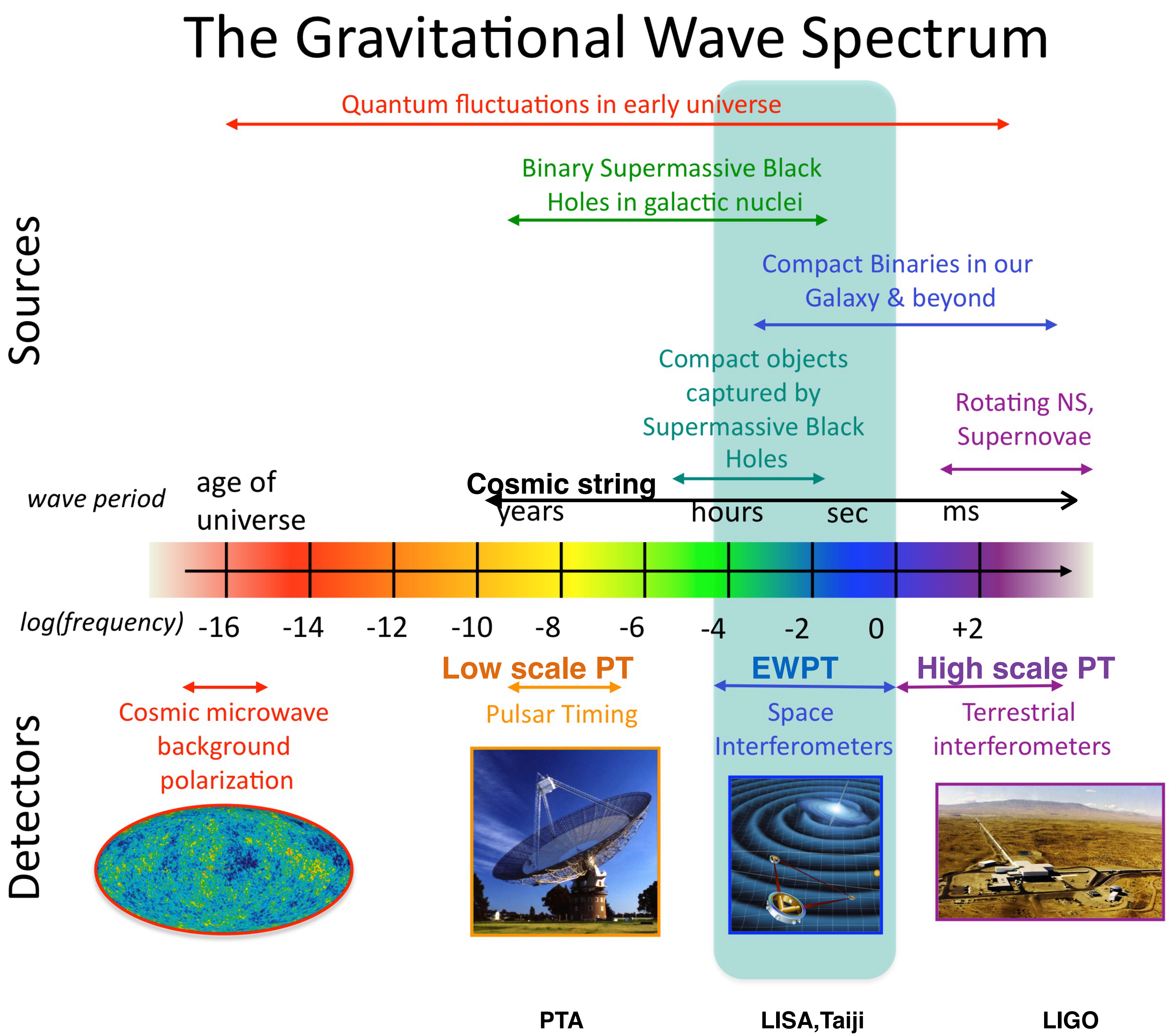
Constraints on first-order phase transitions and relevant particle physics models

Ligong Bian

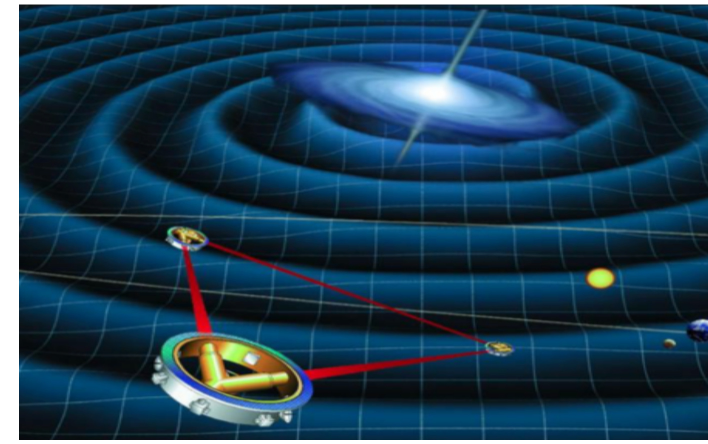
2023/11/9

The 2023 Chung-Ang University Beyond the Standard Model (CAU BSM) Workshop


The Gravitational Wave Spectrum
Quantum fluctuations in early universe
Binary Supermassive Black Holes in galactic nuclei
Compact Binaries in our Galaxy & beyond
Compact objects captured by Supermassive Black Holes
Rotating NS, Supernovae
Sources
wave period
age of universe
Cosmic string
years
hours
sec
ms
log(frequency)
-16
-14
-12
-10
-8
-6
-4
-2
0
+2
Low scale PT
EWPT
High scale PT
Cosmic microwave background polarization
Pulsar Timing
Space Interferometers
Terrestrial interferometers
Detectors
PTA
LISA,Taiji
LIGO

NEW physics&GW from 1st PT

PTA,LIGO,LISA,天琴,太极,⋯



Particle physics model

↓

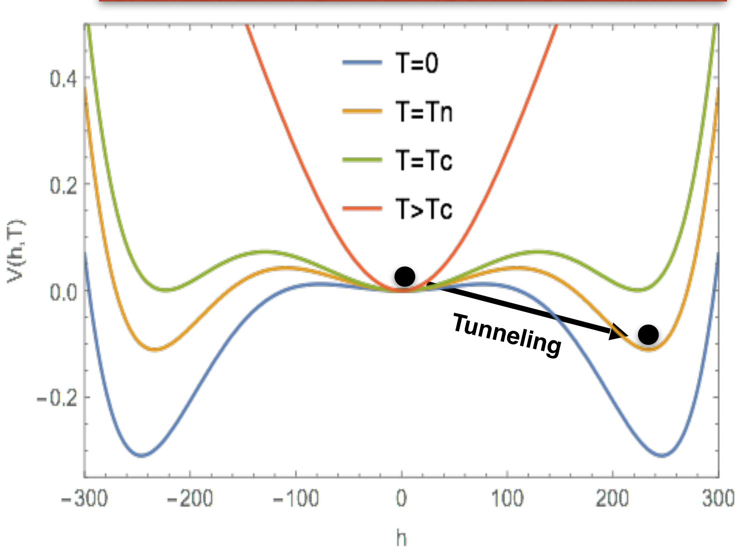
PT parameters

Effective action $\rightarrow \beta,\ H_*$

Energy budget $\rightarrow \alpha,\ \kappa(\alpha, v_w)$

Bubble wall dynamics $\rightarrow v_w$

Thermal field theory



→ **GW power spectrum**

Numerical simulations $\rightarrow$ $h^2\Omega_{\rm GW}(f; H_*, \alpha, \beta, v_w)$

Lattice simulation

→ **LISA sensitivity**

Configuration + noise level $\rightarrow$ $h^2\Omega_{\rm sens}(f)$

↓

Signal-to-noise ratio

SNR

Chiara Caprini *et al* JCAP03(2020)024

SGWB spectrum

- **Bubble collisions**

$$\Omega_{\rm col}h^2 = 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_b^3}{0.42+v_b^2}\right)\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}$$

peak frequency: $$f_{\rm env} = 16.5\times10^{-6}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$$

- **Sound Wave**

$$\Omega h^2_{\rm sw}(f) = 2.65\times10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1}v_b\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

phase transition duration: $$\tau_{sw} = min\left[\frac{1}{H_*},\frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$$

Root-mean-square four-velocity of the plasma: $$\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$$

peak frequency: $$f_{\rm sw} = 1.9\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$$

- **MHD turbulence**

$$\Omega h^2_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}v_b\frac{(f/f_{\rm turb})^3(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

peak frequency: $$f_{\rm turb} = 2.7\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$$

See the talk of David Weir

SFOEWPT simulation

Field basis equation of motion

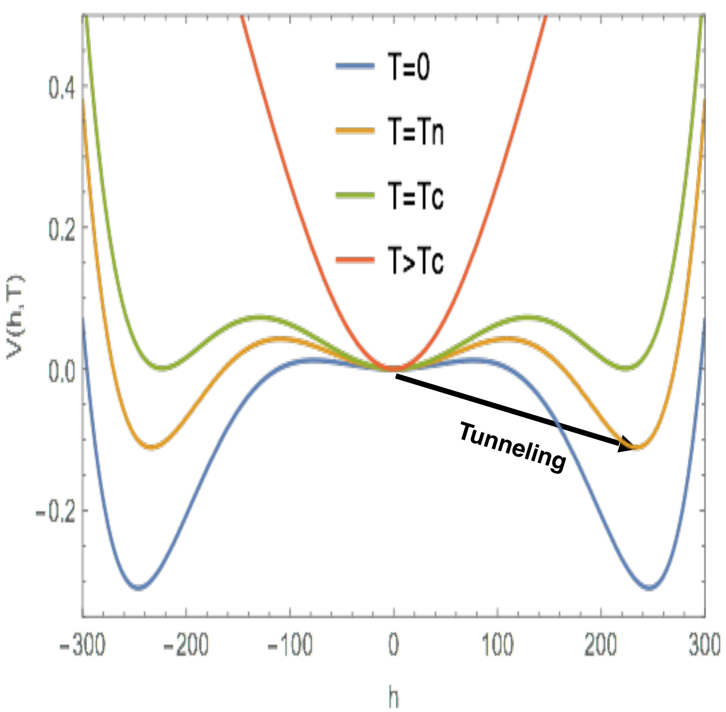
$$
\begin{aligned}
\partial_0^2 \Phi &= D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\
\partial_0^2 B_i &= -\partial_j B_{ij} + g' \,\mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi]\,,
\end{aligned}
$$

$$
\begin{aligned}
&\partial_0 \partial_j B_j - g' \,\mathrm{Im}[\Phi^\dagger \partial_0 \Phi] = 0, \\
&\partial_0 \partial_j W_j^a + g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] = 0.
\end{aligned}
$$

Lattice implementation

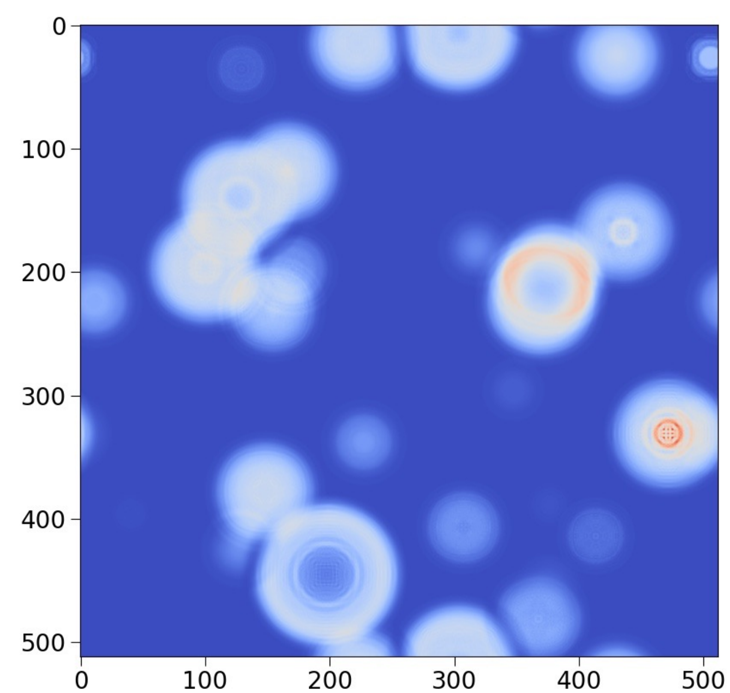
$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t \Big\{ \frac{1}{\Delta x^2} \sum_i [U_i(t,x) V_i(t,x) \Phi(t, x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t, x-i) V_i^\dagger(t, x-i) \Phi(t, x-i)] - \frac{\partial U}{\partial \Phi^\dagger} \Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2, x)] =& \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g'}{\Delta x} \mathrm{Im}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) \Phi(t,x)] \\
& - \frac{2}{g' \Delta x^3} \sum_i \mathrm{Im}[V_k(t,x) V_i(t, x+k) V_k^\dagger(t, x+i) V_i^\dagger(t,x) \\
& + V_i(t, x-i) V_k(t,x) V_i^\dagger(t, x+k-i) V_k^\dagger(t, x-i)] \Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g}{\Delta x} \mathrm{Re}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) i\sigma^m \Phi(t,x)] \\
& - \frac{1}{g \Delta x^3} \sum_i \mathrm{Tr}[i\sigma^m U_k(t,x) U_i(t, x+k) U_k^\dagger(t, x+i) U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x) U_i^\dagger(t, x+k-i) U_k^\dagger(t, x-i) U_i(t, x-i)] \Big\},
\end{aligned}
$$

