New approaches to semi-invisible τ and B decays

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2023 CAU Beyond the Standard Model Workshop

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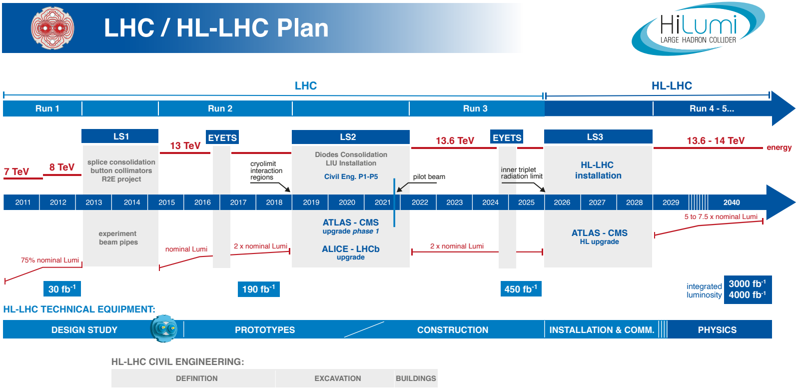
Based on D. Guadagnoli, CBP, F. Tenchini, 2106.16236, PLB (2021)

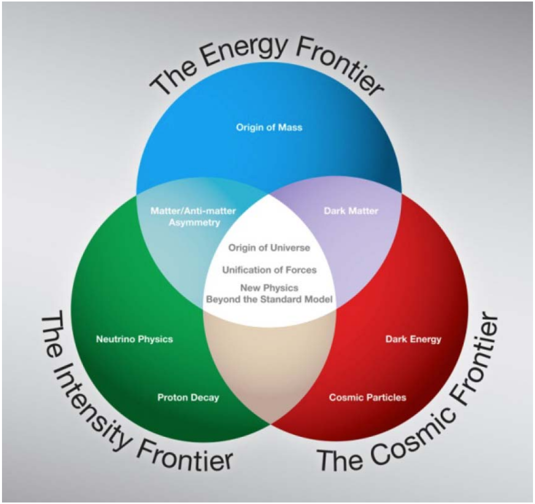
G. de Marino, D. Guadagnoli, CBP, K. Trabelsi, 2209.03387, accepted in PRD

Large Hadron Collider



https://hilumilhc.web.cern.ch/content/hl-lhc-project

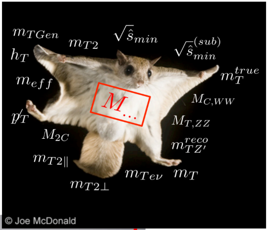
- The third run (Run 3) of the LHC has started from the last year (2022) with upgraded collision energy.

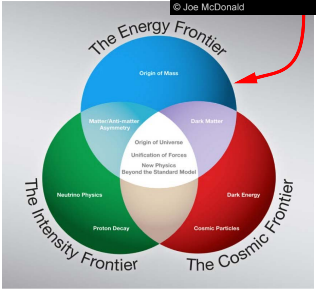


Three frontiers of research in particle physics (U.S. DOE Office of Science)

- The Energy Frontier (to produce new *heavy* particles):

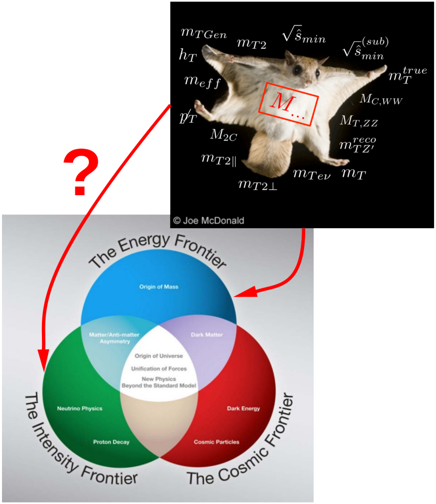
  LHC, **Future Colliders** (CEPC, FCC-ee, FCC-hh, ILC, Muon Collider, . . .)





Upper image taken from Barr *et al.*, 1105.2977 (PRD 2011)

- Basic strategy to search for new heavy particles at high-energy colliders through *dimensionality reduction* of collider data by using **kinematic variables**



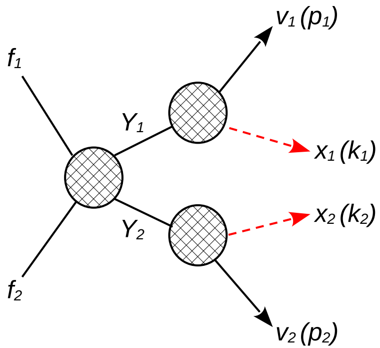
- Applications of the kinematic variables for high-energy colliders to the Intensity Frontier (to search for *rare* new physics process)?
  - Belle/Belle II, LHCb, . . . (*aka* B- and τ-factories)

Outline

1. The M_{T2} and M_2 variables
2. Searching for a new invisible particle: $\tau \to \ell + \phi$
3. Search for rare B decays: $B \to K\tau\mu$
4. Conclusions

Pair production, two invisible particles

- Pair production of heavy resonances: $\boxed{Y_1 Y_2 \to v_1 \chi_1 + v_2 \chi_2}$



  - v: (collections of) visible particles (jets, charged leptons, ...)
  - χ: (stable or long-lived) invisible particles (**neutrinos**, **dark matter**, ...)

