Axion fragmentation

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N. Fonseca, E. Morgante, RS, G. Servant,	1911.08472, JHEP 04 (2020) 010
N. Fonseca, E. Morgante, RS, G. Servant,	1911.08473, JHEP 05 (2020) 080
E. Morgante, W. Ratzinger, RS, B.A. Stefanek,	2109.13823, JHEP 12 (2021) 037
C. Eröncel, RS, G. Servant, P. Sørensen,	2206.14259, JCAP 10 (2022) 053

2023. 2. 22 @ Chung-Ang University BSM workshop

ALP : Axion-like particle

Axion field : ϕ

- Shift symmetry (NG boson) + $\phi G_{\mu\nu}\tilde{G}^{\mu\nu}$-type coupling w/ gauge fields

$$\phi \to \phi + \delta\phi$$

$$\frac{1}{f}\phi G_{\mu\nu}\widetilde{G^{\mu\nu}}$$

Photon,
Gluon,
Hidden gauge bosons.,
…

- Shift symmetry breaking by strong dynamics

$$V(\phi) = \Lambda_b^4 \cos\frac{\phi}{f}$$

Axion-like particle

- Light and stable spin-0 particle is predicted from $\Lambda_b \ll f$.

ALP mass $$m_a = \sqrt{V''} = \frac{\Lambda_b^2}{f}$$

ALP lifetime $$\tau_a \propto \frac{f^2}{m_a^3} = \frac{f^5}{\Lambda_b^6}$$

Axion-like particle & cosmology

Theoretical motivation, interesting phenomenology, …

- Strong CP problem, QCD axion
- Naturalness of electroweak scale, Relaxion
- Axion monodromy
- Axion inflation
- …

Dynamics of axion field is interesting

- Axion & ALP dark mattter
- Relaxion : dynamical expanation of electroweak scale
- …

Solving EOM $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ with some initial condition

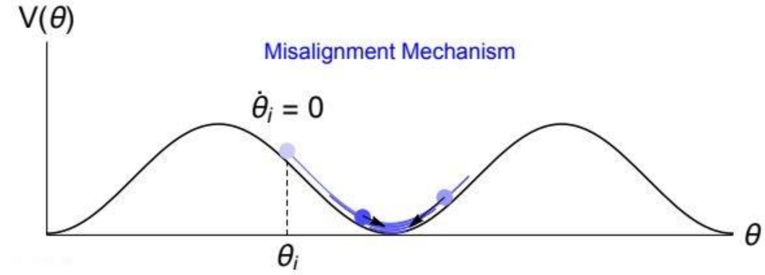
ex) Axion-like particle DM scenario

- Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $\phi = \phi_0 \neq 0$, $\dot{\phi} = 0$

EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin\frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$

$$\Rightarrow \quad \rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4 \theta_i^2}{m_a(T_{osc})}$$

w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

mass — Dilution factor — Number density at $T = T_{osc}$

ex) Axion-like particle DM scenario

- Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]
[Abbott, Sikivie (1983)]
[Dine, Fischler (1983)]

Initial condition $$\phi = \phi_0 \neq 0 \qquad \dot{\phi} = 0$$

What happens if $\dot{\phi} > \Lambda_b^2$?

EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$

[taken from Co, Hall, Harigaya (2019)]

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mass Dilution factor Number density at $T = T_{osc}$

ex) Axion-like particle DM scenario

- Kinetic Misalignment mechanism [Co, Hall, Harigaya (2019)] [Chang, Cui (2019)]

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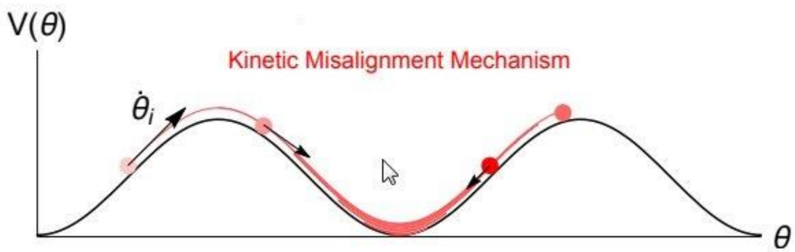
EOM $$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f}\sin\frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $\dot{\phi}^2(T) \sim \Lambda_b^4(T)$

$$\Rightarrow \quad \rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0}\right)^3 \times \frac{\Lambda_b(T_{osc})^4}{m_a(T_{osc})}$$ w/ $\dot{\phi}^2(T_{osc}) \sim \Lambda_b^4(T_{osc})$

mass — Dilution factor — Number density at $T = T_{osc}$

Delay of onset of oscillation → larger ρ_{DM}

Axion fluctuation?

What people usually do

Solving EOM for spatially homogeneous field : $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

However...

Even we start from (almost) homogeneous field configuration,
fluctuations can grow later.

Velocity as U(1) charge

Velocity $\dot{\phi}$ is U(1) charge : $\rho_{\text{shift}} = f \frac{\partial L}{\partial_0 \phi} = f \dot{\phi}$

$\phi \to \phi + f\,\delta$

Shift transf.

Explicit breaking of U(1) : $V(\phi) = \Lambda_b^4 \cos \frac{\phi}{f} + \cdots$

⇨ U(1) charge will be lost = energy dissipation

Axion fragmentation

[Fonseca, Morgante, RS, Servant (2019)]

For related earlier works, see
[Green, Kofman, Starobinsky (1998)]
[Flauger, McAllister, Pajer, Westphal, Xu (2009)]
[Jaeckel, Mehta, Witkowski (2016)]
[Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg (2019)]

1. Introduction
2. Perturbative analysis
3. Non-perturbative analysis
4. Application

EOM of axion

Let us investigate the simplest case.

- $H = 0$ (no cosmic expansion)
- $V(\phi) = \Lambda_b^4 \cos(\phi/f)$

We have only three parameters :

- $\dot{\phi}_0$: initial velocity
- f : decay constant
- Λ_b^4 : height of barrier

EOM of axion :

$$\frac{d^2\phi}{dt^2} - \nabla^2\phi - \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f} = 0$$

EOM of axion

We decompose $\phi(\vec{x}, t) = \bar{\phi}(t) + \left[\int \frac{d^3k}{(2\pi)^3} \delta\phi_k(t) e^{ikx} + h.c.\right]$

EOM of axion :

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At the leading order of $\delta\phi_k$,

$$\frac{d^2\bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f}\sin\frac{\bar{\phi}}{f} = \underbrace{\frac{1}{2}\frac{\Lambda_b^4}{f^3}\sin\frac{\bar{\phi}}{f}\int\frac{d^3x}{V_{vol}}\langle\delta\phi(x)\rangle^2}_{\text{Back reaction}}$$

$$\frac{d^2\delta\phi}{dt^2} - \nabla^2\delta\phi - \frac{\Lambda_b^4}{f^2}\cos\frac{\bar{\phi}}{f}\delta\phi = 0$$

EOM of axion

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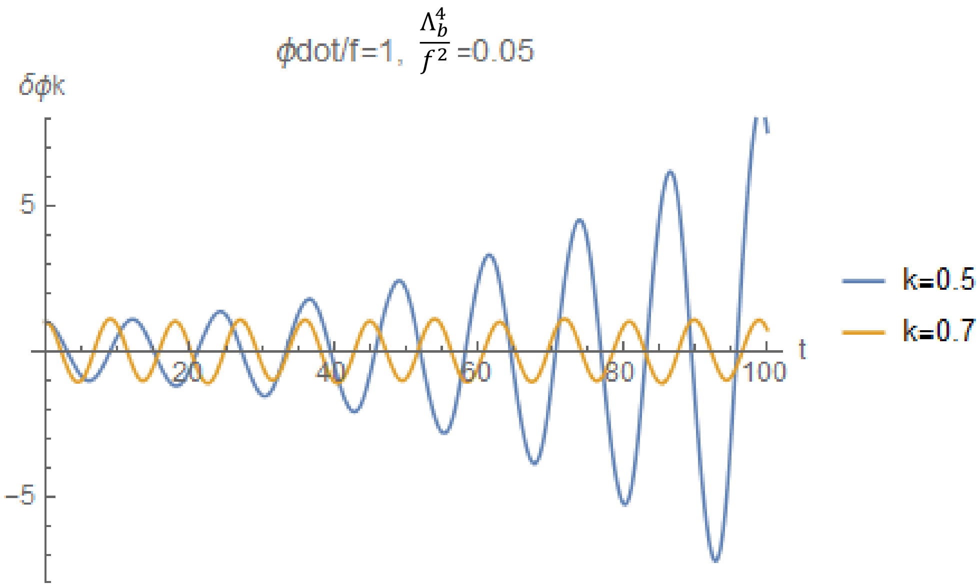
$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)\delta\phi_k = 0$$

Mathieu equation

EOM of axion



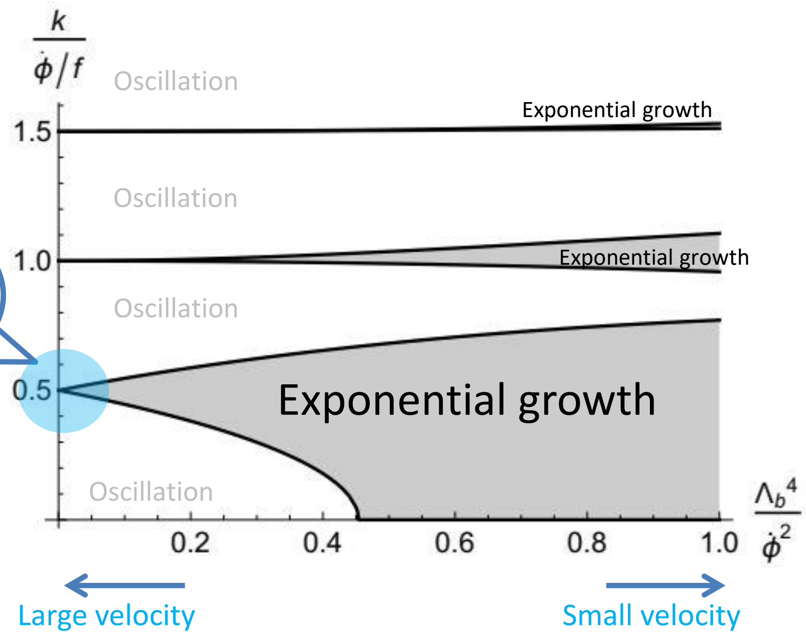
There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2\delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2}\cos\frac{\dot{\bar{\phi}}t}{f}\right)\delta\phi_k \quad = \quad 0$$

