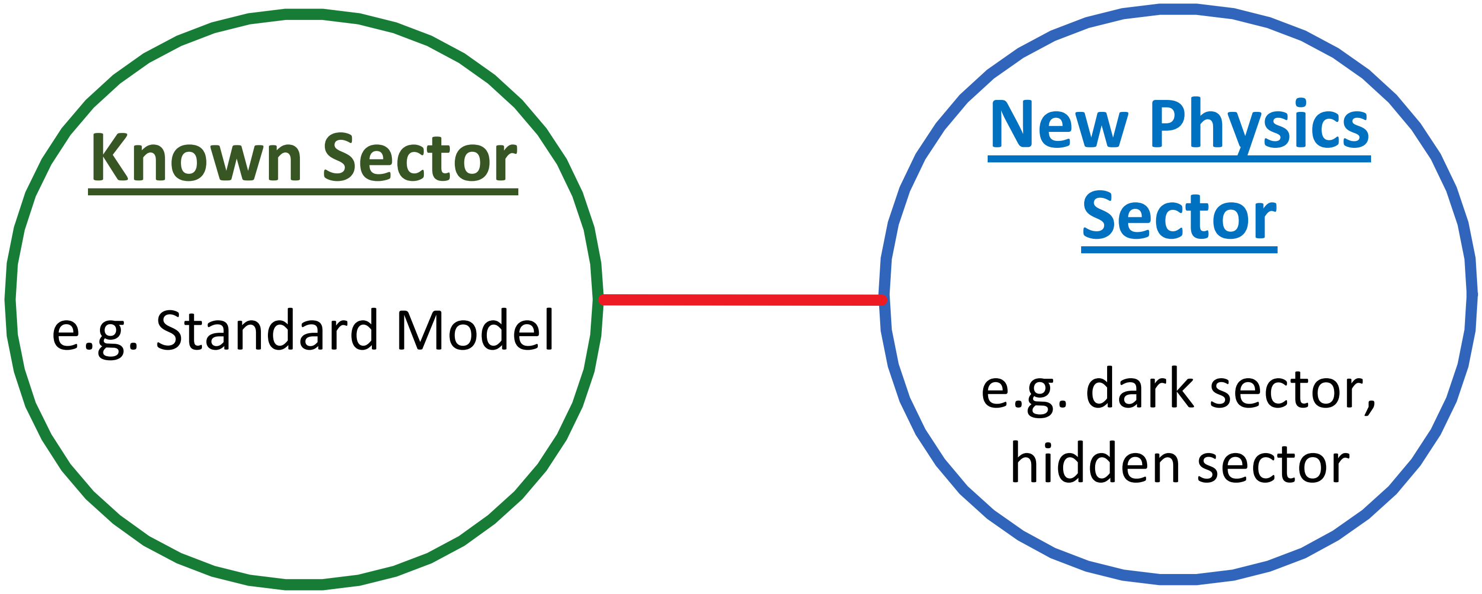


In all the cases considered so far,

New Physics Sector described by a local QFT

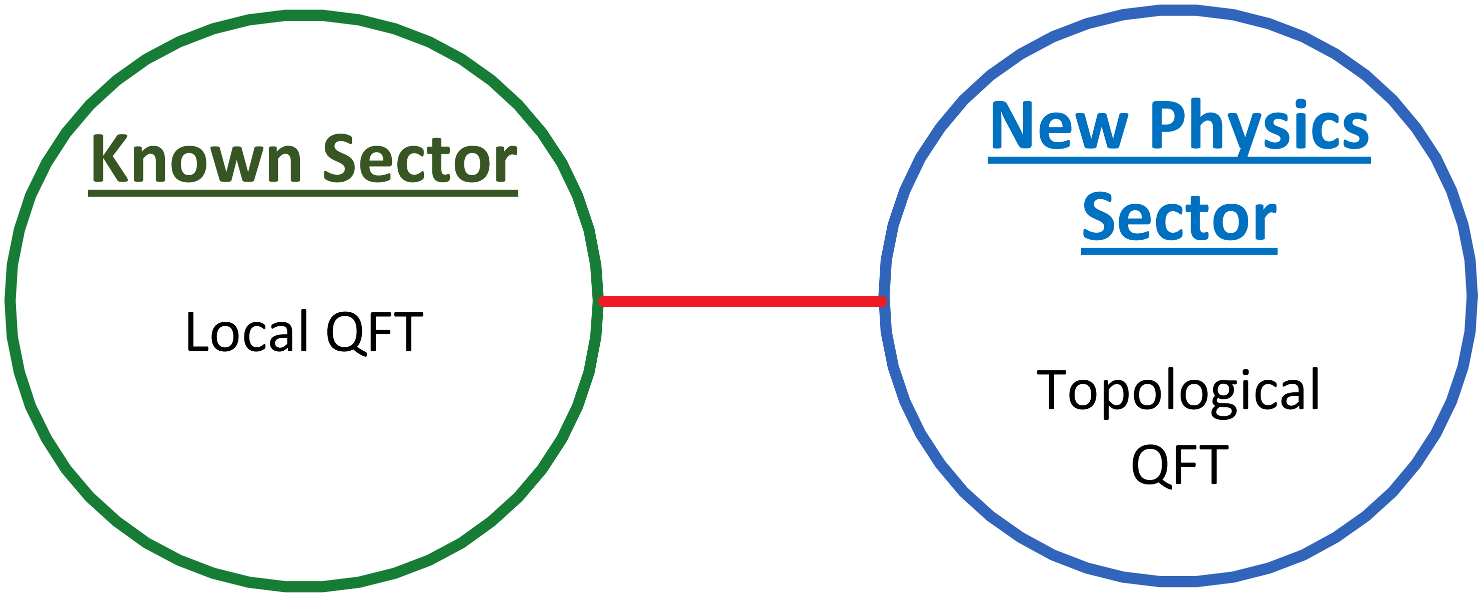
new particles + new interactions
⇒ new/novel dynamics
⇒ solutions to problems in particle physics

A General Setup in Particle Physics



Symmetry
provides an extremely powerful tool.

In this talk,



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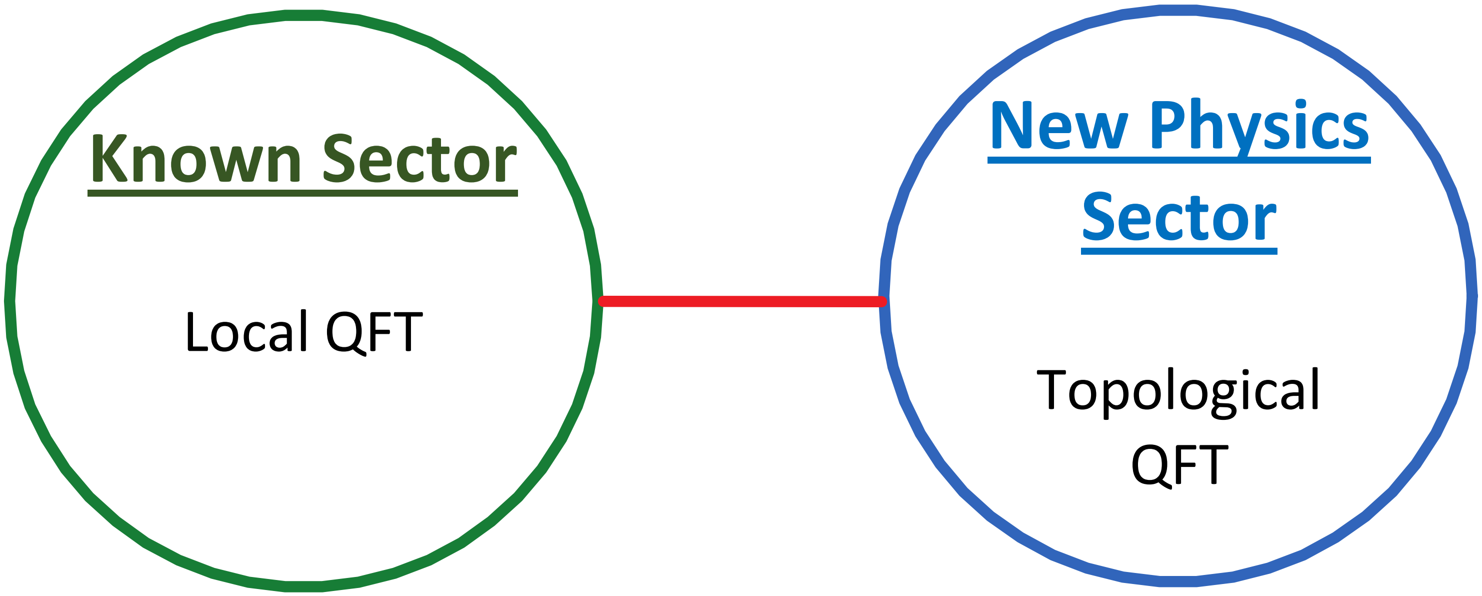

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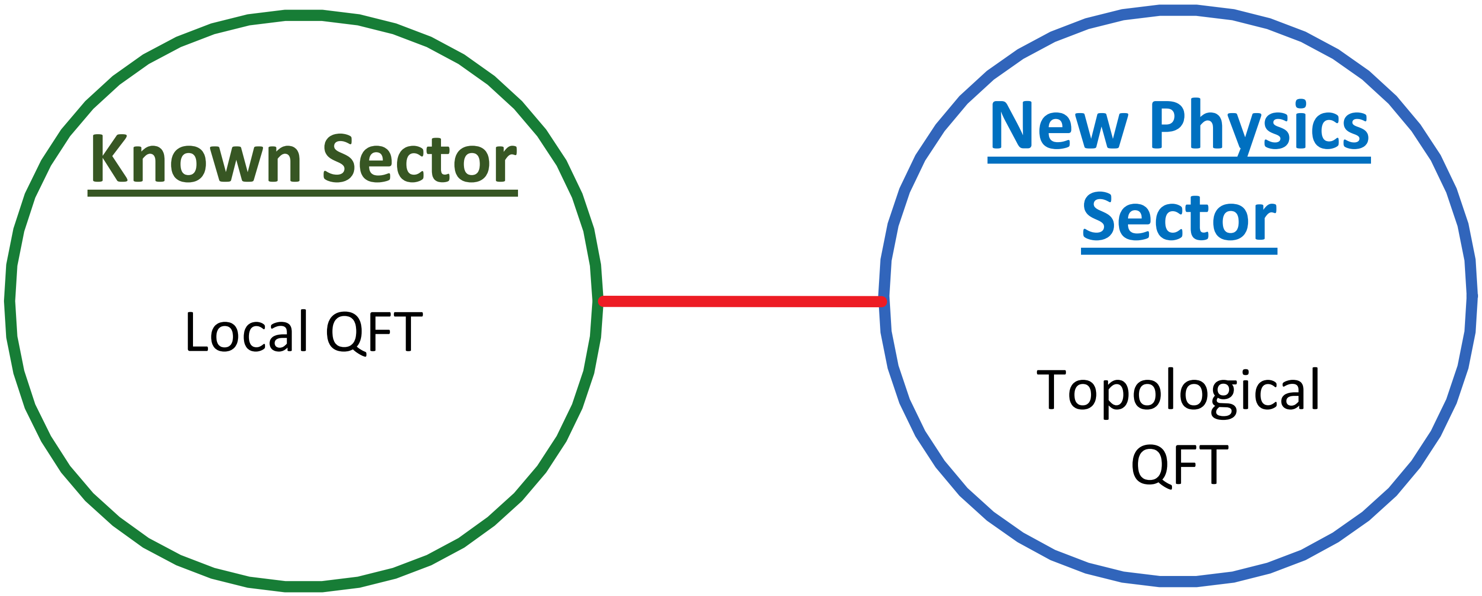

Generalized Global Symmetries
provides an extremely powerful tool.

(Q1) Implications of TQFT-couplings

(Q2) Observable consequences (even in principle)

(Q3) show that TQFT-couplings can exist rather ubiquitously.