- A common decay topology arising in many physics models:

$$\widetilde{q}\widetilde{q} \to q\widetilde{\chi}_1^0 + q\widetilde{\chi}_1^0,\ \widetilde{\ell}\widetilde{\ell} \to \ell\widetilde{\chi}_1^0 + \ell\widetilde{\chi}_1^0,\ \widetilde{g}\widetilde{g} \to q\bar{q}\widetilde{\chi}_1^0 + q\bar{q}\widetilde{\chi}_1^0 \ \textbf{(supersymmetry)},$$

$$t\bar{t} \to b\ell^+\nu + \bar{b}\ell^-\bar{\nu},\ H \to WW^* \to \ell^+\nu + \ell^-\bar{\nu} \ \textbf{(SM)}, \ldots$$

Pair production, two invisible particles: M_{T2}

$$\boxed{\begin{gathered} M_{T2} = \min_{\boldsymbol{k}_{1T},\boldsymbol{k}_{2T}\in\mathbb{R}^2}\left[\max\left\{M_{1T},\, M_{2T}\right\}\right] \\ \text{subject to } \boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} = \not{\boldsymbol{p}}_T \end{gathered}}$$

Lester, Summers, PLB (1999), Barr, Lester, Stephens, J. Phys. G (2003).

where M_{aT} are transverse masses,

$$\begin{aligned} {M_{aT}}^2 &= (E_{aT} + e_{aT})^2 - (\boldsymbol{p}_{aT} + \boldsymbol{k}_{aT})^2 \\ &= m_a^2 + M_\chi^2 + 2(E_{aT}e_{aT} - \boldsymbol{p}_{aT}\cdot\boldsymbol{k}_{aT}) \qquad (a = 1, 2) \end{aligned}$$

- Why *transverse* mass?

  — because we **don't know the longitudinal components** (k_{aL}).

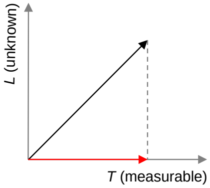
- Why *max*?

  — to access the **heaviest physics scale** of the decay

  (the mass of the heaviest parent particle mass, $\boldsymbol{M_Y}$).

- Why *min*?

  — to get an event-by-event **lower bound on M_Y**.



$$\boxed{M_{T2} \leq M_Y} \qquad (\text{if } M_\chi = M_\chi^{\text{true}})$$

Pair production, two invisible particles: M_{T2}

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https://doi.org/10.1140/epjc/s10052-019-7493-x

THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Experimental Physics

Searches for physics beyond the standard model with the M_{T2} variable in hadronic final states with and without disappearing tracks in proton–proton collisions at $\sqrt{s} = 13$ TeV

CMS Collaboration*

CERN, 1211 Geneva 23, Switzerland

Received: 8 September 2019 / Accepted: 15 November 2019 / Published online: 3 January 2020
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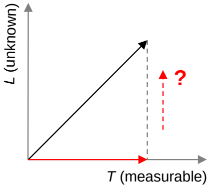
- In the current LHC analyses, it often serves as the main variable in searching for new particles from missing energy events.

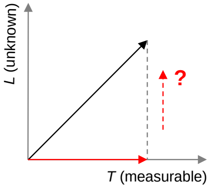
Pair production, two invisible particles: M_2

Q: Should we use the transverse mass?

Can we extend it to $(1+3)$-dim and minimize over three-momenta $(\boldsymbol{k}_{aT}, k_{aL})$?



Pair production, two invisible particles: M_2



Q: Should we use the transverse mass?

Can we extend it to $(1+3)$*-dim and minimize over three-momenta* $(\boldsymbol{k}_{aT}, k_{aL})$*?*

$$\boxed{\begin{array}{l} M_2 = \min\limits_{\boldsymbol{k}_1, \boldsymbol{k}_2 \in \mathbb{R}^3} \left[\max \left\{ M(p_1, k_1), M(p_2, k_2) \right\} \right] \\ \qquad \text{subject to } \boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} = \boldsymbol{P}_T^{\text{miss}} \end{array}}$$

where $M(p_a, k_a)$ are invariant masses,

$$M(p_a, k_a) = (p_a + k_a)^2 = (E_a + e_a)^2 - (\boldsymbol{p}_a + \boldsymbol{k}_a)^2 \quad (a = 1, 2)$$

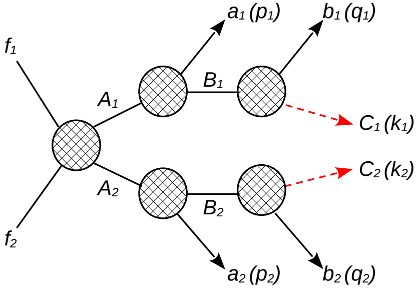
Barr *et al.*, PRD 2011, Cho *et al.*, JHEP 2014

- We can add more kinematic constraints to the definition of M_2

$\Longrightarrow$ a family of M_2 variables.

$$\boxed{\begin{array}{l} M_2 = \min\limits_{\boldsymbol{k}_1, \boldsymbol{k}_2 \in \mathbb{R}^3} \left[\max \left\{ M(p_1, k_1), M(p_2, k_2) \right\} \right] \\ \qquad \text{subject to } \begin{cases} \boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} = \boldsymbol{P}_T^{\text{miss}}, \\ \text{more constraints} \end{cases} \end{array}}$$

Pair production, two invisible particles: M_2



- For two-step symmetric decay chains (e.g., dileptonic $t\bar{t}$) where $M_{A_1} = M_{A_2}, M_{B_1} = M_{B_2}, M_{C_1} = M_{C_2}$,

$$\boxed{\begin{aligned} M_2 = \min_{k_1, k_2 \in \mathbb{R}^3} \left[\max \left\{ M\left(p_1 + q_1, k_1, M_C\right), M\left(p_2 + q_2, k_2, M_C\right) \right\} \right] \\ \text{subject to} \begin{cases} \mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{P}_T^{\text{miss}}, \\ (p_1 + q_1 + k_1)^2 = (p_2 + q_2 + k_2)^2, \\ (q_1 + k_1)^2 = (q_2 + k_2)^2. \end{cases} \end{aligned}}$$

