BSM workshop @ Chung-Ang Univ.

# Quark masses and CKM hierarchies from S4’ modular flavor symmetry

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based on arXiv:2301.07439, 2302.11183 (Q and L)

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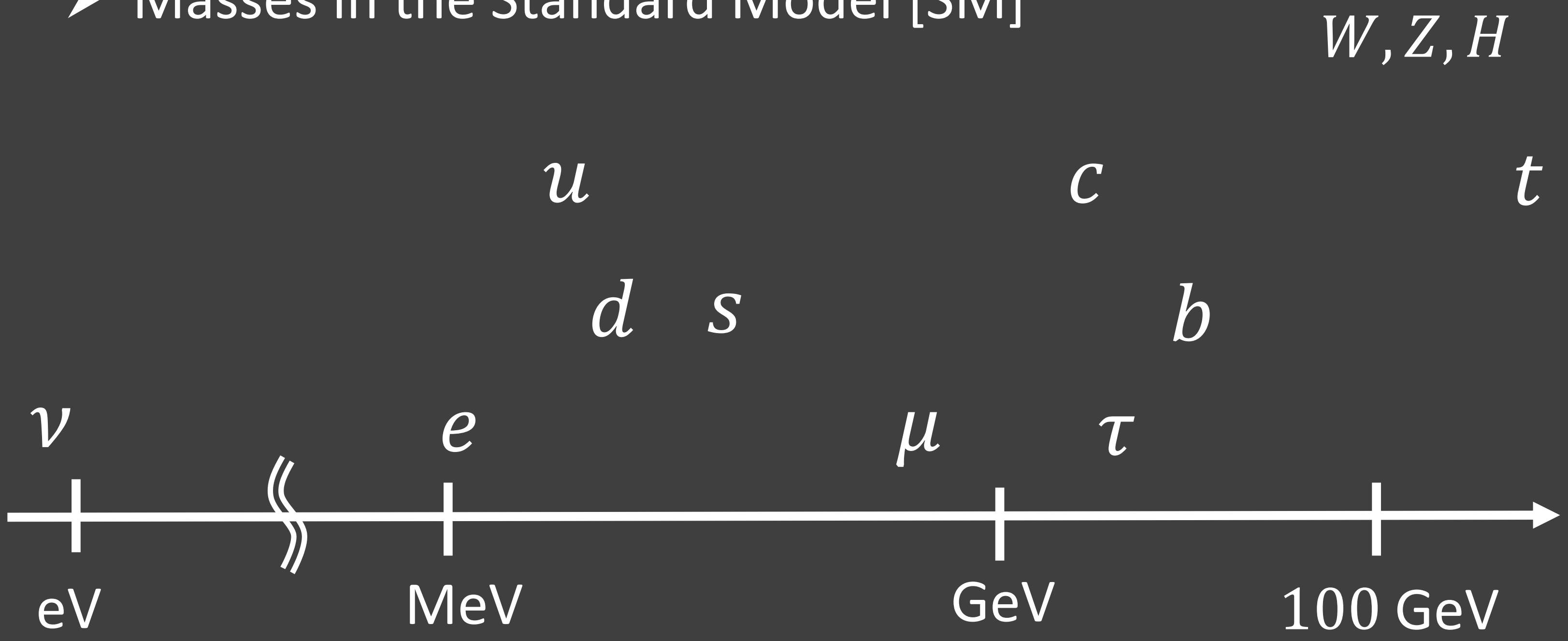
# Quark hierarchies

- Masses in the Standard Model [SM]




Why so hierarchical ?

# Quark hierarchies

- Cabibbo-Kobayashi-Maskawa [CKM] matrix

$M_u \qquad\qquad U_L^\dagger M_u U_R = \mathrm{diag}(m_u, m_c, m_t)$

$M_d \quad \rightarrow \quad V_L^\dagger M_d V_R = \mathrm{diag}(m_d, m_s, m_b)$

$\frac{g}{\sqrt{2}} W_\mu^+ \overline{u_i}\, \delta_{ij} d_j \qquad\qquad \frac{g}{\sqrt{2}} W_\mu^+ \overline{u_i}\, V_{ij}^{CKM} d_j$

diagonal W-couplings     diagonal masses

CKM matrix

$$V^{CKM} = U_L^\dagger V_L \sim \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.041 \\ 0.009 & 0.040 & 0.99 \end{pmatrix}$$

CKM also has hierarchical structure

# Aim of this work

Understand the hierarchies in quark masses and CKM matrix

➢ Modular flavor symmetry

what if Yukawa couplings (masses) are modular form ?