Axion-Maxwell Theory

$$S = \int \frac{1}{2}\, da \wedge * \, da \; + \int \frac{1}{2g^2}\, F \wedge * \, F \; - \int \frac{iK}{8\pi^2} \frac{a}{f} F \wedge F$$

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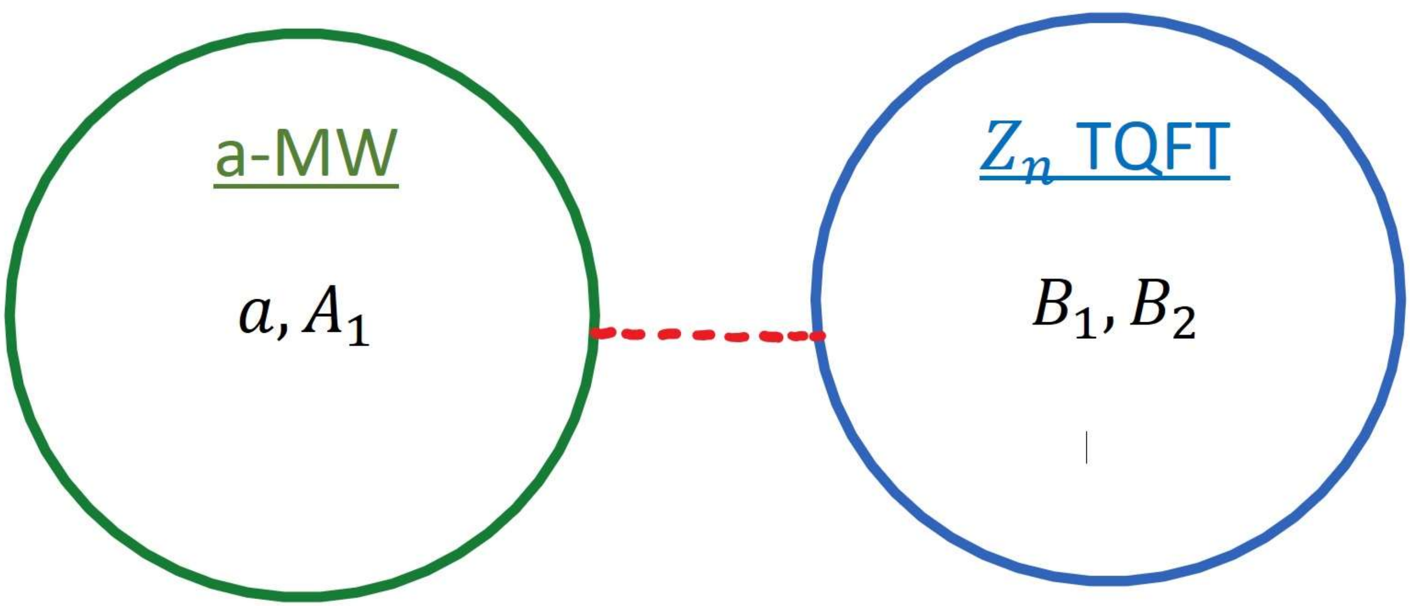
- This very familiar theory enjoys a large set of GGS:
  - 0-form axion shift
  - 2-form axion winding
  - 1-form electric
  - 1-form magnetic
  - ✸ 3-group
  - ✸ Non-invertible symmetries (Cordova, Ohmori '22)

I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT [Brennan,**Hong**,Wang '23]

$$S = \int \frac{1}{2}\, da \wedge * \, da + \int \frac{1}{2g_A^2}\, F_A \wedge * \, F_A - \int \frac{iK_A}{8\pi^2} \frac{a}{f} F_A \wedge F_A$$



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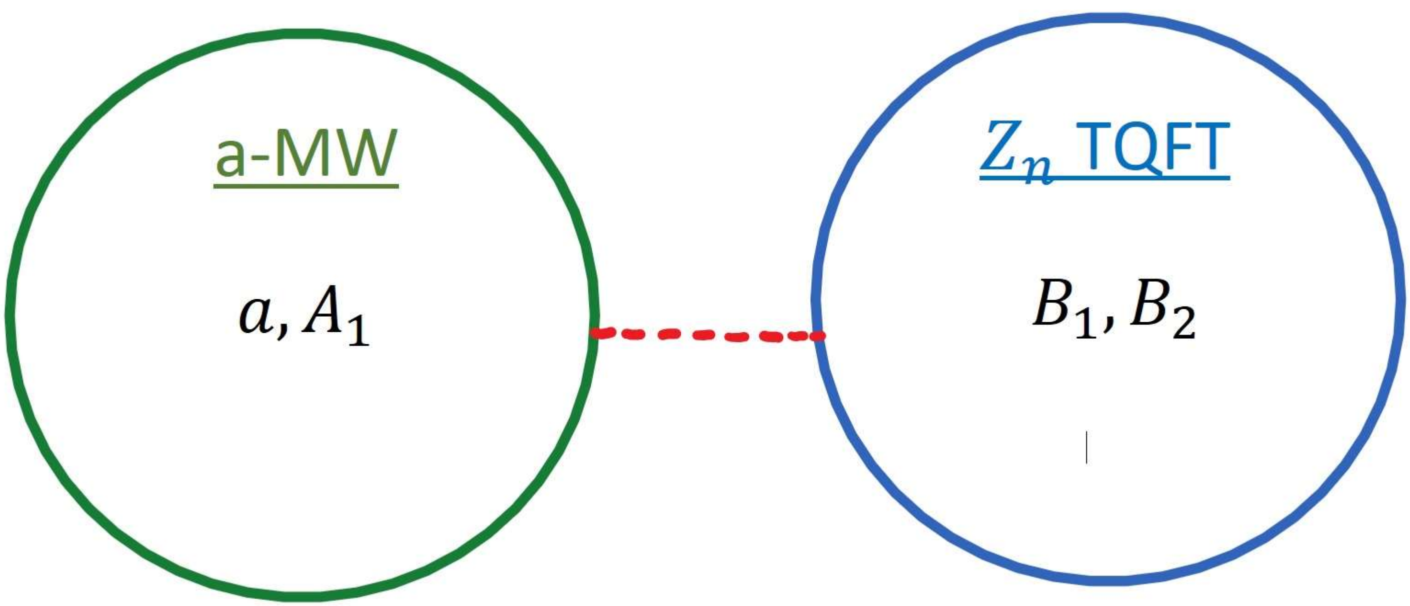
$$+ \int \frac{in}{2\pi}\, B_2 \wedge dB_1 \; - \int \frac{iK_{AB}}{4\pi^2}\frac{a}{f} F_A \wedge F_B - \int \frac{iK_B}{8\pi^2}\frac{a}{f} F_B \wedge F_B$$



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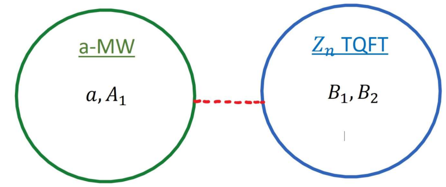

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(Q1) Can there be any **IR-Universal** (local) observable effect?

(Q2) Is this very exotic / pure academic setup?
Or can this arise as IR-EFT of some **standard UV QFT** relevant for particle physics?

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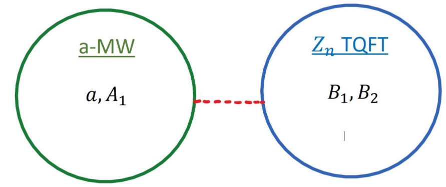

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✓ Illustrate importance of studying carefully the effects of remnant TQFT-couplings (GGS = essential tools)

IR-Universal Observables from TQFT-Coupling

- **Anomaly Inflow**

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

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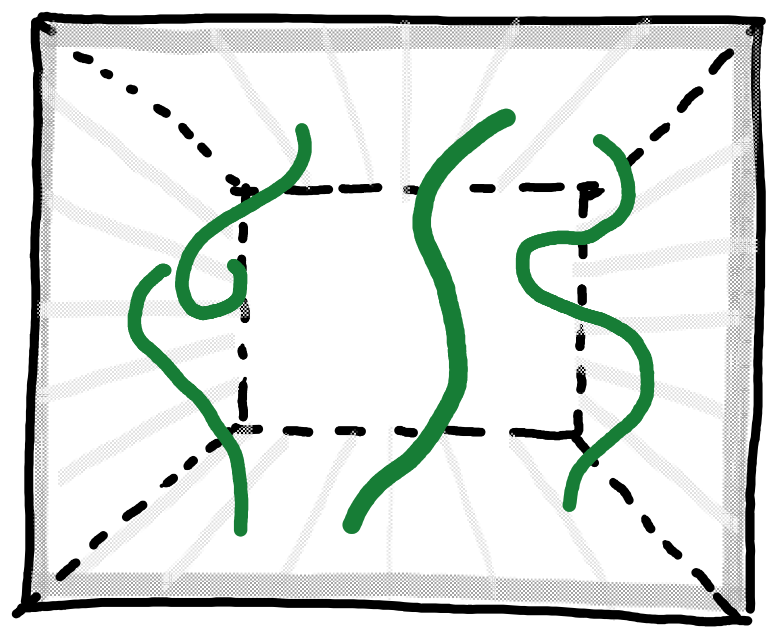

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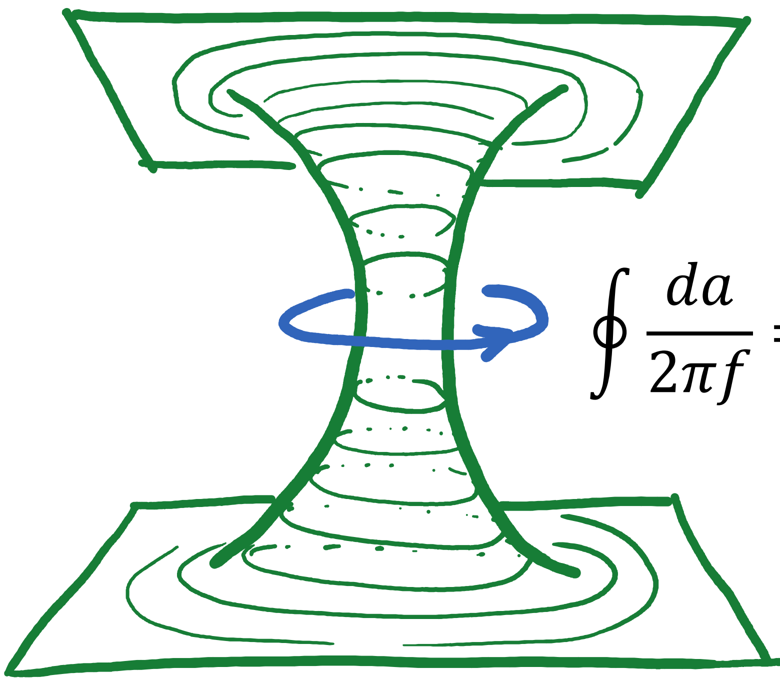
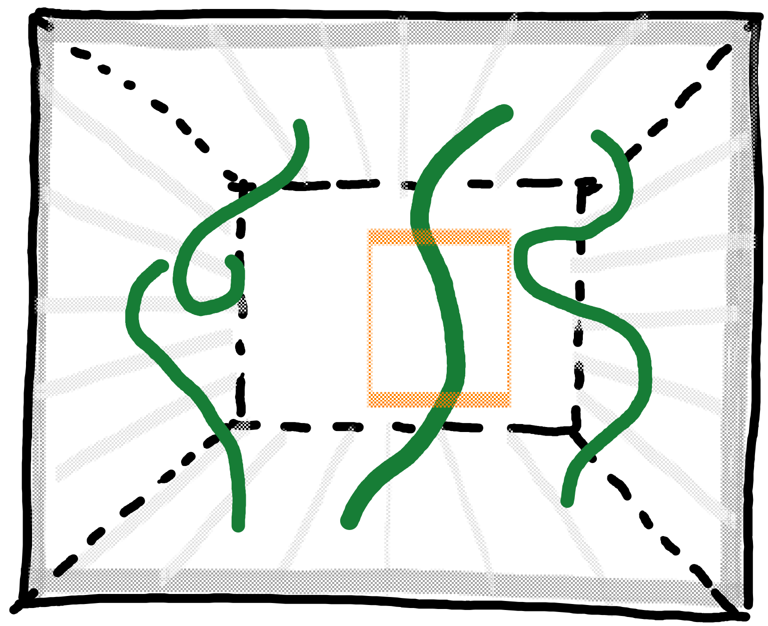


$$\oint \frac{da}{2\pi f} = 1$$



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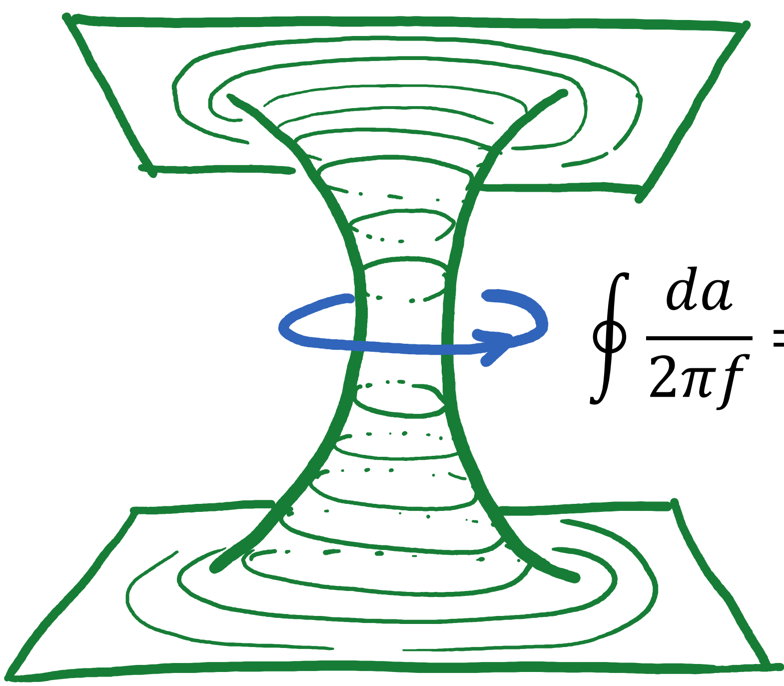


$$\oint \frac{da}{2\pi f} = 1$$

- Consistency with $U(1)_A$ invariance :

