Anomaly Cancellation in EFTs from CDE

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A quick review on anomaly

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a} + \chi^\dagger \bar{\sigma}^\mu P_\mu \chi$$

$$P_\mu \equiv iD_\mu = i\partial_\mu + G^a_\mu t^a$$

$$\left[P_\mu , P_\nu \right] = iF_{\mu\nu}$$

$$U_\alpha = e^{i\alpha} = e^{i\alpha^a t^a}$$

$$\chi \quad \rightarrow \quad \chi_\alpha = U_\alpha \chi$$

$$\chi^\dagger \quad \rightarrow \quad \chi^\dagger_\alpha = \chi^\dagger U^\dagger_\alpha$$

$$P^\mu \quad \rightarrow \quad P^\mu_\alpha = U_\alpha P^\mu U^\dagger_\alpha$$

A quick review on anomaly

$$S_G \qquad S_\chi$$

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$$\int \mathcal{D}G^\mu \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_G + iS_\chi\left[\chi, \chi^\dagger, G^\mu\right]} = \int \mathcal{D}G^\mu e^{iS_G + iW\left[G^\mu\right]}$$

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$$W\left[G^\mu_\alpha\right] \overset{?}{=} W\left[G^\mu\right]$$

A quick review on anomaly

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]}$$

$$= \int \mathcal{D}\chi_\alpha\mathcal{D}\chi_\alpha^\dagger e^{iS_\chi\left[\chi_\alpha,\chi_\alpha^\dagger,G_\alpha^\mu\right]}$$

$$= \int \mathcal{J}_\alpha^{-1}\mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G^\mu\right]} \quad , \quad \left\langle \mathcal{J}_\alpha^{-1} \right\rangle_{S_\chi} \equiv \frac{\int \mathcal{J}_\alpha^{-1}\mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi}}{\int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi}}$$

$$= \left\langle \mathcal{J}_\alpha^{-1} \right\rangle_{S_\chi} e^{iW\left[G^\mu\right]}$$

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$$W\left[G_\alpha^\mu\right] - W\left[G^\mu\right] = -i \log \left\langle \mathcal{J}_\alpha^{-1} \right\rangle_{S_\chi} \equiv \mathcal{A}\left[\alpha\right] + \mathcal{O}\left(\alpha^2\right)$$

"Anomaly Functional"

A quick review on anomaly

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$$\mathcal{L}_\chi = \chi^\dagger \bar{\sigma}^\mu P_\mu \chi + \cdots$$

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"Anomaly Functional"

Motivation

$$\left(W\left[G^{\mu}_{\alpha}\right]-W\left[G^{\mu}\right]\right)\Big|_{\mathcal{O}(\alpha)} \equiv \delta_{\alpha}W\left[G^{\mu}\right] \equiv \mathcal{A}[\alpha] = \int \mathrm{d}^4x\, \alpha^a(x)\mathcal{A}^a(x)$$

Consistent Anomaly

$$\left(\delta_{\alpha_1}\delta_{\alpha_2} - \delta_{\alpha_2}\delta_{\alpha_1}\right)W\left[G^{\mu}\right] = \delta_{-i[\alpha_1\,,\,\alpha_2]}W\left[G^{\mu}\right]$$

Relevant Anomaly

Motivation

$$\left(W\left[G_\alpha^\mu\right]-W\left[G^\mu\right]\right)\Big|_{\mathcal{O}(\alpha)} \equiv \delta_\alpha W\left[G^\mu\right] \equiv \mathcal{A}[\alpha] = \int \mathrm{d}^4x\, \alpha^a(x)\mathcal{A}^a(x)$$

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$$\delta_{\alpha_1}\mathcal{A}[\alpha_2]-\delta_{\alpha_2}\mathcal{A}[\alpha_1] = \mathcal{A}\left[-i[\alpha_1\,,\,\alpha_2]\right]$$

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$$\left(W\left[G_\alpha^\mu\right]-W\left[G^\mu\right]\right)\Big|_{\mathcal{O}(\alpha)} \equiv \delta_\alpha W\left[G^\mu\right] \equiv \mathcal{A}[\alpha] = \int \mathrm{d}^4x\, \alpha^a(x)\mathcal{A}^a(x)$$

Consistent Anomaly $\longrightarrow$ Constraints on regularization scheme

$$\left(\delta_{\alpha_1}\delta_{\alpha_2} - \delta_{\alpha_2}\delta_{\alpha_1}\right)W\left[G^\mu\right] = \delta_{-i[\alpha_1\,,\,\alpha_2]}W\left[G^\mu\right]$$

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Relevant Anomaly $\longrightarrow$ Freedom from renormalization scheme

$$\mathcal{A}[\alpha]=\delta_\alpha\left(-\int \mathrm{d}^4x\mathcal{L}_{\text{ct}}\left[G^\mu\right]\right)$$

Motivation

$$\left(W\left[G^{\mu}_{\alpha}\right]-W\left[G^{\mu}\right]\right)\Big|_{\mathcal{O}(\alpha)}\equiv\delta_{\alpha}W\left[G^{\mu}\right]\equiv\mathcal{A}[\alpha]=\int \mathrm{d}^4x\,\alpha^a(x)\mathcal{A}^a(x)$$

