HEFT vs. SMEFT in particular UV models

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based on 2305.07689, with Sally Dawson, Carlos Quezada-Calonge and Juan José Sanz-Cillero

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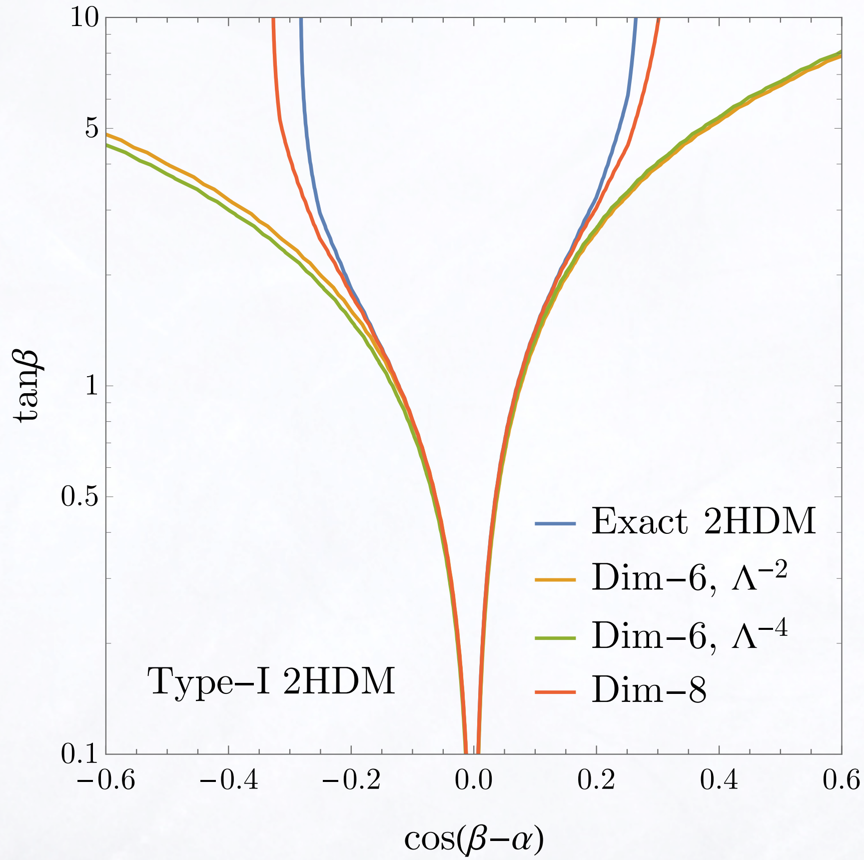
- In HEFT 2022, I showed this fit to Higgs signal strengths:



[Dawson et al, 2205.01561]

- In the SMEFT matching to the **2HDM**, dim. 8 operators are needed in some regions
- Can the HEFT matching do better?

- In the last few years, an EFT approach to BSM physics became very popular
- For BSM physics affecting the Higgs sector, two common EFTs: SMEFT and HEFT
- The SMEFT is a consistent EFT generalization of the SM with a series of higher dimensional operators which are invariant under $\mathrm{SU_c}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$, using solely SM fields:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}, \qquad d > 4$$

In particular, the observed Higgs-like boson is embedded in the SU(2) Higgs doublet

- By contrast, the HEFT is a fusion of chiral perturbation theory (χPT) (in the scalar sector) with SMEFT (in the fermion and gauge sector). Just as in χPT:
  - The 3 Goldstone bosons π^I are imbedded into $\mathbf{U} = \exp\left(i\tau^I \pi^I / v\right)$
  - The remaining scalar (Higgs boson) is a gauge singlet
  - There is an expansion in the number of (covariant) derivatives. At LO:

$$\mathcal{L}_{\mathrm{HEFT}} \supset \tfrac{v^2}{4} \mathcal{F}(h) \mathrm{Tr}\left\{ D_\mu U^\dagger D_\mu U \right\} + \tfrac{1}{2}(\partial_\mu h)^2 - V(h)$$

with: $$\mathcal{F}(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots, \qquad V(h) = \tfrac{1}{2} m_h^2 h^2 \left(1 + d_3 \frac{h}{v} + \frac{d_4}{4}\frac{h^2}{v^2} + \ldots\right)$$

- Ultimately, any EFT deviations from the SM should be explained by a UV model
- Those possible EFT deviations (both SMEFT and HEFT) can be be matched to UV models
- Have the SMEFT and the HEFT ever been matched to the same UV model?

  Yes: in a Z2-symmetric real singlet extension of the SM [Buchala et al, 1608.03564]
- Here, I am interested in taking the **2HDM** as a UV model
  - It looks like it will be essentially like the singlet model, but with more parameters
  - As it turns out, it is quite *different*
  - I will focus on the notion of decoupling, which is crucial to both EFTs
  - I will perform the tree-level matching of both SMEFT and HEFT to the 2HDM
  - I will focus on the tree-level processes $WW \to hh$ and $hh \to hh$
  - The main goal is to compare the performance of the SMEFT and the HEFT matchings

- **2HDM** in a nutshell:
  - take the SM, with its scalar doublet (Φ_1), and add an extra one (Φ_2)
  - impose a Z_2 symmetry, according to which $\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2$
  - allow that symmetry to be softly broken, so that a term proportional to $\Phi_1 \Phi_2$ is possible
  - both Φ_1 and Φ_2 have vevs: $\frac{v_1}{\sqrt{2}}$ and $\frac{v_2}{\sqrt{2}}$; then, define β such that $\tan\beta = v_2/v_1$
  - rotate to the Higgs basis: $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$

