Primordial Cosmic Complexity and Effects of Reheating

2023 Chung-Ang University Beyond the Standard Model Workshop

Feb 23, 2023

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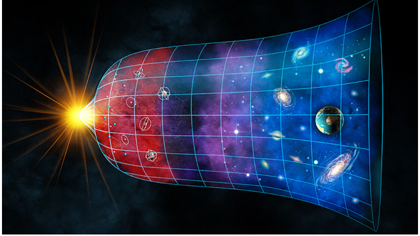
based on arXiv:2212.03059 with Myeonghun Park

Outline

"...the universe may be the ultimate free lunch."–Alan Guth



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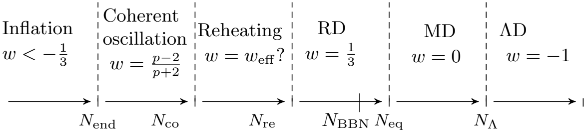
- ☞ Basics of Squeezing formalism.
- ☞ Reheating constraints on Inflationary Models.
- ☞ Reheating constraints to the evolution of Primordial Complexity measures.

Inflation and Reheating



- Inflation is a phase of quasi-exponential expansion in the early Universe that explains the observed small fluctuations in the CMB temperature map.
- During inflation, the vacuum quantum fluctuations amplified by gravitational instability and stretched over cosmological distances that provide the seed for large-scale structure formation.
- The Reheating phase follows the inflation when we recover the Radiation dominated universe.
- Reheating affects the mode re-entry after inflation.

Squeezing and classicalization

1. Squeezing occurs when a certain initial phase space volume evolves in such a way that it shrinks in one direction and simultaneously grows in another direction.
2. Albrecht, Ferreira, Joyce and Prokopec applied the squeezing formalism to Inflationary perturbations.
3. The time evolution operator $\hat{U}$ (corresponding to the system's Hamiltonian) is factorized into a rotation operator $\hat{R}$ and a squeezing operator $\hat{S}$:

$$\hat{U}(\eta) = \hat{S}(r_k(\eta), \varphi_k(\eta))\hat{R}(\theta_k(\eta))$$

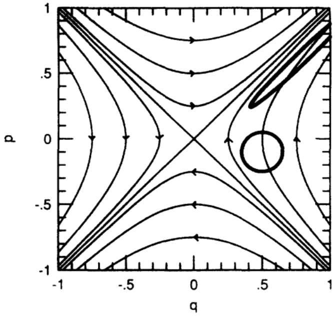


Fig: Albrecht et al. 1994

Squeezing the curvature perturbation

- Perturbation to the scalar field: $\phi(\vec{x}, t) = \phi_0(t) + \delta\phi(\vec{x}, t)$
- Perturbation in the metric:

$$ds^2 = a(\eta)^2 \left[-(1 + 2\Psi(x, \eta)) d\eta^2 + (1 - 2\Psi(x, \eta)) d\vec{x}^2 \right].$$

- The gauge-invariant combination — the curvature perturbation $\mathcal{R} = \Psi + \frac{H}{\dot{\phi}_0} \delta\phi$
- We quantize the curvature perturbation (some suitable re-scaled quantity $v \equiv z\mathcal{R}$, with $z = M_{\rm Pl} a \sqrt{2\epsilon_1}$.)
- As $v(\eta, \vec{x})$ is real, one has $v_{-k} = v_k^* \implies$ In Fourier space, all degrees of freedoms are not independent $\implies$ Partition the system into bipartite system $\mathcal{E} = \mathcal{E}_{\mathbf{k}} \bigotimes \mathcal{E}_{-\mathbf{k}}$.

Squeezing formalism

Equations governing the evolution of the three squeezing parameters (Polarski and Starobinsky 1996):

$$r_k' = -\frac{z'}{z}\cos 2\varphi_k, \tag{1}$$

$$\varphi_k' = -k + \frac{z'}{z}\coth 2r_k \sin 2\varphi_k, \tag{2}$$

$$\theta_k' = k - \frac{z'}{z}\tanh r_k \sin 2\varphi_k \tag{3}$$

We choose the ICs such that the mode functions

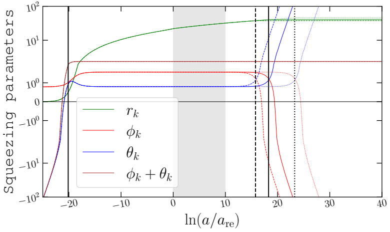
$$f_k(\eta) = \frac{1}{\sqrt{2k}}\left[\mathrm{e}^{-i\theta_k(\eta)}\cosh(r_k(\eta)) - \mathrm{e}^{-i(\theta_k(\eta)+2\varphi_k(\eta))}\sinh(r_k(\eta))\right]$$

start from the Bunch-Davies vacuum:

$$u_k(\eta) = \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right). \tag{4}$$

