POSITIVITY BOUNDS ON HIGGS-PORTAL DARK MATTER

1/32

View from

1st floor@310

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Table of contents

- 1. Effective Field Theory (EFT)
- 2. Positivity Bounds
- 3. Higgs portal DM operators
- 4. Relic Density
- 5. Direct and Indirect Detections
- 6. LHC Search
- 5. Summary

Effective Field Theory (EFT)

• EFT

heavy degrees of freedom decouple for large-distance phenomena or small momentum scale

• EFT interaction terms:

$$\mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \cdots$$

$$\mathcal{L} = \sum_{i=1}^{n_d} \frac{c_i^5}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i^6}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{c_i^7}{\Lambda^3} \mathcal{O}_i^{(7)} + \frac{c_i^8}{\Lambda^4} \mathcal{O}_i^{(8)} + \cdots$$

- EFT is for the energy scale of E << Λ (typical energy scale of the UV physics)
- Many UV models correspond with EFT



 From the general feature of UV theory, can we bound on Wilson coefficients of EFT?



If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

- 1. Special relativity ——>Lorentz invariance
- 2. Conservation of probability ——> Unitarity
- 3. Causality - - > Analyticity

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R.Rattazzi, JHEP 0610, 014(2006)
 One of the way to do this is Positivity bounds

 Positivity bounds: the signs of certain combinations of Wilson coefficients in EFT have to be positive, e.g. W⁴ operators:

$$\frac{F_{T,0}}{\Lambda^4} \operatorname{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\operatorname{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}] = \frac{F_{T,1}}{\Lambda^4} \operatorname{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\operatorname{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]
\frac{F_{T,2}}{\Lambda^4} \operatorname{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\operatorname{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}] = \frac{F_{T,10}}{\Lambda^4} \operatorname{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\operatorname{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]
\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2} W^{I,\mu\nu} = \tilde{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2} \left(\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{I,\rho\sigma}\right)$$

One of the positivity bounds:

$$2F_{T,0} + 2F_{T,1} + F_{T,2} \ge 0$$

KY, C. Zhang, S. Y. Zhou, JHEP 01, 095 (2021)



- Positivity bounds can apply for dim-8 operators [FFFF]=Dim 8 in tree-level ← Froissart Bound (⇔Analyticity)
- Dim-8 operators are more suppressed by Λ than lower dimensional ones, however, for dim-8 aQGC operators, LHC experimentalists have been and currently working on CMS-PAS-SMP-18-001



• In the future, more dim-8 effects may become accessible

(e.g. new observable proposed for DY process: Alioli, Boughezal, Mereghetti, Petriello, Phys. Lett. B **809**, 135703 (2020), X. Li, K. Mimasu, <u>KY</u>, C. Yang, C. Zhang, S. Y. Zhou, JHEP**10**(2022)107)

Positivity Bounds Positivity bounds are important as they offer complementary bounds to the experiments E.g. WZjj (CMS-PAS-SMP-18-001)



Ref: Slides by Francesco Riva **Positivity Bounds** https://indico.ph.tum.de/event/4408/con tributions/3825/attachments/3292/3974/ Berlin-2.pdf Effective Theory Forward Amplitude (IR): $\frac{s^2}{M^4} +$ s^3 s^4 $\mathcal{M} = C_0 + C_1 \frac{s}{M^2} +$ 18 IR E behaviors completion: Lorentz invariance $C_2 > 0, \ C_4 > 0,$ Unitarity Causality Positivity F_{\cdot}

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + C_2 \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \cdots$$

massless scalar 2-2 forward elastic scattering:



9/32

Forward limit positivity bounds are from:

- 1. Lorentz Invariance
- Unitarity ⇒ Optical theorem: e.g., elastic case,

$$\operatorname{Im}\mathcal{M}(k_1, k_2 \to k_1, k_2) = s\sigma_{\operatorname{tot}}(k_1, k_2 \to \operatorname{anything})$$

1. Analyticity* ⇒ Froissart Bound:

$$|\mathcal{M}(s, \underline{\cos \theta} = 1)| < \text{Const. } s(\ln s)^2$$

Froissart, Martin 1960's
(for real s $\rightarrow \infty$)

*Analyticity of the amplitude besides poles and branch cuts on real axis

Positive

massless scalar 2-2 forward elastic scattering amplitude:



Positivity Bounds



Positivity Bounds
UV

$$\frac{1}{2\pi i} \int_{M}^{\infty} ds \left(\frac{M(s + i\epsilon, 0) - M(s - i\epsilon, 0)}{M} \right) / s^{3} \stackrel{(2)\&3)}{= (2i) \text{Im } M(s, 0)} = (2i) s \sigma_{\text{tot}}(s)$$

$$\frac{1}{2\pi i} \int_{-\infty}^{-M} ds \left(\frac{M(-s - i\epsilon, 0) - M(-s + i\epsilon, 0)}{M} \right) / s^{3}$$
1. Crossing Symmetry: M(s,0)=M(-u,0),
2. Schwarz reflection principle: M(s^*,0)=M(s,0)^*
3. Optical theorem: Im M(s,0) = s $\sigma_{\text{tot}}(s)$

$$= \frac{2}{\pi} \int_{M}^{\infty} ds \frac{s\sigma_{tot}(s)}{s^3} > 0$$









Consistent with QED

Positivity bounds: a>0, b>0

Dispersion Relation (for Positivity Bounds)



Dispersion Relation (for Positivity Bounds)

Useful to rewrite Dispersion Relation for Positivity Bounds

$$(Amp \text{ by Dim.8}) (Amp \text{ by Dim.8}) (M^{ijkl}) = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_{X}' \sum_{K=R,I} \frac{d\mu \, m_K_X^{ij} m_K_X^{kl}}{\pi\mu^3} + (j \leftrightarrow l)$$

$$M_{ijkl} = \frac{F_{\alpha} M_{\alpha}^{ijkl}}{\Lambda^4} \quad \text{where} \quad M(ij \to X) \equiv m_{R_X}^{ij} + im_{I_X}^{ij}$$
• When i=k, j=l, RHS complete squares >=0
$$M^{ijij} \geq 0 \quad \text{because} \quad m_K_X^{ij} m_K_X^{ij} \geq 0$$
• More generally,
Elastic Forward Scattering between Superposed States :
$$M(ab \to ab) \quad \text{with} \quad |a\rangle = u^i |i\rangle, \quad |b\rangle = v^i |i\rangle$$

$$\frac{u^i v^j u^{*k} v^{*l} M^{ijkl}}{\prod} = \int_{(\epsilon\Lambda)^2}^{\infty} \sum_{X}' \sum_{K=R,I} \frac{d\mu}{\pi\mu^3} \left[|u \cdot m_{K_X} \cdot v|^2 + |u \cdot m_{K_X} \cdot v^*|^2 \right] \geq 0$$

(generalized) Elastic Positivity Bounds

Higgs Portal DM operators -positivity side-

Derivative Coupling for Higgs and Dark Matter Fields

$$O_{H^2\varphi^2}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$$
$$O_{H^2\varphi^2}^{(2)} = (D_{\mu}H^{\dagger}D^{\mu}H)(\partial_{\nu}\varphi\partial^{\nu}\varphi)$$

- Sensitive to high-energy prosses
- Subject to satisfying positivity bounds
- Spin-2 massive graviton and spin-0 radion mediated DM model is a candidate of this scenario as the partial UV completion

Higgs Portal DM operators -positivity side-

Positivity bounds from the superposed states:

$$O_{H^{2}\varphi^{2}}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$$

$$O_{H^{2}\varphi^{2}}^{(2)} = (D_{\mu}H^{\dagger}D^{\mu}H)(\partial_{\nu}\varphi\partial^{\nu}\varphi)$$

$$O_{\varphi^{4}} = \partial_{\mu}\varphi\partial^{\mu}\varphi\partial_{\nu}\varphi\partial^{\nu}\varphi$$

$$O_{H^{4}}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(D^{\nu}H^{\dagger}D^{\mu}H)$$

$$O_{H^{4}}^{(2)} = (D_{\mu}H^{\dagger}D_{\nu}H)(D^{\mu}H^{\dagger}D^{\nu}H)$$

$$O_{H^{4}}^{(3)} = (D_{\mu}H^{\dagger}D^{\mu}H)(D_{\nu}H^{\dagger}D^{\nu}H)$$

Higgs Portal DM operators -positivity side-

• Results:

Bounds	Channels $(1\rangle + 2\rangle \rightarrow 1\rangle + 2\rangle)$	
$C_{H^4}^{(1)} + C_{H^4}^{(2)} \ge 0$	$ 1\rangle = \phi_1\rangle , \ 2\rangle = \phi_3\rangle$	
$C_{H^4}^{(1)} + C_{H^4}^{(2)} + C_{H^4}^{(3)} \ge 0$	$ 1\rangle = \phi_1\rangle , \ 2\rangle = \phi_1\rangle$	
$C_{H^4}^{(2)} \ge 0$	$\ket{1} = \ket{\phi_1}, \ \ket{2} = \ket{\phi_2}$	
$C_{H^2\varphi^2}^{(1)} \ge 0$	$\left 1\right\rangle = \left \phi_{1}\right\rangle, \left 2\right\rangle = \left \varphi\right\rangle$	
$C_{\varphi^4} \ge 0$	$ 1\rangle = \varphi\rangle , \ 2\rangle = \varphi\rangle$	
$2\sqrt{(C_{H^4}^{(1)} + C_{H^4}^{(2)} + C_{H^4}^{(3)})C_{\varphi^4}}$	$ 1\rangle = 2\sqrt{C_{\varphi^4}} \phi_1\rangle + \sqrt{-(C_{H^2\varphi^2}^{(1)} + C_{H^2\varphi^2}^{(2)})} \varphi\rangle,$	
$\geq -\left(C_{H^{2}\varphi^{2}}^{(1)}+C_{H^{2}\varphi^{2}}^{(2)}\right)$	2 angle = 1 angle Superposition	
$2\sqrt{(C_{11}^{(1)} + C_{11}^{(2)} + C_{11}^{(3)})C_{14}} > C_{12}^{(2)}$	$ 1\rangle = 2\sqrt{C_{\varphi^4}} \phi_1\rangle + \sqrt{C_{H^2\varphi^2}^{(2)}} \varphi\rangle,$	
$-\sqrt{(\circ_{H^4}+\circ_{H^4}+\circ_{H^4})\circ_{\varphi^2}} = -\frac{\circ_{H^2\varphi^2}}{H^2\varphi^2}$	$\left 2\right\rangle = -2\sqrt{C_{\varphi^4}}\left \phi_1\right\rangle + \sqrt{C_{H^2\varphi^2}^{(2)}}\left \varphi\right\rangle$	
Higgs portal DM $O_{H^2\varphi^2}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$ Superposition		
$O^{(2)}_{H^2\varphi^2} = (D_{\mu}H^{\dagger}D^{\mu}H)(\partial_{\nu}\varphi\partial^{\nu}\varphi)$		

Higgs Portal DM operators - dim4 and dim6 -

 Dim-4 and Dim-6 Higgs Portal DM operators relevant to the phenomenology (relic density, direct and indirect detections):

$$-\frac{1}{6\Lambda^{4}} \left(c_{1}m_{\varphi}^{4}\varphi^{4} + 4c_{2}m_{H}^{4}|H|^{4} + 8c_{2}'\lambda_{H}m_{H}^{2}|H|^{6} + 4c_{2}''\lambda_{H}^{2}|H|^{8} + 4c_{3}m_{\varphi}^{2}m_{H}^{2}\varphi^{2}|H|^{2} + 4c_{3}'\lambda_{H}m_{\varphi}^{2}\varphi^{2}|H|^{4} \right) + \frac{1}{6\Lambda^{4}} \left(d_{1}m_{\varphi}^{2}\varphi^{2}(\partial_{\mu}\varphi)^{2} + 4d_{2}m_{H}^{2}|H|^{2}|D_{\mu}H|^{2} + 4d_{2}'\lambda_{H}|H|^{4}|D_{\mu}H|^{2} + 2d_{3}m_{\varphi}^{2}\varphi^{2}|D_{\mu}H|^{2} + 2d_{4}m_{H}^{2}|H|^{2}(\partial_{\mu}\varphi)^{2} + 2d_{4}'\lambda_{H}|H|^{4}(\partial_{\mu}\varphi)^{2} \right)$$

Higgs Portal DM operators

- Massive Graviton and Radion case-
- Higgs/DM and Graviton Interaction:

$$-\frac{c_H}{M} G^{\mu\nu} T^H_{\mu\nu} - \frac{c_\varphi}{M} G^{\mu\nu} T^\varphi_{\mu\nu}$$