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Relevant Anomaly $\longrightarrow$ Freedom from renormalization scheme

$$\mathcal{A}[\alpha]=\delta_{\alpha}\left(-\int \mathrm{d}^4x\mathcal{L}_{\mathrm{ct}}\left[G^{\mu}\right]\right)\;\Rightarrow\;\begin{cases}\mathcal{L} & \rightarrow\quad \mathcal{L}+\mathcal{L}_{\mathrm{ct}}\left[G^{\mu}\right]\\ W & \rightarrow\quad W+\int \mathrm{d}^4x\mathcal{L}_{\mathrm{ct}}\left[G^{\mu}\right]\\ \mathcal{A}[\alpha] & \rightarrow\quad \mathcal{A}[\alpha]+\delta_{\alpha}\left(\int \mathrm{d}^4x\mathcal{L}_{\mathrm{ct}}\left[G^{\mu}\right]\right)=0\end{cases}$$

γ^5 subtlety in dim-reg

$$\mathrm{tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\alpha\gamma^\alpha\right) \equiv \mathrm{tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) d$$

$$\mathrm{tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) \propto -4i\varepsilon^{\mu\nu\rho\sigma}$$

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Motivation

γ^5 subtlety in dim-reg

$$\mathrm{tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\alpha\gamma^\alpha\right) \equiv \mathrm{tr}\left(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right)d$$

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Breitenlohner-Maison/’t Hooft-Veltman (BMHV) scheme

Bélusca-Maïto, Ilakovac, Kühler, Mađor-Božinović, Stöckinger, and Weißwange, arXiv: 2303.09120

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← Anomaly

Filoche, Larue, Quevillon, and Hoa Vuong, arXiv: 2205.02248

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha\bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) \quad , \quad \mathcal{L}_\chi = \chi^\dagger\bar{\sigma}^\mu P_\mu\chi + \cdots$$

Formalism

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) \qquad , \qquad \mathcal{L}_\chi = \chi^\dagger \bar{\sigma}^\mu P_\mu \chi + \cdots$$

$$W\left[G_\alpha^\mu\right] = -i \log\det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) = W\left[G^\mu\right] + \mathrm{Tr}\left[\frac{1}{\bar{\sigma}^\nu P_\nu}\left(\alpha\, \bar{\sigma}^\mu P_\mu - \bar{\sigma}^\mu P_\mu\, \alpha\right)\right] + \mathcal{O}\left(\alpha^2\right)$$

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$$\mathcal{A}[\alpha] \simeq \mathrm{Tr}\left[\frac{1}{\hat{P}_\beta}\left(\alpha \hat{P}_\beta - \hat{P}_\beta \alpha\right)\frac{1-\gamma^5}{2}\right]$$

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]}=\int\mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]}=\det\left(U_\alpha\bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right)\quad,\quad\mathcal{L}_\chi=\chi^\dagger\bar{\sigma}^\mu P_\mu\chi+\cdots$$

$$W\left[G_\alpha^\mu\right]=-i\log\det\left(U_\alpha\bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right)=W\left[G^\mu\right]+\mathrm{Tr}\left[\frac{1}{\bar{\sigma}^\nu P_\nu}\left(\alpha\,\bar{\sigma}^\mu P_\mu-\bar{\sigma}^\mu P_\mu\,\alpha\right)\right]+\mathcal{O}\left(\alpha^2\right)$$

$$\mathrm{tr}\left[\left(\sigma^{\mu_1}\bar{\sigma}^{\nu_1}\right)\left(\sigma^{\mu_2}\bar{\sigma}^{\nu_2}\right)\cdots\left(\sigma^{\mu_k}\bar{\sigma}^{\nu_k}\right)\right]=\mathrm{tr}\left[\left(\gamma^{\mu_1}\gamma^{\nu_1}\right)\left(\tfrac{1-\gamma^5}{2}+\beta\,\tfrac{1+\gamma^5}{2}\right)\left(\gamma^{\mu_2}\gamma^{\nu_2}\right)\left(\tfrac{1-\gamma^5}{2}+\beta\,\tfrac{1+\gamma^5}{2}\right)\cdots\left(\gamma^{\mu_k}\gamma^{\nu_k}\right)\frac{1-\gamma^5}{2}\right]$$

$$\mathcal{A}[\alpha]\simeq\mathrm{Tr}\left[\frac{1}{\hat{P}_\beta}\left(\alpha\hat{P}_\beta-\hat{P}_\beta\alpha\right)\frac{1-\gamma^5}{2}\right]$$

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) \quad , \quad \mathcal{L}_\chi = \chi^\dagger \bar{\sigma}^\mu P_\mu \chi + \cdots$$

$$W\left[G_\alpha^\mu\right] = -i \log\det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) = W\left[G^\mu\right] + \mathrm{Tr}\left[\frac{1}{\bar{\sigma}^\nu P_\nu}\left(\alpha\, \bar{\sigma}^\mu P_\mu - \bar{\sigma}^\mu P_\mu\, \alpha\right)\right] + \mathcal{O}\left(\alpha^2\right)$$

$$\mathrm{tr}\left[\left(\sigma^{\mu_1}\bar{\sigma}^{\nu_1}\right)\left(\sigma^{\mu_2}\bar{\sigma}^{\nu_2}\right)\cdots\left(\sigma^{\mu_k}\bar{\sigma}^{\nu_k}\right)\right] = \mathrm{tr}\left[\left(\gamma^{\mu_1}\gamma^{\nu_1}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\left(\gamma^{\mu_2}\gamma^{\nu_2}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\cdots\left(\gamma^{\mu_k}\gamma^{\nu_k}\right)\frac{1-\gamma^5}{2}\right]$$

$$\bar{\sigma}^\mu P_\mu \quad \to \quad \not{P} \equiv i\not{\partial} + \sum_a \not{G}^a t^a$$

$$\to \quad \hat{P}_\beta \equiv i\not{\partial} + \sum_a \not{G}^a t^a \left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)$$

$$\mathcal{A}[\alpha] \simeq \mathrm{Tr}\left[\frac{1}{\hat{P}_\beta}\left(\alpha \hat{P}_\beta - \hat{P}_\beta \alpha\right)\frac{1-\gamma^5}{2}\right]$$