An alternative was recently proposed in [Banta et al, 2304.09884]

  - in that basis, only H_1 has vev, $\frac{v}{\sqrt{2}} \equiv \frac{\sqrt{v_1^2 + v_2^2}}{\sqrt{2}}$, and $\mathcal{L}_{\text{2HDM}} \ni \mathcal{L}_{\text{kin}} - V$, with

$$\mathcal{L}_{\text{kin}} = (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2),$$

$$\begin{aligned} V = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c.}\right) \\ & + \frac{Z_1}{2}\left(H_1^\dagger H_1\right)^2 + \frac{Z_2}{2}\left(H_2^\dagger H_2\right)^2 + Z_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + Z_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ & + \left\{\frac{Z_5}{2}\left(H_1^\dagger H_2\right)^2 + Z_6 \left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + Z_7 \left(H_2^\dagger H_2\right)\left(H_1^\dagger H_2\right) + \text{h.c.}\right\} \end{aligned}$$

- consider the particular scenario where Y_3, Z_5, Z_6, Z_7 take real values

caveat: this is not a model, but simply one solution of the CP violating model

Then:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_1^{\mathrm{H}} + iG_0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2^{\mathrm{H}} + iA) \end{pmatrix}$$

where all states are mass eigenstates but $h_1^{\mathrm{H}}, h_2^{\mathrm{H}}$. By introducing α , we find:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} s_{\beta-\alpha} & c_{\beta-\alpha} \\ c_{\beta-\alpha} & -s_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} h_1^{\mathrm{H}} \\ h_2^{\mathrm{H}} \end{pmatrix},$$

where h is the (SM) scalar found at the LHC, and H, A, H^+ are extra (BSM) scalars

- take some of the parameters as independent:

$$c_{\beta-\alpha},\ \beta,\ v,\ m_h,\ Y_2,\ m_H,\ m_A,\ m_{H^\pm}$$

- (fermions will not be relevant)

- In order to build an EFT from the **2HDM**, this model needs a separation of scales: $\Lambda \gg v$
- The effects of the heavy states are expected to be suppressed at low-energies
- This is the essence of *decoupling*, formalized in the decoupling theorem [Appelquist, Carazzone, '75]
- But note: that theorem was formulated for a theory without SSB
  - This means the masses of the particles are independent of the coupling constants
  - So, we can take the former to be very large without affecting the latter

    $\Longrightarrow$ Rendering particles very heavy does not spoil *perturbativity*

N.B.: the Appelquist-Carazzone decoupling assumes perturbativity (as we shall assume here)

- In a theory with SSB, it is more subtle
  - The masses may just be the product of a (fixed) vev and a coupling constant. In this case, decoupling is not possible: the only way to render a mass heavy is to increase the coupling constant, which violates perturbativity
  - But if a particle gets *part* of its mass from a Lagrangian parameter, decoupling is possible: that parameter can be taken very large, without requiring the couplings to be large

- What happens in the **2HDM**?
  - We want the BSM scalars to be very heavy: $m_H \simeq m_A \simeq m_{H^+} \gg m_h = 125$ GeV

$$m_h^2 = \frac{c_{\beta-\alpha}^2}{2\,c_{\beta-\alpha}^2 - 1} Y_2 + \frac{2\,(c_{\beta-\alpha}^2 - 1) Z_1 + c_{\beta-\alpha}^2 Z_{345}}{4\,c_{\beta-\alpha}^2 - 2} v^2, \qquad m_A^2 = Y_2 + \frac{Z_{345} - 2Z_5}{2} v^2,$$

$$m_H^2 = \frac{(c_{\beta-\alpha}^2 - 1)}{2\,c_{\beta-\alpha}^2 - 1} Y_2 + \frac{c_{\beta-\alpha}^2 (2Z_1 + Z_{345}) - Z_{345}}{4\,c_{\beta-\alpha}^2 - 2} v^2 \qquad m_{H^+}^2 = Y_2 + \frac{Z_3}{2} v^2$$

with $Z_{345} \equiv Z_3 + Z_4 + Z_5$

  - Clearly, while taking Y_2 large and keeping the Z's fixed, A and H^+ become heavy
  - But given $c_{\beta-\alpha}$ as an independent parameter, taking Y_2 large is not enough: $c_{\beta-\alpha}$ must be small, so that h can be light and H can be heavy
  - Then, the decoupling limit $m_H \simeq m_A \simeq m_{H^+} \gg m_h$ can be obtained in a way consistent with perturbativity if:

$$Y_2 = \Lambda^2, \qquad m_H^2 = \Lambda^2 + \Delta m_H^2, \quad m_A^2 = \Lambda^2 + \Delta m_A^2, \quad m_{H^+}^2 = \Lambda^2 + \Delta m_{H^+}^2,$$

$$\Lambda^2 \gg v^2, \qquad m_h^2 \sim \mathcal{O}(v^2), \qquad \Delta m_H^2, \Delta m_A^2, \Delta m_{H^+}^2 \sim \mathcal{O}(v^2),$$

$$c_{\beta-\alpha} \sim \mathcal{O}(v^2/\Lambda^2)$$

[Gunion, Haber, 0207010]

- This (decoupling) limit can be used to perform *expansions*. To do that, we introduce ξ such that the power-counting that organizes the expansion is:

$$v^2/\Lambda^2 \sim \mathcal{O}(\xi), \qquad c_{\beta-\alpha} \sim \mathcal{O}(\xi)$$

