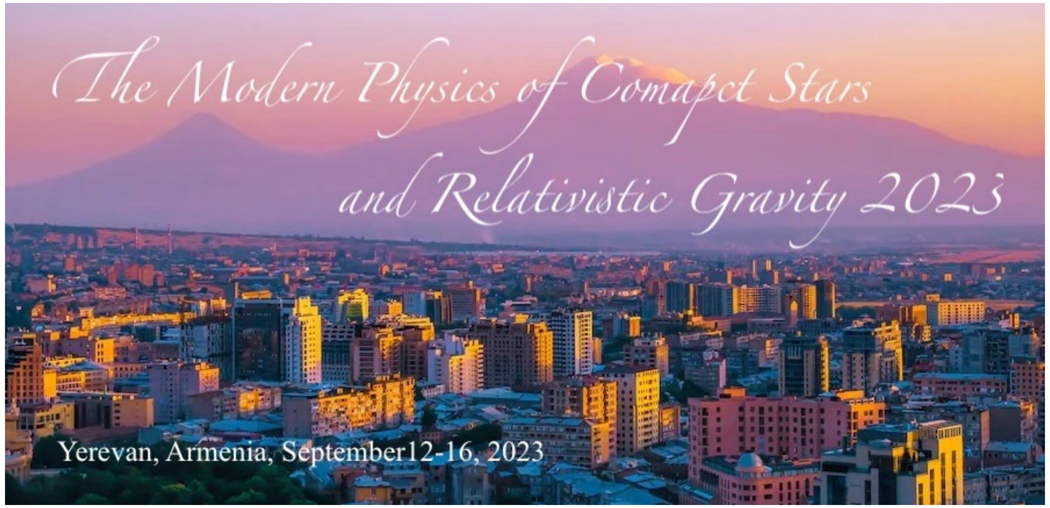

PROPERTIES OF HYPERBOLIC AND BUNCH-DAVIES VACUA IN DE SITTER SPACETIME

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CONTENT

DS SPACETIME

- Background is the $(D+1)$-dimensional de Sitter spacetime, which is described by open coordinates foliated with time slices having constant negative curvature

$\downarrow$

Hyperbolic coordinates

$$R=\frac{D(D+1)}{\alpha^2} \longrightarrow \text{Spacetime curvature}$$

$$\alpha \longrightarrow \text{Curvature radius}$$

- $(D+1)$-dimensional dS spacetime as a hyperboloid

$$\eta_{MN}Z^M Z^N=-\alpha^2,\ M,N=0,1,\ldots,D+1$$

in $(D+2)$-dimensional Minkowski spacetime with the line element

$$ds^2_{D+2}=\eta_{MN}dZ^M dZ^N$$

DS SPACETIME

Hyperbolic coordinates (t, r, ϑ, ϕ)

$$\vartheta = (\theta_1, \ldots \theta_n),\ n = D-2$$

$$0 \leqslant t < \infty \qquad 0 \le r < \infty$$

$$0 \leqslant \theta_k \leqslant \pi \quad k = 1, 2, \ldots, n \qquad 0 \leqslant \phi \leqslant 2\pi$$

$$\boxed{ds^2 = dt^2 - \alpha^2 \sinh^2(t/\alpha)\left(dr^2 + \sinh^2 r\, d\Omega_{D-1}^2\right)}$$

Conformal time ➡ $\sinh(t/\alpha) = -1/\sinh(\eta/\alpha)$

$$ds^2 = \sinh^{-2}(\eta/\alpha)\underbrace{\left[d\eta^2 - \alpha^2\left(dr^2 + \sinh^2 r\, d\Omega_{D-1}^2\right)\right]}$$