Mathieu equation

EOM of axion

$$\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$$



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Mathieu equation

Growth of fluctuation

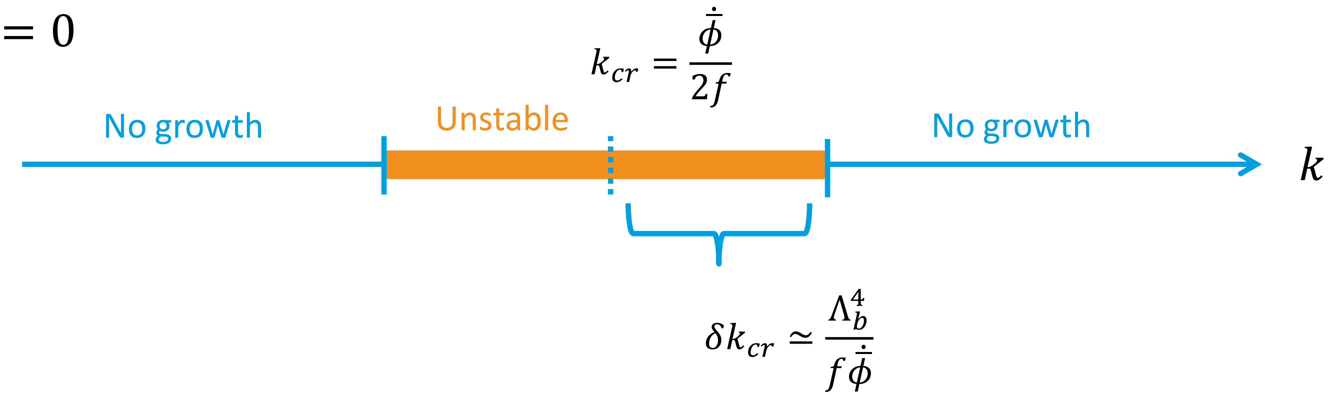
⇩

Back reaction to zeromode

Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$

$t = 0$



By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$$

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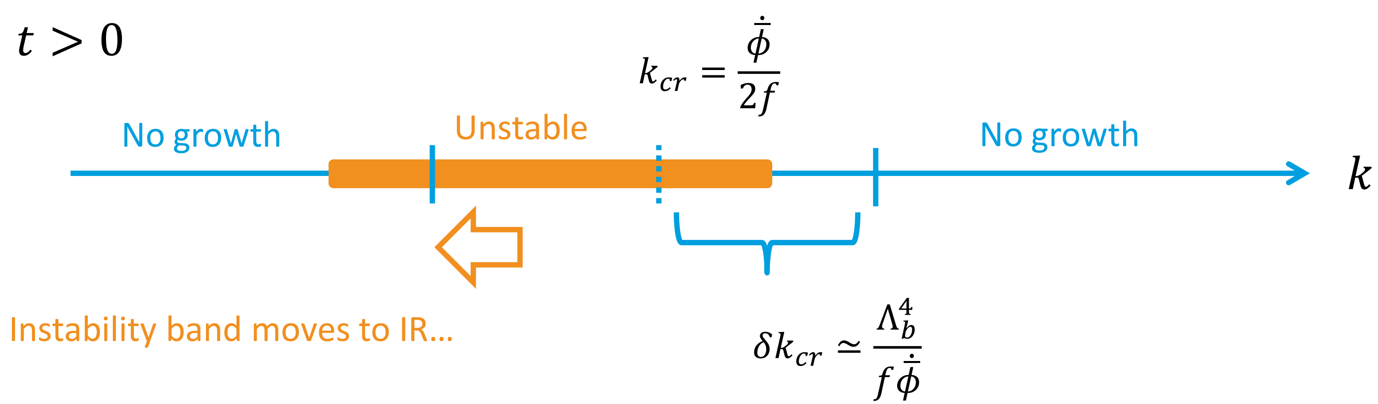

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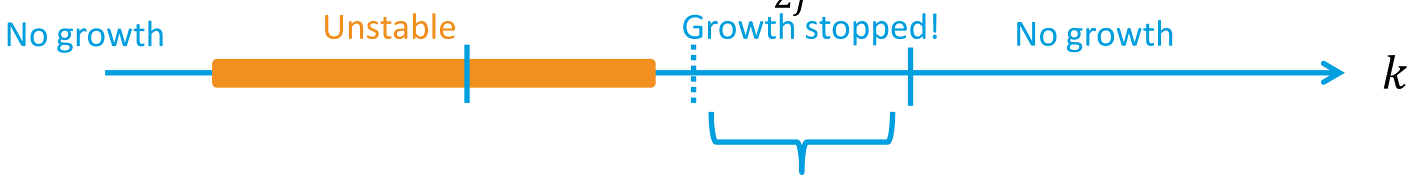
$t = t_{growth}$

$k_{cr} = \frac{\bar{\dot{\phi}}}{2f}$



$\delta k_{cr} \simeq \frac{\Lambda_b^4}{f\bar{\dot{\phi}}}$

By using dimensional analysis

$$\rho_{fluc}(t) \sim k_{cr}^3 \delta k_{cr} \exp\left(\frac{\Lambda_b^4 t}{f\bar{\dot{\phi}}}\right)$$

The growth stops when (of mode with k=kcr)

$$\rho_{fluc}(t_{growth}) \sim \frac{1}{2}\bar{\dot{\phi}}^2 - \frac{1}{2}\left(\bar{\dot{\phi}} - 2f\delta k_{cr}\right)^2$$

Naïve estimation on back reaction

1. ϕ rolls
2. Fluctuation grows & takes energy from ϕ
3. Instability band moves to IR
4. Back to 1. with smaller ϕ

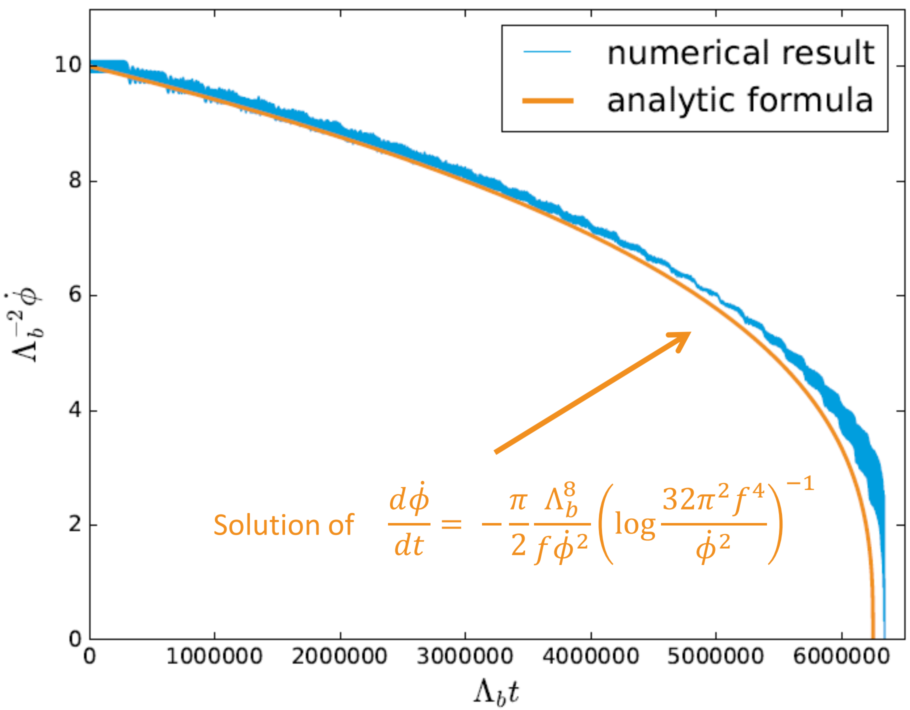
This process repeats until ϕ loses its kinetic energy!

Result in perturbative analysis

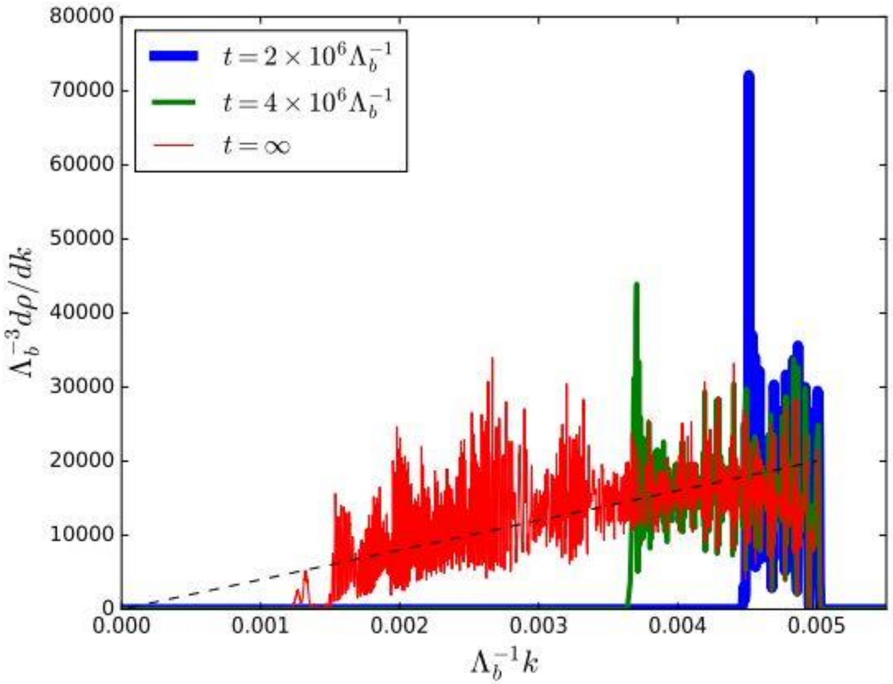
It works!

$$\frac{f}{\Lambda_b} = 1000$$

Time evolution of zeromode velocity



Fluctuation spectrum



[Fonseca, Morgante, RS, Servant (2019)]

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Necessity of non-linear analysis

Perturbative analysis gives intuition.
But reliable?