Behavior of the Squeezing parameters

1. The squeezing parameter r_k grows when they are the outside the horizon.
2. The phase parameters ϕ_k and θ_k grow inside the horizon while the combination $\phi_k + \theta_k$ remains constant.
3. Reheating phase directly do not alter the evolution of the squeezing parameters. It determines when a mode will re-enter the horizon.



Complexity measures-I: OTOC

- The high sensitivity of initial conditions characterizes chaos in classical systems. In a chaotic system, two nearby trajectories with small perturbations in the initial conditions diverge exponentially. This behavior can be easily captured with the Poisson bracket between the position and momentum variables

$$\{q(t), p(0)\}^2 = \left(\frac{\partial q(t)}{\partial q(0)}\right)^2 \sim \sum_i c_n e^{2\lambda_i t},$$

  where the λ_i is the Lyapunov characteristic exponents of the system.
- The simple analog of the Poisson bracket for quantum systems is the *unequal time commutator* $[\hat{q}(t), \hat{p}(0)]$.
- In the semi-classical limit, which reduces to the Poisson bracket $\sim i\hbar\{q(t), p(0)\}$. Thus quantum chaos could be studied from this quantity.
- Being an operator and not a c-number, a more useful quantity to study the chaos in quantum systems is the double unequal-time commutator or the *out-of-time-order correlator* (OTOC)

$$\mathcal{C}^T(t) \equiv -\langle[\hat{q}(t), \hat{p}(0)]^2\rangle_\beta,$$

- OTOC in terms of the squeezing parameters:

$$\mathcal{C}^T_{\mathbf{k}}(\eta) \equiv |\mathfrak{F}_k(\eta, \eta_0)|^2.$$

$$\mathfrak{F}_k(\eta, \eta_0) = \frac{1}{2}\Big[\left(\cosh r_k e^{-i\theta_k} - \sinh r_k e^{-i(\theta_k + 2\phi_k)}\right) \times \left(\cosh r_0 e^{-i\theta_k} + \sinh r_0 e^{-i(\theta_0 + 2\phi_0)}\right) + c.c.\Big],$$

Quantum Discord: von Neumann entanglement entropy

- The concept of quantum discord was introduced to measure the *quantumness of correlations* of two subsystems of a quantum system (Henderson and Vedral 2001; Ollivier and Zurek 2001).
- A relatively close measure to quantum discord is the entanglement between the subsystems. However, quantum discord can be non-zero even if there is no entanglement, while zero discord implies entanglement is also zero.
- Quantum discord seems to be a better tool than quantum entanglement to look for non-classical correlations in a system.
- If a system in the pure state is divided into two subsystems, the quantum discord is identical to the von Neumann entanglement entropy (Bera et al. 2017; Datta et al. 2008)

Quantum Discord: von Neumann entanglement entropy

- The two-mode squeezing operator when acts on the two-mode vacuum state (initial state), $|0\rangle_{\mathbf{k}}|0\rangle_{-\mathbf{k}}$ yields

$$|\Psi_{\rm sqz}\rangle_{\mathbf{k},-\mathbf{k}} = \hat{\mathcal{S}}_{\mathbf{k}}(r_k,\phi_k)\,|0\rangle_{\mathbf{k}}\,|0\rangle_{-\mathbf{k}}\,, \tag{5}$$

$$= \frac{1}{\cosh r_k}\sum_{n=0}^{\infty}(-1)^n e^{in\theta}(\tanh r_k)^n\,|n_k, n_{-k}\rangle \tag{6}$$