Static spacetime foliated by constant negative curvature spatial sections

Inflationary coordinates (planar) $(t_\mathrm{I}, r_\mathrm{I}, \vartheta, \phi)$

$$\vartheta = (\theta_1, \ldots \theta_n),\ n = D-2$$

$$0 \le r_\mathrm{I} < \infty \qquad -\infty < t_\mathrm{I} < +\infty$$

$$0 \leqslant \theta_k \leqslant \pi \quad k = 1, 2, \ldots, n \qquad 0 \leqslant \phi \leqslant 2\pi$$

$$\boxed{ds^2 = dt_\mathrm{I}^2 - e^{2t_\mathrm{I}/\alpha}\left(dr_\mathrm{I}^2 + r_\mathrm{I}^2 d\Omega_{D-1}^2\right)}$$

Inflationary conformal time ➡ $\eta_\mathrm{I} = -\alpha e^{-t_\mathrm{I}/\alpha}$

$$-\infty < \eta_\mathrm{I} < 0$$

$$ds^2 = (\alpha/\eta_\mathrm{I})^2\left(d\eta_\mathrm{I}^2 - dr_\mathrm{I}^2 - r_\mathrm{I}^2 d\Omega_{D-1}^2\right)$$

Conformally flat form

Relations between the inflationary and hyperbolic coordinates ➡

$$t_\mathrm{I} = \alpha \ln\left[\cosh(t/\alpha) + \sinh(t/\alpha)\cosh r\right]$$

$$r_\mathrm{I} = \alpha e^{-t_\mathrm{I}/\alpha}\sinh(t/\alpha)\sinh r$$

GEODESIC DISTANCE

- dS invariant function

$$u(x,x') = \cosh(t/\alpha)\cosh(t'/\alpha) - \sinh(t/\alpha)\sinh(t'/\alpha)w$$

$$w = w(r,r',\theta) = \cosh r\cosh r' - \sinh r\sinh r'\cos\theta$$

$\theta \longrightarrow$ angle between the directions determined by (ϑ,ϕ) and (ϑ',ϕ')

$$u(x,x') = \frac{(\Delta\eta_{\mathrm{I}})^2 - |\Delta\mathbf{r}_{\mathrm{I}}|^2}{2\eta_{\mathrm{I}}\eta_{\mathrm{I}}'} + 1$$

$$\Delta\eta_{\mathrm{I}} = \eta_{\mathrm{I}}' - \eta_{\mathrm{I}}$$

$$|\Delta\mathbf{r}_{\mathrm{I}}|^2 = r_{\mathrm{I}}^2 + r_{\mathrm{I}}'^2 - 2r_{\mathrm{I}}r_{\mathrm{I}}'\cos\theta$$

- Geodesic distance

$$u(x,x') > 1 \Longrightarrow u(x,x') = \cosh\left[d(x,x')/\alpha\right]$$

$$u(x,x') < 1 \Longrightarrow u(x,x') = \cos\left[d(x,x')/\alpha\right]$$

$d(x,x') \longrightarrow$ geodesic distance between two points $x = (t,r,\vartheta,\phi)$ and $x' = (t',r',\vartheta',\phi')$

SCALAR FIELD MODES

- Field equation $\left(\Box+m^2+\xi R\right)\varphi=0$

$\Box \longrightarrow$ d'Alembert operator

- Hyperbolic (H-) vacuum

$$\varphi_\sigma(x)=c_\sigma\frac{P^{iz}_{\nu-1/2}\left(\cosh(t/\alpha)\right)}{\sinh^{(D-1)/2}(t/\alpha)}\frac{P^{1-D/2-l}_{iz-1/2}\left(\cosh r\right)}{\sinh^{D/2-1}r}Y\left(m_p;\vartheta,\phi\right)$$

$$\left|c_\sigma\right|^2=\frac{z\left|\Gamma\left(l+\frac{D-1}{2}+iz\right)\right|^2}{2N\left(m_p\right)\alpha^{D-1}}$$