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) \quad , \quad \mathcal{L}_\chi = \chi^\dagger \bar{\sigma}^\mu P_\mu \chi + \cdots$$

$$W\left[G_\alpha^\mu\right] = -i \log\det\left(U_\alpha \bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) = W\left[G^\mu\right] + \mathrm{Tr}\left[\frac{1}{\bar{\sigma}^\nu P_\nu}\left(\alpha\, \bar{\sigma}^\mu P_\mu - \bar{\sigma}^\mu P_\mu\, \alpha\right)\right] + \mathcal{O}\left(\alpha^2\right)$$

$$\mathrm{tr}\left[\left(\sigma^{\mu_1}\bar{\sigma}^{\nu_1}\right)\left(\sigma^{\mu_2}\bar{\sigma}^{\nu_2}\right)\cdots\left(\sigma^{\mu_k}\bar{\sigma}^{\nu_k}\right)\right] = \mathrm{tr}\left[\left(\gamma^{\mu_1}\gamma^{\nu_1}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\left(\gamma^{\mu_2}\gamma^{\nu_2}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\cdots\left(\gamma^{\mu_k}\gamma^{\nu_k}\right)\frac{1-\gamma^5}{2}\right]$$

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$$\mathcal{A}[\alpha] \simeq \mathrm{Tr}\left[\frac{1}{\hat{P}_\beta}\left(\alpha \hat{P}_\beta - \hat{P}_\beta \alpha\right)\frac{1-\gamma^5}{2}\right]$$

$$\mathcal{A}_\beta^\Lambda[\alpha] \equiv \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right)\frac{1}{\hat{P}_\beta}\left(\alpha \hat{P}_\beta - \hat{P}_\beta \alpha\right)\frac{1-\gamma^5}{2}\right] = \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right)\alpha\gamma^5\right]$$

Formalism

We work with 4D regularization schemes

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha\bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) \quad , \quad \mathcal{L}_\chi = \chi^\dagger\bar{\sigma}^\mu P_\mu\chi + \cdots$$

$$W\left[G_\alpha^\mu\right] = -i\log\det\left(U_\alpha\bar{\sigma}^\mu P_\mu U_\alpha^\dagger\right) = W\left[G^\mu\right] + \mathrm{Tr}\left[\frac{1}{\bar{\sigma}^\nu P_\nu}\left(\alpha\,\bar{\sigma}^\mu P_\mu - \bar{\sigma}^\mu P_\mu\,\alpha\right)\right] + \mathcal{O}\left(\alpha^2\right)$$

$$\mathrm{tr}\left[\left(\sigma^{\mu_1}\bar{\sigma}^{\nu_1}\right)\left(\sigma^{\mu_2}\bar{\sigma}^{\nu_2}\right)\cdots\left(\sigma^{\mu_k}\bar{\sigma}^{\nu_k}\right)\right] = \mathrm{tr}\left[\left(\gamma^{\mu_1}\gamma^{\nu_1}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\left(\gamma^{\mu_2}\gamma^{\nu_2}\right)\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)\cdots\left(\gamma^{\mu_k}\gamma^{\nu_k}\right)\frac{1-\gamma^5}{2}\right]$$

$$\bar{\sigma}^\mu P_\mu \quad \to \quad \not{P} \equiv i\not{\partial} + \sum_a \not{G}^a t^a$$

$$\to \quad \hat{P}_\beta \equiv i\not{\partial} + \sum_a \not{G}^a t^a \left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)$$

$$\mathcal{A}[\alpha] \simeq \mathrm{Tr}\left[\frac{1}{\hat{P}_\beta}\left(\alpha\hat{P}_\beta - \hat{P}_\beta\alpha\right)\frac{1-\gamma^5}{2}\right]$$

$$\mathcal{A}_\beta^\Lambda[\alpha] \equiv \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right)\frac{1}{\hat{P}_\beta}\left(\alpha\hat{P}_\beta - \hat{P}_\beta\alpha\right)\frac{1-\gamma^5}{2}\right] = \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right)\alpha\gamma^5\right]$$

$$\mathcal{A}_\beta[\alpha] \equiv \lim_{\Lambda\to\infty}\left(\mathcal{A}_\beta^\Lambda[\alpha] + \delta_\alpha\int \mathrm{d}^4x\,\mathcal{L}_{\mathrm{ct}}^\Lambda\right)$$

Requirements on the regulator

$$\mathcal{A}_\beta^\Lambda[\alpha] \equiv \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right) \frac{1}{\hat{P}_\beta}\left(\alpha \hat{P}_\beta - \hat{P}_\beta \alpha\right) \frac{1-\gamma^5}{2} \right] = \mathrm{Tr}\left[f\left(-\frac{\hat{P}_\beta^2}{\Lambda^2}\right) \alpha \gamma^5 \right]$$

$$f(0) = 1 \quad , \quad f(+\infty) = 0 \quad , \quad \int_0^{+\infty} f(u)\,\mathrm{d}u < \infty$$

$$u^n \left.\frac{\mathrm{d}^n f}{\mathrm{d}u^n}\right|_{u=0} = u^n \left.\frac{\mathrm{d}^n f}{\mathrm{d}u^n}\right|_{u\to+\infty} = 0$$