Finite-T Veff

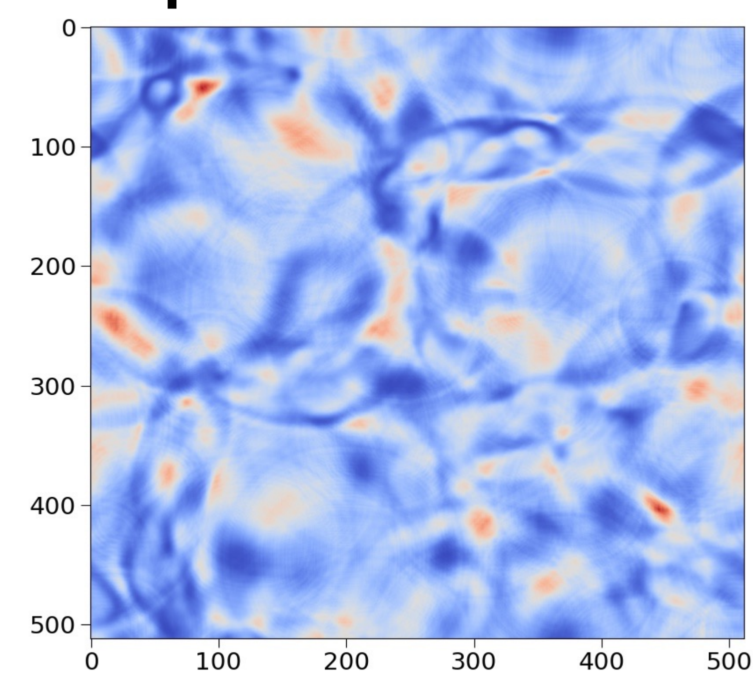


PT temperature (T_*)
duration (β)
strength (α)

Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

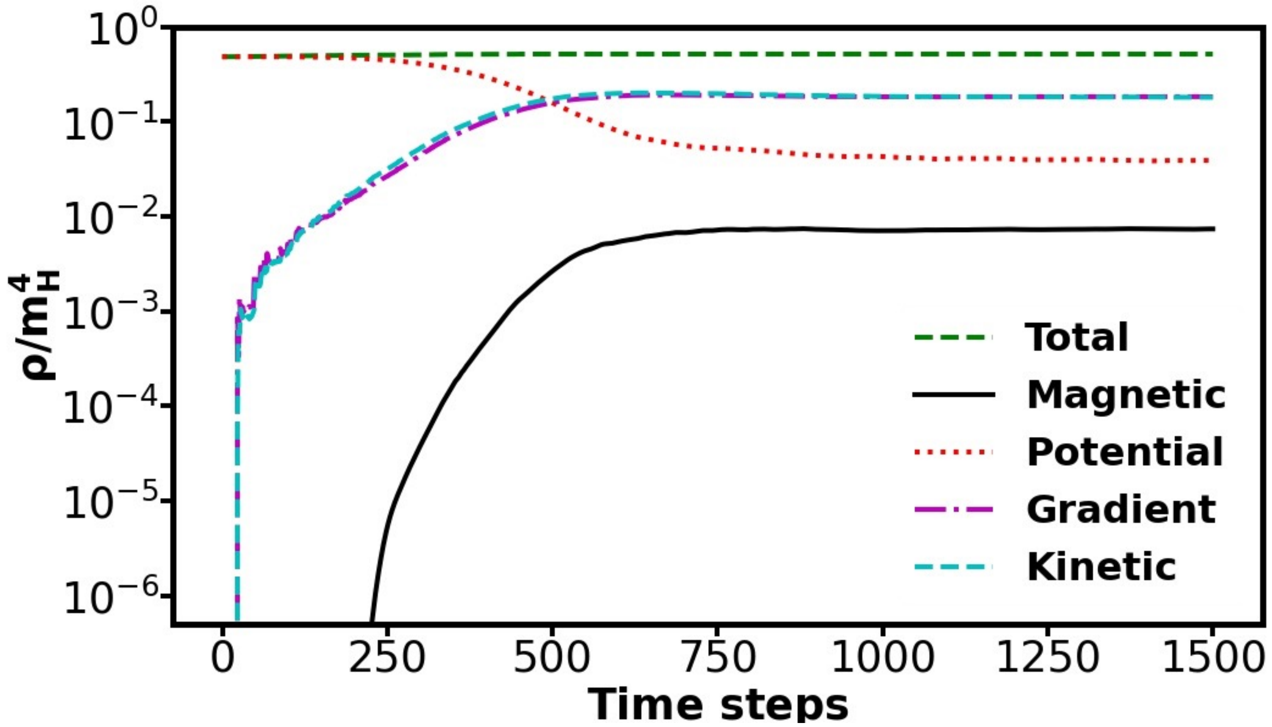
SFOEWPT simulation

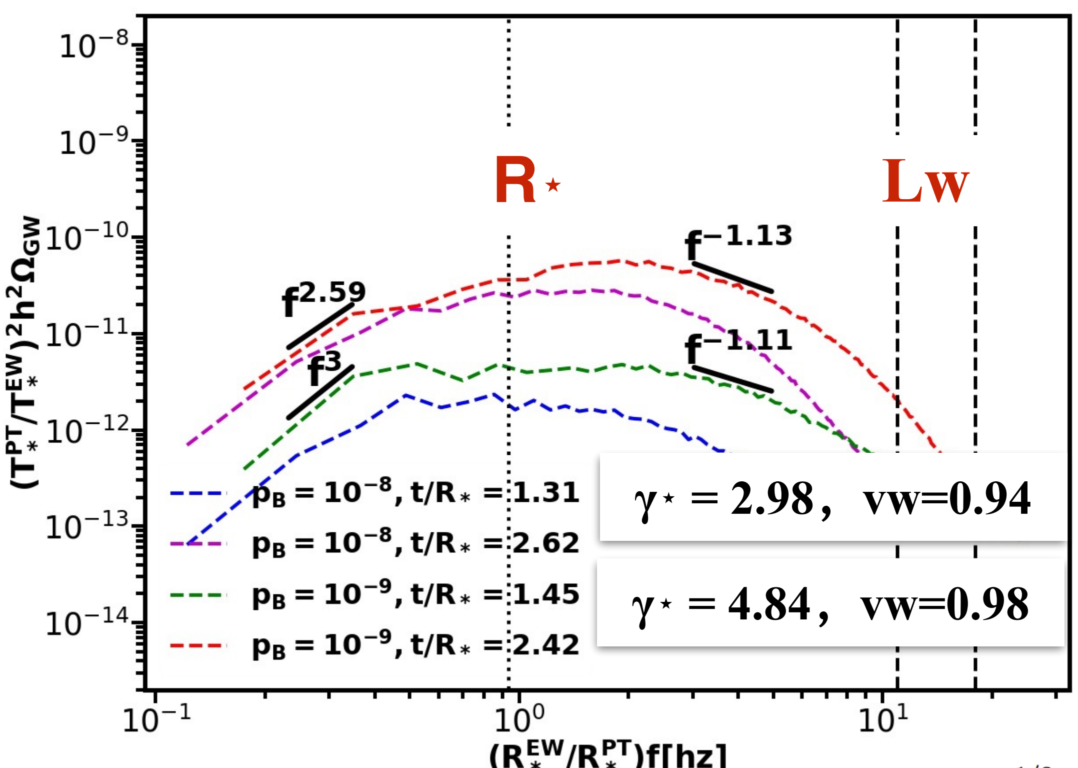
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\rm TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}{\rm Re}[(\partial_i\Phi^\dagger\partial^i\Phi)^2]$$

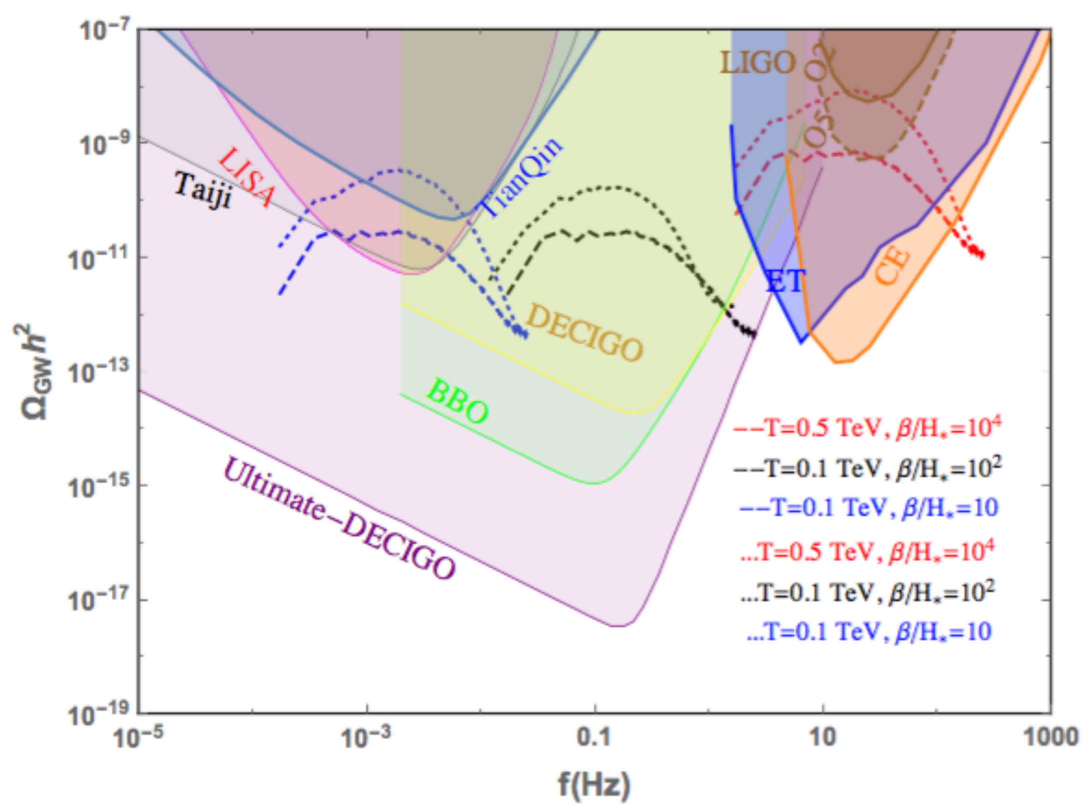
$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

$$\frac{d\Omega_{\rm gw}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





γ⋆ = R⋆ /(2Rc) $R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

▸ Collider&GWs searches of 1st EWPT

■ Higgs&GWs

☑ **SM+Scalar Singlet**

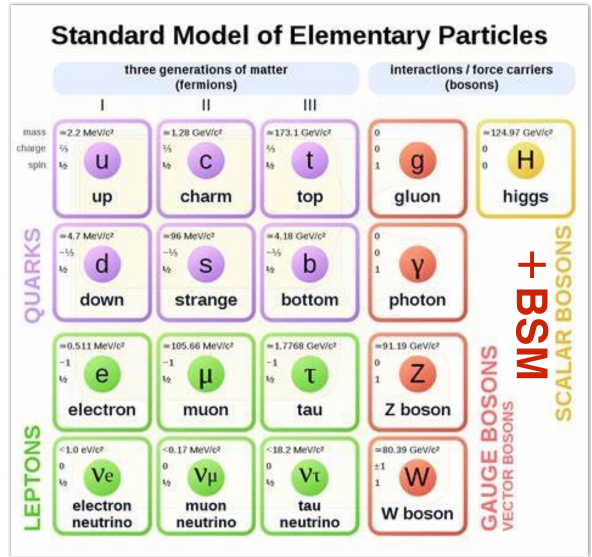
Profumo, Ramsey-Musolf, Wainwright, Winslow 14, **Bian**, Huang, Shu 15， Cheng, **Bian** 17, **Bian**, Tang 18,Chen, Li, Wu, **Bian**,19…

