

# Gravitational Waves and Neutron Star Equation of State

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in collaboration with

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(BUD and KGWG)

# Contents

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## 1. Motivation

- NS tidal deformability and radius estimations

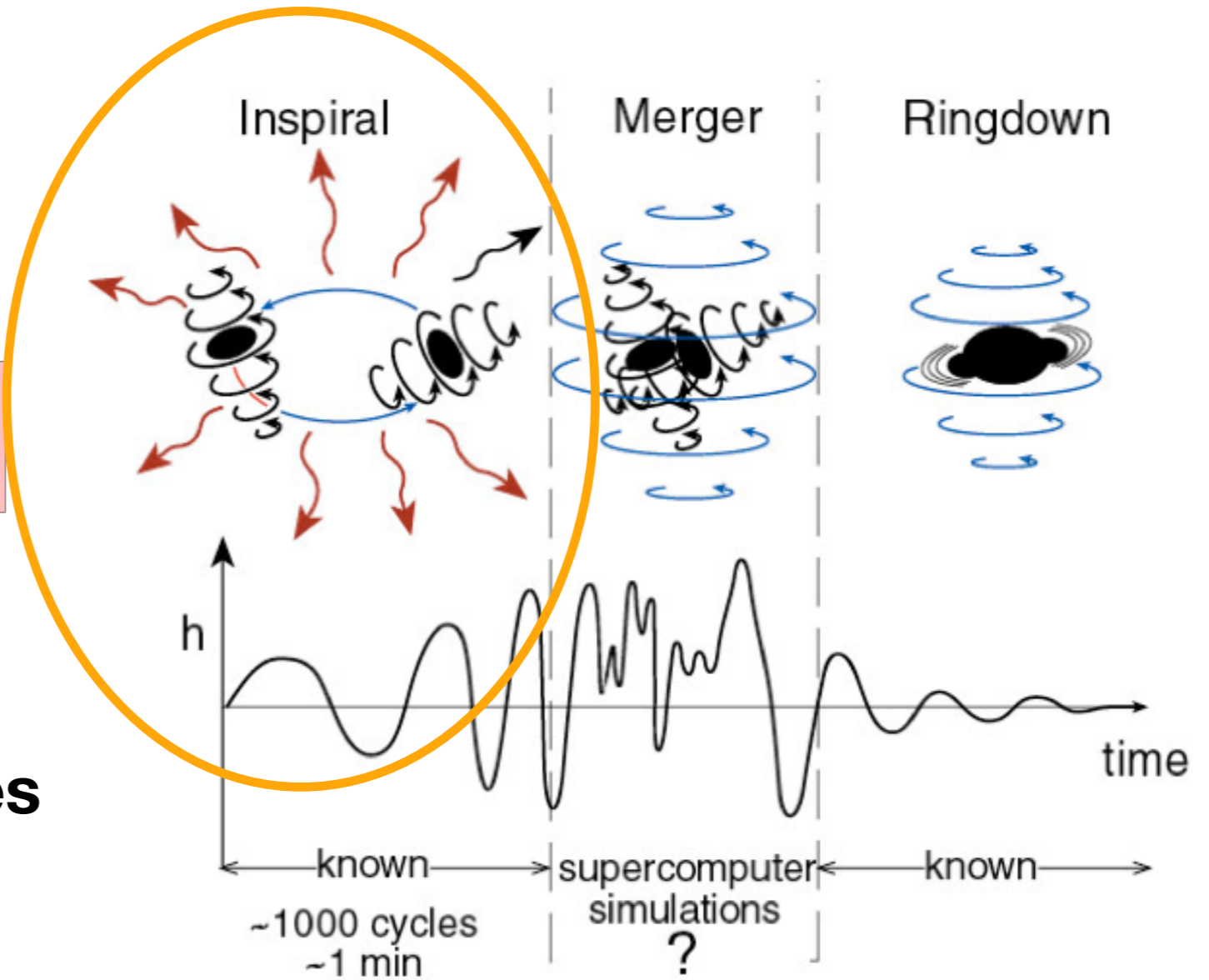
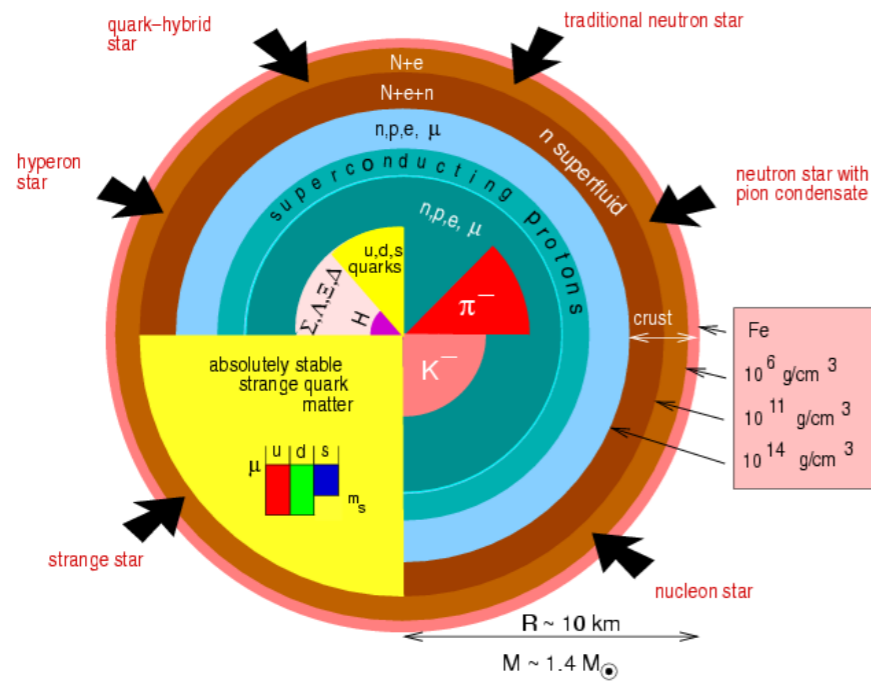
## 2. Observational constraints

## 3. Study on dense matter equation of state

- Posterior distributions of nuclear matter properties

## 4. Summary and future works

# NS Tidal deformability from GW



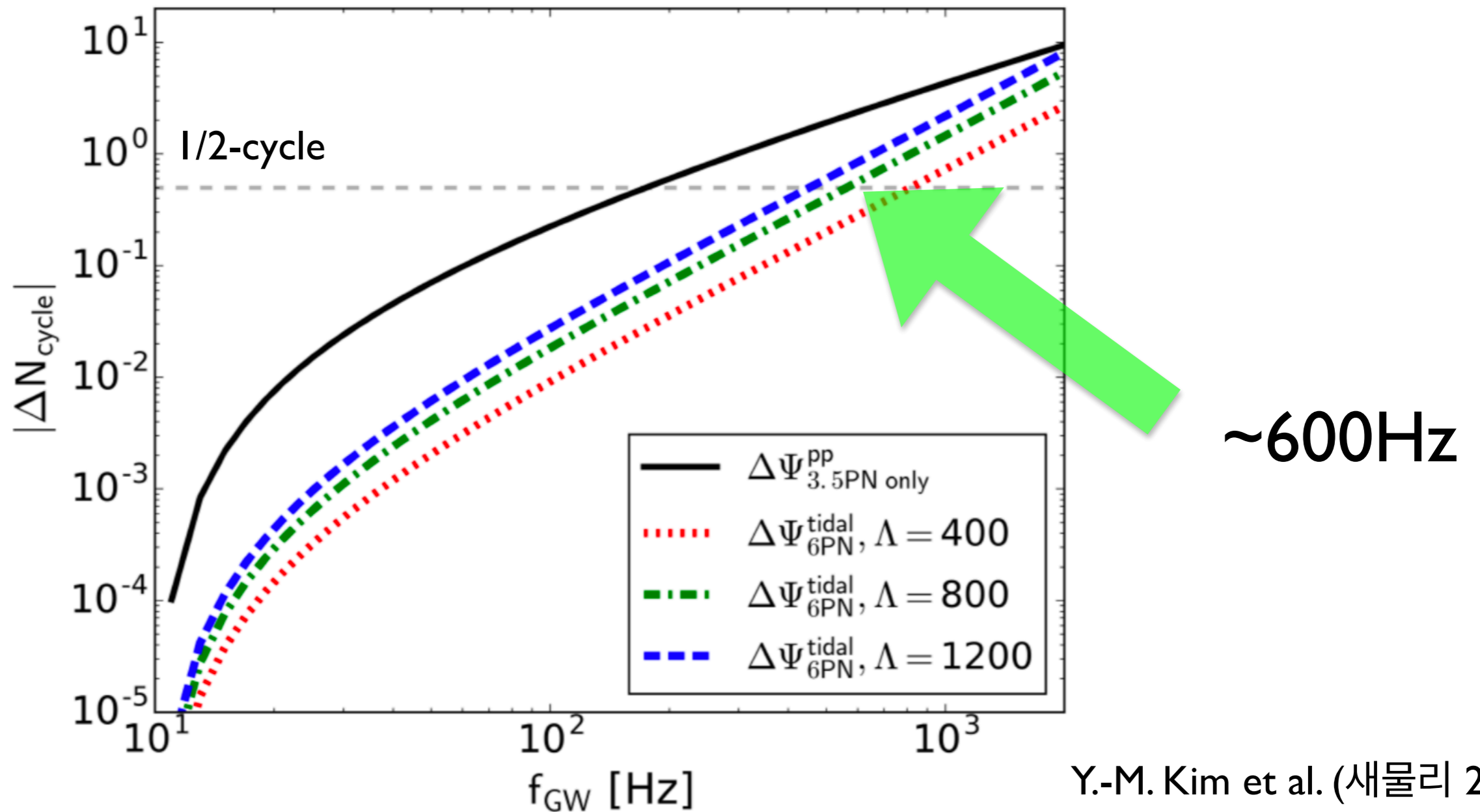
**perturbative approaches**

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\Lambda = \lambda / M^5 \rightarrow G \left( \frac{c^2}{GM} \right)^5 \lambda = \frac{2}{3} \left( \frac{Rc^2}{GM} \right)^5 k_2$$

$\lambda$  : Tidal deformability  
 $k_2$  : Tidal Love number

# Possibility of $\Lambda$ measurement



Y.-M. Kim et al. (새물리 2018)

# First detection of GW from a BNS

GW170817

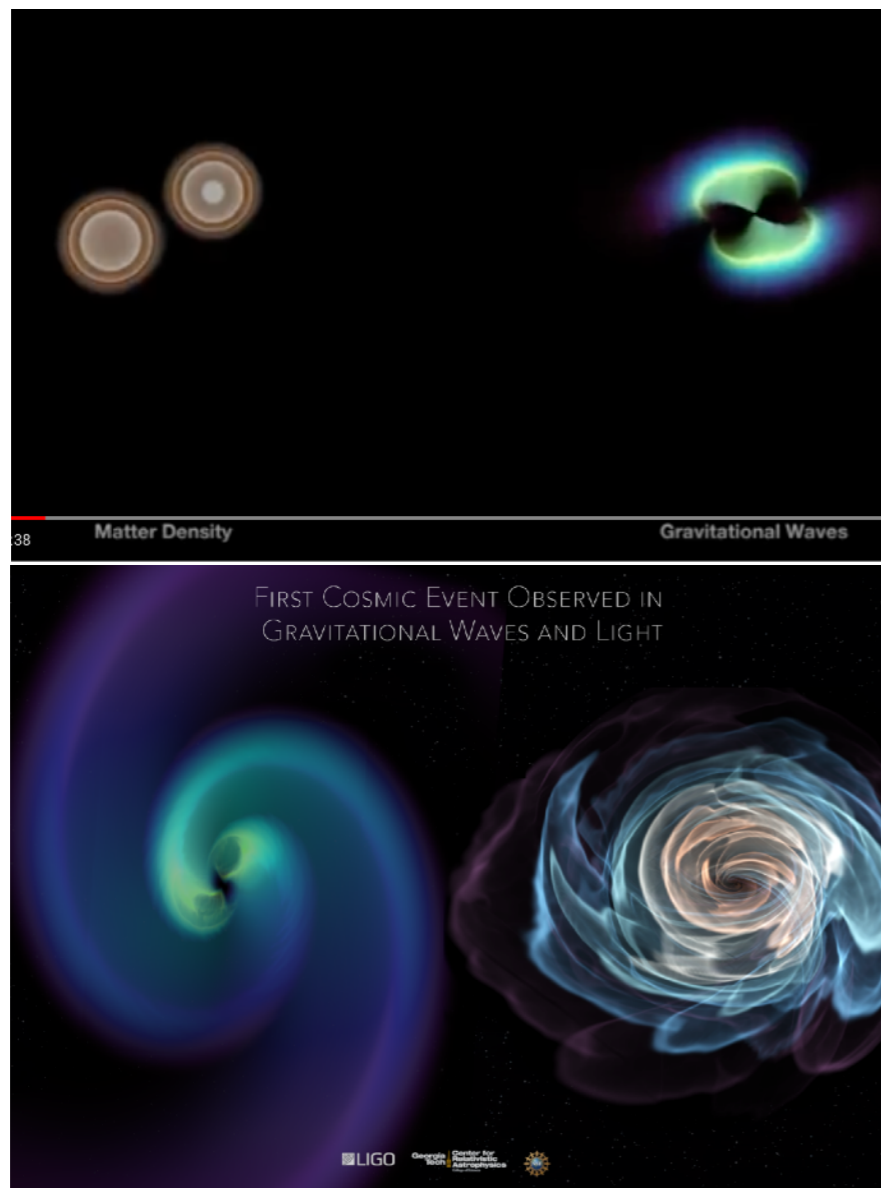
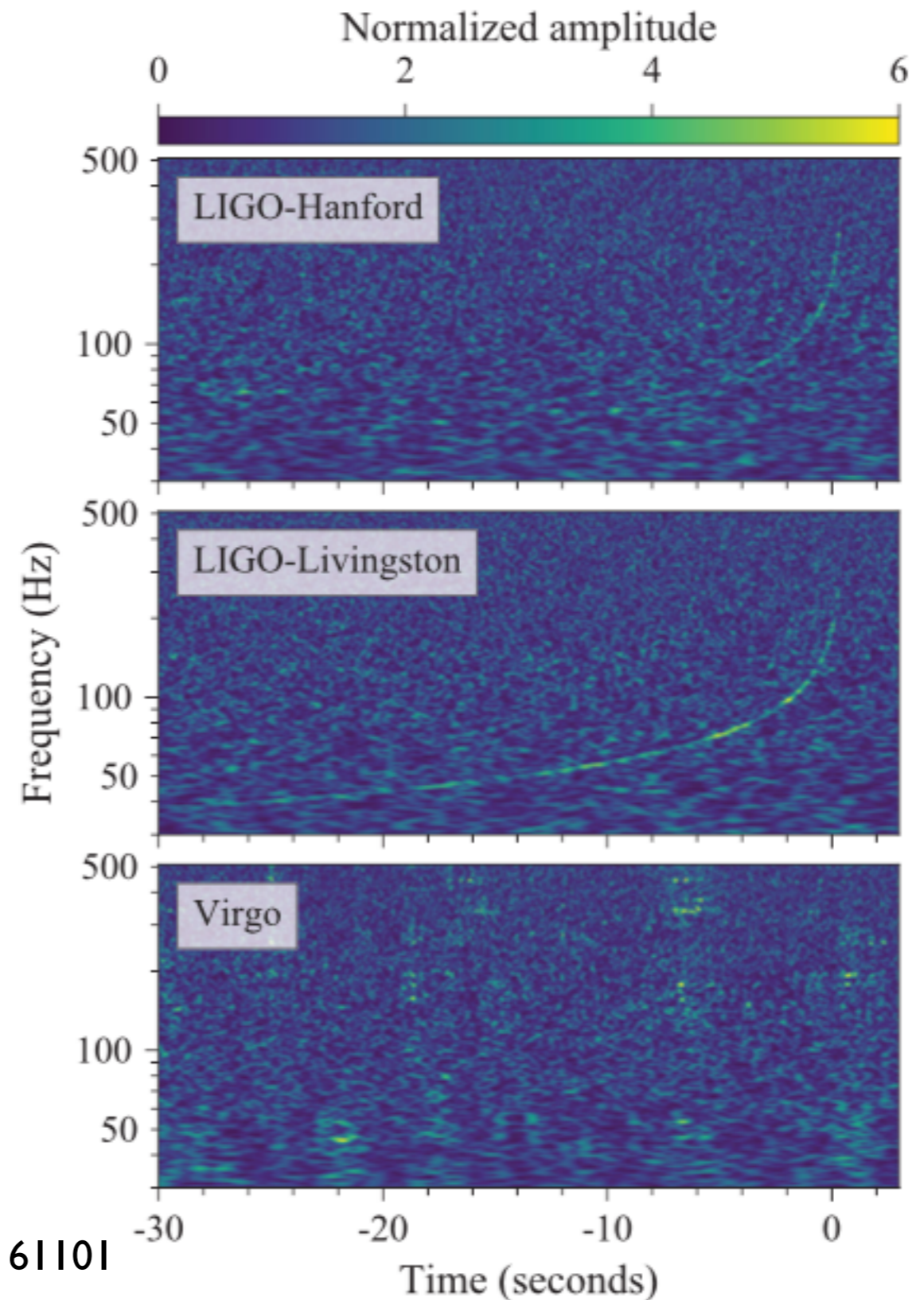


Image credit: Karan Jani/Georgia Tech.

