Light axions in KKLT axiverse

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CAU BSM workshop

Feb. 22, 2023

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Outline

- Some generic features of axions
- Axions & moduli stabilization in string theory
- Light axions in KKLT axiverse

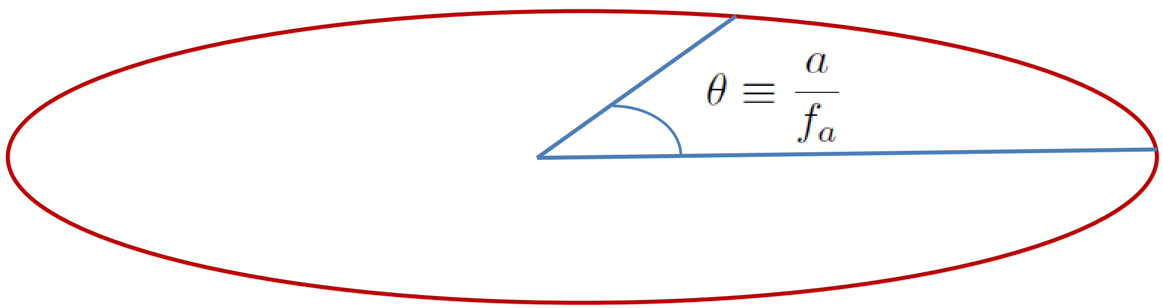
Axions (or axion-like particles=ALP) are light pseudo-scalar bosons postulated in many models for physics beyond the SM:

Strong CP problem, dark matter, inflation, baryogenesis, …

Also string theories generically predict axions in 4D effective theory.

Axions are angular (periodic) fields: $a(x) \cong a(x) + 2\pi f_a$

$$(f_a = \text{axion decay constant or axion scale})$$

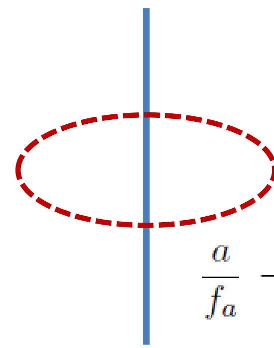


There exist an associated string (soliton, fundamental string, brane wrapping a cycle, ...) and instanton (YM instanton, Euclidean wormhole, world-sheet or brane instantons, ...) which couple to axion as

$$\Delta\mathcal{L}_{\mathrm{axion-instanton}} \propto e^{ia/f_a}$$

$$S_{\mathrm{axion-string}} = \int_{\mathrm{string}} d\Sigma_{\mu\nu}\, B^{\mu\nu}$$

$$(\partial_{[\mu} B_{\nu\rho]} = f_a \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a)$$



$$\frac{a}{f_a} \to \frac{a}{f_a} + 2\pi$$

Axions can be naturally light due to an approximate global U(1) symmetry (Peccei-Quinn (PQ) symmetry) which is non-linearly realized in low energy effective theory, involving a constant shift of axion field:

$$U(1)_{\rm PQ}: \quad a(x) \;\rightarrow\; a(x) + {\rm constant}$$

$$V(a) = -\frac{1}{2}m_a^2 a^2 + ...$$

Axion potential induced by $U(1)_{PQ}$-breaking effects.

$U(1)_{PQ}$ assures that axion couples dominantly to the spin density of matter and gauge fields, not to the number density:

$$\mathcal{L}_{\rm axion} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a}\left(\sum_\psi c_\psi \underbrace{\bar\psi\sigma^\mu\psi}_{\text{matter spin density}} + i\sum_\phi c_\phi(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*)\right)$$

$$+\sum_{A_\mu^A} c_A \frac{g_A^2}{32\pi^2}\frac{a}{f_a}\underbrace{F^{A\mu\nu}\tilde F^A_{\mu\nu}}_{\text{gauge field helicity density}}\cdots \qquad \left(\tilde F^A_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{A\rho\sigma}\right)$$

This makes axions easily satisfy the strong constraints from the 5th force experiments.

Two type of axions

* Type I axions from the phase of $U(1)_{PQ}$-charged complex scalar fields

Peccei, Quinn; Weinberg; Wilczek
KSVZ, DFSZ, ...

$$\phi = \rho(x) e^{ia(x)/f_a} \quad \left(f_a = \langle \rho \rangle \right)$$

* Type II axions from a p-form gauge field which couples to extended object ((p-1)-brane and its magnetic dual) in higher-dimensional spacetime, e.g. string-theoretic axions:

Witten '84, ...

$$\frac{a(x)}{f_a} = \int_{\Sigma_p} C^{(p)}_{[m_1,..,m_p]}(x,y)\, dy^{m_1} ... dy^{m_p} \qquad (\Sigma_p = p\text{-cycle in extra-dim})$$

or $\quad f_a \partial_\mu a = \epsilon_{\mu\nu\rho\sigma} \left(\partial^\nu B^{\rho\sigma} + ... \right)$

Type I axions have a linear PQ-limit $f_a \to 0$ which is well described by 4D effective field theory, while $f_a \to 0$ of Type II axions is not within the regime of 4D effective theory.

