Halo-independent bounds on the non-relativistic effective theory of WIMP-nucleon scattering from direct detection and neutrino observations

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WIMP dark matter searches

- Cold Dark Matter (CDM): provides ~25% of the energy density of the Universe, only detected through gravitational effects.
- WIMPs are the most popular candidates for CDM.
- A popular technique that is used to search for WIMPs is Direct Detection (DD).
- The physics of DD is mainly based on the scattering of WIMPs against nuclear targets.
- Same WIMP–nucleus scattering can trigger capture of WIMPs in celestial bodies.
- Neutrino Telescopes (NTs) (e.g., IceCube, Super-K), searching for ν's produced by the annihilation of captured WIMPs inside Sun or Earth, are also used to search for WIMPs.
- Estimated WIMP signals in both cases depend on the velocity distribution of local WIMPs.

Uncertainties in signal prediction

- Non-detection of any new signal in DD and NT experiments

  $\Rightarrow$ upper-limits on WIMP-nucleus interaction.

- Two classes of major uncertainties in the signal prediction:

  1. The nature of the WIMP–nucleus interaction.
  2. The WIMP speed distribution $f(u)$ (in the Solar reference frame) that determines the WIMP flux.

- WIMP–nucleus interaction:
  Most common choices: standard spin-independent (SI) or spin-dependent (SD) interactions.

- WIMP speed distribution $f(u)$:
  Most common choice: a Maxwell-Boltzmann (MB) speed distribution in the galactic frame (and boosted to the solar frame) with a speed dispersion ~ 300 km/s,
  Standard Halo Model(SHM).

WIMP speed distribution: Halo–independent approach

- The MB distribution (based on Isothermal Model) provides a zero–order approximation to $f(u)$.

  $\rightarrow$ Numerical simulations of Galaxy formation can only tell us about statistical average properties of halos.

  $\rightarrow$ Merger events can add sizeable non–thermal components in $f(u)$.

  $\rightarrow$ The growing number of observed dwarf galaxies suggests that our halo is not perfectly thermalized.

- Halo–independent approach:

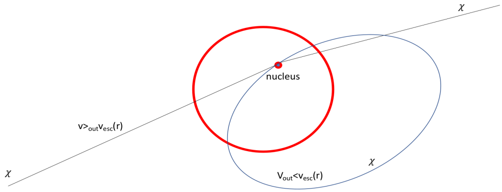
  $\rightarrow$ The strategy is to find the most conservative bounds from null searches with the only constraint:

$$\int_{u=0}^{u_{\max}} f(u) du = 1, \qquad f(u) \Rightarrow \text{any possible speed distribution}$$

WIMP speed distribution: Halo–independent approach

- DD experiments are sensitive to $u > u_{\rm th}^{\rm DD}$ $\quad ({u_{\rm th}^{\rm DD}}^2 = \frac{m_T}{2\mu_{\chi T}^2} E_{R\,{\rm th}})$
  $\Rightarrow$ can not cover the full WIMP speed range $[0, u_{\rm max}]$.
  $u_{\rm max} \equiv$ Galactic escape speed (in solar frame)

- Capture in the Sun is favoured for low (even vanishing) WIMP speeds.



annihilation of captured WIMPs
$\chi\chi \to b\bar{b}, \mu^+\mu^-, W^+W^-\ldots$
$\Rightarrow \nu(\bar{\nu})$

(S. Scopel, CQUeST 2022 workshop)

- Possible solution: DD constraints "+" constraints from expected ν-signal from the annihilation of WIMPs captured in the Sun.

  Extra assumptions! (1) Equilibrium between capture and annihilation
  (2) primary WIMP annihilation channel

WIMP speed distribution: Halo–independent approach

- The complementarity between DD and capture was used to develop a straightforward method that gives conservative constraints on WIMP interaction independent of $f(u)$:
  [Ferrer *et al.* (JCAP09(2015)052)]

  Halo-independent upper-limits.

- The halo-independent method was applied to the case of standard SI/SD scenario without assuming any general structure for the WIMP-nucleus interaction.

Non–relativistic effective theory of WIMP-nucleon scattering

- Non-observation of new physics predicted by popular extensions of the Standard Model (e.g., SUSY)

  ⇒ motivation for the bottom–up approaches that go beyond the standard SI/SD scenario.

- Usually the WIMP scattering process is non–relativistic (NR).

