Jet Bundle Geometry for higher derivative EFTs

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Scalar Effective Field Theories

$$L = V + \frac{1}{2} g_{ij}(\phi)\partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$$

Outline

- Review of geometric formulation of scalar EFTs
- Motivation for use of jet bundles
- Introduction to jet bundles
- Application to toy model

Scalar Theories: Geometric Formulation

- Fields ϕ_i are basically maps $\phi_i : \Sigma \to M$ from spacetime to a field space manifold

  [R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1511.00724]

  [T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2108.03240]

- The manifold may be equipped with a Riemannian metric (geometry)

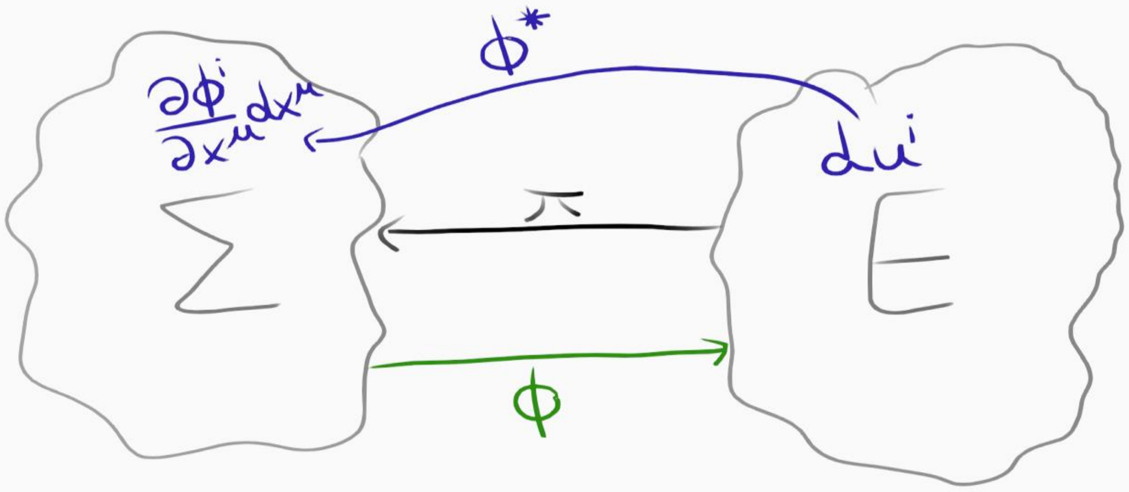
  [R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1605.03602]

$$g = \frac{1}{2} g_{ij}(\phi) d\phi_i \otimes d\phi_j$$

Scalar Theories: Geometric Formulation

- Pulling back to the space-time manifold and contracting with $\eta^{\mu\nu} \frac{\partial}{\partial x^\mu} \otimes \frac{\partial}{\partial x^\nu}$ gives the two derivative terms of the Lagrangian [A. Helset, A. Martin and M. Trott, arXiv:2001.01453]

$$L = V(\phi) + \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$$

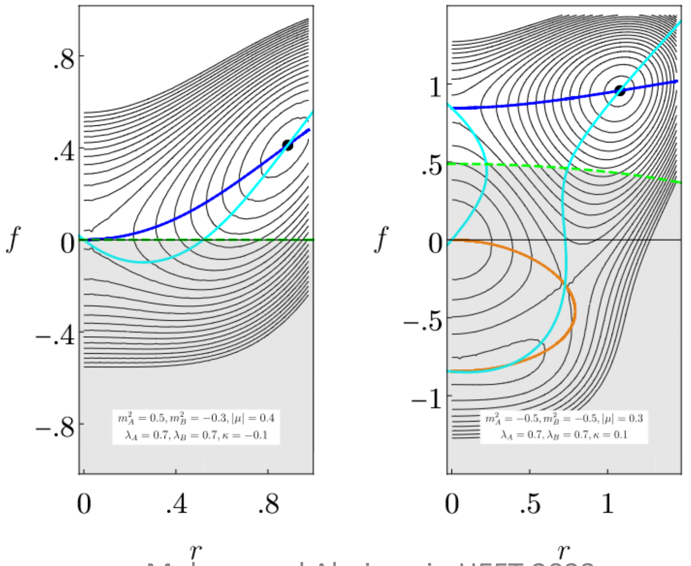


Scalar Theories: Geometric Formulation

- Non-derivative field redefinitions in the Lagrangian may then be interpreted as coordinate transformations on the manifold M

  [R. Alonso, E.E. Jenkins and A.V. Manohar, arXiv:1602.00706]

- Explore geometric invariants as means to distinguish scalar theories from one another, most notably SMEFT and HEFT