$$\nu=\sqrt{D^2/4-\xi D\left(D+1\right)-m^2\alpha^2}$$

$P^\gamma_\rho(x) \longrightarrow$ Associated Legendre function of the 1st kind

$Y(m_p;\vartheta,\phi) \longrightarrow$ Hyperspherical harmonics

- Bunch-Davies (BD) vacuum

$$\varphi_\sigma(x)=\frac{c_{\mathrm{I}\sigma}\eta_{\mathrm{I}}^{D/2}}{r_{\mathrm{I}}^{D/2-1}}H^{(1)}_\nu(\lambda\,|\eta_{\mathrm{I}}|)J_\mu(\lambda r_{\mathrm{I}})Y(m_p;\vartheta,\phi)$$

$$\left|c_{\mathrm{I}\sigma}\right|^2=\frac{\pi e^{i(\nu-\nu^*)\pi/2}}{4N(m_p)\alpha^{D-1}}$$

$H^{(1)}_\nu(y) \longrightarrow$ Hunkel function of the 1st kind

$J_\mu(y) \longrightarrow$ Bessel function

Ref: (H-vacuum) $\longrightarrow$ A. A. Saharian and T. A. Petrosyan, Phys. Rev. D **104**, 065017 (2021).

Ref: (BD-vacuum) $\longrightarrow$ K. A. Milton and A. A. Saharian, Phys. Rev. D **85**, 064005 (2012)

HADAMARD FUNCTION

$$G(x,x') = \sum_{\sigma} \left[\varphi_{\sigma}(x) \varphi_{\sigma}^{*}(x') + \varphi_{\sigma}(x') \varphi_{\sigma}^{*}(x) \right]$$

- H-vacuum

$$G(x,x') = \frac{\alpha^{1-D}}{2(2\pi)^{D/2}} \int_0^{\infty} dz\, z \left| \Gamma\left(\frac{D-1}{2} + iz \right) \right|^2$$
$$\times \frac{\sum_{j=+,-} P_{\nu-1/2}^{jiz}\left(\cosh(t/\alpha)\right) P_{\nu-1/2}^{-jiz}\left(\cosh(t'/\alpha)\right)}{\left[\sinh(t/\alpha)\sinh(t'/\alpha)\right]^{(D-1)/2}} \frac{P_{iz-1/2}^{1-D/2}(w)}{\left(w^2-1\right)^{(D-2)/4}}$$

- BD vacuum

$$G_{\mathrm{BD}}(x,x') = \frac{\Gamma(D/2+\nu)\Gamma(D/2-\nu)}{(2\pi)^{(D+1)/2}\alpha^{D-1}} \frac{P_{\nu-1/2}^{(1-D)/2}(-u(x,x'))}{|u^2(x,x')-1|^{(D-1)/4}}$$

CONFORMALLY COUPLED MASSLESS FIELD

$\xi = (D-1)/(4D)$ $m = 0$ $\longrightarrow$ $\nu = 1/2$

- H-vacuum

$$G(x,x') = 2(-1)^n \frac{\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}}{(2\pi)^{D/2+1}\alpha^{D-1}} \partial_w^n \int_0^\infty dz \sinh(\pi z)$$

$$\times \cos(z\Delta\eta/\alpha)\left|\Gamma\left(\frac{D-1}{2} - n + iz\right)\right|^2 \frac{P_{iz-1/2}^{1-D/2+n}(w)}{\left(w^2-1\right)^{\frac{D-2n-2}{4}}}$$

$n = 0, 1, 2, \ldots$

$$\Updownarrow$$

$$G(x,x') = \left[\Omega_{\rm st}(\eta)\Omega_{\rm st}(\eta')\right]^{\frac{1-D}{2}} G_{\rm st}(x,x')$$

$\Omega_{\rm st}^2(\eta) = \sinh^{-2}(\eta/\alpha)$

$G_{\rm st}(x,x') \longrightarrow$ Hadamard function corresponding to static spacetime