Type I and Type II axions have different pattern of low energy couplings, which might be detectable in future experiments. KC, Im, Kim, Seong, '21

Two well-motivated light axions

* QCD axion solving the strong CP problem, which is also an appealing candidate for dark matter:

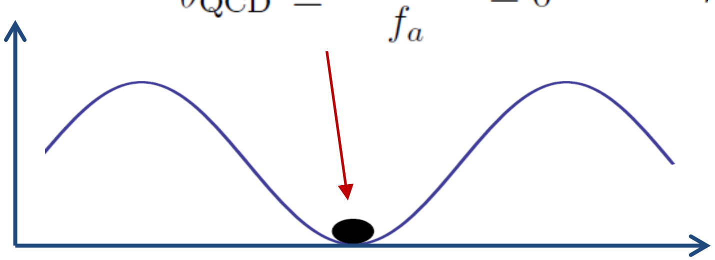
Peccei, Quinn; Weinberg; Wilczek
Preskill, Wise, Wilczek; Dine, Fischler; Abbot, Sikivie

$U(1)_{\rm PQ}$ is broken dominantly by the axion coupling to the gluons:

$$\frac{1}{32\pi^2}\frac{a_{\rm QCD}}{f_a}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$\rightarrow \quad V(a_{\rm QCD}) \simeq -\frac{m_\pi^2 f_\pi^2}{m_u+m_d}\left(m_u^2+m_d^2+2m_u m_d\cos\left(\frac{a_{\rm QCD}}{f_a}\right)\right)^{1/2}$$

$$\theta_{\rm QCD} \equiv \frac{\langle a_{\rm QCD}\rangle}{f_a} = 0 \qquad m_{a_{\rm QCD}} \simeq 5.7\times 10^{-6}\left(\frac{10^{12}\,{\rm GeV}}{f_a}\right){\rm eV}$$



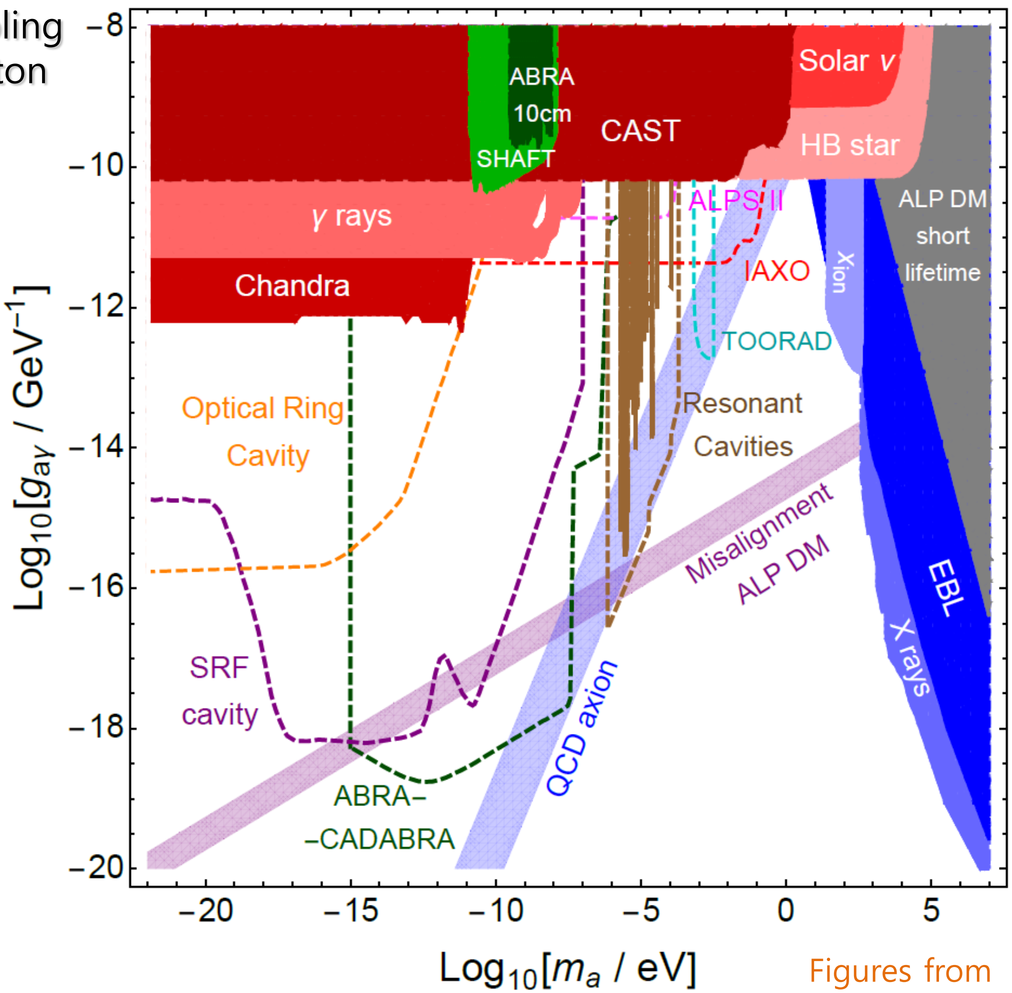
* Ultra-light ALP which is typically lighter than the QCD axion and may constitute (part of) dark matter:

$$\frac{\rho_{\rm ALP}}{\rho_{\rm DM}} \sim \left(\frac{m_a}{10^{-22}{\rm eV}}\right)^{1/2} \left(\frac{f_a}{10^{17}{\rm GeV}}\right)^2$$

Light axions have a bright prospect for experimental detection.

Axion coupling
to the photon

$$\sim 10^{-3}\frac{1}{f_a}$$



Figures from
KC, Im and Shin (2021)

Two theoretical questions on light axions

* PQ-quality problem

Origin of $U(1)_{PQ}$ well-protected from quantum gravity:

Generically quantum gravity violates global symmetry, so it can generate $U(1)_{PQ}$-breaking axion potentials.

On the other hand, gravity-induced axion potential is strongly bounded as

$$\Delta V(a_{\rm QCD}) \lesssim 10^{-10} f_\pi^2 m_\pi^2 \sim \left(10^{-3.5}{\rm GeV}\right)^4$$

for QCD axions solving the strong CP problem

$$\Delta V(a_{\rm ALP}) \lesssim m_a^2 f_a^2 \sim \left(10^{-6.5}{\rm GeV}\right)^4 \left(\frac{m_a}{10^{-21}{\rm eV}}\right)^2 \left(\frac{f_a}{10^{17}{\rm GeV}}\right)^2$$

for ultralight ALP with $m_a \sim 10^{-21}$ eV and $f_a \sim 10^{17}$ GeV

which may require a specific structure in UV theory protecting $U(1)_{PQ}$ from quantum gravity effects.

* Axion scale problem

Origin of the axion scale:

Is there any connection between f_a and other fundamental scales such as $M_{\rm Pl}, M_{\rm SUSY}, M_{\rm weak}, \dots$?

How f_a has evolved in the early Universe?

These are not merely a problem of academic interest, but have important implication for the astrophysical, cosmological, and laboratory properties of axions.

The PQ-quality problem is deeply related to quantum gravity.

(i) String theory is considered as the best candidate for a theory of quantum gravity.
(ii) String theory generically predicts axions in 4D effective theory.
(iii) All couplings & mass scales in string theory depend on moduli VEVs determined by the dynamics stabilizing moduli fields.

➔ String theory with a mechanism to stabilize moduli is the right place to address the PQ-quality problem and the axion scale problem.

Axions in string theory

String theory involves a variety of extended objects (strings & branes) and p-form gauge fields which couple to those extended objects in a way invariant under a (p-1)-form gauge symmetry:

$$C_p \rightarrow C_p + d\Lambda_{p-1} \ \text{ for globally well-defind } \Lambda_{p-1}$$

Upon compactification, zero modes of p-form gauge field correspond to (Type II) axions in 4D effective theory:

$$C_p(x,y) = \theta(x)\,\omega_p(y) \ \text{ for harmonic } p\text{-form } \omega_p \qquad \left(\theta(x) = \frac{a(x)}{f_a} \cong \theta(x) + 2\pi\right)$$

The associated PQ symmetry is locally equivalent to the underlying (p-1)-form gauge symmetry, therefore it may have a good chance to be well-protected from quantum gravity:

$$U(1)_{\text{shift}}: \ \theta(x) \rightarrow \theta(x) + \beta \quad (\beta = \text{constant}) \quad \Rightarrow \quad C_p \rightarrow C_p + \beta\,\omega_p$$

