

# A Green's Basis for the Bosonic SMEFT to Dimension 8

Álvaro Díaz Carmona    [aldiaz@ugr.es](mailto:aldiaz@ugr.es)

Universidad de Granada

HEFT 2023 (Manchester)    19th - 21st June 2023

*This work was based on our paper [10.1007/JHEP05\(2022\)138](https://arxiv.org/abs/10.1007/JHEP05(2022)138)*

*[2112.12724] Chala, Díaz-Carmona, and Guedes 2022.*

*Grant PID2019-106087GB-C21 funded by:*



UNIVERSIDAD  
DE GRANADA



# Outline

Introduction

Building a Green's basis

Removing redundancies

Results and applications

Conclusions

# Introduction

## SMEFT Lagrangian

Consider the SMEFT Lagrangian, up to dimension  $d$ :

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{j=4}^d \sum_i^{n_{\text{op}}} \frac{c_i^{(j)}}{\Lambda^2} \mathcal{O}_i^{(j)}$$

- ▶ Field content of the operators defines classes.
- ▶ Classes are closed sets for algebraic transformations, IBP and other relations.
- ▶ Classes are not closed for the field redefinitions/EoM.
- ▶ **Redundant** operators vanish under EoM. Non-redundant operators are called **physical**.

# Introduction

## Physical basis and Green's Basis

Classes of operators have a finite number of independent members which can be known, for example, through `Sym2Int` [1703.05221] Fonseca 2017 and `Basisgen` [1901.03501] Juan Carlos Criado 2019. This means that a **basis** can be created.

- ▶ For dimension 6, there are **63 physical** and **69 redundant** operators.
- ▶ For dimension 8 there are **993 physical** and **1649 redundant** operators.

We call Green's Basis to a set of off-shell independent operators. It can be applied to renormalisation [2003.12525] Gherardi, Marzocca, and Venturini 2020, [2106.05291] Chala, Guedes, et al. 2021, [2205.03301] Das Bakshi et al. 2022. or to matching [2112.10787] Carmona et al. 2021.

# Introduction

List of physical and Green's bases available

Dimension	Basis
$d_6$	[1008.4884] Grzadkowski et al. 2010
$d_6$	[2003.12525] Gherardi, Marzocca, and Venturini 2020
$d_7$	[1410.4193] Lehman 2014 + [1607.07309] Liao and Ma 2016
$d_7$	[2306.03008] Zhang 2023
$d_8$	[2005.00059] Murphy 2020 & [2005.00008] Li, Ren, Shu, et al. 2021
$d_8$	This work (bosonic) & [2211.01420] Ren and Yu 2022 (GB only!)
$d_9$	[2007.08125] Liao and Ma 2020 & [2007.07899] Li, Ren, Xiao, et al. 2021
$d_9$	...

# Building a Green's basis

## Working in momentum space

For simplicity, we choose working in momentum space, i.e: setting  $\partial_\mu \rightarrow -ip_\mu$ . We can find a 1 to 1 mapping to all the previous concepts and relations. In particular:

$$\left. \begin{aligned} D^2 \phi^i &= \mu^2 \phi^i - 2\lambda(\phi^\dagger \phi) \phi^i \\ p_\phi^2 &= \mu^2 - 2\lambda(\phi^\dagger \phi) \end{aligned} \right\} \text{EoM} \Leftrightarrow \text{on-shell field}$$

- ▶ A minimal set of **physical** independent operators is generated by **onshell** processes.
- ▶ A minimal set of **redundant** independent operators is generated by **offshell** processes.
- ▶ **Onshell** processes are represented by **connected** diagrams.
- ▶ **Offshell** processes are represented by **1PI** diagrams.

# Building a Green's basis

## Off-shell method

In the off-shell approach we study the 1PI amplitudes generated by a set of operators  $\{\mathcal{O}_i\}_{i=1\dots N}$  for some processes:

$$\mathcal{A}(a \rightarrow b) = \sum_{i, \alpha} c_i f_{\alpha}^i(\vec{g}) \kappa_{\alpha} \quad \vec{g} = (g_1, g_2, g_3, \lambda)$$

$\kappa_{\alpha}$  are the kinematic invariants.  $f_{\alpha}^i$  are the amplitude coefficients.