Requirements on the regulator

$$\mathcal{A}_{\beta}^{\Lambda}[\alpha] \equiv \mathrm{Tr}\left[f\left(-\frac{\hat{P}_{\beta}^{2}}{\Lambda^{2}}\right) \frac{1}{\hat{P}_{\beta}} \left(\alpha \hat{P}_{\beta} - \hat{P}_{\beta}\alpha\right) \frac{1-\gamma^{5}}{2} \right] = \mathrm{Tr}\left[f\left(-\frac{\hat{P}_{\beta}^{2}}{\Lambda^{2}}\right) \alpha\gamma^{5} \right]$$

$$f(0)=1 \quad , \quad f(+\infty)=0 \quad , \quad \int_{0}^{+\infty} f(u)\,\mathrm{d}u < \infty$$

$$u^{n} \frac{\mathrm{d}^{n} f}{\mathrm{d}u^{n}}\bigg|_{u=0} = u^{n} \frac{\mathrm{d}^{n} f}{\mathrm{d}u^{n}}\bigg|_{u\to+\infty} = 0$$

Heat kernel: $f(u) = e^{-u}$

Pauli-Villars 1: $f(u) = \dfrac{1}{1+u}$

Pauli-Villars 3: $f(u) = \dfrac{2}{(1+u)(2+u)}$

Universal Evaluation Results

$$\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{i}{16\pi^2}\left\{\left[\Lambda^2\int_0^{+\infty} f(u)\,\mathrm{d}u\right]\mathrm{tr}_0 + \frac{1}{6}\left(\mathrm{tr}_1 + \mathrm{tr}_2 + \mathrm{tr}_3\right)\right\}$$

$$\mathrm{tr}_0 \equiv \mathrm{tr}\left[\hat{P}_\beta^2\gamma^5\alpha\right]$$

$$\mathrm{tr}_1 \equiv \mathrm{tr}\left[\hat{P}_\beta^4\gamma^5\alpha\right]$$

$$\mathrm{tr}_2 \equiv -\frac{1}{2}\mathrm{tr}\left[\left(\hat{P}_\beta^2\gamma_\mu\hat{P}_\beta\gamma^\mu\hat{P}_\beta + \hat{P}_\beta\gamma_\mu\hat{P}_\beta\gamma^\mu\hat{P}_\beta^2\right)\gamma^5\alpha\right]$$

$$\mathrm{tr}_3 \equiv -\frac{1}{2}\mathrm{tr}\left[\hat{P}_\beta\gamma_\mu\hat{P}_\beta^2\gamma^\mu\hat{P}_\beta\gamma^5\alpha\right]$$

$$\hat{P}_\beta \equiv i\partial\!\!\!/ + \sum_a G\!\!\!/^a t^a\left(\tfrac{1-\gamma^5}{2} + \beta\,\tfrac{1+\gamma^5}{2}\right)$$

Universal Evaluation Results

$$\mathcal{A}^{\Lambda}_{\beta}[\alpha] = \int \mathrm{d}^4x \, \frac{1}{16\pi^2} \, \mathrm{tr} \Bigg\{ -2(1-\beta) \left[\Lambda^2 \int_0^\infty \mathrm{d}u \, f(u) \right] G_\mu \left(\partial^\mu \alpha\right)$$

$$+ \frac{1}{3}(1-\beta)\bigg(i\Big[(1+4\beta)\ (\partial_\mu G_\nu) - (1+2\beta)\ (\partial_\nu G_\mu) - i(1+3\beta^2)\left[G_\mu, G_\nu\right] \ , \ G^\mu \Big]$$

$$+ \left(\partial^2 G_\nu\right) + i(1-2\beta)\left[(\partial^\mu G_\mu)\,, G_\nu \right] - G^\mu (1-\beta) G_\nu (1-\beta) G_\mu \bigg) \left(D^\nu_\beta \alpha\right)$$

$$- \frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\bigg(\frac{1}{3}\left\{ (1-\beta) G^\rho \ , \ 2(1+2\beta)\left(\partial^\mu G^\nu\right) - i(1+2\beta+3\beta^2) G^\mu G^\nu \right\} \left(D^\sigma_\beta \alpha\right)$$

$$+ 4\left[\beta\left(\partial^\mu G^\nu\right) - i\beta^2 G^\mu G^\nu \right] \left[\beta\left(\partial^\rho G^\sigma\right) - i\beta^2 G^\rho G^\sigma \right] \alpha \bigg) \Bigg\}, \qquad (4.15)$$

