An EFT hunter's guide to two-to-two scattering

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HEFT 2023, Manchester
June 19, 2023

On-shell EFT

- Standard Lagrangian EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{1}{\Lambda^{d_i-4}} c_i \mathcal{O}_i$$

- Find a basis of operators at given dimension is non-trivial: field redefinition, equations of motions.

- Relation to observables is non-trivial because of gauge redundancy.

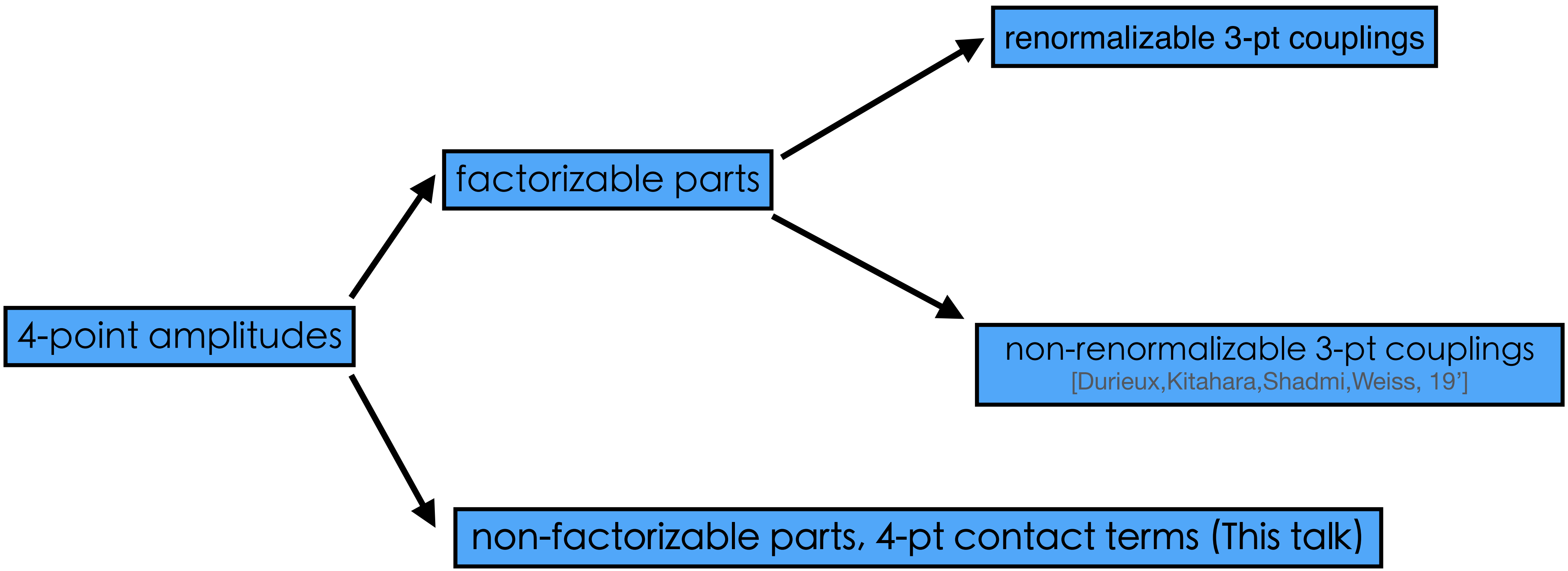
- Alternatively, one could use the amplitude, which is related to the physical observables directly.

2 constructions

- Bottom up ⟶ HEFT
- Top down ⟶ SMEFT

Four-point amplitudes

- Four-point amplitudes are phenomenologically interesting



General structures

Little-group index

Mandelstam invariants

$$\sum_n S_n^{\{I\}} L_n\left[\frac{s_{ij}}{\Lambda^2}\right]$$

Spinor structures Polynomials

Carrying little-group weights and polarizations

Only depends on scattering angle

The massive stripped contact-term(SCT) basis

[Durieux, Kitahara, Machado, Shadmi, Weiss '20]

Bottom-up: HEFT up to E^2

SU(3) X U(1) gauge invariance (W, Z, h are independent) +SM fields

→ HEFT four-point contact-term amplitudes

Example: wwhh

The most general 4-point contact terms

$$[\mathbf{12}]^2, \qquad \langle\mathbf{12}\rangle^2, \qquad [\mathbf{12}]\langle\mathbf{12}\rangle$$

Derive all E^2 contact terms in 2-to-2 amplitude (physical observables)