For each Type II axion, there exists a modulus partner:

$$\tau = \text{volume of the } p\text{-cycle dual to } \omega_p$$

Much of the physical property of axion crucially depends on the VEV of the modulus partner:

Euclidean action of instanton
(quantum gravity or YM)
breaking PQ-symmetry

$$f_a \sim \frac{M_{\rm Pl}}{\langle\tau\rangle} \text{ or } \frac{M_{\rm st}}{\langle\tau\rangle} \qquad\qquad S_{\rm ins} \sim \langle\tau\rangle$$

Nearly saturates
the WGC bound

In models involving U(1) gauge flux, certain combination of Type II axions is eaten by U(1) gauge field while leaving a PQ symmetry providing a more familiar Type I axion in low energy effective theory:

$$\mathcal{L}_{\rm Stueckelberg} \propto qM_S^2\partial_\mu\theta A^\mu \quad \left(q \propto \langle F_{U(1)}\rangle,\ M_S \sim \frac{M_{\rm Pl}}{\tau} \text{ or } \frac{M_{\rm st}}{\tau}\right)$$

$$U(1)_A:\ A_\mu \to A_\mu + \partial_\mu\alpha(x),\ \ \theta(x)\to\theta(x)+q\alpha(x),\ \ \phi\to e^{iq_\phi\alpha}\phi,\ \ \psi\to e^{iq_\psi\alpha}\psi$$

In low energy effective theory below the Stuekelberg mass scale M_S, shift symmetry of the eaten Type II axion is converted to a PQ symmetry involving only the low energy scalar and fermion fields:

$$U(1)_{\rm PQ} = U(1)_{\rm shift}\oplus U(1)_A:\ \phi\to e^{iq_\phi\beta}\phi,\ \ \psi\to e^{iq_\psi\beta}\psi \quad (\beta = \text{constant})$$

Spontaneous breakdown of this PQ symmetry leads to a Type I axion:

$$\phi \ = \ \rho(x)e^{ia(x)/f_a} \quad \Big(f_a = \langle\rho\rangle\Big)$$

Both Type I and II axions in string theory have been discussed a long time ago in the context of heterotic string theory, together with their phenomenological and cosmological features.

Witten '84; KC, Kim '84; Barr '85, …

Axions in other string theories (Type I, IIA, IIB) have the essentially same origin and also have a similar feature.

Sometimes one uses "closed string axion" for Type II axions and "open string axion" for Type I axions, which is not applicable for axions in heterotic string theory.

Yet the PQ symmetry which is locally equivalent to a (p-1)-form gauge symmetry does not assure that the associated axion is light enough to be identified as a QCD axion or ultra-light ALP:

PQ breakings by hidden YM dynamics, world-sheet instantons, …

Dine, Rohm, Seiberg, Witten '85; KC, Kim 85; Wen, Witten 86; …

Also, eventually one needs to stabilize moduli in a way to generate a mass splitting between light axions and their moduli partner:

$$m_\tau \gtrsim \mathcal{O}(10)\,\mathrm{TeV} \gg m_a = \mathcal{O}(10^{-3} - 10^{-32})\,\mathrm{eV}$$

to avoid the cosmological moduli problem

To generate such a mass splitting, the moduli partner should be stabilized by a PQ-conserving (axion-independent) potential.

Light axions with stabilized moduli partners

* KKLT moduli stabilization in Type IIB KC and Jeong '06; Bobkov et al '10; ...

Moduli VEVs at SUSY-breaking dS vacuum, including those of the moduli partner of light axions, are close to the supersymmetric VEVs obtained before introducing SUSY-breaking. This makes it relatively straightforward to examine the moduli potential and its physical consequences.

* Large volume scenario (LVS) in Type IIB Conlon 06; Cicoli et al '12; ...

Moduli VEVs are far away from supersymmetric configuration, making it more difficult to study the moduli potential. Also in some case, stabilizing the moduli partner of light axions involves an assumption about higher-order correction to the Kaehler potential.

* M-theory on G2 Acharya et al '10; ...

The scheme involves a Planckian VEV of composite meson fields in hidden YM sector. Also the scheme does not involve a landscape of flux vacua, which might be necessary for anthropic explanation of the C.C problem.

All previous works on light axions with stabilized moduli are discussing only Type II axions.

There are also recent works discussing some phenomenological features of light axions (again Type II axions) in string axiverse, but without addressing moduli stabilization.