$$\mathcal{O}_1 = (\phi^{\dagger}\phi) D_{\mu}(\phi^{\dagger}\phi) D^{\mu}(\phi^{\dagger}\phi)$$

$$\mathcal{O}_2 = (\phi^{\dagger}\phi)^2 (D^2\phi^{\dagger}\phi + \phi^{\dagger}D^2\phi)$$

$$\mathcal{O}_3 = (\phi^{\dagger}\phi)^2 (D_{\mu}\phi^{\dagger}D^{\mu}\phi)$$

$$\varphi^0(p_1) \rightarrow \varphi^0(p_2)\varphi^+(p_3)\varphi^-(p_4)\varphi^+(p_5)\varphi^-(p_6) \quad \kappa_{ij} = p_i \cdot p_j$$

	$\kappa_{22}$	$\kappa_{32}$	$\kappa_{33}$	$\kappa_{42}$	$\kappa_{43}$	$\kappa_{44}$	$\kappa_{52}$	$\kappa_{53}$	$\kappa_{54}$	$\kappa_{55}$	$\kappa_{62}$	$\kappa_{63}$	$\kappa_{64}$	$\kappa_{65}$	$\kappa_{66}$
$c_1$	0	0	4i	0	6i	4i	0	4i	6i	4i	0	6i	4i	6i	4i
$c_2$	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i	-8i
$c_3$	4i	4i	0	4i	-2i	0	4i	0	-2i	0	4i	-2i	0	-2i	0

# Building a Green's basis

## Independence of operators

We can determine the dependency of a set of operators in momentum space by checking the rank of the matrix  $M_{i\alpha} \equiv f_{\alpha}^i$  for processes involving these operators

$$\text{Rank}(M) = N \Rightarrow \{\mathcal{O}_i\}_{i=1\dots N} \text{ indep.}$$

If a process exists such that the amplitude coefficients matrix has maximum rank, then the operators are independent. For example:

$$\text{Class } B^2\phi^2D^2 \rightarrow \{\mathcal{O}_i\}_{i=1\dots 12} \rightarrow \text{Rank}(M_{\phi^0\phi^0 \rightarrow BB}) = 12$$

Additional (redundant) operators were generated by applying transformations (except EoM) to other known operators.



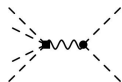
# Removing redundancies

## Equations of motion

To show the redundancies with respect to the physical basis we can apply the equations of motion (EoM). For example:

$$\mathcal{O}_{W\phi^4 D^2}^{(5)} = (\phi^\dagger \phi) \underbrace{D_\nu W^{I\mu\nu}} (D_\mu \phi^\dagger \sigma^I \phi + \text{h.c.})$$

After applying EoM, redundant operators become a linear combination of the physical operators



$$\mathcal{O}_{W\phi^4 D^2}^{(5)} = -g_2 \mathcal{O}_{\phi^6 D^2}^{(2)}$$

which leads to a redefinition of the Wilson Coefficients

$$c_{\phi^6 D^2}^{(2)} \mathcal{O}_{\phi^6 D^2}^{(2)} + c_{W\phi^4 D^2}^{(5)} \mathcal{O}_{W\phi^4 D^2}^{(5)} = \left( c_{\phi^6 D^2}^{(2)} - 2g_2 c_{W\phi^4 D^2}^{(5)} \right) \mathcal{O}_{\phi^6 D^2}^{(2)}$$

# Removing redundancies

## Wilson Coefficient shift

The Wilson Coefficient shift absorbs the coefficients of redundant operators contributing to a certain physical operator.

Wilson Coefficient shift for  $\mathcal{O}_{\phi^6}^{(2)}$ :

$$\begin{aligned}
 c_{\phi^6}^{(2)} \rightarrow & c_{\phi^6}^{(2)} + \frac{1}{4}g_1^2g_2^2c_{B^2D^4} - \frac{g_1^2}{2}c_{B^2\phi^2D^2}^{(8)} - 2g_1\lambda c_{B\phi^2D^4}^{(1)} - g_1c_{B\phi^4D^2}^{(3)} \\
 & + \left(-\frac{1}{8}g_1g_2^2 + 2g_1\lambda\right)c_{B\phi^2D^4}^{(3)} + g_1^2\lambda c_{\phi^2} - 2\lambda c_{\phi^4}^{(6)} + 2\lambda c_{\phi^4}^{(12)} + g_1^2g_2^2c_{W^2D^4} \\
 & - \frac{g_1g_2}{4}c_{WB\phi^2D^2}^{(8)} + \frac{g_1g_2}{2}c_{WB\phi^2D^2}^{(10)} - \frac{3g_1g_2}{8}c_{WB\phi^2D^2}^{(11)} - g_1g_2c_{WB\phi^2D^2}^{(13)} \\
 & - \frac{1}{2}g_1^2g_2c_{W\phi^2D^4}^{(3)} - \frac{g_2}{4}c_{W\phi^4D^2}^{(7)},
 \end{aligned}$$

The list of all shifts is the last requirement for the Green's Basis to be well-defined.