$$A_1 \rightarrow A_1 + d\lambda$$

- $S \supset \dfrac{iK_A}{8\pi^2} \displaystyle\int da \wedge A_1 \wedge F_A$

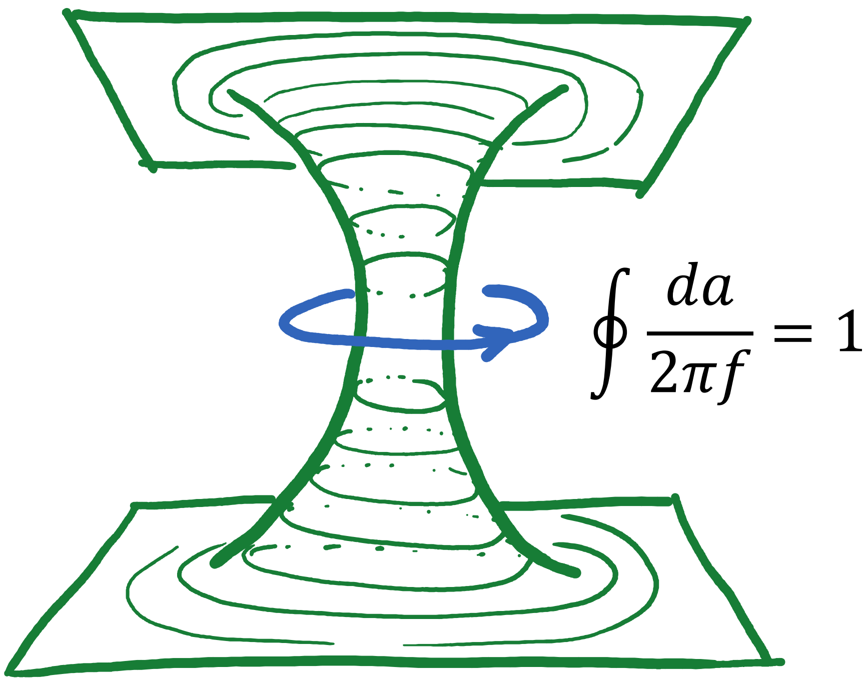
$$\downarrow$$

- $\delta S = i \displaystyle\int \delta^{(2)}\left(M_2^{st}\right) \wedge \left(\lambda \frac{K_A}{4\pi} F_A\right)$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : W/O TQFT-Coupling [Callan and Harvey '85]

$$S \supset \frac{iK_A}{8\pi^2}\int da \wedge A_1 \wedge F_A = i\int A_1 \wedge * J_1$$



- $* J_1 = \frac{K_A}{4\pi}\, da \wedge F_A$
- $\mathrm{d} * J_1(bulk) = \frac{K_A}{4\pi} F_A \wedge \delta^{(2)}(M_2^{st})$
- $\vec{J}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)

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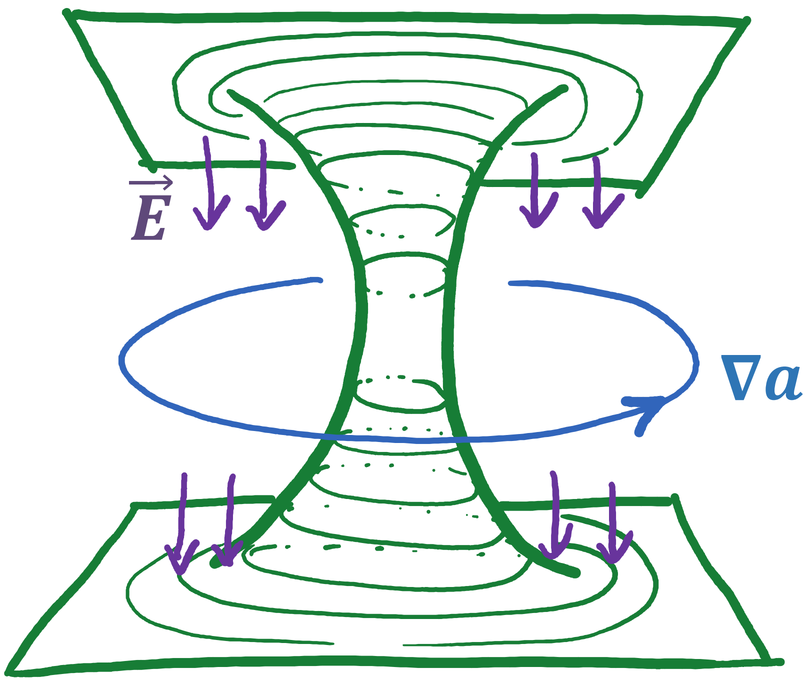

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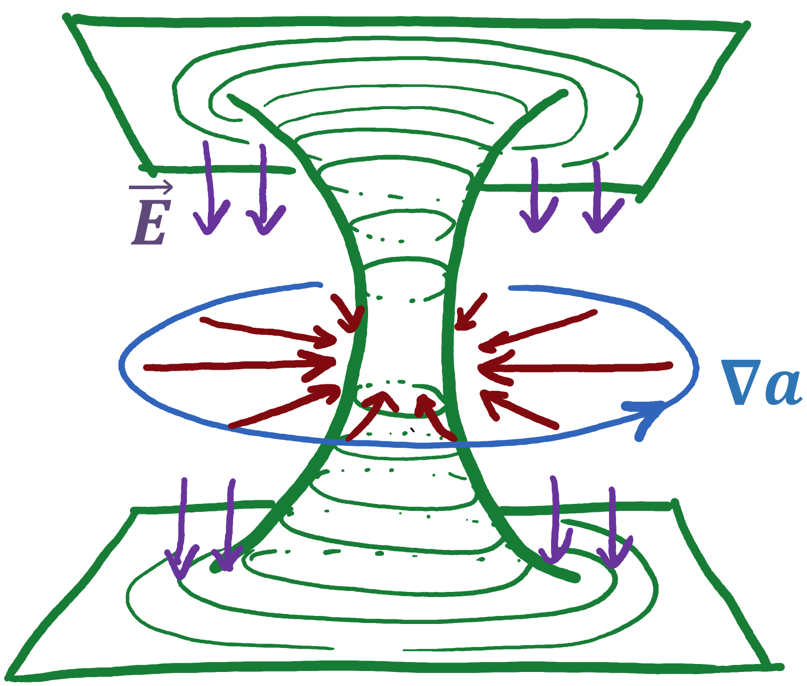

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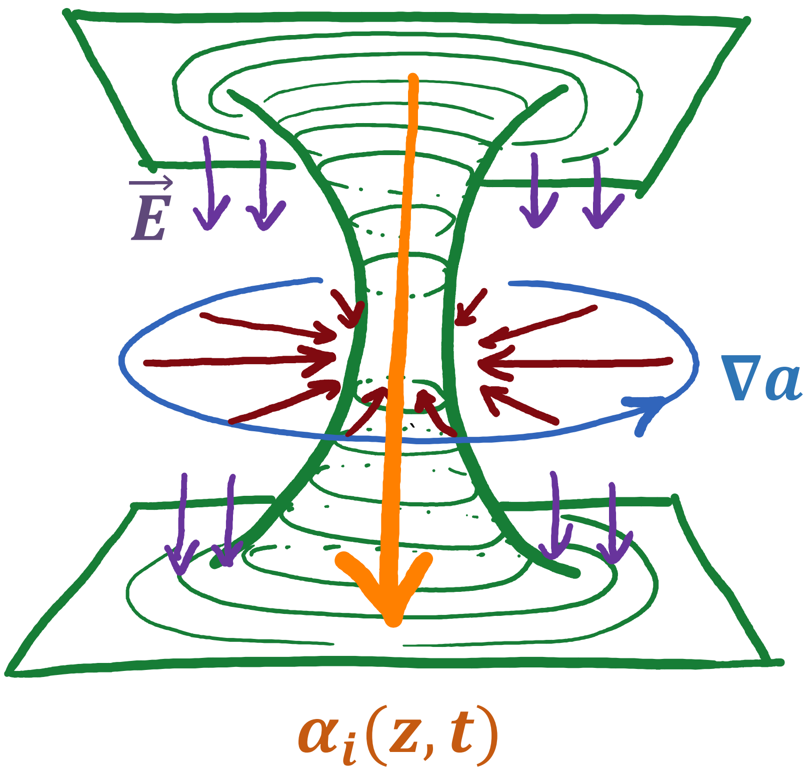

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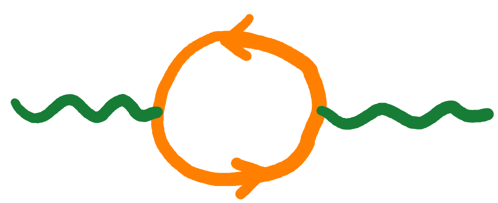
- $\vec{J}_1 \sim \nabla a \times \vec{E}$ (Hall-like current)
- 2d chiral fermions $\{\alpha_i\,(z,t)\}$

$$\mathrm{d} * J_1(2d) = -\frac{K_A}{4\pi} F_A$$



$$\sum_i Q_i^2 = K_A$$

IR-Universal Observables from TQFT-Coupling

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1. $A_1 \to A_1 + d\lambda_A$

$$\delta_A S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_A \left(\frac{K_A}{4\pi} F_A + \frac{K_{AB}}{2\pi} F_B\right)$$

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2. $B_1 \to B_1 + d\lambda_B \,, \qquad \lambda_B = \frac{2\pi}{n} \kappa \,, \qquad \kappa = 0, 1, \cdots, n-1$

$$\delta_B S = i \int \delta^{(2)}\left(M_2^{st}\right) \wedge \lambda_B \left(\frac{K_{AB}}{2\pi} F_A + \frac{K_B}{4\pi} F_B \right)$$

IR-Universal Observables from TQFT-Coupling

✸ **Anomaly Inflow** : With **TQFT-Coupling** [Brennan,**Hong**,Wang '23]

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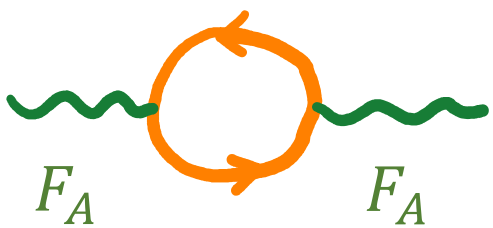

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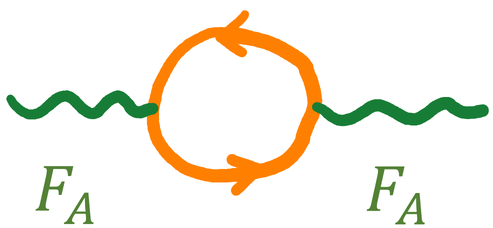

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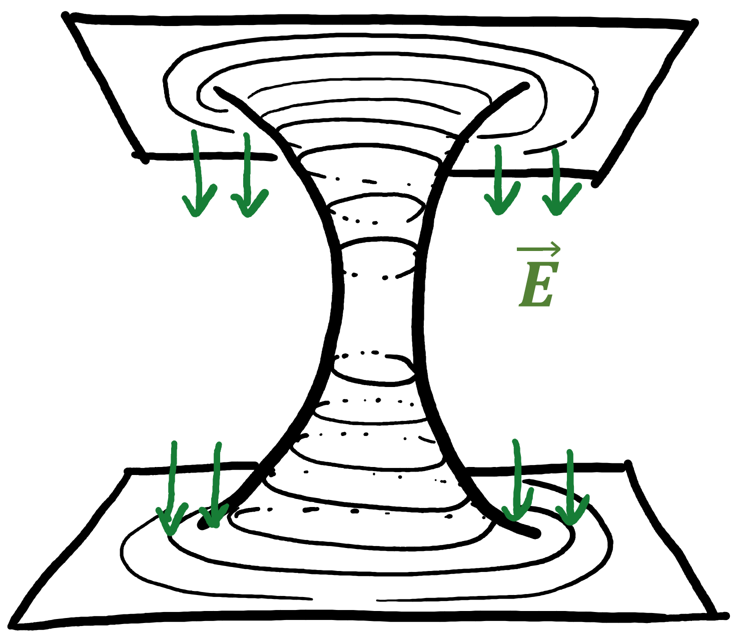

