Rotating Neutron Stars

Universal Relations for the Increase in the Mass and Radius of a Rotating Neutron Star

Andreas Konstantinou
akonstan@ualberta.ca

University of Alberta
Department of Physics

Supervisor: Sharon Morsink

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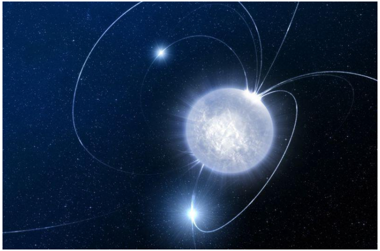

Figure: Neutron star. Credits: ESO/LUÍS CALÇADA

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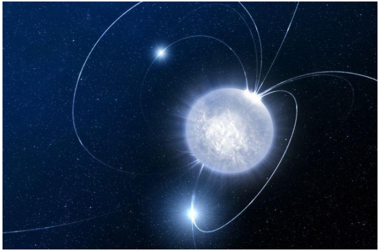

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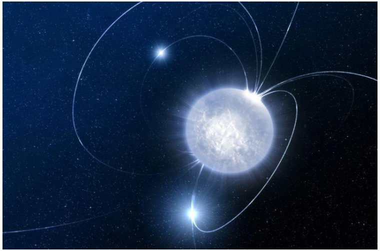
- **Part III** : Results & Applications
- **Part IV** : Why Universality?
- **Part V** : Conclusions



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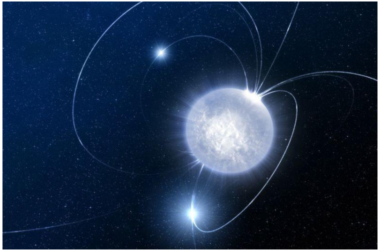

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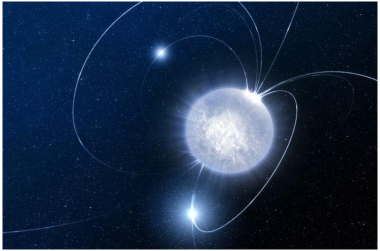

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Part I : Basics of Neutron Stars

- **Atmosphere**
  -slim plasma envelope

- **Outer crust**($\sim 10^{11}$ g cm^{-3})
  -Z ions and free e
- **Inner crust** ($\sim 10^{14}$ g cm^{-3})
  - free n and e
  -neutron-rich atomic nuclei

- **Outer core** ($\sim 10^{14}$ g cm^{-3})
  -free n, p, e and μ
  -superfluid-Fermi liquid.
- **Inner core** (larger than 10^{15} g cm^{-3})
  -hyperons, free quarks, exotic matter (???)

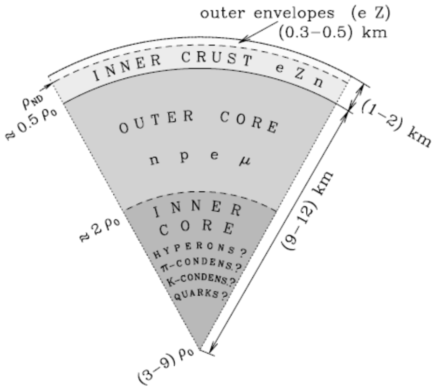

Figure: Internal structure of a neutron star ($\rho_0 = 2.28\ 10^{14}$ g cm^{-3}) [Haensel et al., 2007]

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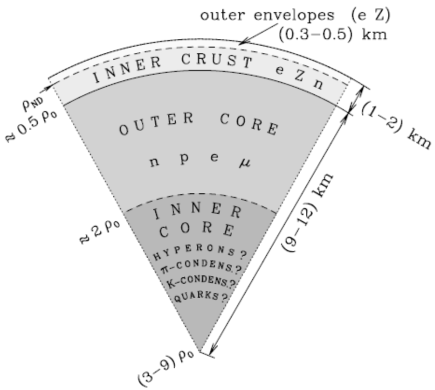