  - **"Constrained"** numerical minimization

(Cho *et al*, 'OPTIMASS', JHEP 2016, **CBP**, 'YAM2', CPC 2021)

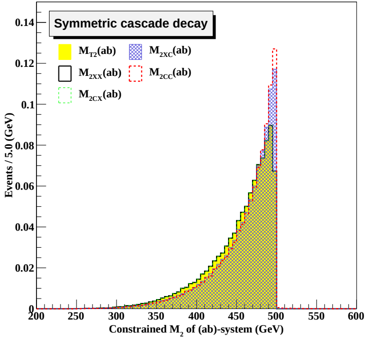
Pair production, two invisible particles: M_2

- The distribution of M_2 is bounded by M_Y ($\because$ min). Furthermore,

$$\boxed{M_{T2} \leq M_2 \leq M_Y}$$

- The addition of kinematic constraints generally **increases** the value of M_2

$\Longrightarrow$ the distribution becomes **sharper** near the upper edge.



taken from Cho *et al.*, JHEP 2014

$$M_{T2} = M_{2XX}(\text{least constrained}) \longrightarrow M_{2CC}(\text{most constrained})$$

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$\tau \to \ell + \phi$

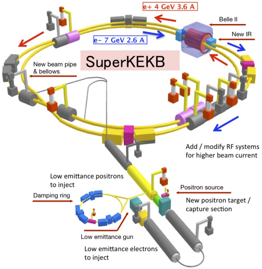
- Consider a new invisible particle ϕ in the MeV–GeV range.
  - E.g., axion-like particle,

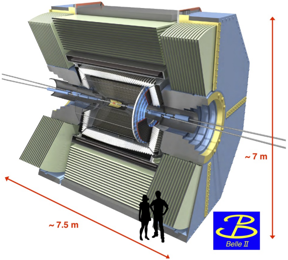
$$\mathcal{L}_{\text{int}} = \frac{\partial_\mu a}{2 f_a} \bar{\ell}_i \gamma^\mu (c_V + c_A \gamma^5) \ell_j$$

- Such a light invisible particle can be searched for from lepton flavor violating $\boxed{\tau \to \ell + \phi}$ $(\ell = e, \mu)$.
  - searches performed at

    Mark III (SLAC) (1985), **ARGUS** (DESY) (1995), and **Belle II** (2212.03634)

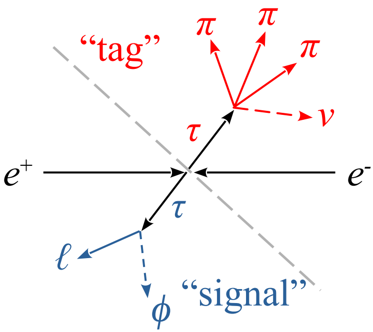
    by using $\boxed{e^+e^- \to \tau^+\tau^-}$ data.

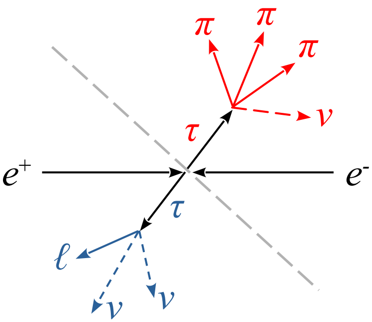




  - At SuperKEKB, $\sigma(e^+e^- \to \tau^+\tau^-) = 0.9$ nb $\Longrightarrow$ $\sim 5 \times 10^{10}$ τ pairs for $L = 50$ ab^{-1}

$\tau \to \ell + \phi$





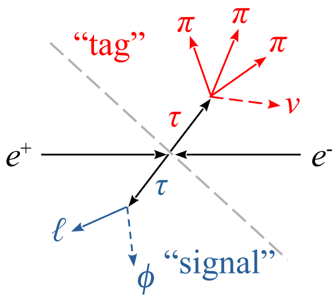
- The irreducible SM background:

$$\tau + \tau \to \ell \nu \bar{\nu} + 3\pi\nu$$

- Signal and background have the identical event topology:

$$\tau_{\text{sig}}(\to \text{visible} + \text{invisible}) + \tau_{\text{tag}}(\to \text{visible} + \text{invisible})$$

$\tau \to \ell + \phi$



- If we *could* reconstruct the momentum of $\tau_{\rm sig}$,

$$\boxed{|\boldsymbol{p}_\ell| = \frac{m_\tau^2 - m_\phi^2}{2m_\tau}} = \text{const. for a given } m_\phi \quad (\because \text{ two-body kinematics})$$

in the rest frame of $\tau_{\rm sig}$.

  - At lepton colliders, $\sqrt{s}$ is fixed (At Belle, $\sqrt{s} = 10.58$ GeV).
  In the center-of-mass (CM) frame of e^+e^- collision, $E_\tau \approx \sqrt{s}/2$, and
  $$p_\tau = \frac{\sqrt{s}}{2}\left(1, \hat{\boldsymbol{p}}_\tau \sqrt{1 - \frac{4m_\tau^2}{s}}\right) \quad (\hat{\boldsymbol{p}}_\tau : \text{ the flying direction of } \tau)$$
  - $\boxed{\hat{\boldsymbol{p}}_\tau \approx ?}$ $\Longrightarrow$ How to get approximate τ-rest frame?