Universal Evaluation Results

$$\begin{aligned}\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x\, \frac{1}{16\pi^2}\Bigg\{ -\sum_{a,b} \mathrm{tr}\left(t^a t^b\right)(1-\beta_a)\Bigg[2\left(\Lambda^2\int_0^\infty \mathrm{d}u\, f(u)\right) G_\mu^a\left(\partial^\mu\alpha^b\right) \\
+\frac{1}{3}\Big\{ f^{aef}\Big[(1+4\beta_a)\left(\partial_\mu G_\nu^e\right) - (1+2\beta_a)\left(\partial_\nu G_\mu^e\right) + \left(1+3\beta_a^2\right) f^{egh} G_\mu^g G_\nu^h\Big] G^{f\mu} \\
-\left(\partial^2 G_\nu^a\right) + (1-2\beta_a) f^{aef}\left(\partial^\mu G_\mu^e\right) G_\nu^f \Big\}\left(\partial_\nu \alpha^b + \beta_b f^{bcd} G_\nu^c \alpha^d\right)\Bigg] \\
-\sum_{a,b,c,d} \mathrm{tr}\left(t^a t^b t^c t^d\right)\frac{1}{3}(1-\beta_a)(1-\beta_b)(1-\beta_c)\, G_\mu^a G_\nu^b G^{c\mu}\left(\partial_\nu\alpha^d + \beta_d f^{def} G_\nu^e \alpha^f\right) \\
-\sum_{a,b,c} \mathrm{tr}\left(\{t^a, t^b\}\, t^c\right)\frac{1}{4}\varepsilon_{\mu\nu\rho\sigma}\Bigg[\beta_a\beta_b\left(F_{\text{lin}}^{a\mu\nu} + \beta_a f^{ade} G^{d\mu} G^{e\nu}\right)\left(F_{\text{lin}}^{b\rho\sigma} + \beta_b f^{bfg} G^{f\rho} G^{g\sigma}\right)\alpha^c \\
+\frac{1}{3}(1-\beta_b)\left(2(1+2\beta_a) F_{\text{lin}}^{a\mu\nu} + \left(1+2\beta_a+3\beta_a^2\right) f^{ade} G^{d\mu} G^{e\nu}\right) G^{b\rho} \\
\times\left(\partial^\sigma\alpha^c + \beta_c f^{cfg} G^{f\sigma}\alpha^g\right)\Bigg]\Bigg\}, \qquad (4.17)\end{aligned}$$

Some special cases of β

Covariant

$$\mathcal{A}^{\Lambda}_{\beta=1}[\alpha] = \int \mathrm{d}^4x \frac{-1}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left(F_{\mu\nu}F_{\rho\sigma}\alpha\right) = \int \mathrm{d}^4x \frac{-1}{64\pi^2} \mathrm{tr}\left(\left\{t^a, t^b\right\} t^c\right) \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^b_{\rho\sigma} \alpha^c$$

Some special cases of β

Covariant

$$\mathcal{A}_{\beta=1}^{\Lambda}[\alpha] = \int d^4x \frac{-1}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left(F_{\mu\nu}F_{\rho\sigma}\alpha\right) = \int d^4x \frac{-1}{64\pi^2} \mathrm{tr}\left(\left\{t^a, t^b\right\} t^c\right) \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^b \alpha^c$$

Consistent

$$\mathcal{A}_{\beta=0}^{\Lambda}[\alpha] = \int d^4x \frac{-1}{48\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left\{\left[\partial_\mu\left(G_\nu F_{\rho\sigma} + iG_\nu G_\rho G_\sigma\right)\right]\alpha\right\} - \delta_\alpha\left(\int d^4x \mathcal{L}_{\mathrm{ct}}^{\Lambda}\right)$$

$$= \int d^4x \frac{1}{48\pi^2} \mathrm{tr}\left(\left\{t^a, t^b\right\} t^c\right) \varepsilon^{\mu\nu\rho\sigma} \left(\partial_\mu \alpha^a\right)\left[\left(\partial_\nu G_\rho^b\right) + \frac{1}{4} f^{bde} G_\nu^d G_\rho^e\right] G_\sigma^c - \delta_\alpha\left(\int d^4x \mathcal{L}_{\mathrm{ct}}^{\Lambda}\right)$$

Applications: Minimally Coupled

Some special cases of β

Covariant

$$\mathcal{A}^{\Lambda}_{\beta=1}[\alpha] = \int d^4x \frac{-1}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left(F_{\mu\nu}F_{\rho\sigma}\alpha\right) = \int d^4x \frac{-1}{64\pi^2} \mathrm{tr}\left(\left\{t^a, t^b\right\}t^c\right) \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^b_{\rho\sigma} \alpha^c$$

Consistent

$$\mathcal{A}^{\Lambda}_{\beta=0}[\alpha] = \int d^4x \frac{-1}{48\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left\{\left[\partial_\mu\left(G_\nu F_{\rho\sigma} + iG_\nu G_\rho G_\sigma\right)\right]\alpha\right\} - \delta_\alpha\left(\int d^4x \mathcal{L}^{\Lambda}_{\mathrm{ct}}\right)$$

$$= \int d^4x \frac{1}{48\pi^2} \mathrm{tr}\left(\left\{t^a, t^b\right\}t^c\right) \varepsilon^{\mu\nu\rho\sigma} \left(\partial_\mu \alpha^a\right)\left[\left(\partial_\nu G^b_\rho\right) + \frac{1}{4} f^{bde} G^d_\nu G^e_\rho\right] G^c_\sigma - \delta_\alpha\left(\int d^4x \mathcal{L}^{\Lambda}_{\mathrm{ct}}\right)$$

- Application Example: Simple non-Abelian Sector

Wess-Zumino Consistency $\Rightarrow$ $\beta = 0$

Applications: Minimally Coupled

- Application Example: $U(1)_V \times U(1)_A$

$$\mathcal{L} = \bar{\psi} i\partial\psi = \bar{\psi}_L i\partial\psi_L + \bar{\psi}_R i\partial\psi_R$$

$$U(1)_V: \quad \psi \to e^{i\alpha}\psi$$

$$\begin{matrix} +1 \\ +1 \end{matrix} \quad \begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$U(1)_A: \quad \psi \to e^{i\alpha\gamma^5}\psi$$

$$\begin{matrix} -1 \\ +1 \end{matrix} \quad \begin{cases} \psi_L \to e^{-i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