Demirates et al ‘18; Halverson et al ‘19; Mehta et al ‘20; Demirates et al ‘21, …

Ignoring moduli stabilization allows to study a bigger set of models over a larger region in moduli space. However most of the moduli VEVs assumed in these studies might be in a runaway region.

Light axions in KKLT axiverse

KC, Im, Jeong, Yun, in preparation

Extension of the previous works on light axions in KKLT setup:

* Incorporation of Type I axions

* Study of certain phenomenological features of light axions over a bigger set of models in which all moduli are stabilized:

  axion scales and couplings, axion and moduli cosmology, $\theta_{\rm QCD} = \langle a_{\rm QCD} \rangle / f_a$ induced by sub-leading PQ-breaking for QCD axion

KKLT scenario is the best studied model of moduli stabilization for de Sitter vacua.

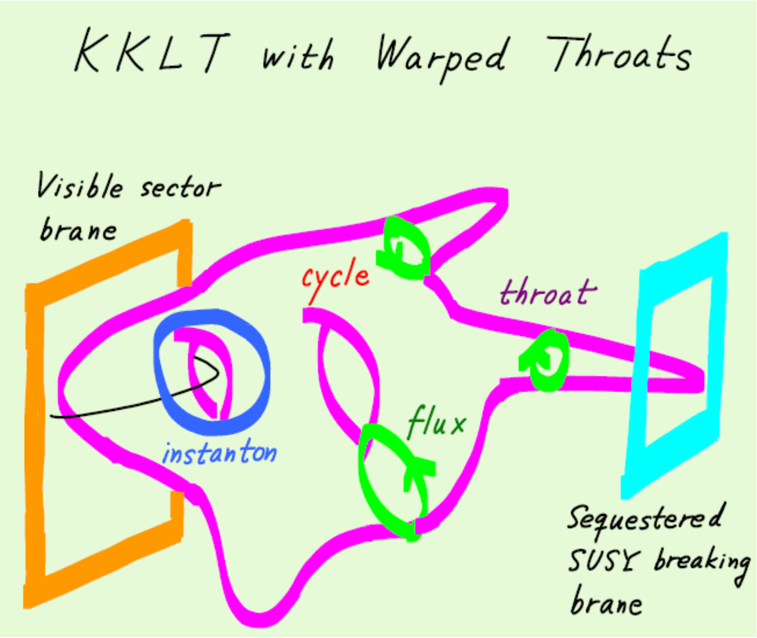
Kachru, Kallosh, Linde, Trivede, '03

Three key ingredients:

1) 3-form fluxes stabilizing the dilaton and complex structure moduli, which also produce warped throats while providing a huge landscape of flux vacua which would allow an anthrophic selection of small C.C.

2) Hidden gaugino condensations on D7 branes wrapping 4-cycles (or D3 brane instantons), stabilizing the Kahler moduli

3) SUSY breaking anti-D3 brane at the tip of warped throat, providing an uplifting potential for dS vacuum



Axions from 4-form gauge field in Type IIB string theory and their Kaehler moduli partners:

$$T_i = \tau_i + i\theta_i \quad \left(i = 1, 2, .., h_{1,1}\right)$$

$$C_4(x, y) = \sum_i \theta_i(x)\omega_4^{(i)}(y)$$

$$\omega_4^{(i)} = \text{harmonic 4-forms on CY 3-fold}$$

$$\tau_i = \text{volume of 4-cycle dual to } \omega_4^{(i)}$$

PQ symmetries and their violations

3-form gauge symmetry:

$$C_4 \to C_4 + d\Lambda_3 \quad \text{for globally well-defined } \Lambda_3$$

PQ symmetries which are locally equivalent to 3-form gauge symmetry:

$$U(1)^{(i)}_{\text{shift}}:\; C_4 \to C_4 + \beta_i\, \omega_4^{(i)} \;\; \text{for constant } \beta_i$$

$$\Rightarrow \quad \theta_i(x) \to \theta_i(x) + \beta_i$$

PQ symmetries can be violated only by non-local effect on 4-cycles, which is exponentially suppressed by (in the absence of 4-form flux)

$$\Big(\exp\left(-[\text{4-cycle volume}]\right)\Big)^p = \exp(-p\tau_i)$$

$$p = \text{model-dependent rational number}$$

In the original KKLT setup, all Kaehler moduli τ_i are stabilized by PQ-breaking hidden gaugino condensations or D3-brane instantons.