$\tau \to \ell + \phi$: ARGUS and thrust method

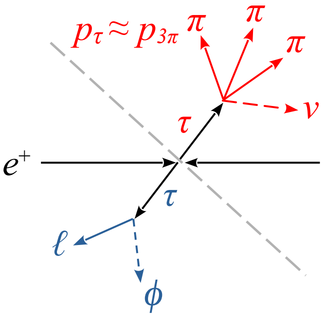
- **ARGUS** method: $\hat{\boldsymbol{p}}_\tau^{\text{sig}} = -\hat{\boldsymbol{p}}_{3\pi}$ $(\because \hat{\boldsymbol{p}}_\tau^{\text{sig}} \overset{\text{CM}}{=} -\hat{\boldsymbol{p}}_\tau^{\text{tag}} \approx -\hat{\boldsymbol{p}}_{3\pi})$

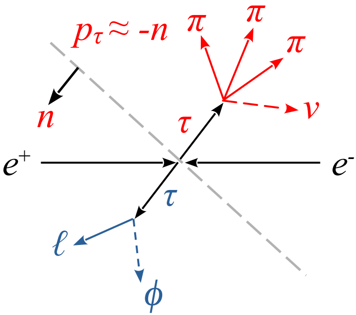
- **Thrust** method (the current state-of-the-art): $\hat{\boldsymbol{p}}_\tau^{\text{sig}} = \hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is the thrust axis of

$$T = \max_{\hat{\boldsymbol{n}}} \frac{\sum_i |\hat{\boldsymbol{n}} \cdot \boldsymbol{p}_i|}{\sum_i |\boldsymbol{p}_i|}$$

($T \to 1$ for back-to-back and $T \to 0.5$ for spherically symmetric events)

  - The thrust axis $\hat{\boldsymbol{n}}$ is used to define the *hemisphere* of each tau decay products.





$\tau \to \ell + \phi$: ARGUS and thrust method

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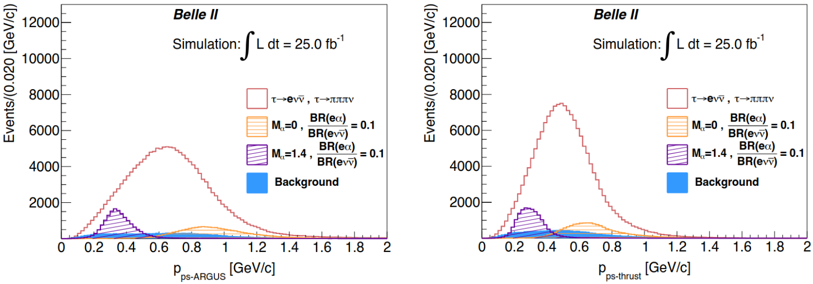
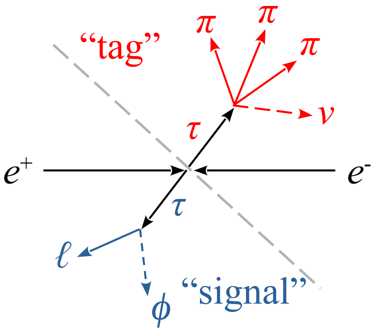
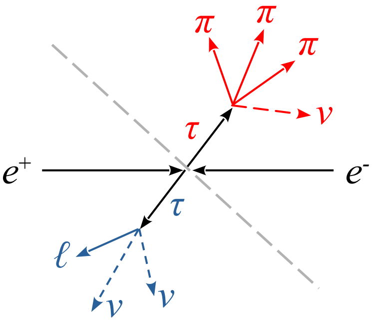


from Tenchini *et al.* (Belle II), ICHEP 2020

$\tau \to \ell + \phi$





- Signal has **two** invisibles: ϕ and ν, while background has **three** invisibles: 3 ν's. We should solve the problem of

$$\boxed{2 \text{ vs } 3}$$

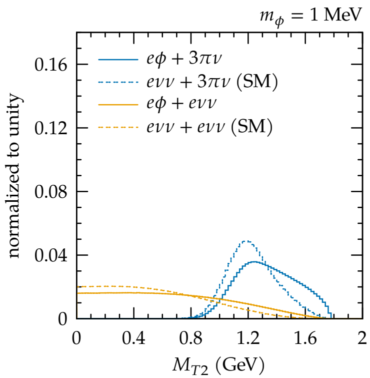
(or $\boxed{3 \text{ vs } 4}$ if $\tau_{\text{tag}} \to \ell\nu\bar{\nu}$)

- Kinematic features *sensitive to the number of invisibles*?

$\tau \to \ell + \phi$: M_{T2}

- The shape of the M_{T2} distribution depends on the number of invisibles!

(Agashe, Kim, Walker, Zhu, PRD 2011, Giudice, Gripaios, Mahbubani, PRD 2012)



- The *smaller* the number of invisibles, the *more* the M_{T2} distribution is populated *towards the upper edge*.