  In general the WIMP–nucleon interaction can be parameterized with an effective Hamiltonian $\mathcal{H}$, complies with Galilean symmetry:

$$\mathcal{H} = \sum_{\tau=0,1} \sum_{i} c_i^{\tau} \mathcal{O}_i$$

  $\mathcal{O}_i$: Galilean–invariant operators.

  c_i^{τ}: Wilson coefficients, with τ $(= 0{,}1)$ the isospin.

  $c_i^p = c_i^0 + c_i^1$, $c_i^n = c_i^0 - c_i^1$.

[Fitzpatrick *et al.* (JCAP02(2013)004)], [Anand *et al.* (PRC 89, 065501 (2014))]

Non–relativistic effective theory

NR Galilean invariant operators for a WIMP of spin 1/2
(up to linear terms in the WIMP velocity $\vec{v}$)

[Fitzpatrick *et al.* (JCAP02(2013)004)], [Anand *et al.* (PRC 89, 065501 (2014))]

$\mathcal{O}_1 = 1_\chi 1_N$ (standard SI)	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$ (standard SD)	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$	$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$
$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

$m_N \equiv$ nucleon mass ; $\vec{q} \equiv$ transferred momentum ; $\vec{v}^\perp . \vec{q} = 0$

- $\mathcal{O}_i$'s are the most general building blocks of the low-energy theory.
- **Discussion of the halo-independent method when the WIMP–nucleus interaction is driven by each $\mathcal{O}_i$ is crucial for understanding the more general scenarios involving the sum of several NR operators**.

WIMP–nucleus scattering

- Differential cross-section of WIMP-nucleus scattering $\frac{d\sigma_T}{dE_R}$:
  $\rightarrow$ required for calculating both the DD signal and that from WIMP capture in the Sun.

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v^2}\left[\frac{1}{2j_\chi+1}\frac{1}{2j_T+1}|\mathcal{M}_T|^2\right]$$

[Fitzpatrick *et al.* (JCAP02(2013)004)], [Anand *et al.* (PRC 89, 065501 (2014))]

$$|\mathcal{M}_T|^2 = 4\pi(2j_\chi+1)\sum_{\tau=0,1}\sum_{\tau'=0,1}\sum_k R_k^{\tau\tau'}\left[(c_i^\tau)^2, (v^\perp)^2, \frac{q^2}{m_N^2}\right] W_{Tk}^{\tau\tau'}(q)$$

$$(v^\perp)^2 = v^2 - v_{\min}^2\ , \qquad v_{\min}^2 = \frac{q^2}{4\mu_{\chi T}^2} = \frac{m_T E_R}{2\mu_{\chi T}^2}, \qquad q^2 = 2m_T E_R$$

WIMP response functions: $R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}(v^2 - v_{\min}^2)$

Nuclear response functions (form factor): $W_{Tk}^{\tau\tau'}(q)$

$k = M,\ \Phi'',\ \tilde{\Phi}',\ \Sigma'',\ \Sigma',\ \Delta$
index representing different effective nuclear operators

Direct detection events & capture rate

- Number of expected events in a DD experiment:

$$R_{\rm DD} = M\tau_{\rm exp}\left(\frac{\rho_\odot}{m_\chi}\right)\int du\, f(u)\, u \sum_T N_T \int_{E_{R\rm th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\;\; \epsilon(E_R)\,\frac{d\sigma_T}{dE_R}$$

- We assume equilibrium between WIMP capture and annihilation in the Sun ($\Gamma_\odot = C_\odot/2$).
  In this case the ν-flux from WIMP annihilations in the Sun is determined by $C_\odot$:

$$C_\odot = \left(\frac{\rho_\odot}{m_\chi}\right)\int du\, f(u)\,\frac{1}{u}\int_0^{R_\odot} dr\, 4\pi r^2\, w^2 \times \sum_T \eta_T(r)\,\Theta(u_T^{\rm C-max} - u)\int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R\,\frac{d\sigma_T}{dE_R}$$

$w^2 = u^2 + v_{\rm esc}^2(r)$

$u_T^{\rm C-max} = v_{\rm esc}(r)\sqrt{\frac{4m_\chi m_T}{(m_\chi - m_T)^2}}$ (maximum WIMP speed for which capture through the scattering off target T is kinematically possible)

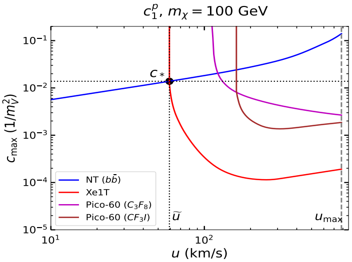
Single–stream halo–independent bound

$c_{i\,\max}(u) \equiv$ upper-limit on c_i when all WIMPs are in a single speed stream u

$$\left(u_{\rm th}^{\rm DD\,2} = \frac{m_T}{2\mu_{\chi T}^2} E_{R\,\rm th}\right)$$