- BD vacuum

$$G_{\rm BD}(x,x') = \frac{\alpha^{1-D}\Gamma((D-1)/2)}{(2\pi)^{\frac{D+1}{2}}\left[1-u(x,x')\right]^{\frac{D-1}{2}}}$$

- Further simplifications for odd and even values of D

ODD VALUES OF D

$\boxed{n=(D-3)/2}$

H-vacuum →

$$G(x,x')=\frac{\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}}{(-2\pi)^{\frac{D+1}{2}}\alpha^{D-1}}\left(\frac{\partial_\zeta}{\sinh\zeta}\right)^{\frac{D-3}{2}}\frac{2\zeta/\sinh\zeta}{\zeta^2-(\Delta\eta)^2/\alpha^2}$$

$w=\cosh\zeta$

BD vacuum →

$$G_{\mathrm{BD}}(x,x')=\frac{\alpha^{1-D}}{(2\pi)^{\frac{D+1}{2}}}\partial_u^{\frac{D-3}{2}}\frac{1}{1-u(x,x')}$$

Using the relations

$$\partial_u=-\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\frac{\partial_\zeta}{\sinh\zeta}\qquad 1-u(x,x')=\frac{\cosh\zeta-\cosh(\Delta\eta/\alpha)}{\sinh(\eta/\alpha)\sinh(\eta'/\alpha)}$$

We obtain

$$G_{\mathrm{BD}}(x,x')=\frac{\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}}{(-2\pi)^{\frac{D+1}{2}}\alpha^{D-1}}\left(\frac{\partial_\zeta}{\sinh\zeta}\right)^{\frac{D-3}{2}}\frac{1}{\cosh\zeta-\cosh(\Delta\eta/\alpha)}$$

Final expression →

$$G(x,x')-G_{\mathrm{BD}}(x,x')=\frac{\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}}{(-2\pi)^{(D+1)/2}\alpha^{D-1}}\times\left(\frac{\partial_\zeta}{\sinh\zeta}\right)^{\frac{D-3}{2}}\left[\frac{2\zeta/\sinh\zeta}{\zeta^2-(\Delta\eta)^2/\alpha^2}-\frac{1}{\cosh\zeta-\cosh(\Delta\eta/\alpha)}\right]$$

EVEN VALUES OF D

$\boxed{n = D/2 - 1}$

- H-vacuum

$$G(x,x') = -\frac{\alpha^{1-D}}{(-2\pi)^{D/2}}\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}$$

$$\times\partial_w^{D/2-1}\int_0^\infty dz \tanh(\pi z)\cos(z\Delta\eta/\alpha)P_{iz-1/2}(w)$$

- BD vacuum

$$G_{\mathrm{BD}}(x,x') = -\frac{\alpha^{1-D}}{(-2\pi)^{D/2}}\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}$$

$$\times\partial_w^{D/2-1}\int_0^\infty dz \cos(z\Delta\eta/\alpha)P_{iz-1/2}(w)$$

Final expression ➡

$$G(x,x') - G_{\mathrm{BD}}(x,x') = \frac{2\alpha^{1-D}}{(-2\pi)^{D/2}}\left[\sinh(\eta/\alpha)\sinh(\eta'/\alpha)\right]^{\frac{D-1}{2}}$$

$$\times\int_0^\infty dz\frac{\cos(z\Delta\eta/\alpha)}{e^{2\pi z}+1}\frac{P_{iz-1/2}^{D/2-1}(w)}{(w^2-1)^{\frac{D-2}{4}}}$$

VEV OF FIELD SQUARED

$$\langle \varphi^2 \rangle = \underbrace{\langle \varphi^2 \rangle_{\mathrm{BD}}}_{\text{divergent}} + \underbrace{\frac{1}{2}\lim_{x' \to x}\left[G(x,x') - G_{\mathrm{BD}}(x,x')\right]}_{\text{finite}}$$