- The reduced density operators for the individual models are:

$$\hat{\rho}_k = \sum_{n=0}^{\infty}\frac{1}{(\cosh r_k)^2}(\tanh r_k)^{2n}\,\langle n_k|n_k\rangle\,, \tag{7}$$

$$\hat{\rho}_{-k} = \sum_{n=0}^{\infty}\frac{1}{(\cosh r_{-k})^2}(\tanh r_{-k})^{2n}\,\langle n_{-k}|n_{-k}\rangle \tag{8}$$

Quantum Discord: von Neumann entanglement entropy

$$S(\hat{\rho}_k) = -\mathrm{Tr}\left[\hat{\rho}_k \ln \hat{\rho}_k\right] = S(\hat{\rho}_{-k}),$$

$$= -\sum_{n=0}^{\infty} p_n \ln p_n,$$

$$= \frac{(\tanh r_k)^{2n}}{(\cosh r_{-k})^2} \ln \frac{(\tanh r_k)^{2n}}{(\cosh r_{-k})^2},$$

$$= \frac{(\tanh r_k)^{2n}}{(\cosh r_{-k})^2}\left[\ln\left(\tanh^{2n} r_k\right) - \ln\left(\cosh^2 r_k\right)\right],$$

$$\boxed{S(\hat{\rho}_k) = \cosh^2 r_k \ln\bigl(\cosh^2 r_k\bigr) - \sinh^2 r_k \ln\bigl(\sinh^2 r_k\bigr).}$$

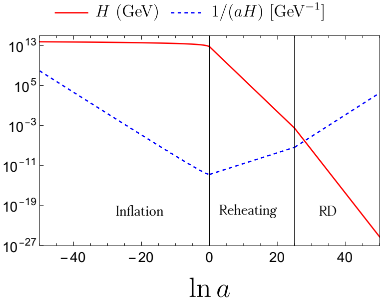
Background Model

- We take the ubiquitous inflationary model:

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2,$$

- The Hubble expansion is given by

$$H = \begin{cases} m_\phi \sqrt{\frac{1}{3} - \frac{2}{3}\ln a}, & a \leq 1 \\ \frac{m_\phi}{\sqrt{3}} e^{-\frac{3}{2}(w_{\rm re}+1)\ln a}, & 1 \leq a \leq a_{\rm re} \\ \frac{m_\phi}{\sqrt{3}} e^{-\frac{\ln a_{\rm re}}{2}(3w_{\rm re}-1)} e^{-2\ln a}, & a \geq a_{\rm re} \end{cases}$$



- Reheating efolds number is given by (Dai et al. 2014):

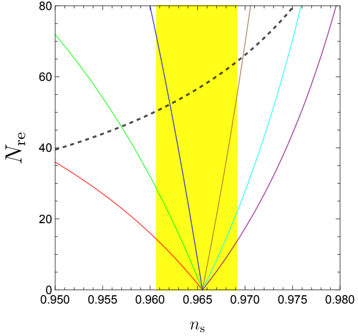
$$N_{\rm re} = \frac{4}{3w_{\rm re} - 1}\left[N_k - 61.6 - \frac{1}{2}\ln\left(\frac{\pi^2 M_{\rm Pl}^2 r_k A_s}{2V_{\rm end}^{1/2}}\right)\right],$$