For a DD and a NT experiment

$$(c^{\rm NT})^2{}_{\max}(u) \quad \leq \quad c_*^2 \qquad [0 \leq u \leq \tilde{u}]$$
$$(c^{\rm DD})^2{}_{\max}(u) \quad \leq \quad c_*^2 \qquad [\tilde{u} \leq u \leq u_{\max}]$$



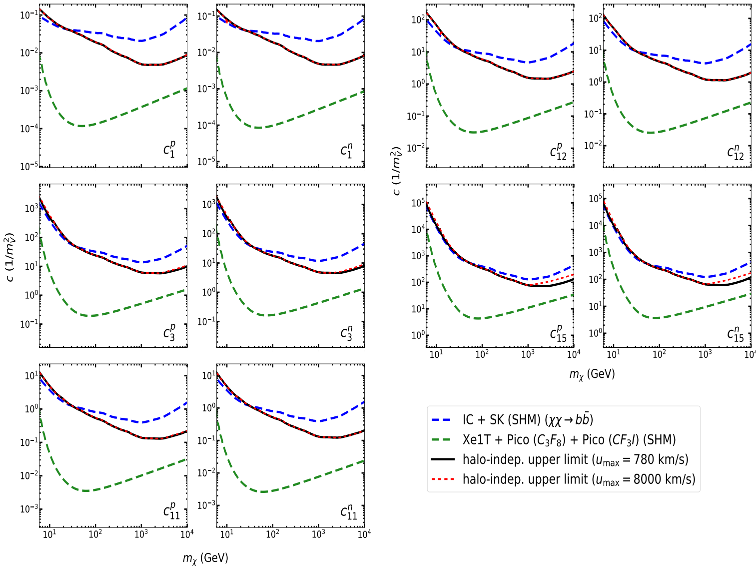
NT: IceCube/Super-K $[\chi\chi \to b\bar{b}]$

The halo-independent upper-limit:

$$c^2 \leq 2\, c_*^2$$

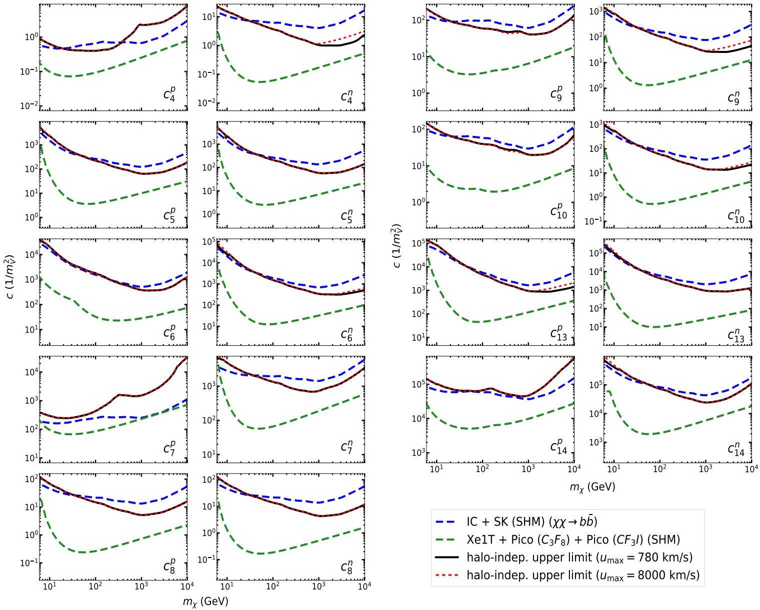
- Halo-independent bound is obtained for each pairs of NT + DD
- The most constraining limit is taken

Halo–independent bounds on couplings



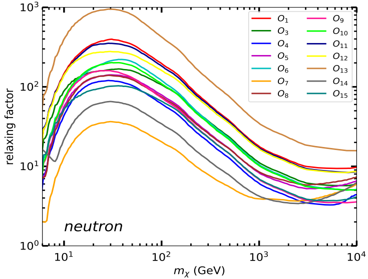
[S. Kang, AK, S. Scopel, (2212.05774)]

Halo–independent bounds on couplings



[S. Kang, AK, S. Scopel, (2212.05774)]

Relaxing factor (WIMP-neutron couplings)

