Electroweak input schemes and universal corrections in SMEFT

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[arXiv:2305.03763]

EW Input Schemes

$$\alpha \rightarrow \{\alpha, M_W, M_Z\}$$
$$\alpha_\mu \rightarrow \{G_F, M_W, M_Z\}$$
$$LEP \rightarrow \{G_F, \alpha, M_Z\}$$

EW Input Schemes

Defined by the choice of measured quantities used to replace the gauge couplings and Higgs vev

$$\mathcal{L}_{bare}(g_1, g_2, v_T) \qquad \langle H^\dagger H \rangle \equiv \frac{v_T^2}{2}$$

$$\mathcal{L}\,(\alpha, M_W, M_Z)$$
α-scheme

$$\mathcal{L}\,(G_F, M_W, M_Z)$$
α_μ-scheme

$$\mathcal{L}\,(G_F, \alpha, M_Z)$$
LEP-scheme

- M_W, M_Z defined on-shell
- α defined from 2-pt functions in 5 flavour QEDxQCD - Log[mf] terms resummed in definition
- G_F defined from muon decay

$$\alpha \to \{\alpha, M_W, M_Z\}$$
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EW Input Schemes

In practice perform field rotations to get the bare Lagrangian in terms of

$$\mathcal{L}_{bare}(M_{W_{bare}}, M_{Z_{bare}}, v_T)$$

$$v_\alpha \equiv \frac{2 M_W s_w}{\sqrt{4\pi\alpha}}$$

$$v_\mu \equiv (\sqrt{2} G_F)^{-\frac{1}{2}}$$

$$\mathcal{L}\left(v_\alpha, M_W, M_Z\right)$$
α-scheme

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Does the scheme make a difference ?

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Does the scheme make a difference ?

- To explore this we calculated heavy boson decays to NLO in the three schemes.

$$W \to l\nu \qquad Z \to l^+ l^- \qquad H \to bb$$

- All Wilson coefficients with no flavour assumptions (~50 operators) !
- A number of these results are new and analytic files are available in the electronic submission on the arXiv along with the analytic files needed to convert between schemes [arXiv:2305.03763] .

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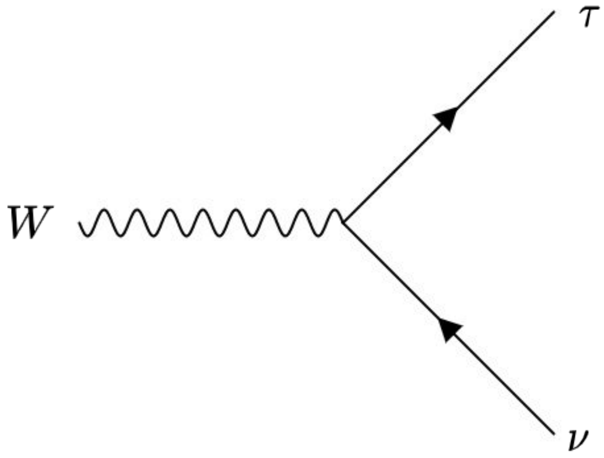
$W \to \tau\nu$

$$\Gamma_{W\tau\nu} = \frac{M_W}{12\pi}\frac{M_W^2}{v_T^2}\left(1 + 2v_T^2 C_{\substack{Hl\\33}}^{(3)}\right) \quad + \quad \ldots$$