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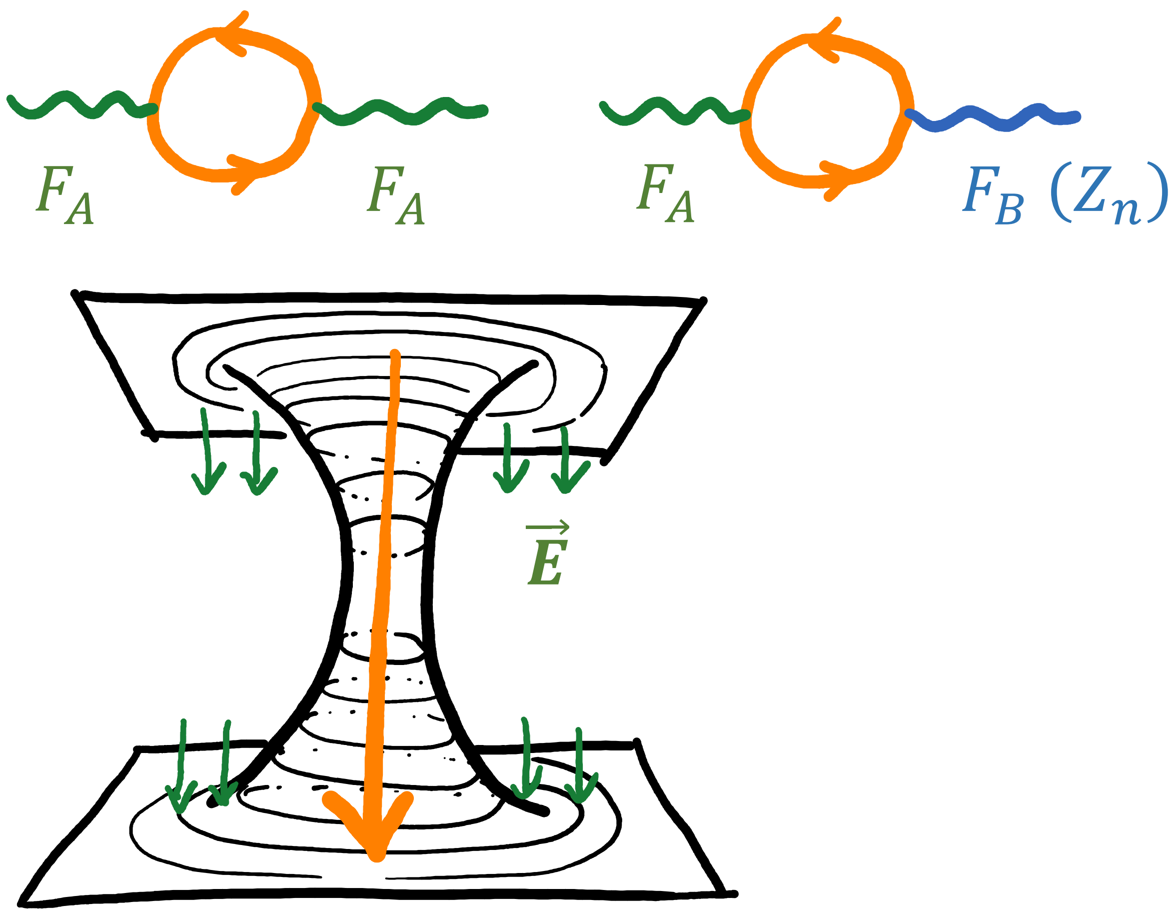

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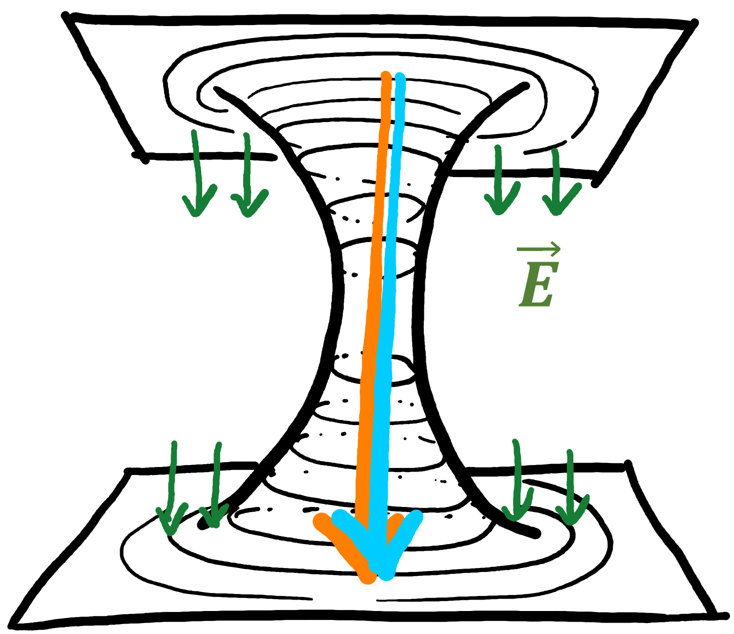

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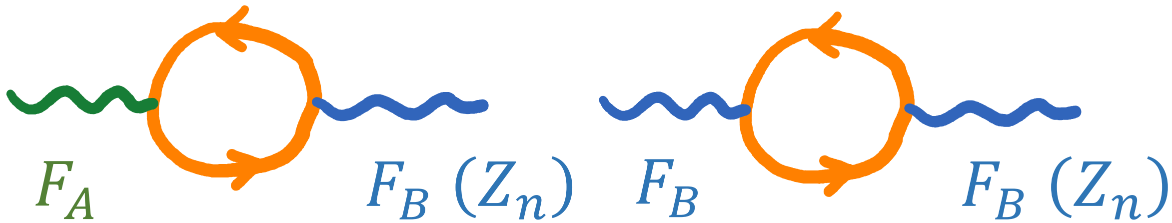

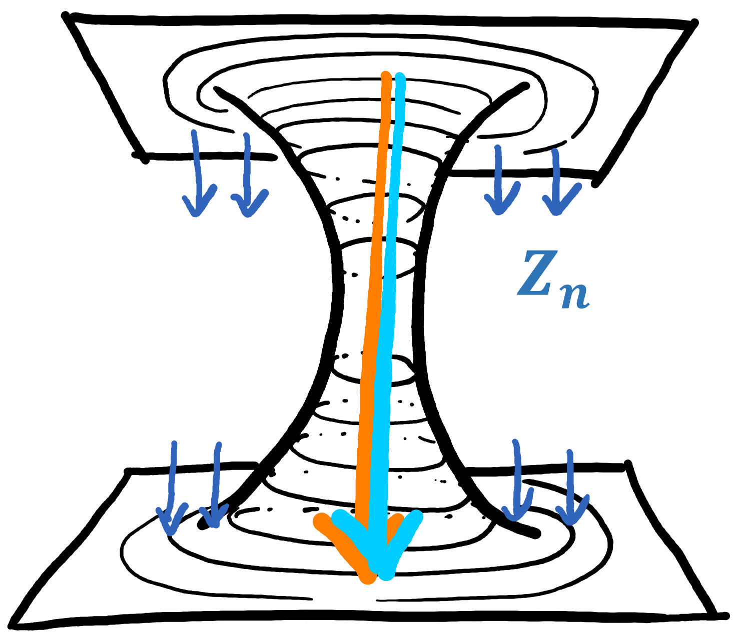

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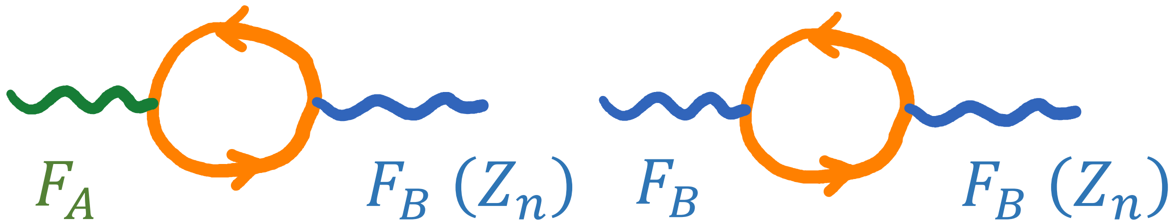
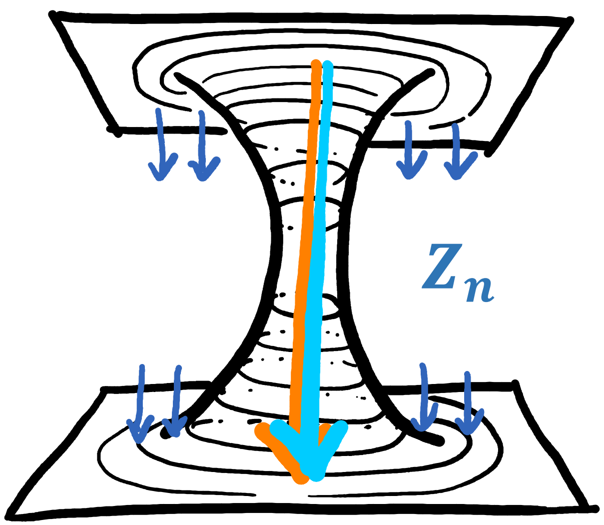




◇ $O(1)$ change of local physics from TQFT-coupling
(via 4D → 2D dimensional reduction)

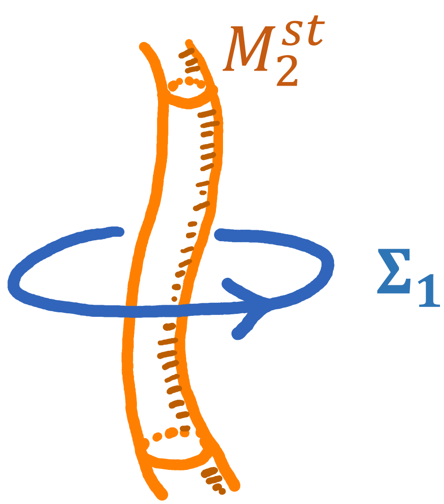
◇ Charge and SC current very differrent
: vorton stability, cosmic string network evolution,
cosmological plasma collider signals

◇ Solving axion DW problem from TQFT-coupling ?

IR-Universal Observables from TQFT-Coupling

W/O TQFT-Coupling

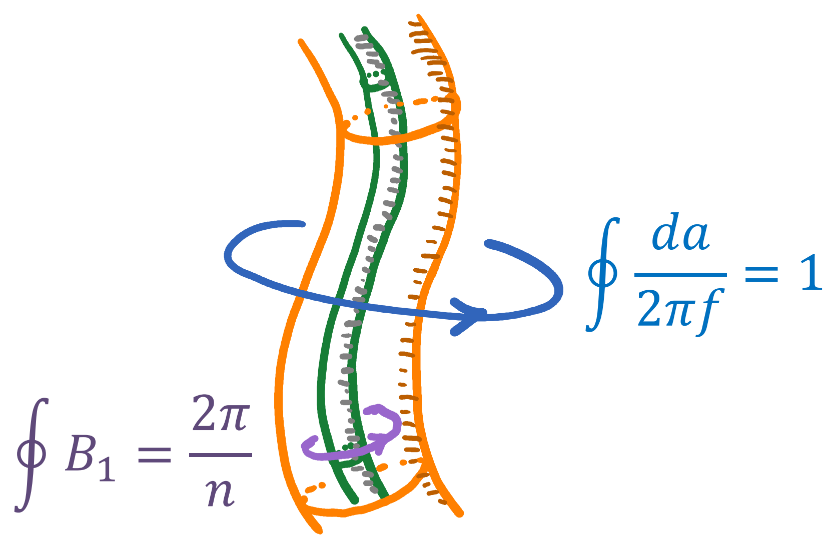
- Axion strings: Global strings



With TQFT-Coupling

- Axion strings: Global strings
- BF strings: $W_2(\Sigma_2, \ell) = e^{i\ell \oint_{\Sigma_2} B_2}$

  Local or (Quasi) Aharonov-Bohm
- Coaxial Hybrid strings ?



Extended KSVZ with TQFT-Coupling [Brennan, **Hong**, Wang '23]

$$\mathcal{L} = -\frac{1}{2g_A^2} F_A \wedge * F_A + \overline{\psi_1} i\gamma^\mu D_\mu \psi_1 + \overline{\chi_1} i\gamma^\mu D_\mu \chi_1 - \lambda_1 \Phi_1^\dagger \psi_1 \chi_1$$

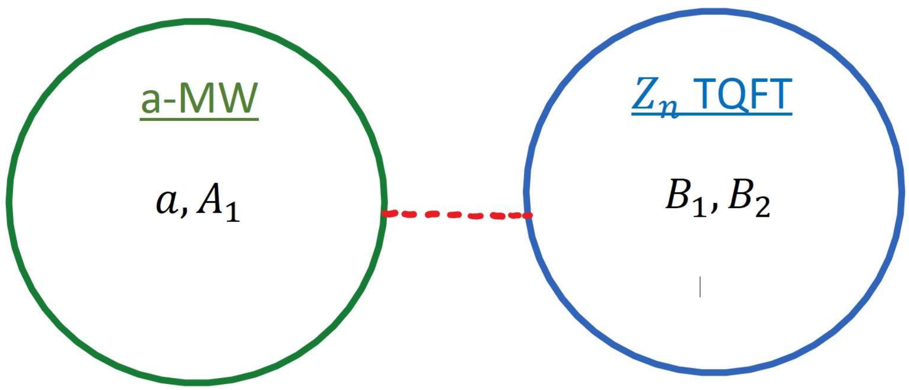
$$-\frac{1}{2g_B^2} F_B \wedge * F_B + \overline{\psi_2} i\gamma^\mu D_\mu \psi_2 + \overline{\chi_2} i\gamma^\mu D_\mu \chi_2 - \lambda_2 \Phi_2 \psi_2 \chi_2 + V(\Phi_1, \Phi_2)$$

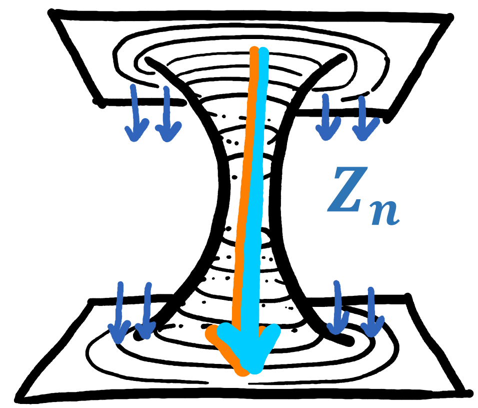
	$U(1)_{PQ}$	$U(1)_A$	$U(1)_B$
Φ_1	1	0	n
Φ_2	0	0	n
ψ_1	1	1	q
χ_1	0	-1	n-q
ψ_2	0	1	q-n
χ_2	0	-1	-q