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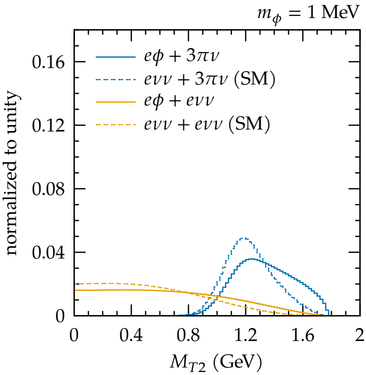


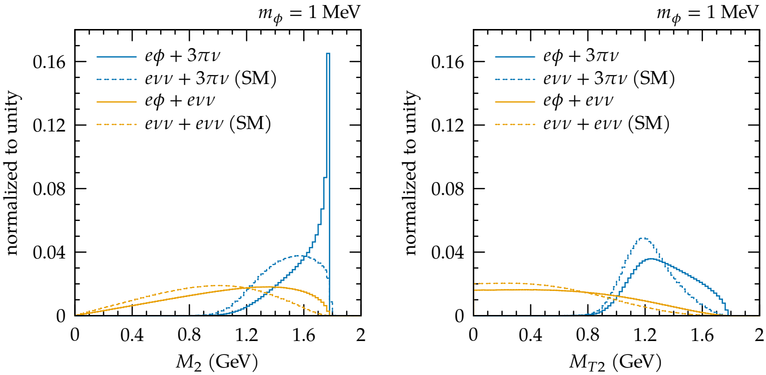
  - The *smaller* the number of invisibles, the *more* the M_{T2} distribution is populated *towards the upper edge*.

- How about M_2? — How to define M_2 for $e^+e^- \to \tau\tau \to \ell\phi + 3\pi\nu$?

$\tau \to \ell + \phi$: M_2

- M_2 for lepton collider (where $\sqrt{s}$ is fixed) Guadagnoli, CBP, Tenchini, PLB 2021:

$$\boxed{\begin{aligned} M_2 = & \min_{\boldsymbol{k}_1, \boldsymbol{k}_2 \in \mathbb{R}^3} \left[\max \left\{ M(p_1, k_1), M(p_2, k_2) \right\} \right] \\ & \text{subject to} \begin{cases} \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{P}^{\text{miss}}, \\ (p_1 + p_2 + k_1 + k_2)^2 = s. \end{cases} \end{aligned}}$$



- M_2 is an "invisible-savvy" variable.

$\tau \to \ell + \phi$: MAOS method

$$\boxed{\begin{aligned} M_2 = & \min_{\boldsymbol{k}_1, \boldsymbol{k}_2 \in \mathbb{R}^3} \left[\max \left\{ M(p_1, k_1), M(p_2, k_2) \right\} \right] \\ & \text{subject to } \begin{cases} \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{P}^{\text{miss}}, \\ (p_1 + p_2 + k_1 + k_2)^2 = s. \end{cases} \end{aligned}}$$

- The solution to the minimization can be used to an estimate of the invisible momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2$,

$$\boxed{\boldsymbol{k}_a^{\text{maos}} \approx \boldsymbol{k}_a^{\text{true}}}$$

$\Longrightarrow$ ***M_2*-assisted on-shell (MAOS)** invisible momenta

(Cho, Choi, Kim, CBP, PRD 2009, Kim, Matchev, Moortgat, Pape, JHEP 2017)

- The "M_2"-based MAOS momenta are *more efficient* than the "M_{T2}"-based counterparts.

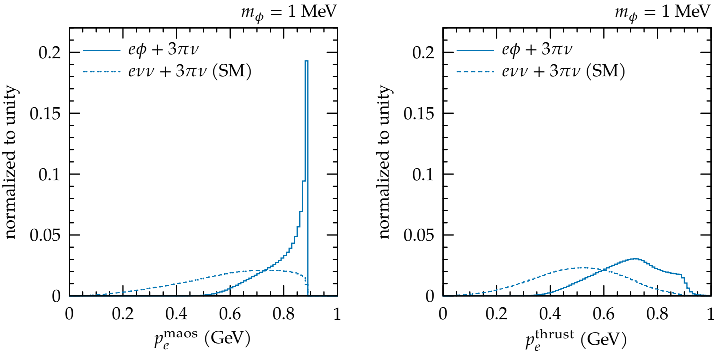
(Kim, Matchev, Moortgat, Pape, JHEP 2017)

$\tau \to \ell + \phi$: MAOS method

- With the MAOS momenta we can reconstruct $|\boldsymbol{p}_\ell|$ in the rest frame of $\tau_{\rm sig}$.

$$|\boldsymbol{p}_\ell| = \frac{m_\tau^2 - m_\phi^2}{2m_\tau}$$

for the signal events.



- The MAOS method performs much better than the thrust method.

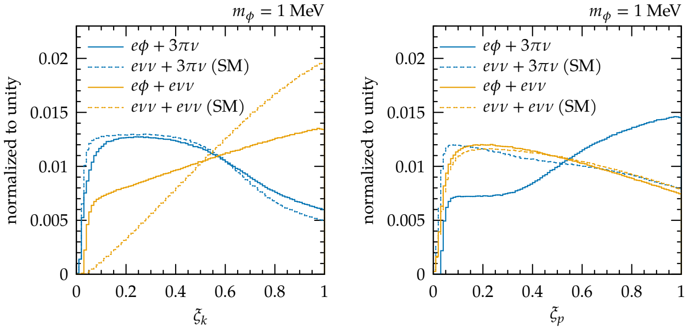
$\tau \to \ell + \phi$: Invisible-savvy variables

- Using $\boldsymbol{k}_{1,2}^{\text{maos}}$, we can construct the ratio

$$\boxed{\xi_k = \frac{\min\{|\boldsymbol{k}_1|,\,|\boldsymbol{k}_2|\}}{\max\{|\boldsymbol{k}_1|,\,|\boldsymbol{k}_2|\}} \in [0,\,1]}$$

- The distribution of ξ_k is populated around 1 for symmetric decay chains:
  this is the case for the $\ell\nu\nu + \ell\nu\nu$ background.

- We can also construct ξ_p of visible particle momenta.