PhysRevLett.119.161101

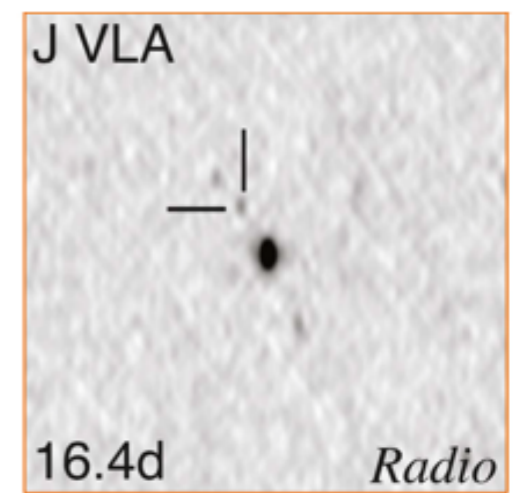
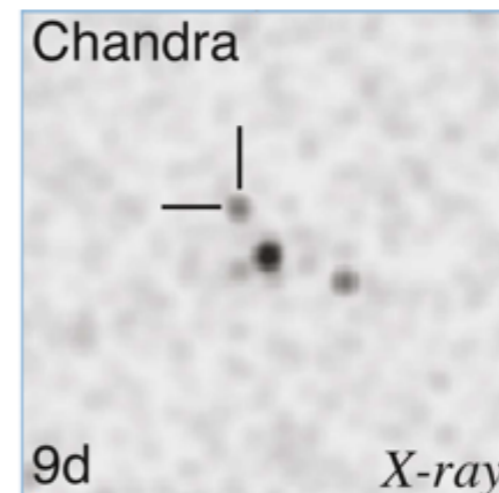
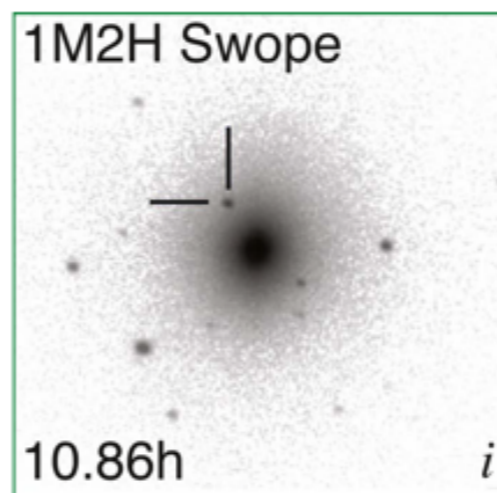
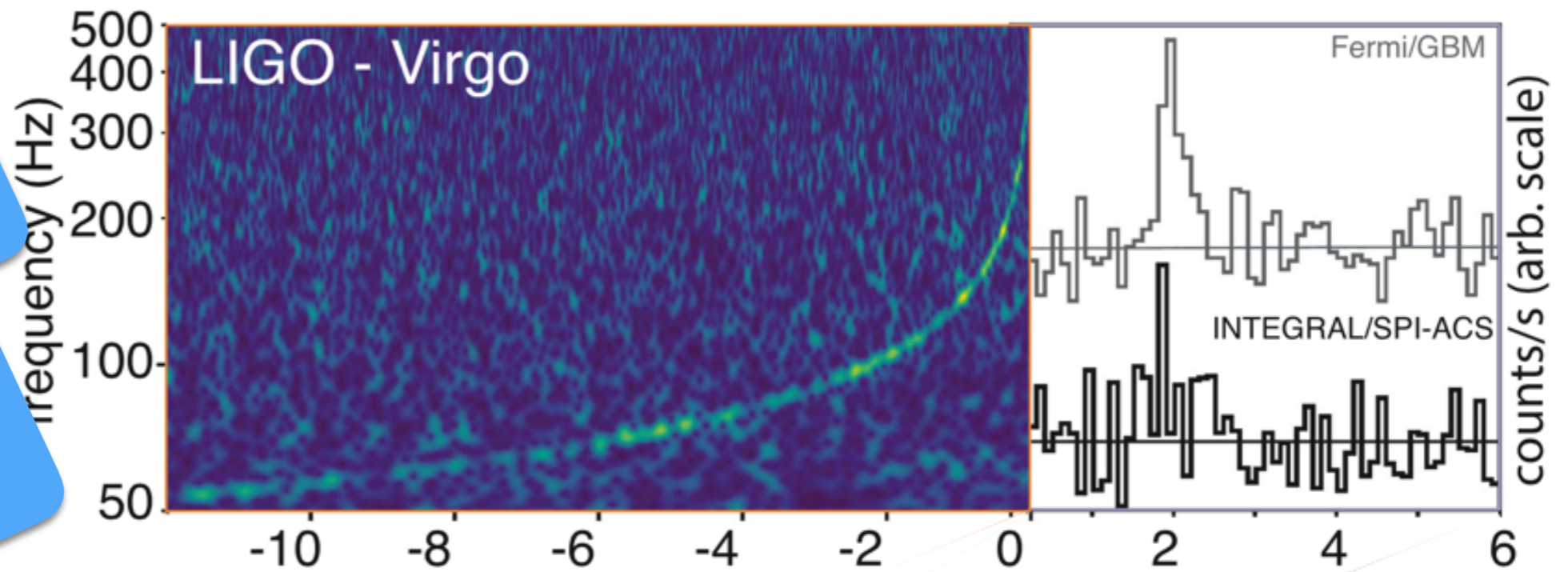


# First event of Multi-messenger Astronomy

GW170817

GRB170817A

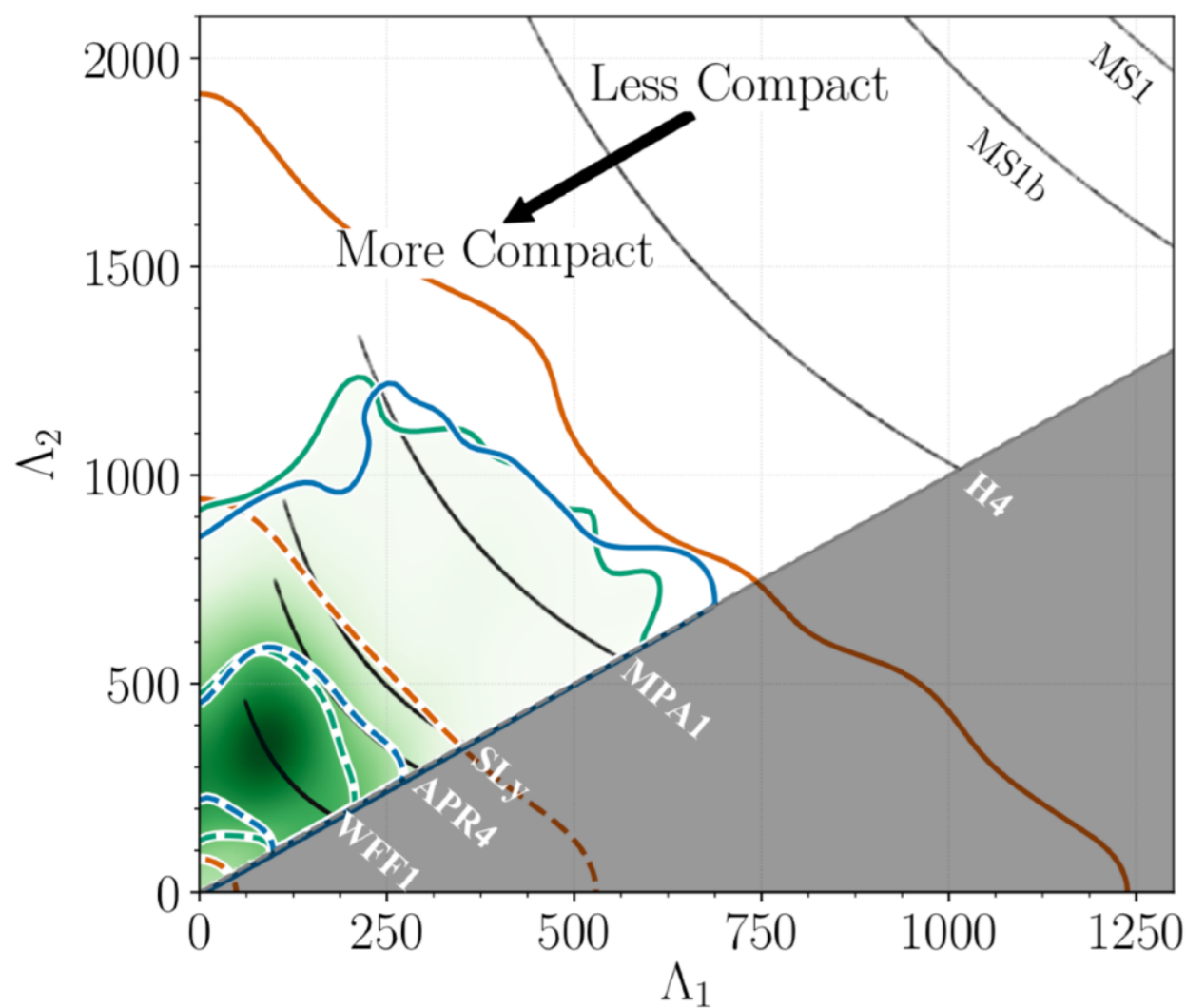
SSS17al  
AT2017gfo



ApJL.848.L12(2017)

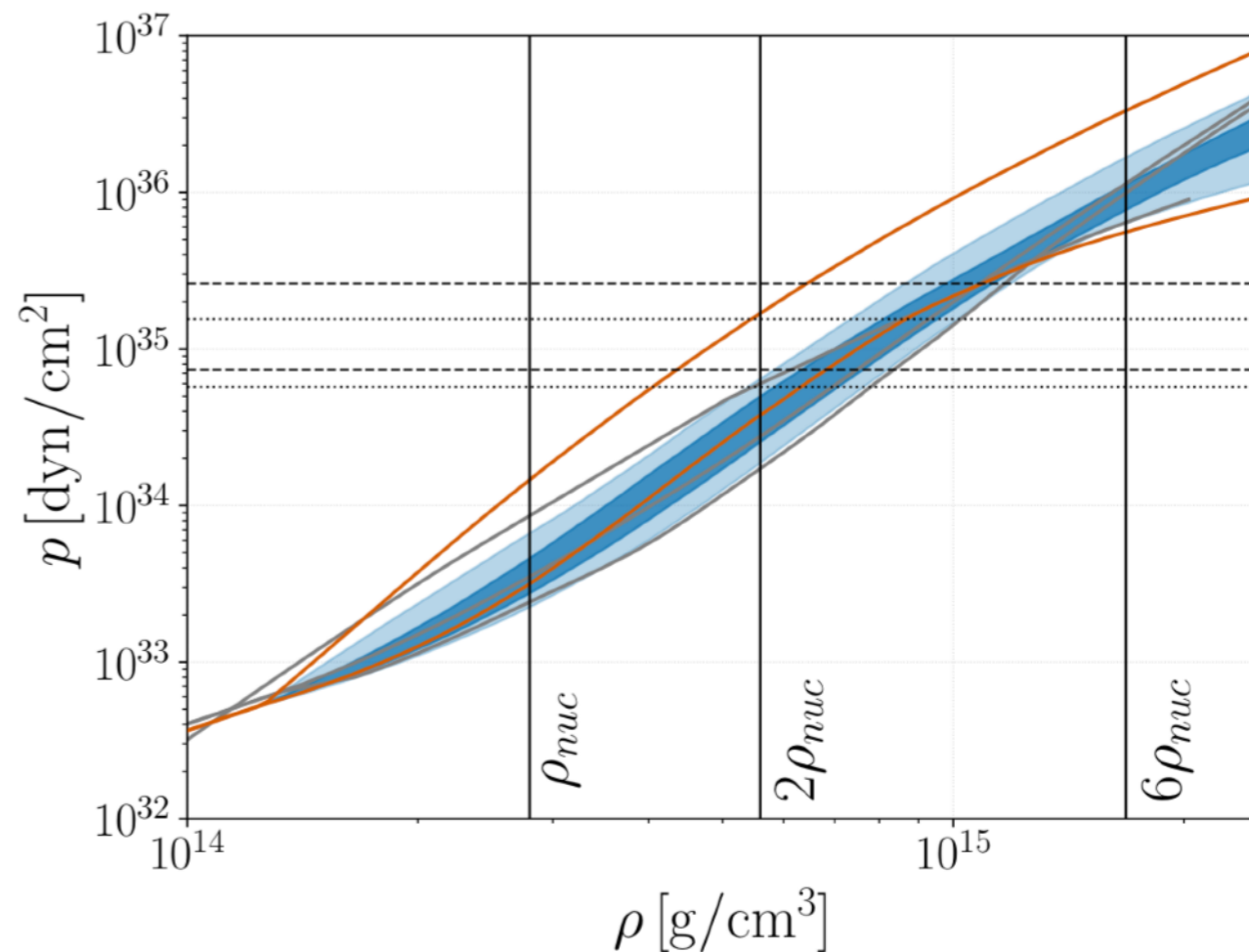
# Measurements of $\Lambda$ in GW170817

$$\Lambda(1.4M_{\odot}) = 190^{+390}_{-120}$$



$$P(2 \rho_{\text{nuc}}) = 3.5^{+2.7}_{-1.7} \times 10^{34} \text{ dyne/cm}^2$$

$$P(6 \rho_{\text{nuc}}) = 9.0^{+7.9}_{-2.6} \times 10^{35} \text{ dyne/cm}^2$$

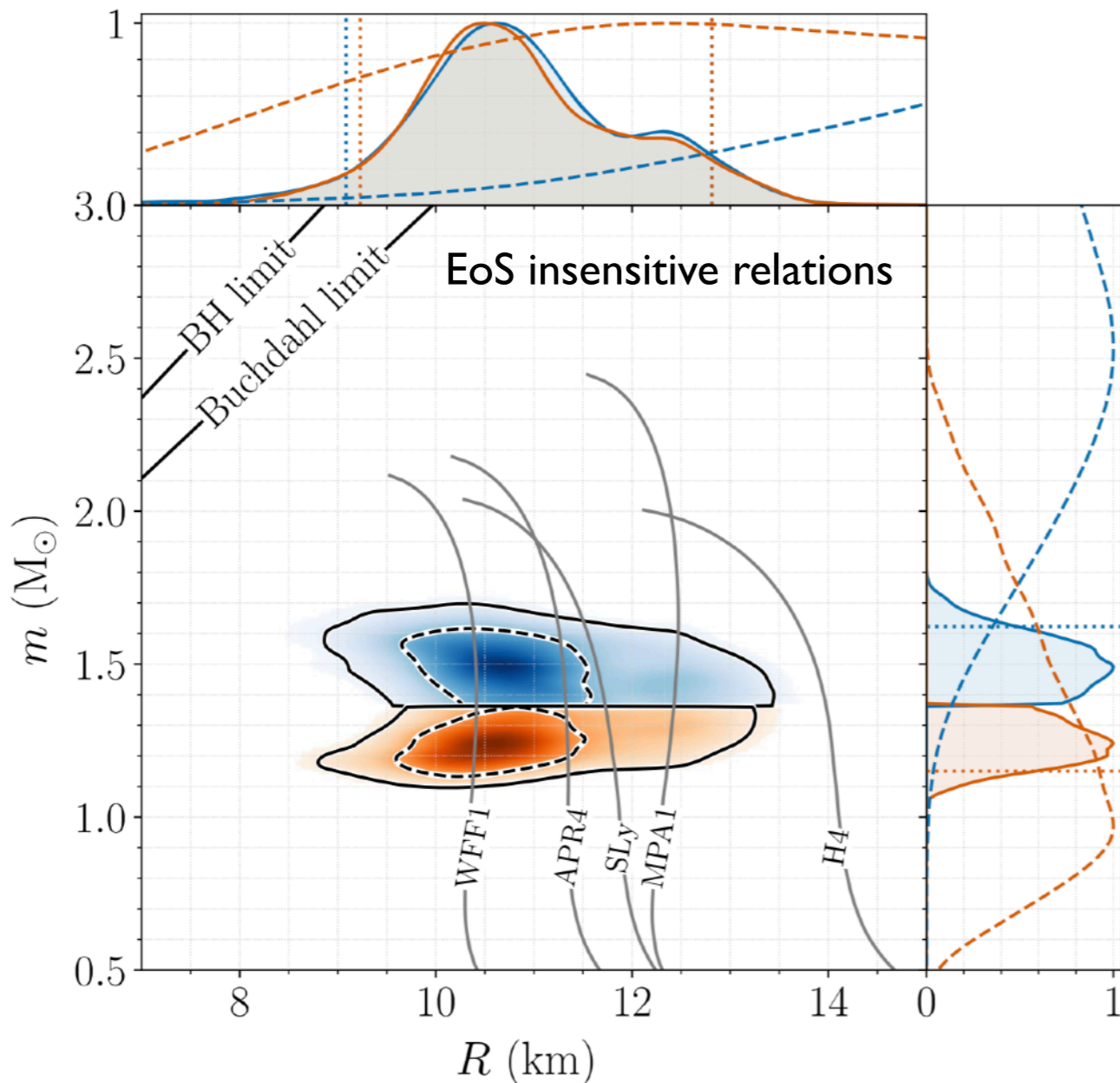


Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PhysRevLett.121.161101)

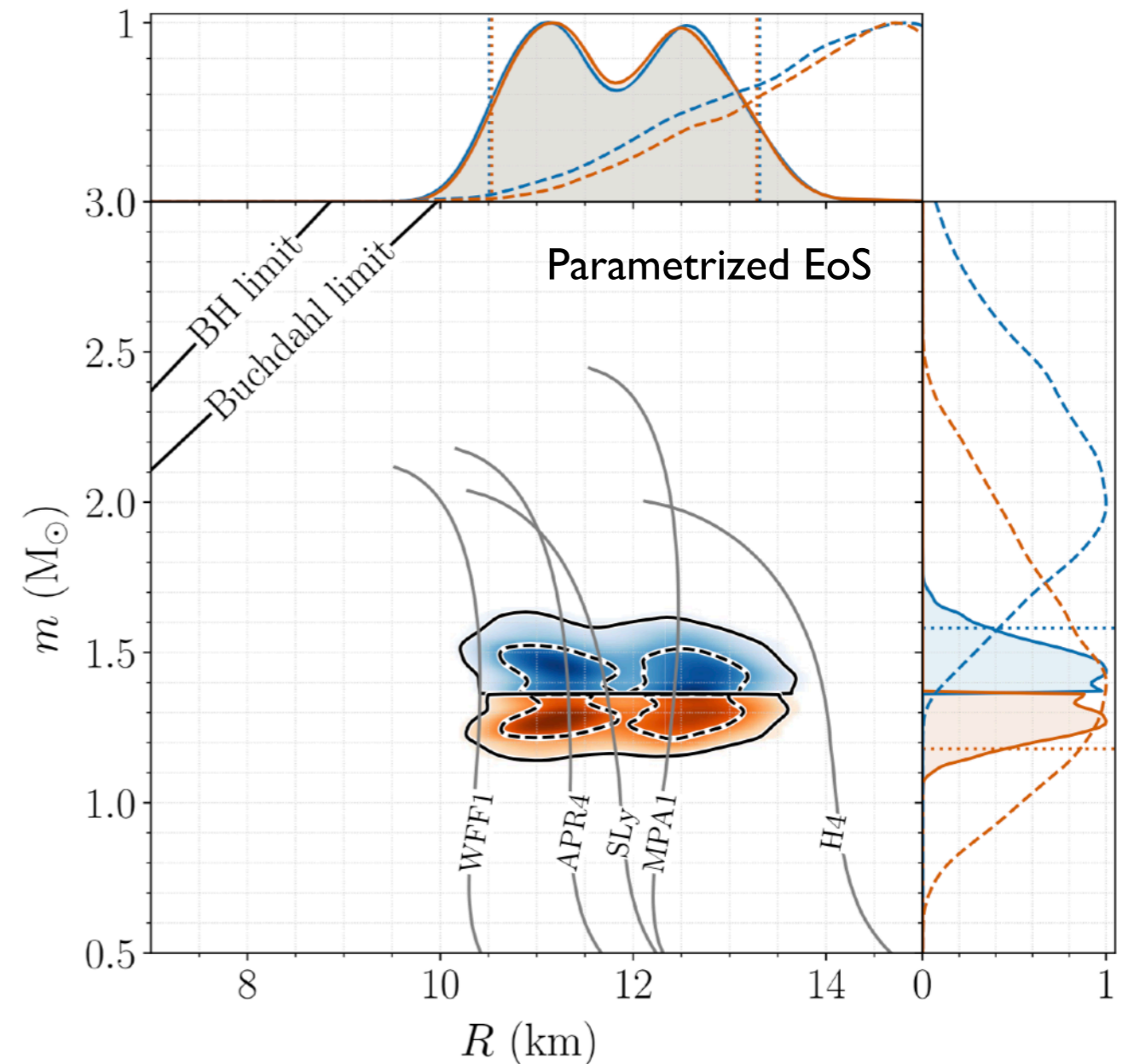
$$\rho_{\text{nuc}} = 2.8 \times 10^{14} \text{ g/cm}^3$$

# Measurements of Radii in GW170817

Abbott et al. (LSC and Virgo), PhysRevLett.121.161101



$M1 = (1.36, 1.62) M_{\text{sun}}, R1 = 10.8^{+2.0}_{-1.7} \text{ km}$   
 $M2 = (1.15, 1.36) M_{\text{sun}}, R2 = 10.7^{+2.1}_{-1.5} \text{ km}$   
 @90% CL