☑ **SM+Scalar Doublet**

Dorsch, Huber, Mimasu,No.14,Bernon, **Bian,** Jiang 17, **Bian**, Liu 18, Huang, Yu, 18,…

☑ **SM+Scalar Triplet**

Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19, Ramsey-Musolf etal 21, Zhou, **Bian**,Du,22,…



☑ **Composite Higgs**

Bruggisser, Harling, Matsedonskyi, Servant,18, **Bian**,Wu,Xie 19, **Bian**,Wu,Xie 20,…

☑ **NMSSM**

Bi, **Bian**, Huang, Shu, Yin 15, **Bian,** Guo, Shu 17,Baum, Carena, Shah, Wagner, Wang 20, …

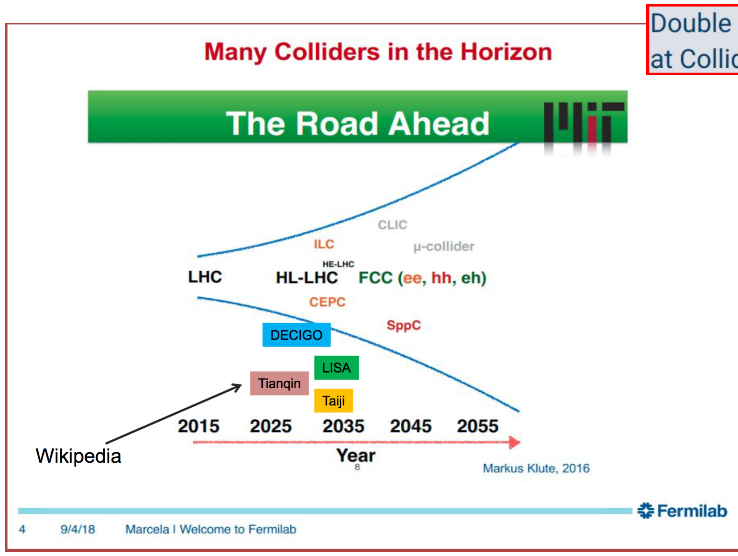
☑ **SMEFT**

Cao, Huang, Xie, & Zhang 17, Zhou, **Bian**, Guo 19, Cai,Hashino,Wang,Yu,22…

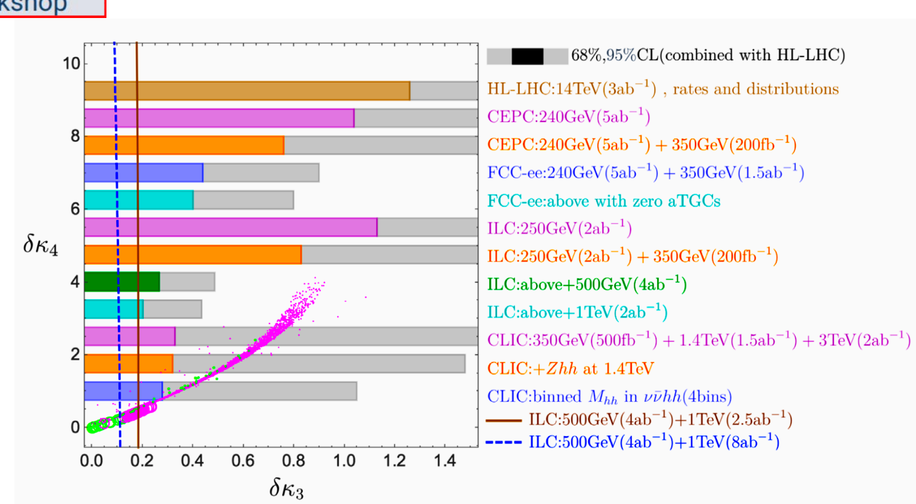
Symmetry breakdown

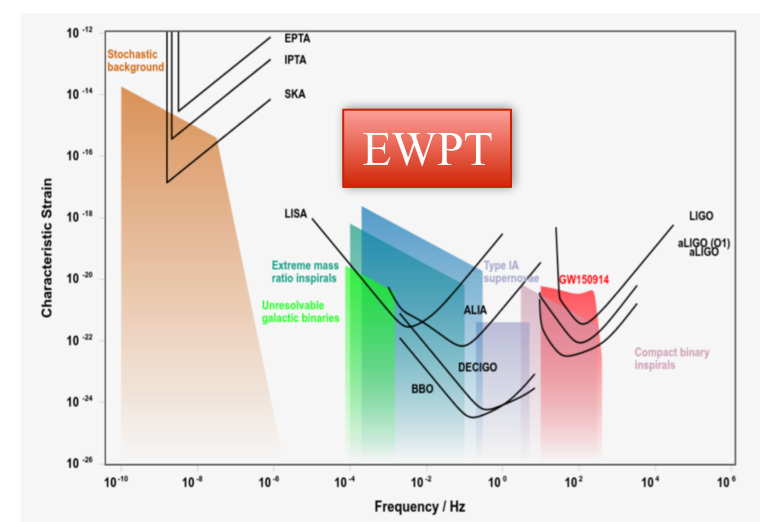
Symmetry breaking process

Double Higgs Production at Colliders Workshop



$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$





C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

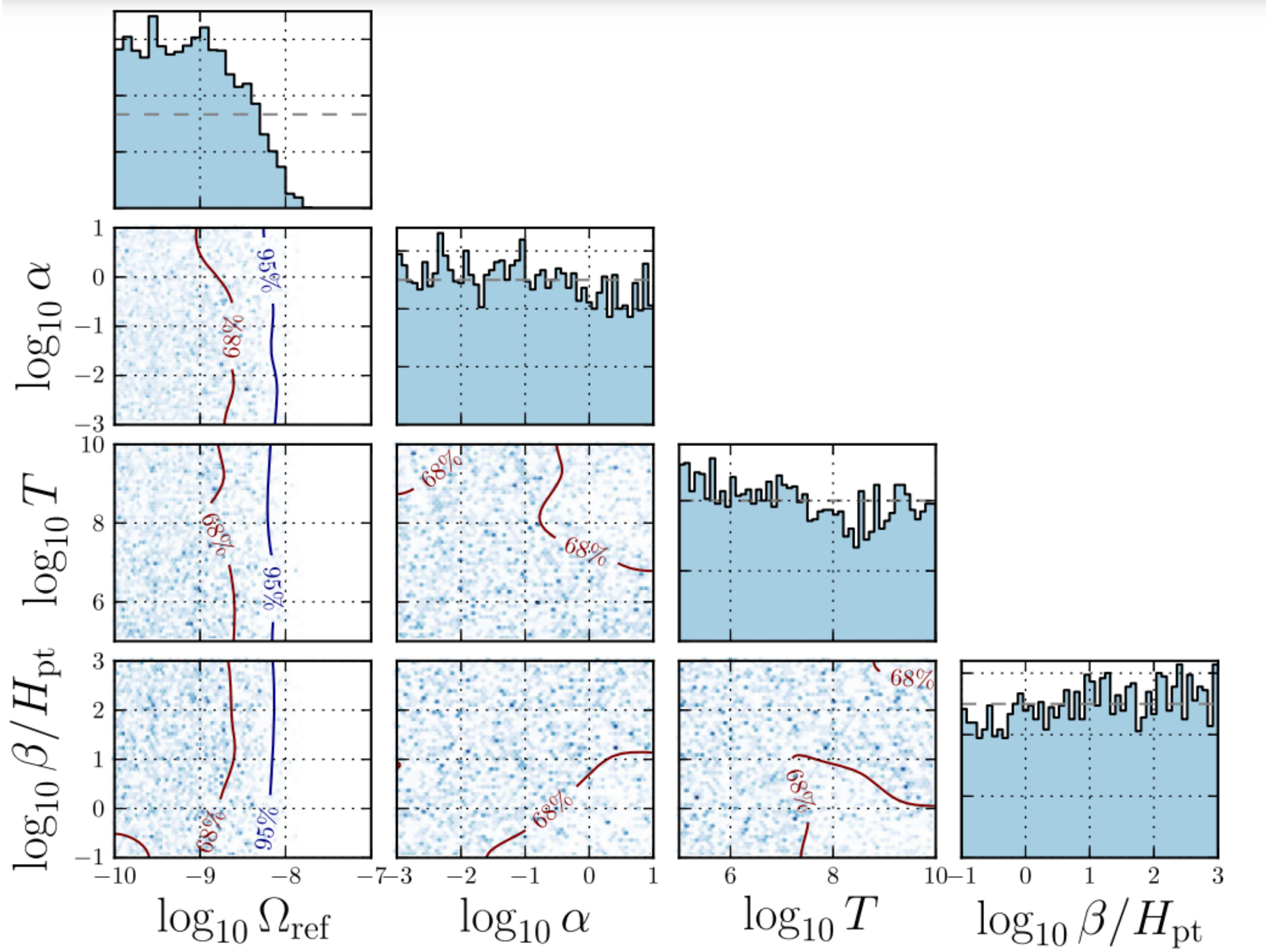
SNR > 10 for two-step and one-step SFOEWPT

PTA,LIGO,LISA,天琴,太极,⋯

▶ LIGO-Virgo search for FOPT

High-scale PT Romero, Martinovic,Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3



▸ PPTA search for FOPT

- **PPTA DR2 dataset constrain low-scale phase transition，dark sector and QCD scale FOPT**

PHYSICAL REVIEW LETTERS **127,** 251303 (2021)

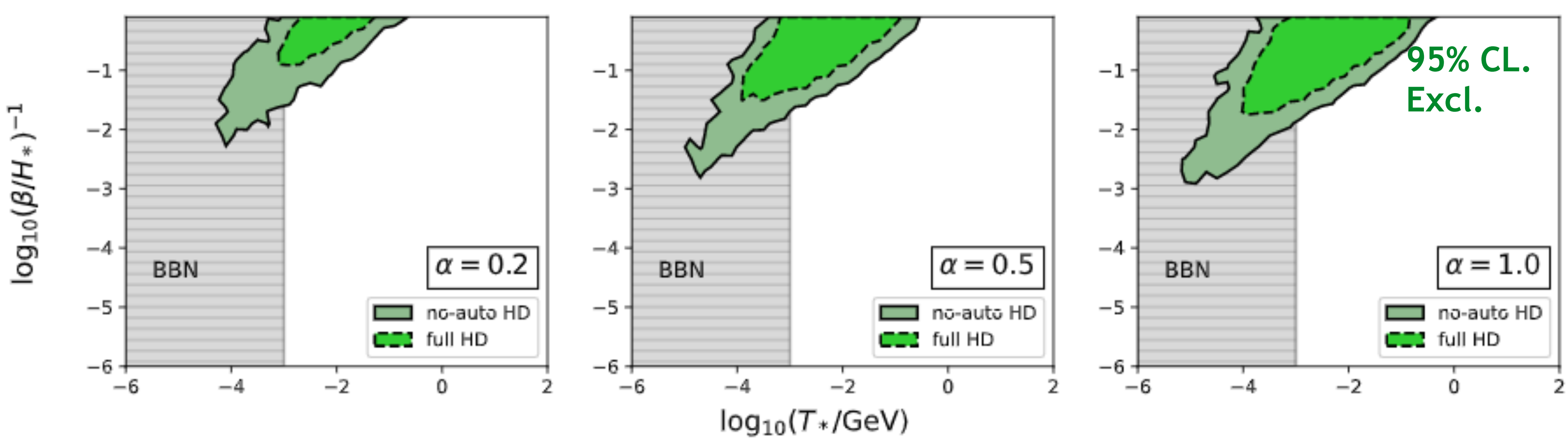
Editors' Suggestion | Featured in Physics

Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

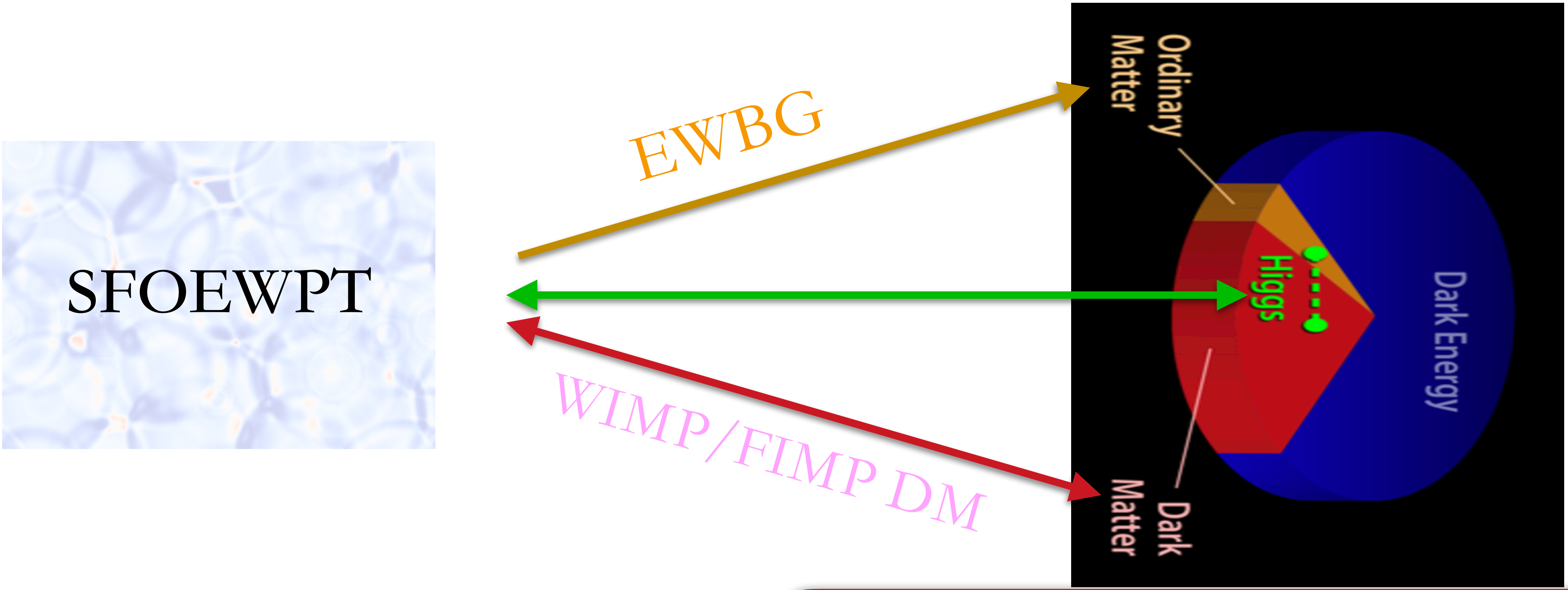
Xiao Xue,[1,2,3] Ligong Bian,[4,5,*] Jing Shu,[1,2,6,7,8,†] Qiang Yuan,[9,10,7,‡] Xingjiang Zhu,[11,12,13,§] N. D. Ramesh Bhat,[14] Shi Dai,[15] Yi Feng,[16] Boris Goncharov,[11,12] George Hobbs,[17] Eric Howard,[17,18] Richard N. Manchester,[17] Christopher J. Russell,[19] Daniel J. Reardon,[12,20] Ryan M. Shannon,[12,20] Renée Spiewak,[21,20] Nithyanandan Thyagarajan,[22] and Jingbo Wang[23]

TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1-σ interval)	
					T_*/MeV, $\alpha \times 10^3$, β/H_*	$A_{\rm comred}$, $\gamma_{\rm comred}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}$, $3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}$, 271^{+165}_{-92}, $9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}$, $3.8^{+27.9}_{-3.4}$, 854^{+9622}_{-782}	$-14.51^{+0.64}_{-0.68}$, $3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}$, $3.2^{+19.9}_{-2.8}$, 1053^{+11256}_{-962}	$-14.45^{+0.62}_{-0.64}$, $3.27^{+1.37}_{-1.54}$



▸ Two step EWPT DM+ BAU



Electroweak baryogenesis and dark matter from a singlet Higgs #2

James M. Cline (McGill U.), Kimmo Kainulainen (Jyvaskyla U. and Helsinki Inst. of Phys. and Helsinki U.) (Oct, 2012)

Published in: *JCAP* 01 (2013) 012 • e-Print: 1210.4196 [hep-ph]

pdf DOI cite 217 citations

SM+real singlet

Gravitational wave, collider and dark matter signals from a scalar singlet electroweak baryogenesis #4

Ankit Beniwal (Adelaide U. and Adelaide U., Sch. Chem. Phys.), Marek Lewicki (Adelaide U. and Warsaw U. and Adelaide U., Sch. Chem. Phys.), James D. Wells (DESY and Michigan U., MCTP), Martin White (Adelaide U., Sch. Chem. Phys. and Adelaide U.), Anthony G. Williams (Adelaide U., Sch. Chem. Phys. and Adelaide U.) (Feb 20, 2017)

Published in: *JHEP* 08 (2017) 108 • e-Print: 1702.06124 [hep-ph]

pdf DOI cite 131 citations

Impact of a complex singlet: Electroweak baryogenesis and dark matter #6

Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC) (Feb 26, 2015)

Published in: *Phys.Rev.D* 93 (2016) 6, 065032 • e-Print: 1502.07574 [hep-ph]

pdf DOI cite 102 citations

CxSM

Unified explanation for dark matter and electroweak baryogenesis with direct detection and gravitational wave signatures #5

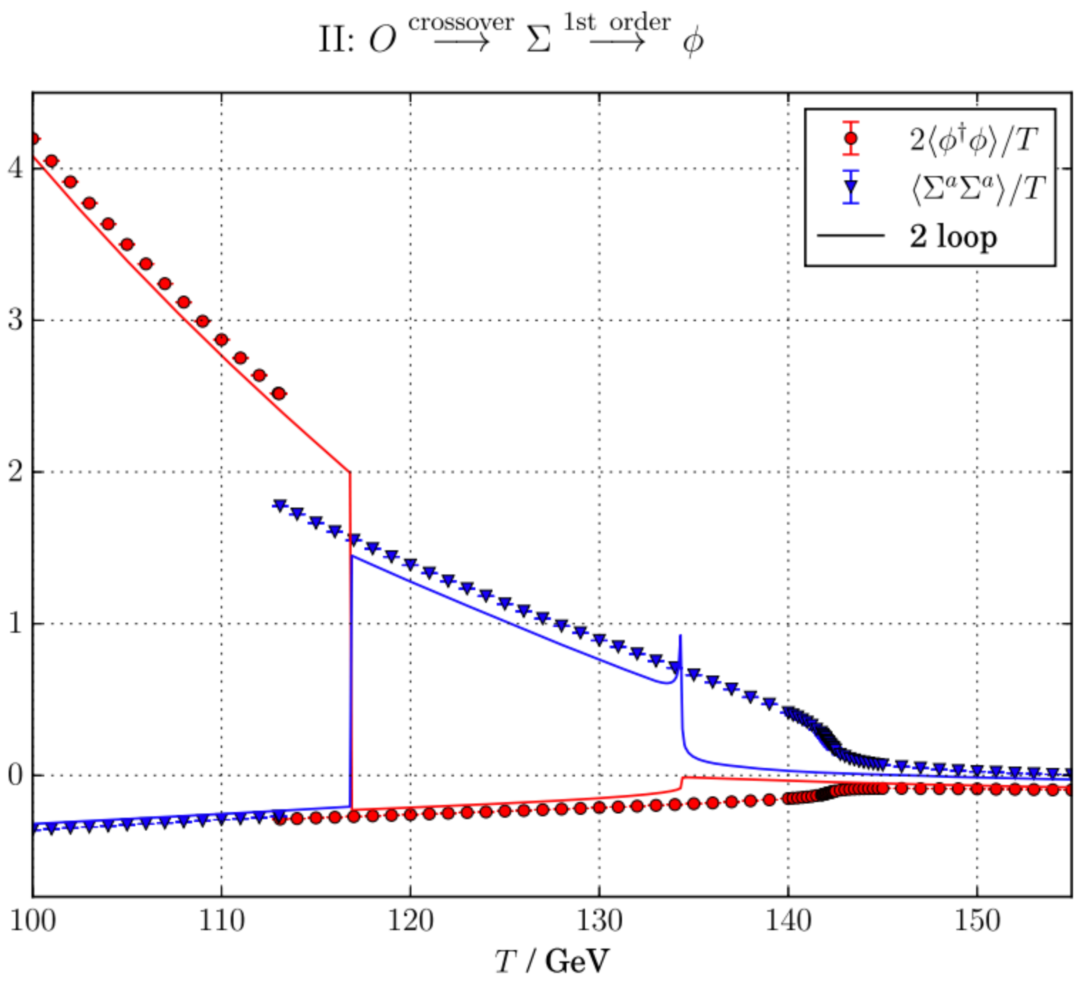
Mikael Chala (DESY), Germano Nardini (U. Bern, AEC), Ivan Sobolev (Lomonosov Moscow State U. and Moscow, INR) (May 27, 2016)

Published in: *Phys.Rev.D* 94 (2016) 5, 055006 • e-Print: 1605.08663 [hep-ph]