# Results and applications

## Results

- In [2112.12724] Chala, Díaz-Carmona, and Guedes 2022, we feature the explicit form of 89 physical and 86 redundant independent operators, classified by their field content.
- The relation of the redundant operators to the physical ones is explicitly shown.

$\phi^4 D^4$	$(D_\nu \phi^\dagger D_\nu \phi)(D^\mu \phi^\dagger D^\mu \phi)$	$\mathcal{O}_{\phi^4}^{(1)}$	$(D_\mu \phi^\dagger D_\nu \phi)(D^\mu \phi^\dagger D^\nu \phi)$	$\mathcal{O}_{\phi^4}^{(2)}$
	$(D^\mu \phi^\dagger D_\mu \phi)(D^\nu \phi^\dagger D_\nu \phi)$	$\mathcal{O}_{\phi^4}^{(3)}$	$D_\nu \phi^\dagger D^\nu \phi (\phi^\dagger D^2 \phi + \text{h.c.})$	$\mathcal{O}_{\phi^4}^{(4)}$
	$D_\nu \phi^\dagger D^\nu \phi (\phi^\dagger i D^2 \phi + \text{h.c.})$	$\mathcal{O}_{\phi^4}^{(5)}$	$(D_\nu \phi^\dagger \phi)(D^2 \phi^\dagger D_\nu \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(6)}$
	$(D_\nu \phi^\dagger \phi)(D^2 \phi^\dagger i D_\nu \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(7)}$	$(D^2 \phi^\dagger \phi)(D^2 \phi^\dagger \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(8)}$
	$(D^2 \phi^\dagger \phi)(i D^2 \phi^\dagger \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(9)}$	$(D^2 \phi^\dagger D^2 \phi)(\phi^\dagger \phi)$	$\mathcal{O}_{\phi^4}^{(10)}$
	$(\phi^\dagger D^2 \phi)(D^2 \phi^\dagger \phi)$	$\mathcal{O}_{\phi^4}^{(11)}$	$(D_\nu \phi^\dagger \phi)(D^\nu \phi^\dagger D^2 \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(12)}$
$(D_\nu \phi^\dagger \phi)(D^\nu \phi^\dagger i D^2 \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(13)}$			
$X^3 \phi^2$	$f^{ABC}(\phi^\dagger \phi) G_{\mu\nu}^A G_{\rho\sigma}^B G_{\tau\kappa}^C$	$\mathcal{O}_{G^3 \phi^2}^{(1)}$	$f^{ABC}(\phi^\dagger \phi) G_{\mu\nu}^A G_{\rho\sigma}^B G_{\tau\kappa}^C$	$\mathcal{O}_{G^3 \phi^2}^{(1)}$
	$\epsilon^{IJK}(\phi^\dagger \phi) W_{\mu\nu}^I W_{\rho\sigma}^J W_{\tau\kappa}^K$	$\mathcal{O}_{W^3 \phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger \phi) W_{\mu\nu}^I W_{\rho\sigma}^J \tilde{W}_{\tau\kappa}^K$	$\mathcal{O}_{W^3 \phi^2}^{(2)}$
	$\epsilon^{IJK}(\phi^\dagger \phi) B_{\mu\nu}^I W_{\rho\sigma}^J W_{\tau\kappa}^K$	$\mathcal{O}_{W^2 B \phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger \phi) (\tilde{B}_{\mu\nu} W_{\rho\sigma}^I W_{\tau\kappa}^K + B^{\mu\nu} W_{\rho\sigma}^I \tilde{W}_{\tau\kappa}^K)$	$\mathcal{O}_{W^2 B \phi^2}^{(2)}$
$X^2 \phi^4$	$(\phi^\dagger \phi)^2 G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{G^2 \phi^4}^{(1)}$	$(\phi^\dagger \phi)^2 \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{G^2 \phi^4}^{(2)}$
	$(\phi^\dagger \phi)^2 W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2 \phi^4}^{(1)}$	$(\phi^\dagger \phi)^2 \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2 \phi^4}^{(2)}$