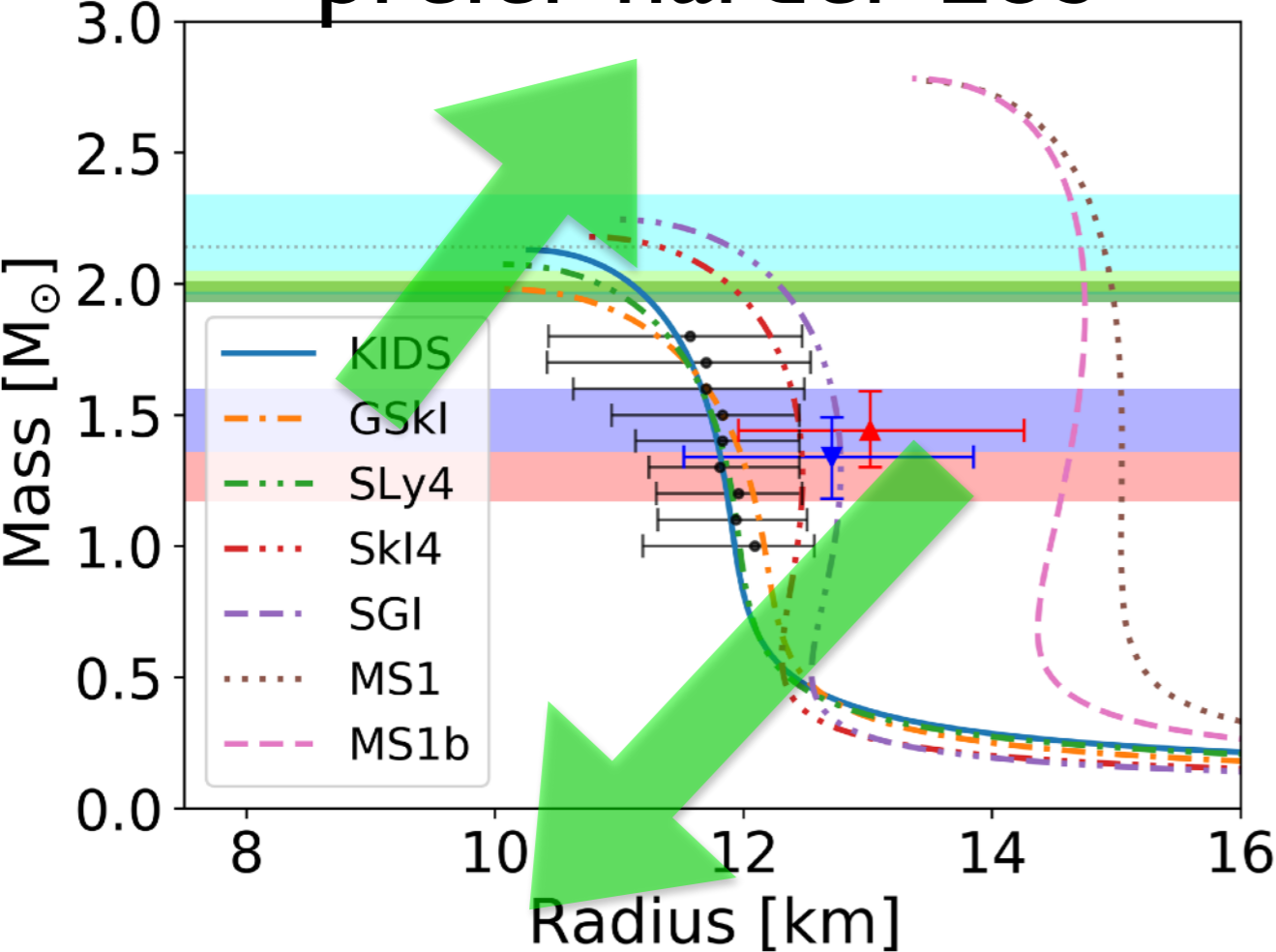
$M1 = (1.36, 1.58) M_{\text{sun}}, R1 = 11.9^{+1.4}_{-1.4} \text{ km}$   
 $M2 = (1.18, 1.36) M_{\text{sun}}, R2 = 11.9^{+1.4}_{-1.4} \text{ km}$   
 @90% CL



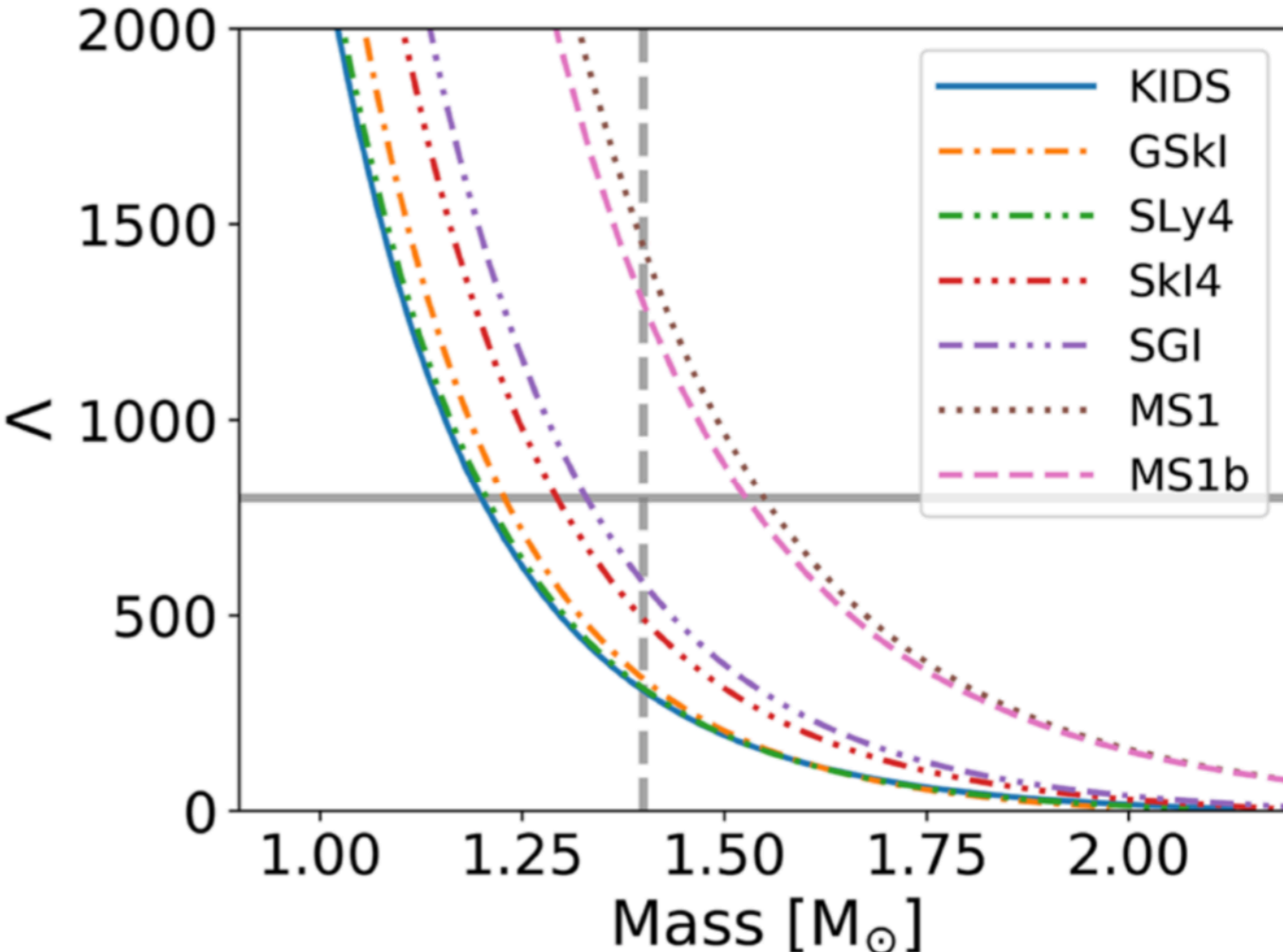
# Tidal deformability for various EoSs

Y.-M. Kim et al. (EPJA 2020)

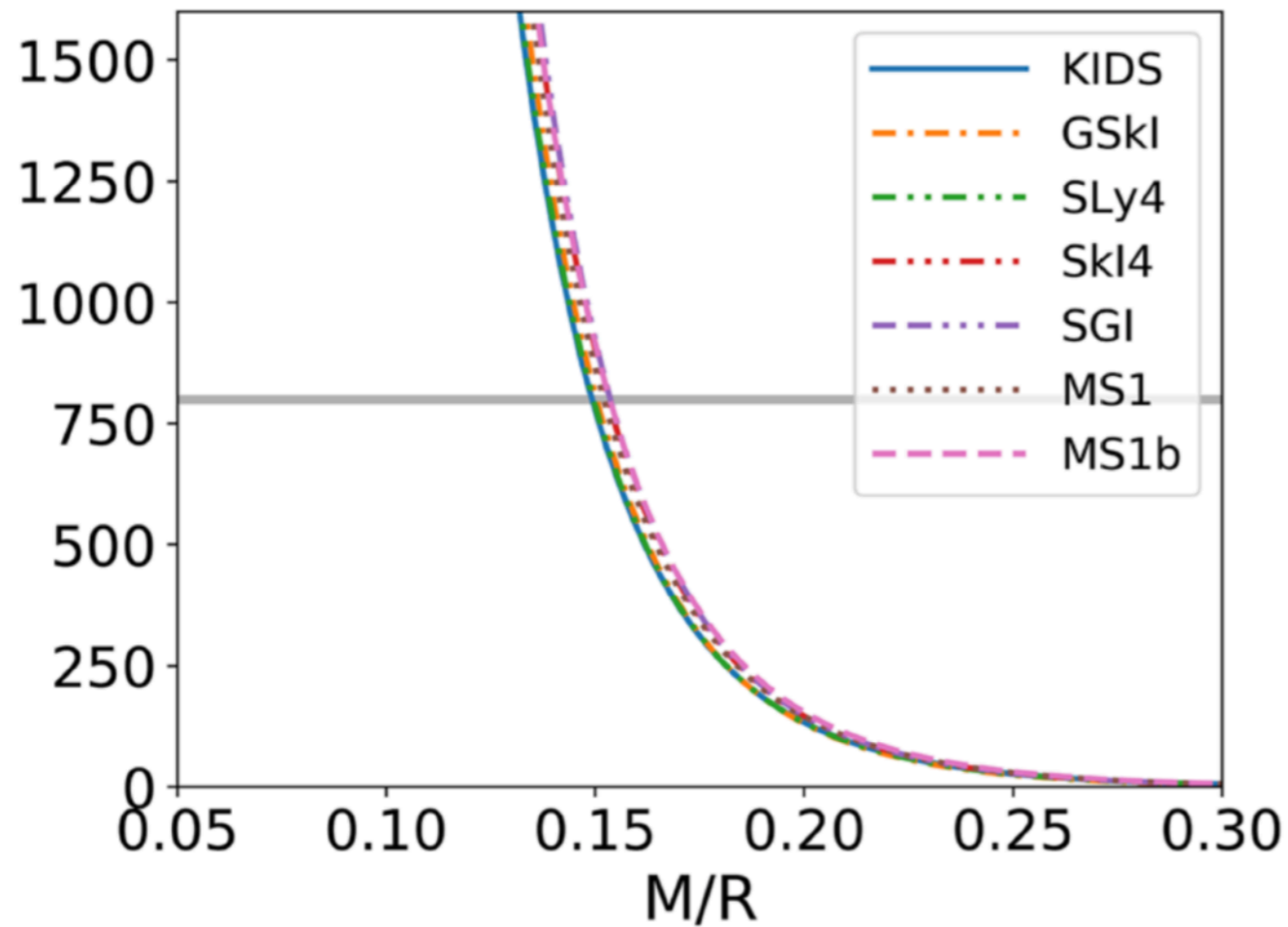
prefer harder EoS



prefer soft EoS



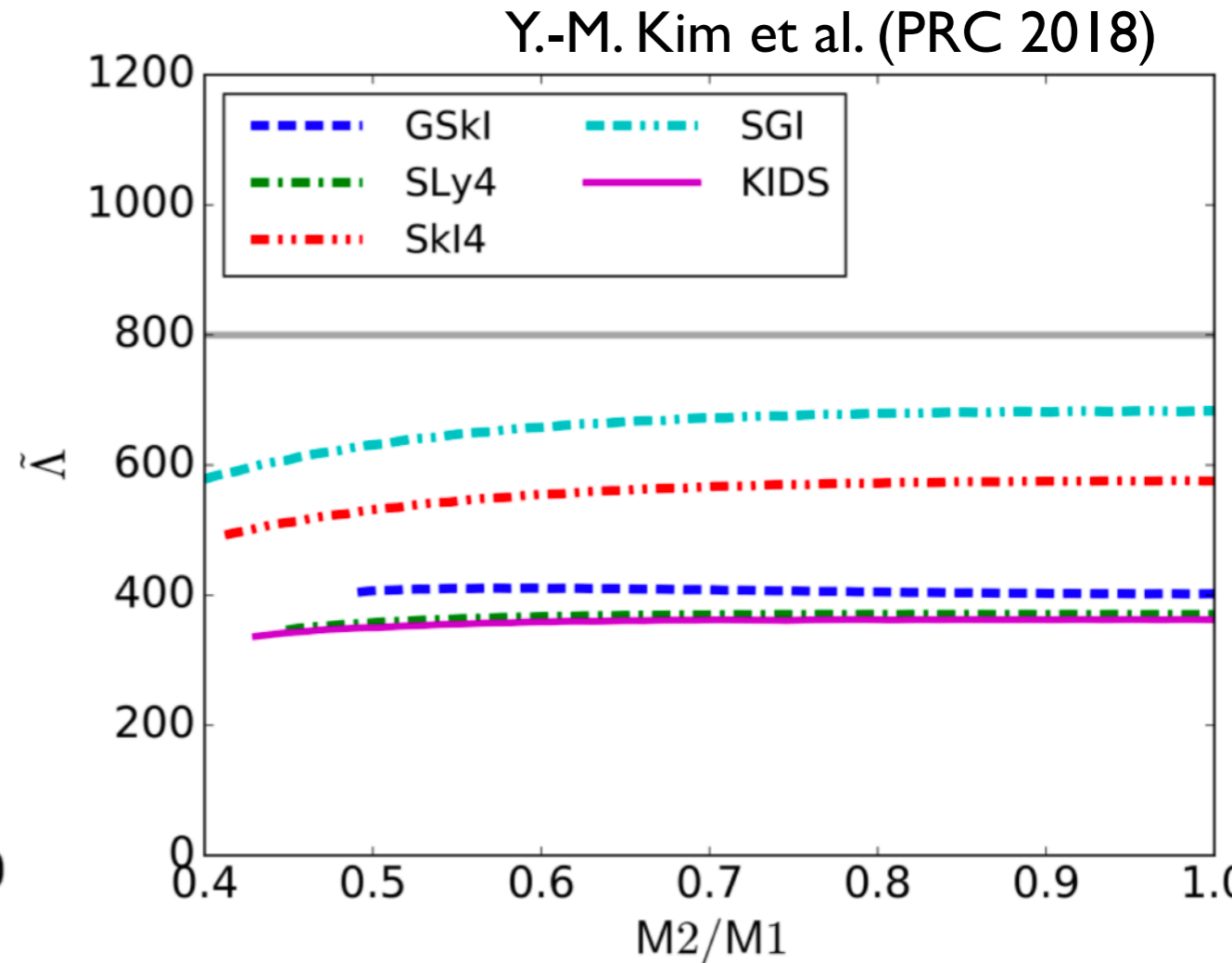
# Universal relations for $\Lambda$



Inensitive to EoS

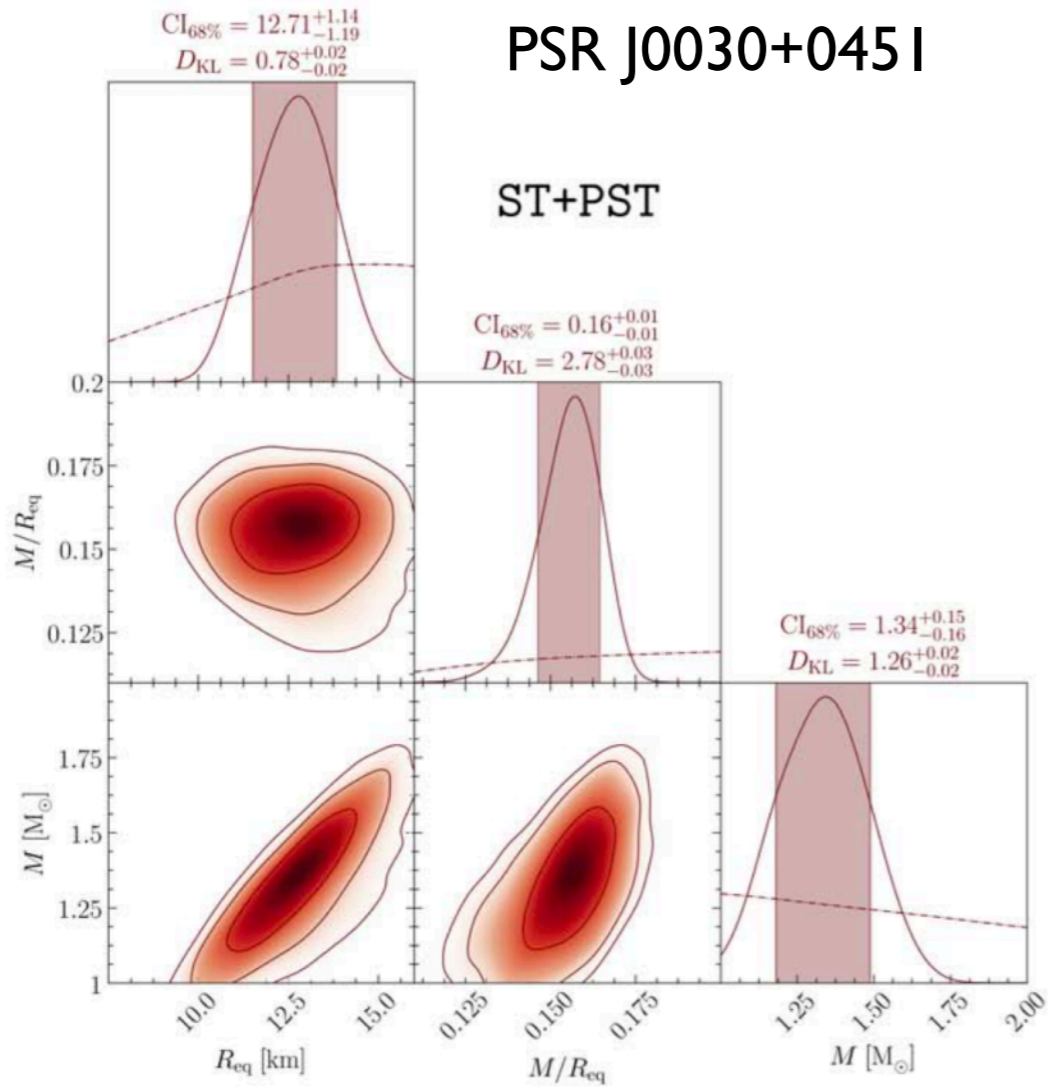
$$C = a_0 + a_1(\ln\Lambda) + a_2(\ln\Lambda)^2$$