- Application Example : $U(1)_V \times U(1)_A$

$$\mathcal{L} = \bar{\psi} i\partial\!\!\!/ \psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R$$

$$i\partial\!\!\!/ \to i\partial\!\!\!/ + V\!\!\!\!/ + A\!\!\!/ \gamma^5$$

$$U(1)_V: \quad \psi \to e^{i\alpha}\psi$$

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$$\begin{matrix} -1 \\ +1 \end{matrix} \quad \begin{cases} \psi_L \to e^{-i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

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$$U(1)_V: \quad \psi \to e^{i\alpha}\psi \qquad \begin{matrix} +1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$U(1)_A: \quad \psi \to e^{i\alpha\gamma^5}\psi \qquad \begin{matrix} -1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{-i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{-1}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} \begin{Bmatrix} \left[(1+\beta_V)(1+\beta_A) + \frac{1}{3}(1-\beta_V)(1-\beta_A)\right] 2F_V^{\mu\nu} F_A^{\rho\sigma} \alpha_V \\ + \left[(1+\beta_V)^2 + \frac{1}{3}(1-\beta_V)^2\right] F_V^{\mu\nu} F_V^{\rho\sigma} \alpha_A \\ + \left[(1+\beta_A)^2 + \frac{1}{3}(1-\beta_A)^2\right] F_A^{\mu\nu} F_A^{\rho\sigma} \alpha_A \end{Bmatrix}$$

Applications: Minimally Coupled

- Application Example: $U(1)_V \times U(1)_A$

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi = \bar{\psi}_L i \partial\!\!\!/ \psi_L + \bar{\psi}_R i \partial\!\!\!/ \psi_R$$

$$i\partial\!\!\!/ \to i\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/ \gamma^5$$

$U(1)_V$: $\psi \to e^{i\alpha}\psi$

$$\begin{matrix} +1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$U(1)_A$: $\psi \to e^{i\alpha\gamma^5}\psi$

$$\begin{matrix} -1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{-i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{-1}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} \left\{ \begin{aligned} &\left[(1+\beta_V)(1+\beta_A) + \frac{1}{3}(1-\beta_V)(1-\beta_A) \right] 2F_V^{\mu\nu} F_A^{\rho\sigma} \alpha_V \\ &+ \left[(1+\beta_V)^2 + \frac{1}{3}(1-\beta_V)^2 \right] F_V^{\mu\nu} F_V^{\rho\sigma} \alpha_A \\ &+ \left[(1+\beta_A)^2 + \frac{1}{3}(1-\beta_A)^2 \right] F_A^{\mu\nu} F_A^{\rho\sigma} \alpha_A \end{aligned} \right\}$$

Wess-Zumino Consistency $\Rightarrow$ $\beta_V = \beta_A$ or $\beta_V = 1$ or $\beta_A = 1$

- Application Example : $U(1)_V \times U(1)_A$

$$\mathcal{L} = \bar{\psi} i\partial\!\!\!/ \psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R$$

$$i\partial\!\!\!/ \to i\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/ \gamma^5$$

$$U(1)_V: \quad \psi \to e^{i\alpha}\psi \qquad \begin{matrix} +1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$U(1)_A: \quad \psi \to e^{i\alpha\gamma^5}\psi \qquad \begin{matrix} -1 \\ +1 \end{matrix} \begin{cases} \psi_L \to e^{-i\alpha}\psi_L \\ \psi_R \to e^{i\alpha}\psi_R \end{cases}$$

$$\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{-1}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} \left\{ \begin{aligned} &\left[(1+\beta_V)(1+\beta_A) + \frac{1}{3}(1-\beta_V)(1-\beta_A) \right] 2F_V^{\mu\nu} F_A^{\rho\sigma} \alpha_V \\ &+ \left[(1+\beta_V)^2 + \frac{1}{3}(1-\beta_V)^2 \right] F_V^{\mu\nu} F_V^{\rho\sigma} \alpha_A \\ &+ \left[(1+\beta_A)^2 + \frac{1}{3}(1-\beta_A)^2 \right] F_A^{\mu\nu} F_A^{\rho\sigma} \alpha_A \end{aligned} \right\}$$

Wess-Zumino Consistency $\Rightarrow$ $\beta_V = \beta_A$ or $\beta_V = 1$ or $\beta_A = 1$

$$\mathcal{A}_{\beta_V=1,\,\beta_A=-1}^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{-1}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} \left(F_V^{\mu\nu} F_V^{\rho\sigma} + \frac{1}{3} F_A^{\mu\nu} F_A^{\rho\sigma} \right) \alpha_A$$

Accommodating EFT Interactions

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}_\mu P^\mu U_\alpha^\dagger\right)$$

$$\mathcal{L}_\chi = \sum_{i=1}^{n} \chi_i^\dagger \bar{\sigma}_\mu P^\mu \chi_i$$

Accommodating EFT Interactions

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi \mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}_\mu P^\mu U_\alpha^\dagger\right)$$

$$\mathcal{L}_\chi = \sum_{i=1}^{n} \chi_i^\dagger \bar{\sigma}_\mu P^\mu \chi_i + \sum_{i,j=1}^{n} \chi_i^\dagger \bar{\sigma}_\mu V_{ij}^\mu \chi_j + \left[\chi_i \left(S_{ij} + i\sigma_\mu \bar{\sigma}_\nu T_{ij}^{\mu\nu}\right)\chi_j + \text{h.c.}\right]$$