	$(\phi^\dagger \phi)(\phi^\dagger \phi)(\phi^\dagger \phi) W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W \phi^4}^{(3)}$	$(\phi^\dagger \phi)(\phi^\dagger \phi)(\phi^\dagger \phi) \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W \phi^4}^{(2)}$
	$(\phi^\dagger \phi)(\phi^\dagger \phi) W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{W B \phi^4}^{(1)}$	$(\phi^\dagger \phi)(\phi^\dagger \phi) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{W B \phi^4}^{(2)}$
	$(\phi^\dagger \phi)^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{B \phi^4}^{(1)}$	$(\phi^\dagger \phi)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{B \phi^4}^{(2)}$
$X \phi^2 D^4$	$i(D_\nu \phi^\dagger \sigma^I D^2 \phi - D^2 \phi^\dagger \sigma^I D_\nu \phi) D_\mu W^{I\mu\nu}$	$\mathcal{O}_{W \phi^2 D^4}^{(1)}$	$(D_\nu \phi^\dagger \sigma^I D^2 \phi + D^2 \phi^\dagger \sigma^I D_\nu \phi) D_\mu W^{I\mu\nu}$	$\mathcal{O}_{W \phi^2 D^4}^{(2)}$
	$i(D_\nu D_\rho \phi^\dagger \sigma^I D^\nu \phi - D^\nu \phi^\dagger \sigma^I D_\rho D_\nu \phi) D_\mu W^{I\mu\rho}$	$\mathcal{O}_{W \phi^2 D^4}^{(3)}$		
	$i(D_\nu \phi^\dagger D^2 \phi - D^2 \phi^\dagger D_\nu \phi) D_\mu B^{\mu\nu}$	$\mathcal{O}_{B \phi^2 D^4}^{(1)}$	$(D_\nu \phi^\dagger D^2 \phi + D^2 \phi^\dagger D_\nu \phi) D_\mu B^{\mu\nu}$	$\mathcal{O}_{B \phi^2 D^4}^{(2)}$
	$i(D_\nu D_\rho \phi^\dagger D^\nu \phi - D^\nu \phi^\dagger D_\rho D_\nu \phi) D_\mu B^{\mu\rho}$	$\mathcal{O}_{B \phi^2 D^4}^{(3)}$		
$X \phi^4 D^2$	$i(\phi^\dagger \phi)(D^\mu \phi^\dagger \sigma^I D^\nu \phi) W_{\mu\nu}^I$	$\mathcal{O}_{W \phi^4 D^2}^{(1)}$	$i(\phi^\dagger \phi)(D^\mu \phi^\dagger \sigma^I D^\nu \phi) \tilde{W}_{\mu\nu}^I$	$\mathcal{O}_{W \phi^4 D^2}^{(2)}$
	$i\epsilon^{IJK}(\phi^\dagger \phi)(\phi^\dagger \phi)(D^\mu \phi^\dagger \sigma^I D^\nu \phi) W_{\mu\nu}^K$	$\mathcal{O}_{W \phi^4 D^2}^{(3)}$	$i\epsilon^{IJK}(\phi^\dagger \phi)(\phi^\dagger \phi)(D^\mu \phi^\dagger \sigma^I D^\nu \phi) \tilde{W}_{\mu\nu}^K$	$\mathcal{O}_{W \phi^4 D^2}^{(4)}$
	$(\phi^\dagger \phi) D_\nu W^{I\mu\nu} (D_\mu \phi^\dagger \sigma^I \phi + \text{h.c.})$	$\mathcal{O}_{W \phi^4 D^2}^{(5)}$	$(\phi^\dagger \phi) D_\nu W^{I\mu\nu} (D_\mu \phi^\dagger \sigma^I \phi + \text{h.c.})$	$\mathcal{O}_{W \phi^4 D^2}^{(6)}$
	$\epsilon^{IJK} (D_\nu \phi^\dagger \sigma^I \phi)(\phi^\dagger \sigma^I D_\nu \phi) W^{K\mu\nu}$	$\mathcal{O}_{W \phi^4 D^2}^{(7)}$	$i(\phi^\dagger \phi)(D^\mu \phi^\dagger D^\nu \phi) B_{\mu\nu}$	$\mathcal{O}_{B \phi^4 D^2}^{(1)}$
	$i(\phi^\dagger \phi)(D^\mu \phi^\dagger D^\nu \phi) \tilde{B}_{\mu\nu}$	$\mathcal{O}_{B \phi^4 D^2}^{(2)}$	$(\phi^\dagger \phi) D_\nu B^{\mu\nu} (D_\mu \phi^\dagger \phi + \text{h.c.})$	$\mathcal{O}_{B \phi^4 D^2}^{(3)}$