K. Yagi and N. Yunes, Phys. Rep. 681 (2017) I



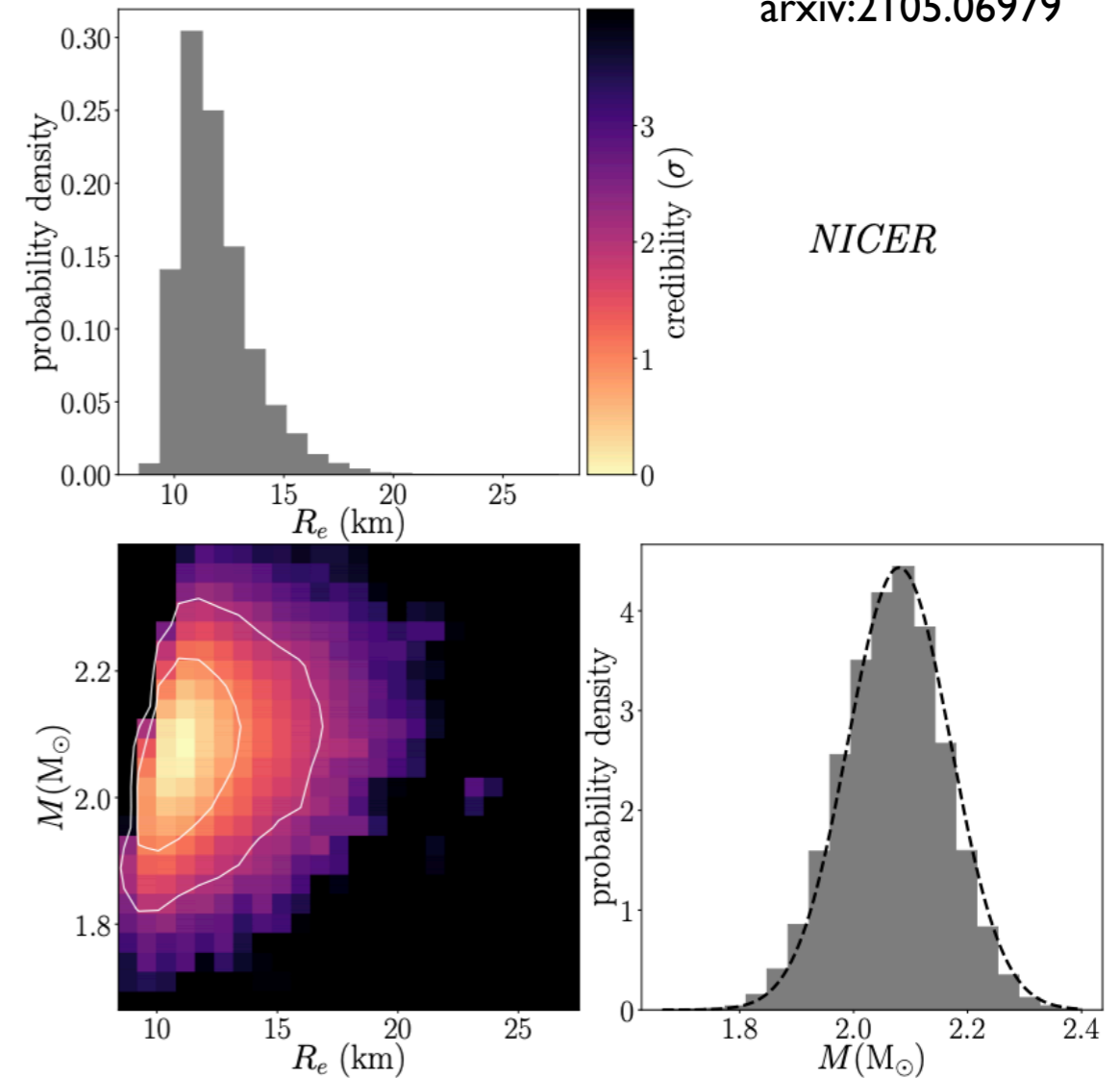
Inensitive to mass-ratio

# NICER observations



## PSR J0740+6620

arxiv:2105.06979



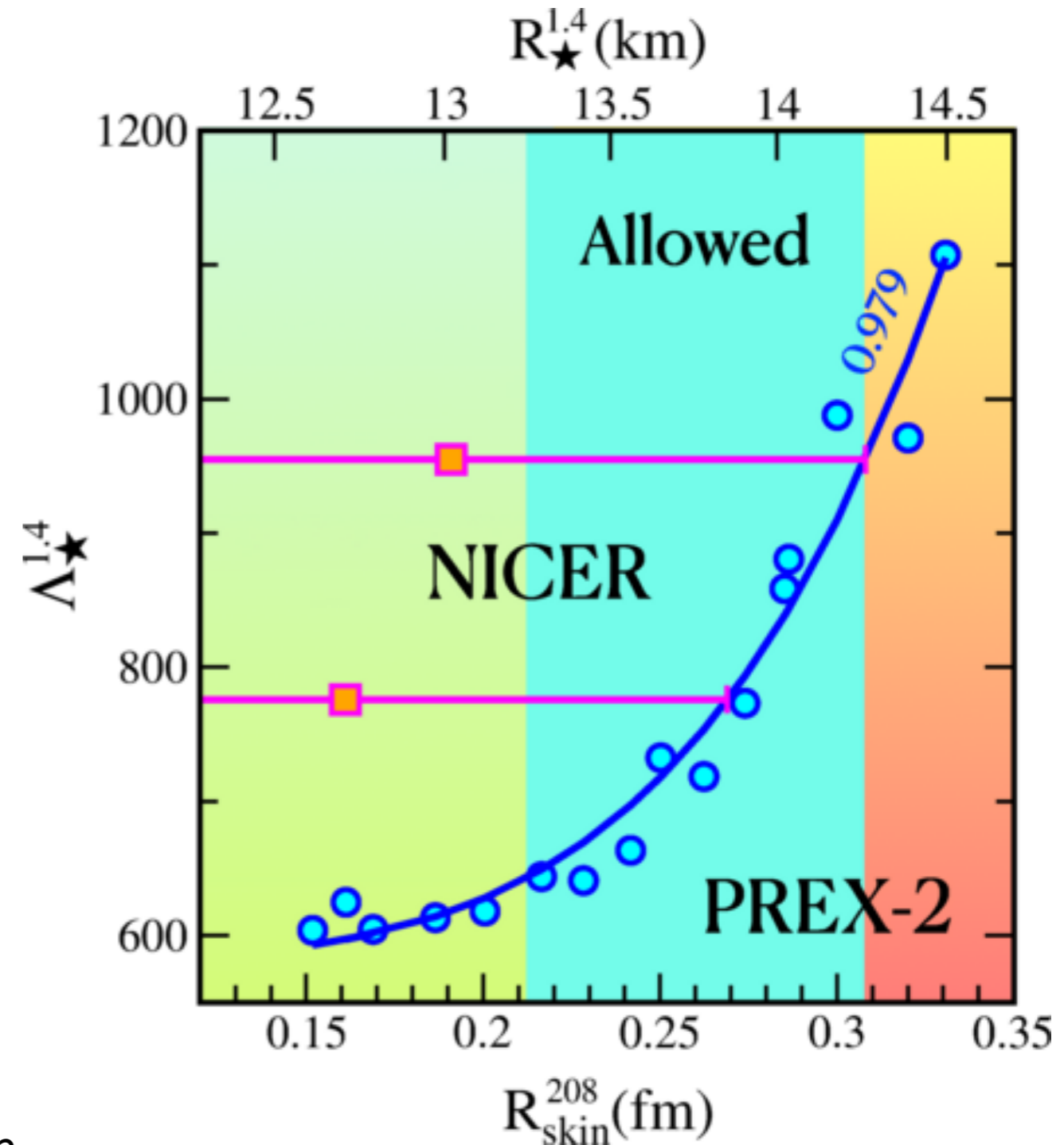
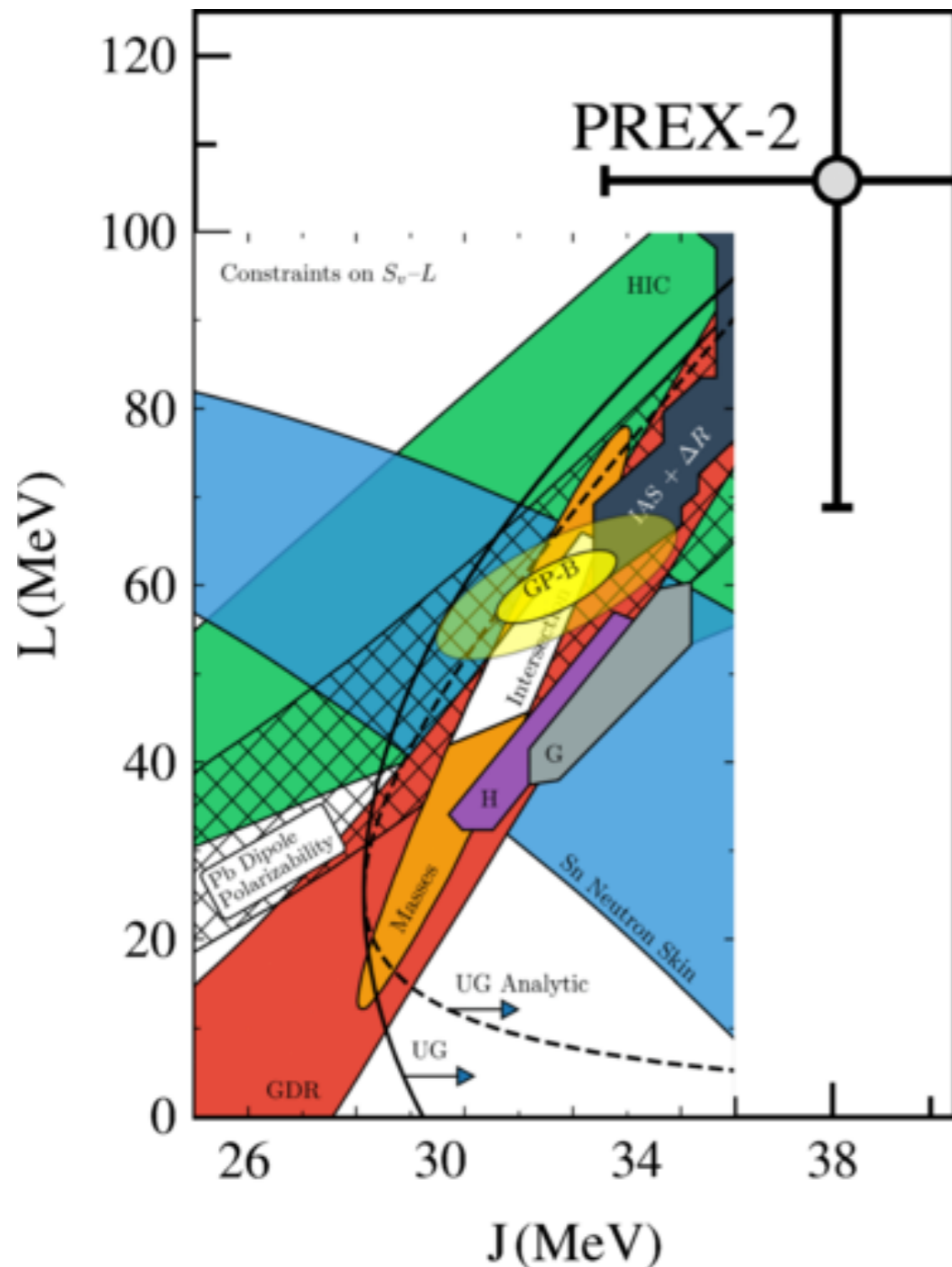
Riley\_2019\_ApJL\_887\_L21

	Mass ( $M_{sun}$ )	Radius (km)
Riley <i>et al.</i> <sup>[25]</sup>	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$
Miller <i>et al.</i> <sup>[26]</sup>	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$

# Implication of PREX 2 and NICER Obs.

PREX 2 - PhysRevLett.126.172502 (2021) Neutron skin thickness,  $R_n - R_p = 0.278 \pm 0.078$  (exp.)  $\pm 0.012$  (theo.) fm

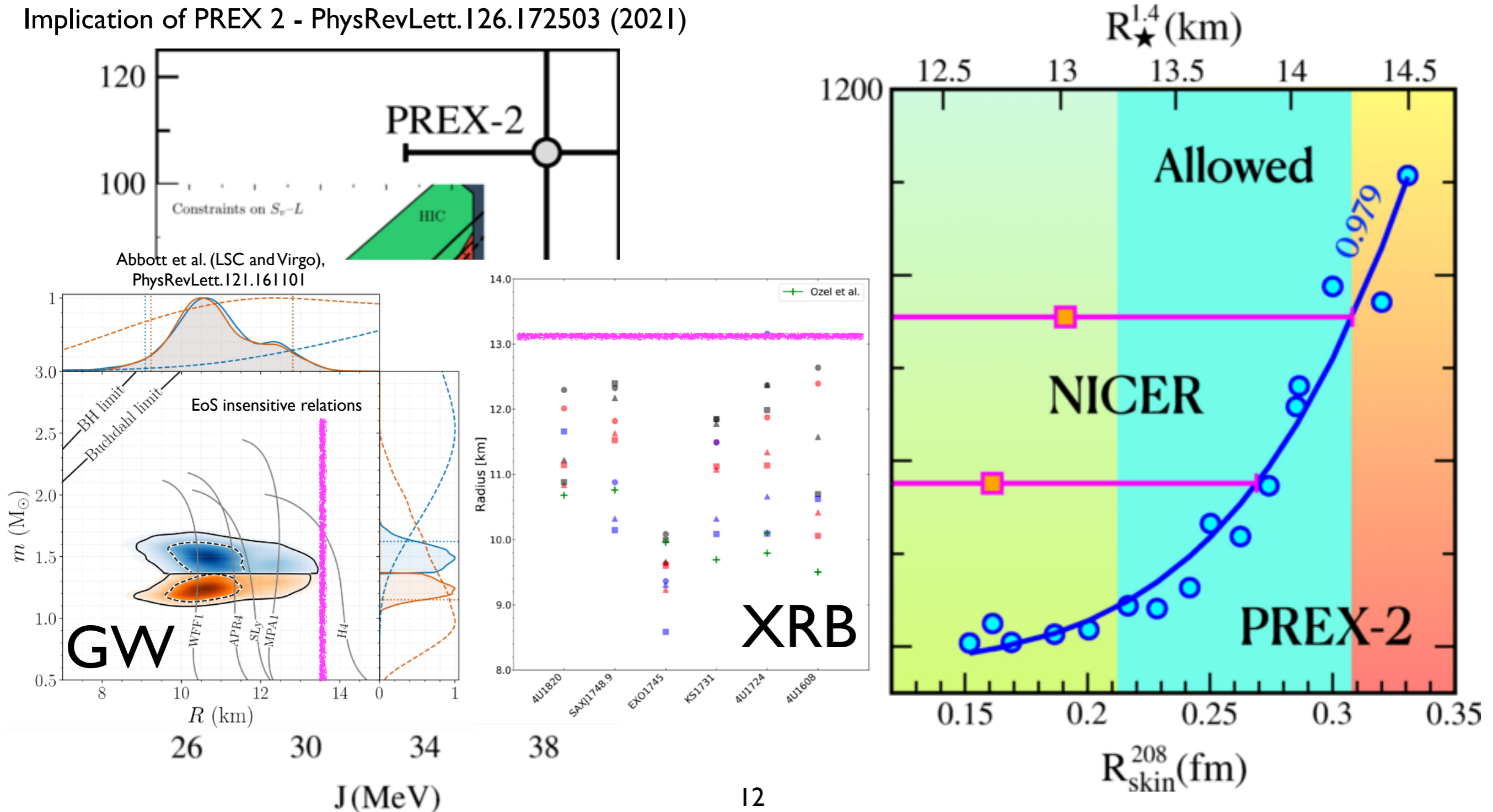
Implication of PREX 2 - PhysRevLett.126.172503 (2021)



# Implication of PREX 2 and NICER Obs.

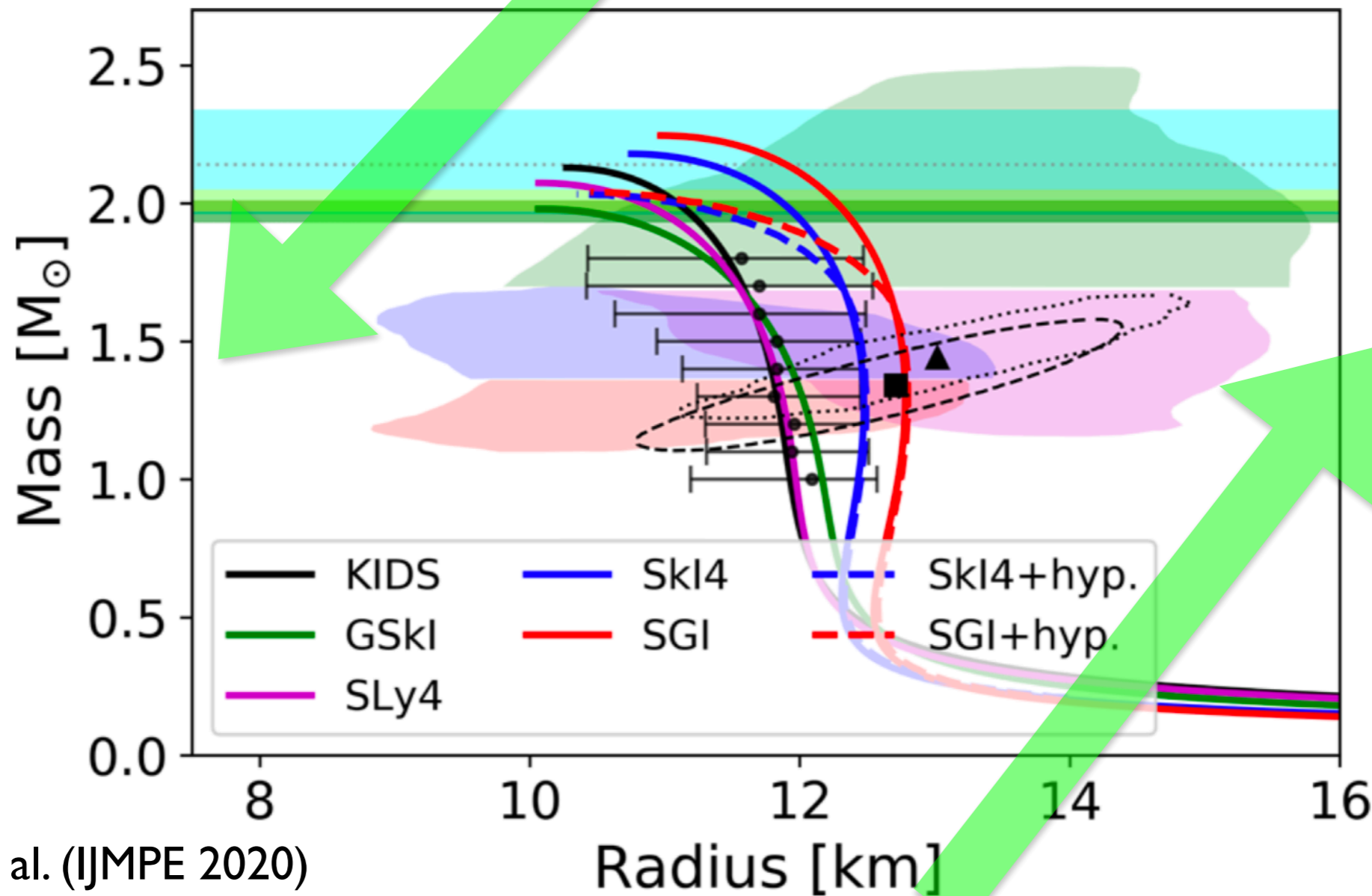
PREX 2 - PhysRevLett.126.172502 (2021) Neutron skin thickness,  $R_n - R_p = 0.278 \pm 0.078$  (exp.)  $\pm 0.012$  (theo.) fm

Implication of PREX 2 - PhysRevLett.126.172503 (2021)



# Constraints on Equation of State

prefer soft EoS: GW170817, strangeness



M. Kim et al. (IJMPE 2020)

prefer harder EoS:  $M_{\text{max}}$ , NICER, PREX

# Study on EoS w/ Bayesian in MMA

EoSs  
(model-  
dependent  
parameters)



NS properties  
( $M, R, \Lambda$ , etc.)  
Finite Nuclei  
(B.E.,  $R_{ch}$ ,  $\Delta R_{np}$ ,  
etc)

Sampling

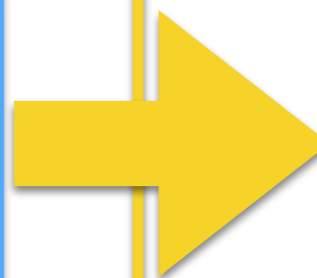
Likelihood

MCMC  
or

Nested Sampling

Obs. Data  
(GW, X-ray,  
etc.)