Accommodating EFT Interactions

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha \bar{\sigma}_\mu P^\mu U_\alpha^\dagger\right)$$

$$\mathcal{L}_\chi = \sum_{i=1}^{n} \chi_i^\dagger \bar{\sigma}_\mu P^\mu \chi_i + \sum_{i,j=1}^{n} \chi_i^\dagger \bar{\sigma}_\mu V_{ij}^\mu \chi_j + \left[\chi_i\left(S_{ij} + i\sigma_\mu\bar{\sigma}_\nu T_{ij}^{\mu\nu}\right)\chi_j + \text{h.c.}\right]$$

$$= \frac{1}{2}\begin{pmatrix}\chi^\dagger & -\chi^T i\sigma^2\end{pmatrix}\begin{bmatrix}\bar{\sigma}^\mu\left(i\partial_\mu + G_\mu + V_\mu\right) & \left(S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu}\right)^\dagger \\ S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu} & \sigma^\mu\left(i\partial_\mu + G_\mu^T + V_\mu^T\right)\end{bmatrix}\begin{pmatrix}\chi \\ i\sigma^2\chi^*\end{pmatrix}$$

Accommodating EFT Interactions

$$e^{iW\left[G^\mu_\alpha\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G^\mu_\alpha\right]} = \det\left(U_\alpha\bar{\sigma}_\mu P^\mu U^\dagger_\alpha\right)$$

$$\mathcal{L}_\chi = \sum_{i=1}^{n}\chi_i^\dagger\bar{\sigma}_\mu P^\mu\chi_i + \sum_{i,j=1}^{n}\chi_i^\dagger\bar{\sigma}_\mu V^\mu_{ij}\chi_j + \left[\chi_i\left(S_{ij} + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu}_{ij}\right)\chi_j + \text{h.c.}\right]$$

$$= \frac{1}{2}\begin{pmatrix}\chi^\dagger & -\chi^T i\sigma^2\end{pmatrix}\begin{bmatrix}\bar{\sigma}^\mu\left(i\partial_\mu + G_\mu + V_\mu\right) & \left(S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu}\right)^\dagger \\ S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu} & \sigma^\mu\left(i\partial_\mu + G^T_\mu + V^T_\mu\right)\end{bmatrix}\begin{pmatrix}\chi \\ i\sigma^2\chi^*\end{pmatrix}$$

$$\hat{\boldsymbol{P}}_\beta \equiv \begin{bmatrix} i\not{\partial} + \not{G}\left(\frac{1-\gamma^5}{2} + \beta_G\frac{1+\gamma^5}{2}\right) + \not{V}\left(\frac{1-\gamma^5}{2} + \beta_V\frac{1+\gamma^5}{2}\right) & S^\dagger\left(\frac{1+\gamma^5}{2} + \beta_S\frac{1-\gamma^5}{2}\right) + \sigma^{\mu\nu}T^\dagger_{\mu\nu}\left(\frac{1+\gamma^5}{2} + \beta_T\frac{1-\gamma^5}{2}\right) \\ S\left(\frac{1-\gamma^5}{2} + \beta_S\frac{1+\gamma^5}{2}\right) + \sigma^{\mu\nu}T_{\mu\nu}\left(\frac{1-\gamma^5}{2} + \beta_T\frac{1+\gamma^5}{2}\right) & i\not{\partial} - \gamma^\mu G^T_\mu\left(\frac{1+\gamma^5}{2} + \beta_G\frac{1-\gamma^5}{2}\right) - \gamma^\mu V^T_\mu\left(\frac{1+\gamma^5}{2} + \beta_V\frac{1-\gamma^5}{2}\right)\end{bmatrix}$$

Accommodating EFT Interactions

$$e^{iW\left[G_\alpha^\mu\right]} = \int \mathcal{D}\chi\mathcal{D}\chi^\dagger e^{iS_\chi\left[\chi,\chi^\dagger,G_\alpha^\mu\right]} = \det\left(U_\alpha\bar{\sigma}_\mu P^\mu U_\alpha^\dagger\right)$$

$$\mathcal{L}_\chi = \sum_{i=1}^n \chi_i^\dagger\bar{\sigma}_\mu P^\mu\chi_i + \sum_{i,j=1}^n \chi_i^\dagger\bar{\sigma}_\mu V_{ij}^\mu\chi_j + \left[\chi_i\left(S_{ij} + i\sigma_\mu\bar{\sigma}_\nu T_{ij}^{\mu\nu}\right)\chi_j + \text{h.c.}\right]$$

$$= \frac{1}{2}\begin{pmatrix}\chi^\dagger & -\chi^T i\sigma^2\end{pmatrix}\begin{bmatrix}\bar{\sigma}^\mu\left(i\partial_\mu + G_\mu + V_\mu\right) & \left(S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu}\right)^\dagger \\ S + i\sigma_\mu\bar{\sigma}_\nu T^{\mu\nu} & \sigma^\mu\left(i\partial_\mu + G_\mu^T + V_\mu^T\right)\end{bmatrix}\begin{pmatrix}\chi \\ i\sigma^2\chi^*\end{pmatrix}$$

$$\hat{\boldsymbol{P}}_\beta \equiv \begin{bmatrix} i\partial\!\!\!/ + G\!\!\!\!/\left(\frac{1-\gamma^5}{2} + \beta_G\frac{1+\gamma^5}{2}\right) + V\!\!\!\!/\left(\frac{1-\gamma^5}{2} + \beta_V\frac{1+\gamma^5}{2}\right) & S^\dagger\left(\frac{1+\gamma^5}{2} + \beta_S\frac{1-\gamma^5}{2}\right) + \sigma^{\mu\nu}T_{\mu\nu}^\dagger\left(\frac{1+\gamma^5}{2} + \beta_T\frac{1-\gamma^5}{2}\right) \\ S\left(\frac{1-\gamma^5}{2} + \beta_S\frac{1+\gamma^5}{2}\right) + \sigma^{\mu\nu}T_{\mu\nu}\left(\frac{1-\gamma^5}{2} + \beta_T\frac{1+\gamma^5}{2}\right) & i\partial\!\!\!/ - \gamma^\mu G_\mu^T\left(\frac{1+\gamma^5}{2} + \beta_G\frac{1-\gamma^5}{2}\right) - \gamma^\mu V_\mu^T\left(\frac{1+\gamma^5}{2} + \beta_V\frac{1-\gamma^5}{2}\right)\end{bmatrix}$$