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$$\Gamma^{\alpha}_{W,\text{LO}} = \quad 234.6\,\text{MeV} + v_\alpha^2 \Gamma^{\alpha(4,0)}_{W\tau\nu_\tau}\left\{2.000 C^{(3)}_{\substack{Hl\\33}} + 3.733 C_{HWB} + 1.742 C_{HD}\right\}$$

$$\Gamma^{\alpha_\mu}_{W,\text{LO}} = 227.2\,\text{MeV} + v_\mu^2 \Gamma^{\mu(4,0)}_{W\tau\nu_\tau}\left\{2.000 C^{(3)}_{\substack{Hl\\33}} - 1.000 \sum_{j=1,2} C^{(3)}_{\substack{Hl\\jj}} + 1.000 C_{\substack{ll\\1221}}\right\}$$

$$\Gamma^{\text{LEP}}_{W,\text{LO}} = \quad 222.7\,\text{MeV} + v_\mu^2 \Gamma^{\text{LEP}(4,0)}_{W\tau\nu_\tau}\left\{2.000 C^{(3)}_{\substack{Hl\\33}} - 2.379 C_{HWB} - 1.656 \sum_{j=1,2} C^{(3)}_{\substack{Hl\\jj}} + 1.656 C_{\substack{ll\\1221}} - 1.078 C_{HD}\right\}$$

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The number show the percentage correction to the LO result to reach NLO

$W \to \tau\nu$	SM	C_{HD}	C_{HWB}	$C^{(3)}_{\substack{Hl\\jj}}$	$C_{\substack{ll\\1221}}$	$C^{(3)}_{\substack{Hl\\33}}$
α	−4.2%	−1.7%	−3.0%	—	—	2.2%
α_μ	−0.3%	—	—	2.5%	−0.2%	2.2%
LEP	2.0%	8.1%	3.2%	5.1%	2.5%	4.6%

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- 4% difference between α and αμ schemes in the SM
- LEP is not as consistent in the SM (depends on M_W structure) but equally predictable

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- Significant differences in some corrections
- Some being very large - up to 70% (Z Decay)

What is the Goal ?

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Can we understand the SM and SMEFT corrections ?

- Patterns
- Common Structure

Are we able to do anything with this knowledge?

- Resummation of geometric series
- Universal corrections from top loops

Understanding the scheme differences - SM

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$$\mathcal{A} = \mathcal{A}_{bare} + Z_{wf} + \delta X_i \quad ; \quad X_i \in \{\text{inputs}\}$$

Study the structure of CT

Scheme Independent

Scheme Dependent

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$$\frac{1}{v_{T,0}^2} = \frac{1}{v_\sigma^2}\left[1 - v_\sigma^2 \Delta v_\sigma^{(6,0,\sigma)} - \frac{1}{v_\sigma^2}\Delta v_\sigma^{(4,1,\sigma)} - \Delta v_\sigma^{(6,1,\sigma)}\right], \qquad \sigma \in \{\alpha, \alpha_\mu\}$$

CT are not physical

$$v_\alpha \equiv \frac{2 M_W s_w}{\sqrt{4\pi\alpha}}$$

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$$\frac{v_\alpha^2}{v_\mu^2} = 1 + \Delta r$$

Physical (finite and tadpole free) – Can write CT in a more intuitive form

Use Large Top mass limit as these are the dominant corrections

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Use Large Top mass limit as these are the dominant corrections

$$\frac{1}{v_{T,0}^2}\bigg|_{m_t \to \infty} = \frac{1}{v_\sigma^2}\left[1 + \frac{1}{v_\sigma^2}\left(\Delta r_t^{(4,1)} \delta_{\alpha\sigma} + 2\Delta M_{W,t}^{(4,1)}\right)\right]$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta \rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\% \quad ; \quad \frac{\Delta \rho_t^{(4,1)}}{v_\alpha^2} = \frac{3}{16\pi^2}\frac{m_t^2}{v_\alpha^2} \approx 1\%$$

- Scheme dependence manifest in terms of physical Δr
- Origin of consistent difference of 4% between α and α_μ schemes