- The expansion is in powers of ξ. The trivial order, $\mathcal{O}(\xi^0)$, yields SM couplings for the Higgs boson: the alignment limit, $\mathbf{c}_{\beta-\alpha} = \mathbf{0}$

- Both the SMEFT and the HEFT matchings to the **2HDM** will follow this expansion
  - It is true that the SMEFT and the HEFT are in general different
  - Yet, if they match a perturbative 2HDM with heavy masses, they follow the same power-counting
  - Given our choice of independent parameters, that power-counting is in powers of ξ

- To obtain the an EFT from the **2HDM**, the heavy states must be integrated out
- To obtain the SMEFT matching, we must start from the SM before SSB
- So, the integration out of the **2HDM** states must happen before SSB
- Identifying Y_2 with the SMEFT heavy scale Λ^2, and making sure the fields are canonically normalized, we obtain the Lagrangian for the SMEFT matching:

[Dawson et al, 2205.01561]

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{C_{\mathcal{H}}}{\Lambda^2}(\mathcal{H}^\dagger\mathcal{H})^3 + \frac{C_{\mathcal{H}^8}}{\Lambda^4}(\mathcal{H}^\dagger\mathcal{H})^4 + \frac{C^{(1)}_{\mathcal{H}^6}}{\Lambda^4}(\mathcal{H}^\dagger\mathcal{H})^2\,(D_\mu\mathcal{H})^\dagger\,(D^\mu\mathcal{H}) + \ldots + \mathcal{O}(1/\Lambda^6),$$

where $\mathcal{H}$ is the SMEFT Higgs doublet, and, up to $\mathcal{O}(\xi^2)$,

$$\begin{aligned}
\frac{C_{\mathcal{H}}}{\Lambda^2} &= c^2_{\beta-\alpha}\,(\sqrt{2}G_F)^2\left[\Lambda^2 - 4\,(m_h^2 - \Delta m_H^2)\right] + \mathcal{O}(\xi^3),\\
\frac{C_{\mathcal{H}^8}}{\Lambda^4} &= 2\,c^2_{\beta-\alpha}\,(\sqrt{2}G_F)^3\,\left(m_h^2 - \Delta m_H^2\right) + \mathcal{O}(\xi^3),\\
\frac{C^{(1)}_{\mathcal{H}^6}}{\Lambda^4} &= -c^2_{\beta-\alpha}\,(\sqrt{2}G_F)^2 + \mathcal{O}(\xi^3)
\end{aligned}$$

  - Only three operators are relevant for $WW \to hh$ and $hh \to hh$ at tree-level
  - Even at $\mathcal{O}(1/\Lambda^4)$, there is no dependence on β or odd powers of $c_{\beta-\alpha}$

- In the HEFT matching to the **2HDM**, we also use the power-counting in ξ, but we start from the mass states
- The 3-point functions are obtained trivially from the **2HDM** ones. For >3-point, however, we need to integrate out the three heavy states:
  - We write the Lagrangian by separating the light (i.e. SM) fields from the heavy (i.e. BSM) ones:

$$\mathcal{L}_{\text{2HDM}} \supset \frac{1}{2}(\partial_\mu H^a)^2 - \frac{1}{2}(M^2)^{ab} H^a H^b + J_0 + J_1^a H^a \\ + J_2^{ab} H^a H^b + J_3^{abc} H^a H^b H^c + J_4^{abcd} H^a H^b H^c H^d,$$

  where J_k only has light (i.e. SM) fields, and H^a only heavy (i.e. BSM) ones:

$$H^a = (H,\, A,\, H_3,\, H_4), \;\; \text{with} \;\; H^\pm \equiv (H_3 \mp i H_4)/\sqrt{2}$$

  - Each physical heavy scalar H^a is integrated out at tree-level by solving its EoM
  - Replacing those solutions back in the 2HDM Lagrangian yields the HEFT Lagrangian for the **2HDM**. Comparing with the general HEFT Lagrangian,

$$\mathcal{L}_{\text{HEFT}} \supset \frac{v^2}{4}\mathcal{F}(h)\text{Tr}\left\{D_\mu U^\dagger D_\mu U\right\} + \frac{1}{2}(\partial_\mu h)^2 - V(h), \qquad \mathcal{F}(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \dots, \qquad V(h) = \frac{1}{2}m_h^2 h^2 \left(1 + d_3\frac{h}{v} + \frac{d_4}{4}\frac{h^2}{v^2} + \dots\right)$$

  we find the HEFT matching expressions:

N.B.: the matching in general requires higher order terms in the derivative expansion. I do not show them.

$$
\begin{aligned}
\Delta a^2 \equiv a^2 - 1 &= -c_{\beta-\alpha}^2\,, \\
\Delta b \equiv b - 1 &= -3c_{\beta-\alpha}^2 + 4c_{\beta-\alpha}^2 \frac{\Delta m_H^2}{\Lambda^2} + \mathcal{O}(\xi^4)\,, \\
\Delta d_3 \equiv d_3 - 1 &= -2c_{\beta-\alpha}^2 \frac{\Lambda^2}{m_h^2} + \frac{1}{2} c_{\beta-\alpha}^2 + c_{\beta-\alpha}^3 \left[-\cot(2\beta) \left(1 - \frac{2\Delta m_H^2}{m_h^2} \right) + 2c_{\beta-\alpha} \cot^2(2\beta) \frac{\Lambda^2}{m_h^2} \right] + \mathcal{O}(\xi^4)\,, \\
\Delta d_4 \equiv d_4 - 1 &= -12c_{\beta-\alpha}^2 \frac{\Lambda^2}{m_h^2} + c_{\beta-\alpha}^2 \left(\frac{16\Delta m_H^2}{m_h^2} - 11 \right) \\
&\quad + c_{\beta-\alpha}^2 \left[2c_{\beta-\alpha}^2 \frac{\Lambda^2}{m_h^2} \left(22\cot^2(2\beta) - 17 \right) - 22c_{\beta-\alpha} \cot(2\beta) \left(1 - \frac{2\Delta m_H^2}{m_h^2} \right) \right. \\
&\qquad \left. + 16 \frac{\Delta m_H^2}{\Lambda^2} \left(\frac{2 - \Delta m_H^2}{m_h^2} \right) \right] + \mathcal{O}(\xi^4)\,.
\end{aligned}
$$