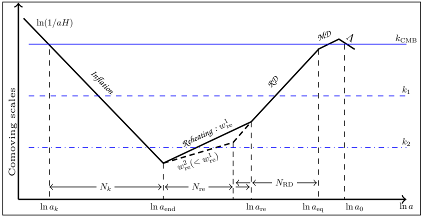
$$N_k = \frac{4}{2(1 - n_s)},$$

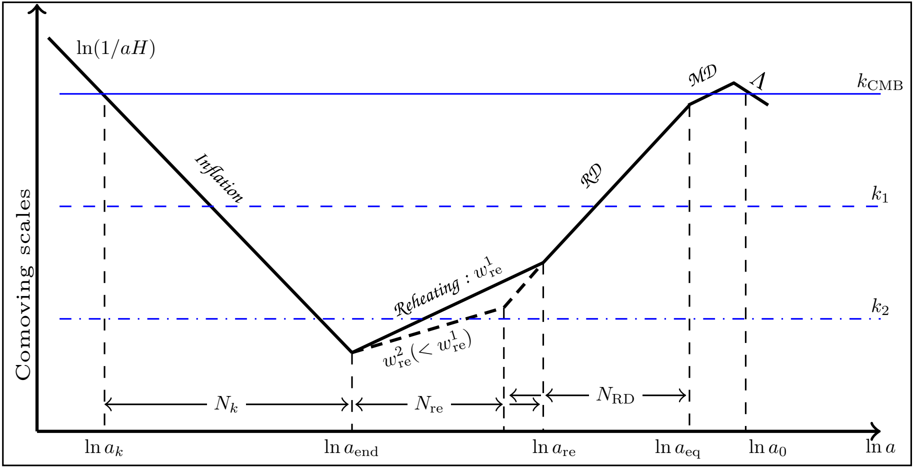
$$r_k = 4(1 - n_s)$$

Reheating constrains

1. The scalar spectral index (n_s) fixes the inflationary (N_k) and reheating efolds.
2. It also determines the type of reheating equation of state: $w_{\rm re} < 1/3$,$w_{\rm re} = 1/3$ or $w_{\rm re} > 1/3$.
3. If a mode renter the horizon during radiation dominated epoch, its horizon reentry will same for all equation of the state parameters from one of the three classes.

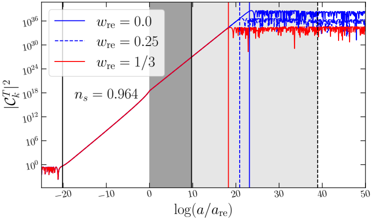


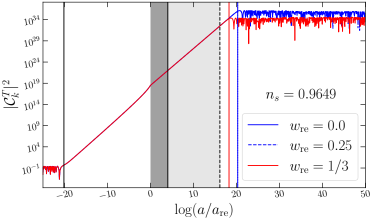


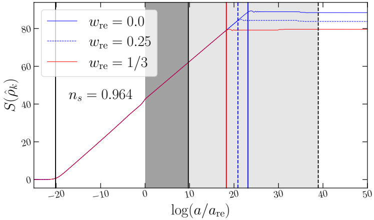


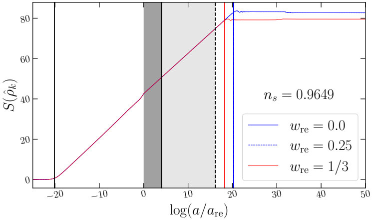
$\ln(1/aH)$
Comoving scales
$k_{\rm CMB}$
k_1
k_2
Inflation
Reheating : $w_{\rm re}^1$
$w_{\rm re}^2(< w_{\rm re}^1)$
RD
MD
Λ
N_k
$N_{\rm re}$
$N_{\rm RD}$
$\ln a_k$
$\ln a_{\rm end}$
$\ln a_{\rm re}$
$\ln a_{\rm eq}$
$\ln a_0$
$\ln a$

Reheating constraints in Primordial Complexity

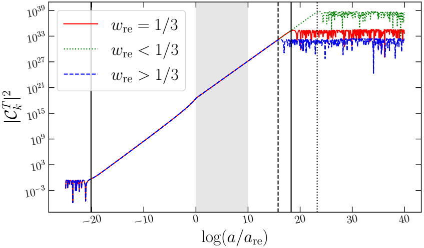








Summary Plot



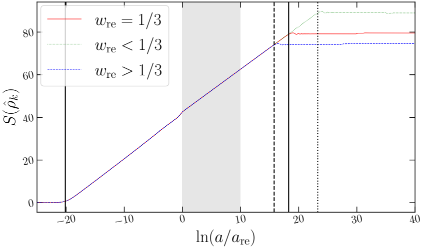


Figure: Reheating groups the Complexity measures into three classes

Summary and Outlook

- ✍ A finite reheating epoch determines when a mode will re-enter the horizon after inflation.
- ✍ Taking care of the reheating constrains, the re-entry of the modes can be classified into three classes.
- ✍ This classification shows similar signatures in different primordial Complexity measures.
- ✍ Taking the central value of the scalar spectral index ($n_s = 0.9649$) from Planck and the equation of state during reheating $w_{\rm re} = 0.25$ as benchmark values, we found that the behavior of the complexities for all modes smaller than $1.27 \times 10^{16}\ \mathrm{Mpc}^{-1}$ can be classified as above.

—— Thank You! ——