HEFT up to E^2

$(\mathbf{12}) = [\mathbf{12}],\ \text{or}\ \langle\mathbf{12}\rangle$

helicity category in the massless limit

Massive amplitudes	E^2 contact terms
$\mathcal{M}(WWhh)$	$C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}],\ C^{\pm\pm}_{WWhh}(\mathbf{12})^2$
$\mathcal{M}(ZZhh)$	$C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}],\ C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(1\mathbf{2})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^cfhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^cfWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle,\ C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}],\ C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cfZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle,\ C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}],\ C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^cf\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(q^cqgh)$	$C^{\pm\pm\pm}_{qqgh}\ (\mathbf{1}3)(\mathbf{2}3)$
$\mathcal{M}(f^cff^cf)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34}),\ C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}],\ C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}],\ C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24}),\ C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle,\ C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle,\ C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

[HL, Ma, Shadmi, Waterbury '23]

HEFT up to E^2

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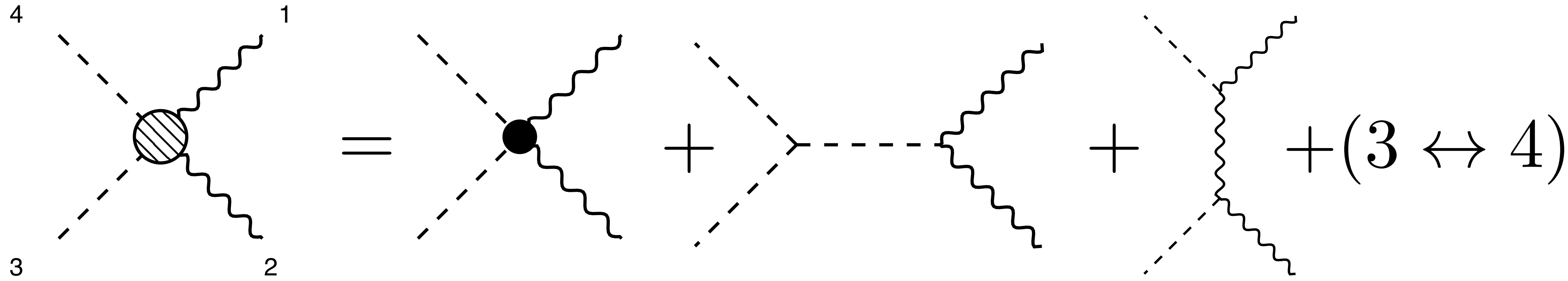
$\mathcal{M}(gghh)$	$C^{\pm\pm}_{gghh}(12)^2$
$\mathcal{M}(\gamma\gamma hh)$	$C^{\pm\pm}_{\gamma\gamma hh}(12)^2$
$\mathcal{M}(\gamma Zhh)$	$C^{\pm}_{\gamma Zhh}(\mathbf{12})^2$
$\mathcal{M}(hhhh)$	C_{hhhh}
$\mathcal{M}(f^c fhh)$	$C^{\pm\pm}_{ffhh}(\mathbf{12})$
$\mathcal{M}(f^c fWh)$	$C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle,\ C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}],\ C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c fZh)$	$C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle,\ C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}],\ C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c f\gamma h)$	$C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(q^c qgh)$	$C^{\pm\pm\pm}_{qqgh}(\mathbf{13})(\mathbf{23})$
$\mathcal{M}(f^c f f^c f)$	$C^{\pm\pm\pm\pm,1}_{ffff}(\mathbf{12})(\mathbf{34}),\ C^{--++}_{ffff}\langle\mathbf{12}\rangle[\mathbf{34}],\ C^{-+-+}_{ffff}\langle\mathbf{13}\rangle[\mathbf{24}],\ C^{-++-}_{ffff}\langle\mathbf{14}\rangle[\mathbf{23}]$ $C^{\pm\pm\pm\pm,2}_{ffff}(\mathbf{13})(\mathbf{24}),\ C^{++--}_{ffff}[\mathbf{12}]\langle\mathbf{34}\rangle,\ C^{+-+-}_{ffff}[\mathbf{13}]\langle\mathbf{24}\rangle,\ C^{+--+}_{ffff}[\mathbf{14}]\langle\mathbf{23}\rangle$

[HL, Ma, Shadmi, Waterbury ’23]