Initial kinetic energy : $\dot{\phi}_0^2/2$

Typical wavenumber : $\dot{\phi}_0/f$

Energy conservation : $(\delta\phi)^2 \times \left(\dot{\phi}_0/f\right)^2 \sim \dot{\phi}_0{}^2$

⇒ Typical field variation : $\delta\phi \sim f$ **NOT SMALL !!**

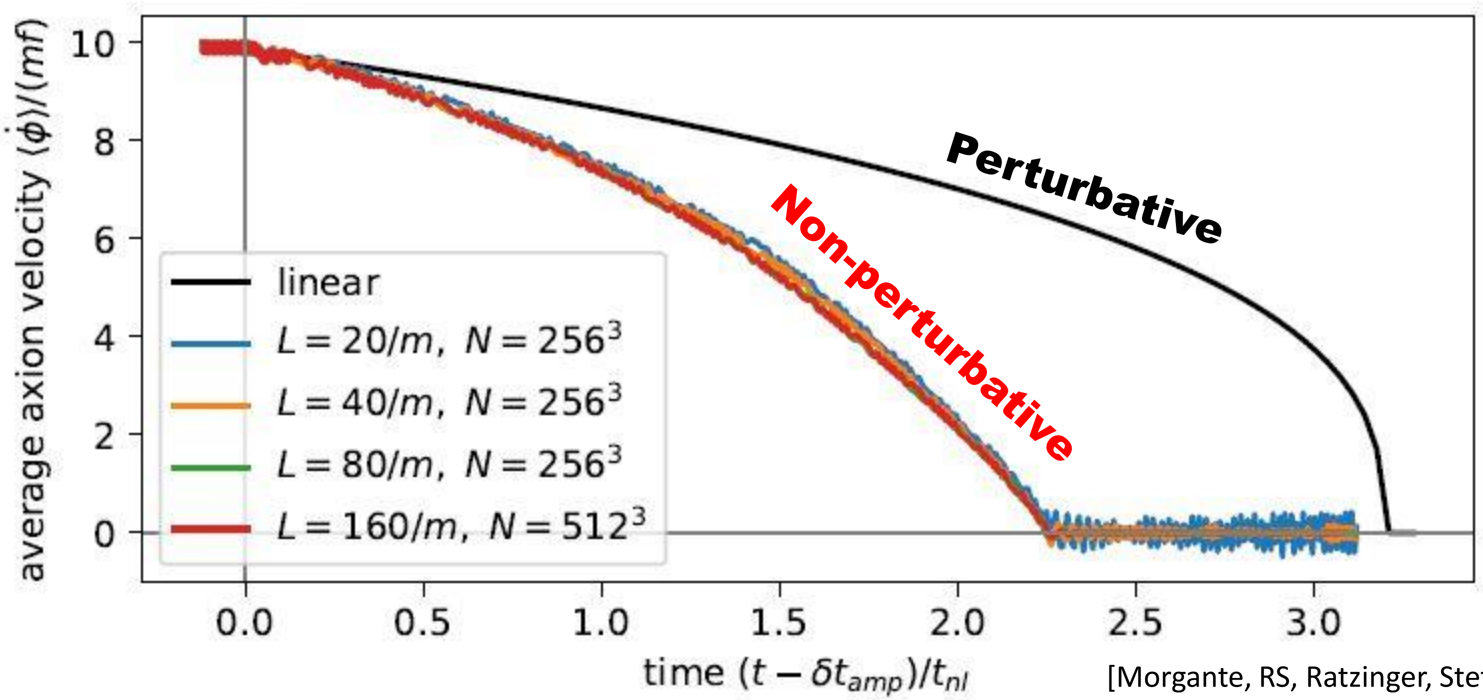
Classical lattice simulation

$$\ddot{\phi} = \nabla^2\phi + \frac{\Lambda_b^4}{f}\sin\frac{\phi}{f}$$

⇒

$$\begin{aligned}\frac{d^2\phi_{i,j,k}}{dt^2} =& \frac{1}{a^2}\left(\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}\right)\\ &+ \frac{1}{a^2}\left(\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}\right)\\ &+ \frac{\Lambda_b^4}{f}\sin\frac{\phi_{i,j,k}}{f}.\end{aligned}$$

Velocity of zeromode

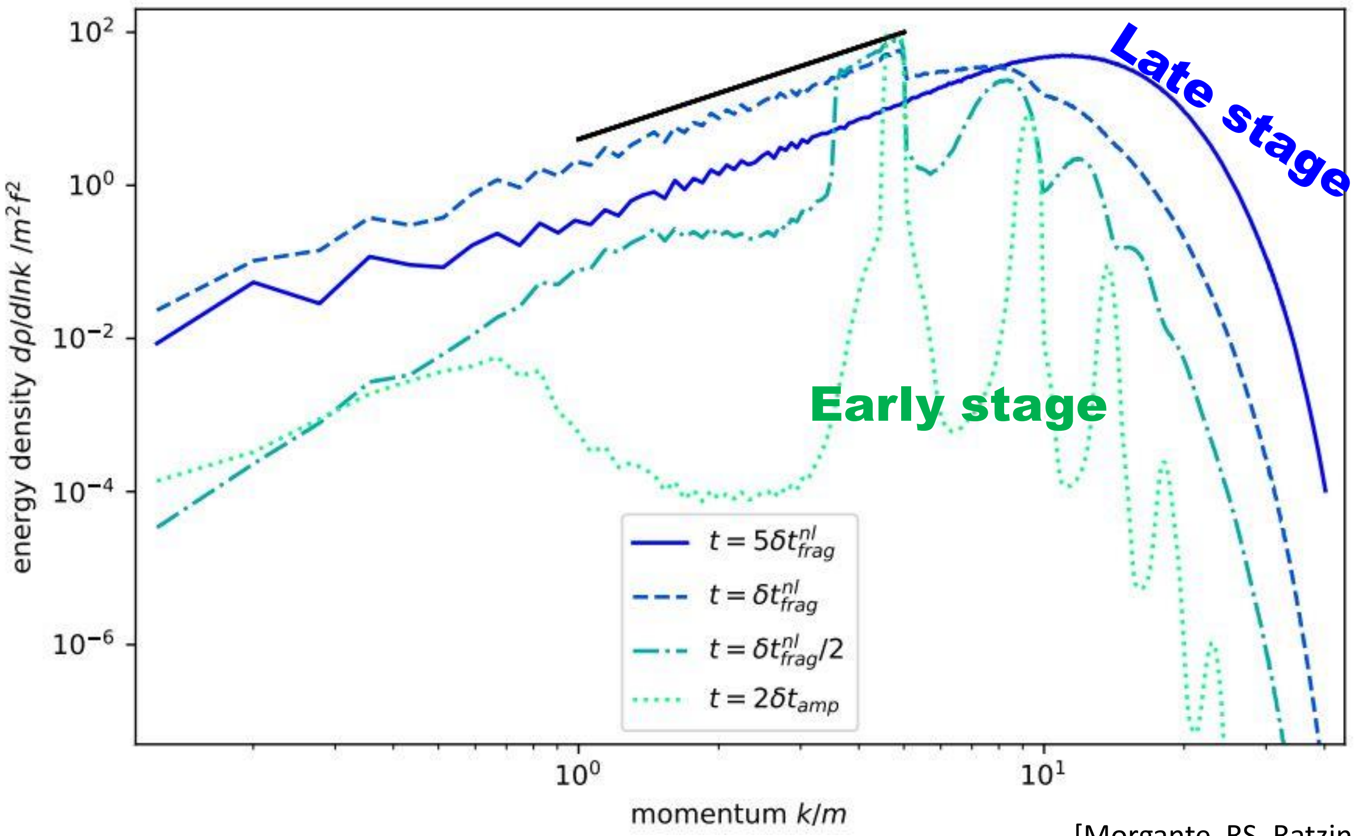


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\left(t_{nl} = \frac{f\dot{\phi}_0^3}{\Lambda_b^8}\right)$$

- Confirmed energy dissipation in non-perturbative calculation.
- Dissipation effect is stronger than perturbative analysis.

Growth of spectrum (late stage)



$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4}\log\frac{16f^4}{\dot{\phi}^2}$$

[Morgante, RS, Ratzinger, Stefanek (2021)]

- Parametric resonance in early stage
- Broad spectrum from non-linear effect

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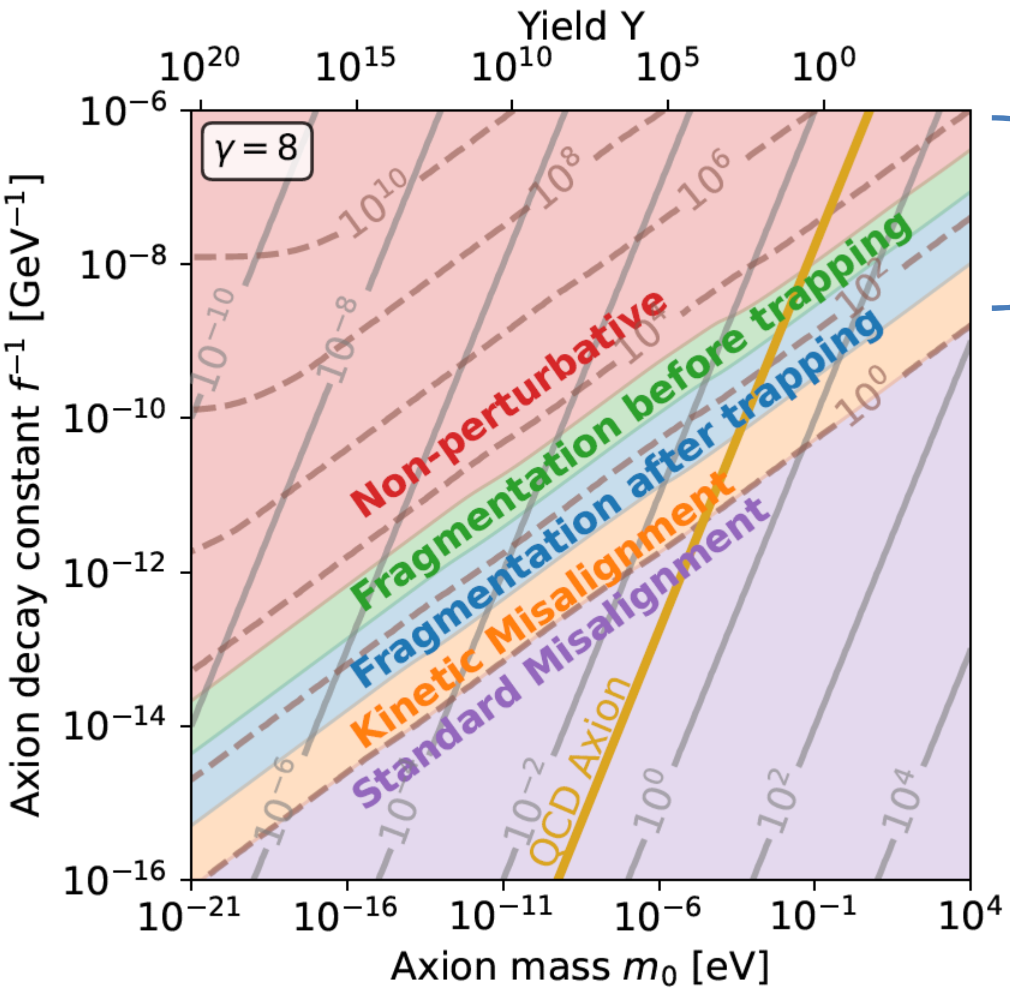
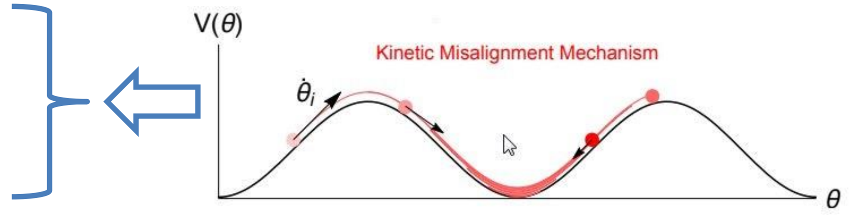
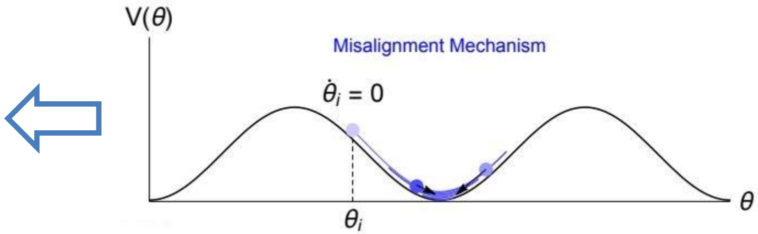
Implication to ALP dark matter