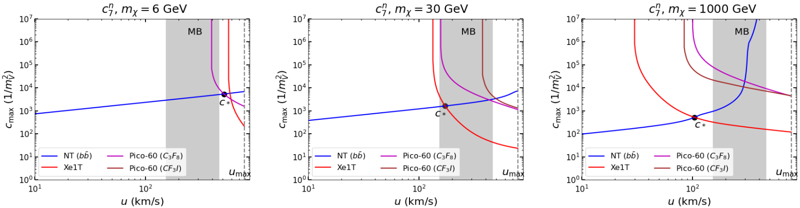


relaxing factor$=\frac{(c_i)_{\text{halo-indep.}}}{(c_i)_{\text{SHM}}}$ $\left(\equiv \frac{\sqrt{2}\, c_*}{(c_i)_{\text{SHM}}}\right)$

$(c_i)_{\text{halo–indp.}} \equiv$ halo-independent upper-limit on coupling c_i

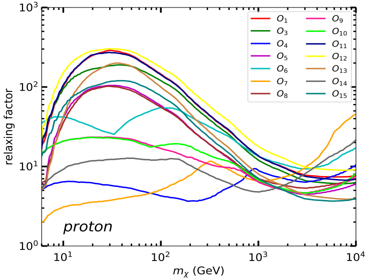
$(c_i)_{\text{SHM}} \equiv$ strongest upper-limit on c_i for a standard MB speed distribution

Explanation for the general pattern of the relaxing factor:



Small relaxing factor $\Rightarrow$ MB (SHM) is not a very optimistic assumption

Relaxing factor (WIMP-proton couplings)



- Moderate relaxing factors (in the intermediate m_χ range) for "SD" operators

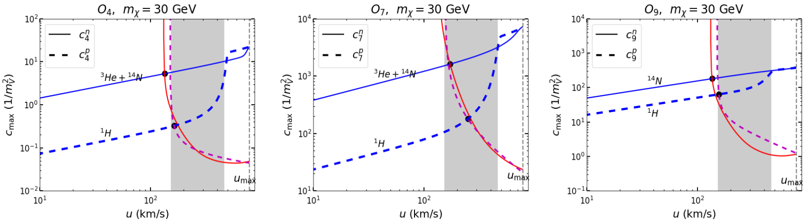
  - smallest relaxing factors for $\mathcal{O}_4$ and $\mathcal{O}_7$
  ["SD" with no momentum suppression]

  - followed by $\mathcal{O}_9$, $\mathcal{O}_{10}$ and $\mathcal{O}_{14}$
  ["SD" with q^2 suppression]

  - followed by $\mathcal{O}_6$
  ["SD" with q^4 suppression]

continued....

Explanation for the low relaxing factors (in the intermediate m_χ range) for "SD" WIMP-proton couplings:



- WIMP capture is strongly enhanced due to scattering off abundant ^{1}**H** [more prominent for $\mathcal{O}_7$ ("SD", no momentum suppression, velocity–dependent)]

  $\Rightarrow$ c_* (peak value of the convolution of NT and DD limits) is low
  $\Rightarrow$ smaller relaxing factor

Summary

- Combining direct detection and ν-search results we obtain halo-independent bounds on each coupling of the NR effective $\mathcal{H}$ that drives the WIMP(spin 1/2)–nuclei scattering
- One single coupling at a time
  (a first step towards more general scenarios involving several NR operators at a time)
- For most of the couplings the relaxation of the halo-independent bounds compared to those obtained for the SHM is relatively moderate in the low and high m_χ regimes
- More moderate values of the bound relaxation is observed for "SD"–type WIMP-proton couplings with comparatively small momentum suppression

  $\Rightarrow$ **SHM is not a very optimistic choice**
- Other cases are sensitive on the WIMP speed distribution

Thank you

Backup slides

Details of the Operator structure

operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

index k corresponding to each operator $\mathcal{O}_i$, for the velocity-independent and the velocity-dependent components parts of the WIMP response function. The power of q in the WIMP response function is in parenthesis.

Single stream method

Considering one effective coupling (c_i) at a time,
expected number of events in a DD experiment/the expected WIMP capture rate in the Sun:

$$R_{\rm exp}(c_i^2) = \int du\, f(u)\, H_{\rm exp}(c_i^2, u) \leq R_{\rm max}$$

$R_{\rm max} \equiv$ corresponding experimental bound

Define

$$c_{i\,{\rm max}}^2(u) = \frac{R_{\rm max}}{H(c_i = 1, u)}$$

Using $H(c_i^2, u) = c_i^2 H(c_i = 1, u)$,

$$H(c_{i\,{\rm max}}^2(u), u) = R_{\rm max}$$

$c_{i\,{\rm max}}(u) \equiv$ upper-limit on c_i when all WIMPs are in a single speed stream u.