BD vacuum is maximally symmetric $\Rightarrow$ $\langle \varphi^2 \rangle_{\mathrm{BD}}$ does not depend on spacetime coordinates

1. Even D

$$\lim_{w \to 1} \frac{P^{D/2-1}_{iz-1/2}(w)}{(w^2-1)^{\frac{D-2}{4}}} = \frac{2^{1-D/2}\Gamma\left(iz+(D-1)/2\right)}{\Gamma(D/2)\Gamma\left(iz-(D-3)/2\right)}$$

$$\Downarrow$$

$$\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_{\mathrm{BD}} - \frac{2[\alpha \sinh(t/\alpha)]^{1-D}}{(4\pi)^{D/2}\Gamma(D/2)} \int_0^\infty dz \frac{z^{D-2}}{e^{2\pi z}+1} \prod_{l=0}^{D/2-2}\left[\left(\frac{l+1/2}{z}\right)^2 + 1\right]$$

$\leftarrow$ negative

Expressed in terms of Riemann zeta function $\longrightarrow$ $\boxed{\zeta(s) = \frac{1}{\left(1-2^{1-s}\right)\Gamma(s)}\int_0^\infty \frac{x^s-1}{e^x+1}\mathrm{d}x}$

VEV OF FIELD SQUARED

2. Odd D

$$\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_{\rm BD} - \frac{2[\alpha \sinh(t/\alpha)]^{1-D}}{(4\pi)^{D/2}\Gamma(D/2)} \int_0^\infty dz \frac{z^{D-2}}{e^{2\pi z}-1} \prod_{l=0}^{(D-3)/2} \left[\left(\frac{l}{z} \right)^2 + 1 \right]$$

$$\Updownarrow$$

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^s - 1}{e^x - 1} \mathrm{d}x, \quad \mathrm{Re}\, s > 1$$

$$\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_{\rm BD} - \frac{(2\pi)^{-\frac{D+1}{2}} b_D}{12[\alpha \sinh(t/\alpha)]^{D-1}}$$

$$b_3 = 1,\ b_5 = \frac{11}{30},\ b_7 = \frac{191}{630},\ b_9 = \frac{2497}{6300},\ b_{11} = \frac{14797}{20790}$$

3. General D

$$\langle \varphi^2 \rangle = \langle \varphi^2 \rangle_{\rm BD} - \frac{B_D}{[\alpha \sinh(t/\alpha)]^{D-1}}$$

$$B_D = \frac{2(4\pi)^{-\frac{D}{2}}}{\Gamma(D/2)} \int_0^\infty dz \frac{z^{D-2} A_D(z)}{e^{2\pi z} + (-1)^D}$$

$$A_D(z) = \prod_{l=0}^{l_m} \left[\left(\frac{l + 1/2 - \{D/2\}}{z} \right)^2 + 1 \right]$$

$$l_m = D/2 - 2 + \{D/2\}$$

$\{w\} \longrightarrow$ fractional part of w

VEV OF FIELD SQUARED: RINDLER COORDINATES

- Minkowski spacetime

Rindler coordinates → $(\tau_{\mathrm{R}}, \chi, \mathbf{x}_{\mathrm{R}}), \quad \mathbf{x}_{\mathrm{R}} = (x_{\mathrm{R}}^2, \ldots, x_{\mathrm{R}}^D)$

Line element → $ds_{\mathrm{M}}^2 = \chi^2 d\tau_{\mathrm{R}}^2 - d\chi^2 - d\mathbf{x}_{\mathrm{R}}^2$

- Massless scalar field

VEV of the field squared → $\langle \varphi^2 \rangle_{\mathrm{FR}} = \langle \varphi^2 \rangle_{\mathrm{M}} - \frac{B_D}{\chi^{D-1}}$

Fulling-Rindler vacuum   Minkowski vacuum

Ref: (for FR vacuum) → A. A. Saharian, Class. Quantum Grav. **19**, 5039 (2002)