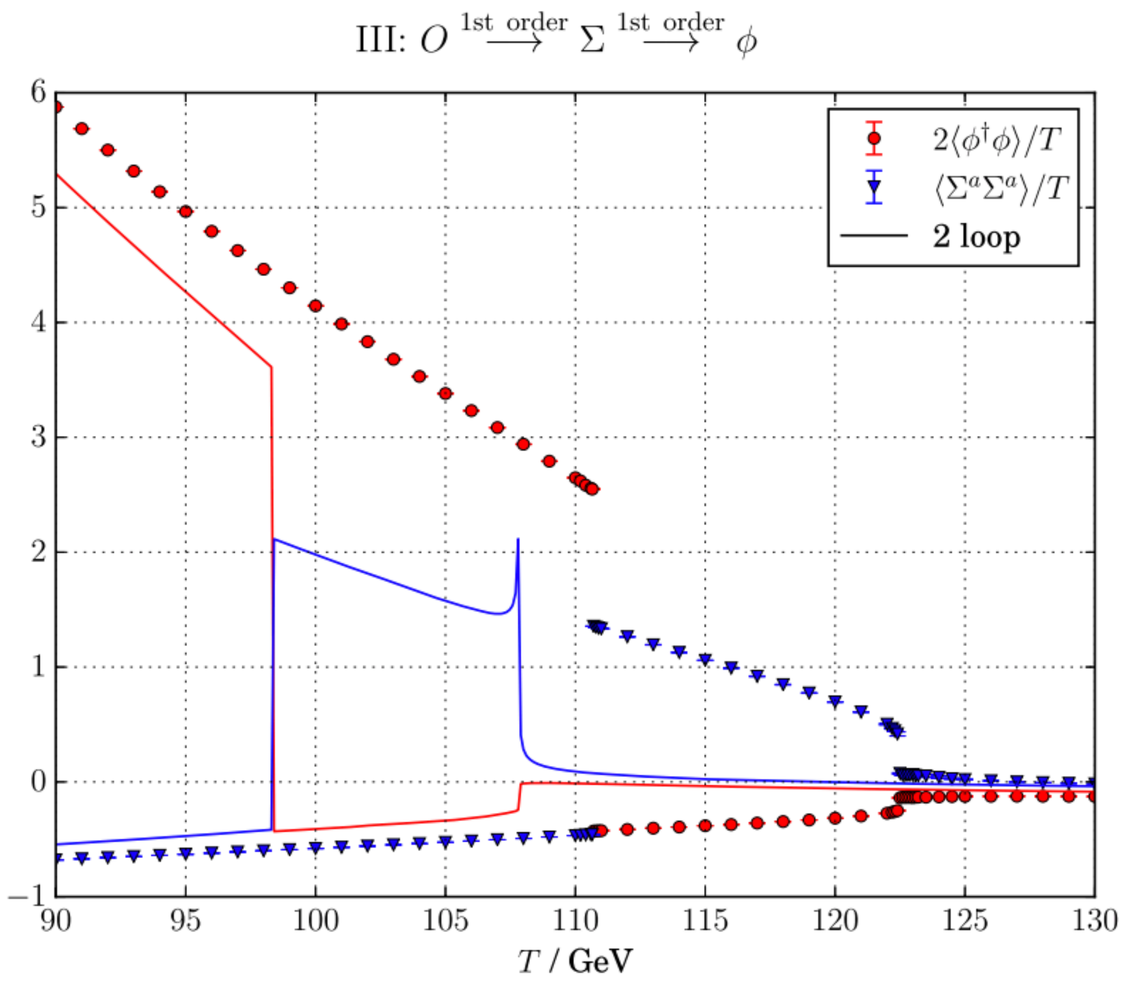
pdf DOI cite 125 citations

Composite DM

Two step lattice simulation



(a) BM1: $(M_\Sigma, a_2, b_4) = (160\text{ GeV}, 1.1, 0.25)$

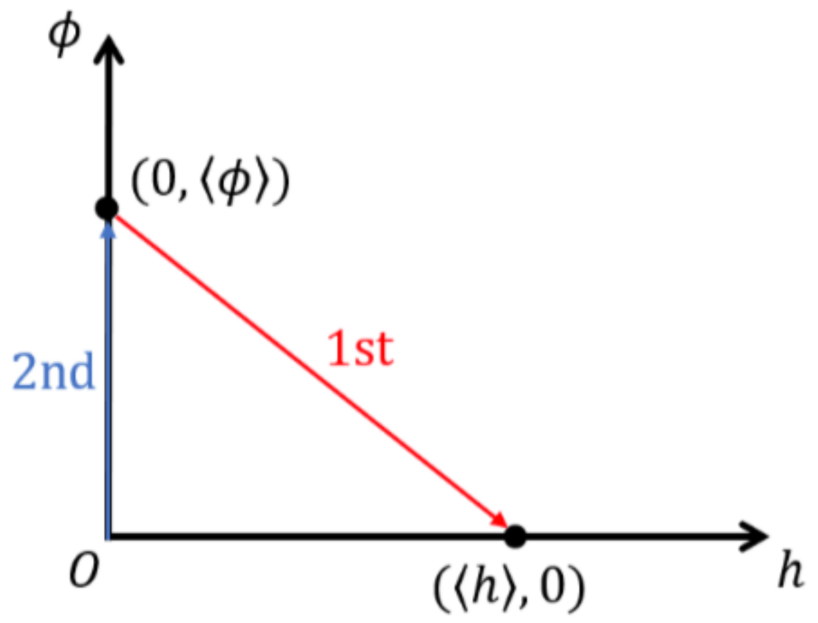


(b) BM2: $(M_\Sigma, a_2, b_4) = (255\text{ GeV}, 2.3, 0.25)$

PHYSICAL REVIEW LETTERS 126, 171802 (2021)

▸ Two-step FOPT potential

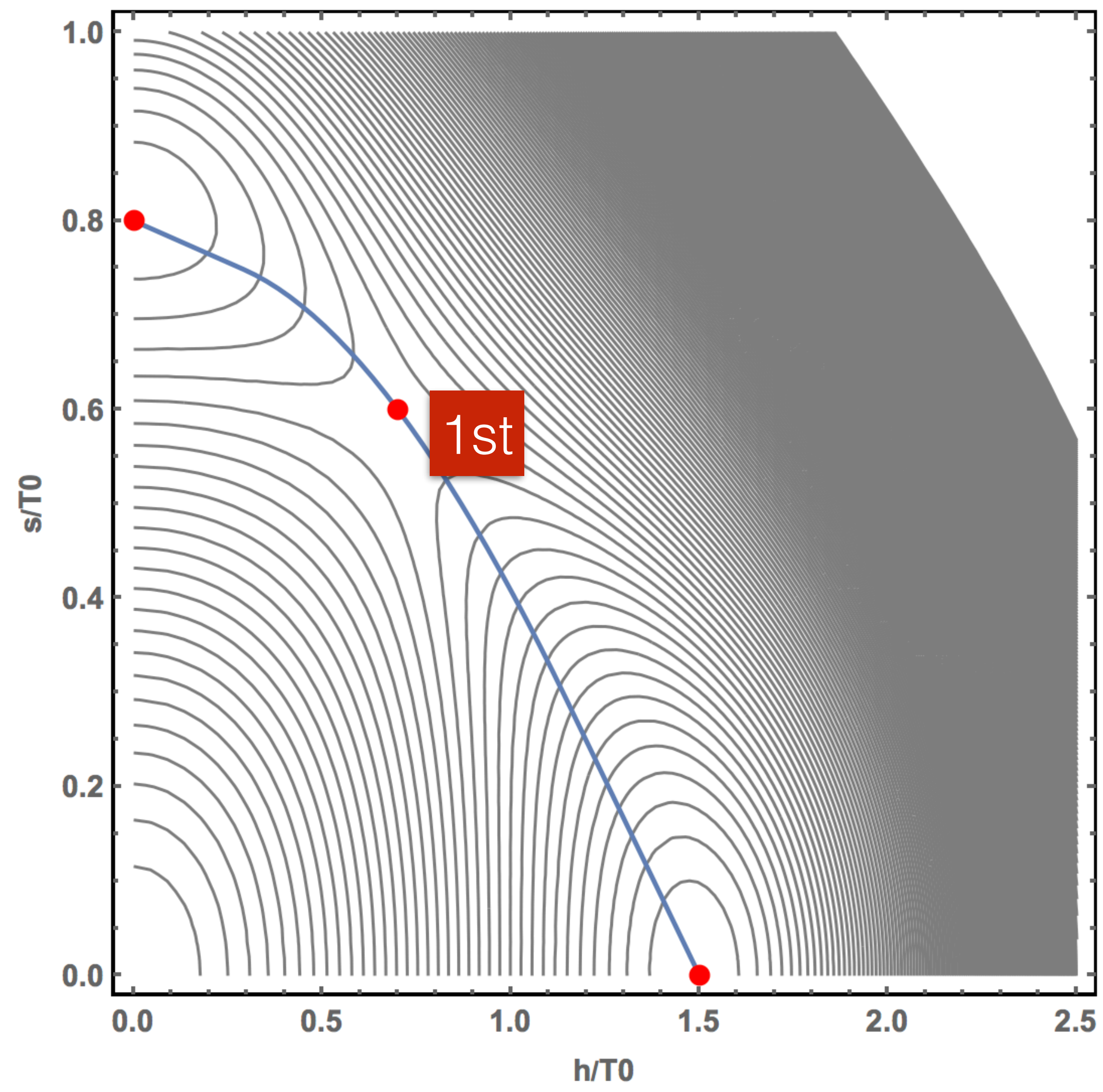
Type-a



$$V_a(\phi,h,T) = \frac{1}{2}(\mu_\phi^2 + c_\phi T^2)\phi^2 + \frac{1}{2}\lambda_{h\phi}h^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}(-\mu_h^2 + c_h T^2)h^2 + \frac{1}{4}\lambda_h h^4$$

$$c_\phi = \lambda_\phi/4 + \lambda_{h\phi}/3$$

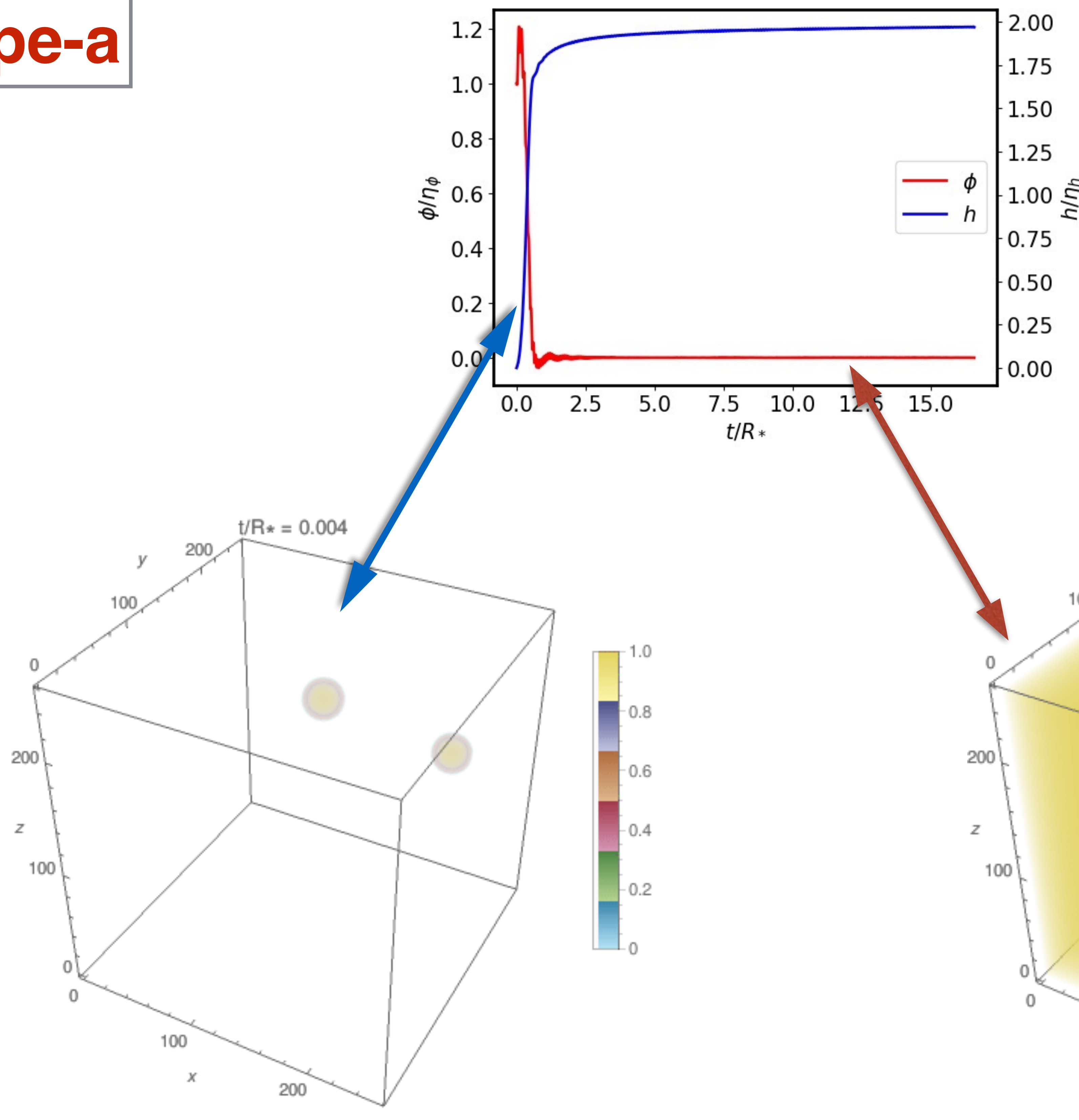
$$c_h = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_{h\phi}/12$$



Motivated for DM&EWBG, see:1804.06813,1702.06124,1609.07143, 1605.08663, 1605.08663,etc