- Scheme independent part (4,1) without a σ contains the divergences

Understanding the scheme differences - SM

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- Universal correction $\Delta r_t^{(4,1)}$ can be resummed in the α scheme - Phys. Lett. B 227 (1989) 167.
-

$$\frac{1}{v_{T,0}^2} \approx \frac{1}{v_\alpha^2}\left[1 + \frac{1}{v_{T,0}^2}\Delta r_t^{(4,1)}\right] \approx \frac{1}{v_\alpha^2}\left[1 + \frac{1}{v_\alpha^2}\Delta r_t^{(4,1)} + \frac{1}{v_\alpha^2 v_{T,0}^2}\left(\Delta r_t^{(4,1)}\right)^2 + \dots\right]$$
$$= \frac{1}{v_\alpha^2}\left[1 - \frac{1}{v_\alpha^2}\Delta r_t^{(4,1)}\right]^{-1} \equiv \frac{1}{\tilde{v}_\alpha^2}$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta \rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\%$$

- Theory predictions for observables become more compatible

Generalisation to SMEFT

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- Make contact with a process
  - Choose W decay as it replicates the SM resummation

$$\frac{M_{W,0}^2}{v_{T,0}^2} z_W \Big|_{m_t \to \infty} \equiv \frac{M_W^2}{v_\sigma^2} \left[1 + v_\sigma^2 K_W^{(6,0,\sigma)} + \frac{1}{v_\sigma^2} K_W^{(4,1,\sigma)} + K_W^{(6,1,\sigma)}\right]$$

  - $K_W^{(i,j,\sigma)}$ are the Large top mass corrections to W decay
  - They are finite and tadpole free

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$$\Delta v_{\sigma,t}^{(4,1,\sigma)} = -K_W^{(4,1,\sigma)} + 2\Delta M_{W,t}^{(4,1)},$$
$$\Delta v_\sigma^{(6,0,\sigma)} = -K_W^{(6,0,\sigma)},$$
$$\Delta v_{\sigma,t}^{(6,1,\sigma)} = -K_W^{(6,1,\sigma)} + \left[2\Delta M_{W,t}^{(6,1,\sigma)} + 2\Delta M_{W,t}^{(4,1)} K_W^{(6,0,\sigma)} + \Delta z_{W,t}^{(6,1,\sigma)}\right]$$

Finite scheme dependent corrections

Generalisation to SMEFT

An Example - Higgs Decay

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$$\Gamma_{hb\bar{b}} = \frac{3m_b^2 M_h}{8\pi v_T^2}\left[1 + v_T^2\left(2C_{H\Box} - \frac{1}{2}C_{HD} - \sqrt{2}\frac{v_T}{m_b}C_{\substack{dH\\33}}\right)\right]$$

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$$\Delta K_h^{(6,1,\sigma)} = K_W^{(6,1,\sigma)} + 2K_h^{(4,1)}K_W^{(6,0,\sigma)} + \frac{1}{\sqrt{2}}\frac{v_\sigma}{m_b}K_W^{(4,1,\sigma)}C_{\substack{dH\\33}}$$

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Universal Corrections in SMEFT

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Write LO amplitude in terms of v_T, s_w and M_W

Make the following replacements to get a LO_K approximation:

$$\frac{1}{v_T^2} \to \frac{1}{v_\sigma^2}\left[1 + v_\sigma^2 K_W^{(6,0,\sigma)} + \frac{K_W^{(4,1,\sigma)}}{v_\sigma^2} + K_W^{(6,1,\sigma)}\right]$$

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$K_W^{(i,j,\sigma)}$ = Large mt corrections to W decay.