[Ferrer *et al.* (JCAP09(2015)052)]

Methodology

$$R(c_i^2) = \int_0^{u_{\max}} du\, f(u)\, H(c_i^2, u) \leq R_{\max}$$

Since $H(c_i^2, u) = c_i^2 H(c_i = 1, u)$, one can write

$$\begin{aligned} R(c_i^2) &= \int_0^{u_{\max}} du\, f(u)\, H(c_i^2, u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\ \max}^2(u)} H(c_{i\ \max}^2(u), u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\ \max}^2(u)} R_{\max} \quad \leq \quad R_{\max} \end{aligned}$$

upper bound on the coupling c_i :

$$c_i^2 \leq \left[\int_0^{u_{\max}} du \frac{f(u)}{c_{i\ \max}^2(u)} \right]^{-1}$$

Methodology

$$c_i^2 \leq \left[\int_0^{u_{\max}} du \frac{f(u)}{c_{i\ \max}^2(u)}\right]^{-1}$$

$$\begin{aligned} (c^{\mathrm{NT}})^2{}_{\max}(u) &\leq c_*^2 && \text{for } 0 \leq u \leq \tilde{u} \\ (c^{\mathrm{DD}})^2{}_{\max}(u) &\leq c_*^2 && \text{for } \tilde{u} \leq u \leq u_{\max} \end{aligned}$$

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u)\right]^{-1} = \frac{c_*^2}{\delta} \qquad \text{with} \quad \delta = \int_0^{\tilde{u}} du\, f(u)$$

$$c^2 \leq c_*^2 \left[\int_{\tilde{u}}^{u_{\max}} du f(u)\right]^{-1} = \frac{c_*^2}{1-\delta} \qquad \text{with} \quad 1-\delta = \int_{\tilde{u}}^{u_{\max}} du\, f(u)$$

$\Rightarrow \delta = 1/2$

$$c^2 \leq 2\, c_*^2$$

Methodology

For a choice of a large $u_{\rm max}$ it may happen that

$$({c^{\rm DD})^2}_{\rm max}(u_{\rm max}) > c_*^2$$

[Mainly due to the suppression of the scattering amplitude by the nuclear form factor at large recoil energies (large WIMP speeds)]

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u) \right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq ({c^{\rm DD})^2}_{\rm max}(u_{\rm max}) \left[\int_{\tilde{u}}^{u_{\rm max}} du f(u) \right]^{-1} = \frac{({c^{\rm DD})^2}_{\rm max}(u_{\rm max})}{1-\delta}$$

$$c^2 \leq ({c^{\rm DD})^2}_{\rm max}(u_{\rm max}) + c_*^2$$

- A larger escape speed $u_{\rm max}$ (much larger than $\sim$ 800 km/s) is also considered.

Equilibrium between WIMP capture and annihilation in the Sun

Searches for solar ν's at neutrino telescopes (NTs) put bounds on $\Gamma_\odot$

$$\frac{d\phi_\nu}{dE_\nu} = \frac{\Gamma_\odot}{4\pi d_\odot^2} \sum_f B_f \left(\frac{dN_\nu}{dE_\nu}\right)_f$$

$$\begin{aligned} \Gamma_\odot &= (C_\odot/2)\tanh^2(t_\odot/\tau_\odot) \\ \frac{t_\odot}{\tau_\odot} &= 330 \left(\frac{C_\odot}{\mathrm{s}^{-1}}\right)^{1/2} \left(\frac{\langle\sigma v\rangle}{\mathrm{cm}^3\mathrm{s}^{-1}}\right)^{1/2} \left(\frac{m_\chi}{10\,\mathrm{GeV}}\right)^{3/4} \end{aligned}$$

For the present sensitivities of IceCube and Super-Kamiokande and assuming $\langle\sigma v\rangle \simeq 3 \times 10^{-26}$ cm^3 s^{-1}

$$\frac{t_\odot}{\tau_\odot} \gg 1 \ \ [\mathrm{Equilibrium}] \ \ \Rightarrow \Gamma_\odot \simeq C_\odot/2$$

$\Rightarrow$ The upper-limits on $\Gamma_\odot$, provided by NTs (assuming a particular WIMP annihilation channel), are converted directly into the upper-limits on $C_\odot$ and hence on the WIMP-nucleon couplings that drive $C_\odot$