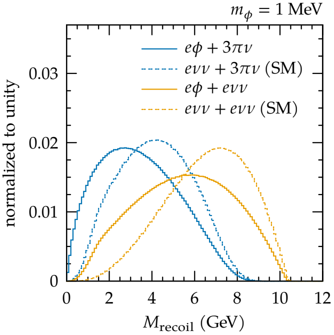


- Earlier literature proposed "max / min" ($\in [1,\,\infty]$, leading to a long distribution tail) to distinguish 2 and 3 (Agashe, Kim, Walker, Zhu, PRD 2011)

$\tau \to \ell + \phi$: Invisible-savvy variables

- We also include variables that do not require MAOS momenta.
  - $M_{\text{recoil}}{}^2 = (p^{\text{CMS}} - p_1 - p_2)^2$: invariant mass of the full invisible system

    $(p^{\text{CMS}} = p_1 + p_2 + k_1 + k_2)$.
    - Backgrounds have more invisibles than the signal $\Longrightarrow M_{\text{recoil}}(\text{bkg}) > M_{\text{recoil}}(\text{sig})$
  - $E_{\text{miss}} = |\boldsymbol{P}^{\text{miss}}| = |\boldsymbol{k}_1 + \boldsymbol{k}_2| = |\boldsymbol{p}^{\text{CMS}} - \boldsymbol{p}_1 - \boldsymbol{p}_2|$
    - the more symmetric the two decay chains (as in $\ell\nu\nu + \ell\nu\nu$), the more the invisible momenta tends to cancel



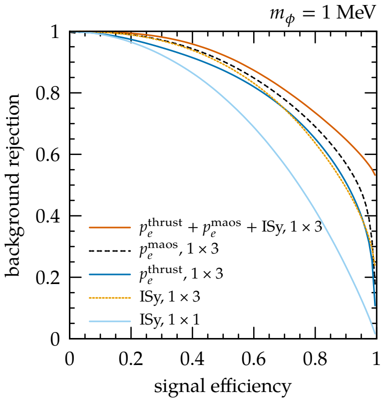
$\tau \to \ell + \phi$

- We collectively denote the kinematic variables sensitive to the number of invisibles

$$M_2,\ \xi_k,\ \xi_p,\ M_{\text{recoil}},\ E_{\text{miss}}$$

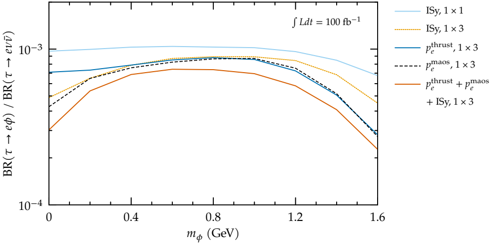
as "Invisible-Savvy" or '**ISy**' classifiers.

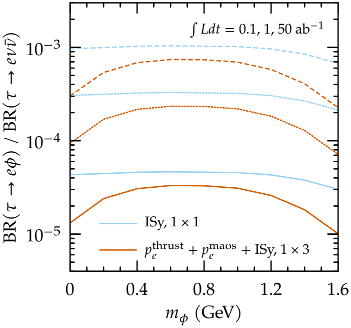
  - We also include p_ℓ^{maos} and p_ℓ^{thrust} in our analysis.



($\mathbf{1 \times 3}$: $\tau_{\text{tag}} \to \mathbf{3}\pi + \nu$, $\mathbf{1 \times 1}$: $\tau_{\text{tag}} \to \ell + \nu\bar{\nu}$)

$\tau \to \ell + \phi$





- With 3 benchmark Belle II luminosities we get

$$\mathrm{BR}(\tau \to e\phi) \leq$$

$$5.4 \times 10^{-5} \quad \left(L = 0.1\ \mathrm{ab}^{-1}\right)$$

$$1.7 \times 10^{-5} \quad \left(L = 1\ \mathrm{ab}^{-1}\right)$$

$$2.4 \times 10^{-6} \quad \left(L = 50\ \mathrm{ab}^{-1}\right)$$

for $m_\phi = 1$ MeV (95% C.L.).

- improvement by a factor of 3 than p_ℓ^{thrust}.

Outline

$B \to K\tau\mu$

- "***B* anomalies**" suggest new physics dominantly coupled to the third generation of down-type fermions (Glashow, Guadagnoli, Lane, PRL 2015).

$$\mathcal{H}_{\rm NP} = \frac{1}{\Lambda_{\rm NP}} \bar{b}_L \gamma^\lambda b_L \bar{\tau}_L \gamma_\lambda \tau_L.$$

- Flavor mixing $\Longrightarrow$ dominant effects in $b \to s$ transitions and in final states with τ including lepton-flavor violating ones.

- These observations were made properly $SU(2)_L$-compliant

  (Bhattacharya, Datta, London, Shivashankara, PLB 2015),

  thus paving the way for joint explanations of $b \to s$ and $b \to c$ data

  (See also Greljo, Isidori, Marzocca, JHEP 2015).

- Another avenue is a minimally broken $U(2)^5$ global symmetry

  (Barbieri *et al.*, EPJC 2011, JHEP 2012).

- One of the most dramatic signatures of new physics explaining the B anomalies is

$$\boxed{B^\pm \to K^\pm \tau\mu}$$

  which can be searched at Belle and Belle II.

$B \to K\tau\mu$

- At Belle, B mesons are produced in a pair, $e^+e^- \to \Upsilon(4S) \to B^+B^-$, and

$$\textbf{signal-side}: \quad B^+_{\text{sig}} \to K^+_{\text{sig}}\tau\ell_{\text{sig}}$$

$$\textbf{tag-side}: \quad B^-_{\text{tag}} \to D^0(\to K^-_{\text{tag}}\pi^+)\pi^-$$

  - τ decays into the final states including invisible neutrino(s): e.g., $\tau \to \pi\nu/\ell\nu\nu$.