Altarelli, Feruglio, 2010

$$Y = Y(\tau) \rightarrow (c\tau + d)^k \rho(r) Y(\tau)$$

→
- non-Abelian discrete symmetry
- Froggatt-Nielsen [FN] mechanism by residual symmetry

→ explain the quark hierarchies

# Outline

1. Introduction
2. Modular flavor symmetry
3. Models with modular $S_4'$ flavor symmetry
4. Summary

# Modular group

- modular group $\Gamma \Leftrightarrow$ special linear group

$$\Gamma := SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \; a,b,c,d \in \mathbb{Z}\,, ad - bc = 1 \right\}$$

generators

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT$$

- action to modulus $\tau$ : complex scalar with $\mathrm{Im}\,\tau > 0$

$$\tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d} \qquad \tau \xrightarrow{S} -1/\tau \qquad \tau \xrightarrow{T} \tau + 1 \qquad \tau \xrightarrow{R} \tau$$

# Finite modular group $\Gamma_N$

- Congruence group $\Gamma(N)$ level $N \in \mathbb{N}$

$$\Gamma(N) \coloneqq \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \bmod N \right\}$$

ex) $T^N = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in \Gamma(N) \Leftrightarrow \tau \to \tau + N$

- finite modular group $\Gamma_N \coloneqq \Gamma/\Gamma(N)$

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad TR = RT, \quad T^N = 1$$

→ isomorphic to non-Abelian discrete symmetries for $N \leq 5$

$$\Gamma_2 \simeq S_3', \quad \Gamma_3 \simeq A_4', \quad \Gamma_4 \simeq S_4', \quad \Gamma_5 \simeq A_5'$$

*e.g. $\Gamma_4/\mathbb{Z}_2^R \simeq S_4$

# $\Gamma_4 \simeq S_4'$ modular symmetry Novichkov, Penedo, Petkov, Titov, 18’

- Representations under $S_4 = S_4'/\mathbb{Z}_2^R$
  - two singlets $1, 1'$, one doublet $2$ and two triplets $3, 3'$
  - there are $\mathbb{Z}_2^R$-odd representations denoted by $\hat{r}$ under $S_4'$

- Modular form of rep. $r$ and weight $k \in \mathbb{N}$

  is a holomorphic function of $\tau$ transforms as

$$Y_r^{(k)} = Y_r^{(k)}(\tau) \to (c\tau + d)^k \rho(r) Y_r^{(k)}(\tau) \qquad \tau \xrightarrow{\Gamma} \frac{a\tau + b}{c\tau + d}$$

$\rho(r)$: representation matrix of $r$

  - the number of rep. is fixed for a given weight $k$
  - one $\hat{3}$ at $k = 1$, one $2$ and one $3'$ at $k = 2$ and so on

# Residual $\mathbb{Z}_4^T$ symmetry

Novichkov, Penedo, Petkov, 21'

$$S^2 = R, \quad (ST)^3 = R^2 = 1, \quad T^4 = 1$$

- At $\tau \sim i\infty$

$\tau$ is insensitive to $\tau \xrightarrow{T} \tau + 1$ ➡ $\mathbb{Z}_4^T$ symmetry is unbroken

- Modular forms at $\mathrm{Im}\tau \gg 1$

$$Y_{\hat{3}}^{(1)}(\tau) \sim \begin{pmatrix} \sqrt{2}\epsilon(\tau) \\ \epsilon(\tau)^2 \\ -1 \end{pmatrix} \quad \begin{matrix} \mathbb{Z}_4^T\text{-charge} \\ 1 \\ 2 \\ 0 \end{matrix} \qquad \epsilon(\tau) \sim 2\exp\left(\frac{2\pi i\tau}{4}\right) \ll 1$$

powers of $\epsilon \ll 1$ is controlled by $\mathbb{Z}_4^T$ charge

➡ Froggatt-Nielsen [FN] mechanism $\left(\frac{\langle\phi\rangle}{\Lambda}\right)^n \Leftrightarrow \epsilon(\tau)^n$

natural and predictive realization of FN mech.