▸ Two-step PT with the second-step being FOPT

Type-a



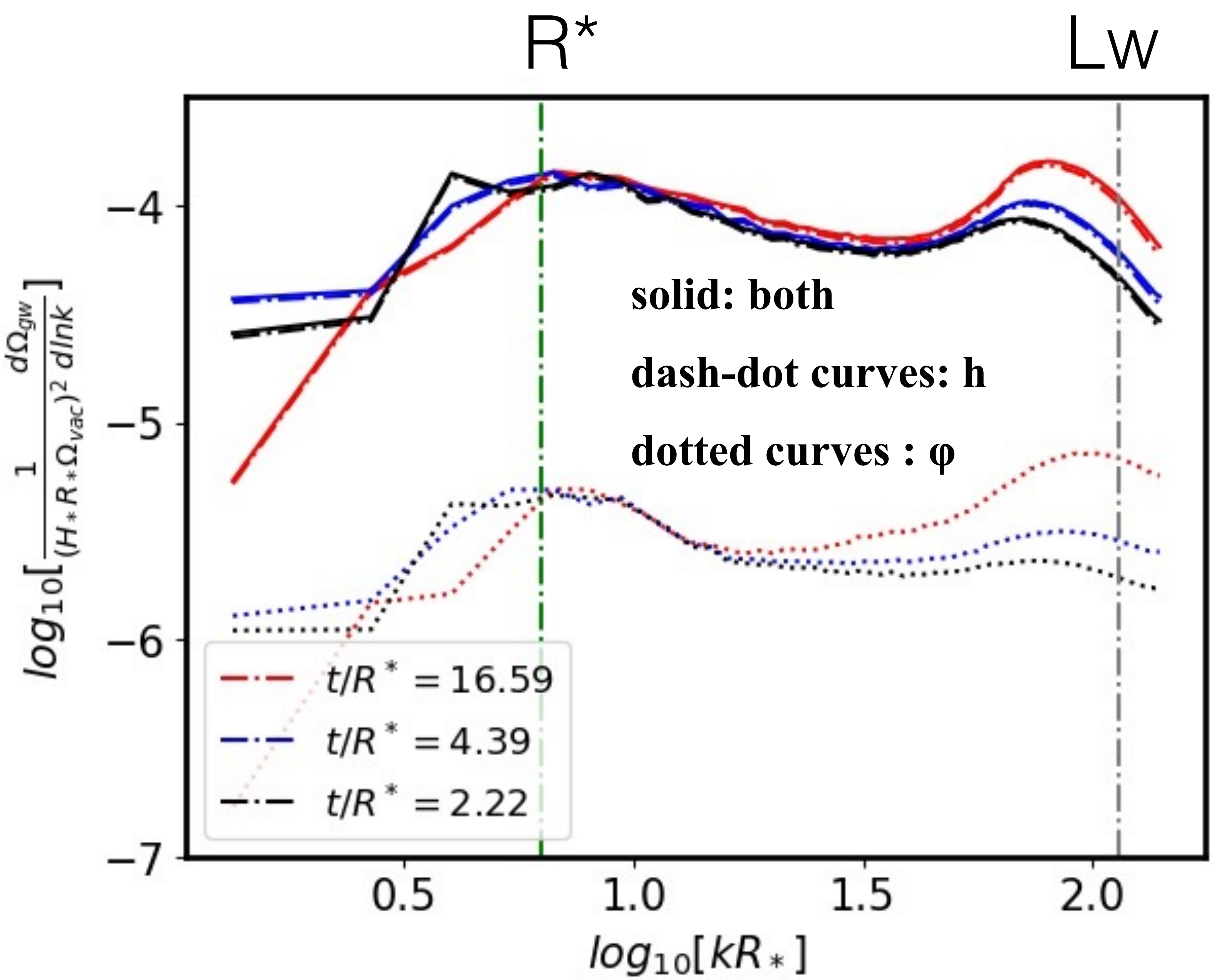
$$h(t=0,r) = \eta_h/2\left[1 - \tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

$$\phi(t=0,r) = \eta_\phi/2\left[1 + \tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with the second-step being FOPT

Type-a



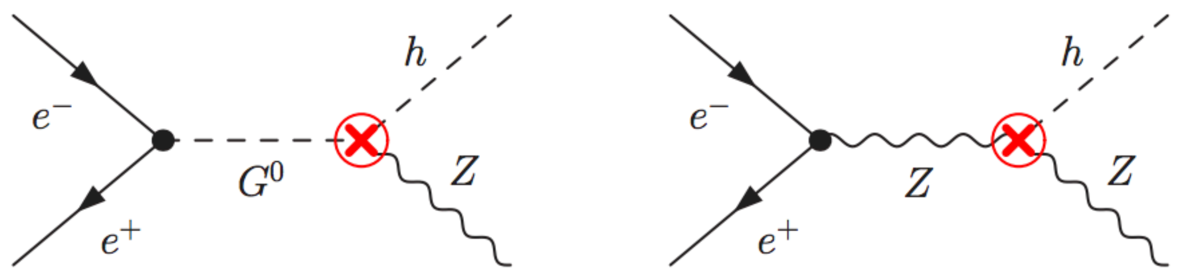
Zhao, Di, Bian, Cai,2204.04427

▸ Collider search for 2step FOPT

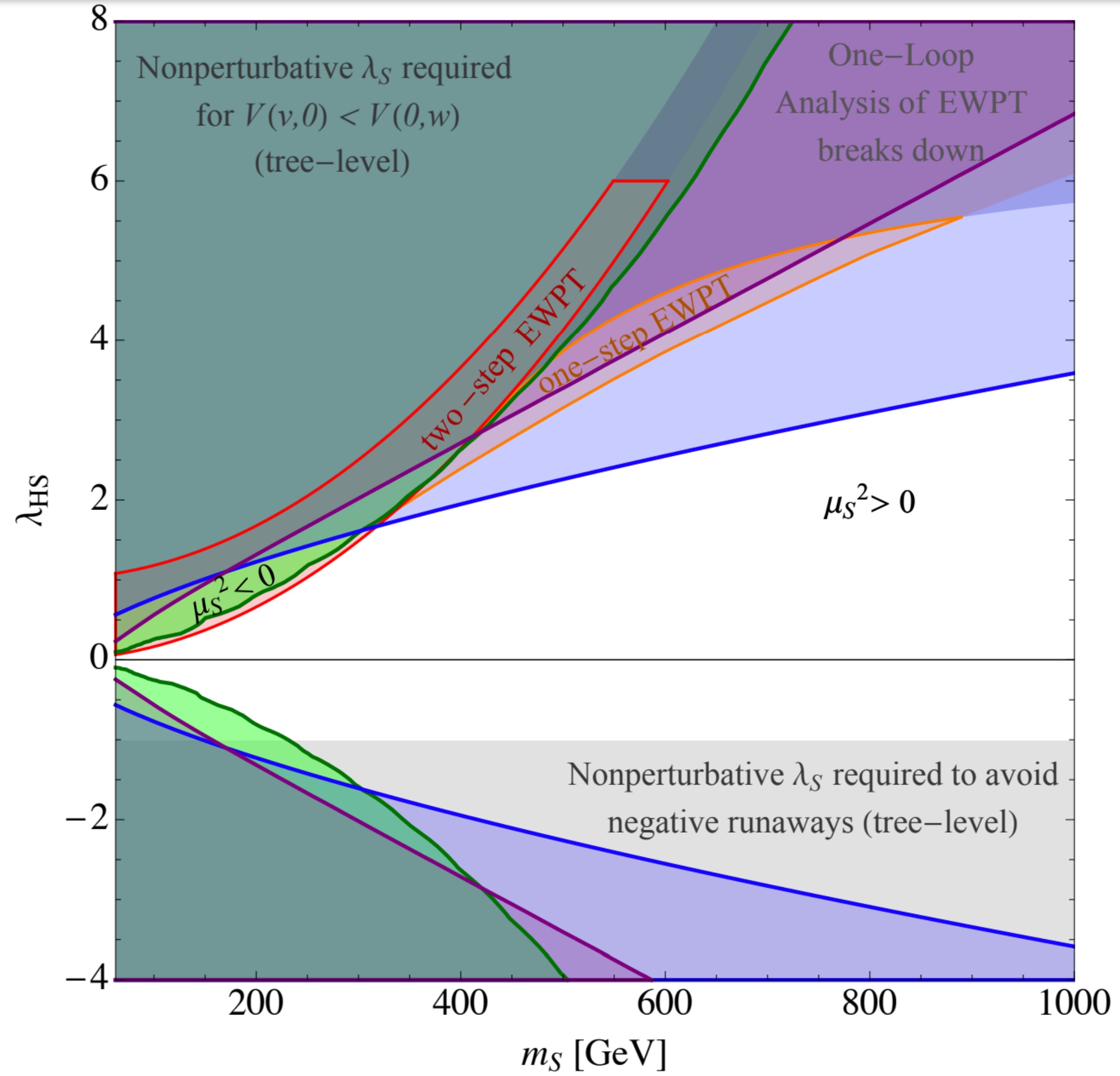
◉ Zh@ILC/CEPC

$$V_0 = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}\mu_S^2 S^2 + \lambda_{HS}|H|^2 S^2 + \frac{1}{4}\lambda_S S^4$$

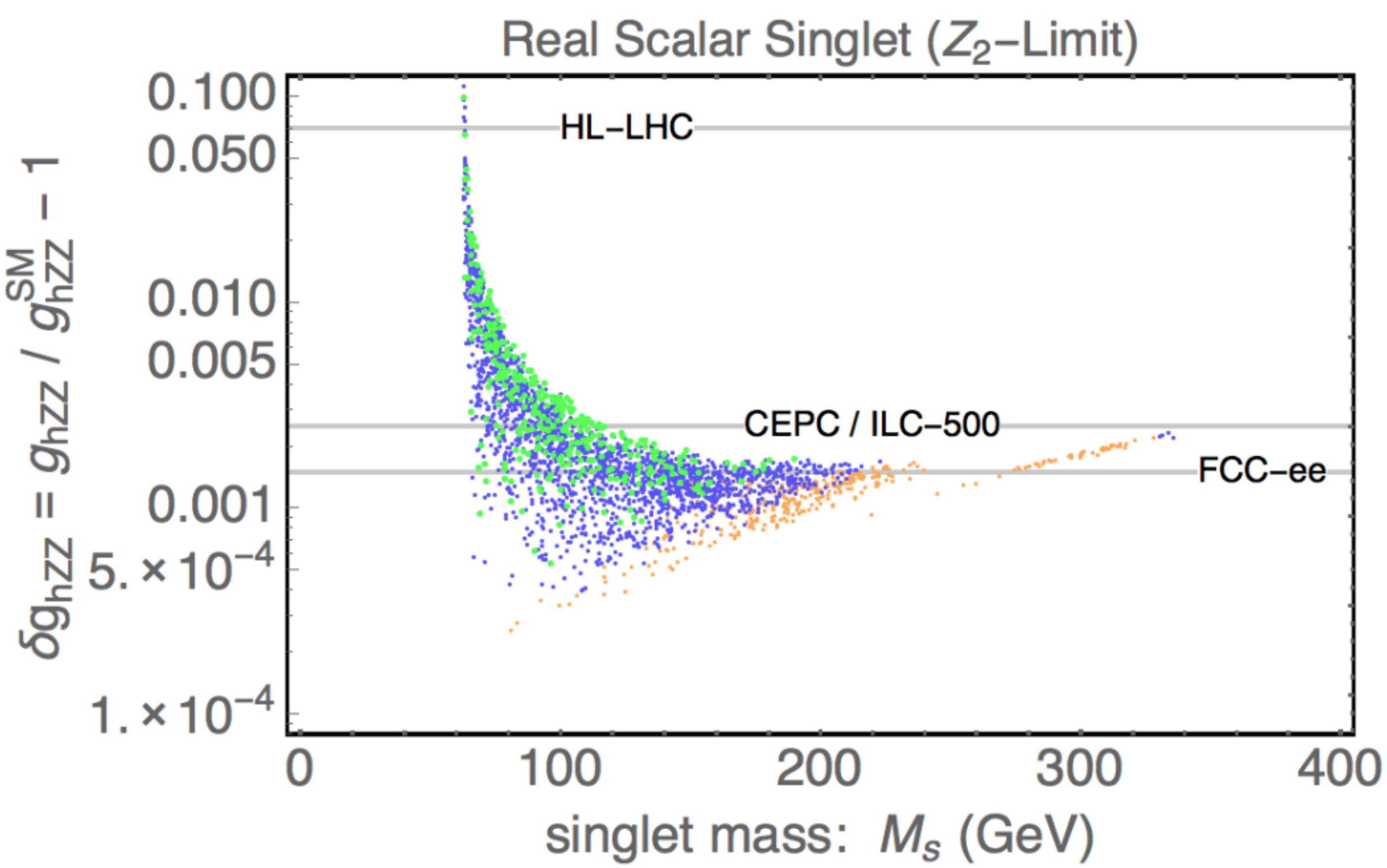
$$V_{\rm eff}(h,T) = V_0(h) + V_0^{CW}(h) + V_T(h,T) + V_r(h,T)$$