Higgs/DM and Radion Interaction:

$$\mathcal{L}_r = \frac{c_H^r}{\sqrt{6}M} r \, T^H + \frac{c_\varphi^r}{\sqrt{6}M} r \, T^\varphi$$

- After Integrating out Massive Graviton/Radion, we can identify coefficients of dim-4, 6, and 8 operators as an example
- We found that they satisfied the positivity conditions as far as $c_H c_{\varphi} \ge 0$ (attractive force for the graviton)

Relic Density

• Higgs-portal interactions linear in the Higgs boson h

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c_3')\lambda_H v^3 m_{\varphi}^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

• Feynman diagrams for DM annihilation processes when $c'_3=c_3$ and $c'_4=c_4$ ($\varphi \phi \rightarrow h \rightarrow ff$ are absent):



Note that the tree-level direct detection bounds are absent in this case

Relic Density



Relic Density - Graviton and Radion case-





Indirect Detection

Note on some cases:

• When $c'_3 = c_3$ and $c'_4 = c_4$, $\varphi \varphi \rightarrow h \rightarrow ff$ are absent:

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c_3')\lambda_H v^3 m_{\varphi}^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

- In this case $\varphi \phi \rightarrow hh$, *WW*, and *ZZ* can be constrained by indirect detection
- If we assume that only massive graviton is involved, $\varphi \phi \rightarrow hh$ also vahish at *s*-wave, but $\varphi \phi \rightarrow WW/ZZ$ are *s*-wave dominant

LHC Search

ATLAS measurement with 139/fb at the 13 TeV LHC





- For our dim-8 operators,
 H in Fig. is integrated out
- $\chi \to \varphi$
- Higgs takes vev
- Covariant Derivative contains vector bosons

$$O_{H^2\varphi^2}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_{\mu}H^{\dagger}D^{\mu}H)(\partial_{\nu}\varphi\partial^{\nu}\varphi)$$

LHC Search

• 95% upper limits: 0.11 pb G. Aad *et al.* [ATLAS], JHEP 08, 104 (2022)

$\sqrt{s} = 13 \text{ TeV LHC}, L_{\text{int}} = 139 \text{ fb}^{-1}$	$\sigma^{\text{VBF}} \times B_{\text{inv}} = 0.11 \text{ pb} (m_H = 1 \text{ TeV})$
$\Lambda = 1 \text{ TeV}, m_{\varphi} = 375 \text{ GeV}$	cross section from EFT operators
$(C^{(1)}_{H^2 \varphi^2}, C^{(2)}_{H^2 \varphi^2}) = (40, 40)$	0.28 pb Excluded
$(C^{(1)}_{H^2arphi^2},C^{(2)}_{H^2arphi^2})=(32,32)$	0.11 pb Excluded
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (40, 0)$	0.012 pb
$(C^{(1)}_{H^2\varphi^2}, C^{(2)}_{H^2\varphi^2}) = (0, 40)$	0.097 pb



$$O_{H^2\varphi^2}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_{\mu}H^{\dagger}D^{\mu}H)(\partial_{\nu}\varphi\partial^{\nu}\varphi)$$

LHC Search

High Luminocity LHC (HL-LHC) Search



- Amplitude for $W^+W^-/ZZ \rightarrow \varphi \varphi$
 - $O^{(2)}_{H^2 \varphi^2}$ shows only Mondelstam s and mass dependencies • $O_{H^2(o^2)}^{(1)}$ causes *t* dependency also

may help to distinguish between $O_{H^2 \varphi^2}^{(1)}$ and $O_{H^2 \varphi^2}^{(2)}$ $O_{H^2 \varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H) (\partial_\nu \varphi \partial^\nu \varphi)$

Checking angular distributions $O_{H^2\omega^2}^{(1)} = (D_{\mu}H^{\dagger}D_{\nu}H)(\partial^{\mu}\varphi\partial^{\nu}\varphi)$

X. Li, K. Mimasu, KY, C. Yang, C. Zhang, S. Y. Zhou, JHEP10(2022)107 31/32

Summary and Outlook

- We consider Higgs portal dark matter derivative coupled dim-8 interactions and apply the positivity conditions to them
- We also included dim-4 and dim-6 Higgs portal interactions
- We see constraints from positivity and relic density, direct and indirect detections, and the relation to the massive graviton&radion case as an example of the partial UV completion
- For HL-LHC search, utilizing the kinematical distributions may be useful