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# Quark hierarchies

see also models based on $A_4$ (2212. 13336), $\Gamma_6$ (2301.03737), $A_4 \times A_4 \times A_4$ (2302.03326)

- masses and CKM matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$

$$(m_d, m_s, m_b) \sim \epsilon^p (\epsilon^2, \epsilon^2, 1)\, m_t / t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & 1 & \epsilon^2 \\ 1 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$$

$\epsilon \sim 0.05$

$p = 0{,}1$

$t_\beta = v_u / v_d$

- $N = 4$ is the minimal number for the hierarchy with $\epsilon^3$
- $\mathcal{O}(0.1)$ deviations could be explained by modular forms

- representations of quarks for $p = 1$

there is only one combination of reps. for the quark hierarchy *

| | | |
|---|---|---|
| $u^c = 3$ | $d^c = 1' \oplus 1' \oplus 1'$ | $Q = 1 \oplus 2$ |
| RH up quark | RH down quark | LH doublet quark |

* assume no coexistence of $\mathbb{Z}_2^R$-even and –odd states in same quark

# Yukawa couplings

$u^c = 3 \quad d^c = 1' \oplus 1' \oplus 1' \quad Q = 1 \oplus 2$

$d_1^c \quad d_2^c \quad d_3^c \qquad q_1 \quad q_2$

- modular transformation of quarks

we assume quarks transform as $q_r^{(k)} \to (c\tau + d)^{-k} \rho(r) q_r^{(k)}$

- modular invariant Yukawa couplings

$$-\mathcal{L}_{up} = H_u \left\{ \alpha_1 q_1 \left( Y_3^{(k_{u_1})} u^c \right)_1 + \alpha_2 \left( q_2 Y_3^{(k_{u_2})} u^c \right)_1 + \alpha_3 \left( q_2 Y_{3'}^{(k_{u_2})} u^c \right)_1 \right\}$$

$$-\mathcal{L}_{down} = H_d \sum_{i=1}^{3} \left\{ \beta_{1i} q_1 \left( Y_{1'}^{(k_{d_{1i}})} d_i^c \right)_1 + \beta_{2i} \left( q_2 Y_2^{(k_{d_{2i}})} d_i^c \right)_1 \right\}$$

$(\cdots)_1$ : singlet combination

$\alpha_i, \beta_{ai}$ : $\mathcal{O}(1)$ coefficients

$k_{u_a} = k_u + k_{q_a}, k_{d_{ai}} = k_{d_i} + k_{q_a}$

# Canonical normalization

- modular invariant kinetic term

kinetic term $\frac{i\bar{q}\gamma^\mu\partial_\mu q}{(-i\tau+i\bar{\tau})^{k_q}} \rightarrow i\bar{q}\gamma^\mu\partial_\mu q$

canonical basis

Yukawa coup. $Y^{(k_Y)}(\tau) \rightarrow (2\mathrm{Im}\tau)^{k_Y/2}\, Y^{(k_Y)}$

- When $\epsilon(\tau) \ll 1$

$\epsilon(\tau) \sim 0.05 \rightarrow t \coloneqq 2\mathrm{Im}\,\tau \sim 5$ gives additional structure

another FN-like mechanism controlled by modular weights

# Yukawa structures

$$k_u = 2, \quad (k_{d_1}, k_{d_2}, k_{d_3}) = (4, 2, 0), \quad (k_{q_1}, k_{q_2}) = (2, 4)$$

- textures of Yukawa matrices $t := 2\mathrm{Im}\tau \sim 5, \epsilon \sim 0.06$

$$Y_u \sim \begin{pmatrix} \epsilon^2/t & \epsilon/t & \epsilon/t \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon^3 & 1 \end{pmatrix} \qquad Y_d \sim \begin{pmatrix} \epsilon^2 & 0 & 0 \\ \epsilon^2 t & \epsilon^2 & \epsilon^2/t \\ t & 1 & 1/t \end{pmatrix}$$