As a consequence, the size of PQ-breaking is tied to moduli masses which are also connected to SUSY-breaking, therefore it can not be small enough to give light axions:

$$e^{-p\tau_i} \sim \frac{m_{\tau_i}}{M_{\rm Pl}}$$

$$m_{\theta_i} \sim m_{\tau_i} \sim m_{3/2}\ln(M_{\rm Pl}/m_{3/2}) \gg 1\,{\rm TeV}$$

The original KKLT setup can be generalized to incorporate light axions (both Type I and Type II) while stabilizing all moduli partners.

In the generalized setup, Kaehler moduli are stabilized by one of the following three ingredients:

KC, Im, Jeong, Yun, in preparation

(1) PQ-violating hidden gaugino condensations producing (not from D3 instantons)

$$W_{\rm np} = \sum_h A_h e^{-\sum_i \ell_h^i T_i / C_h} \qquad \Big(h = 1, .., N_h\Big)$$

(2) PQ-conserving Fayet-Illiopoulos D-term potential induced by U(1) gauge flux, which is accompanied by Stuekelberg mixing of axions with U(1) gauge fields:

$$V_D = \sum_\alpha \frac{1}{2} g_\alpha^2 \Big(\sum_i q_i^\alpha \frac{\partial K}{\partial T_i}\Big)^2 \qquad \Big(\sum_i \ell_h^i q_i^\alpha = 0, \quad \alpha = 1, .., N_{U(1)}\Big)$$

$$U(1)_\alpha: \; A_\mu^\alpha \to A_\mu^\alpha + \partial_\mu \lambda_\alpha, \;\; \theta_i \to \theta_i + q_i^\alpha \lambda_\alpha(x), \;\; \phi \to e^{i q_\phi^\alpha \lambda_\alpha} \phi, \;\; \psi \to e^{i q_\psi^\alpha \lambda_\alpha} \psi$$

(3) PQ-conserving uplifting potential from SUSY-breaking at the tip of warped throat

In the original KKLT setup, uplifting potential gives only a small correction to the moduli masses induced by gaugino condensation or D3 instantons, while in our setup, it provides dominant contribution to some moduli masses.

Kaehler moduli stabilization by $V = V_F + V_D + V_{\rm lift}$

$$V_F = e^K\left(K^{i\bar{j}}D_iW(D_jW)^* - 3|W|^2\right) \quad (i,j = 1,2,..,h_{1,1})$$

Volume of Calabi-Yau 6-manifold

$$K(T_i + T_i^*) = -2\ln \mathcal{V}_{\rm CY} \quad \left(\mathcal{V}_{\rm CY} = \frac{1}{6}d_{ijk}t^i t^j t^k, \ \ \tau_i = \frac{\partial \mathcal{V}_{\rm CY}}{\partial t^i} = \frac{1}{2}d_{ijk}t^j t^k\right)$$

$$W = \omega_{\rm flux} + \sum_{h=1}^{N_h} A_h \exp(-\ell_h^i T_i/C_h) + \mathcal{O}(e^{-T_i}) \quad \left(T_i = \tau_i + \theta_i, \ \ D_iW = \frac{\partial W}{\partial T_i} + \frac{\partial K}{\partial T_i}W\right)$$

3-form fluxes

gaugino condensations

D3-brane instantons which do not play any role in moduli stabilization, but provide a tiny mass to ultralight ALPs

$$V_D = \sum_{\alpha=1}^{N_{U(1)}} \frac{1}{2}g_\alpha^2\left(q_i^\alpha \frac{\partial K}{\partial T_i}\right)^2$$

D-term potential induced by U(1) gauge flux

d_{ijk} = integer-valued intersection numbers

ℓ_h^i = integer-valued wrapping numbers of $D7$ branes supporting hidden-gaugino condensations

$$V_{\rm lift} = e^{-4\Omega}e^{2K/3}$$

Uplifting potential induced by SUSY-breaking at the tip of warped throat

C_h = dual Coxeter number of the hidden YM gauge group

q_i^α = gauge flux-induced $U(1)_\alpha$-charge of θ_i $\quad \left(\sum_i \ell_h^i q_i^\alpha = 0\right)$

$e^{-2\Omega}$ = exponentially small warp factor induced by 3-form flux

Fine-tuning for small C.C

$$\frac{m_{3/2}}{M_{\rm Pl}} \sim \omega_{\rm flux} \sim e^{-2\Omega} \qquad \tau \sim \frac{8\pi^2}{g_{\rm SM}^2}$$

SM on D7 branes wrapping 4-cycle

Moduli VEVs at meta-stable dS vacuum is close to supersymmetric solution:

KC and Jeong '06

Shift of moduli VEV due to the uplifting potential

$$\frac{\delta\tau}{\tau} \sim \frac{m_{3/2}^2}{m_{\tau_H}^2} \ll 1 \quad \left(m_{\tau_H} \sim m_{3/2} \ln(M_{\rm Pl}/m_{3/2}) \ \text{ or } \ \frac{g_{\rm GUT}^2}{8\pi^2} M_{\rm Pl} \right)$$

$$\sum_j q_j^\alpha \frac{\partial K}{\partial T_j} = D_i W = 0 \quad \Rightarrow \quad \sum_i q_i^\alpha t^i = 0 \quad (\alpha = 1, .., N_{U(1)})$$

$$\sum_i \ell_h^i \tau_i = C_h \ln(M_{\rm Pl}/m_{3/2}) \quad (h = 1, .., N_h)$$

$$\sum_{x=1}^{h_{1,1}-N_{U(1)}} \sum_i Q_i^x (L^{-1})_x^m t^i = 0 \quad (m = 1, .., h_{1,1} - N_{U(1)} - N_h)$$

Smith normal form: $q_i^\alpha = \sum_\beta U_\beta^\alpha \tilde{Q}_\beta Q_i^\beta$ $\quad (\alpha, \beta = 1, .., N_{U(1)})$ $\qquad \sum_i Q_i^x \ell_h^i = \sum_{h'} V_h^{h'} \tilde{L}_{h'} L_{h'}^x$ $\quad (x = 1, .., h_{1,1} - N_{U(1)},\ h' = 1, .., N_h)$

$$Q = [Q_i^\alpha, Q_i^x] \in GL(h_{1,1}, \mathbf{Z}), \quad U \in GL(N_{U(1)}, \mathbf{Z}), \quad \tilde{Q}_\alpha | \tilde{Q}_{\alpha+1}$$

$$L = [L_h^x, L_m^x] \in GL(h_{1,1} - N_{U(1)}, \mathbf{Z}), \quad V \in GL(N_h, \mathbf{Z}), \quad \tilde{L}_h | \tilde{L}_{h+1}$$

Moduli VEVs

$$D_i W = 0 \quad \Rightarrow \quad D_\alpha = \sum_i q_i^\alpha \partial_i K = 0$$

$\Rightarrow$ moduli VEVs are independent of q_i^α

$$\tau_i = \frac{1}{2} \sum_{jk,h,h'} d_{ijk} \xi_h \xi_{h'} \ell_h^j \ell_{h'}^k \quad \left(t^i = \sum_h \xi_h \ell_h^i \right)$$

$$\sum_{h'h''} \left(\sum_{ijk} d_{ijk} \ell_h^i \ell_{h'}^j \ell_{h''}^k \right) \xi_{h'} \xi_{h''} = 2C_h \ln(M_{\rm Pl}/m_{3/2}) \quad (h = 1, .., N_h)$$

Single gaugino condensation: $N_h = 1$

$$\tau_i = C \ln(M_{\rm Pl}/m_{3/2}) \frac{\sum_{jk} d_{jk} \ell^j \ell^k}{\sum_{i'j'k'} d_{i'j'k'} \ell^{i'} \ell^{j'} \ell^{k'}}$$

From N_h gaugino condensations, N_h combinations of Kaehler moduli and their axion partners get a heavy mass

$$m_\tau \sim m_\theta \sim m_{3/2} \ln(M_{\rm Pl}/m_{3/2})$$

From V_D and the related Stuekelberg mixing, $N_{U(1)}$ combinations of Kaehler moduli, U(1) gauge bosons, and axions get a superheavy mass

$$m_\tau \sim m_\theta \sim m_{A_\mu} \sim \frac{M_{\rm Pl}}{\tau}$$

From $V_{\rm lift}$, the remained $h_{1,1} - N_h - N_{U(1)}$ Kaehler moduli get

$$m_\tau \sim m_{3/2},$$

while their axion partners remain massless.

Decay constants and couplings of $h_{1,1}-N_h-N_{U(1)}$ Type II light axions:

$$\theta_i = \sum_{x,m} Q_i^x (L^{-1})_x^m \tilde{\theta}_m + \text{heavy axions} \quad \left(\tilde{\theta}_m \cong \tilde{\theta}_m + 2\pi : \text{light Type II axions}\right)$$

$$\frac{1}{2}\sum_{ij}(f^2)^{ij} D_\mu \theta_i D^\mu \theta_j + \frac{1}{32\pi^2}\sum_i k^i_{\rm QCD}\theta_i G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \ldots$$