VEV OF ENERGY-MOMENTUM TENSOR

$$\Delta\langle T_{ik}\rangle \equiv \langle T_{ik}\rangle - \langle T_{ik}\rangle_{\mathrm{BD}}$$

$$\Delta\langle T_{ik}\rangle = \frac{1}{2}\lim_{x'\to x}\partial_{i'}\partial_k\left[G(x,x') - G_{\mathrm{BD}}(x,x')\right] + \left[\left(\xi - \frac{1}{4}\right)g_{ik}\nabla_p\nabla^p - \xi\nabla_i\nabla_k - \xi R_{ik}\right]\left(\langle\varphi^2\rangle - \langle\varphi^2\rangle_{\mathrm{BD}}\right)$$

Finite

∇_i ⟶ covariant derivative operator

$R_{ik} = Dg_{ik}/\alpha^2$ ⟶ Ricci tensor for the dS spacetime

- Renormalization is not required

Problem symmetry ⟹ $\Delta\langle T_{ik}\rangle$ is a function of the time coordinate alone; $\Delta\langle T_1^1\rangle = \Delta\langle T_2^2\rangle = \cdots = \Delta\langle T_D^D\rangle$ ⟺ Isotropic vacuum stresses

$\xi = (D-1)/(4D)$, $m = 0$ ⟼ $\Delta\langle T_i^i\rangle = 0$ ⟹ $\Delta\langle T_0^0\rangle = -D\Delta\langle T_1^1\rangle$

Continuity equation ⟹ $\nabla_k\langle T_i^k\rangle = 0$ ⟹ $\Delta\langle T_1^1\rangle = \dfrac{\partial_{t/\alpha}\left[\sinh^D(t/\alpha)\Delta\langle T_0^0\rangle\right]}{D\sinh^{D-1}(t/\alpha)\cosh(t/\alpha)}$

VEV OF ENERGY-MOMENTUM TENSOR

$$\Delta\left\langle T_0^0\right\rangle = -D\Delta\left\langle T_1^1\right\rangle$$

$$\Delta\left\langle T_1^1\right\rangle = \frac{\partial_{t/\alpha}\left[\sinh^D(t/\alpha)\Delta\left\langle T_0^0\right\rangle\right]}{D\sinh^{D-1}(t/\alpha)\cosh\left(t/\alpha\right)}$$

$$\longmapsto \quad \left\langle T_i^k\right\rangle = \left\langle T_i^k\right\rangle_{\mathrm{BD}} + C_D\frac{\mathrm{diag}\left(1,-1/D,\cdots,-1/D\right)}{\left[\alpha\sinh(t/\alpha)\right]^{D+1}}$$

- BD vacuum

$$\left\langle T_i^k\right\rangle_{\mathrm{BD}} = \frac{C_D^{(\mathrm{BD})}\delta_i^k}{(D+1)\alpha^{D+1}}$$

$$C_D^{(\mathrm{BD})} = \alpha^{D+1}\left\langle T_i^i\right\rangle_{\mathrm{BD}} \longrightarrow$$ Determined by the trace anomaly

1. Even D $\longrightarrow$ $C_D^{(\mathrm{BD})}=0$ $\Longleftrightarrow$ Trace anomaly is absent

2. Odd D $\longrightarrow$ $C_3^{(\mathrm{BD})}=1/(240\pi^2),\ C_5^{(\mathrm{BD})}=-5/(4032\pi^3),\ C_7^{(\mathrm{BD})}=23/(34560\pi^4)$

Ref: (for odd D) $\longrightarrow$ E. J. Copeland, D. J. Toms, Class. Quantum Grav. 3, 431 (1986)