[T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2008.08597]

Motivation for Jet Bundles

- Two challenges are faced when using the procedure outlined earlier

  1. It is unclear how to write a higher derivative term geometrically
     [N. Craig, Y. Lee, X. Lu and D. Sutherland, arXiv:2305.09722]
  2. Field redefinitions involving derivatives are not possible to write as coordinate transformations
     [T. Cohen, N. Craig, X. Lu and D. Sutherland, arXiv:2202.06965]

- Invariance of objects like the Ricci scalar is specific to coordinate transformations and not to general transformations of a metric

Motivation for Jet Bundles

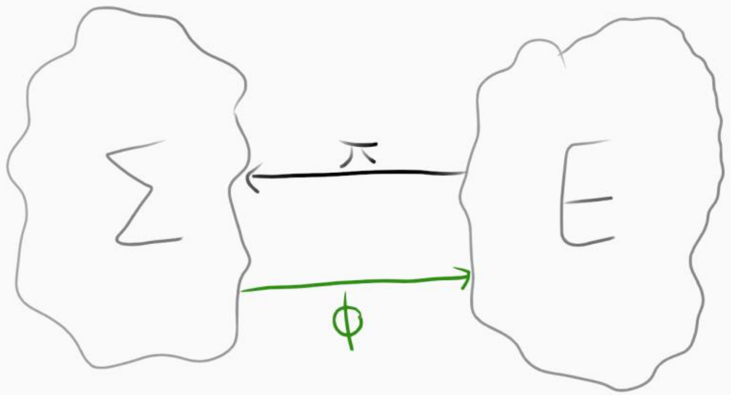
- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4)$ Geometry on M

- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4) + O(\partial^6)$ Geometry on $J^1\pi$

- $L = V(\phi) + \frac{1}{2} g_{ij}(\phi) \partial_\mu \phi^i \partial^\mu \phi^j + O(\partial^4) + O(\partial^6) + O(\partial^8)$ Geometry on $J^2\pi$

What is a Jet?

- Start with a bundle $E \to \Sigma$



- For a coordinate system (x^μ, u^i) on E we say that two sections ϕ, ψ are 1-equivalent at $p \in M$ if

$$\phi(p) = \psi(p) \qquad \left.\frac{\partial(\phi \circ u^i)}{\partial x^\mu}\right|_p = \left.\frac{\partial(\psi \circ u^i)}{\partial x^\mu}\right|_p$$

What is a Jet?

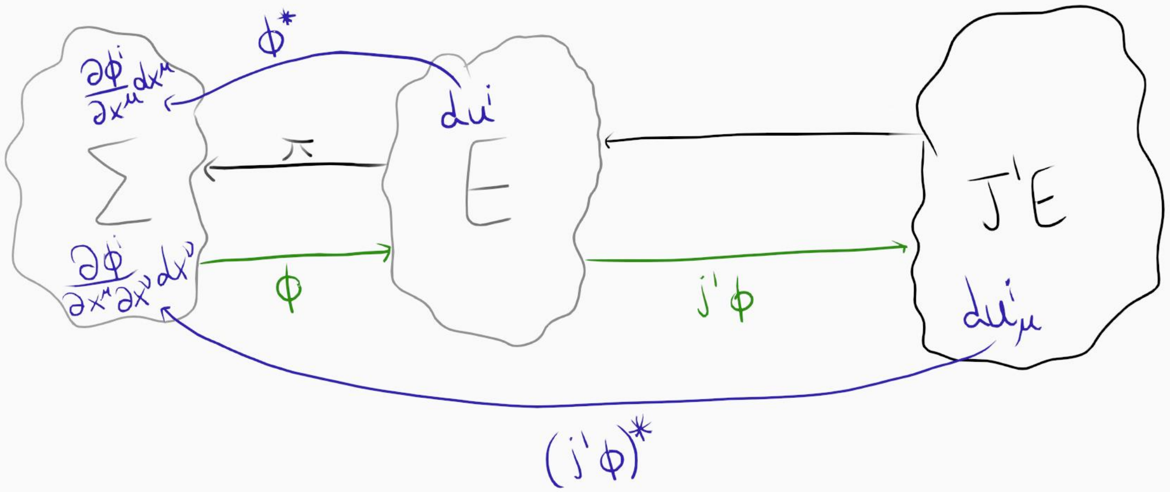
- The equivalence class at p containing ϕ is called the 1-jet of ϕ and is denoted $j_p^1\phi$

- The set of all 1-jets of local sections has a natural structure as a smooth manifold and is referred to as the jet manifold $J^1\pi$