- We considered the HEFT matching up to $\mathcal{O}(\xi^3)$, whereas the SMEFT one up to $\mathcal{O}(\xi^2)$

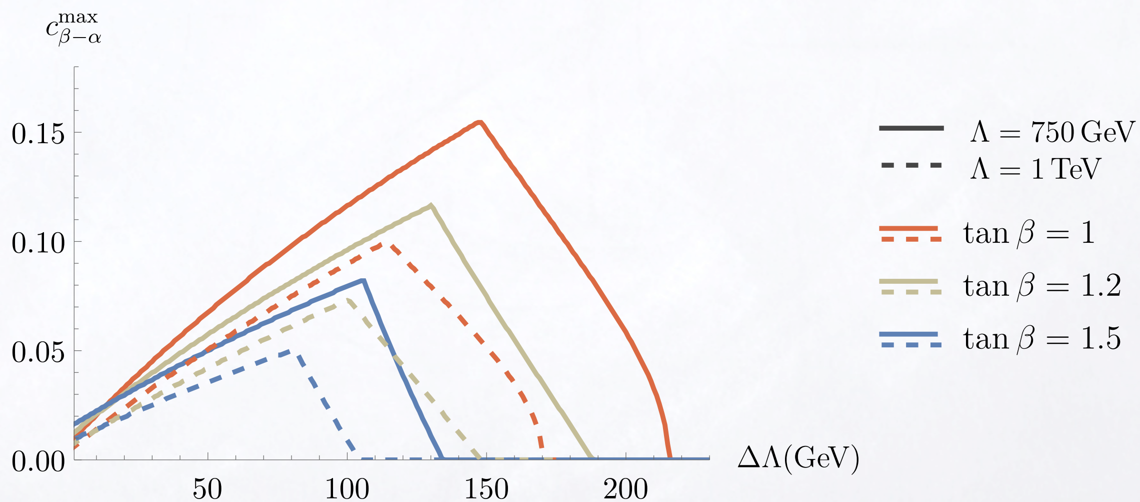
- For our numerical results, we assume that the heavy masses are degenerate, such that:

$$m_H = m_A = m_{H^+} = \Lambda + \Delta\Lambda$$

Since $Y_2 = \Lambda^2$ and $m_A^2 = Y_2 + f_A(Z_j)\,v^2$, $m_{H^+}^2 = Y_2 + f_{H^+}(Z_j)\,v^2$, the new parameter $\Delta\Lambda$ measures the amount of mass in m_A, m_{H^+} that comes from the vev

- We require the **2HDM** to obey theoretical constraints of perturbativity, boundedness from below and EW precision measurements via S, T, U
  - What is the impact of these contraints on the 2HDM parameter space?

- For these large values of Λ , the 2HDM is forced to be close to the alignment limit
- Larger values of Λ (or of $\tan\beta$) would require even narrower a window of $c_{\beta-\alpha}^{\max}$
- In all curves, the segment with positive slope is constrained by boundedness from below, whereas that with negative slope by perturbativity



- The **2HDM** parameter space is also contrained from experiments, especially Higgs couplings measurements, b meson decays and searches for heavy Higgses $\Longrightarrow$ $\tan\beta = 1.2$

- We now compare the (tree-level) SMEFT and HEFT matchings to the **2HDM** at $\mathcal{O}(\xi^2)$
  - Recall that, since we require the 2HDM to have decoupling, the SMEFT and the HEFT matchings follow the same power-counting
  - Hence, even if they are structurally different, their results end up being very similar
  - For example, the couplings hVV and $h\bar{f}f$ are the **same** in both approaches to $\mathcal{O}(\xi^2)$, as are the one-loop processes $gg \to h$ and $h \to \gamma\gamma$
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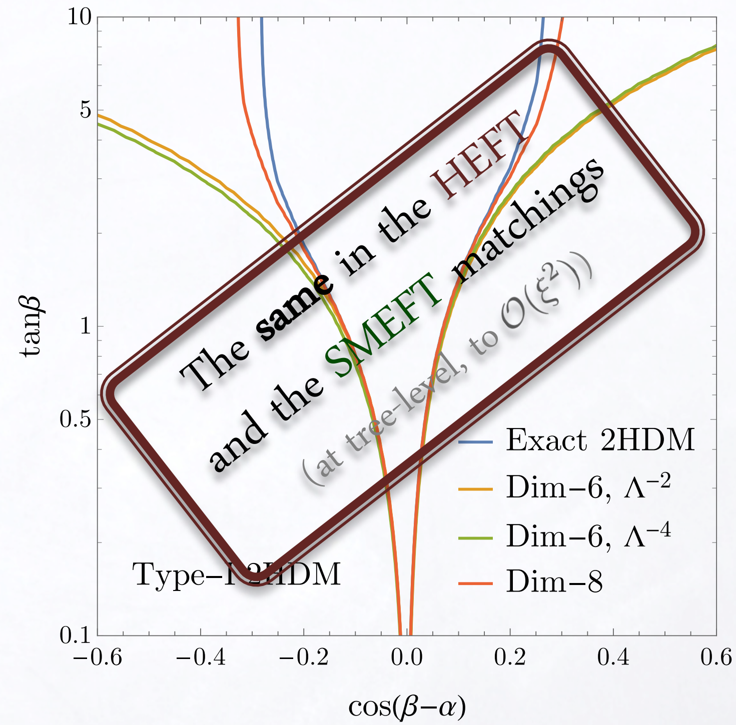