$$\mathcal{A}_\beta^\Lambda[\alpha] = \frac{1}{2}\text{Tr}\left[f\left(-\frac{\hat{\boldsymbol{P}}_\beta^2}{\Lambda^2}\right)\frac{1}{\hat{\boldsymbol{P}}_\beta}\left(\boldsymbol{\alpha}\hat{\boldsymbol{P}}_\beta - \hat{\boldsymbol{P}}_\beta\boldsymbol{\alpha}\right)\frac{1-\boldsymbol{\Gamma}^5}{2}\right] \quad , \quad \boldsymbol{\alpha} \equiv \begin{pmatrix}\alpha & \\ & -\alpha^T\end{pmatrix} \quad , \quad \boldsymbol{\Gamma}^5 \equiv \begin{pmatrix}\gamma^5 & \\ & -\gamma^5\end{pmatrix}$$

Accommodating EFT Interactions

$$\mathcal{A}_\beta^\Lambda[\alpha] = \int \mathrm{d}^4x \frac{i}{16\pi^2} \frac{1}{2} \left\{ \left[\Lambda^2 \int_0^{+\infty} f(u)\,\mathrm{d}u \right] \mathbf{tr}_0 + \frac{1}{6}\left(\mathbf{tr}_1 + \mathbf{tr}_2 + \mathbf{tr}_3 \right) \right\}$$

$$\mathbf{tr}_0 \equiv \mathrm{tr}\left[\hat{\boldsymbol{P}}_\beta^2 \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_1 \equiv \mathrm{tr}\left[\hat{\boldsymbol{P}}_\beta^4 \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_2 \equiv -\frac{1}{2} \mathrm{tr}\left[\left(\hat{\boldsymbol{P}}_\beta^2 \gamma_\mu \hat{\boldsymbol{P}}_\beta \gamma^\mu \hat{\boldsymbol{P}}_\beta + \hat{\boldsymbol{P}}_\beta \gamma_\mu \hat{\boldsymbol{P}}_\beta \gamma^\mu \hat{\boldsymbol{P}}_\beta^2 \right) \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_3 \equiv -\frac{1}{2} \mathrm{tr}\left[\hat{\boldsymbol{P}}_\beta \gamma_\mu \hat{\boldsymbol{P}}_\beta^2 \gamma^\mu \hat{\boldsymbol{P}}_\beta \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

Accommodating EFT Interactions

$$\mathcal{A}_{\beta}^{\Lambda}[\alpha] = \int \mathrm{d}^4 x \frac{i}{16\pi^2} \frac{1}{2} \left\{ \left[\Lambda^2 \int_0^{+\infty} f(u)\,\mathrm{d}u \right] \mathbf{tr}_0 + \frac{1}{6} \left(\mathbf{tr}_1 + \mathbf{tr}_2 + \mathbf{tr}_3 \right) \right\}$$

$$\mathbf{tr}_0 \equiv \mathrm{tr}\left[\hat{\boldsymbol{P}}_{\beta}^2 \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_1 \equiv \mathrm{tr}\left[\hat{\boldsymbol{P}}_{\beta}^4 \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_2 \equiv -\frac{1}{2} \mathrm{tr}\left[\left(\hat{\boldsymbol{P}}_{\beta}^2 \gamma_\mu \hat{\boldsymbol{P}}_{\beta} \gamma^\mu \hat{\boldsymbol{P}}_{\beta} + \hat{\boldsymbol{P}}_{\beta} \gamma_\mu \hat{\boldsymbol{P}}_{\beta} \gamma^\mu \hat{\boldsymbol{P}}_{\beta}^2 \right) \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathbf{tr}_3 \equiv -\frac{1}{2} \mathrm{tr}\left[\hat{\boldsymbol{P}}_{\beta} \gamma_\mu \hat{\boldsymbol{P}}_{\beta}^2 \gamma^\mu \hat{\boldsymbol{P}}_{\beta} \boldsymbol{\Gamma}^5 \boldsymbol{\alpha} \right]$$

$$\mathcal{A}_{\beta=0}^{\Lambda}[\alpha] = \mathcal{A}_{\beta=0}^{\Lambda,\mathrm{mc}}[\alpha] - \delta_\alpha \left(\int \mathrm{d}^4 x \mathcal{L}_{\mathrm{ct}}^{\Lambda} \right)$$

Summary

- Various types of anomalies, abelian/non-abelian, covariant/consistent, relevant/irrelevant, can be accommodated by a single functional trace evaluation, which we call the "Anomaly Functional" $\mathcal{A}[\alpha]$.

- We introduced a set of 4D regularization schemes parameterized by β, each choice of which yields an unambiguous evaluation result.

- Our regularization schemes enable an efficient and universal evaluation, which reproduces various well-known results, as a sanity check.

- We applied our formalism to new anomaly calculations --- EFTs with higher-dimensional operators, and found that these interactions only yield irrelevant anomalies.