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  - The search reduces to a "bump hunt" by employing $\boxed{\boldsymbol{M}_{\rm recoil}}$ for τ:

$$p_{e^+e^-} = p_{B_{\rm sig}} + p_{B_{\rm tag}} = (p_{K_{\rm sig}\ell_{\rm sig}} + p_\tau) + p_{B_{\rm tag}}$$

$$\Longrightarrow \quad M^2_{\rm recoil} = p_\tau^2 = (p_{e^+e^-} - p_{K_{\rm sig}\ell_{\rm sig}} - p_{B_{\rm tag}})^2$$

$$= \boxed{m^2_{B_{\rm tag}} + m^2_{K_{\rm sig}\ell_{\rm sig}} - 2\left(E_{B_{\rm tag}}E_{K_{\rm sig}\ell_{\rm sig}} + |\boldsymbol{p}_{B_{\rm tag}}||\boldsymbol{p}_{K_{\rm sig}\ell_{\rm sig}}|\cos\theta\right)}$$

$$\cos\theta = \hat{\boldsymbol{p}}_{B_{\rm tag}} \cdot \hat{\boldsymbol{p}}_{K_{\rm sig}\ell_{\rm sig}}$$

    - All quantities are in the CM frame. $\Longrightarrow E_{B_{\rm sig}} = E_{B_{\rm sig}} = \sqrt{s}/2$ and $\boldsymbol{p}_{B_{\rm sig}} = -\boldsymbol{p}_{B_{\rm tag}}$.
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  - "Semi-leptonic" (SL) tag: $\boldsymbol{p}_{B_{\text{tag}}}$ **CANNOT** be reconstructed due to invisible neutrino.

$$\boxed{\textbf{tag-side}: \quad B^-_{\text{tag}} \to D^0(\to K^-_{\text{tag}}\pi^+)\ell^-\bar{\nu}}$$

How can we get the $\cos\theta$ value in the SL tag?

$B \to K\tau\mu$

- $\cos\theta \Longleftarrow \boldsymbol{p}_{B_{\text{tag}}} = \boldsymbol{p}_2 + \boldsymbol{k}_2 \Longleftarrow \boldsymbol{k}_2$

$$B_{\text{sig}} B_{\text{tag}} \to V_1(p_1)\chi_1(k_1) + V_2(p_2)\chi_2(k_2)$$

  - V_i are visible, χ_i are invisible (sets of) particles.

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- **Strategy**: construct M_2,

$$M_2 = \min_{\boldsymbol{k}_1, \boldsymbol{k}_2} \left[\max \left\{ M_{(1)},\, M_{(2)} \right\} \right] \quad (M_{(i)}^2 = (p_i + k_i)^2)$$

subject to constraints,

  - and use the MAOS momenta $\boldsymbol{k}_{1,2}^{\text{maos}}$ as the estimator of $\boldsymbol{k}_{1,2}$

$$\Longrightarrow \boldsymbol{p}_{B_{\text{tag}}} = \boldsymbol{p}_2 + \boldsymbol{k}_2^{\text{maos}} \Longrightarrow \cos\theta$$

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  - *Which constraints for M_2?*

- At lepton colliders (such as Belle),

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{P}^{\text{miss}}, \quad (p_1 + p_2 + k_1 + k_2)^2 = s.$$

$$\boxed{\begin{aligned} M_{2s} &= \min_{\boldsymbol{k}_1, \boldsymbol{k}_2}\left[\max\left\{M_{(1)},\, M_{(2)}\right\}\right] \\ &\text{subject to } \begin{cases} \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{P}^{\text{miss}}, \\ (p_1 + p_2 + k_1 + k_2)^2 = s. \end{cases} \end{aligned}}$$

$B \to K\tau\mu$: M_{2sB}

- Furthermore, because we know m_B,

$$(p_1 + k_1)^2 = (p_2 + k_2)^2 = m_B^2,$$

$$\boxed{\begin{aligned} M_{2sB} = \min_{\mathbf{k}_1, \mathbf{k}_2} \left[\max \left\{ M_{(1)},\ M_{(2)} \right\} \right] \\ \text{subject to} \begin{cases} \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{P}^{\text{miss}}, \\ (p_1 + p_2 + k_1 + k_2)^2 = s, \\ (p_1 + k_1)^2 = (p_2 + k_2)^2 = m_B^2. \end{cases} \end{aligned}}$$

- The constraints reduce to *zero* the number of d.o.f.

$$\boxed{4 + 4 = 8} \text{ unknowns}$$

$$\xrightarrow{\text{choose } M_{\chi_i}} 6 \xrightarrow{\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{P}^{\text{miss}}} 3 \xrightarrow{(p_1 + p_2 + k_1 + k_2)^2 = s} 2 \xrightarrow{(p_1 + k_1)^2 = (p_2 + k_2)^2 = m_B^2} \boxed{0}$$

  - M_{2sB} is not a distribution — its minimum is a solver of the constraint equations.