D. J. Saunders, The Geometry of Jet Bundles, doi:10.1017/CBO9780511526411

Jet Bundle Properties

- For an arbitrary bundle $E \to \Sigma$ with coordinates (x^μ, u^i) the jet manifold naturally acquires a coordinate system $\left(x^\mu, u^i, u^i_\mu\right)$
- u^i_μ are independent coordinates on J^1E, but pullback under $j^1\phi$ to $\frac{\partial \phi^i}{\partial x^\mu}$



Riemannian Geometry on Jet Bundle

- The jet manifold may always be equipped by a Riemannian metric since it is a smooth manifold
- For a base space Σ with dimension m and a total space E of dimension $n+m$ the first jet manifold has dimension $m+n+nm$
- Components of the metric are functions of ϕ^i, ϕ^i_μ i.e $g_{ij} = g_{ij}(\phi^k, \phi^k_\mu)$

Riemannian Metric on Jet Bundle

$$L = \eta^{-1}((j^1\phi)^* g)$$

$$g = g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{ij} d\phi^i \otimes d\phi^j + g_{ij}^{\mu\nu} d\phi^i_\mu \otimes d\phi^j_\nu$$

$$+ g_{i\mu} d\phi^i \otimes dx^\mu + g_{ij}^{\mu} d\phi^i_\mu \otimes d\phi^j + g_{i\nu}^{\mu} d\phi^i_\mu \otimes dx^\nu$$

$$g_{ij}\eta^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \subset L$$

$$g_{\mu\nu}\eta^{\mu\nu} = V(\phi) + \cdots \subset L$$

$$g_{ij}^{\mu\nu}\eta^{\rho\sigma}\partial_\rho\partial_\mu\phi^i\partial_\sigma\partial_\nu\phi^j \subset L$$

Lagrangian from Metric: SMEFT Example

- Look at an operator

$$O = \frac{1}{\Lambda^2}\left(\partial_\mu\partial_\nu\phi \cdot \partial^\mu\partial^\nu\phi\right)F\left(\frac{\phi\cdot\phi}{\Lambda^2}\right)$$

- Obtained by pulling back

$$\frac{1}{\Lambda^2}\eta^{\mu\nu}\delta_{ij}F\left(\frac{\phi\cdot\phi}{\Lambda^2}\right)d\phi_\mu^i \otimes d\phi_\nu^j \sim g_{ij}^{\mu\nu}d\phi_\mu^i \otimes d\phi_\nu^j$$

- And contracting with $\eta^{\rho\sigma}\frac{\partial}{\partial x^\rho} \otimes \frac{\partial}{\partial x^\sigma}$

Understanding Jets: Toy Example

- Simplest EFT: QM on a line with two $\mathbb{Z}_2$ symmetries: time reversal and parity
- Bundle $E \to \Sigma$ with coordinates (t, ϕ)
- Jet Bundle $J^1 E \to E$ with coordinates $(t, \phi, \dot{\phi})$
- Most general 4-derivative lagrangian in a non-redundant basis

$$L = V(\phi) + A(\phi)\dot{\phi}^2 + B(\phi)\dot{\phi}^4 + C(\phi)\ddot{\phi}^2$$

Understanding Jets: Toy Example

$$L = \eta^{-1}((j^1\phi)^* g)$$

- Most general metric g generating L

$$g_{\dot\phi\dot\phi} = C(\phi) \qquad g_{t\dot\phi} = \dot\phi\tilde{A}(\phi) + \phi\dot\phi^2\tilde{D}(\phi)$$

$$g_{\phi\dot\phi} = \dot\phi\phi D(\phi) \qquad g_{t\phi} = \dot\phi\tilde{A}(\phi) + \dot\phi^3\tilde{B}(\phi)$$

$$g_{\phi\phi} = A(\phi) + \dot\phi^2 B(\phi) \qquad g_{tt} = V(\phi) + \dot\phi^2\tilde{A}(\phi) + \dot\phi^4\tilde{B}(\phi)$$

Understanding Jets: Toy Example

- The functions $D(\phi), \widetilde{D}(\phi)$ may be removed by IBP relations
- The functions denoted with $\tilde{A}, \tilde{B}$ may be eliminated by shifting A, B
- The relation between metrics and Lagrangians is not bijective, but is *surjective*!
- The full Lagrangian up to four derivatives has been captured by the metric!!
- Works for SMEFT/HEFT as well!!!

Conclusion and outlook

- Write higher derivative terms geometrically (Completed)
- Geometric invariants on jet manifolds and their significance?
- Derivative field redefinitions?
- How does gauging work? **(Very important)**
- Higher jet orders?

Thank you!