Craig,Englert, and McCullough,1305.5251

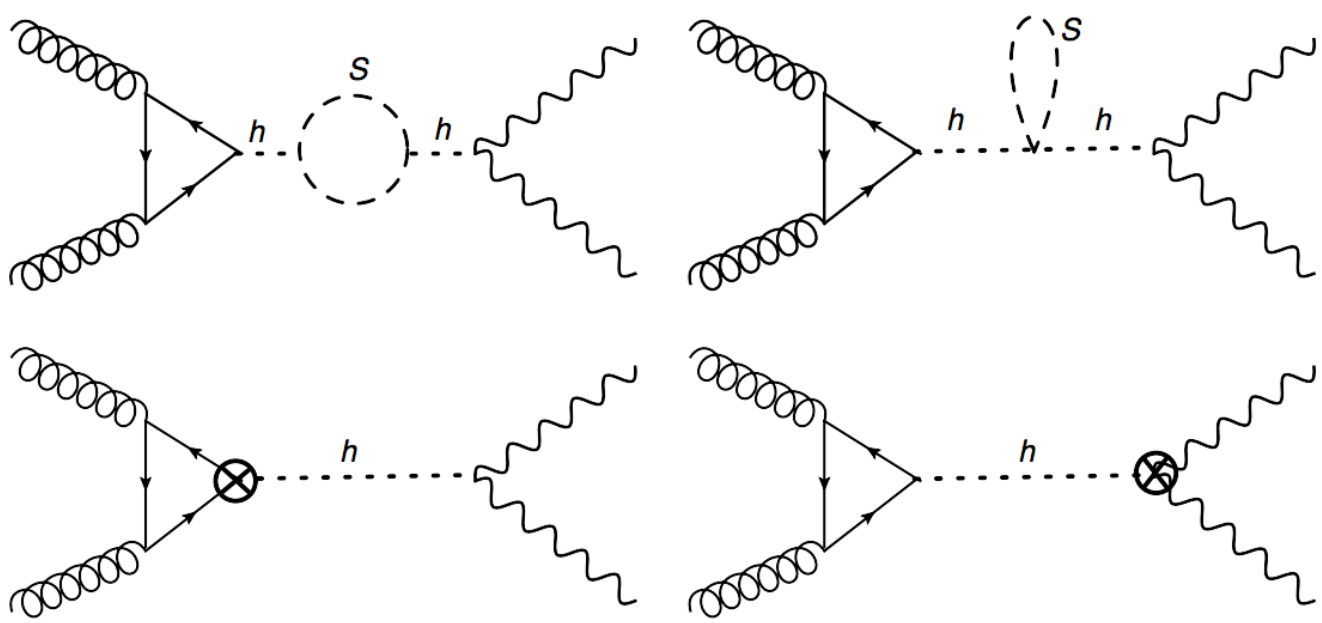


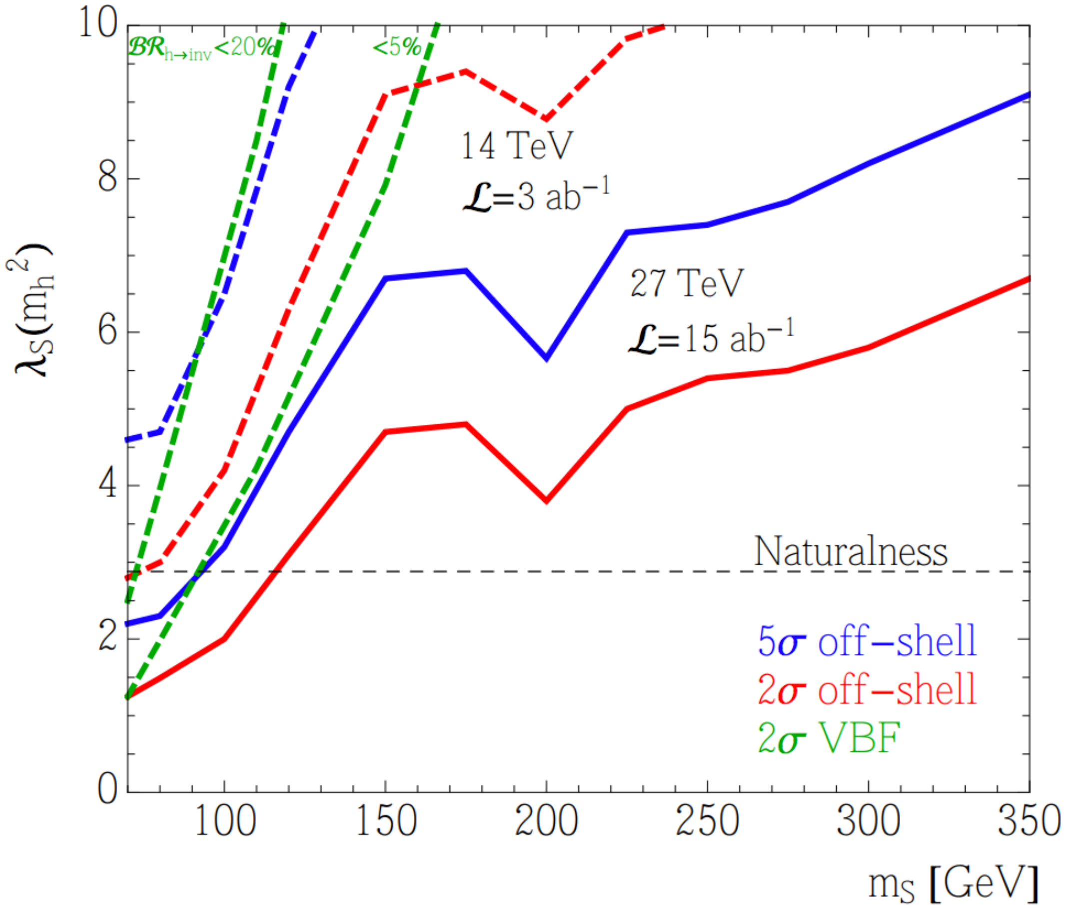
Curtin, Meade, Yu,1409.0005



Huang, Long, and Wang,1608.06619

◉ Off-shell Higgs@LHC

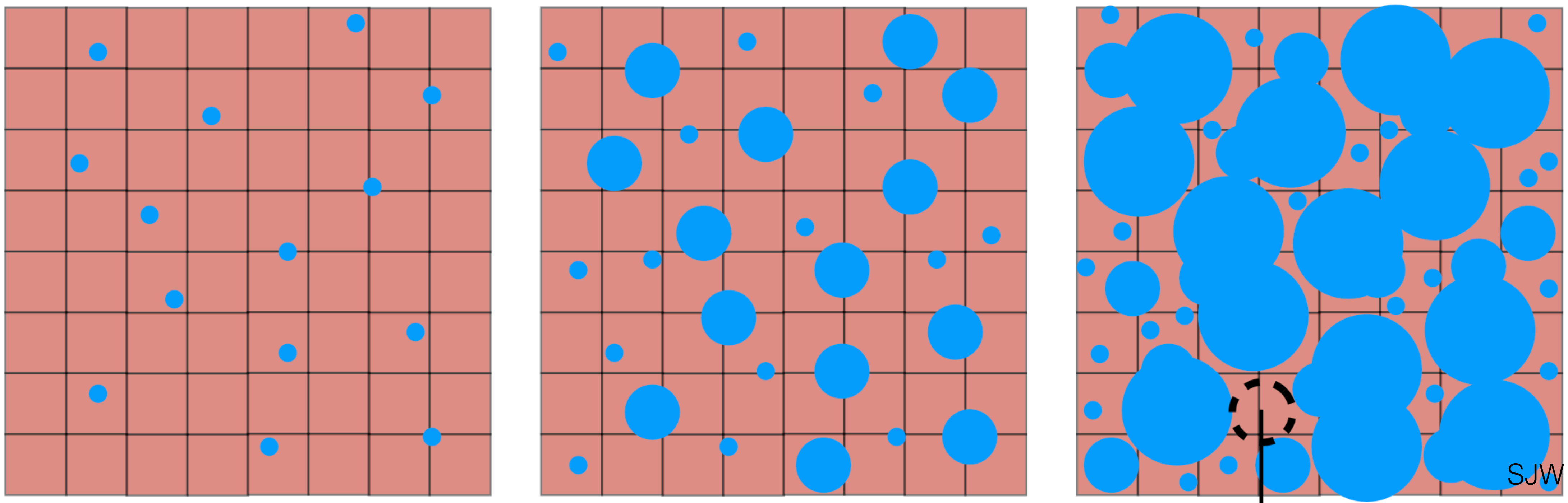




Goncalves,Han,and Mukhopadhyay, 1710.02149

See also: Lee, Park,and Qian, 1812.02679

PBH from delayed vacuum decay



Probability for a Hubble volume not to decay until time t_n

$$V_H(t) = \frac{4}{3}\pi H(t_{\rm PBH})^{-3}\frac{a(t)^3}{a(t_{\rm PBH})^3}$$

$$P(t_n) = \exp\left[-\frac{4\pi}{3}\int_{t_i}^{t_n}\frac{a^3(t)}{a^3(t_{\rm PBH})}H^{-3}(t_{\rm PBH})\Gamma(t)dt\right]$$