$B \to K\tau\mu$: M_{2sV}

- At present and upcoming high-intensity colliders, accurate vertex information is available. $\Longrightarrow$ constraints on the flight direction of parent B,

$$\hat{\boldsymbol{v}}_{\text{sig}} = \frac{\boldsymbol{r}_{\text{sig}} - \boldsymbol{r}_0}{\left|\boldsymbol{r}_{\text{sig}} - \boldsymbol{r}_0\right|}$$

  - $\boldsymbol{r}_0$: the location of the primary vertex (interaction point),
    $\boldsymbol{r}_{\text{sig}}$: the location of the B_{sig}-decay vertex

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  - $\boldsymbol{r}_0$: the location of the primary vertex (interaction point),
    $\boldsymbol{r}_{\text{sig}}$: the location of the B_{sig}-decay vertex
  - The constraint could be implemented as

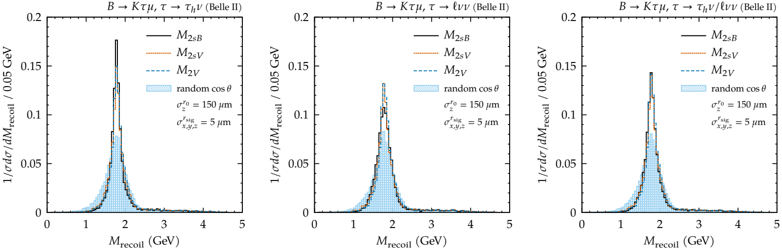
$$\arccos\left(\hat{\boldsymbol{p}}_{B_{\text{sig}}} \cdot \hat{\boldsymbol{v}}_{\text{sig}}\right) \le \delta_{\text{sig}} \quad (\boldsymbol{p}_{B_{\text{sig}}} = \boldsymbol{p}_1 + \boldsymbol{k}_1)$$

    - δ_{sig} parametrizes the experimental uncertainties of $\boldsymbol{r}_0$ and $\boldsymbol{r}_{\text{sig}}$.
    - Constraint on $\hat{\boldsymbol{v}}_{\text{tag}}$ is redundant since $\boldsymbol{p}_{B_{\text{tag}}} = -\boldsymbol{p}_{B_{\text{sig}}}$ in the CM frame.

$$\boxed{\begin{aligned} M_{2sV} &= \min_{\boldsymbol{k}_1, \boldsymbol{k}_2} \left[\max\left\{M_{(1)}, M_{(2)}\right\}\right] \\ \text{subject to} &\begin{cases} \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{P}^{\text{miss}}, \\ (p_1 + p_2 + k_1 + k_2)^2 = s, \\ \arccos\left(\hat{\boldsymbol{p}}_{B_{\text{sig}}} \cdot \hat{\boldsymbol{v}}_{\text{sig}}\right) \le \delta_{\text{sig}}. \end{cases} \end{aligned}}$$

    - We can omit the "s" constraint if it's unavailable (such as in LHCb) $\Longrightarrow M_{2V}$.
    - For simplicity, we replace the true $\hat{\boldsymbol{v}}_{\text{sig}}$ with a vector estimated by smearing $\boldsymbol{r}_0$ and $\boldsymbol{r}_{\text{sig}}$ and take $\delta_{\text{sig}} \to 0$ (inequality $\to$ equality constraint).

$B \to K\tau\mu$



- At Belle and Belle II, the beam has a non-negligible size mostly in the z axis:
  At Belle, $\sigma_z^{\rm IP} \sim 4$ mm $\Longrightarrow \hat{v}_{\rm sig}$ constraint is ineffectual.
  At Belle II, $\sigma_z^{\rm IP} \simeq 350\ \mu$m (to further improve to 150 μm).
  - M_{2sV} would be useful when the precise vertex information is available.
- With some simplifications, we get a 90% CL upper bound:

$$\boxed{\mathcal{B}(B^\pm \to K^\pm\tau^\pm\mu^\mp) \le 1.2 \times 10^{-5}}$$

using M_{2sB} alone at Belle II ($L = 710$ fb^{-1}).

  - *Cf.* If we use the *true* momenta of invisible particles, $\mathcal{B}(B^\pm \to K^\pm\tau^\pm\mu^\mp) \le 0.6 \times 10^{-5}$.

Outline

1. The M_{T2} and M_2 variables

2. Searching for a new invisible particle: $\tau \to \ell + \phi$

3. Search for rare B decays: $B \to K\tau\mu$

4. Conclusions

Conclusions

- The M_{T2} and its generalizations M_2 were conceived fro high-$\boldsymbol{p}_T$ events such as the pair productions of heavy particles.
  - We port these ideas to low-energy processes at high-intensity colliders.
- We devise a novel search strategy that we apply to pair productions of τ and B mesons,

$$\begin{aligned} &\tau \to \ell\phi \quad (\phi : \text{ light invisible particle, } m_\phi \text{ in MeV–GeV}) \\ &B \to K\tau\mu \quad (\text{rare } B \text{ decay}) \end{aligned}$$

  at Belle II.
- Our strategy has a vast domain of applicability: $B \to K\nu\nu$, $B \to \tau\mu$, etc. at Belle II and LHCb.

Thank you for your attention!

Backup

- In the leptonic decays of τ, we have two neutrinos (*cf.* $\tau \to \pi\nu$),

$$B \to K\tau\mu \to K\ell\mu + \nu\bar{\nu}$$

  - A simple ansatz is to take $\underline{m_{\nu\bar{\nu}} = 0}$.

- We can get an $\underline{\text{approximate } m_{\nu\bar{\nu}}}$ value, assuming the back-to-back momentum of B–$\bar{B}$ pair is negligible ($m_B = 5.279$ GeV $\sqrt{s} = 10.58$ GeV at Belle II).

  1. B is assumed to be at rest, $p_B = (m_B, \mathbf{0})$.
  2. Boost the B momentum to the LAB frame ($p_{e^-} = 7$ GeV, $p_{e^+} = 4$ GeV at Belle II).
  3. Then, $m^2_{\nu\bar{\nu}} = (p_B - p_{K\ell\mu})^2$.