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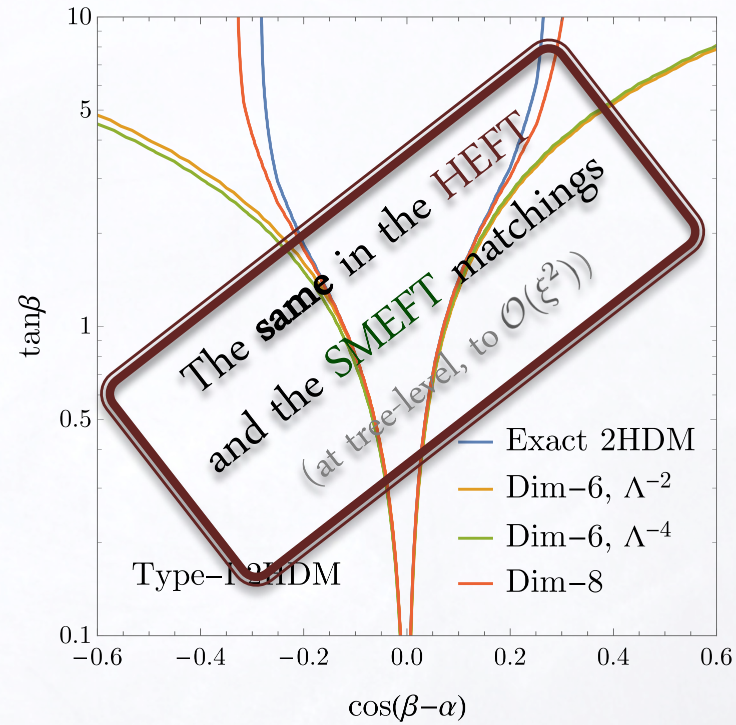
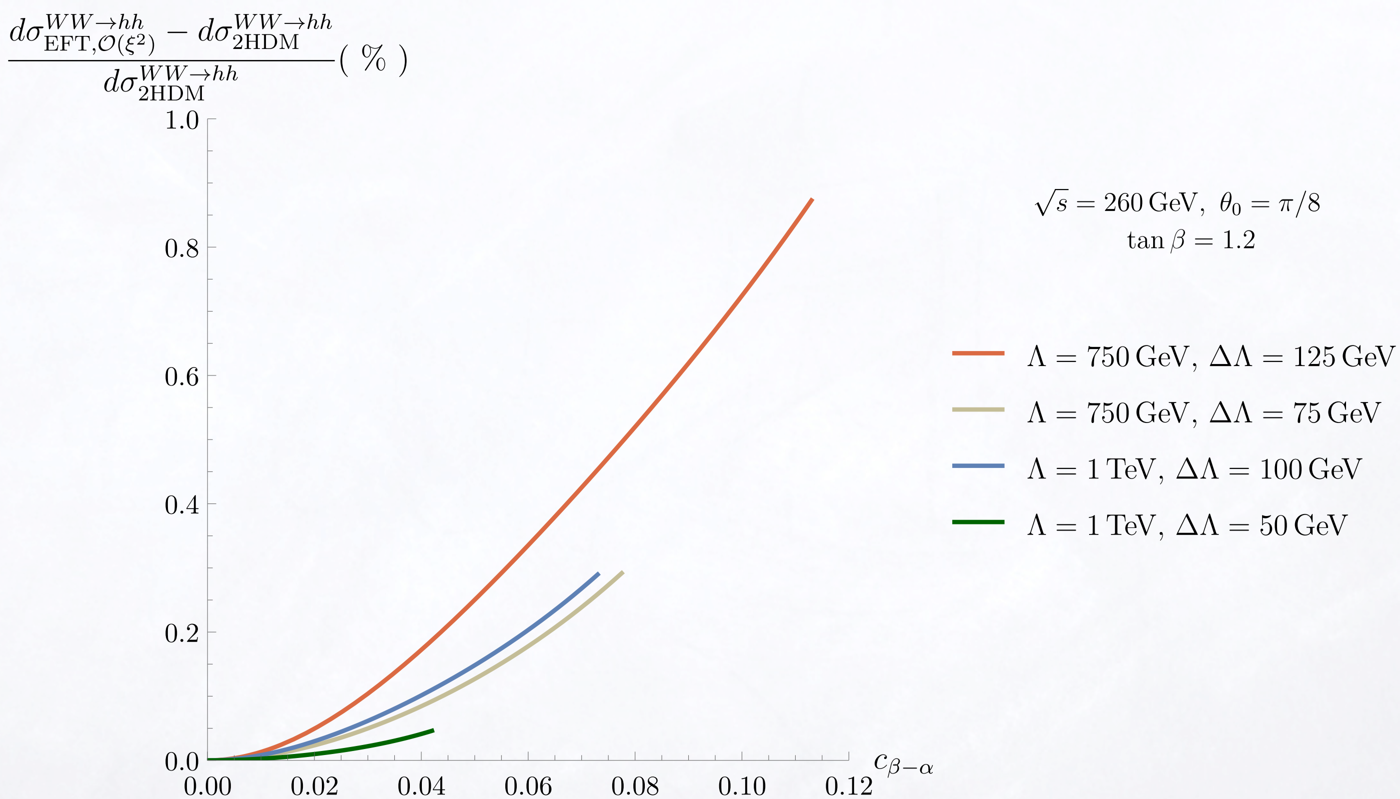