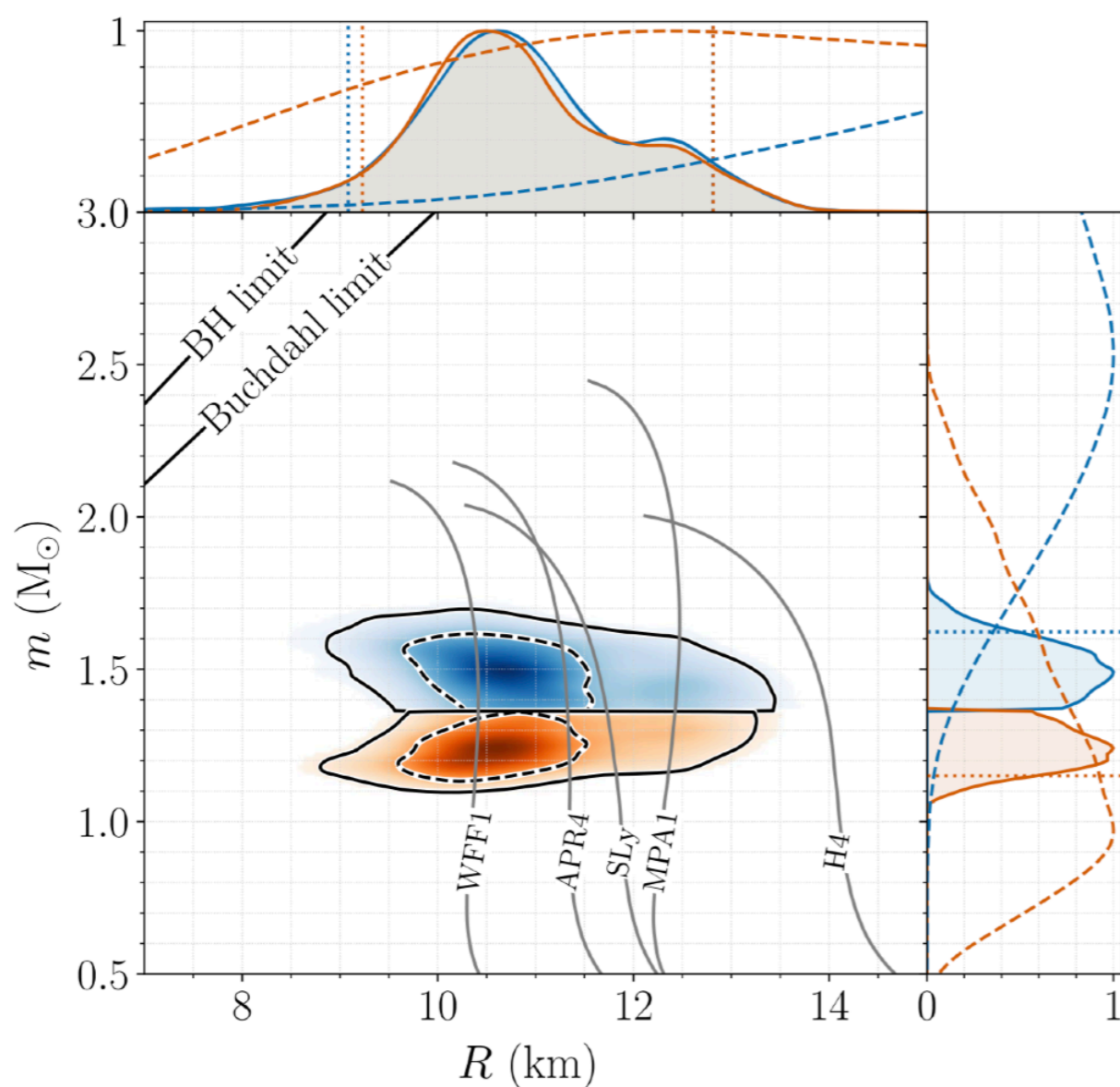
Exp. Data  
(Raon, DJBUU)



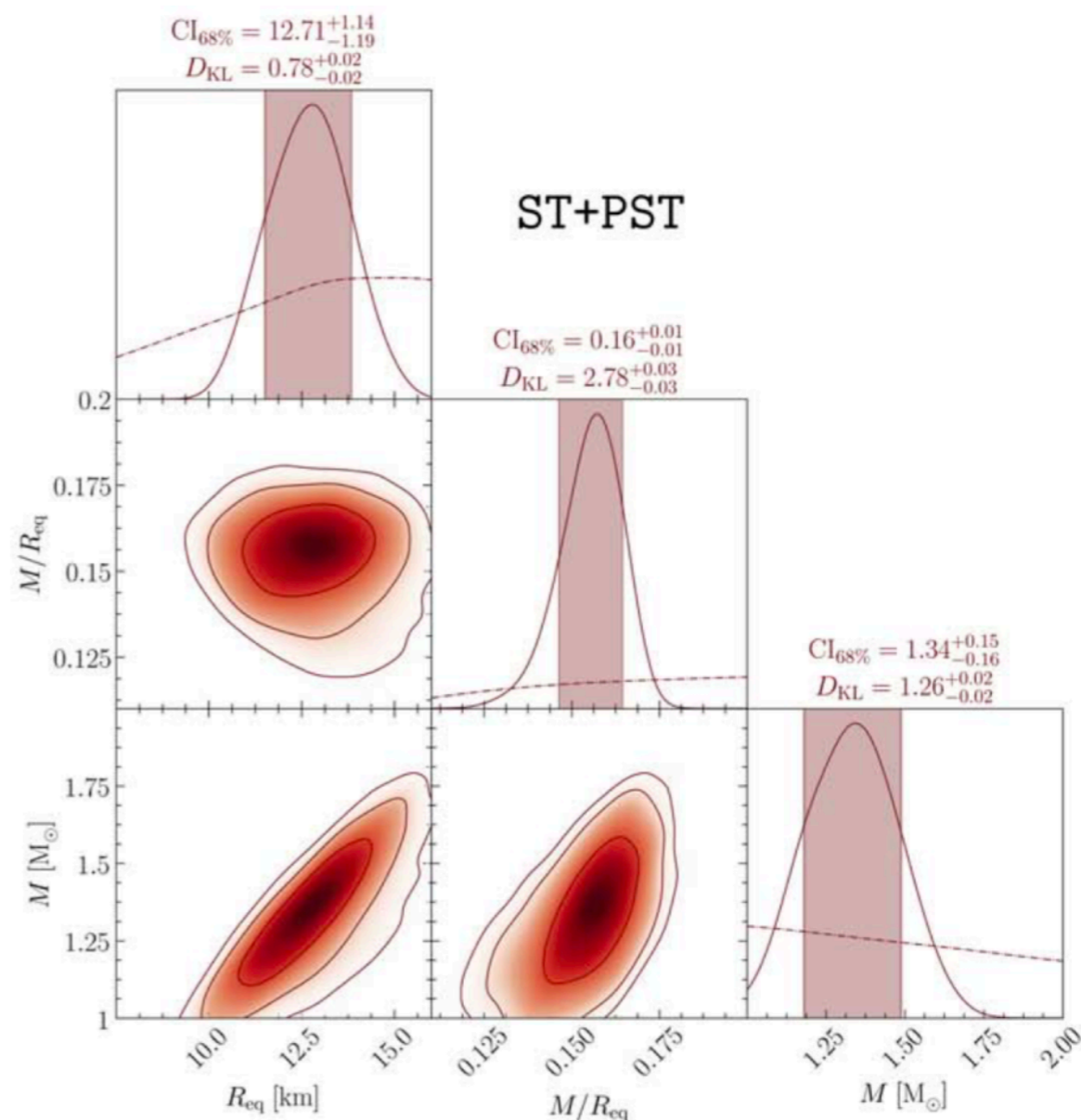
Posteriors :  
 $M, R, \Lambda$ , etc.  
 $L, K_{sym}, Q_{sym}$ , etc.

# GW170817 vs. PSR J0030+0451

GW170817 (EoS-insensitive)



PSR J0030+0451



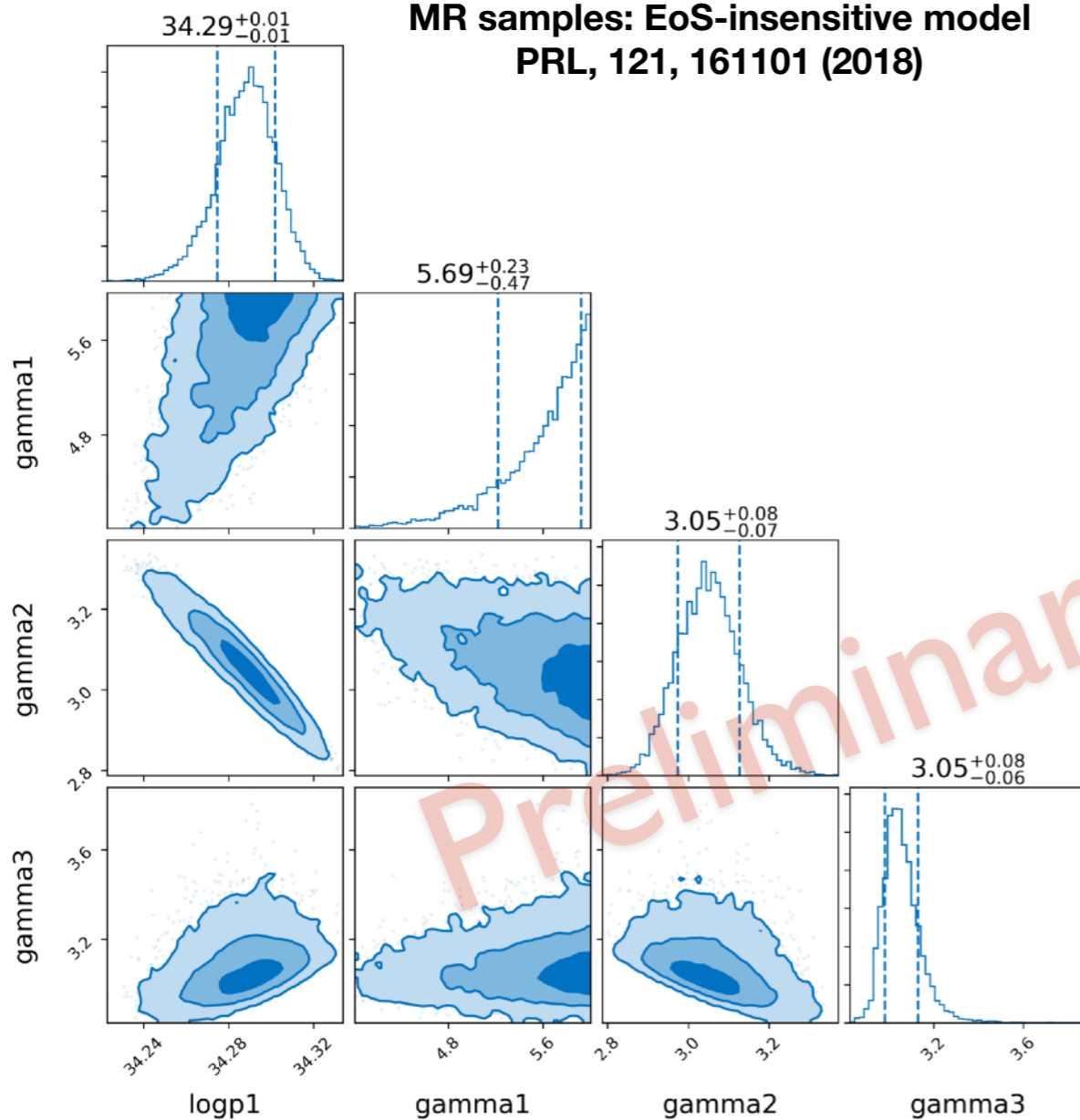


# Posteriors w/ Piece-wise Polyotropic EoSs

Reference for Piece-wise Polyotropic EoSs : Read et al. PRD 79, 124032 (2009)

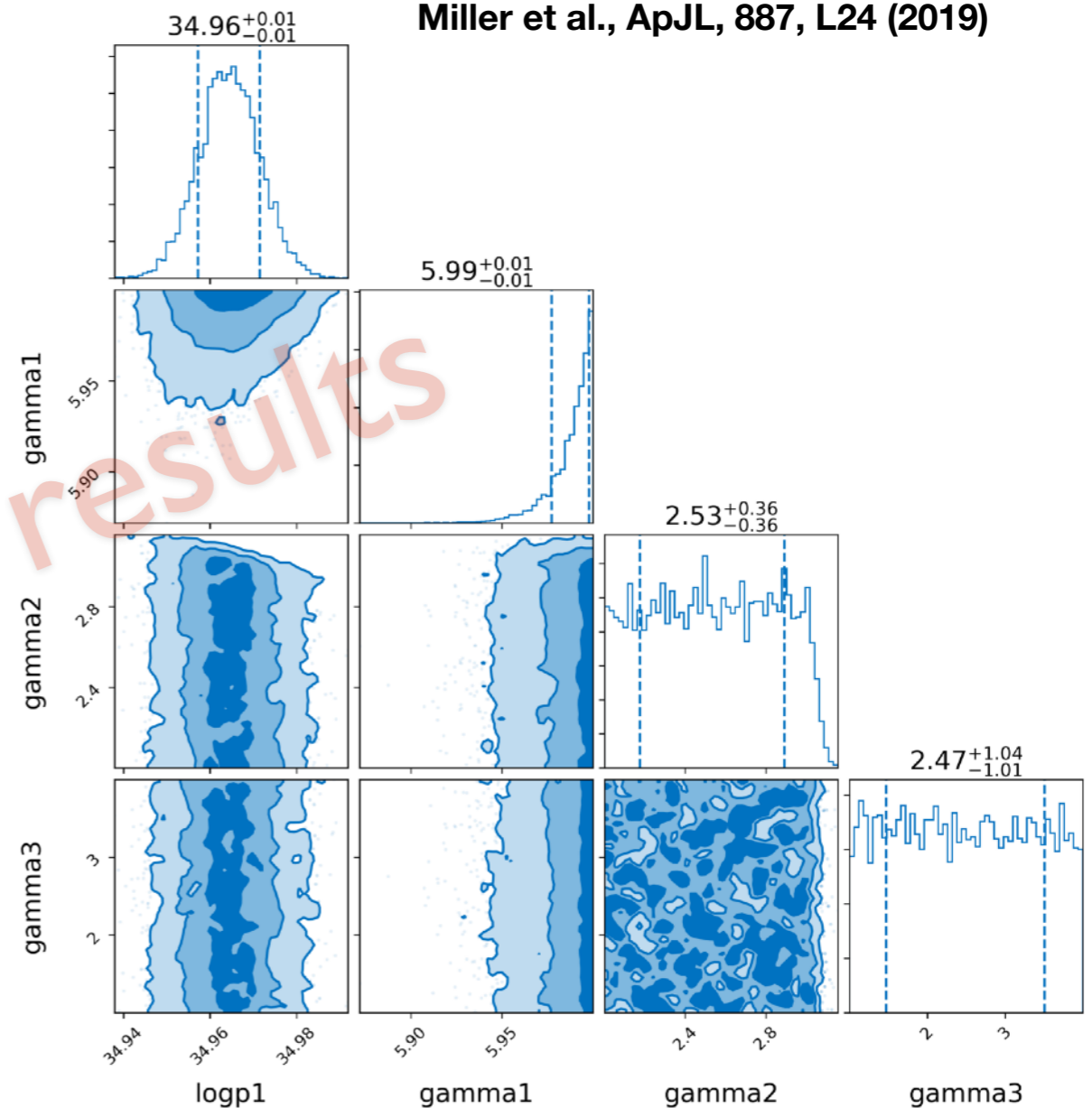
## GW170817

MR samples: EoS-insensitive model  
PRL, 121, 161101 (2018)



## PSR J0030+0451

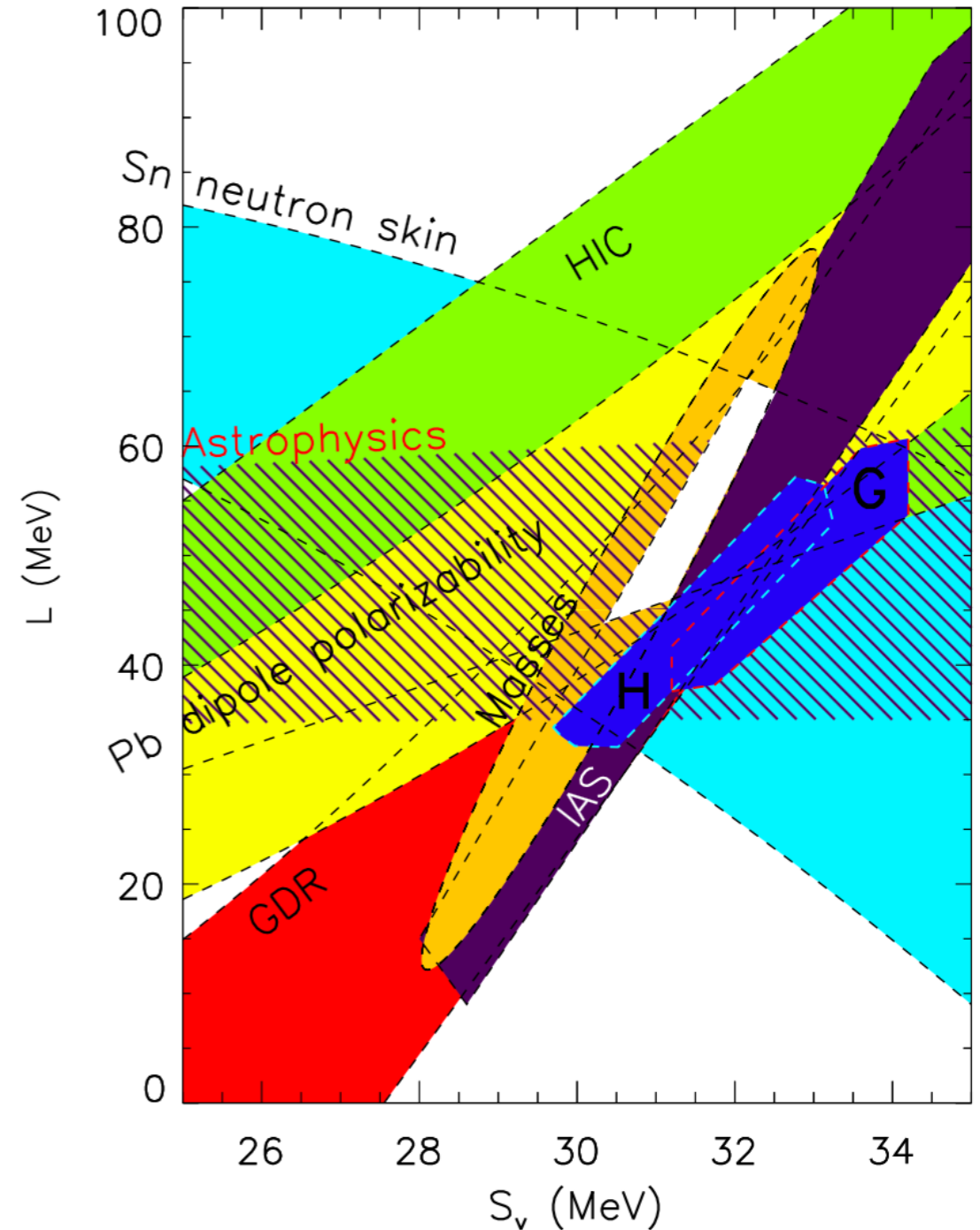
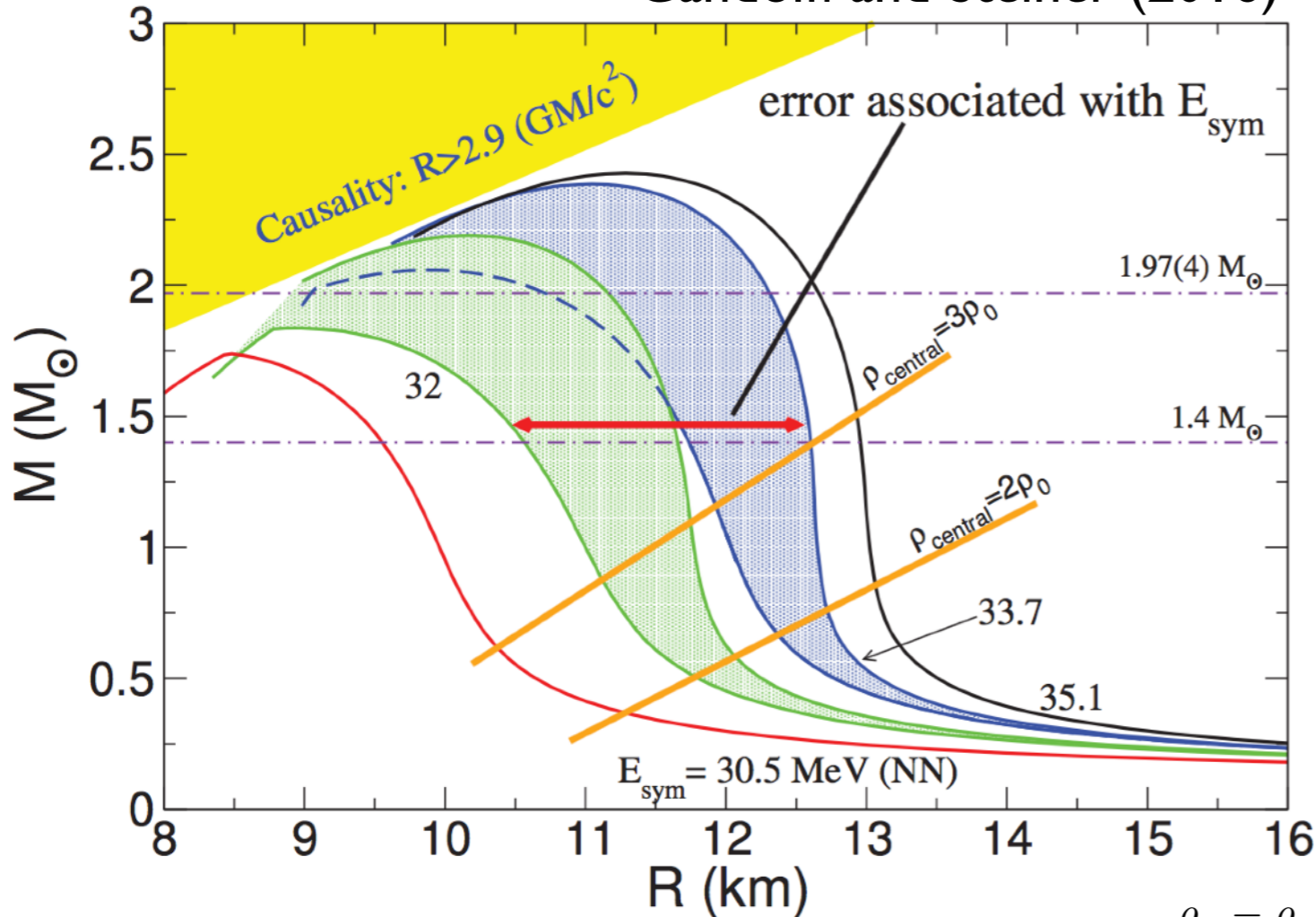
MR samples: 2 spot model  
Miller et al., ApJL, 887, L24 (2019)