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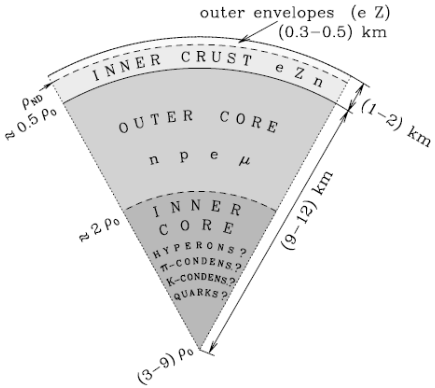

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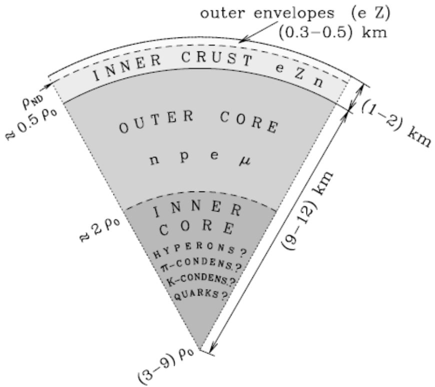

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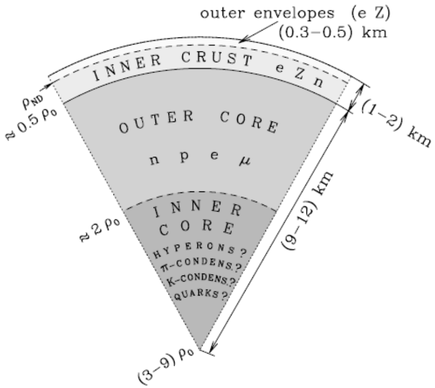

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General Relativistic System

- For a spherical symmetric (non-rotating) NS we use the TOV equations

$$\frac{dP(r)}{dr} = -\frac{G(\epsilon(r)c^2 + P(r))(m(r) + 4r^3 P(r)/c^2)}{rc^2[r - 2Gm(r)/c^2]} \tag{1}$$

$$\frac{dm(r)}{dr} = 4\pi r^2 \epsilon(r) \tag{2}$$

$$EOS : P(\epsilon) \tag{3}$$

- where P: pressure, ϵ: energy density, m: mass coordinate and r: the radial distance

EQUATION OF STATE (EOS)

- The blue area in the plots is our expectations based on **observations** and **experiments**

- **Phenomenological EOS**:
  -Piecewise polytropes
  -Speed of sound model
- **EOS models**:
  -Nuclear EOS : APR, BBB, HLPS
  -Hybrid EOS: QHC21, ABPR
  -Hyperons: H0
  -π^0 condensate: L

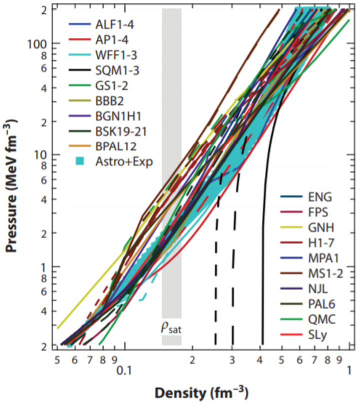

Figure: EOS models
[Özel and Freire, 2016]

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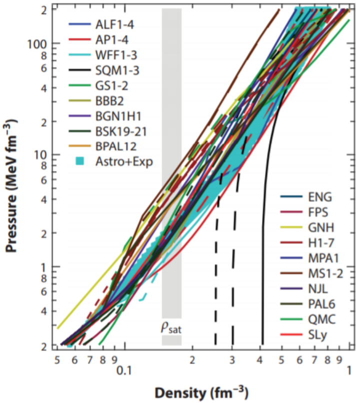

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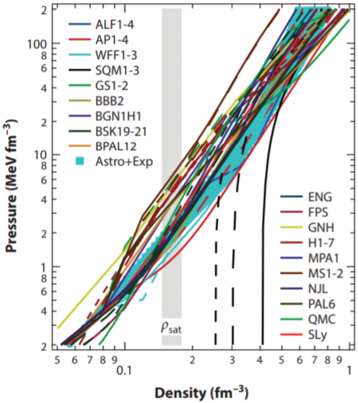