* there are vanishing elements in $Y_d$ because there is no $1'$ at $k = 2, 4$

$$\rightarrow \quad (m_u, m_c, m_t) \sim (\epsilon^3/t, \epsilon, 1)\, m_t \qquad (m_d, m_s, m_b) \sim (\epsilon^2/t^2, \epsilon^2, t) m_t \qquad V^{CKM} \sim \begin{pmatrix} 1 & 1/t & \epsilon^2/t \\ 1/t & 1 & \epsilon^2 \\ \epsilon^2/t & \epsilon^2 & 1 \end{pmatrix}$$

- Cabbibo angle and $m_s/m_d$ are explained by powers of $t$
- $|V_{ts,cb}| \sim 0.04 \gg \epsilon^2 \sim 0.004$ are explained by $Y_2^{(8)} \sim (1, 10/\sqrt{3}\epsilon^2)$

# Fitted results

$\tan\beta = 36, \tau = 2.5 + 2.2i, |\alpha_3| = 0.0041$

at GUT scale
S.Antusch, V.Maurer 1306.6879

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_3'\end{pmatrix} = \begin{pmatrix}-0.14\\ -1.7\\ e^{0.0074i}\\ -0.69\end{pmatrix}$$

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\beta_{11}\\ \beta_{21}^1\\ \beta_{21}^2\\ \beta_{22}\\ \beta_{23}\end{pmatrix} = \begin{pmatrix}-3.1\\ 0.14\\ 1.6\\ -0.13\\ 0.28\end{pmatrix}$$

→

| obs. | value | center | error |
|---|---|---|---|
| $y_u/10^6$ | 2.8 | 2.7 | 1.3 |
| $y_c/10^3$ | 1.487 | 1.422 | 0.095 |
| $y_t$ | 0.5139 | 0.5139 | 0.0084 |
| $y_d/10^4$ | 1.9935 | 1.9935 | 0.0087 |
| $y_s/10^3$ | 3.946 | 3.946 | 0.014 |
| $y_b$ | 0.2282 | 0.2282 | 0.0001 |
| $s_{12}$ | 0.2274 | 0.2274 | 0.0007 |
| $s_{23}/10^2$ | 3.945 | 3.942 | 0.065 |
| $s_{13}/10^3$ | 3.43 | 3.43 | 0.13 |
| $\delta_{\text{CP}}$ | 1.215 | 1.208 | 0.054 |

our model exp. error

- overall size of the coefficients is $\mathcal{O}(0.001)$
- complex phase is necessary in the coefficients
- the sizes of coefficients are in $[0.13, 3.1]$, ratio is 23

} can be explained by another $S_3$

# Quark and lepton hierarchies 2302.11183

$u^c = 1 \oplus 1 \oplus \hat{1}'$ RH up quark

$d^c = 1 \oplus 1 \oplus 1$ RH down quark

$Q = 3$ LH doublet quark

$L = 1 \oplus 1 \oplus 1$ LH doublet lepton

$e^c = 3$ RH charged lepton

- masses and CKM /PMNS matrix

$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1)\, m_t$$
$$(m_d, m_s, m_b) \sim (m_e, m_\mu, m_\tau) \sim (\epsilon^3, \epsilon^2, \epsilon)\, m_t / t_\beta$$

$$V^{CKM} \sim \begin{pmatrix} 1 & \epsilon & \epsilon^2 \\ \epsilon & 1 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} \qquad V^{PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

good agreement with the measured values

# Summary

- Modular flavor symmetry realizes
  - generalized non-Abelian discrete symmetry
  - hierarchical Yukawa matrix via Froggatt-Nielsen mechanisms

- $S_4'$ model
  - maybe minimal possibility to realize the quark hierarchy
  - successfully explains the quark masses and CKM mixing
  - $S_4' \times S_3$ can explain the $\mathcal{O}(0.1)$ coefficients and spontaneous CPV
  - we found $S_4'$ model for both quarks and leptons

## Thank you

# backups

# # of representations under $S_4'$

| weight | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $1$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| $1'$ | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| $2$ | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 2 |
| $3$ | 1 | 0 | 1 | 1 | 2 | 1 | 2 | 2 | 3 | 2 | 3 |
| $3'$ | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 |