VEV OF ENERGY-MOMENTUM TENSOR

- H- vacuum

1. Even D $\longrightarrow$ $$C_D = -\frac{2^{1-D}\pi^{-D/2}}{\Gamma(D/2)}\int_0^\infty dz \frac{z^D A_D(z)}{e^{2\pi z}+1}$$

$$A_D(z) = \prod_{l=0}^{l_m}\left[\left(\frac{l+1/2-\{D/2\}}{z}\right)^2+1\right]$$

$\zeta(x)$ $\longrightarrow$ Riemann zeta function

Special case: $$C_4 = -\frac{3}{2^9\pi^7}\left[\pi^2\zeta(3)+15\zeta(5)\right]$$

2. Odd D $\longrightarrow$ $$C_D = -\frac{2^{1-D}\pi^{-D/2}}{\Gamma(D/2)}\int_0^\infty dz \frac{z^D A_D(z)}{e^{2\pi z}-1}$$

Special cases: $$C_3 = -\frac{1}{480\pi^2},\ C_5 = -\frac{31}{60480\pi^3}$$

3. General D $\longrightarrow$ $$C_D = -\frac{2^{1-D}\pi^{-D/2}}{\Gamma(D/2)}\int_0^\infty dz \frac{z^D A_D(z)}{e^{2\pi z}+(-1)^D} < 0$$

$\Downarrow$

Energy density in the H-vacuum is smaller than that for the BD vacuum

Ref: $\longrightarrow$ J. D. Pfautsch, Phys. Lett. B 117, 283 (1982)
(for $D=3$)

DENSITY OF STATES

- H-vacuum

Vacuum energy density → $\langle T_0^0 \rangle = \langle T_0^0 \rangle_{\mathrm{BD}} - \frac{\sinh^{-D-1}(t/\alpha)}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \int_0^\infty dE \frac{E^D A_D(\alpha E)}{e^{2\pi\alpha E} + (-1)^D}$ $\Big|\ E = z/a$

BD vacuum is a thermal state with respect to H-vacuum with the temperature $T = 1/(2\pi\alpha)$

	Thermal distribution
Even D	Fermi-Dirac type
Odd D	Bose-Einstein type

Number of states in the energy range $(E, E+dE)$ → $\rho(E)dE$

Density of states → $\rho(E) = \frac{2E^{D-1} A_D(\alpha E)}{(4\pi)^{D/2}\Gamma(D/2)}$

DENSITY OF STATES: RINDLER COORDINATES

Density of states for spin-0 massless particles in Minkowski spacetime → $\rho_{\mathrm{M}}(E) = \rho(E) / A_D(\alpha E)$

Minkowski vacuum is a thermal state with respect to Fulling-Rindler vacuum

For even D the average number of particles is given by Fermi-Dirac distribution

dS spacetime ⟷ Minkowski spacetime (in Rindler coordinates)

Conformal relation

Coordinate transformations $(\tau_{\mathrm{R}}, \chi, \mathbf{x}_{\mathrm{R}}) \to (\eta, r, \vartheta, \phi)$

$$\tau_{\mathrm{R}} = \frac{\eta}{\alpha},\ \chi = \frac{\alpha}{\cosh r - \sinh r \cos\theta_1},$$
$$x_{\mathrm{R}}^l = \chi w^l \sinh r,\ l = 2,\ldots,D$$

$$w^2 = \sin\theta_1 \cos\theta_2, \ldots, w^{D-2} = \cos\theta_{D-2} \prod_{i=1}^{D-3} \sin\theta_i$$
$$w^{D-1} = \cos\phi \prod_{i=1}^{D-2} \sin\theta_i,\ w^D = \sin\phi \prod_{i=1}^{D-2} \sin\theta_i$$

$$ds_{\mathrm{R}}^2 = \chi^2 \left(d\eta^2 / \alpha^2 - dr^2 - \sinh^2 r d\Omega_{D-1}^2 \right)$$
$$ds^2 = dt^2 - \alpha^2 \sinh^2(t/\alpha)(dr^2 + \sinh^2 r d\Omega_{D-1}^2)$$

$$ds^2 = \Omega_{\mathrm{R}}^2 ds_{\mathrm{R}}^2$$
$$\Omega_{\mathrm{R}}^2 = \frac{(\cosh r - \sinh r \cos\theta_1)^2}{\sinh^2(\eta/\alpha)}$$