Bottom-up construction to full WWhh



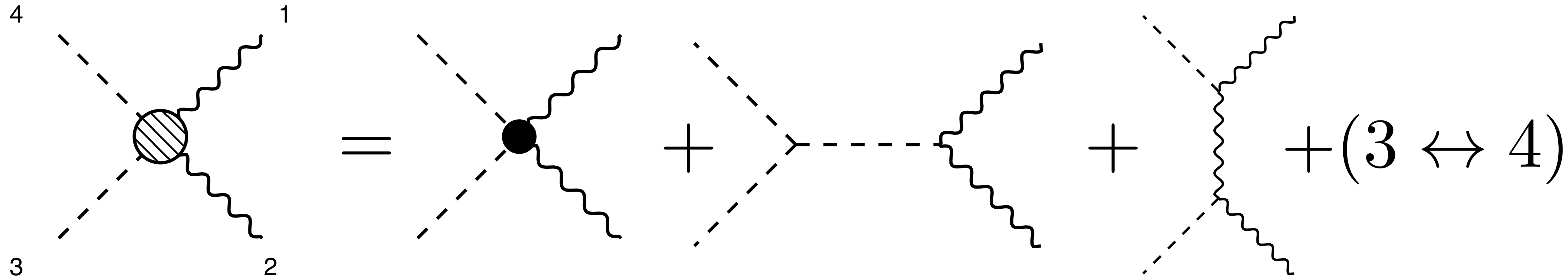
Three-point terms $C^{00}_{WWh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W},\ m_h C_{hhh}$

Four-point contact terms $C^{00,\mathrm{fac}}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2} + C^{00}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$

→ determined by factorizable part

Perturbative unitarity ⟷ Gauge invariance

Bottom-up construction to full WWhh



renormalizable, nonrenormalizable, s-channel

$$\mathcal{M}(W^+, W^-, h, h) = c^{00,\text{fac}}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2} + c^{00}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2} - \frac{m_h m_W c^{00}_{WWh} c_{hhh}}{s_{12} - m_h^2}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2}$$

$$+ \frac{{c^{00}_{WWh}}^2}{s_{13} - m_W^2}\left(\frac{\langle\mathbf{131}]\langle\mathbf{242}]}{2m_W^2} - [\mathbf{12}]\langle\mathbf{12}\rangle\right) + \frac{{c^{00}_{WWh}}^2}{s_{14} - m_W^2}\left(\frac{\langle\mathbf{141}]\langle\mathbf{232}]}{2m_W^2} - [\mathbf{12}]\langle\mathbf{12}\rangle\right)$$

t-channel, u-channel

WWhh: perturbative unitarity

Take high-energy limit for the longitudinal W

$$\boldsymbol{\mathcal{M}}(W^+, W^-, h, h) \supset -\left(c_{WWhh}^{00,\text{fac}} + \frac{{c_{WWh}^{00}}^2}{2} \right) \frac{s_{12}}{2m_W^2}$$

Perturbative unitarity requires

$$c_{WWhh}^{00,\text{fac}} + \frac{{c_{WWh}^{00}}^2}{2} = 0$$

Factorizable part is fixed by the 3-point coupling

WWhh: gauge invariance

Take high-energy limit for the transverse W

$$\mathcal{M}(W^+, W^-, h, h) \to c^{00,fac}_{WWhh} \frac{[1\xi_2]\langle\xi_1 2\rangle}{[2\xi_2]\langle 1\xi_1\rangle} + \frac{{c^{00}_{WWh}}^2}{2}\left(\frac{[13\xi_1\rangle[\xi_2 42\rangle}{s_{13}[2\xi_2]\langle 1\xi_1\rangle} + \frac{[14\xi_1\rangle[\xi_2 32\rangle}{s_{14}[2\xi_2]\langle 1\xi_1\rangle}\right)$$

Depends on arbitrary spinors $\xi_1,\ \xi_2$

Requiring independent → $c^{00,\text{fac}}_{WWhh} + \frac{{c^{00}_{WWh}}^2}{2} = 0$