ALP dark matter : Fragmentation could happen before axion starts to oscillate







[Eröncel, RS, Sørensen, Servant (2022)]

Possible signals

- Axion mini-cluster

See Eröncel-Servant (2207.10111)

- Gravitational Wave (tensor perturbation in metric)

$$\nu \sim \frac{k}{a_{emit}} \frac{a_{emit}}{a_0} \quad \text{(Typically, } k \sim m\text{)}$$

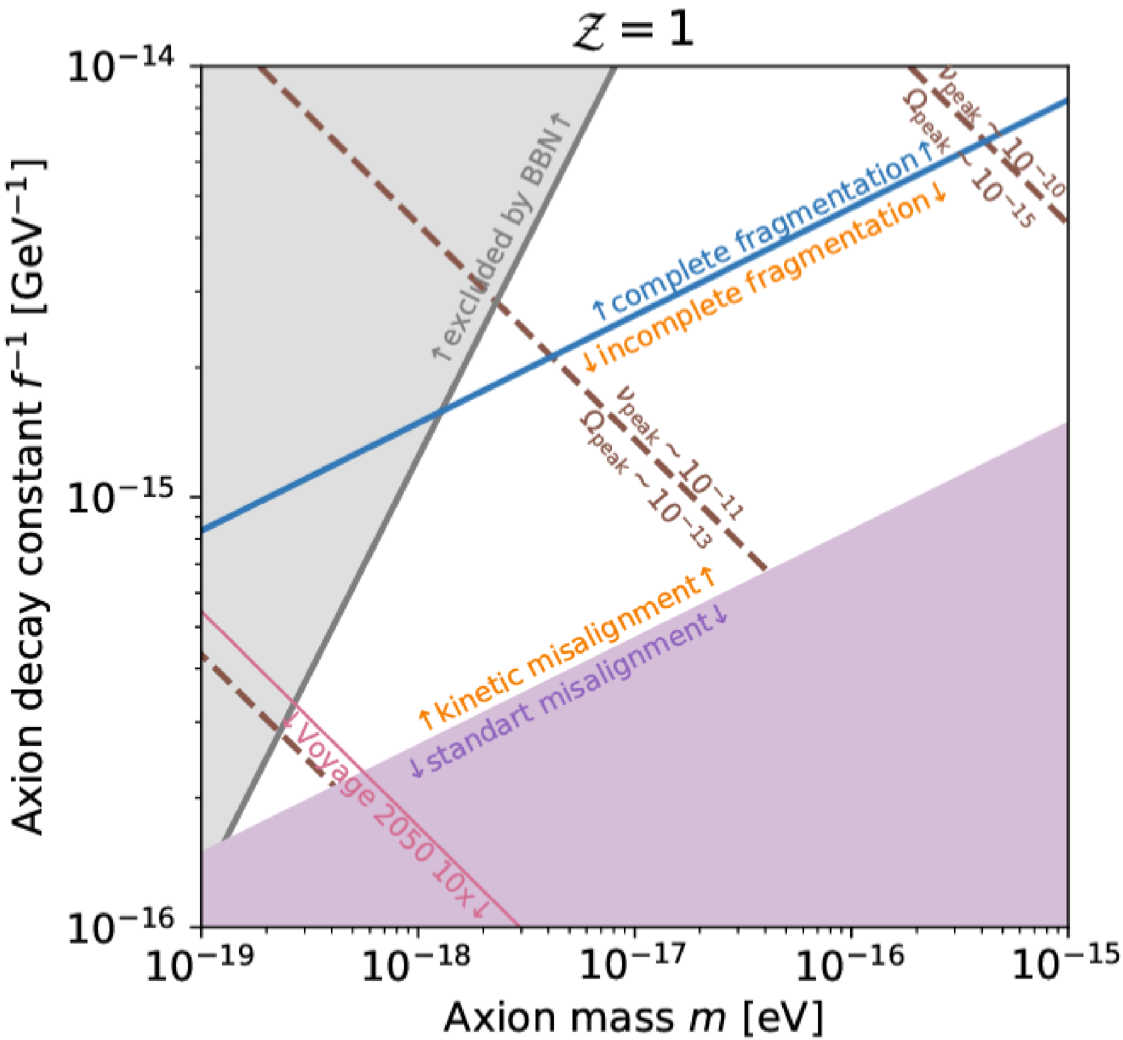
Wave number at emission — Redshift

$$\Omega_{GW}^{peak} \sim \frac{64\pi^2}{3M_{pl}^4 H_{emit}^2} \frac{\rho_{\theta,emit}^2}{\left(k_{peak}/a_{emit}\right)^2} \frac{\alpha^2}{\beta} \quad \text{(Typically, } \alpha < 1,\ \beta > 1\text{)}$$

See [Chatrchyan, Jaeckel (2020)]

c.f.) $\ddot{h} + 3H\dot{h} \sim \frac{1}{M_{pl}^2}\rho_\phi, \quad \rho_{GW} \sim M_{pl}^2 \dot{h}^2$

Possible signals : gravitational waves



[Eröncel, RS, Sørensen, Servant (2022)]

Detailed analysis is future work

Summary

- Large axion velocity → growth of fluctuation
- Zeromode kinetic energy dissipates into fluctuations
- Generic phenomena w/ periodic potential and large velocity
- Applications
  - ALP dark matter
  - Relaxion scenario (1911.08473, Fonseca-Morgante-Sato-Servant)
    Relaxion fragmentation can be a source of friction to stop relaxion.
  - Any other interesting application?

Backup

Naïve estimation on back reaction

Time scale of growth of single mode :

$$t_{stop} \sim \frac{f\bar{\dot{\phi}}}{\Lambda_b^4} \log \frac{f^4}{\bar{\dot{\phi}}^2}$$

Energy stored in fluctuations :

$$\rho_{fluc}(t_{stop}) \sim \bar{\dot{\phi}}^2 \times \frac{\delta k_{cr}}{k_{cr}},$$

$$\Rightarrow \quad \frac{d}{dt}\dot{\phi}^2 \sim -\frac{\rho_{fluc}(t_{stop})}{t_{stop}} \quad \sim -\frac{\Lambda_b^8}{f\dot{\phi}}\left(\log \frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

$$\Rightarrow \quad \frac{d}{dt}\dot{\phi} \sim -\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log \frac{f^4}{\dot{\phi}^2}\right)^{-1}$$

c.f.) WKB approx. with $\dot{\phi} \gg \Lambda_b^2$ gives $\quad \frac{d\dot{\phi}}{dt} = -\frac{\pi}{2}\frac{\Lambda_b^8}{f\dot{\phi}^2}\left(\log \frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$

(see 1911.08472 for details)

Time scale of fragmentation :

$$\Delta t_{frag} \sim f\frac{\dot{\phi}_0^3}{\Lambda_b^8} \log \frac{f^4}{\dot{\phi}_0^2}$$

Field excursion:

$$\Delta\phi_{frag} \sim \dot{\phi}_0 \Delta t_{frag} \sim f\frac{\dot{\phi}_0^4}{\Lambda_b^8} \log \frac{f^4}{\dot{\phi}_0^2}$$

Non-zero slope & Hubble expansion

What happens for non-zero μ^3 & non-zero H?

- Fragmentation $\ddot{\phi}_{frag} = -\frac{\pi\Lambda_b^8}{2f\dot{\phi}^2}\left(\log\frac{32\pi^2 f^4}{\dot{\phi}^2}\right)^{-1}$
- Acceleration by slope μ^3
- Hubble expansion $3H\dot{\phi}$

Fragmentation works if

- During inflation ($3H\dot{\phi} \sim \mu^3$)

  $3H\dot{\phi} < \sim|\ddot{\phi}_{frag}|$ If not, axion keeps rolling with slow-roll velocity

- Not during inflation ($3H\dot{\phi} \ll \mu^3$)

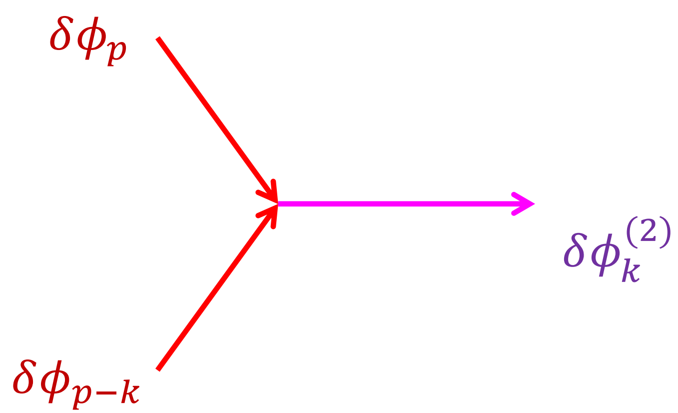
  $\mu^3 < \sim|\ddot{\phi}_{frag}|$ If not, axion is just accelerated by slope