Outline

Coupling a Cosmic String to a TQFT

(with T. Daniel Brennan and Liantao Wang)





I. TQFT-Coupling 1: Axion-Portal to a Z_n TQFT

II. TQFT-Coupling 2: Z_M Discrete Gauging

Outline

Coupling a Cosmic String to a TQFT

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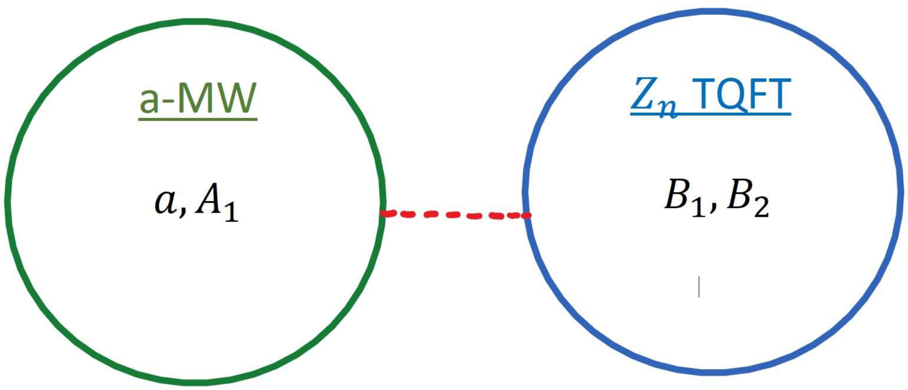

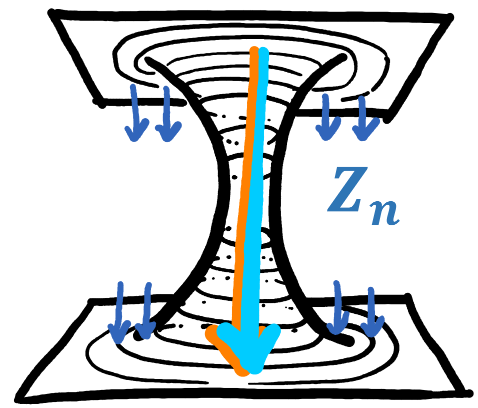

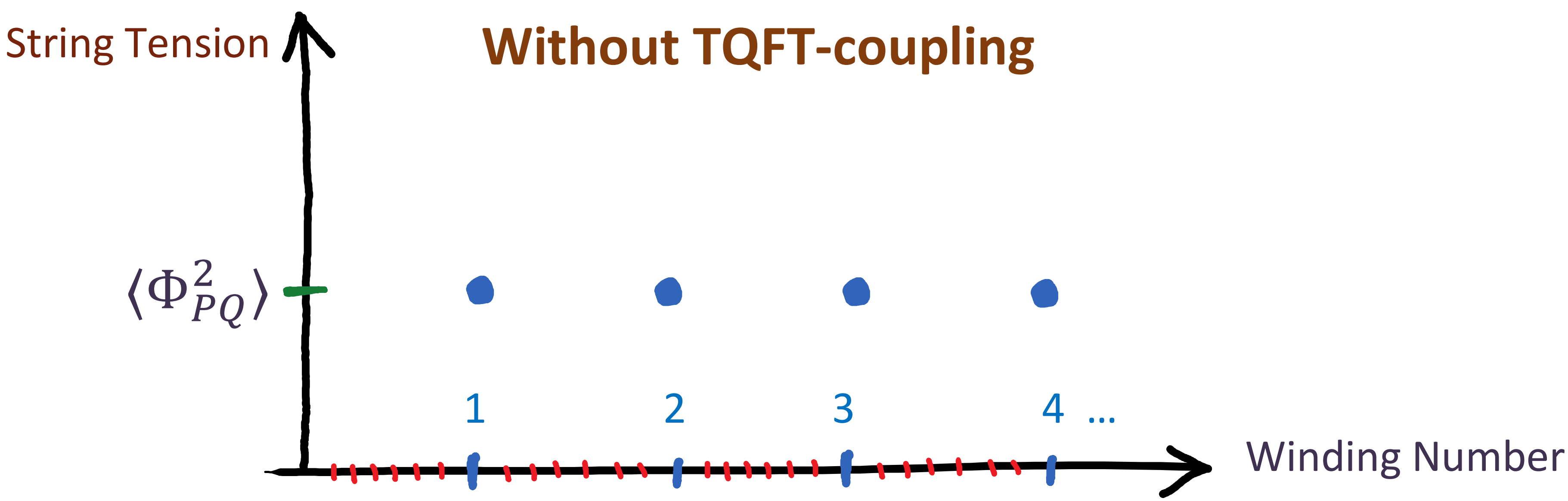

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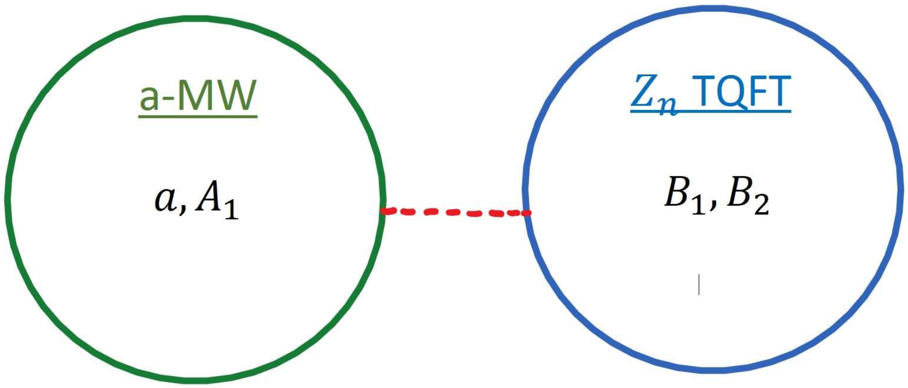
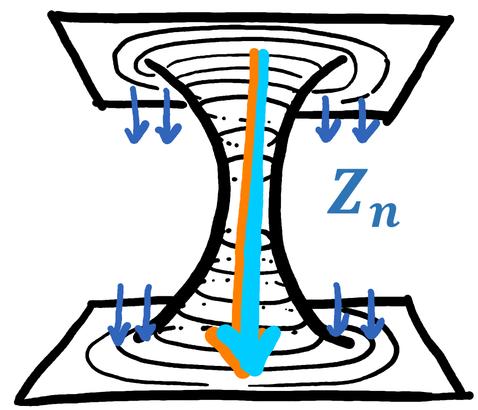
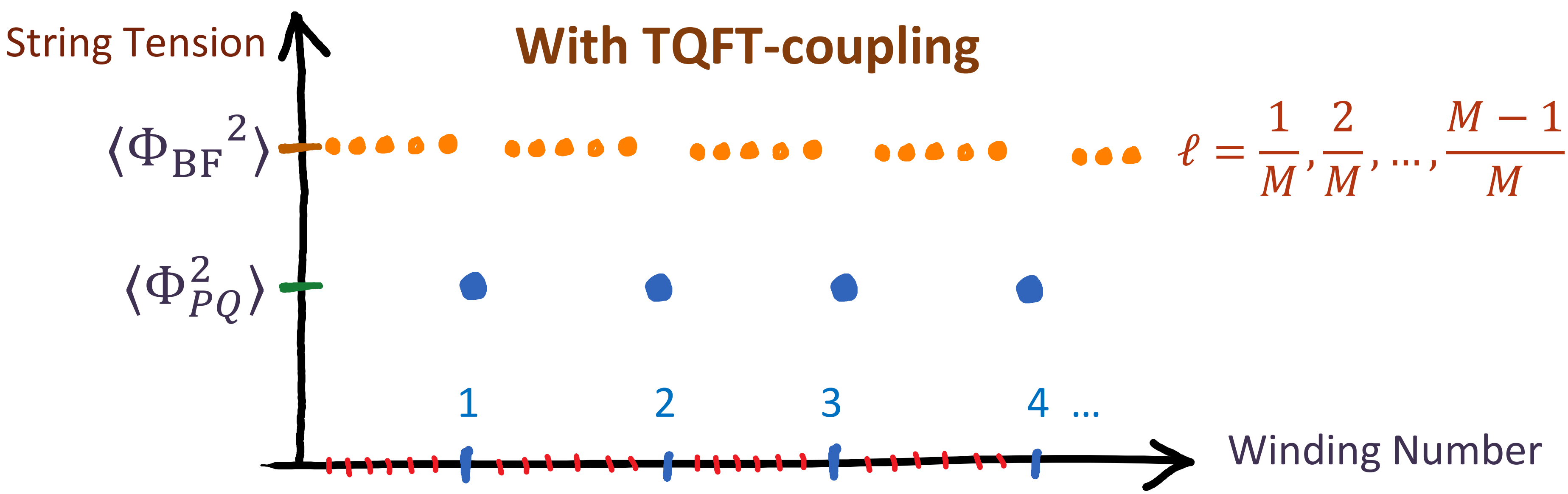






Outline

I. Non-Invertible Symmetry: Practical Introduction

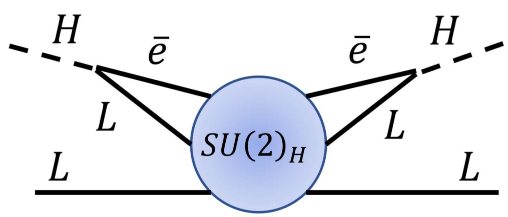
II. Non-Invertible Symmetry in SM and SM+U(1)

III. Small M_ν from Generalized Symmetry Breaking

Neutrino Masses from

Generalized Symmetry Breaking

(with Clay Córdova, Seth Koren, and Kantaro Ohmori)



$$D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right)$$

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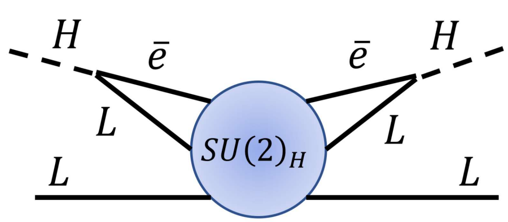

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Non-Invertible Symmetry

Consider a massless QED: ψ_-, ψ_+ charged under gauged $U(1)$

∃ global $U(1)_A$ with Adler-Bell-Jackiw (ABJ) anomaly

$$\psi_- \to e^{i\alpha}\psi_-, \quad \psi_+ \to e^{i\alpha}\psi_+ \quad \Rightarrow \quad d * J_1 = \frac{N_f}{8\pi^2}\, F \wedge F$$

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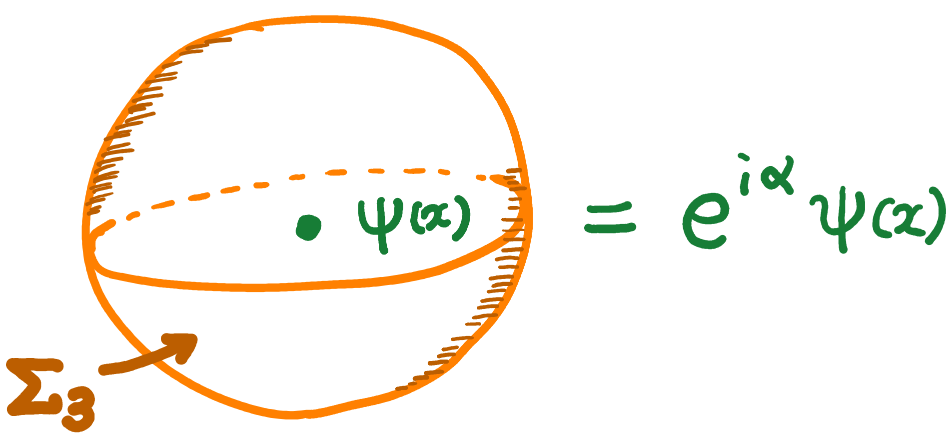
Under $\alpha = \frac{2\pi}{k}, \quad S \to S + \frac{2\pi i N_f}{k} \int_{M_4} \frac{F \wedge F}{8\pi^2}$



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"Symmetry Defect Operator"