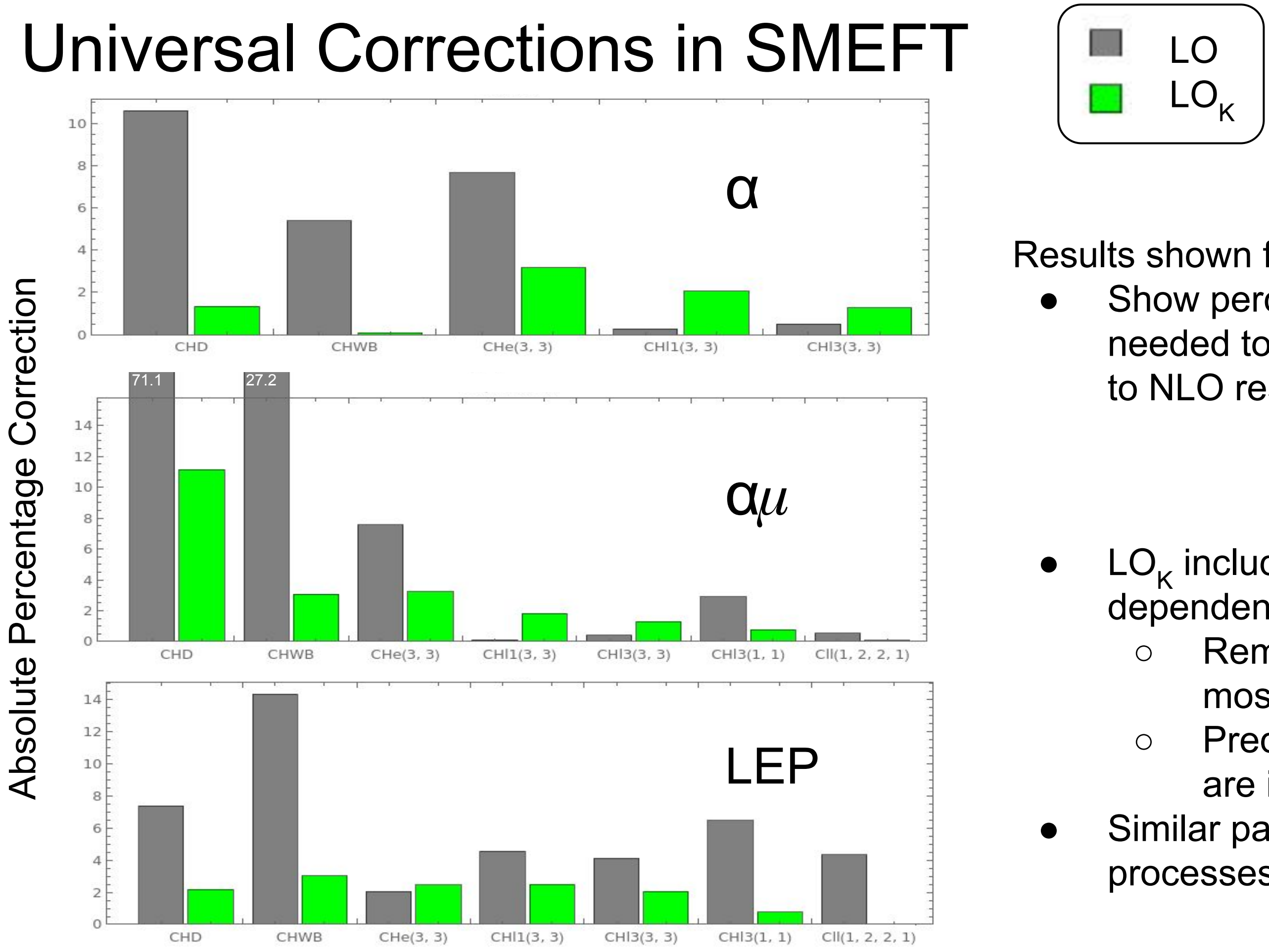
$$s_w^2 \to s_w^2\left(1 - \frac{1}{v_\sigma^2}\Delta r_t^{(4,1)} + \Delta v_\sigma^{(6,0,\sigma)}\Delta r_t^{(4,1)} - 2C_{\substack{Hq\\33}}^{(3)}\Delta r_t^{(4,1)}\right)$$

$$\frac{\Delta r_t^{(4,1)}}{v_\alpha^2} = -\frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} \approx -3.5\%$$

$$c_w^2 \to c_w^2\left(1 - \frac{1}{v_\sigma^2}\Delta\rho_t^{(4,1)} + \Delta v_\sigma^{(6,0,\sigma)}\Delta\rho_t^{(4,1)} - 2C_{\substack{Hq\\33}}^{(3)}\Delta\rho_t^{(4,1)}\right)$$

$$\frac{\Delta\rho_t^{(4,1)}}{v_\alpha^2} = \frac{3}{16\pi^2}\frac{m_t^2}{v_\alpha^2} \approx 1\%$$