2 to 1 process

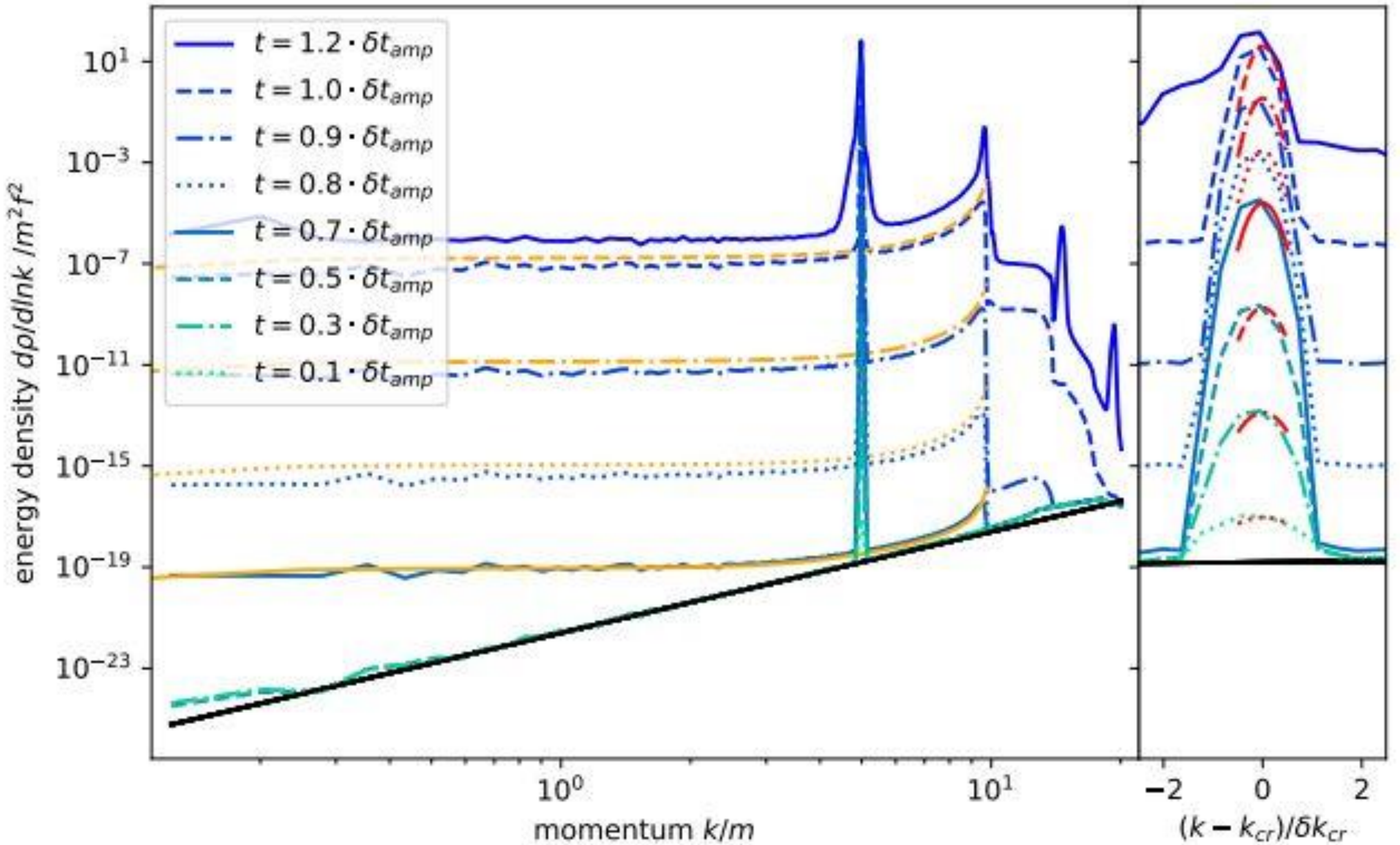
$$\phi(x,t) = \phi(t) + \delta\phi(x,t) + \delta\phi^{(2)}(x,t) + \ \ldots$$

$$\ddot{\phi} - \nabla^2\phi = V'(\phi) \quad \Rightarrow \quad \delta\ddot{\phi}^{(2)} + (k^2 + V'')\delta\phi^{(2)} = -\frac{1}{2}V'''\int d^3p\ \delta\phi_p\delta\phi_{k-p}$$

- $\delta\phi_p$ with $|p| = \dot{\phi}/2f$ is amplified by resonance
- $\delta\phi$ becomes source term for $\delta\phi^{(2)}$



Growth of spectrum (early stage)



[Morgante, RS, Ratzinger, Stefanek (2021)]

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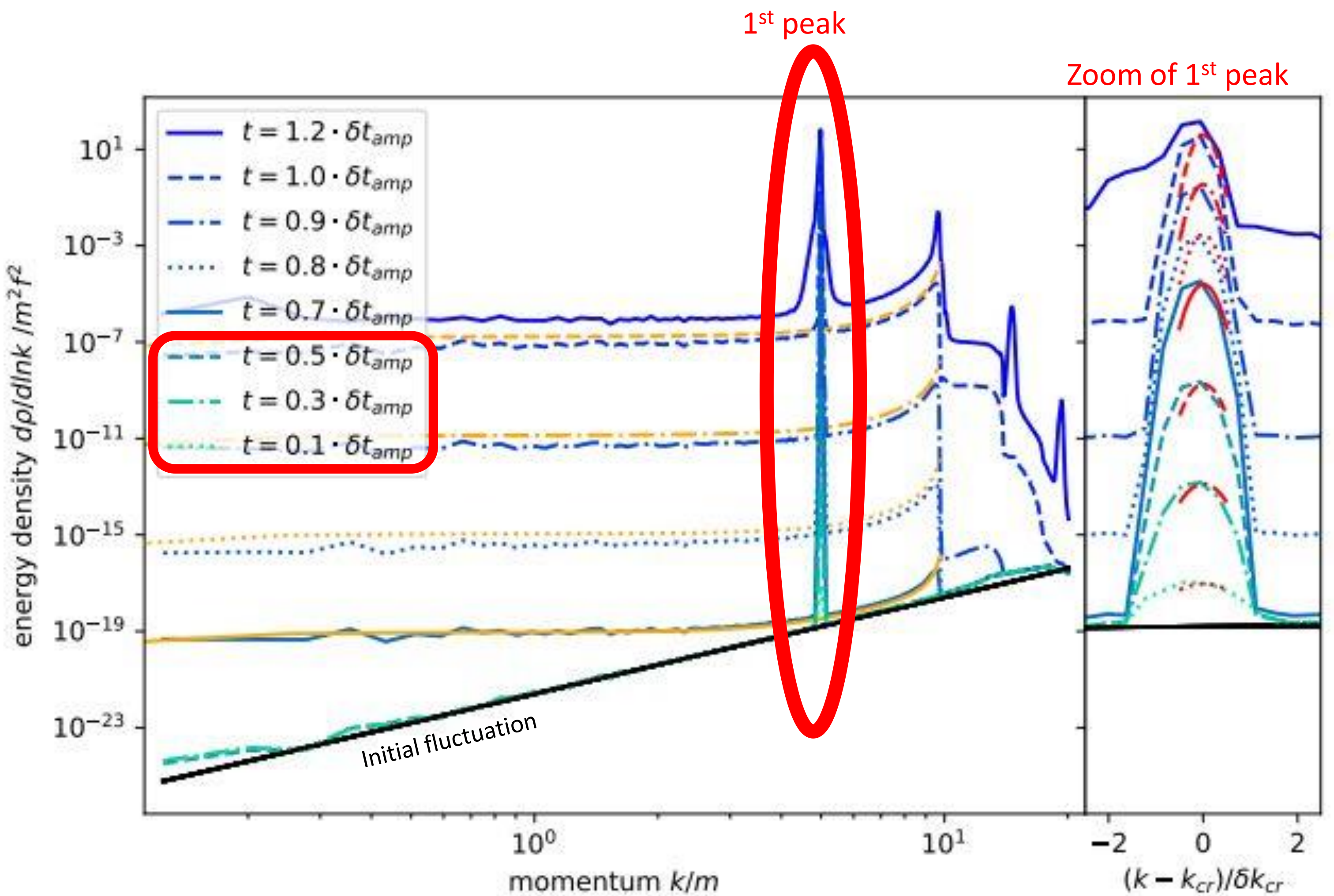

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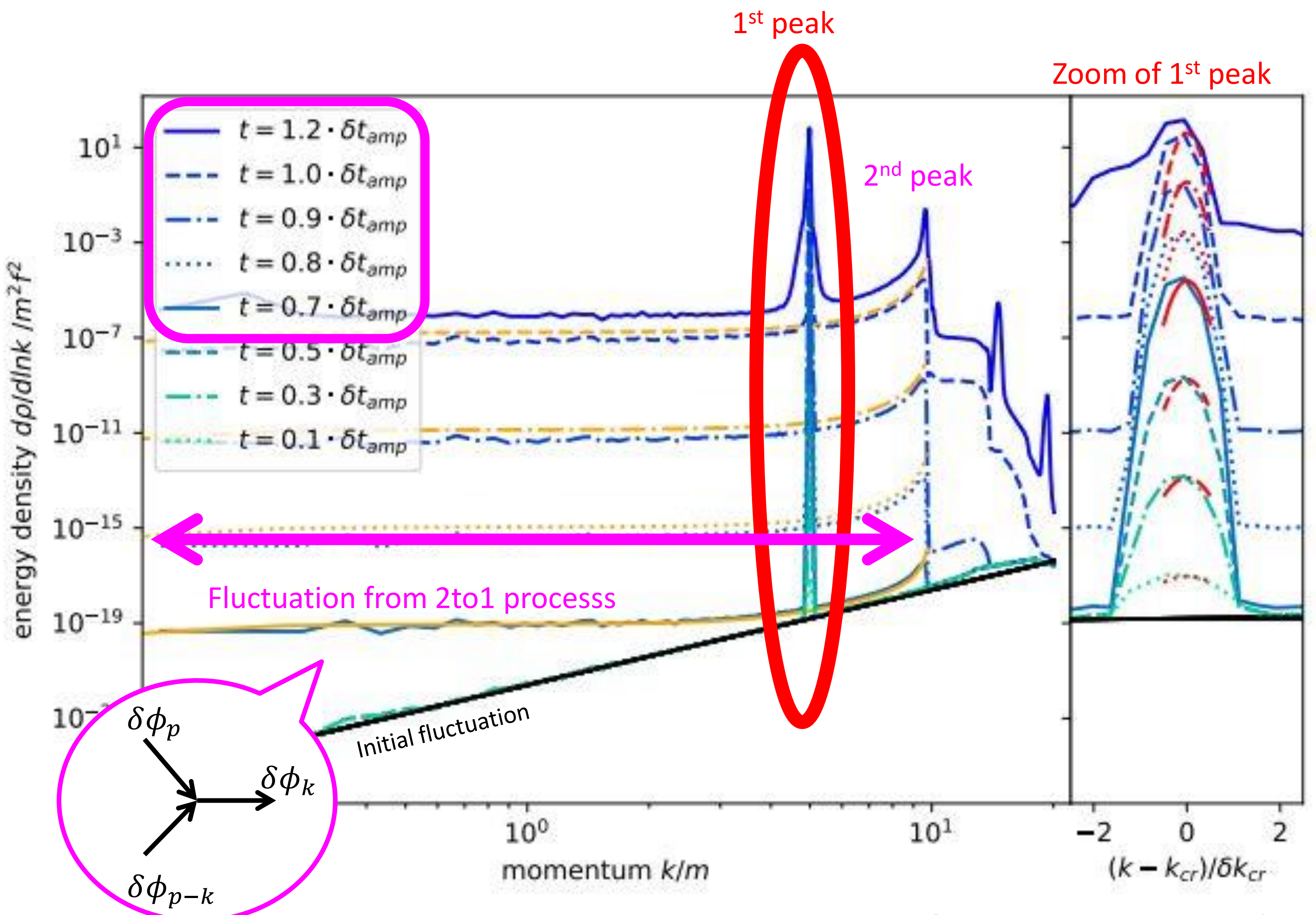