* reps. for odd weights are hatted ones

there are $2k+1$ independent modular forms at a weight $k$

# Yukawa matrices

- model for $p = 1$

$$Y_u = \begin{pmatrix} \alpha_1[Y_3^{(4)}]_1 & \alpha_1[Y_3^{(4)}]_3 & \alpha_1[Y_3^{(4)}]_2 \\ -2\alpha_2[Y_3^{(6)}]_1 & \alpha_2[Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY(6)}]_2 & \alpha_2[Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY(6)}]_3 \\ -2\alpha_3^{iY}[Y_{3'}^{iY(6)}]_1 & \alpha_3^{iY}[Y_{3'}^{iY(6)}]_3 - \sqrt{3}\alpha_2[Y_3^{(6)}]_2 & \alpha_3^{iY}[Y_{3'}^{iY(6)}]_2 - \sqrt{3}\alpha_2[Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11}Y_{1'}^{(6)} & 0 & 0 \\ -\beta_{21}^{iY}[Y_2^{iY(8)}]_2 & -\beta_{22}[Y_2^{(6)}]_2 & -\beta_{23}[Y_2^{(4)}]_2 \\ \beta_{21}^{iY}[Y_2^{iY(8)}]_1 & \beta_{22}[Y_2^{(6)}]_1 & \beta_{23}[Y_2^{(4)}]_1 \end{pmatrix},$$

- model for $p = 0$

$$Y_u = \begin{pmatrix} \alpha_1[Y_3^{(6)}]_1 & \alpha_1[Y_3^{(6)}]_3 & \alpha_1[Y_3^{(6)}]_2 \\ -2\alpha_2[Y_3^{(6)}]_1 & \alpha_2[Y_3^{(6)}]_3 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY(6)}]_2 & \alpha_2[Y_3^{(6)}]_2 + \sqrt{3}\alpha_3^{iY}[Y_{3'}^{iY(6)}]_3 \\ -2\alpha_3^{iY}[Y_{3'}^{iY(6)}]_1 & \alpha_3^{iY}[Y_{3'}^{iY(6)}]_3 - \sqrt{3}\alpha_2[Y_3^{(6)}]_2 & \alpha_3^{iY}[Y_{3'}^{iY(6)}]_2 - \sqrt{3}\alpha_2[Y_3^{(6)}]_3 \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \beta_{11}Y_{\hat{1}}^{(9)} & 0 & 0 \\ \beta_{21}[Y_{\hat{2}}^{(9)}]_1 & \beta_{22}[Y_{\hat{2}}^{(7)}]_1 & \beta_{23}[Y_{\hat{2}}^{(5)}]_1 \\ \beta_{21}[Y_{\hat{2}}^{(9)}]_2 & \beta_{22}[Y_{\hat{2}}^{(7)}]_2 & \beta_{23}[Y_{\hat{2}}^{(5)}]_2 \end{pmatrix}.$$

# Model for $p = 0$

$u^c = 3 \quad d^c = \hat{1}' \oplus \hat{1}' \oplus \hat{1}' \quad Q = 1 \oplus 2$

$d_1^c \quad d_2^c \quad d_3^c \qquad q_1 \quad q_2$

$$k_u = 2, \quad (k_{d_1}, k_{d_2}, k_{d_3}) = (5\,3, 1), \quad (k_{q_1}, k_{q_2}) = (4, 4)$$

$$\tan\beta = 1.6, \tau = 1.5 + 2.7i, |\alpha_3| = 0.0013$$

at GUT scale

S.Antusch, V.Maurer 1306.6879

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_3'\end{pmatrix} = \begin{pmatrix}-0.27\\ -1.7\\ e^{-3.1i}\\ -1.4\end{pmatrix}$$

$$\frac{1}{|\alpha_3|}\begin{pmatrix}\beta_{11}\\ \beta_{21}\\ \beta_{22}\\ \beta_{23}\end{pmatrix} = \begin{pmatrix}-6.9\\ 0.13\\ 0.28\\ 0.41\end{pmatrix}$$