Universal Corrections in SMEFT



$$\alpha \to \{\alpha, M_W, M_Z\}$$
$$\alpha_\mu \to \{G_F, M_W, M_Z\}$$
$$LEP \to \{G_F, \alpha, M_Z\}$$

$$(i, j, \sigma) \to (\dim i, j \text{ loops}, v_\sigma)$$

Results shown for Z decay

- Show percentage correction needed to go from LO/LO$_K$ to NLO result

- LO$_K$ includes the largest scheme dependent correction.
  - Remaining corrections are mostly numerically smaller
  - Predictions in the three schemes are in better alignment
- Similar pattern seen for the either two processes.

Conclusion

Conclusion

- We have conducted a systematic study of 3 commonly used EW input schemes.

- Proposed a method for obtaining dominant scheme dependent corrections from LO result

- No strong arguments in favour of any one scheme after these universal corrections are included
    - Motivates fits in different schemes as a consistency check on uncertainties

Backup slides

Backup Slide - LEP scheme Universal correction

To obtain the LO_K prediction in the LEP scheme we first get the LO_K prediction in the αμ scheme and then replace M_M with its SMEFT expansion in the LEP

$$M_W = \hat{M}_W\left[1 + v_\mu^2\hat{\Delta}_W^{(6,0,\mu)} + \frac{1}{v_\mu^2}\hat{\Delta}_W^{(4,1,\mu)} + \hat{\Delta}_W^{(6,1,\mu)}\right]$$

$$\hat{M}_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha v_\mu^2}{M_Z^2}}\right)$$

$$\hat{\Delta}_W^{(6,0,\mu)} = -\frac{\hat{s}_w^2}{2\hat{c}_{2w}}\hat{\Delta}r^{(6,0)},$$

$$\hat{\Delta}_W^{(4,1,\mu)} = -\frac{\hat{s}_w^2}{2\hat{c}_{2w}}\hat{\Delta}r^{(4,1)},$$

$$\hat{\Delta}_W^{(6,1,\mu)} = -\frac{\hat{s}_w^2}{2\hat{c}_{2w}}\hat{\Delta}r^{(6,1)} - \frac{\hat{s}_w^4}{4\hat{c}_{2w}^2}\left(1 + \frac{4\hat{c}_w^2}{\hat{c}_{2w}}\right)\hat{\Delta}r^{(6,0)}\hat{\Delta}r^{(4,1)}$$