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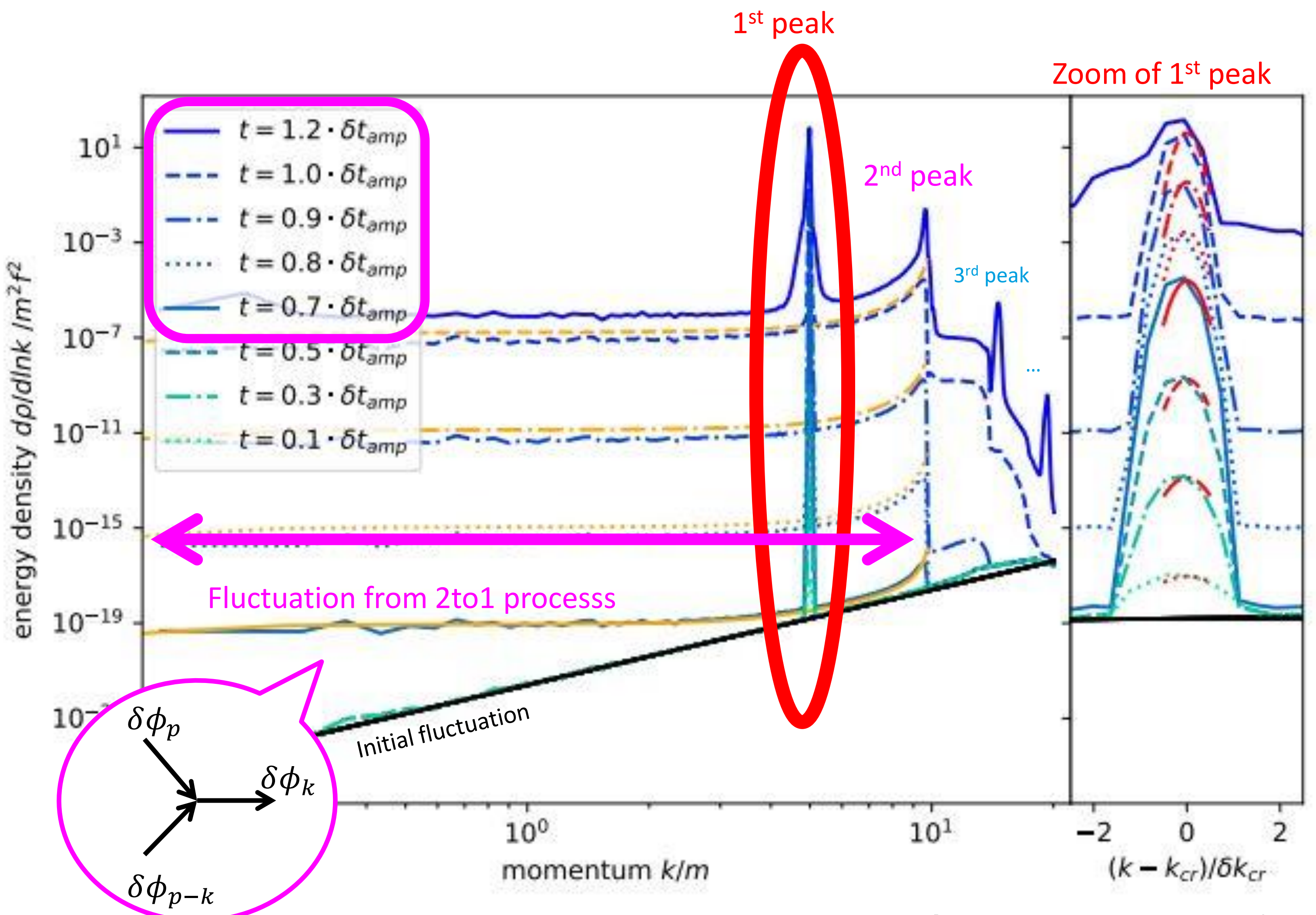

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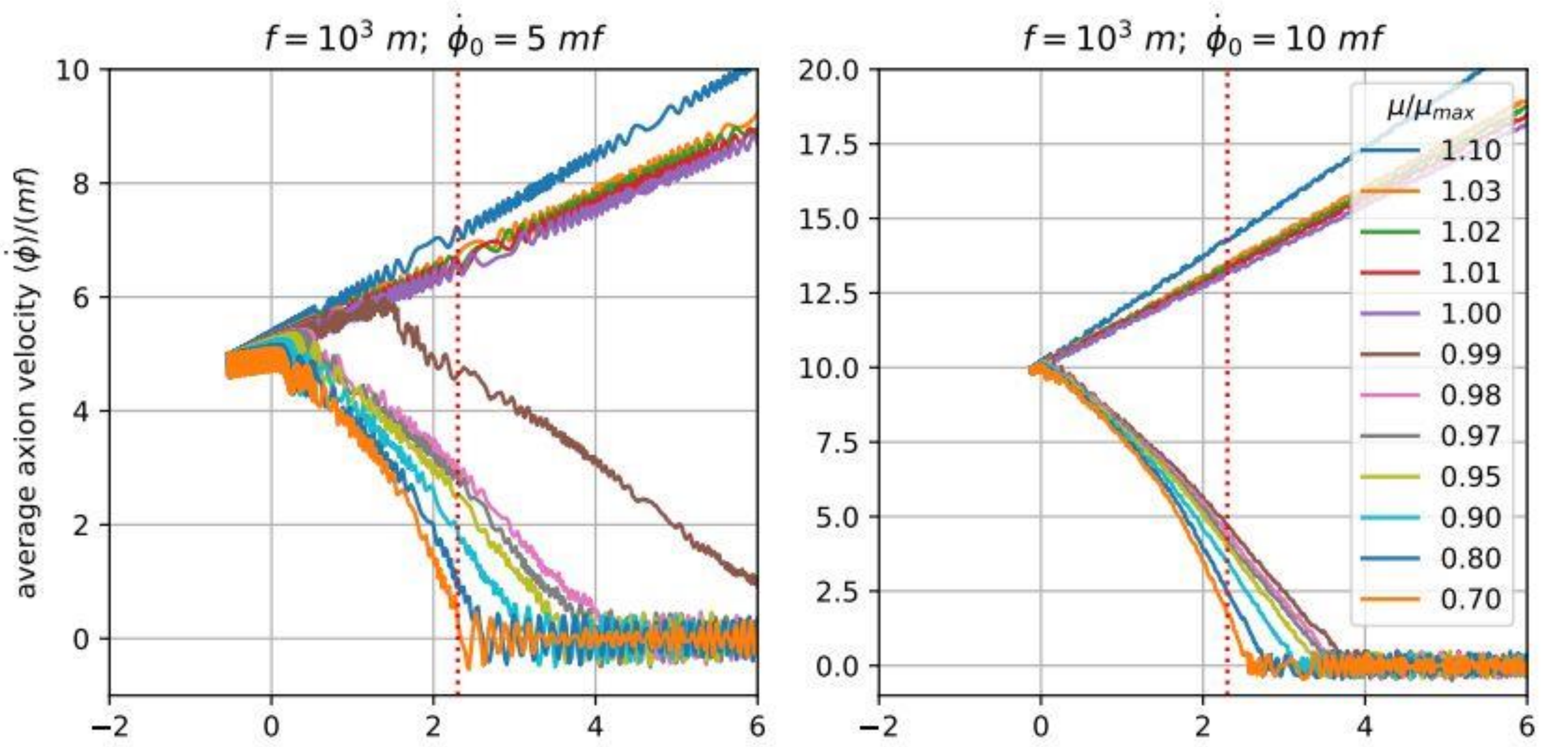
Lattice calc. w/ slope term

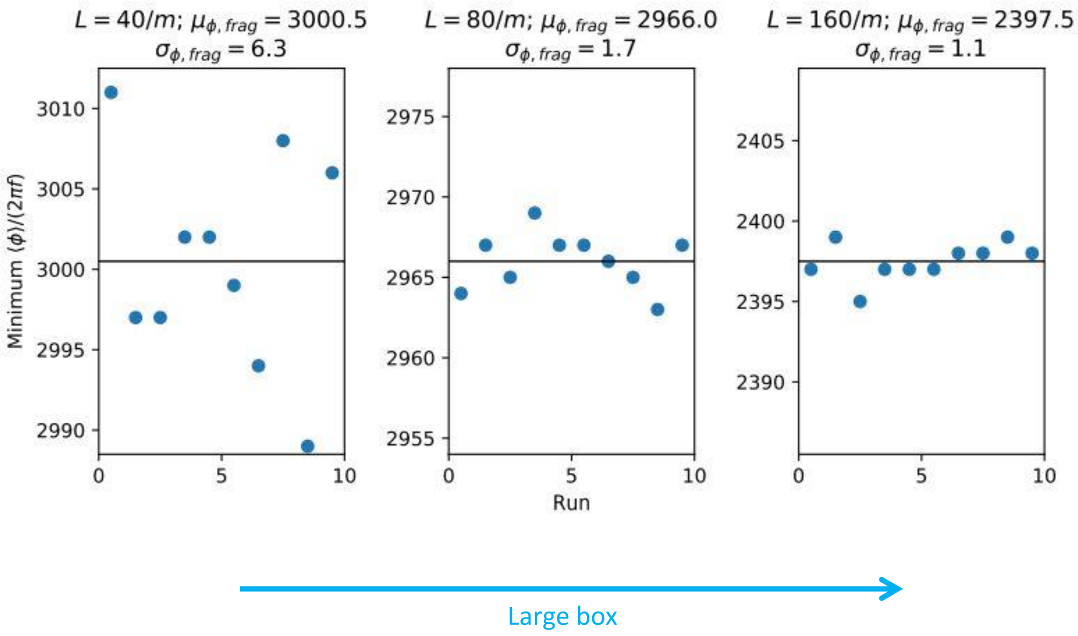


Domain wall?

Field variance after fragmentation is not so small : $\delta\phi \sim f$

Multiple run with finite size box

- $\delta\phi$ in multiple run = $\delta\phi$ of causally disconnected area
- Extrapolation to $V^{1/3} \approx \delta t_{\rm frag}$



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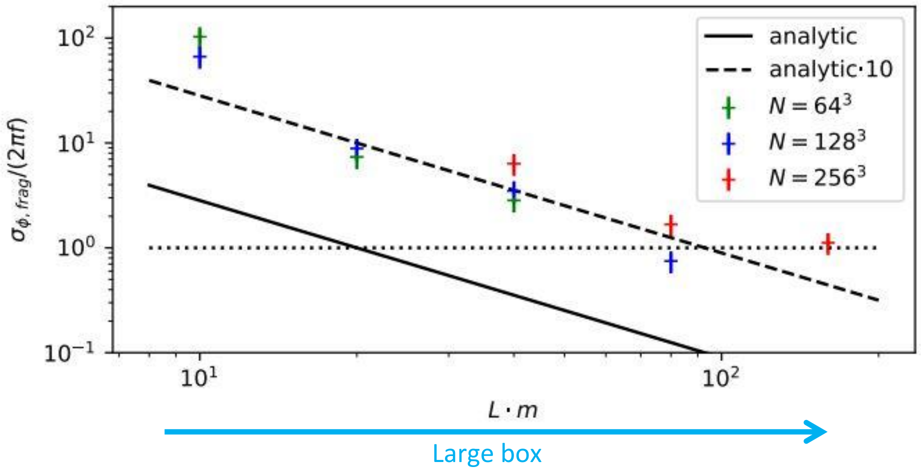


Empirical formula of variance:

$$\frac{\delta\phi}{2\pi f} \sim O(10) \times V^{-1/2} \times \left(\frac{f\dot{\phi}_0}{\Lambda_b^2}\right)^{3/2}$$