↓

amounts to gauge invariance of high-energy theory

⇒ Gauge invariance and perturbative unitarity are equivalent

HEFT with E^3 and E^4

HEFT, minimum SMEFT dim's

# of independent structures

PF = parity flip
angle <—> square

4.1.8 $W^+W^+W^-W^-$

$$
\begin{array}{rll}
[\mathbf{12}][\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, [\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (4;8) & \#=2\\
[\mathbf{12}]^2[\mathbf{34}]\langle\mathbf{34}\rangle;\ \mathrm{PF} & (6;8) & \#=2\\
\{[\mathbf{12}][\mathbf{34}][\mathbf{13}]\langle\mathbf{24}\rangle, [\mathbf{14}][\mathbf{23}][\mathbf{13}]\langle\mathbf{24}\rangle\} + (1\leftrightarrow 2) + (3\leftrightarrow 4);\ \mathrm{PF} & (6;8) & \#=4\\
[\mathbf{34}]^2[\mathbf{12}]\langle\mathbf{12}\rangle;\ \mathrm{PF} & (6;8) & \#=2\\
[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle + (1\leftrightarrow 2) & (6;8) & \#=1\\
[\mathbf{12}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{34}\rangle + (1\leftrightarrow 2) + (3\leftrightarrow 4);\ \mathrm{PF} & (6;8) & \#=2\\
[\mathbf{13}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{24}\rangle + (3\leftrightarrow 4) & (6;8) & \#=1\\
\{[\mathbf{13}]^2[\mathbf{24}]^2 + (1\leftrightarrow 2), [\mathbf{13}][\mathbf{14}][\mathbf{23}][\mathbf{24}]\};\ \mathrm{PF} & (8;8) & \#=4\\
[\mathbf{12}]^2\langle\mathbf{34}\rangle^2;\ \mathrm{PF} & (8;8) & \#=2\\
[\mathbf{24}]^2\langle\mathbf{13}\rangle^2 + [\mathbf{14}]^2\langle\mathbf{23}\rangle^2 + (3\leftrightarrow 4) & (8;8) & \#=1
\end{array} \tag{4.8}
$$

At E^5 several new $vvvv$ SCTs become independent in the $(+000)$, $(++\,+0)$, and $(++\,-0)$ helicity categories.

[HL, Ma, Shadmi, Waterbury '23]

$$[\mathbf{12}][\mathbf{34}][\mathbf{142}\rangle\langle\mathbf{34}\rangle \quad [\mathbf{12}][\mathbf{34}][\mathbf{23}][\mathbf{134}\rangle \quad [\mathbf{12}]^2\langle\mathbf{34}\rangle[\mathbf{423}\rangle$$

2 constructions

- So far: HEFT from bottom-up approach

- Now: SMEFT from Top-down approach

Top-down

- Starting point: massless on-shell amplitudes

[Durieux, Machado, ’19], [Ma, Shu, Xiao ’19]

- From massless amplitudes of the unbroken theory -> SMEFT low-energy contact terms

Bolding

[Balkin, Durieux, Kitahara, Shadmi, Weiss ’21]

- Higgsing/bolding to obtain the low-energy contact terms
  - Massless SCTs featuring just fermions and vectors are simply bolded into massive SCTs
  - Massless SCT featuring a scalar with a momentum insertion can give a massive SCT featuring a vector

$$p_i \rightarrow \boldsymbol{i}\rangle[\boldsymbol{i}$$

  - When the scalar is additional Higgs that gets vev, need to take the soft limit by setting the momentum to zero

$$p_i \rightarrow 0$$

Mapping between HEFT and SMEFT example: WWhh

WWhh

Three-point terms $C^{00}_{WWh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W}$

Independent

Four-point terms $C^{00,\text{fac}}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{m_W^2}+C^{00}_{WWhh}\frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$

- The factorizable part is determined by the three-point couplings. The non-factorizable (local) four-point term can be determined by matching to the SMEFT

- In SMEFT, h and W_L come from Higgs doubet H, the 4H operator generates W_L W_L hh at the leading order

$$H=\left(G^+,\frac{v+h}{\sqrt{2}}\right)^T$$

Massless dim-6 SMEFT

- Massless SMEFT contact term [Ma, Shu, Xiao '19]
- Each kinematic structure in the physical amplitudes can be associated with a specific operator in the Warsaw basis