$$\Rightarrow \quad \frac{1}{2}\sum_{mn}(\tilde{f}^2)^{mn}\partial_\mu\tilde{\theta}_m\partial^\mu\tilde{\theta}_n + \frac{1}{32\pi^2}\sum_m \tilde{k}^m_{\rm QCD}\tilde{\theta}_m G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$

$$(f^2)^{ij} = \frac{1}{\mathcal{V}_{\rm CY}}\left(\frac{t^i t^j}{\mathcal{V}_{\rm CY}} - 2(X^{-1})^{ij}\right) \quad \left(\mathcal{V}_{\rm CY} = \frac{1}{6}d_{ijk}t^i t^j t^k, \;\; X_{ij} = d_{ijk}t^k\right)$$

$$\left(M_{\rm Pl} = 1/\sqrt{8\pi G_N} = 1\right)$$

$$(\tilde{f}^2)^{mn} = (f^2)^{ij}\left(Q_i^x (L^{-1})_x^m\right)\left(Q_j^{x'}(L^{-1})_{x'}^{m'}\right)$$

$$-\left((f^2)^{ij}\left(Q_i^x(L^{-1})_x^m\right)Q_j^\alpha\right)\left[\left((f^2)^{ij}Q_i^\alpha Q_j^\beta\right)^{-1}\right]_{\alpha\beta}\left((f^2)^{ij}Q_i^\beta\left(Q_j^{x'}(L^{-1})_{x'}^{m'}\right)\right)$$

$$\tilde{k}^m_{\rm QCD} = \sum_{ix} Q_i^x (L^{-1})_x^m k^i_{\rm QCD}$$

Our setup involves $N_{U(1)}$ PQ symmetry for Type I axions, which originate from the shift symmetries of the eaten Type II axions:

$$U(1)_{\rm PQ} = U(1)_{\rm shift} \oplus U(1)_A : \ \phi \to e^{iq_\phi \beta}\phi, \ \ \psi \to e^{iq_\psi \beta}\psi \quad (\beta = \text{constant})$$

Axion scale of those Type I axions is determined by SUSY-breaking at lower energy scales:

$$\Delta W = \lambda \frac{\phi_1^n \phi_2}{M_{\rm Pl}^{n-2}} \quad \left(q_{\phi_2} = -n q_{\phi_1}, \ n \geq 3\right)$$

$$\Rightarrow \quad V_{\rm eff}(|\phi|) = \frac{\lambda^2}{M_{\rm Pl}^{2(n-2)}}\left(|\phi_1|^{2n} + n^2|\phi_1|^{2(n-2)}|\phi_2|^2\right)$$

$$+ \left(\lambda_\phi m_{3/2}\frac{\phi_1^n \phi_2}{M_{\rm Pl}^{n-2}} + \text{h.c}\right) + \sum_\phi m_\phi^2 |\phi|^2 + \ldots \quad \left(m_\phi^2 \ll m_{3/2}^2\right)$$

$$\Rightarrow \quad f_a \sim \langle \phi \rangle \sim \left(\frac{m_{3/2} M_{\rm Pl}^{n-2}}{\lambda}\right)^{1/(n-1)}$$

Light axions in KKLT axiverse

* $h_{1,1} - N_h - N_{U(1)}$ Type II light axions with $f_a \sim \frac{M_{\rm Pl}}{\tau} \sim 10^{16}\,{\rm GeV}$

Generically gaugino condensations have a tendency to enlarge the decay constants of light axions (alignment, clockwork), while the Stuekelberg mixings have an opposite tendency to decrease the decay constants of light axions (anti-clockwork).

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* $N_{U(1)}$ Type I light axions with $f_a \sim \sqrt{m_{3/2} M_{\rm Pl}} \sim 10^{11} - 10^{13}\,{\rm GeV} \quad (n=3)$

Introducing D7 branes supporting the SM gauge fields, a combination of these light axions couples to gluons and can be identified as a QCD axion, which is mostly a Type I axion (if exists) with the smallest decay constant.

Other combinations are lighter ALPs which have a mass of $\mathcal{O}\left(e^{-\tau} m_{3/2}\right)$ induced by D3 brane instantons.

The models discussed so far provide a mini-landscape of axion models that can be realized within Type IIB string theory, in which all moduli are successfully stabilized while leaving (multiple) light axions.

The resulting moduli VEVs can be explicitly computed in terms of the integer-valued model parameters defining the mini-landscape and the gravitino mass, with which one can study various physical properties of light axions over the mini-landscape, e.g. the pattern or distribution of axion couplings, decay constants, ALP masses, and $\theta_{\rm QCD} \equiv \langle a_{\rm QCD} \rangle / f_a$.

Thank you for your attention.