$$\partial_W \equiv M_W\frac{\partial}{\partial M_W}$$

$$\hat{\Delta}r^{(6,0)} = \Delta r^{(6,0)}\Big|_{M_W=\hat{M}_W},$$

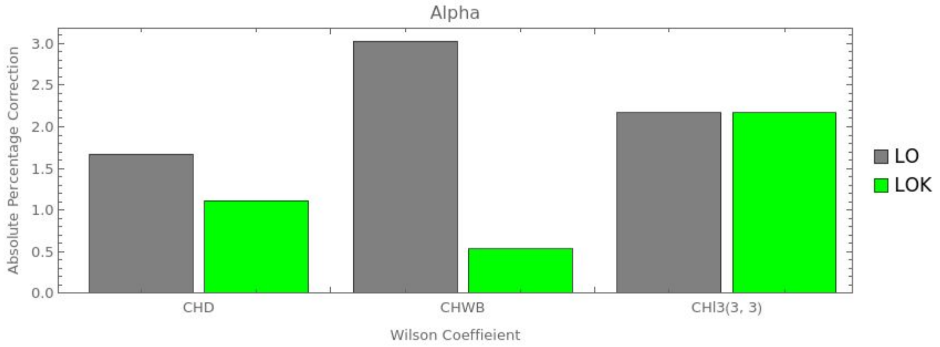
$$\hat{c}_{2w} = 2\hat{c}_w^2 - 1$$

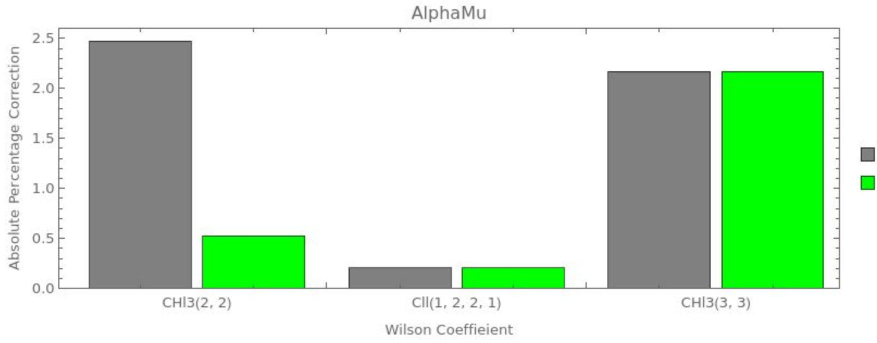
$$\hat{\Delta}r^{(4,1)} = \Delta r^{(4,1)}\Big|_{M_W=\hat{M}_W},$$

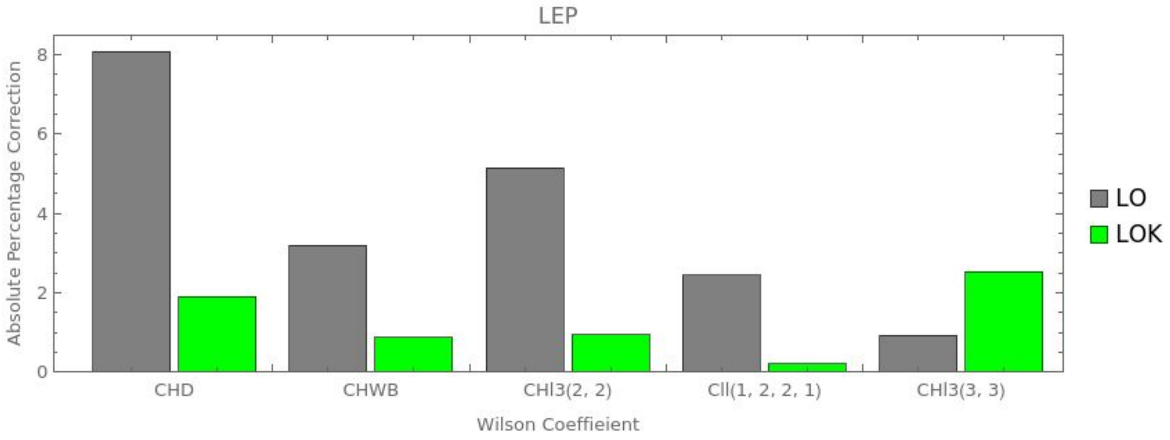
$$\hat{\Delta}r^{(6,1)} = \Delta r^{(6,1)} - \frac{\hat{s}_w^2}{2\hat{c}_{2w}}\left[\Delta r^{(6,0)}\partial_W\Delta r^{(4,1)} + \Delta r^{(4,1)}\partial_W\Delta r^{(6,0)}\right]\Bigg|_{M_W=\hat{M}_W}$$

$$\frac{v_\alpha^2}{v_\mu^2} = 1 + \Delta r$$

Backup Slide - W decay LO_K







Backup Slide - H decay LO_K