  - Actually, the tree-level scatterings $WW \to hh$ and $hh \to hh$ are also the same at $\mathcal{O}(\xi^2)$!
    - This holds even if the individual Feynman diagrams different

i.e. there is a field redefinition from the HEFT to the SMEFT matching

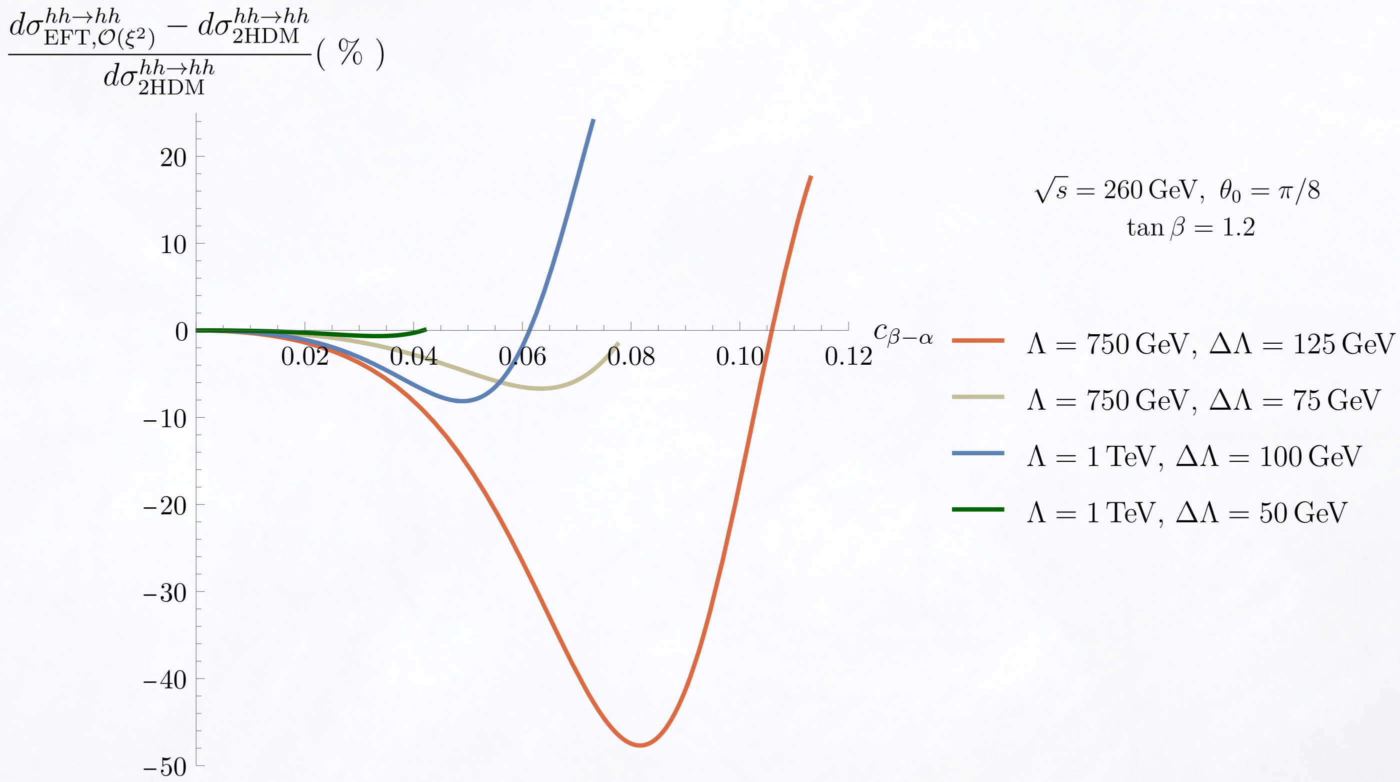
    - In the following, we refer to the two identical matchings at $\mathcal{O}(\xi^2)$ simply as the EFT matching

- Let's start with $WW \to hh$. Using the short notation $d\sigma \equiv \frac{d\sigma}{d\theta}\,|_{\theta=\theta_0}$, and showing only the range of (positive values of) $c_{\beta-\alpha}$ allowed by the theoretical constraints, we find:



- The EFT matching reproduces the 2HDM quite well, with relative differences below 1%

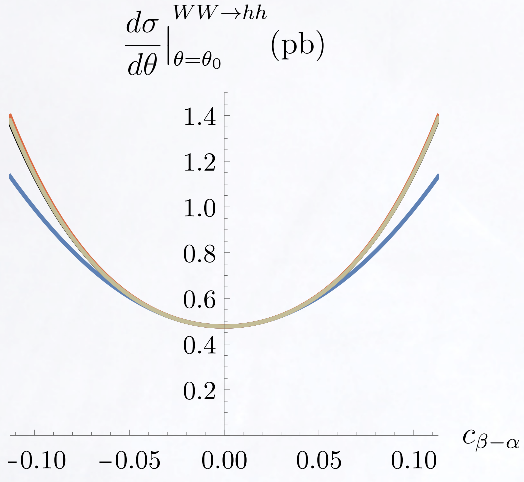
- The case $hh \to hh$ is very different:

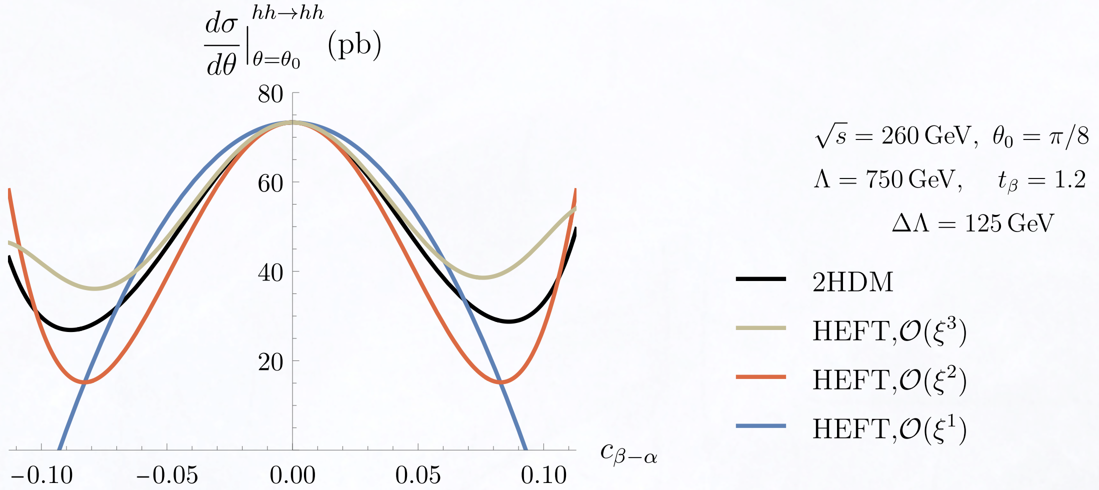


- There are regions where the relative differences (in modulus) is >40%
- In these regions, therefore, $\mathcal{O}(\xi^2)$ is not enough to faithfully replicate the 2HDM results

in terms of SMEFT operators, this means that even dim. 8 operators are not enough!

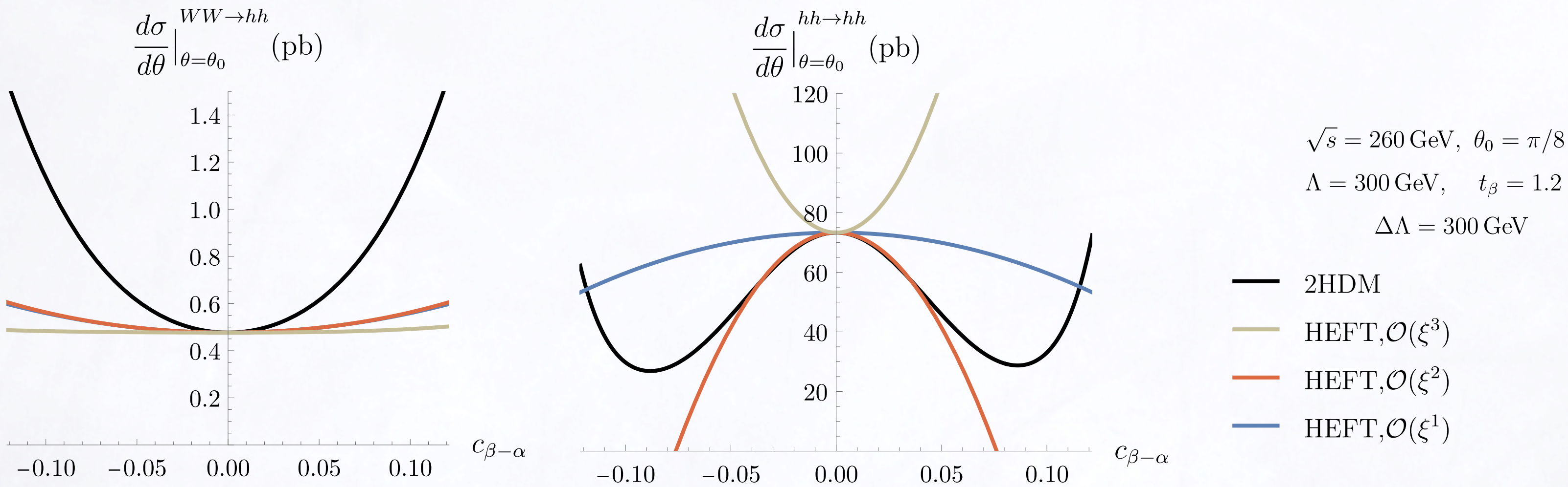
- We can present the results for both $WW \to hh$ and $hh \to hh$ in a different way:





- The plots show the HEFT matching now, which we performed up to $\mathcal{O}(\xi^3)$, but which we are only assured of being equal to the SMEFT one up to $\mathcal{O}(\xi^2)$
- In both plots, the $\mathcal{O}(\xi^1)$ curve does not replicate the 2HDM result away from $\mathbf{c_{\beta-\alpha} = 0}$
- But whereas in $WW \to hh$ the $\mathcal{O}(\xi^2)$ curve does, in $hh \to hh$ not quite
  - For larger values of $c_{\beta-\alpha}$ in $hh \to hh$, the ξ expansion is quite slow

- What happens if decoupling is lost?