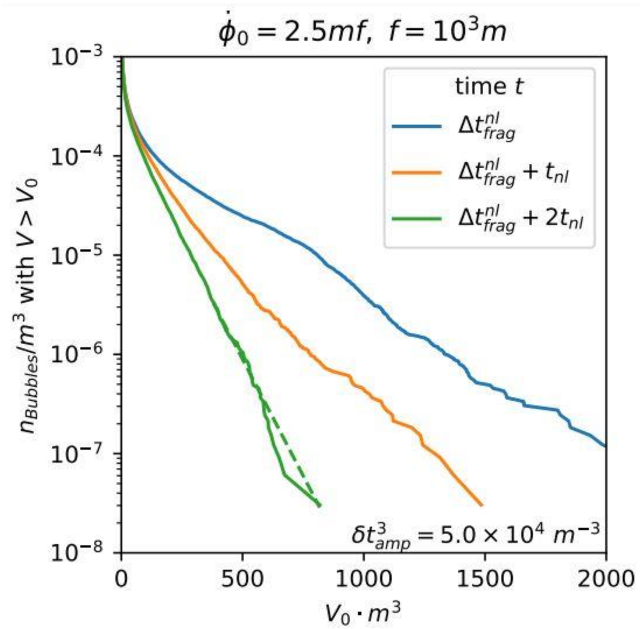
Naïve extrapolation to $V^{1/3} \sim t_{amp}$:

$$\frac{\sigma}{2\pi f} \sim O(10) \times \left(\log \frac{8\pi f^2}{\dot{\phi}_0}\right)^{-\frac{3}{2}} \sim 0.01 - 0.1$$

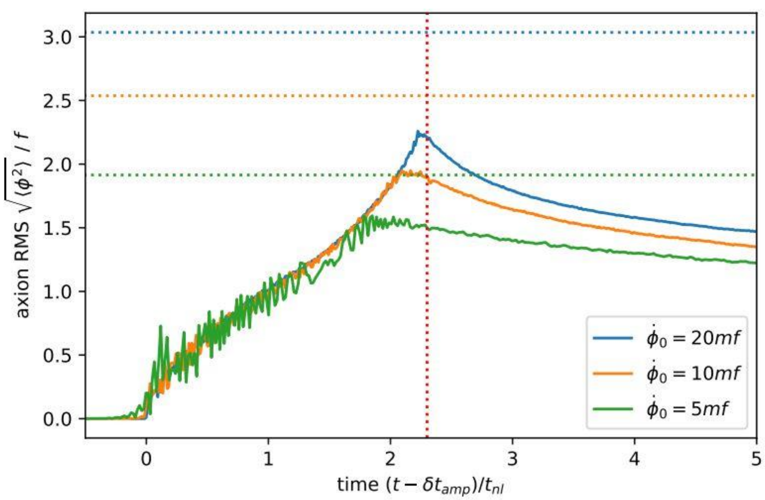
Domain wall formation probability is $\sim e^{-100} - e^{-10}$

Energy cascade into UV

Number counting of “bubble”



Time evolution of variance $\langle \delta\phi^2 \rangle$



- Fluctuation with long wave-length is exponentially suppressed.
- The size of variance decreases in time.

How to get initial velocity

[taken from slide by P. Sørensen (2021)]

Implementations: How to get the kick

Strategy: Radial dynamics:

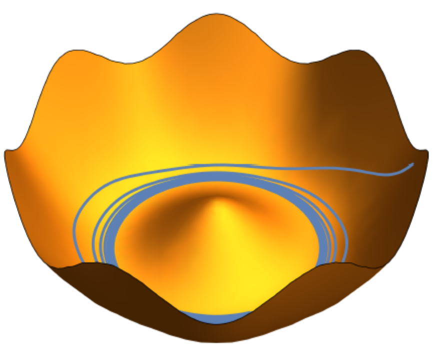
$$P = \frac{S}{\sqrt{2}} e^{i\theta}$$

Afflek-dine-like setup (Afflek and Dine, 1985 and Co et al., 2019), with a nearly-quadratic potential + higher dimensional operators:

$$V = (m_S^2 - c_H H^2)|P|^2 + \frac{A m_s + aH}{n} \frac{P^n}{M^{n-3}} + h.c. + \frac{|P|^{2n-2}}{M^{2n-6}}$$

Large initial radial VEV:

$$S(H) = (HM^{n-3})^{\frac{1}{n-2}} \left(\frac{2^{n-2}}{n-1} \right)^{\frac{1}{2n-4}}$$



Solve EOM for θ:

$$\begin{aligned} n_{PQ} &= S^2 \dot{\theta}_{\text{kick}} \\ &= 2^{1-\frac{n}{2}} \frac{A N_{dw} S^n \sin(n\theta/N_{dw})}{m_{s,\text{eff}} M^{n-3}}. \end{aligned}$$

Elliptic orbit

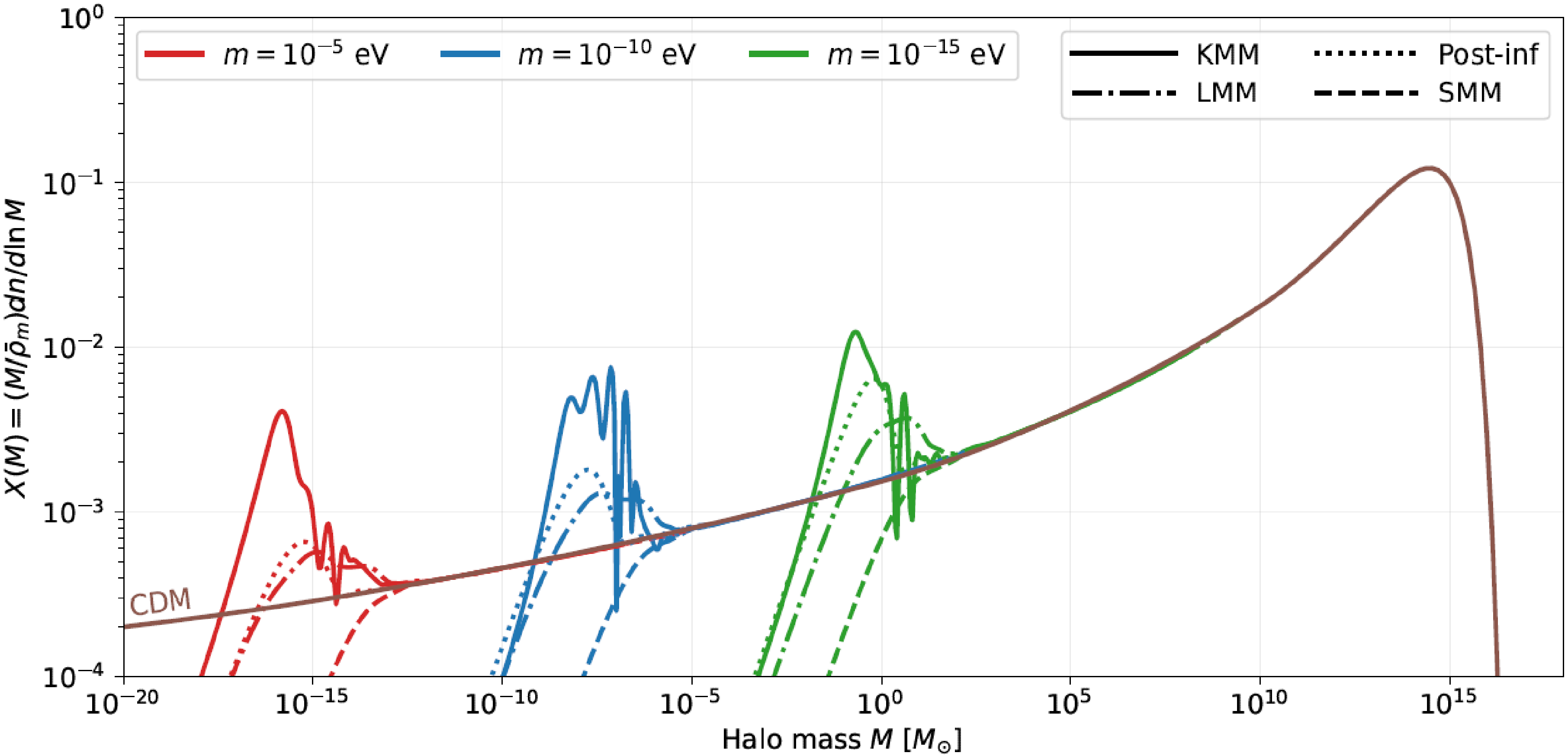
$\rightarrow$ radial oscillations must be damped

Possible signals : ALP mini-cluster

clump of axion DMs

Small m → Large mini-cluster

Perturbative analysis + Press-Schechter formalism



[Eröncel, Servant (2022)]