# Nuclear Symmetry Energy

*J.M. Lattimer / Nuclear Physics A 928 (2014) 276–295*

Gandolfi and Steiner (2016)



$$\frac{E}{A} = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) + \dots,$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{sym}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2$$

$$L = 3\rho_0 \frac{\partial E_{sym}(\rho)}{\partial \rho} \Big|_{\rho=\rho_0} \quad K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}(\rho)}{\partial \rho^2} \Big|_{\rho=\rho_0}.$$

# KIDS Energy Density Functional

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KIDS Energy density functional form

Phys. Rev. C 97, 014312 (2018)

$$\mathcal{E}(\rho, \delta) = \frac{E(\rho, \delta)}{A} = \mathcal{T}(\rho, \delta) + \sum_{i=0}^{N-1} c_i(\delta) \rho^{1+i/3} \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

$$\mathcal{T}(\rho, \delta) = \frac{3}{5} \left[ \frac{\hbar^2}{2m_p} \left( \frac{1-\delta}{2} \right)^{5/3} + \frac{\hbar^2}{2m_n} \left( \frac{1+\delta}{2} \right)^{5/3} \right] (3\pi^2 \rho)^{2/3}$$

$c_i(\delta) = \alpha_i + \beta_i \delta^2$  to be determined by fitting to the observables

at zero temperature  $k_F = (3\pi^2 \rho/2)^{1/3}$   $k_{F_\tau} = k_F (1 + \tau\delta)^{1/3}$

# KIDS Parametrization

$$x = (\rho - \rho_0)/(3\rho_0)$$

$$S(\rho) = \frac{1}{2} \frac{\partial^2}{\partial \delta^2} \mathcal{E}(\rho, \delta) \Big|_{\delta=0} = \mathcal{T}_{\text{sym}}(\rho) + \sum_{i=0}^{N-1} \beta_i \rho^{1+i/3}$$

$$\mathcal{E}(\rho, 0) = E_0 + \frac{1}{2} K_0 x^2 + \frac{1}{6} Q_0 x^3 + O(x^4),$$

$$S(\rho) = J + Lx + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{6} Q_{\text{sym}} x^3 + \frac{1}{24} R_{\text{sym}} x^4 + O(x^5),$$

$$K_T = K_{\text{sym}} - (6 + Q_0/K_0)L$$

$$\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \rho_0 & \rho_0^{4/3} & \rho_0^{5/3} \\ 0 & 4\rho_0^{4/3} & 10\rho_0^{5/3} \\ 0 & -8\rho_0^{4/3} & -10\rho_0^{5/3} \end{pmatrix}^{-1} \begin{pmatrix} E_0 - \mathcal{Z}_0 \\ K_0 + 2\mathcal{Z}_0 \\ Q_0 - 8\mathcal{Z}_0 \end{pmatrix}$$

$\alpha_3 = 0$ ,  $\rho_0 = \text{nuclear saturation density}$   
 $\mathcal{Z}_0 = \text{kinetic } E \text{ at } \rho_0, \rho_n = \rho_p$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \rho_0 & \rho_0^{4/3} & \rho_0^{5/3} & \rho_0^2 \\ 3\rho_0 & 4\rho_0^{4/3} & 5\rho_0^{5/3} & 6\rho_0^2 \\ 0 & 4\rho_0^{4/3} & 10\rho_0^{5/3} & 18\rho_0^2 \\ 0 & -8\rho_0^{4/3} & -10\rho_0^{5/3} & 0 \end{pmatrix}^{-1} \begin{pmatrix} J - \mathcal{Z}_{\text{sym},0} \\ L - 2\mathcal{Z}_{\text{sym},0} \\ K_{\text{sym}} + 2\mathcal{Z}_{\text{sym},0} \\ Q_{\text{sym}} - 8\mathcal{Z}_{\text{sym},0} \end{pmatrix}$$

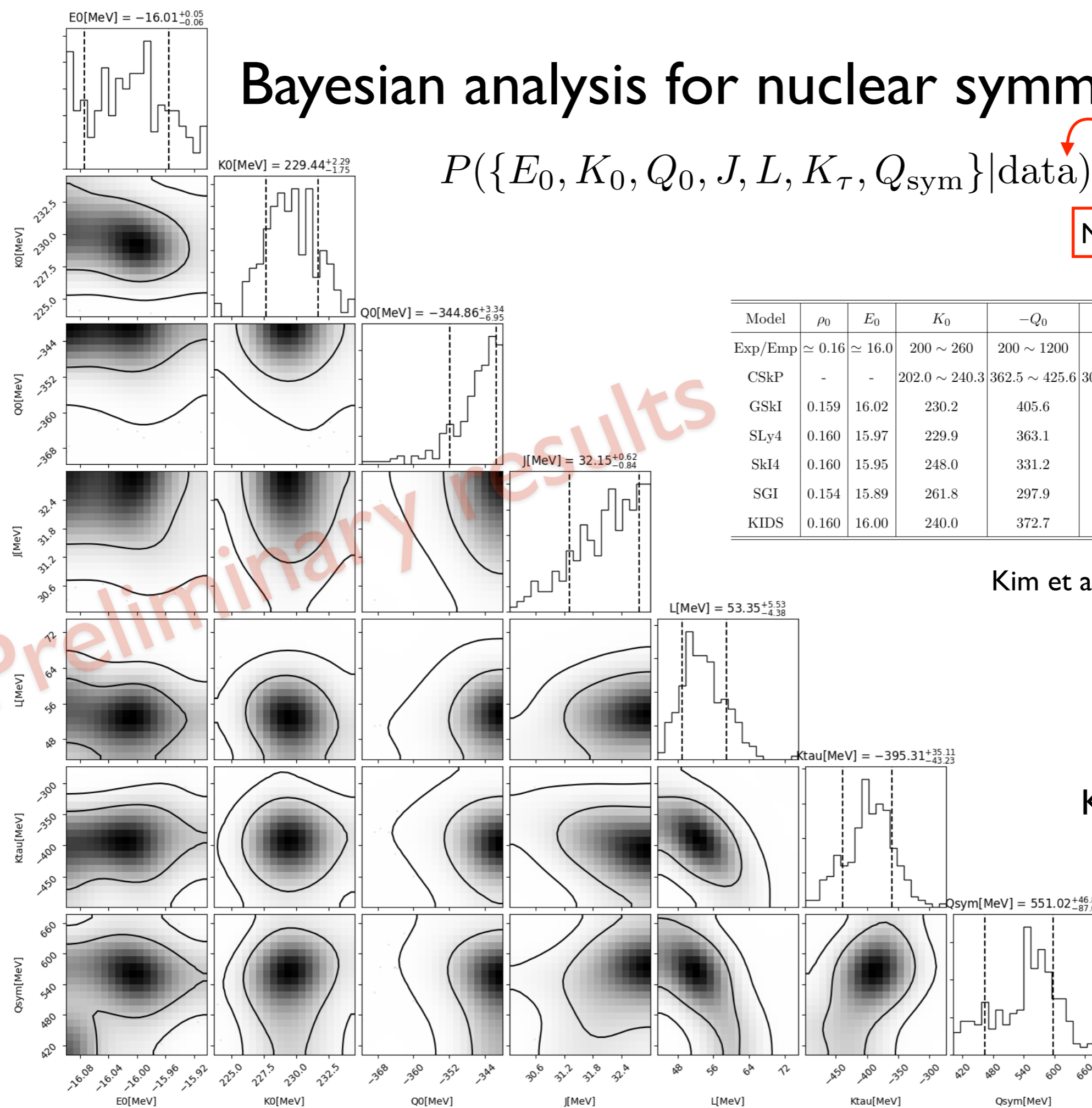
$\mathcal{Z}_{\text{sym},0} : \text{kinetic } E \text{ at } \rho_0, \rho_p = 0$

It works well. - PhysRevC.100.014312

# Bayesian analysis for nuclear symmetry energy

$$P(\{E_0, K_0, Q_0, J, L, K_\tau, Q_{\text{sym}}\} | \text{data})$$

Nuclei properties (or APR)



Model	$\rho_0$	$E_0$	$K_0$	$-Q_0$	$J$	$L$	$-K_\tau$	$M_{\text{max}}$
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	200 ~ 260	200 ~ 1200	30 ~ 35	40 ~ 76	372 ~ 760	$\geq 1.93 \sim 2.05$
CskP	-	-	202.0 ~ 240.3	362.5 ~ 425.6	30.0 ~ 35.5	48.6 ~ 67.1	360.1 ~ 407.1	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

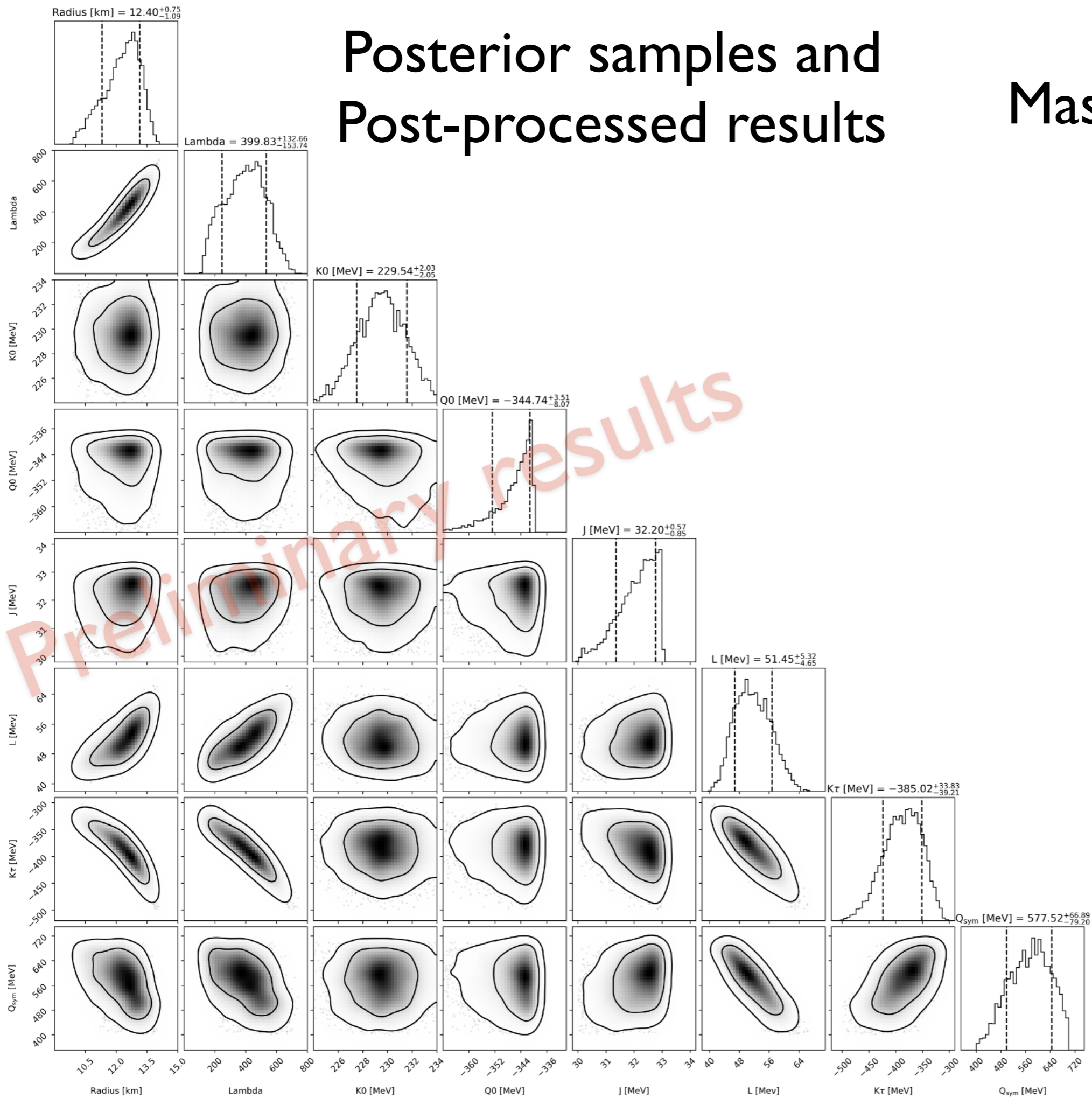
Kim et al., PhysRevC.98.065805

$$K_\tau = K_{\text{sym}} - (6 + Q_0/K_0)L$$

Y.-M. Kim, in progress

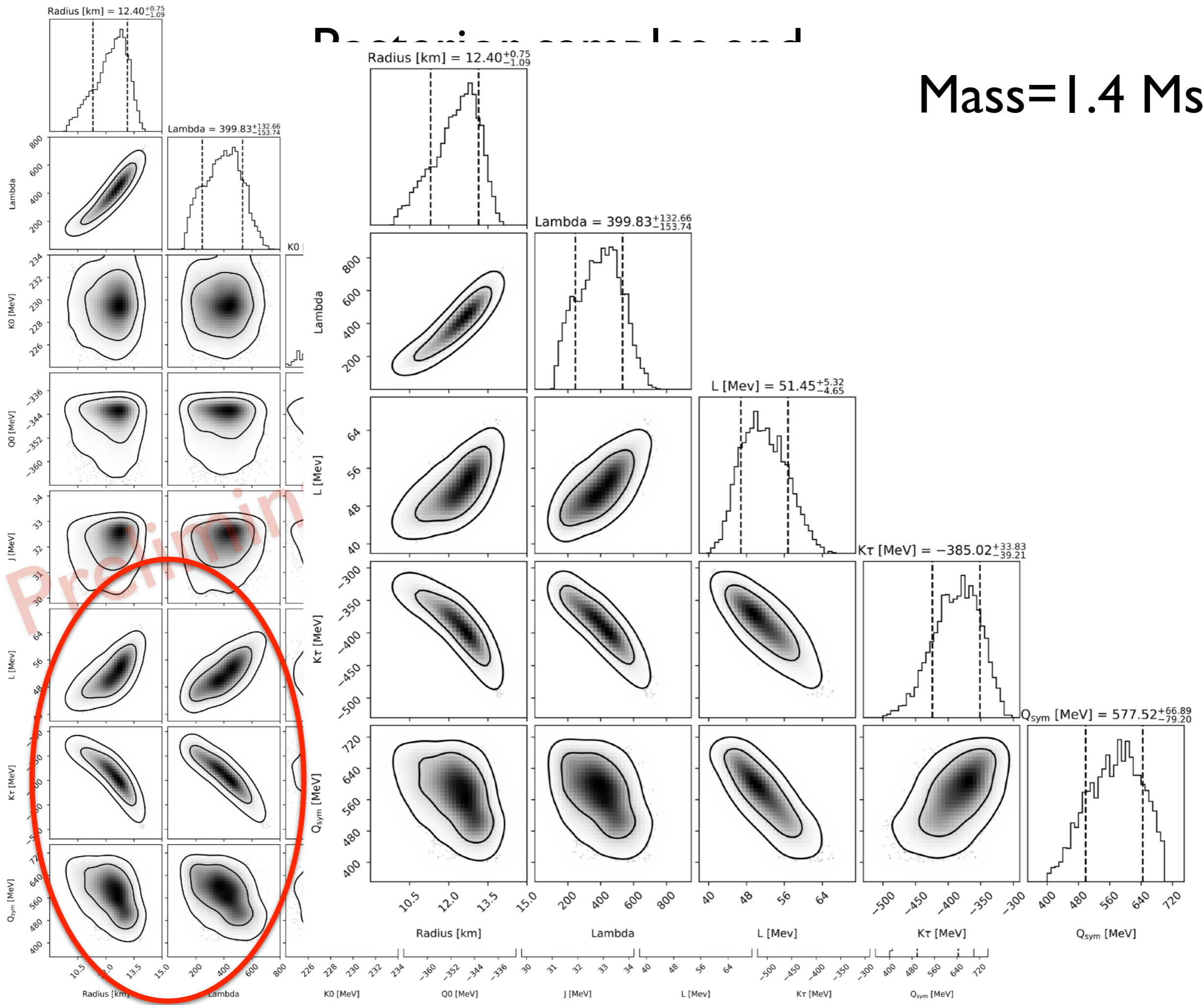
# Posterior samples and Post-processed results

Mass = 1.4 Msun

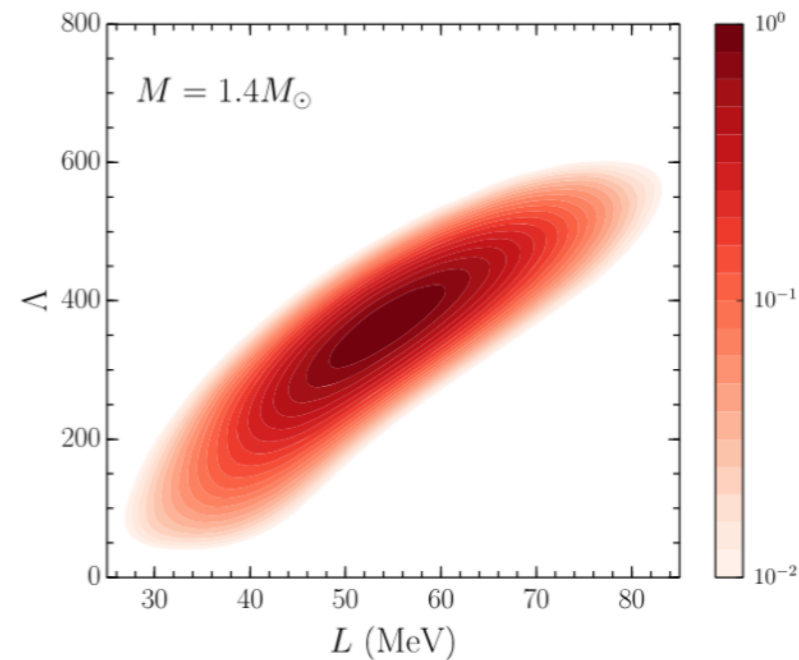
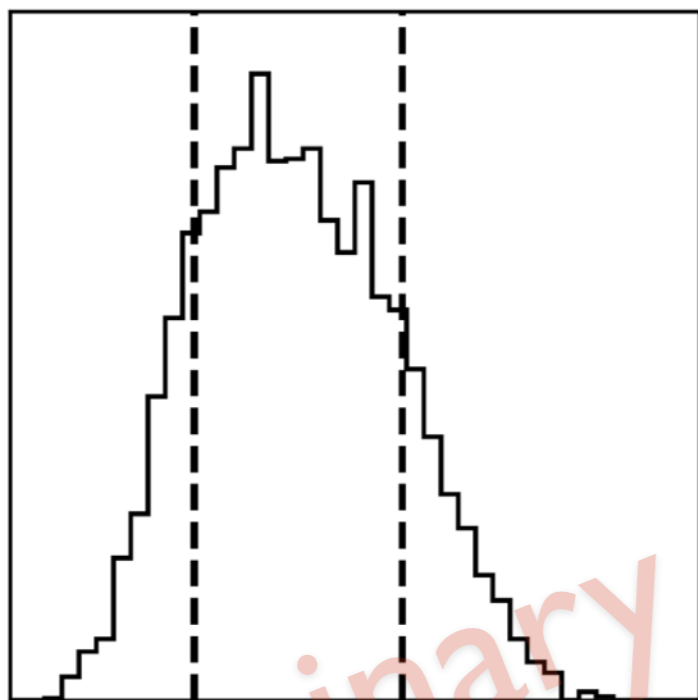