H-vacuum ⟷ Fulling-Rindler vacuum

conformal counterpart

FLAT SPACETIME LIMIT

dS spacetime (hyperbolic coordinates) — flat spacetime limit — Milne universe

$$ds^2 = dt^2 - \alpha^2 \sinh^2(t/\alpha)(dr^2 + \sinh^2 r d\Omega_{D-1}^2) \xrightarrow[\alpha \to \infty]{} ds_{\text{Milne}}^2 = dt^2 - t^2(dr^2 + \sinh^2 r d\Omega_{D-1}^2)$$

H-vacuum $\longrightarrow$ Conformal vacuum

BD vacuum $\longrightarrow$ Minkowski vacuum

Assuming that the renormalizing VEVs in the Minkowski vacuum are zero,

$$\left\langle \varphi^2 \right\rangle_{\text{M, Ren}} = 0, \quad \left\langle T_i^k \right\rangle_{\text{M, Ren}} = 0 \quad \Longrightarrow \quad \left\langle \varphi^2 \right\rangle_{\text{Milne}} = -\frac{B_D}{t^{D-1}}, \quad \left\langle T_i^k \right\rangle_{\text{Milne}} = \frac{C_D}{t^{D+1}} \text{diag}(1, -1/D, \cdots, -1/D)$$

Ref: (for conformal vacuum) $\longrightarrow$ A. A. Saharian, T. A. Petrosyan, Symmetry 12, 619 (2020).

ASYMPTOTICS

- Early stages of the cosmological expansion $t \to 0$

$$\langle \varphi^2 \rangle - \langle \varphi^2 \rangle_{\mathrm{BD}} \approx -\frac{B_D}{t^{D-1}}$$

$$\langle T_{ik} \rangle - \langle T_{ik} \rangle_{\mathrm{BD}} \approx \frac{C_D}{t^{D+1}} \mathrm{diag}\left(1, -1/D, \cdots, -1/D\right)$$

- Late stages of the expansion $t \gg \alpha$

$$\langle \varphi^2 \rangle - \langle \varphi^2 \rangle_{\mathrm{BD}} \approx e^{-(D-1)t/\alpha}$$

$$\langle T_{ik} \rangle - \langle T_{ik} \rangle_{\mathrm{BD}} \approx e^{-(D+1)t/\alpha}$$

Scalar field with $m^2 + \xi R > 0$ ⟶ BD vacuum is the future attractor for cosmological solutions

Ref: ⟶ P. R. Anderson, W. Eaker, S. Habib, C. Molina-Paris, E. Mottola, Int. J. Theor. Phys. 40, 2217 (2001)

SUMMARY

- VEVs of the field squared and of the energy-momentum tensor are investigated for a scalar field prepared in the H-vacuum of dS spacetime, for the general number of spacetime dimensions
- Hadamard function is decomposed into two contributions:

BD vacuum contribution + Correlation of the vacuum fluctuations in two vacua

- Renormalization of the VEVs for the H-vacuum is reduced to the renormalization for the BD vacuum state
- Important special case is considered:

Conformally coupled massless field

- BD vacuum is a thermal state with respect to H-vacuum
- Thermal distribution is of Fermi-Dirac type for even D, and of Bose-Einstein type for odd D
- At late stages of expansion the difference between the VEVs in the H- and BD vacua is exponentially suppressed
- At early stages of expansion the VEV of energy-momentum tensor is large and the backreaction of quantum effects on the spacetime geometry should be taken into account

Ref: → A. A. Saharian, T. A. Petrosyan, V. S. Torosyan, Ann. Phys. **437**, 168728 (2022)

THANK YOU