→

| obs. | value | center | error |
|---|---|---|---|
| $y_u/10^6$ | 2.9 | 2.9 | 1.3 |
| $y_c/10^3$ | 1.560 | 1.508 | 0.095 |
| $y_t$ | 0.5464 | 0.5464 | 0.0084 |
| $y_d/10^6$ | 9.00 | 9.06 | 0.87 |
| $y_s/10^4$ | 1.73 | 1.79 | 0.14 |
| $y_b/10^2$ | 1.011 | 0.994 | 0.013 |
| $s_{12}$ | 0.2274 | 0.2274 | 0.0007 |
| $s_{23}/10^2$ | 3.991 | 3.989 | 0.065 |
| $s_{13}/10^3$ | 3.47 | 3.47 | 0.13 |
| $\delta_{CP}$ | 1.204 | 1.208 | 0.054 |

our model exp. error

- similar to the model for $p = 1$, but with $\tan\beta \sim 1$
- the sizes of coefficients are in $[0.13, 6.9]$, ratio is 50

# $S_3$ symmetry

the coefficients have the “hierarchy” structure

$$\alpha_1 \ll \alpha_2, \alpha_3 \qquad \beta_{11} \gg \beta_{21}, \beta_{22}, \beta_{23}$$

first low is smaller (larger) than others in up (down) quarks

- $S_3$ model with another modulus $\tau_2$

$d^c, q_1$: singlet 1 $\qquad$ $u^c, q_2$: non-trivial singlet $1'$

$$\rightarrow \quad Y_u \sim \begin{pmatrix} \epsilon_2 & \epsilon_2 & \epsilon_2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{matrix} \alpha_1 \\ \\ \alpha_2, \alpha_3 \end{matrix} \qquad Y_d \sim \begin{pmatrix} 1 & 1 & 1 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \end{pmatrix} \begin{matrix} \beta_{11} \\ \\ \beta_{21}, \beta_{22}, \beta_{23} \end{matrix}$$

the hierarchy is explained by $\epsilon_2(\tau_2) \sim 0.1$

# Spontaneous CP violation

- from $S_4'$

$\epsilon(\tau) \sim 2\exp\left(\frac{2\pi\tau i}{4}\right)$ is a complex parameter

However, it induces unphysical phases in CKM matrix up to $\epsilon^3$

→ spontaneous CP violation is not enough

- from $S_3$

CPV from $\epsilon_2 \sim 0.1$ does not physical phase up to $\epsilon_2$

However, $\epsilon_2^2 \sim 0.01$ ($\mathbb{Z}_2^T$ neutral) is enough for CKM phase

→ moderate CP violation from $S_3$

# Inputs in the model for Q and L

We found the benchmark point which can explained the experimental values within $0.9\sigma$. The values are given by $\tan\beta = 14.1755$, $\tau = 4.0000 + 3.0744i$, $|\alpha_3| = 8.6886 \times 10^{-4}$,

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}\alpha_1\\ \alpha_2^1\\ \alpha_2^2\\ \alpha_3^1\\ \alpha_3^2\end{pmatrix} = \begin{pmatrix}-1.0336\\ 1.2757\\ -2.2480\\ e^{0.5391i}\\ 4.8979\end{pmatrix}, \quad \frac{1}{|\alpha_3^1|}\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3^1\\ \beta_3^2\end{pmatrix} = \begin{pmatrix}2.5696\\ -3.3719\\ 4.0290\\ -0.8087\end{pmatrix}, \tag{30}$$

$$\frac{1}{|\alpha_3^1|}\begin{pmatrix}\gamma_1\\ \gamma_2^1\\ \gamma_2^2\\ \gamma_3^1\\ \gamma_3^2\end{pmatrix} = \begin{pmatrix}4.0267\\ 4.6090\\ 0.8186\\ 4.4630 \times e^{-2.9326i}\\ -0.8028\end{pmatrix}, \quad \frac{1}{|\alpha_3^1|}\begin{pmatrix}c_{11}\\ c_{22}\\ c_{33}\\ c_{12}\\ c_{13}\\ c_{23}\end{pmatrix} = \begin{pmatrix}-1.3694\\ 1.2175\\ 1.3739\\ 1.4887\\ -6.0655\\ -1.1843\end{pmatrix}$$

→ coefficients are in [0.8, 6.1]