- The choice $\Lambda = 300\,\text{GeV}$ is a blatant violation of the decoupling assumption $\Lambda^2 \gg v^2$
- Hence, even if $m_H = m_A = m_{H^+} = \Lambda + \Delta\Lambda = 600\,\text{GeV}$, the expansion does not converge

- I discussed the matching of both the SMEFT and the HEFT to the **2HDM**
- Requiring the **2HDM** to have decoupling (and perturbativity), we obtained an expansion in ξ which we applied to both the SMEFT and the HEFT matchings
- Choosing $c_{\beta-\alpha}$ as independent, we must take into account that $c_{\beta-\alpha} \sim \mathcal{O}(\xi)$
- I performed the SMEFT and the HEFT matchings to $\mathcal{O}(\xi^2)$ at tree-level...
- ... and found no differences between the two approaches
- I studied $WW \to hh$ and $hh \to hh$ at $\mathcal{O}(\xi^2)$. Whereas the former replicates the **2HDM** results for all the allowed range of $c_{\beta-\alpha}$, the latter does not
- The expansion in ξ clearly does not converge if decoupling is lost
- For the future:

a) other processes/orders, *b*) loops, *c*) other models, *d*) alternative power-countings

- On the power-counting:
  - The scaling $c_{\beta-\alpha} \sim \mathcal{O}(\xi)$ complies with perturbativity in the case of heavy H, A, H^+
    - A scaling like $c_{\beta-\alpha} \sim \mathcal{O}(\xi^0)$ would not comply with perturbativity: even though the latter would require $c_{\beta-\alpha}$ to be small, that scaling would ignore this $\implies$ bad power-counting
  - Had we chosen Z_6 as independent instead of $c_{\beta-\alpha}$, one would simply require $\dfrac{Z_6}{4\pi} \lesssim \mathcal{O}(1)$, in which case the expansion would simply be in inverse powers of Λ^2
    - In that case, we would find $c_{\beta-\alpha} = Z_6 \dfrac{v^2}{Y_2} + \mathcal{O}(\dfrac{v^4}{Y_2^2}) \sim \mathcal{O}(\xi^1)$, which justifies the scaling of $c_{\beta-\alpha}$ when we take it as independent
    - That would be enough to ensure that h is light and H is heavy:

$$(m_h^H)^2 = \frac{2\,Y_2 + v^2(2Z_1 + Z_{345}) \pm \sqrt{\left[2\,Y_2 + v^2(Z_{345} - 2Z_1)\right]^2 + 16 v^4 Z_6^2}}{4}$$

    - The two scenarios (taking $c_{\beta-\alpha}$ or Z_6 as independent) are equivalent

- The need to scale $c_{\beta-\alpha}$ can be seen though the h^3 coupling:
  - As any 3-point function, it is not affected by the integration out of heavy states
    - $\underbrace{\mathcal{L} = \_\_H^2}_{\text{2-point}} + \underbrace{\_\_Hh^2 + \_\_HW^\mu W_\mu + \ldots}_{\text{3-point}} \implies H = \underbrace{\_\_h^2 + \_\_W^\mu W_\mu + \ldots}_{\text{2-point}}$
    - So, replacing the the solution of the EoM in $\mathcal{L}$ generates at least 4-point functions
  - Therefore, the h^3 interaction in the HEFT matching is obtained simply by:

    a) considering the h^3 interaction in the 2HDM,

    b) applying the EFT expansion
  - Now, the h^3 interaction in the 2HDM has the Feynman rule:

$$\frac{3i\,\csc^2(2\beta)}{2v}\Big\{s_{\beta-\alpha}\cos(4\beta)\Big[-3c^4_{\beta-\alpha}m_H^2 - 2c^2_{\beta-\alpha}Y_2 + \left(3c^4_{\beta-\alpha} + c^2_{\beta-\alpha} + 1\right)m_h^2\Big]$$
$$+c^3_{\beta-\alpha}\sin(4\beta)\Big[\left(1-3c^2_{\beta-\alpha}\right)m_h^2 + \left(3c^2_{\beta-\alpha} - 2\right)m_H^2 + 2Y_2\Big]$$
$$+s_{\beta-\alpha}\Big[2c^2_{\beta-\alpha}Y_2 - c^4_{\beta-\alpha}m_H^2 + \left(c^4_{\beta-\alpha} - c^2_{\beta-\alpha} - 1\right)m_h^2\Big]\Big\}.$$

  - But since this rule scales with *positive* powers of m_H, we can't just expand in $1/m_H$

- This is to be contrasted with the Z2 real singlet extension of the SM, where the h^3 Feynman rule is $i\frac{m^2}{2vv_s}(s^3v - c^3v_s)$
  - This leads Buchalla et al to perform an expansion just in powers of the physical heavy mass M
  - They show that, at least at tree-level, the effective Lagrangian has no *positive* powers of M
  - The key seems related with an *exact* Z2: imposing an exact Z2 on the 2HDM would lead to a similar behavior
- In the 2HDM (with softly broken Z2), we really need to scale $c_{\beta-\alpha}$

- We considered the HEFT matching up to $\mathcal{O}(\xi^3)$, whereas the SMEFT one up to $\mathcal{O}(\xi^2)$. This is because the HEFT approach is much simpler to implement (for our purposes)

  - In the SMEFT approach, higher order terms contain the scalar doublet, which includes the vev. Hence, 2-point functions are in general affected

    (which means that kinetic terms and relations between masses and Lag. parameters need to be redefined)

    In the HEFT approach, this never happens, for the integration out of heavy states affects only >3-point functions, as seen before

  - Besides, 3-point function in the HEFT approach are trivially obtained, but not in the SMEFT one

  - For simple processes (as the ones considered here), the HEFT results can be obtained starting from the Feynman diagrams for the 2HDM, and applying the ξ expansion