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H_i^c H_j^c H_k^c H^l H^m H^n)$	$T^{+\,lmn}_{ijk}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$s_{12} T^{+\,kl}_{ij}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H_i^c H_j^c H^k H^l)$	$(s_{13} - s_{23}) T^{-\,kl}_{ij}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$
$\mathcal{A}(B^\pm B^\pm H_i^c H^j)$	$(12)^2 \delta_i^j$	$(\mathcal{O}_{HB} \pm i\mathcal{O}_{H\tilde{B}})/2$	$c^{\pm\pm}_{BBHH}$
$\mathcal{A}(B^\pm W^{I\pm} H_i^c H^j)$	$(12)^2 (\sigma^I)_i^j$	$\mathcal{O}_{HWB} \pm i\mathcal{O}_{H\tilde{W}B}$	$c^{\pm\pm}_{BWHH}$
$\mathcal{A}(W^{I+} W^{J+} H_i^c H^j)$	$(12)^2 \delta^{IJ} \delta_i^j$	$(\mathcal{O}_{HW} \pm i\mathcal{O}_{H\tilde{W}})/2$	$c^{\pm\pm}_{WWHH}$
$\mathcal{A}(g^{A\pm} g^{B\pm} H_i^c H^j)$	$(12)^2 \delta^{AB} \delta_i^j$	$(\mathcal{O}_{HG} \pm i\mathcal{O}_{H\tilde{G}})/2$	$c^{\pm\pm}_{GGHH}$
$\mathcal{A}(L_i^c e H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij}$	$\mathcal{O}_{eH}/2$	c^{++}_{LeHHH}
$\mathcal{A}(Q^c_{a,i} d^b H_j^c H^k H^l)$	$[12] T^{+\,kl}_{ij} \delta_a^b$	$\mathcal{O}_{dH}/2$	c^{++}_{QdHHH}
$\mathcal{A}(Q^c_{a,i} u^b H_j^c H_k^c H^l)$	$[12] \varepsilon_{im} T^{+\,ml}_{jk} \delta_a^b$	$\mathcal{O}_{uH}/2$	c^{++}_{QuHHH}
$\mathcal{A}(e^c e H_i^c H^j)$	$\langle 142] \delta_i^j$	$\mathcal{O}_{He}/2$	c^{-+}_{eeHH}
$\mathcal{A}(u_a^c u^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hu}/2$	c^{-+}_{uuHH}
$\mathcal{A}(d_a^c d^b H_i^c H^j)$	$\langle 142] \delta_i^j \delta_a^b$	$\mathcal{O}_{Hd}/2$	c^{-+}_{ddHH}
$\mathcal{A}(u_a^c d^b H^i H^j)$	$\langle 142] \epsilon^{ij} \delta_a^b$	$\mathcal{O}_{Hud}/2$	c^{-+}_{udHH}
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} + \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(+)}_{LLHH}$
$\mathcal{A}(L_i^c L^j H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik}$	$\left(\mathcal{O}^{(1)}_{HL} - \mathcal{O}^{(3)}_{HL}\right)/8$	$c^{+-,(-)}_{LLHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{+\,jl}_{ik} \delta_a^b$	$\left(3\mathcal{O}^{(1)}_{HQ} + \mathcal{O}^{(3)}_{HQ}\right)/8$	$c^{+-,(+)}_{QQHH}$
$\mathcal{A}(Q^c_{a,i} Q^{b,j} H_k^c H^l)$	$[142\rangle T^{-\,jl}_{ik} \delta_a^b$	$(\mathcal{O}^{(1)}_{HQ} - \mathcal{O}^{(3)}_{HQ})/8$	$c^{+-,(-)}_{QQHH}$
$\mathcal{A}(L_i^c e B^+ H^j)$	$[13][23] \delta_i^j$	$-i\mathcal{O}_{eB}/(2\sqrt{2})$	c^{+++}_{LeBH}
$\mathcal{A}(Q^c_{a,i} d^b B^+ H^j)$	$[13][23] \delta_i^j \delta_a^b$	$-i\mathcal{O}_{dB}/(2\sqrt{2})$	c^{+++}_{QdBH}
$\mathcal{A}(Q^c_{a,i} u^b B^+ H_j^c)$	$[13][23] \epsilon_{ij} \delta_a^b$	$-i\mathcal{O}_{uB}/(2\sqrt{2})$	c^{+++}_{QuBH}
$\mathcal{A}(L_i^c e W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j$	$-i\mathcal{O}_{eW}/(2\sqrt{2})$	c^{+++}_{LeWH}
$\mathcal{A}(Q^c_{a,i} d^b W^{I+} H^j)$	$[13][23] (\sigma^I)_i^j \delta_a^b$	$-i\mathcal{O}_{dW}/(2\sqrt{2})$	c^{+++}_{QdWH}
$\mathcal{A}(Q^c_{a,i} u^b W^{I+} H_j^c)$	$[13][23] (\sigma^I)_{ik} \epsilon_j^k \delta_a^b$	$-i\mathcal{O}_{uW}/(2\sqrt{2})$	c^{+++}_{QuWH}
$\mathcal{A}(Q^c_{a,i} d^b g^{A+} H^j)$	$[13][23] \delta_i^j (\lambda^A)_a^b$	$-i\mathcal{O}_{dG}/(2\sqrt{2})$	c^{+++}_{QdGH}
$\mathcal{A}(Q^c_{a,i} u^b g^{A+} H_j^c)$	$[13][23] \epsilon_{ij} (\lambda^A)_a^b$	$-i\mathcal{O}_{uG}/(2\sqrt{2})$	c^{+++}_{QuGH}
$\mathcal{A}(W^{I\pm} W^{J\pm} W^{K\pm})$	$(12)(23)(31) \epsilon^{IJK}$	$(\mathcal{O}_W \pm i\mathcal{O}_{\tilde{W}})/6$	$c^{\pm\pm\pm}_{WWW}$
$\mathcal{A}(g^{A\pm} g^{B\pm} g^{C\pm})$	$(12)(23)(31) f^{ABC}$	$(\mathcal{O}_G \pm i\mathcal{O}_{\tilde{G}})/6$	$c^{\pm\pm\pm}_{GGG}$