# Applications

## Integrating out a scalar singlet to one loop

SM extended with a heavy singlet scalar  $\mathcal{S}$  with a  $\mathbb{Z}_2$  symmetry such that  $\mathcal{S} \rightarrow -\mathcal{S}$ . The New Physics Lagrangian is:

$$\mathcal{L}_{NP} = \frac{1}{2}(D_\mu \mathcal{S})(D^\mu \mathcal{S}) - \frac{1}{2}m_S^2 \mathcal{S}^2 - \lambda_{\mathcal{S}\phi} \mathcal{S}^2 \phi^\dagger \phi - \lambda_S \mathcal{S}^4$$

and can be matched at 1L with our Green's basis just presented using MatchMakerEFT [2112.10787] Carmona et al. 2021

$$\frac{c_{\phi^6}^{(1)}}{\Lambda^4} = \frac{1}{1920 m_S^4 \pi^2} \lambda_{\mathcal{S}\phi}^2 (5\lambda_{\mathcal{S}\phi} - 8\lambda), \quad \frac{c_{\phi^4}^{(3)}}{\Lambda^4} = \frac{1}{960 m_S^4 \pi^2} \lambda_{\mathcal{S}\phi}^2$$

# Results and applications

## Reduction of Lagrangian to a physical basis (I)

Functional methods perform matching by integrating out the heavy fields. A basis is not needed in this case but can be very useful.

For example, if we extend the SM with a real vector triplet  $\mathcal{W} \sim (1, 3)_0$ , where

$$\mathcal{L}_{NP} = \frac{1}{2} \left[ D_\mu \mathcal{W}_\nu^\dagger D^\nu \mathcal{W}^\mu - D_\mu \mathcal{W}_\nu^\dagger D^\mu \mathcal{W}^\nu + m_{\mathcal{W}}^2 \mathcal{W}_\mu^\dagger \mathcal{W}^\mu \right. \\ \left. + (g_{\mathcal{W}}^\phi \mathcal{W}^\mu \phi^{I\dagger} \sigma^I i D_\mu \phi + \text{h.c.}) \right]$$

and we want to integrate it out at Tree-Level.

# Applications

## Reduction of Lagrangian to a physical basis (II)

Matching with MatchingTools [\[1710.06445\]](#) Juan C. Criado 2018 yields the following output:

$$\begin{aligned}
 \mathcal{L}_{\text{EFT}}^{(8)} = & \frac{(g_{\mathcal{W}}^{\phi})^2}{m_{\mathcal{W}}^4} \left[ 2(D_{\mu}\phi^{\dagger}D_{\nu}\phi)(D^{\mu}\phi^{\dagger}D^{\nu}\phi) + 4(D_{\nu}\phi^{\dagger}D^{\nu}D^{\mu}\phi)(D_{\mu}\phi^{\dagger}\phi) \right. \\
 & - 2(D_{\mu}\phi^{\dagger}D_{\nu}\phi)(\phi^{\dagger}D^{\mu}D^{\nu}\phi) - 4(D_{\mu}\phi^{\dagger}\phi)(D^{\mu}D_{\nu}\phi^{\dagger}D^{\nu}\phi) \\
 & + 2(D_{\mu}\phi^{\dagger}D_{\nu}\phi)(D^{\mu}D^{\nu}\phi^{\dagger}\phi) - 4(D_{\mu}\phi^{\dagger}D^{\mu}\phi)(D_{\nu}\phi^{\dagger}D^{\nu}\phi) \\
 & + 2(D_{\mu}\phi^{\dagger}D_{\nu}\phi)(D^{\nu}\phi^{\dagger}D^{\mu}\phi) + \frac{1}{2}(\phi^{\dagger}D_{\mu}D_{\nu}\phi)(\phi^{\dagger}D^{\mu}D^{\nu}\phi) \\
 & - 2(D_{\nu}D_{\rho}\phi^{\dagger}D^{\nu}D^{\rho}\phi)(\phi^{\dagger}\phi) + (D_{\mu}D_{\nu}\phi^{\dagger}\phi)(\phi^{\dagger}D^{\mu}D^{\nu}\phi) \\
 & - 4(\phi^{\dagger}D_{\rho}\phi)(D_{\nu}\phi^{\dagger}D^{\rho}D^{\nu}\phi) + 2(\phi^{\dagger}D_{\nu}D_{\mu}\phi)(D^{\mu}\phi^{\dagger}D^{\nu}\phi) \\
 & + \frac{1}{2}(D_{\mu}D_{\nu}\phi^{\dagger}\phi)(D^{\mu}D^{\nu}\phi^{\dagger}\phi) + 4(D_{\rho}D_{\nu}\phi^{\dagger}D^{\rho}\phi)(D^{\nu}\phi^{\dagger}\phi) \\
 & - 2(D_{\nu}D_{\mu}\phi^{\dagger}\phi)(D^{\mu}\phi^{\dagger}D^{\nu}\phi) - \frac{1}{2}(\phi^{\dagger}D_{\nu}D_{\mu}\phi)(\phi^{\dagger}D^{\mu}D^{\nu}\phi) \\
 & \left. + 2(D_{\rho}D_{\nu}\phi^{\dagger}D^{\nu}D^{\rho}\phi)(\phi^{\dagger}\phi) - (D^{\nu}D^{\mu}\phi^{\dagger}\phi)(\phi^{\dagger}D_{\mu}D_{\nu}\phi) - \frac{1}{2}(D_{\nu}D_{\mu}\phi^{\dagger}\phi)(D^{\mu}D^{\nu}\phi^{\dagger}\phi) \right].
 \end{aligned}$$

# Applications

## Reduction of Lagrangian to a physical basis (III)

By using a simple script we can match  $\mathcal{L}_{\text{EFT}}^{(8)}$  at tree-level to the SMEFT where we embedded our Green's basis.

$$\begin{aligned} \mathcal{L}_{\text{EFT}}^{(8)} = \frac{(g_W^\phi)^2}{m_W^4} & \left[ 2\mathcal{O}_{\phi^4}^{(1)} + 2\mathcal{O}_{\phi^4}^{(2)} - 4\mathcal{O}_{\phi^4}^{(3)} - \frac{1}{4}g_2^2\mathcal{O}_{W^2\phi^4}^{(1)} \right. \\ & + \frac{1}{2}g_1g_2\mathcal{O}_{WB\phi^4}^{(1)} + \frac{3}{4}g_1^2\mathcal{O}_{B^2\phi^4}^{(1)} - 2g_2\mathcal{O}_{W\phi^4D^2}^{(1)} \\ & \left. + 6g_1\mathcal{O}_{B\phi^4D^2}^{(1)} + 2g_1\mathcal{O}_{B\phi^4D^2}^{(3)} \right]. \end{aligned}$$

We get an equivalent Lagrangian with simpler and fewer operators.

# Conclusions

## A Green's Basis for the Bosonic SMEFT to Dimension 8

- ▶ The SMEFT extends the SM with **all possible operators**, but they are related through algebraic relations, IBP or Equations of Motion.
- ▶ We have presented a Green's basis for bosonic interactions up to dimension 8. Computation and algebraic manipulation was simplified by **working in momentum space**.
- ▶ The Green's basis operators are explicitly translated to a physical basis via the Equations of Motion. **Renormalization** can also be achieved.
- ▶ This basis is essential for **off-shell matching methods**, and can be extended for fermionic operators.



# Conclusions

A Green's Basis for the Bosonic SMEFT to Dimension 8

# Thanks for your attention!

*This work was based on our paper [10.1007/JHEP05\(2022\)138](https://arxiv.org/abs/2112.12724)*

*[2112.12724] Chala, Díaz-Carmona, and Guedes 2022.*

Álvaro Díaz Carmona    [aldiaz@ugr.es](mailto:aldiaz@ugr.es)