PBH abundance $\Omega_{\rm PBH}^{\rm form} = P(t_n)$

Postponed Hubble volume V_H

↓

Un-diluted false vacuum energy

↓

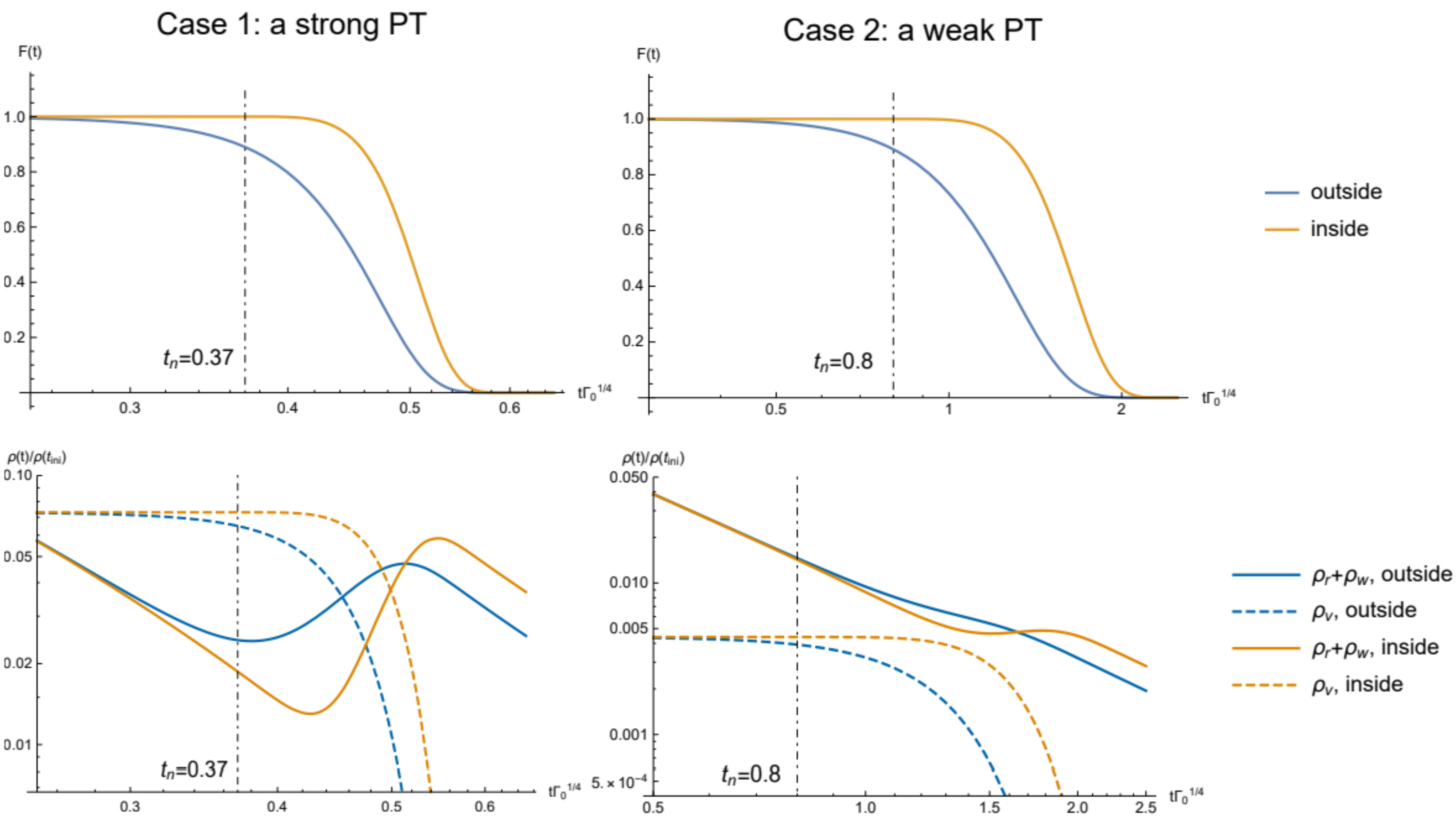
PBH

Collapse of the Hubble horizon

PBH from delayed vacuum decay

$\beta/H_* = 14.8,\ \alpha_* = 6$

$\beta/H_* = 3.7,\ \alpha_* = 0.5$



Curvature from delayed vacuum decay

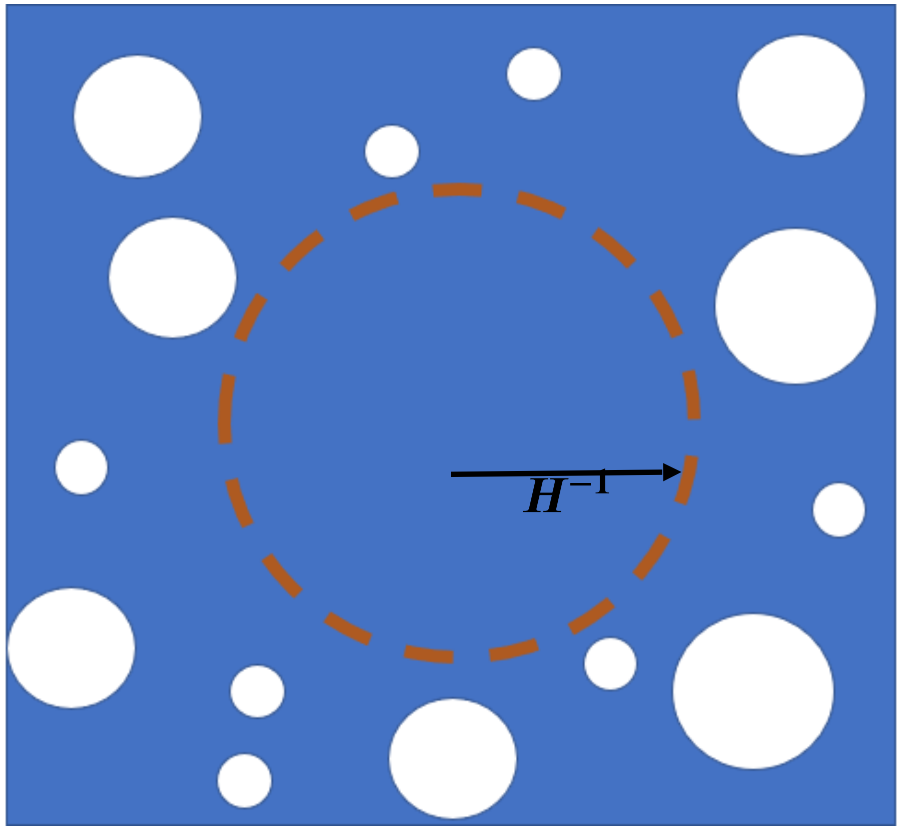
Hubble-sized perturbations

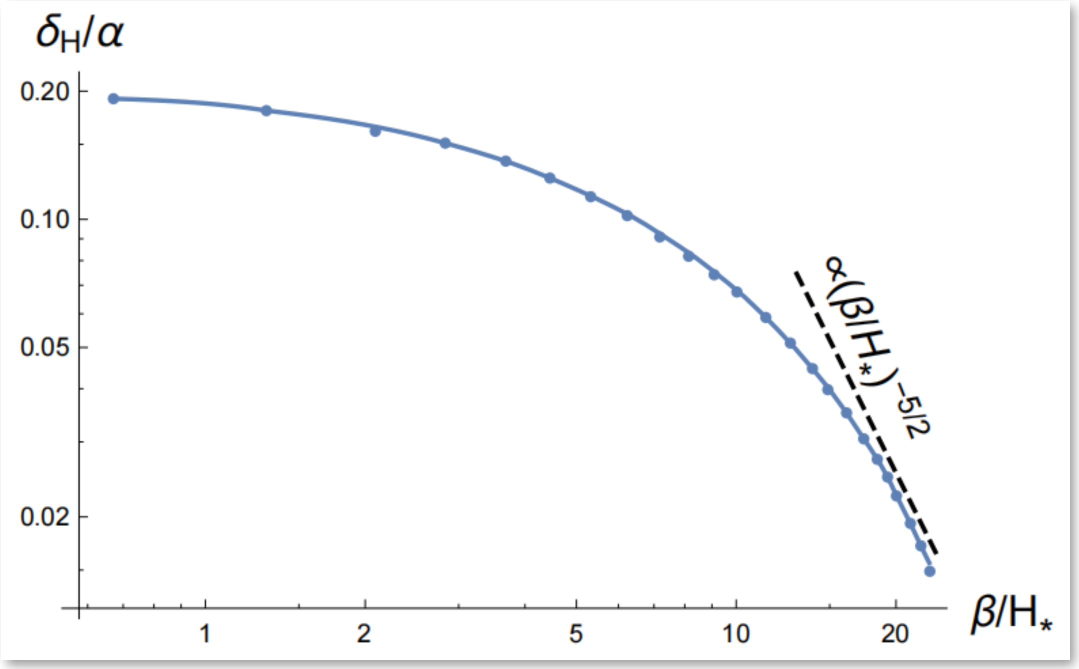
Press-Schechter formalism

$$\delta_H^2 = \frac{16}{81}\int_0^\infty \frac{dk}{k}\,(kR_H)^4\,W^2(k,R_H)\,\mathcal{P}_{\mathcal{R}}(k)$$

Causality requires

$P_R \propto k^3$





Vacuum decay is postponed in a Hubble horizon

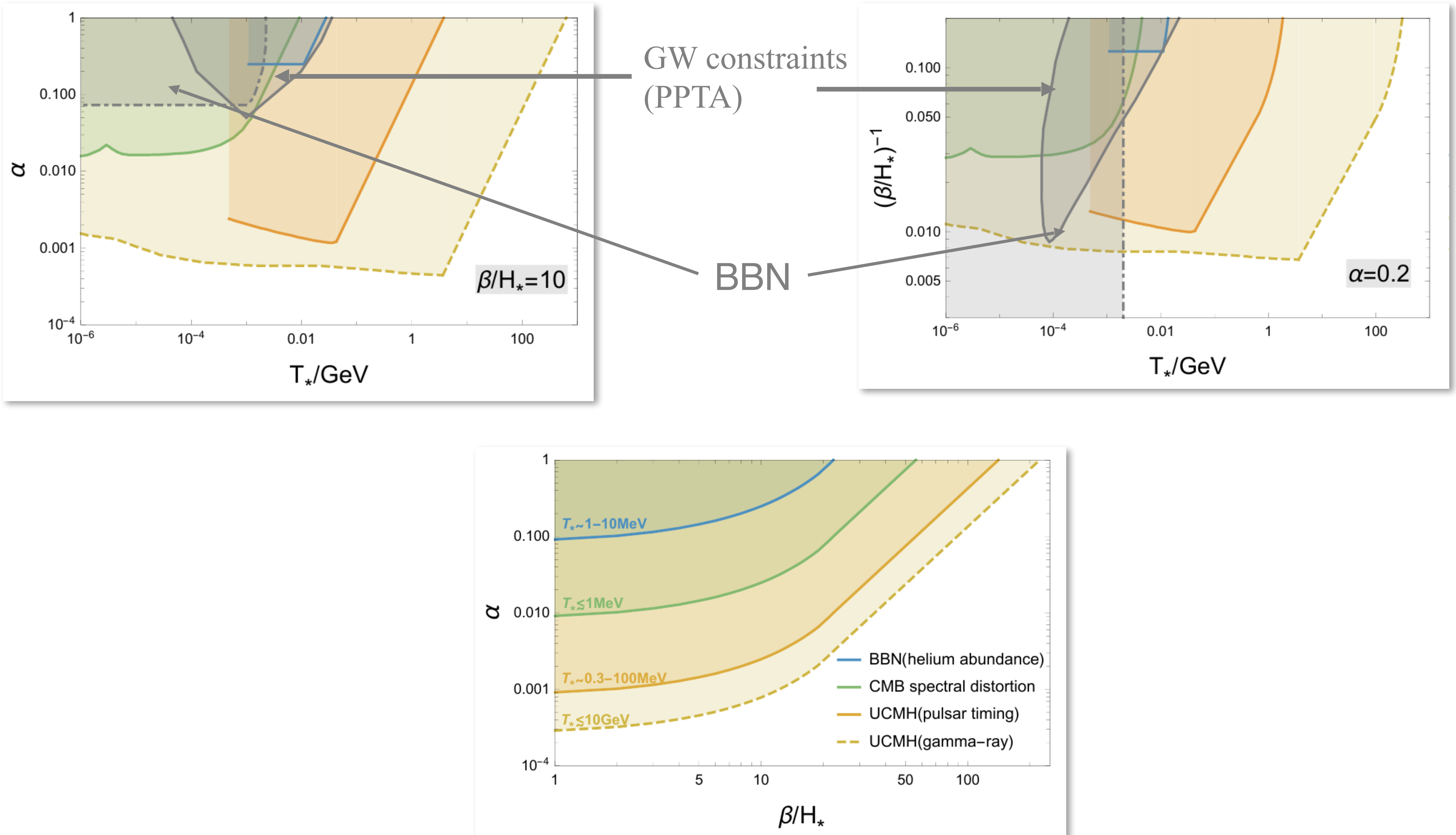
$$\delta_H \equiv \sqrt{(\delta\rho(H_*^{-1})/\rho)^2} \propto \alpha(\beta/H_*)^{-5/2}$$

at the Hubble scale

J Liu, **L Bian**, RG Cai, Zong-Kuan Guo, SJ Wang, Phys.Rev.Lett. 130 (2023) 5, 051001, 2208.14086

Constraints on PT models

low-scale and slow 1st PTs motived for dark PT and EWBG



Thanks

PBH from delayed vacuum decay

Spatial fraction of the false vacuum

$$F(t)=\exp\left[-\frac{4\pi}{3}\int_{t_i}^{t}\mathrm{d}t'\Gamma\left(t'\right)a^3\left(t'\right)r^3\left(t,t'\right)\right]$$

Bubble nucleation rate

$$\Gamma(t)=\Gamma_0 e^{\beta t}$$

Comoving radius of vacuum bubble

$$r(t,t')\equiv\int_{t'}^{t}a^{-1}(\tau)d\tau$$

False vacuum energy density

$$\rho_{\mathrm{v}}=F(t)\Delta V$$

Radiation evolution Eq.

$$\frac{d(\rho_{\mathrm{r}}+\rho_{\mathrm{w}})}{dt}+4H(\rho_{\mathrm{r}}+\rho_{\mathrm{w}})=\left(-\frac{d\rho_{\mathrm{v}}}{dt}\right)$$

Relativistic plasma and bubble walls $\rho_{\mathrm{w}},\rho_{\mathrm{r}}\propto a^{-4}(t)$

FLRW Eq.

$$H^2=\frac{\rho_{\mathrm{v}}+\rho_{\mathrm{r}}+\rho_{\mathrm{w}}}{3}$$

PBH threshold

$$\delta(t_{\mathrm{PBH}})=\frac{\rho_v(t_{\mathrm{PBH}};t_n)+\rho_r(t_{\mathrm{PBH}};t_n)}{\rho_v(t_{\mathrm{PBH}};t_i)+\rho_r(t_{\mathrm{PBH}};t_i)}-1\geq\delta_c\Rightarrow t_{\mathrm{PBH}}$$

$$\rho_{\mathrm{PBH}}\propto a^{-3}$$

PBH mass

$$M_{\mathrm{PBH}}\approx\frac{4\pi}{3}\gamma H^{-3}(t_{\mathrm{PBH}})\rho_c=4\pi\gamma H^{-1}(t_{\mathrm{PBH}})$$