[HL, Ma, Shadmi, Waterbury '23]

WWhh

$$T^{\pm\,kl}_{ij} \equiv \frac{1}{2}(\delta^k_i \delta^l_j \pm \delta^k_j \delta^l_i)$$

Amplitude	Contact term	Warsaw basis operator	Coefficient
$\mathcal{A}(H^c_i H^c_j H^c_k H^l H^m H^n)$	$T^{+\,lmn}_{ijk}$	$\mathcal{O}_H/6$	$c_{(H^\dagger H)^3}$
$\mathcal{A}(H^c_i H^c_j H^k H^l)$	$s_{12} T^{+\,kl}_{ij}$	$\mathcal{O}_{HD}/2 + \mathcal{O}_{H\Box}/4$	$c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{A}(H^c_i H^c_j H^k H^l)$	$(s_{13} - s_{23}) T^{-\,kl}_{ij}$	$\mathcal{O}_{HD}/2 - \mathcal{O}_{H\Box}/4$	$c^{(-)}_{(H^\dagger H)^2}$

$$\mathcal{A}(H^c_i H^c_j H^k H^l) = \frac{c_+}{\Lambda^2} s_{12} T^{+\,kl}_{ij} + \frac{c_-}{\Lambda^2}(s_{13} - s_{23}) T^{-\,kl}_{ij}$$

$$[\mathbf{12}]\langle\mathbf{12}\rangle \equiv \frac{1}{2}[1^{\{I_1}2^{\{J_1}]\langle 1^{I_2\}}2^{J_2\}}\rangle \to \frac{1}{2}[12]\langle 12\rangle = -\frac{1}{2}s_{12}$$

$$\boldsymbol{\mathcal{M}}(W^+_L W^-_L hh) = \frac{c_+ - 3c_-}{2} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

$$\Longrightarrow \quad c^{00}_{WWhh} = \frac{c_+ - 3c_-}{2} = 4(C_{H\Box} - \frac{1}{4}C_{HD})$$