# APPENDIX

# Removing redundances

## Other transformations and identities

By using algebraic transformations and other relations, the redundant operators are expressed in terms of physical operators:

$$[D_\mu, D_\nu] \phi = -i \frac{g_1}{2} B_{\mu\nu} \phi - i \frac{g_2}{2} \sigma^I W_{\mu\nu}^I \phi$$

$$[D_\mu, D_\nu] W^{I\rho\lambda} = g_2 \epsilon^{IJK} W_{\mu\nu}^J W^{K\rho\lambda}$$

$$[D_\mu, D_\nu] G^{A\rho\lambda} = g_3 f^{ABC} G_{\mu\nu}^B G^{C\rho\lambda}$$

$$0 = \partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu}$$

Bianchi

$$0 = D_\mu (\mathcal{O}_1 \mathcal{O}_2) = D_\mu \mathcal{O}_1 \mathcal{O}_2 + \mathcal{O}_1 D_\mu \mathcal{O}_2$$

IBP

# Applications

## Integrating out a scalar quadruplet to one loop

SM extended with a  $SU(2)_L$  quadruplet scalar  $\Theta$  with  $Y = 1/2$  :

$$\mathcal{L}_{NP} = D_\mu \Theta^\dagger D^\mu \Theta - m_\Theta^2 \Theta^\dagger \Theta - \lambda_\Theta (\phi^\dagger \sigma^I \phi) C_{I\beta}^\alpha \tilde{\phi}^\beta \epsilon_{\alpha\gamma} \Theta^\gamma + h.c.$$

and can be matched at 1L as well.

$$\frac{c_{B^4}^{(1)}}{\Lambda^4} = \frac{7g_1^4}{92160 m_\Theta^4 \pi^2},$$

$$\frac{c_{B^4}^{(2)}}{\Lambda^4} = \frac{g_1^4}{92160 m_\Theta^4 \pi^2},$$

$$\frac{c_{\phi^4}^{(1)}}{\Lambda^4} = \frac{4480 |\lambda_\Theta|^2 - 3g_1^4}{40320 m_\Theta^4 \pi^2},$$

$$\frac{c_{B^2\phi^4}^{(1)}}{\Lambda^4} = \frac{1960 g_1^2 |\lambda_\Theta|^2 - 3g_1^6}{322560 m_\Theta^4 \pi^2},$$

$$\frac{c_{\phi^6}^{(1)}}{\Lambda^4} = \frac{|\lambda_\Theta|^2}{3 m_\Theta^2} + \frac{-6440 g_1^2 |\lambda_\Theta|^2 + 103040 |\lambda_\Theta|^2 \lambda}{80640 m_\Theta^4 \pi^2},$$

$$\frac{c_{\phi^6}^{(2)}}{\Lambda^4} = -\frac{|\lambda_\Theta|^2}{2 m_\Theta^2} + \frac{+3640 g_1^2 |\lambda_\Theta|^2 - 655200 |\lambda_\Theta|^2 \lambda}{483840 m_\Theta^4 \pi^2},$$

$$\frac{c_{\phi^4}^{(2)}}{\Lambda^4} = \frac{3g_1^4 + 1120 |\lambda_\Theta|^2}{40320 m_\Theta^4 \pi^2}, \quad \frac{c_{\phi^4}^{(3)}}{\Lambda^4} = -\frac{|\lambda_\Theta|^2}{18 m_\Theta^4 \pi^2},$$

$$\frac{c_{B\phi^4 D^2}^{(1)}}{\Lambda^4} = -\frac{g_1^5}{13440 m_\Theta^4 \pi^2}.$$

# Appendix

## Subsequent lines of work

- ▶ Computing the anomalous dimension matrix. Bosonic sector has been computed in [2205.03301] Das Bakshi et al. 2022:

$$\dot{c}_{W^2 H^4}^{(i)} \begin{pmatrix} c_{BH^4 D^2}^{(1)} & c_{BH^4 D^2}^{(2)} & c_{WH^4 D^2}^{(1)} & c_{WH^4 D^2}^{(2)} \\ -\frac{g_1 g_2^2}{8} & 0 & \frac{g_1^2 g_2}{3} + \frac{41 g_2^3}{24} - 5g_2 \lambda & 0 \\ 0 & -\frac{g_1 g_2^2}{8} & 0 & \frac{g_1^2 g_2}{3} + \frac{15 g_2^3}{8} - 5g_2 \lambda \\ -\frac{g_1 g_2^2}{8} & 0 & -\frac{5 g_1^2 g_2}{24} & 0 \\ 0 & -\frac{g_1 g_2^2}{8} & 0 & -\frac{5 g_1^2 g_2}{24} \end{pmatrix}$$

- ▶ Including fermionic operators to make a complete dimension 8 Green's basis with onshell relations.