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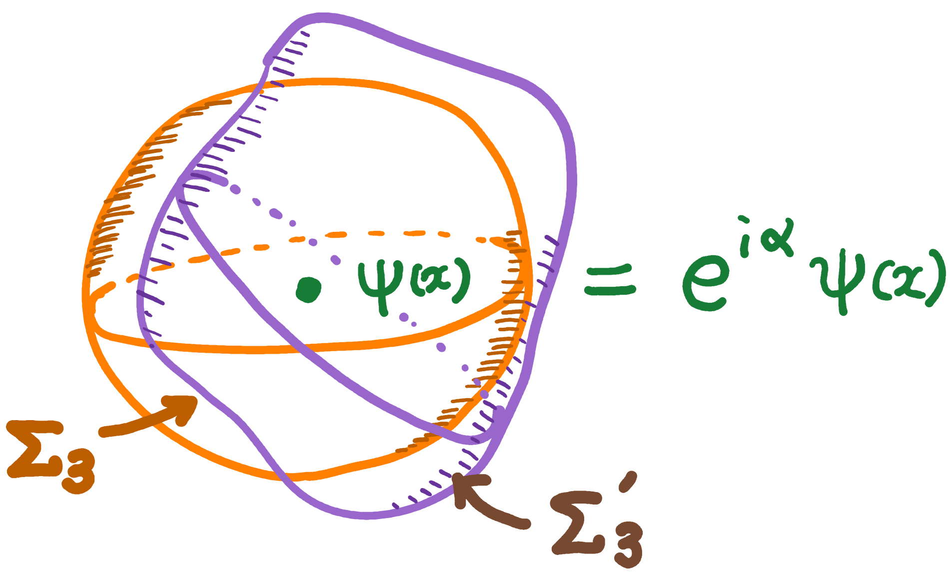

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"Old days": $U(1)_A \to Z_{N_f}$

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"Old days": $U(1)_A \to Z_{N_f}$ vs $\int_{M_4} \frac{F \wedge F}{8\pi^2} = 0$ for $U(1)$

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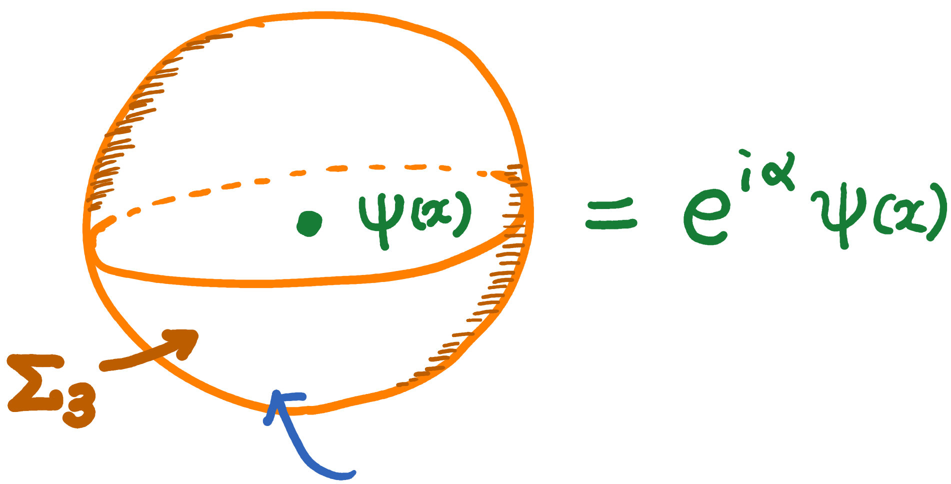
"Old days": $U(1)_A \to Z_{N_f}$ $\quad$ vs $\quad$ $\int_{M_4} \frac{F \wedge F}{8\pi^2} = 0$ for $U(1)$

Now, we say that $U(1)_A \to Z_{N_f}$ (invertible) + {non-invertible}

2205.05086 (Yichul Choi, Ho Tat Lam, Shu-Heng Shao),
2205.06243 (Clay Córdova, Kantaro Ohmori)

Non-Invertible Symmetry

Under $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi i N_f}{k} \int_{M_4} \frac{F \wedge F}{8\pi^2}$



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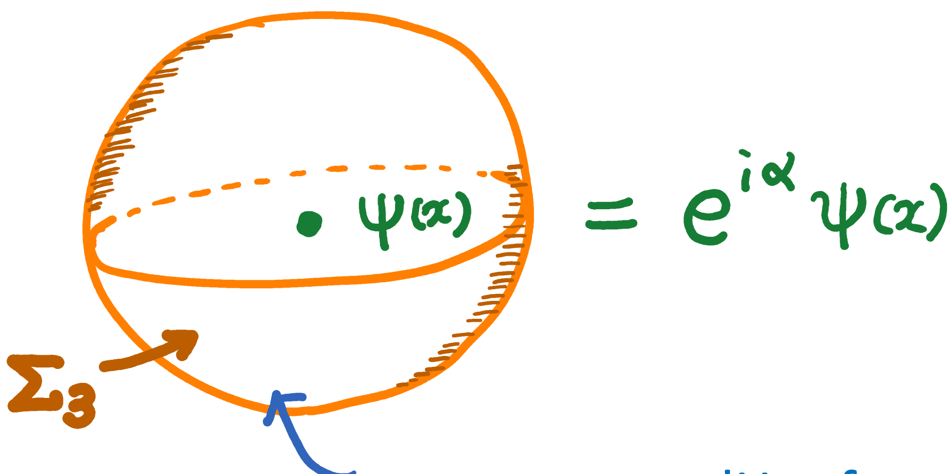

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t 'Hooft anomaly of $Z_N^{(1)}$ 1-form global symmetry

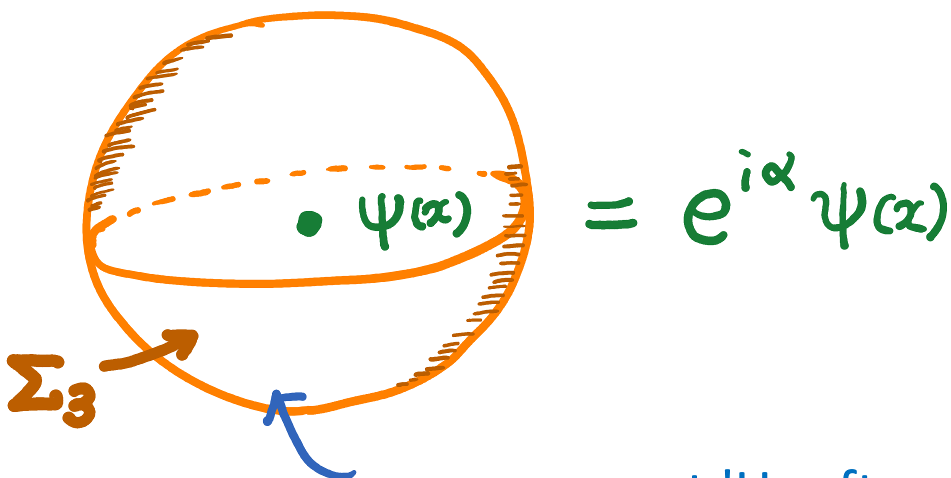
$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC$$

$$S_{inflow} = -\frac{2\pi i p}{N} \int_{M_4} \frac{\mathcal{P}(B_2)}{2}$$

$$C \to C + \frac{1}{N}\epsilon_1, \int \frac{\epsilon_1}{2\pi} \in Z$$

Non-Invertible Symmetry

Under $\alpha = \frac{2\pi}{k}$, $\quad S \to S + \frac{2\pi i N_f}{k} \int_{M_4} \frac{F \wedge F}{8\pi^2} - \frac{2\pi i p}{N} \int_{M_4} \frac{F \wedge F}{8\pi^2}$



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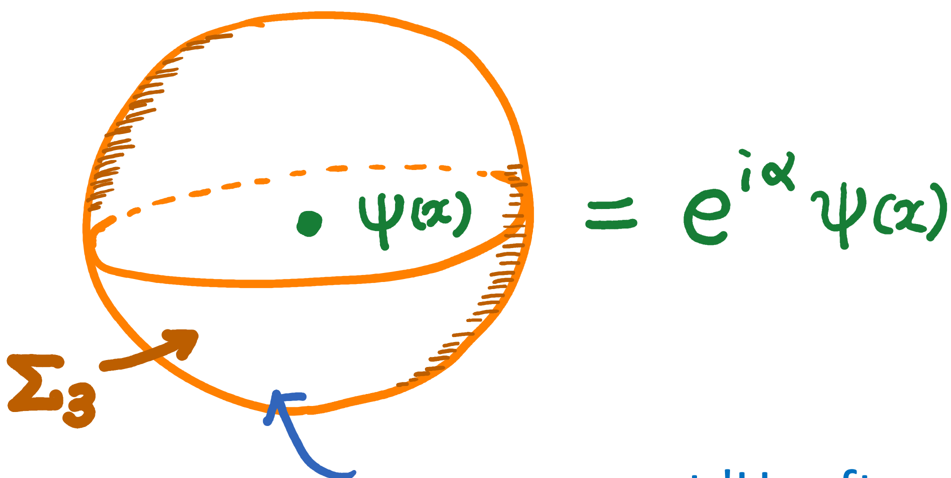

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$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC + \frac{i}{2\pi} \int_{\Sigma_3} C \wedge dA$$

$$S_{inflow} = -\frac{2\pi i p}{N} \int_{M_4} \frac{\mathcal{P}(B_2)}{2} \to -\frac{2\pi i p}{N} \int_{M_4} \frac{F \wedge F}{8\pi^2}$$

$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right) \text{ with } \frac{p}{N} = \frac{N_f}{k}$$

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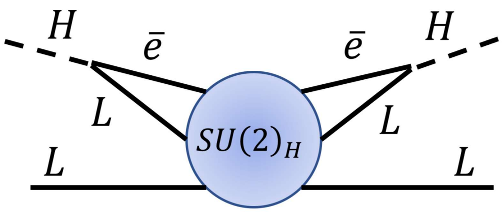

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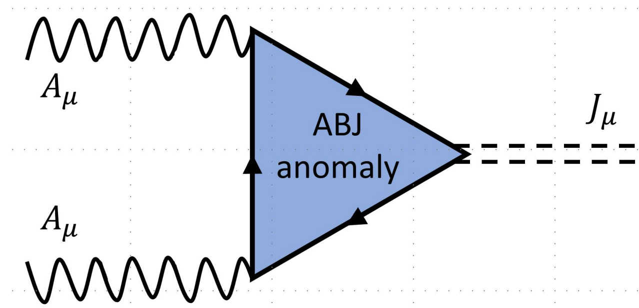
No non-invertible symmetry in SM

1. Classical Symmetry of SM

$$U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times \frac{U(1)_B}{Z_3}$$



2. ABJ (Adler-Bell-Jackiw) anomalies

	$SU(2)_L^2$	$U(1)_Y^2$	$SU(3)_C^2$
$U(1)_B$	$N_g N_c$	$-18 N_g N_c$	0
$U(1)_{L_k}$	1	-18	0
$U(1)_L$	N_g	$-18 N_g$	0

$(L = L_e + L_\mu + L_\tau)$

No non-invertible symmetry in SM

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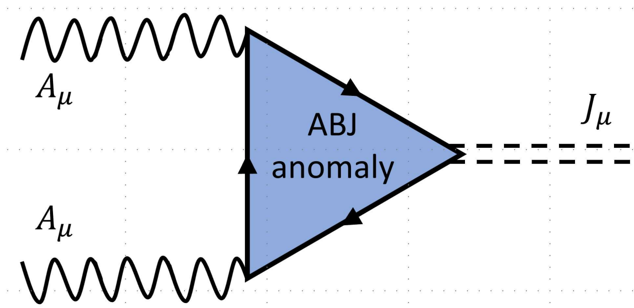

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$U(1)_{L_k}$	1	-18	0
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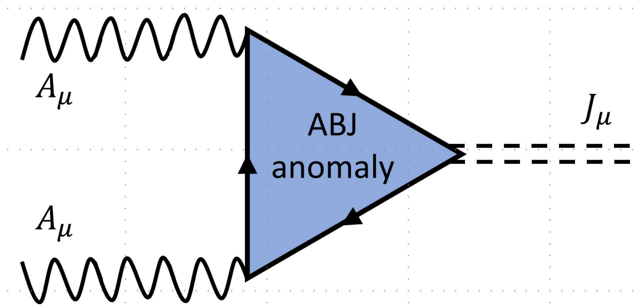
$(L = L_e + L_\mu + L_\tau)$

3. Quantum Invertible Symmetry : $U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau} \times \frac{U(1)_{B-N_C L}}{Z_{N_C}}$

No non-invertible symmetry in SM

1. Classical Symmetry of SM

$$U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times \frac{U(1)_B}{Z_3}$$



2. ABJ (Adler-Bell-Jackiw) anomalies

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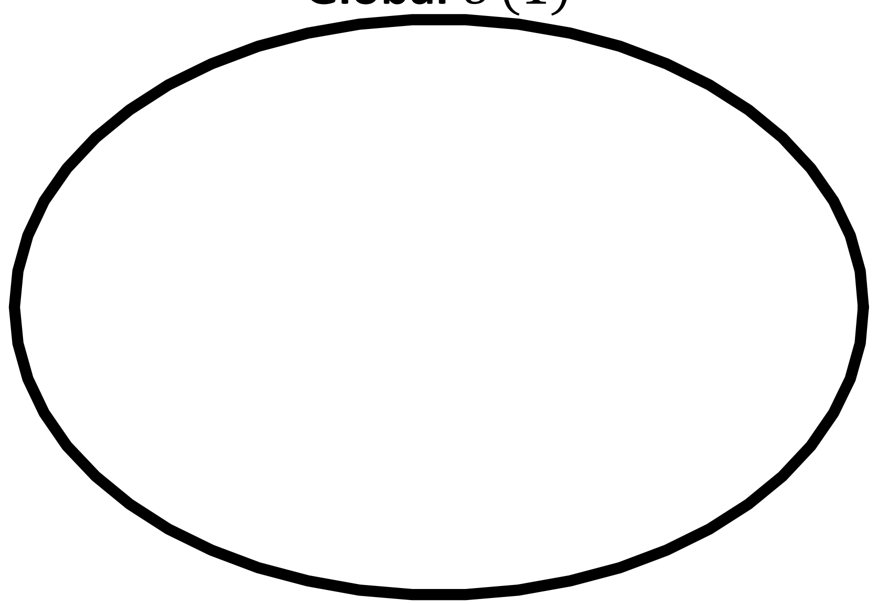
Global $U(1)$