HEFT and SMEFT mapping

- In ±± helicity of VVhh, 10 HEFT parameters and 8 SMEFT parameters

Massive $d=6$ amplitudes	SMEFT Wilson coefficients
$\mathcal{M}(W_L^+W_L^-hh) = C^{00}_{WWhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{WWhh} = (c^{(+)}_{(H^\dagger H)^2} - 3c^{(-)}_{(H^\dagger H)^2})/2$
$\mathcal{M}(W_\pm^+W_\pm^-hh) = C^{\pm\pm}_{WWhh}(\mathbf{12})^2$	$C^{\pm\pm}_{WWhh} = 2c^{\pm\pm}_{WWHH}$
$\mathcal{M}(Z_LZ_Lhh) = C^{00}_{ZZhh}\langle\mathbf{12}\rangle[\mathbf{12}]$	$C^{00}_{ZZhh} = -2c^{(+)}_{(H^\dagger H)^2}$
$\mathcal{M}(Z_\pm Z_\pm hh) = C^{\pm\pm}_{ZZhh}(\mathbf{12})^2$	$C^{\pm\pm}_{ZZhh} = c_W^2 c^{\pm\pm}_{WWHH} + s_W^2 c^{\pm\pm}_{BBHH} + c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(g_\pm g_\pm hh) = C^{\pm\pm}_{gghh}(12)^2$	$C^{\pm\pm}_{gghh} = c^{\pm\pm}_{GGHH}$
$\mathcal{M}(\gamma_\pm\gamma_\pm hh) = C^{\pm\pm}_{\gamma\gamma hh}(12)^2$	$C^{\pm\pm}_{\gamma\gamma hh} = s_W^2 c^{\pm\pm}_{WWHH} + c_W^2 c^{\pm\pm}_{BBHH} - c_W s_W c^{\pm\pm}_{BWHH}$
$\mathcal{M}(\gamma_\pm Zhh) = C^{\pm}_{\gamma Zhh}(\mathbf{12})^2$	$C^{\pm}_{\gamma Zhh} = s_W c_W c^{\pm\pm}_{WWHH} - s_W c_W c^{\pm\pm}_{BBHH} + \frac{1}{2}(s_W^2 - c_W^2)c^{\pm\pm}_{BWHH}$
$\mathcal{M}(hhhh) = C_{hhhh}$	$C_{hhhh} = -3c_{(H^\dagger H)^2} + 45\, v^2 c_{(H^\dagger H)^3}$
$\mathcal{M}(f^c_\pm f_\pm hh) = C^{\pm\pm}_{ffhh}(\mathbf{12})$	$C^{\pm\pm}_{ffhh} = 3c^{\pm\pm}_{\Psi\psi HHH}v/(2\sqrt{2})$
$\mathcal{M}(f^c_+ f'_- W_L h) = C^{+-0}_{ffWh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{ffWh} = (c^{+-,(+)}_{\Psi\Psi HH} - c^{+-,(-)}_{\Psi\Psi HH})/2$
$\mathcal{M}(f^c_- f'_+ W_L h) = C^{-+0}_{ffWh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0}_{ffWh} = c^{-+}_{\psi_R\psi'_R HH}$
$\mathcal{M}(f^c_\pm f'_\pm W_\pm h) = C^{\pm\pm\pm}_{ffWh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffWh} = c^{\pm\pm\pm}_{\Psi\psi WH}/2$
$\mathcal{M}(f^c_+ f_- Z_L h) = C^{+-0}_{ffZh}[\mathbf{13}]\langle\mathbf{23}\rangle$	$C^{+-0}_{e_Le_LZh} = -i\sqrt{2}c^{+-,(+)}_{\Psi\Psi HH},\ C^{+-0}_{\nu_L\nu_L Zh} = -i(c^{+-,(+)}_{\Psi\Psi HH} + c^{+-,(-)}_{\Psi\Psi HH})/\sqrt{2}$
$\mathcal{M}(f^c_- f_+ Z_L h) = C^{-+0}_{ffZh}\langle\mathbf{13}\rangle[\mathbf{23}]$	$C^{-+0,\mathrm{CT}}_{ffZh} = -i\sqrt{2}c^{-+}_{\psi\psi HH}$
$\mathcal{M}(f^c_\pm f_\pm Z_\pm h) = C^{\pm\pm\pm}_{ffZh}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ffZh} = -(s_W c^{\pm\pm\pm}_{\Psi\psi BH} + c_W c^{\pm\pm\pm}_{\Psi\psi WH})/\sqrt{2}$
$\mathcal{M}(f^c_\pm f_\pm \gamma_\pm h) = C^{\pm\pm\pm}_{ff\gamma h}(\mathbf{13})(\mathbf{23})$	$C^{\pm\pm\pm}_{ff\gamma h} = (-s_W c^{\pm\pm\pm}_{\Psi\psi WH} + c_W c^{\pm\pm\pm}_{\Psi\psi BH})/\sqrt{2}$
$\mathcal{M}(q^c_\pm q_\pm g^A_\pm h) = C^{\pm\pm\pm}_{qqgh}\lambda^A(13)(\mathbf{23})$	$C^{\pm\pm\pm}_{qqgh} = c^{\pm\pm\pm}_{\Psi\psi GH}/\sqrt{2}$

[HL, Ma, Shadmi, Waterbury ’23]

Derive low-energy contact-term amplitudes
example: WWWW

WWWW

HE

$$W_+^a W_+^b W_+^c W_+^d$$

$$\mathcal{A}_1/c_1 = [12]^2[34]^2\delta^{ab}\delta^{cd} + [13]^2[24]^2\delta^{ac}\delta^{bd} + [14]^2[23]^2\delta^{ad}\delta^{bc}$$

$$\mathcal{A}_2/c_2 = ([13]^2[24]^2 + [14]^2[23]^2)\delta^{ab}\delta^{cd} + ([12]^2[34]^2 + [14]^2[23]^2)\delta^{ac}\delta^{bd} + ([12]^2[34]^2 + [13]^2[24]^2)\delta^{ad}\delta^{bc}$$

LE

$$W_+^- W_+^- W_+^+ W_+^+$$

$$2c_2[\mathbf{12}]^2[\mathbf{34}]^2 + (c_1 + c_2)([\mathbf{13}]^2[\mathbf{24}]^2 + [\mathbf{14}]^2[\mathbf{23}]^2)$$

$$= (c_1 + 3c_2)([\mathbf{13}]^2[\mathbf{24}]^2 + [\mathbf{14}]^2[\mathbf{23}]^2) - 4c_2[\mathbf{13}][\mathbf{14}][\mathbf{23}][\mathbf{24}]$$