# Posterior probability

Mass = 1.4 Msun



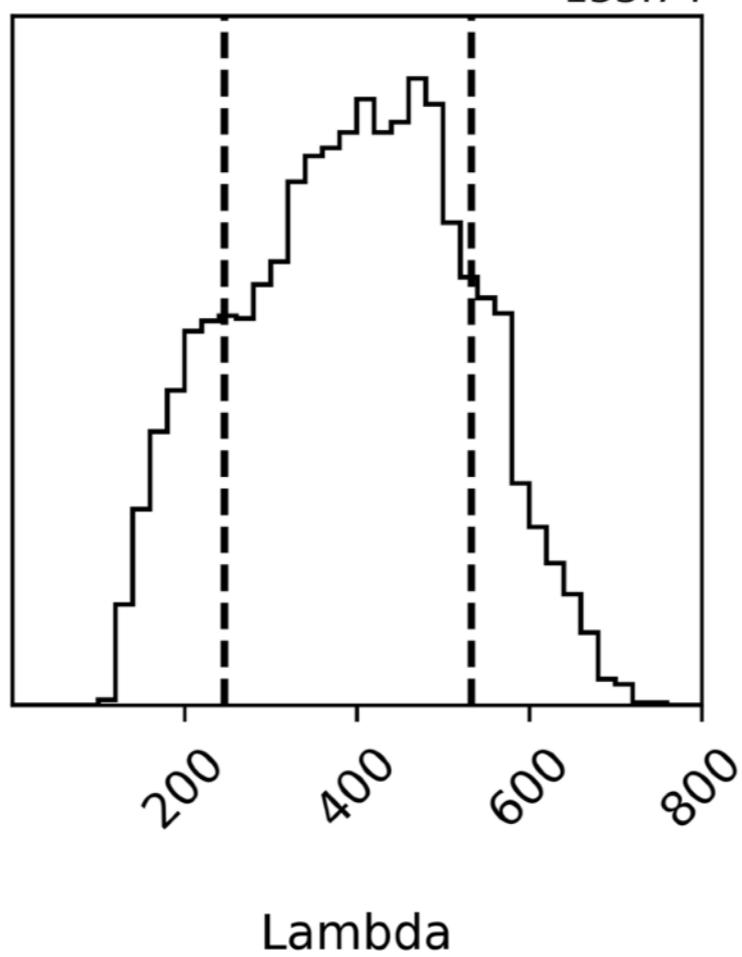
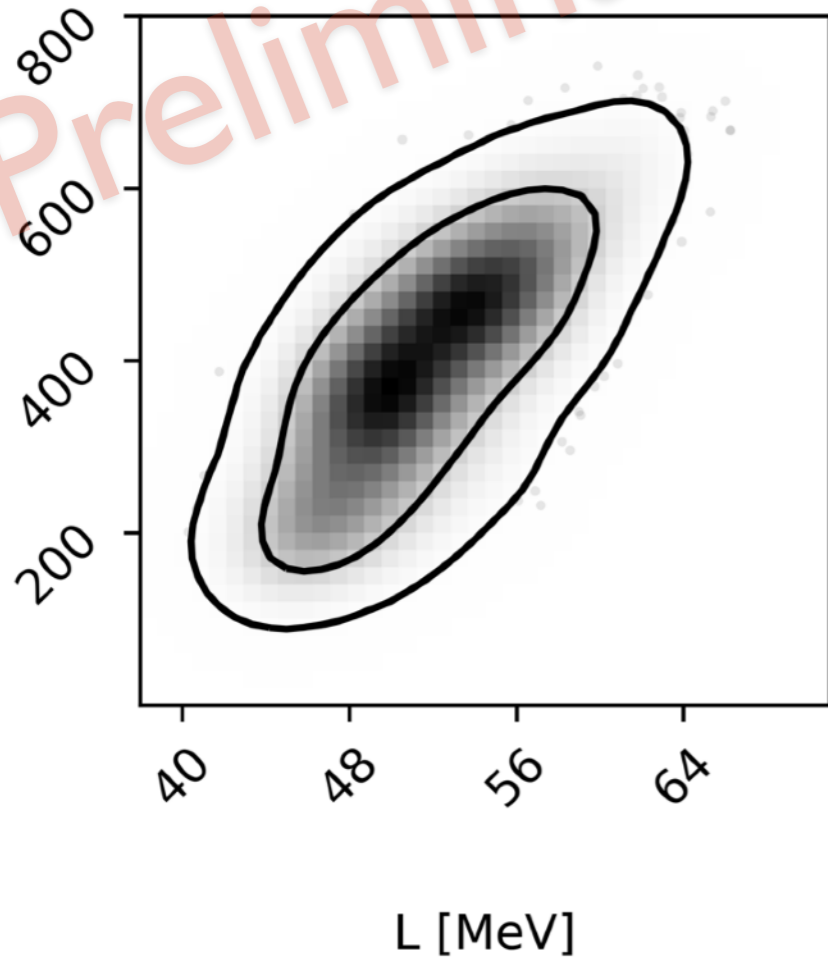
$$L \text{ [MeV]} = 51.45^{+5.32}_{-4.65}$$



isospin-asymmetry energy slope parameter  $L$

[D] Y.Lim and J. Holt, PRL.121.062701  
(arXiv:1803.02803v2)

$$\text{Lambda} = 399.83^{+132.66}_{-153.74}$$



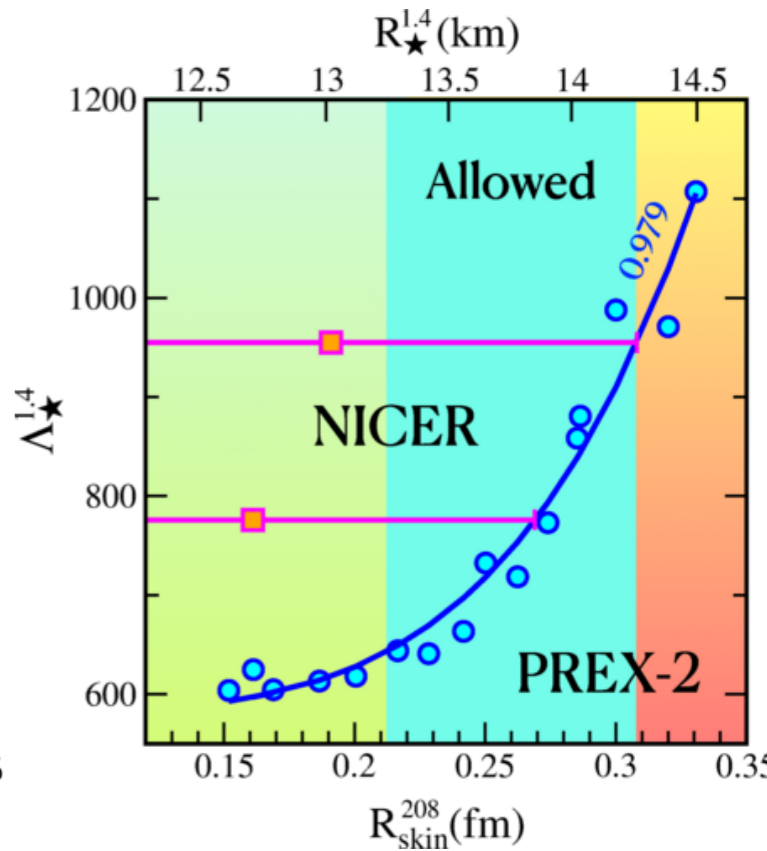
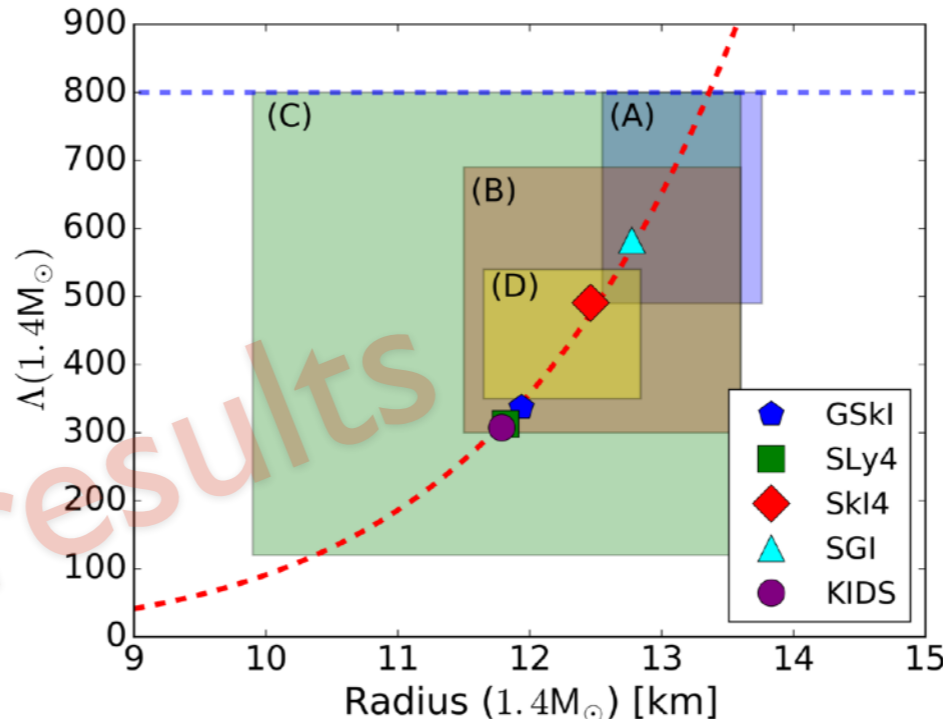
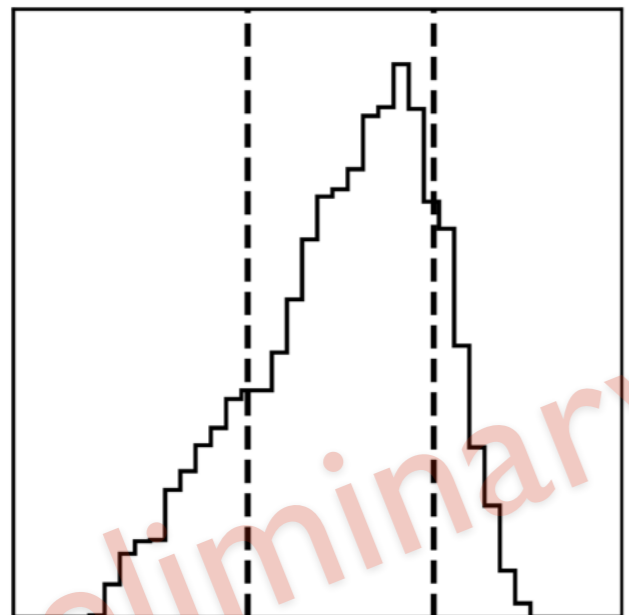


# Lambda-Radius relation

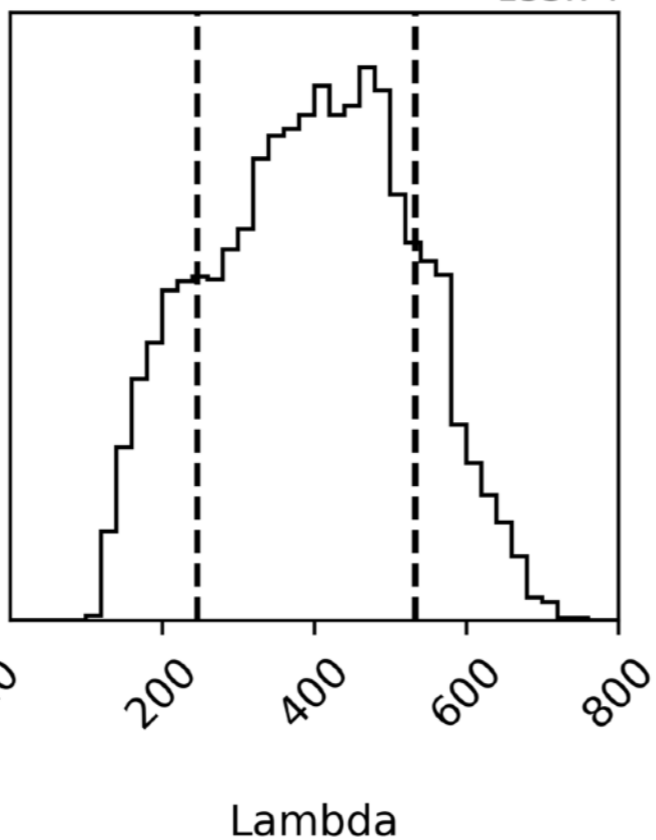
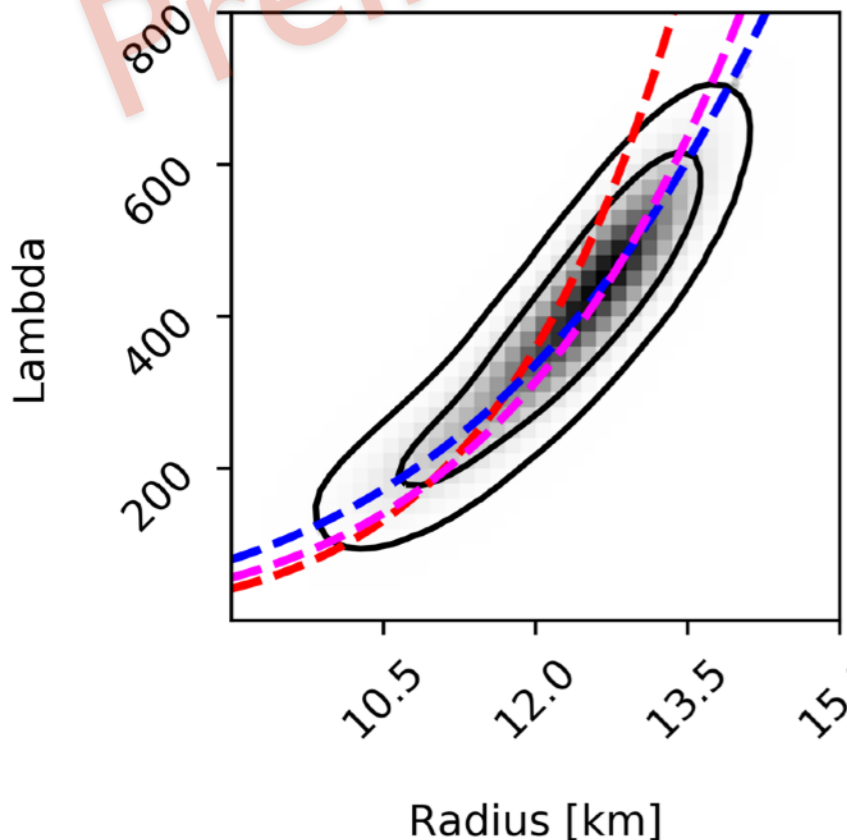
Implication of PREX 2 - PhysRevLett.126.172503 (2021)

Kim et al., PhysRevC.98.065805 (2018)

Radius [km] =  $12.40^{+0.75}_{-1.09}$



Lambda =  $399.83^{+132.66}_{-153.74}$



Red line:  $\Lambda(1.4M_{\odot}) = 2.88 * 10^{-6} (R/km)^{7.5}$   
 [C] PhysRevLett.120.172703.pdf

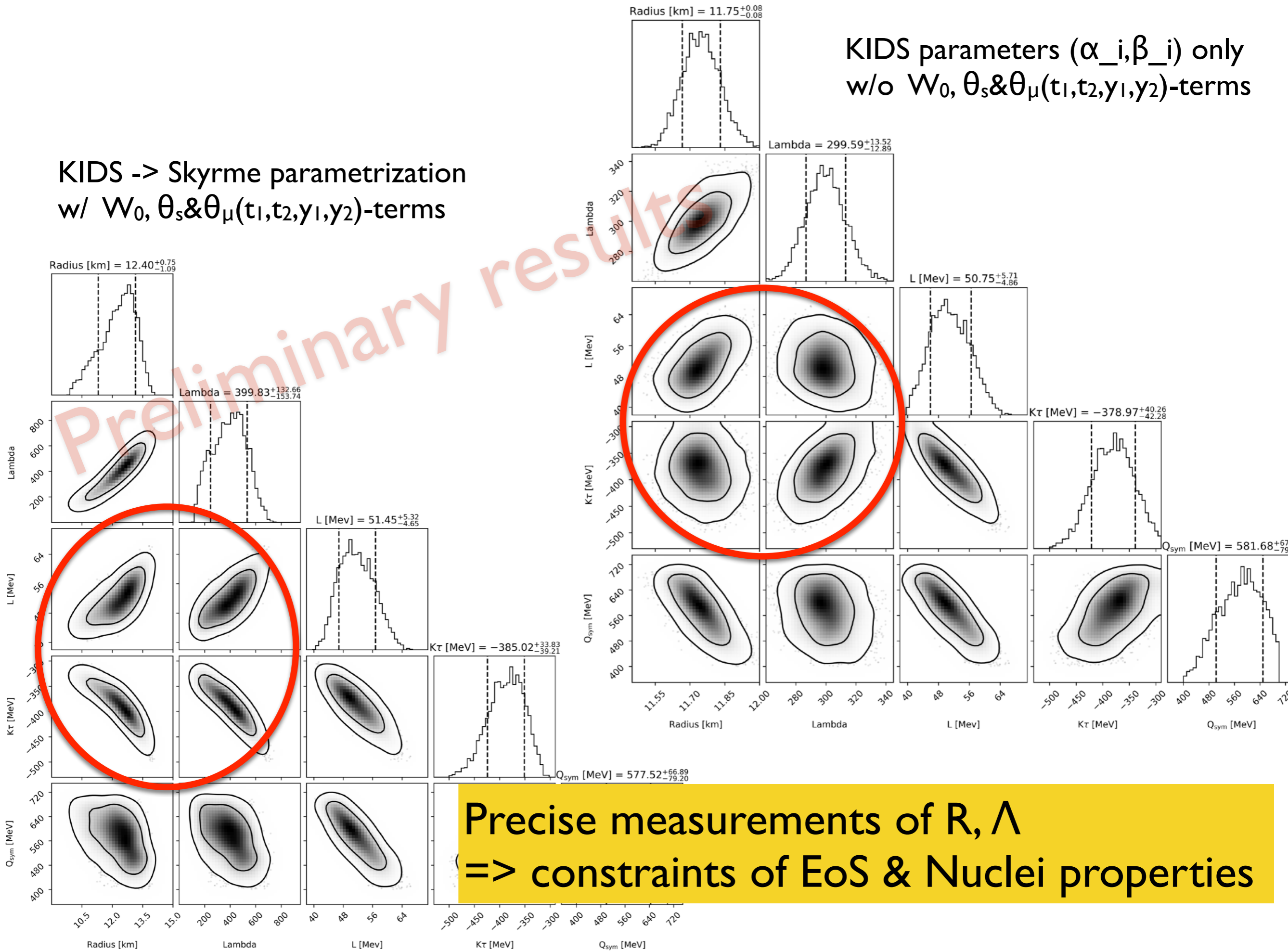
Blue line:  $\Lambda(1.4M_{\odot}) = 1.35 * 10^{-3} (R/km)^{5.0}$   
 Implication of PREX 2 - PhysRevLett.126.172503 (2021)  
 $\Rightarrow \Lambda \sim R^{4.8}$