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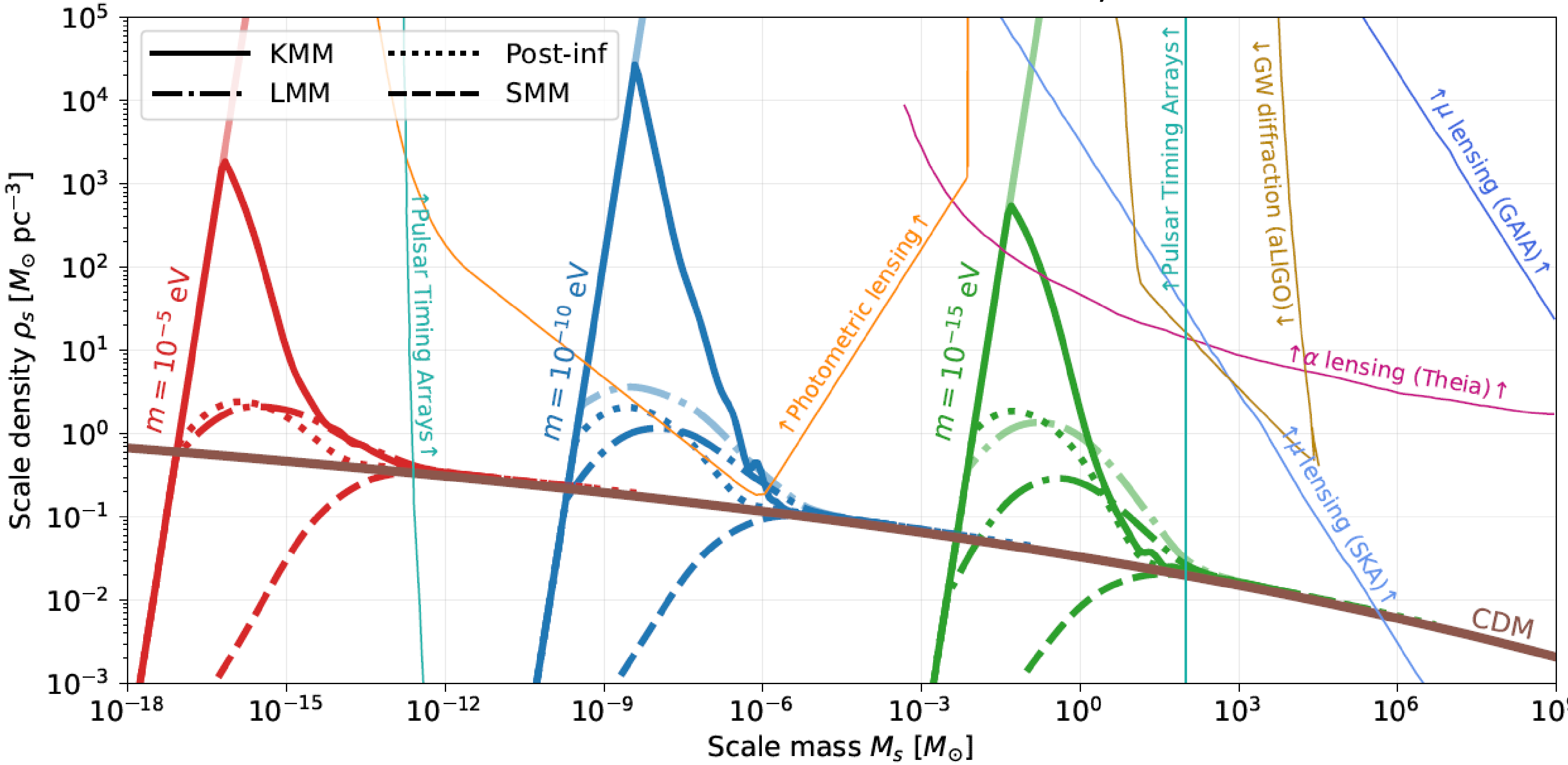
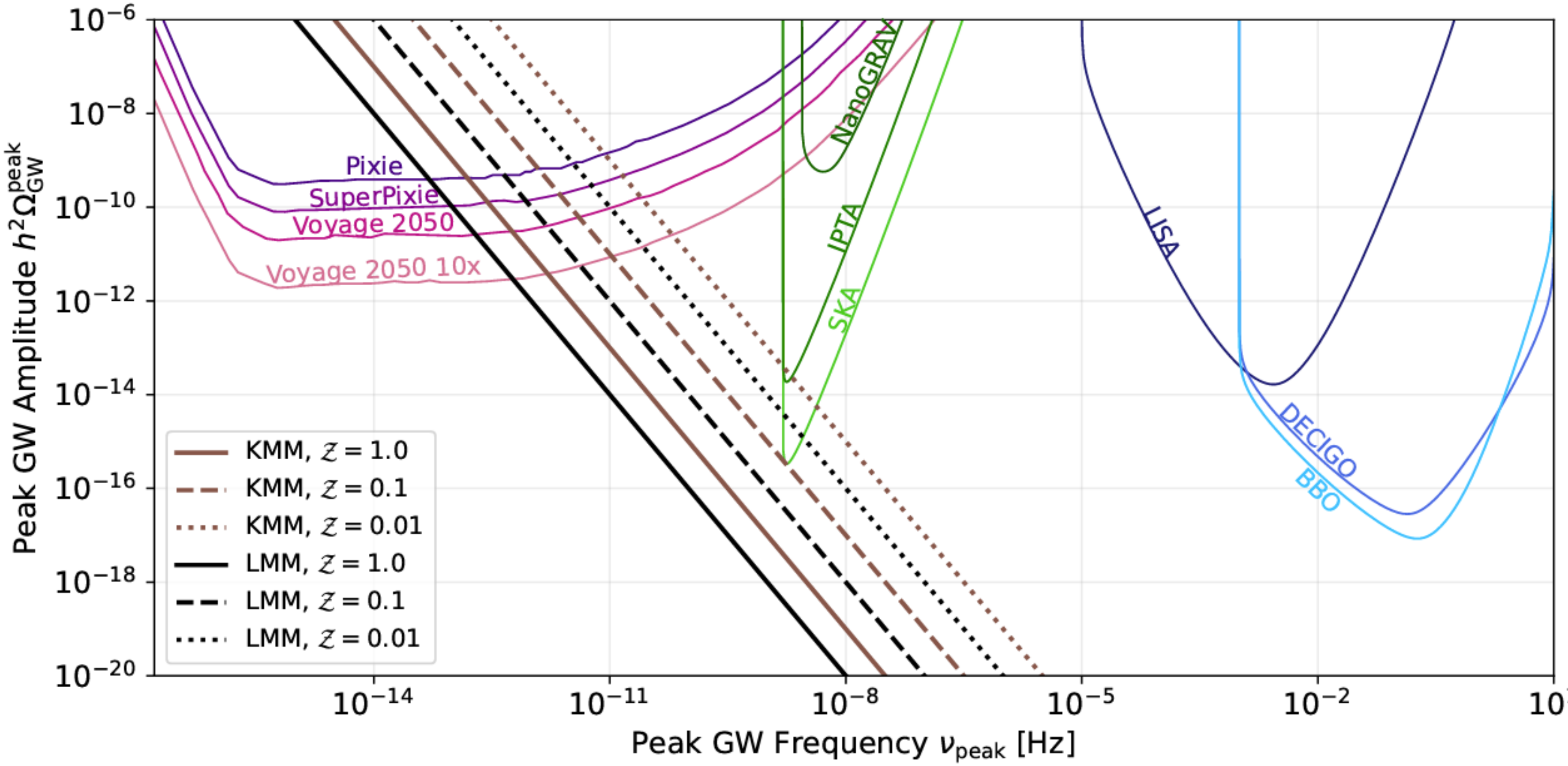


[Eröncel, Servant (2022)]



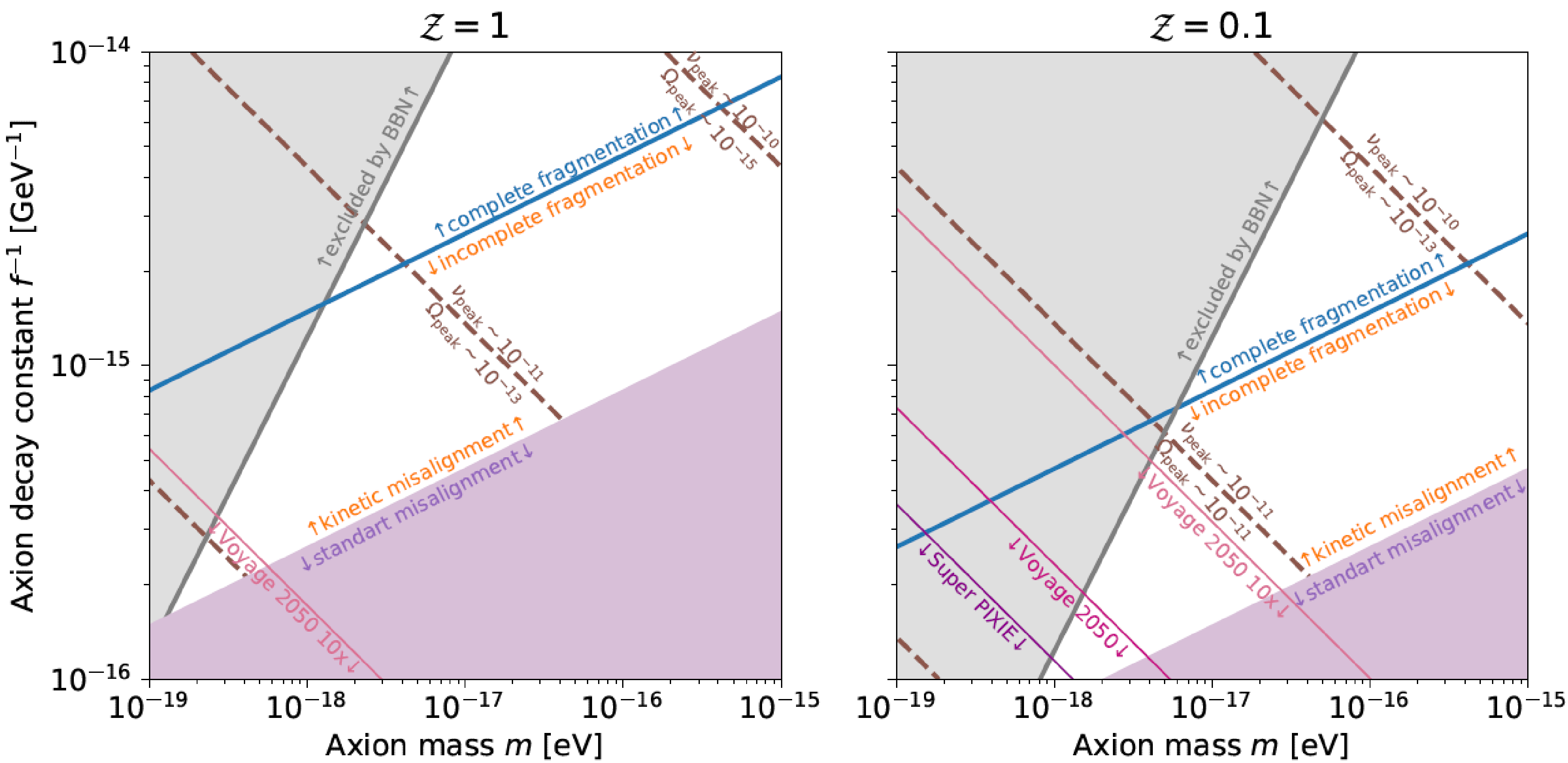
Peak GW Amplitude $h^2\Omega_{GW}^{peak}$
Peak GW Frequency ν_{peak} [Hz]
Pixie
SuperPixie
Voyage 2050
Voyage 2050 10x
NanoGRAV
IPTA
SKA
LISA
DECIGO
BBO
KMM, $\mathcal{Z} = 1.0$
KMM, $\mathcal{Z} = 0.1$
KMM, $\mathcal{Z} = 0.01$
LMM, $\mathcal{Z} = 1.0$
LMM, $\mathcal{Z} = 0.1$
LMM, $\mathcal{Z} = 0.01$

$$\nu_{\rm peak} \sim 8 \times 10^{-11}\,{\rm Hz} \left(\frac{m_*}{m_0}\right)^{2/3} \left(\frac{m_0}{10^{-16}\,{\rm eV}}\right)^{1/3} \left(\frac{f}{10^{14}\,{\rm GeV}}\right)^{-2/3} \mathcal{Z}^{-1/3}.$$

$$\frac{a_*}{a_0} = \left(\frac{3\pi}{8} \frac{\Omega_{\rm DM}}{\mathcal{Z}} \frac{M_{\rm pl}^2 H_0^2}{m_0 m_* f^2}\right)^{1/3}.$$

$$\Omega_{\rm GW,0}^{\rm peak} \sim 1.5 \times 10^{-15} \left(\frac{m_*}{m_0}\right)^{2/3} \left(\frac{m_0}{10^{-16}\,{\rm eV}}\right)^{-2/3} \left(\frac{f}{10^{14}\,{\rm GeV}}\right)^{4/3} \mathcal{Z}^{-4/3}.$$

Possible signals : gravitational waves



Detailed analysis is future work

[Eröncel, RS, Sørensen, Servant (2022)]