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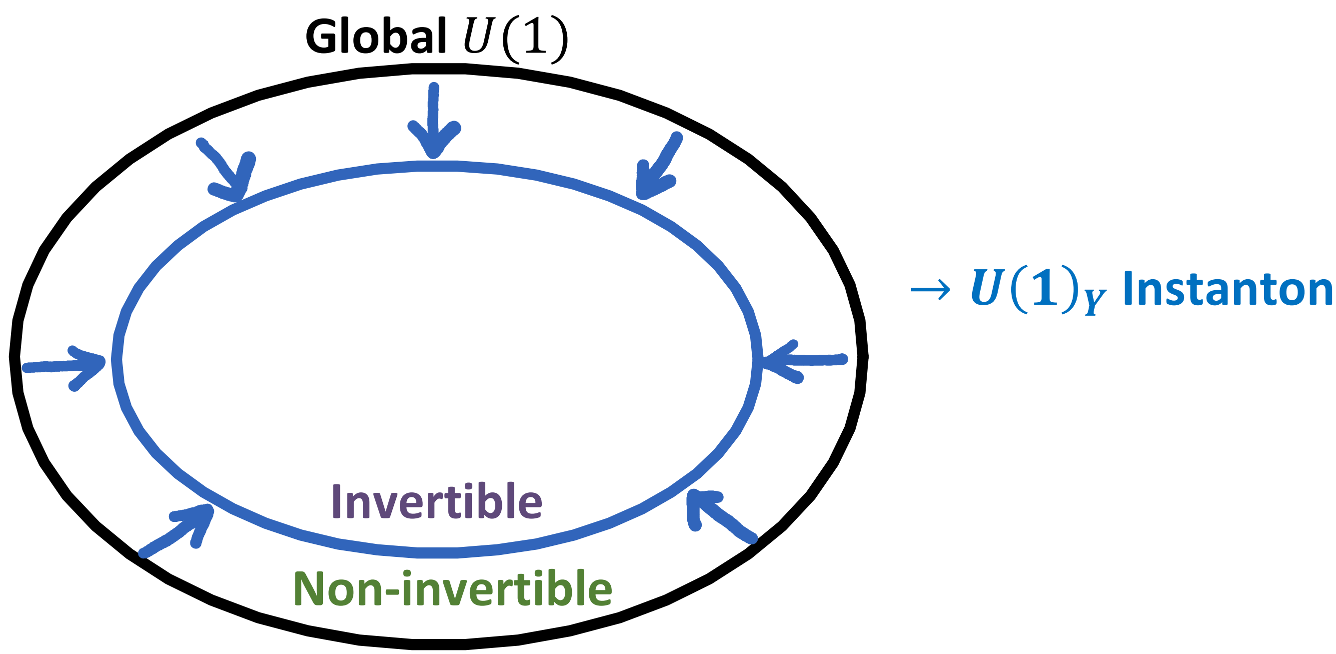

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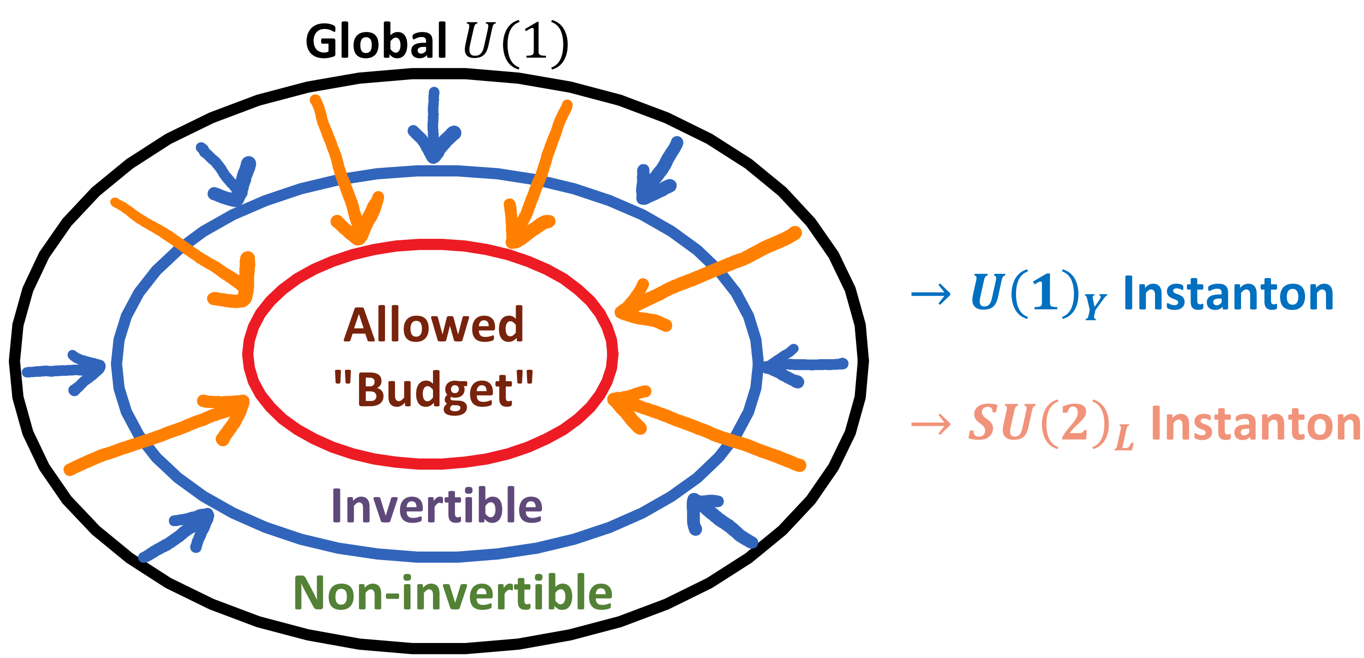

Non-invertible symmetry in SM+$U(1)$

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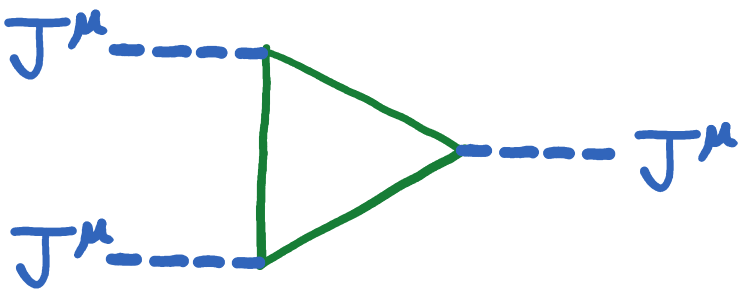

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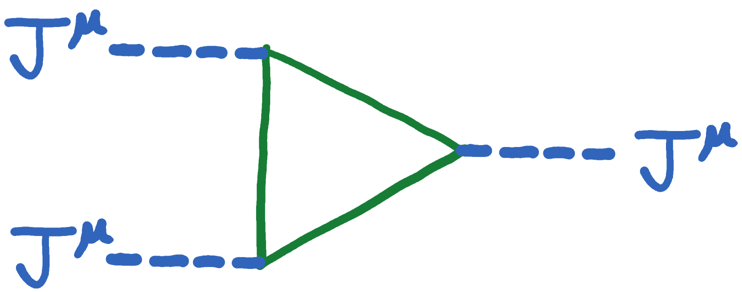

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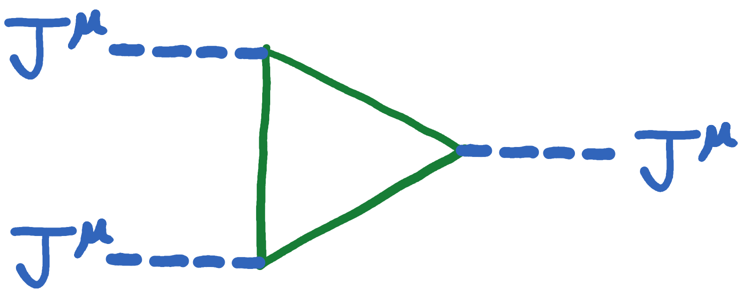

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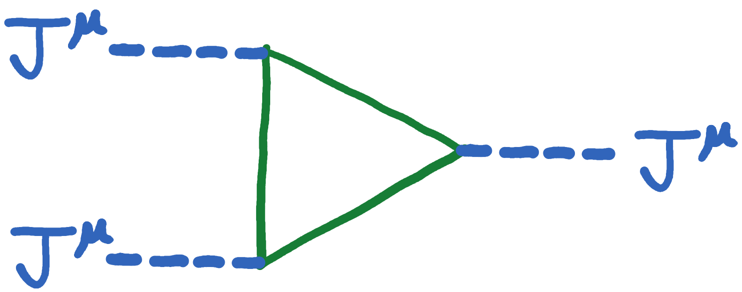

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3. Symmetry of $G_{SM}\times U(1)_{L_\mu-L_\tau}$:

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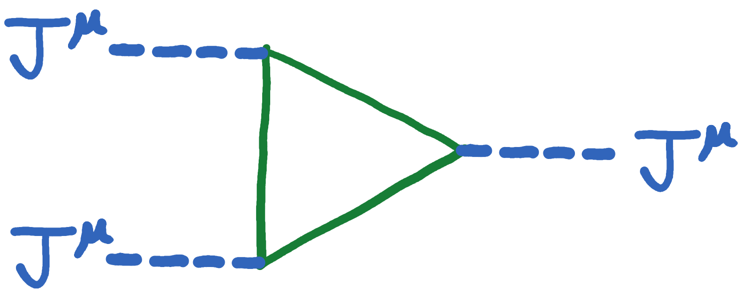

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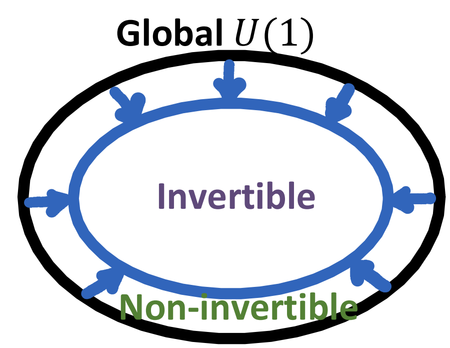

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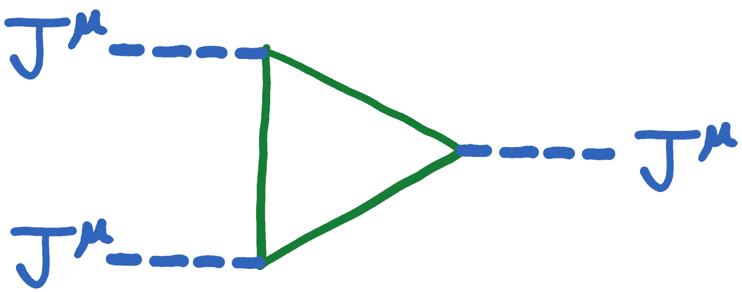

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Outline

I. Non-Invertible Symmetry: Practical Introduction

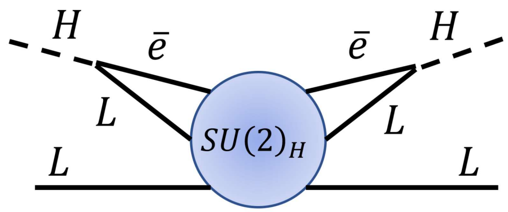
II. Non-Invertible Symmetry in SM and SM+U(1)

III. Small M_ν from Generalized Symmetry Breaking

Neutrino Masses from

Generalized Symmetry Breaking

(with Clay Córdova, Seth Koren, and Kantaro Ohmori)



$$D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}\left(\frac{F}{2\pi}\right)$$

Small M_ν from Generalized Symmetry Breaking

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   (See our paper for Dirac case and many more details)

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- From invertible symmetry: $(HL_e)^2$ forbidden
- From non-invertible symmetry: Both $(HL_e)^2, \left(HL_\mu\right)(HL_\tau)$ forbidden

Small M_ν from Generalized Symmetry Breaking

4. UV Completion

- non-perturbative breaking of non-invertible symmetry
- Embed $U(1)_{L_\mu - L_\tau} \subset \; SU(2)_H \times U(1)_Z$