Magenta line:  $\Lambda(1.4M_{\odot}) = 1.05 * 10^{-4} (R/km)^{6.0}$

Y.-M. Kim, in progress

KIDS -> Skyrme parametrization  
w/  $W_0, \theta_s & \theta_\mu(t_1, t_2, y_1, y_2)$ -terms

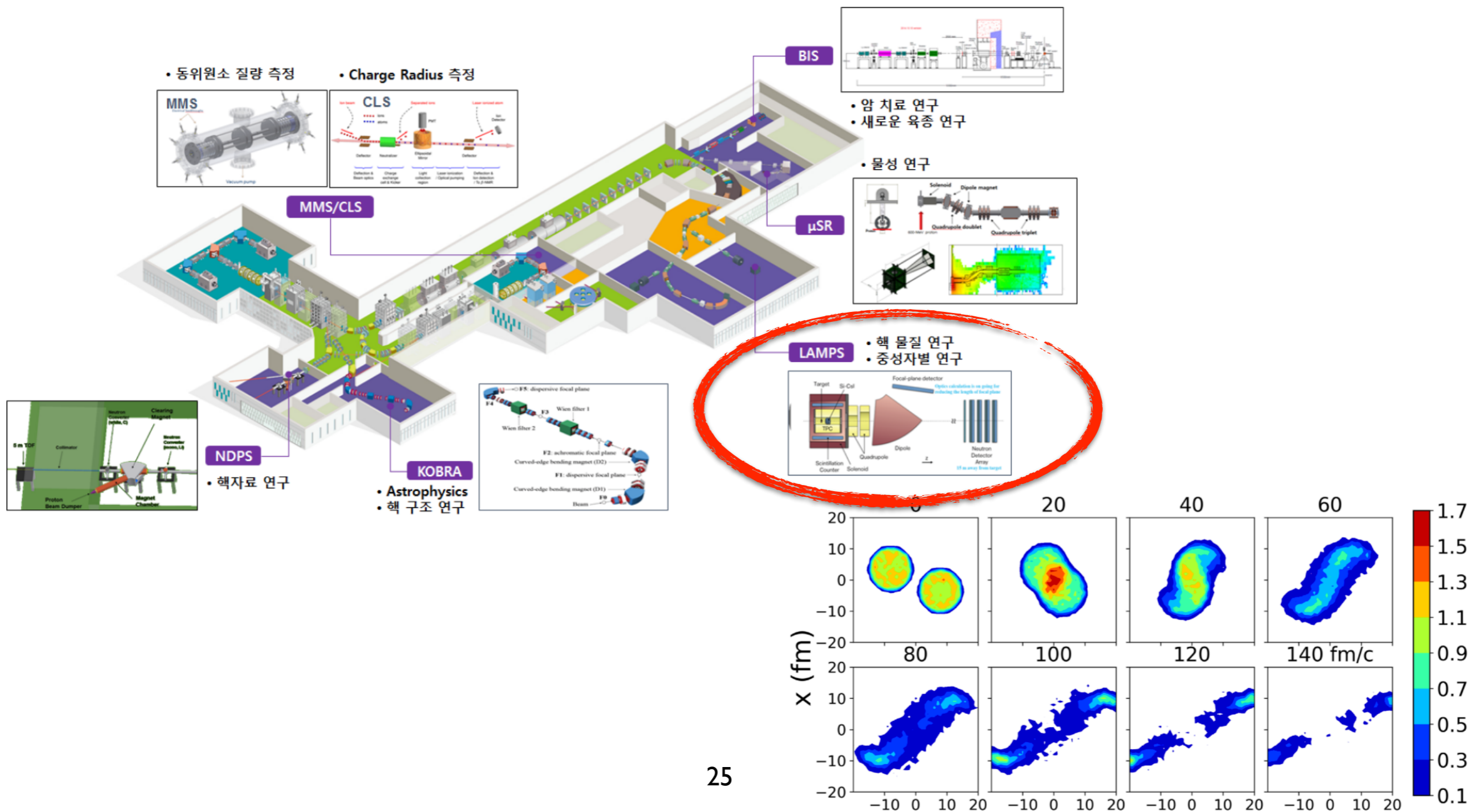
KIDS parameters ( $\alpha_i, \beta_i$ ) only  
w/o  $W_0, \theta_s & \theta_\mu(t_1, t_2, y_1, y_2)$ -terms



Precise measurements of  $R, \Lambda$   
=> constraints of EoS & Nuclei properties

# Application to HIC @ RAON

## Experimental Systems



# KIDS-DJBUU

---

$$U_{0,q} = \frac{1}{2} [(2t_0 + y_0)\rho - (t_0 + 2y_0)\rho_q],$$

$$U_{3,q} = \frac{1}{24} \sum_{n=1}^3 \frac{n}{3} \left[ (2t_{3n} + y_{3n})\rho^2 - (t_{3n} + 2y_{3n}) \sum_q \rho_q^2 \right] \rho^{n/3-1}$$

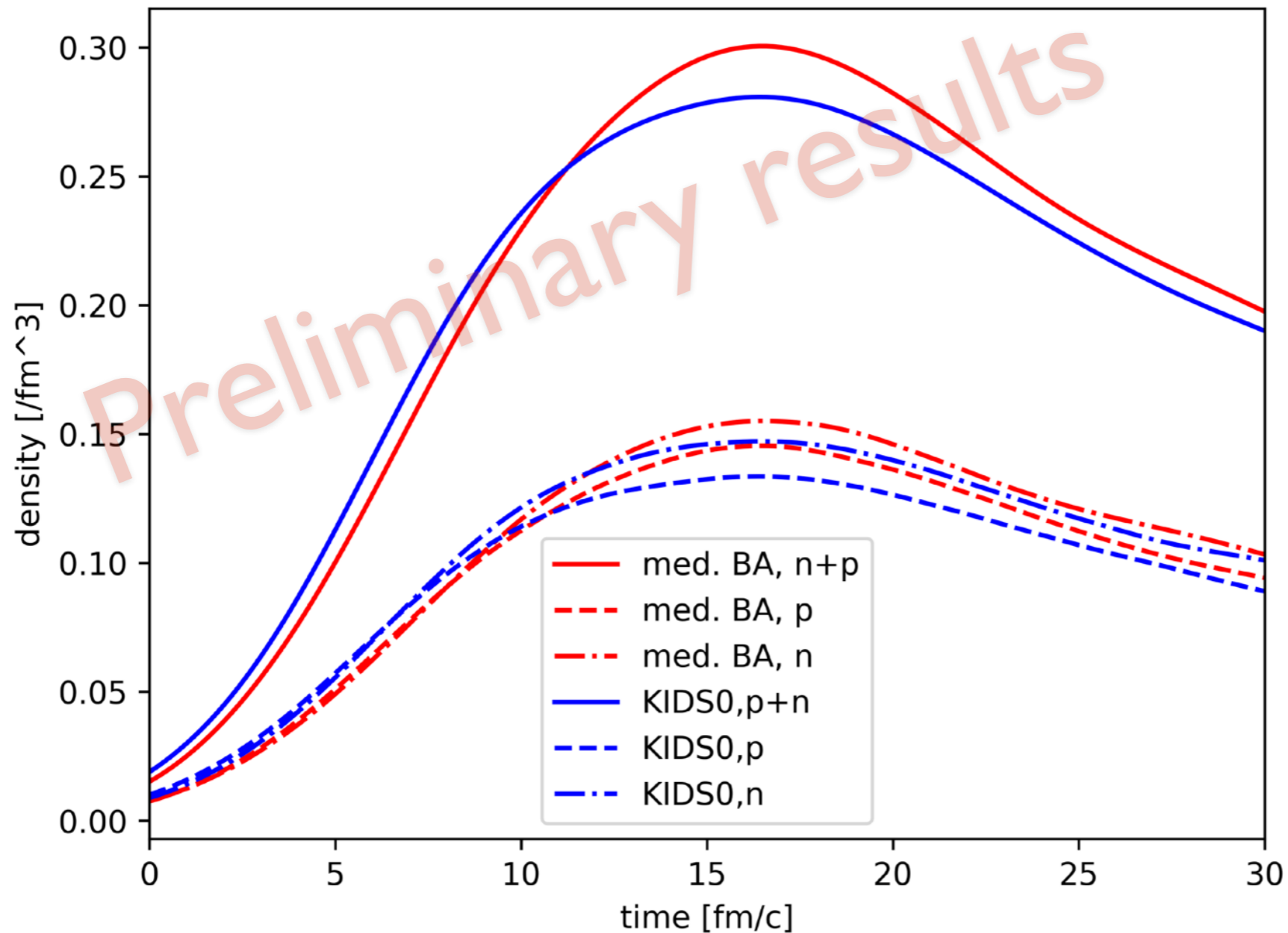
$$+ \frac{1}{12} \sum_{n=1}^3 [(2t_{3n} + y_{3n})\rho - (t_{3n} + 2y_{3n})\rho_q] \rho^{n/3},$$

$$U_{\text{eff},q} = \frac{1}{8\hbar^2} [(2t_1 + y_1) + (2t_2 + y_2)] \int d^3p' (\mathbf{p} - \mathbf{p}')^2 f(\mathbf{r}, \mathbf{p}') \\ + \frac{1}{8\hbar^2} [-(t_1 + 2y_1) + (t_2 + 2y_2)] \int d^3p' (\mathbf{p} - \mathbf{p}')^2 f_q(\mathbf{r}, \mathbf{p}').$$

$n! = 2$ ,  $t_i, y_i$  can be determined by NS observations  
 $n = 2$ ,  $t_2, y_2, t_{32}, y_{32}$  should be determined by Nuclei

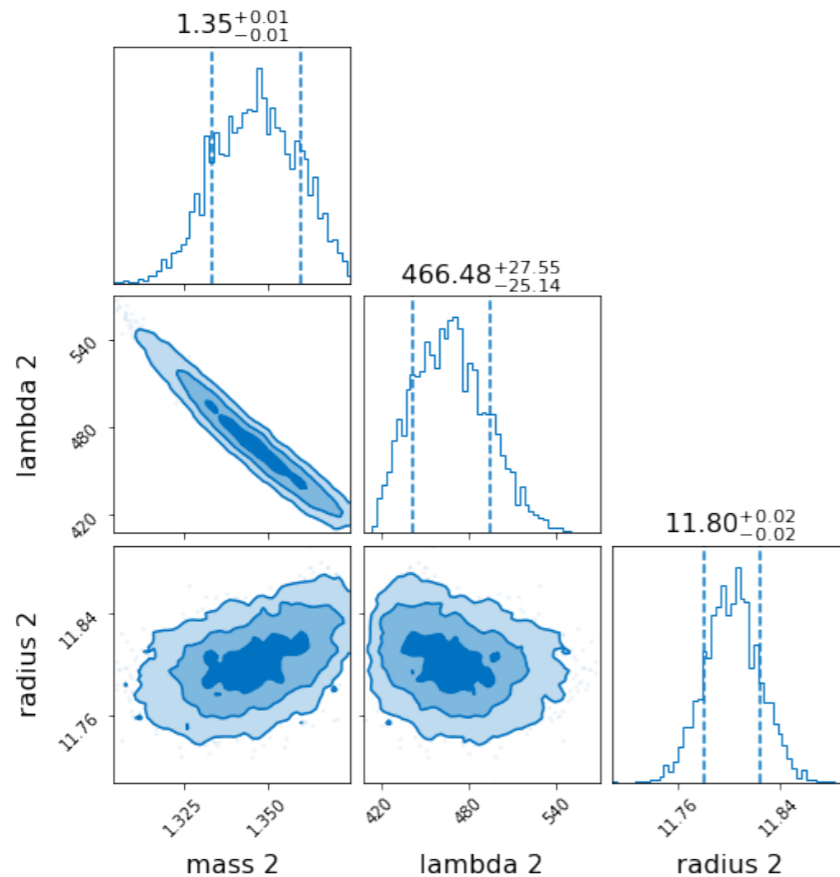
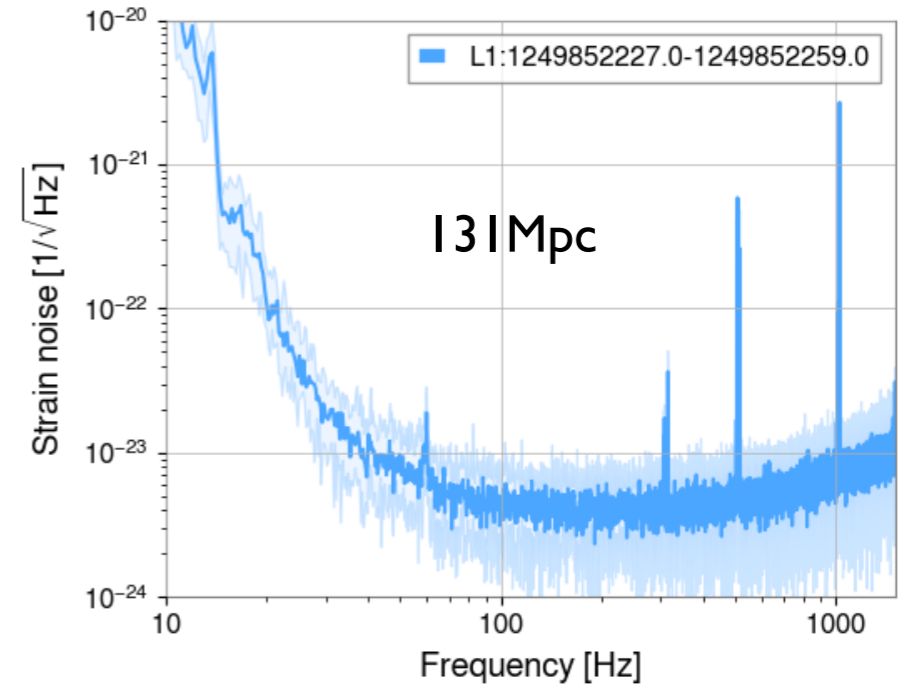
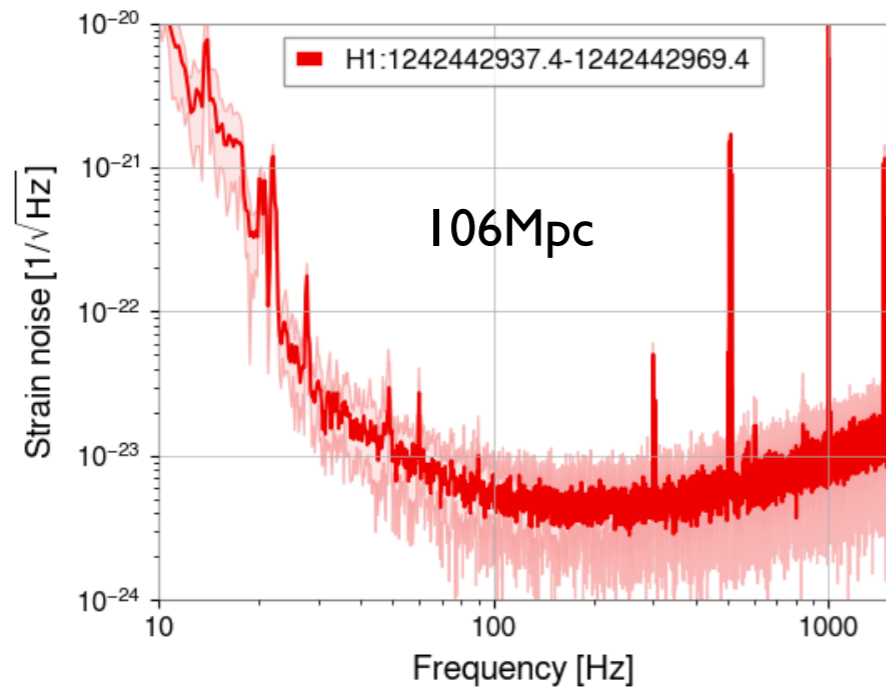
# Application to Heavy Ion Collision

DJBUU simulation w/ KIDS  
based on M.Kim's  
preliminary code

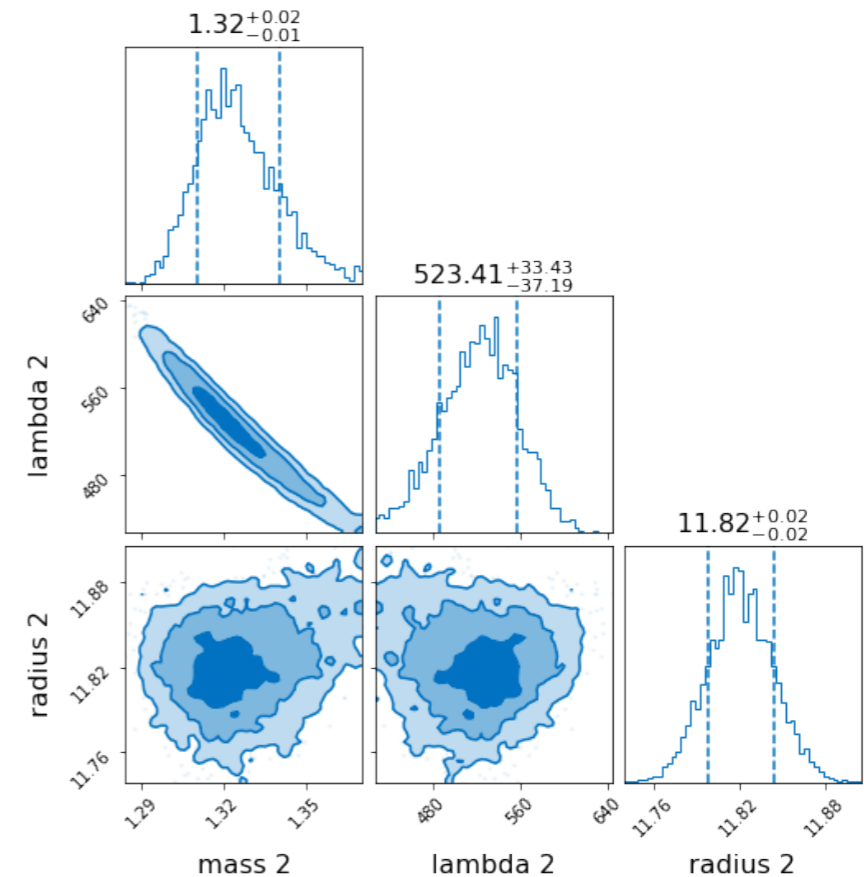


Study on Model Uncertainty or Uncertainty Quantification

# Posteriors depending on DET status



$m_1 = 1.4 \text{ Msun}$   
 $\Lambda_1 = 400$   
 $m_2 = 1.35 \text{ Msun}$   
 $\Lambda_2 = 450$



# Final Goal: EoS w/ Bayesian in MMA

EoS  
(model-  
dependent  
parameters)



NS properties  
( $M, R, \Lambda$ , etc.)  
Finite Nuclei  
(B.E.,  $R_{ch}$ ,  $\Delta R_{np}$ ,  
etc)

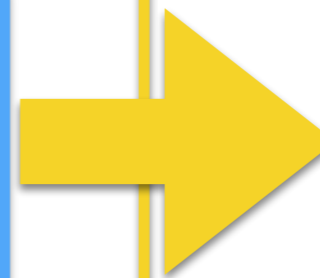
Sampling

Likelihood

MCMC  
or  
Nested Sampling

Obs. Data  
(GW, X-ray,  
etc.)

Exp. Data  
(Raon, DJBUU)



Posteriors :  
 $M, R, \Lambda$ , etc.  
 $L, K_{sym}, Q_{sym}$ , etc.

# Summary and future plans

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1. Neutron Star is an ultimate testing place for physics of dense matter.
  - Multi-Messenger Astrophysics : analysis on GW, LMXB
  - RAON : Heavy Ion simulation (DJBUU)
2. Precise estimation of NS masses and radii is important for studying dense nuclear / stellar matter
  - More observations from BNSs and/or NSBHs via GWs
3. Future plans
  - Bayesian inference w/ hyperons, etc., and RMF model, FRG, etc.
  - Uncertainty Quantification for BNS merger and HIC
    - Acceleration with Deep Learning and/or multi-cores/GPU
  - Consideration of the spin and the magnetic field of NS
  - Towards next generation GW detectors



Thank you for your attention.