WWWW

	LE	SMEFT dim
$W^-_+ W^-_+ W^+_+ W^+_+$	$[\mathbf{13}]^2[\mathbf{24}]^2 + [\mathbf{23}]^2[\mathbf{14}]^2$ $[\mathbf{13}][\mathbf{14}][\mathbf{23}][\mathbf{24}]$	8
$W^-_+ W^-_+ W^+_- W^+_-$	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2$	8
$W^-_+ W^-_- W^+_+ W^+_-$	$[\mathbf{13}]^2\langle\mathbf{24}\rangle^2$ + Sym.	8
$W^-_+ W^-_L W^+_+ W^+_L$	$[\mathbf{13}][\mathbf{12}][\mathbf{34}]\langle\mathbf{24}\rangle$ + Sym. $[\mathbf{13}][\mathbf{14}][\mathbf{23}]\langle\mathbf{24}\rangle$ + Sym.	8
$W^-_+ W^-_L W^+_- W^+_L$	$[\mathbf{12}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{34}\rangle$ + Sym.	8
$W^-_L W^-_L W^+_L W^+_L$	$[\mathbf{12}][\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle$ $[\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle$ + Sym.	8

WWWW

HE

$$W_+^a W_+^b H_k H_l H^{\dagger,i} H^{\dagger,j} \qquad [12]^2(s_{34}) \quad \delta^{ab}(\delta_k^i\delta_l^j + \delta_k^j\delta_l^i)$$

LE

$$W_+^- W_+^- W_L^+ W_L^+ \longrightarrow v^2[\mathbf{12}]^2[\mathbf{34}]\langle\mathbf{34}\rangle/\Lambda^6$$

HEFT with E^3 and E^4

Green: Produced by SMEFT at 4-point amplitudes

Red: Produced by SMEFT at 6-point amplitudes

4.1.8 $W^+W^+W^-W^-$

green	$[\mathbf{12}][\mathbf{34}]\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, [\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle + (3 \leftrightarrow 4)$	$(4;8)$	$\# = 2$
red	$[\mathbf{12}]^2[\mathbf{34}]\langle\mathbf{34}\rangle;\ \mathrm{PF}$	$(6;8)$	$\# = 2$
green	$\{[\mathbf{12}][\mathbf{34}][\mathbf{13}]\langle\mathbf{24}\rangle, [\mathbf{14}][\mathbf{23}][\mathbf{13}]\langle\mathbf{24}\rangle\} + (1 \leftrightarrow 2) + (3 \leftrightarrow 4);\ \mathrm{PF}$	$(6;8)$	$\# = 4$
red	$[\mathbf{34}]^2[\mathbf{12}]\langle\mathbf{12}\rangle;\ \mathrm{PF}$	$(6;8)$	$\# = 2$
red	$[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle + (1 \leftrightarrow 2)$	$(6;8)$	$\# = 1$
green	$[\mathbf{12}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{34}\rangle + (1 \leftrightarrow 2) + (3 \leftrightarrow 4);\ \mathrm{PF}$	$(6;8)$	$\# = 2$
red	$[\mathbf{13}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{24}\rangle + (3 \leftrightarrow 4)$	$(6;8)$	$\# = 1$
green	$\{[\mathbf{13}]^2[\mathbf{24}]^2 + (1 \leftrightarrow 2), [\mathbf{13}][\mathbf{14}][\mathbf{23}][\mathbf{24}]\};\ \mathrm{PF}$	$(8;8)$	$\# = 4$
green	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2;\ \mathrm{PF}$	$(8;8)$	$\# = 2$
green	$[\mathbf{24}]^2\langle\mathbf{13}\rangle^2 + [\mathbf{14}]^2\langle\mathbf{23}\rangle^2 + (3 \leftrightarrow 4)$	$(8;8)$	$\# = 1$

Summary

- We derive the four-point contact terms of the standard model particles, keeping terms with up to quartic energy growth. It will complete the 4-point amplitudes together with the factorizable parts.

- Imposing just the unbroken low-energy symmetry, and treating the electroweak gauge bosons and the Higgs as independent degrees of freedom, we obtain the most general HEFT four-point contact-term amplitudes.

- For terms with quadratic energy growth, we also derive the low-energy SMEFT predictions, via on-shell Higgsing of the massless SMEFT contact terms, we also provide the mapping to the Warsaw basis.

- Our results provide a formulation of EFT analyses directly in terms of observable quantities.

Thanks!