	$SU(2)_H$	$U(1)_Z$	$L_\mu - L_\tau$	$U(1)_L$
Φ	2	-1	$\begin{bmatrix}\Phi_e\\ \Phi_\tau\end{bmatrix} = \begin{bmatrix}0\\ -1\end{bmatrix}$	0
$L_{\mu e}$	2	$+1$	$\begin{bmatrix}L_\mu\\ L_{e_1}\end{bmatrix} = \begin{bmatrix}+1\\ 0\end{bmatrix}$	$+1$
$L_{E\tau}$	2	-1	$\begin{bmatrix}L_{e_2}\\ L_\tau\end{bmatrix} = \begin{bmatrix}0\\ -1\end{bmatrix}$	$+1$
ψ_L	$-$	0	0	-1
$\bar{e}_{\mu e}$	2	-1	$\begin{bmatrix}\bar{e}_1\\ \bar{\mu}\end{bmatrix} = \begin{bmatrix}0\\ -1\end{bmatrix}$	-1
$\bar{e}_{E\tau}$	2	$+1$	$\begin{bmatrix}\bar{\tau}\\ \bar{e}_2\end{bmatrix} = \begin{bmatrix}+1\\ 0\end{bmatrix}$	-1
$\psi_{\bar{e}}$	$-$	0	0	$+1$

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- non-perturbative breaking of non-invertible symmetry
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$$+\lambda_{L_2} \widetilde{\Phi} L_{E\tau} \psi_L + \lambda_{e_1} \widetilde{\Phi} \bar{e}_{\mu e} \psi_{\bar{e}} + \lambda_{e_2} \Phi \bar{e}_{E\tau} \psi_{\bar{e}} + \lambda_\psi \widetilde{H} \psi_L \psi_{\bar{e}}$$

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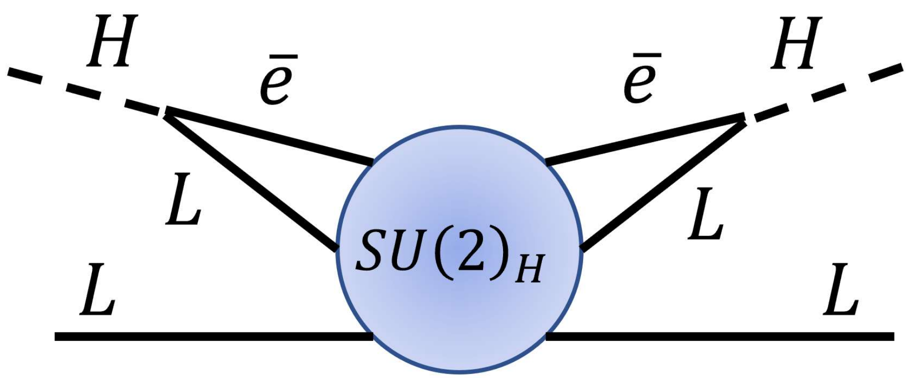
- ABJ-anomalies ($SU(2)_H$ instanton):
  - $U(1)_L \to Z^L_{N_g - 1} = Z^L_2$
  - $U(1)_{B-N_c L} \to Z^{B-N_c L}_{N_c(N_g-1)} \supset Z^L_2$

Small M_ν from Generalized Symmetry Breaking

4. UV Completion



$$U(1)_L \, SU(2)_H^2 = N_g - 1$$

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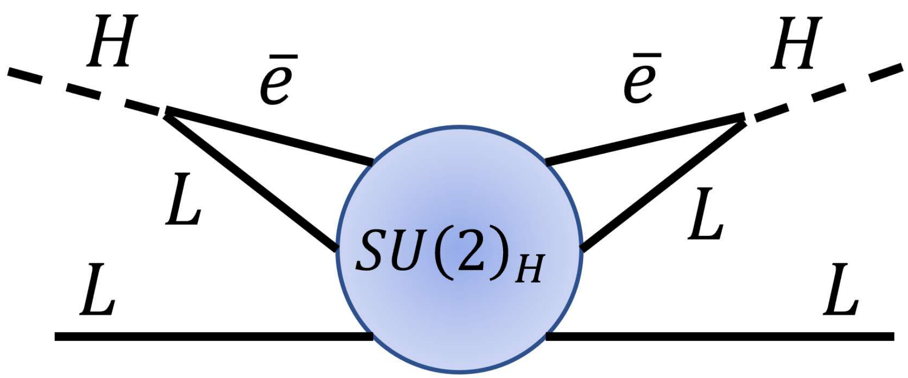
Small M_ν from Generalized Symmetry Breaking

4. UV Completion



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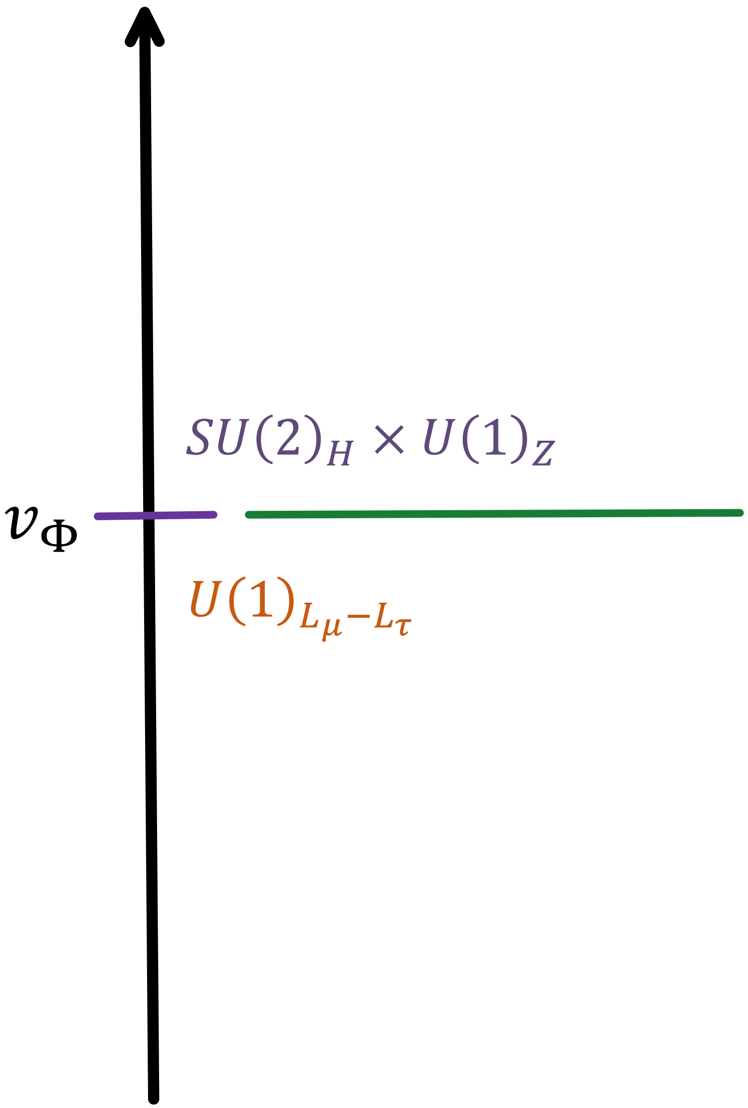
- $\mathcal{L} \sim y_\tau y_\mu \frac{v^2}{v_\Phi} e^{-\frac{2\pi}{\alpha_H}} [\nu_\mu \nu_\tau - \frac{1}{2} \sin 2\theta_L \nu_e \nu_e]$

Small M_ν from Generalized Symmetry Breaking



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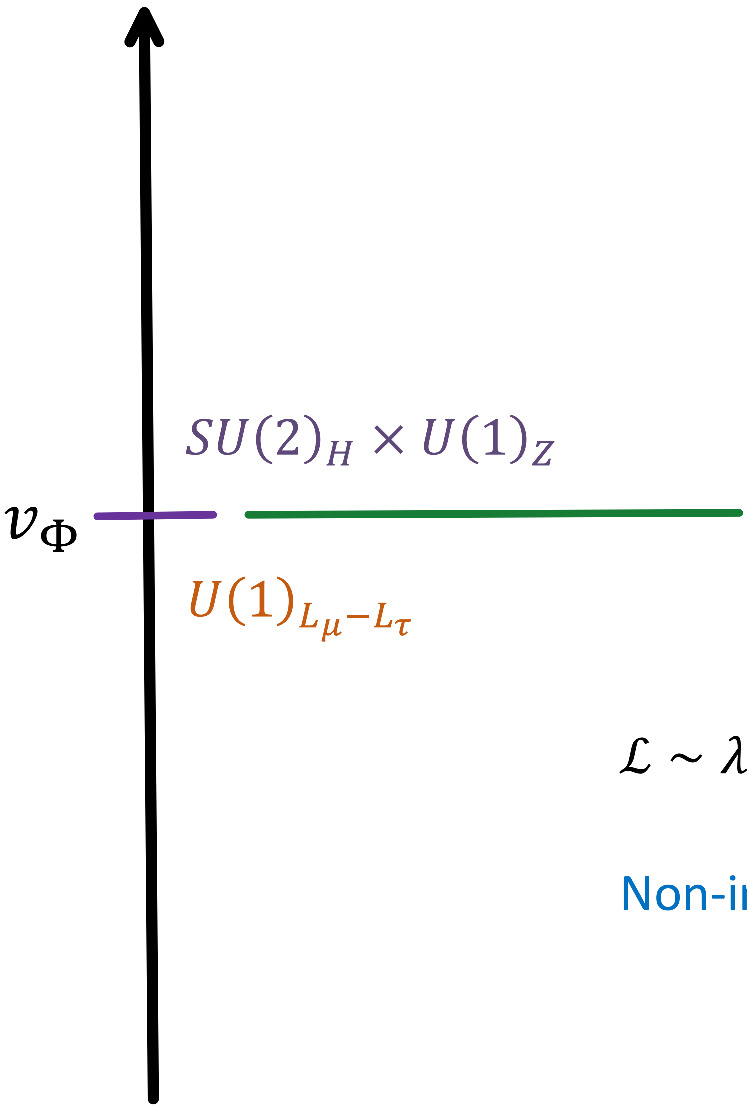

$\mathcal{L} \sim \lambda_{ij}(HL_i)(HL_j) \sim M_\nu^{ij}\, \nu_i \nu_j$ forbidden by

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Small M_ν from Generalized Symmetry Breaking

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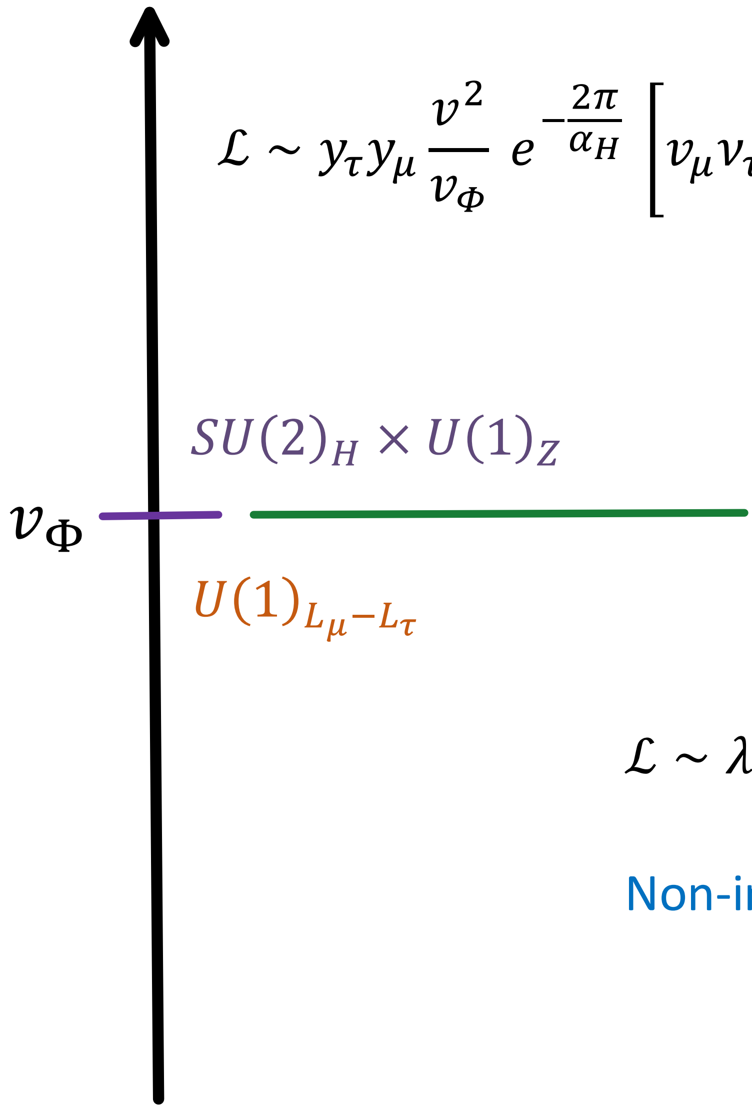

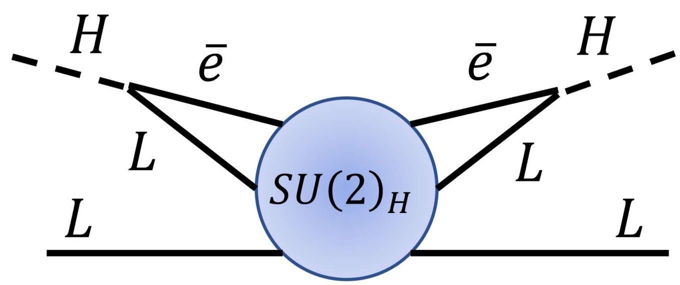

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THANK YOU

FOR

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