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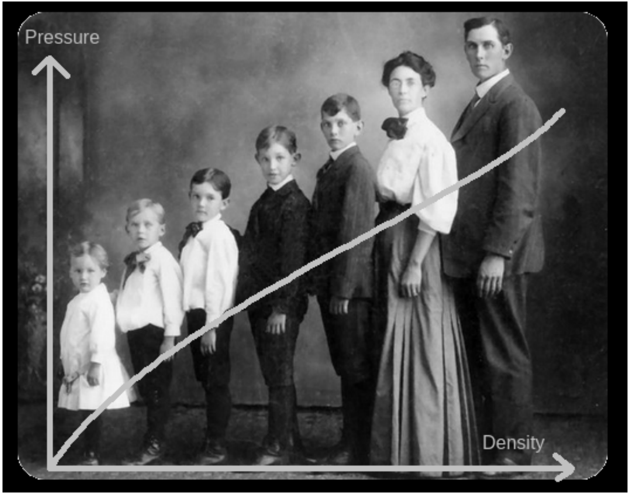
The Equation of State Families



PIECEWISE POLYTROPES

- [Read et al., 2009],

$$P(\rho) = \begin{cases} P_{crust} & \rho \leq \rho_{crust} \\ K_1\rho^{\Gamma_1} & \rho_{crust} \leq \rho \leq \rho_1 \\ K_2\rho^{\Gamma_2} & \rho_1 \leq \rho \leq \rho_2 \\ K_3\rho^{\Gamma_3} & \rho_2 \leq \rho \end{cases} \quad (4)$$

- [Hebeler et al., 2013] **a) Causality**
  b) Supports the **heaviest** observed NS

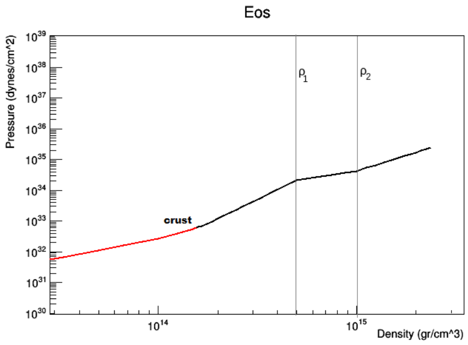


Figure: Piecewise Polytrope

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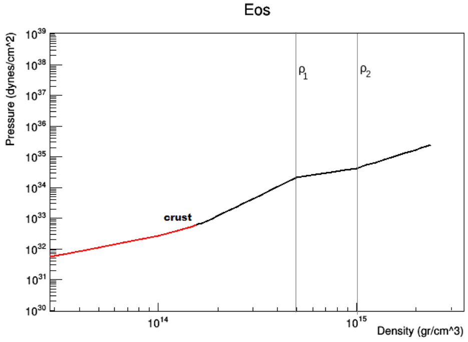

Figure: Piecewise Polytrope

Speed of Sound Model

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- [Greif et al., 2019],

$$P(\epsilon) = \begin{cases} P_{crust} & n \leq 0.5n_0 \\ P_{cEFT} & 0.5n_0 \leq n \leq 1.1n_0 \\ \int_0^{\epsilon} (c_s(\epsilon')/c)^2 d\epsilon' & 1.1n_0 \leq n \end{cases} \quad (5)$$

- Causality $0 \leq c_s \leq$ c
- Supports the **heaviest** observed NS
- perturbative quantum chromodynamics (**pQCD**) : $\frac{c_s}{c}$ approaches $\frac{1}{\sqrt{3}}$ from below when $\sim 10^{16}$ g/cm^3)

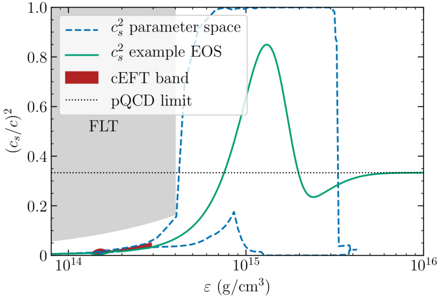


Figure: Speed of sound profile from [Greif et al., 2019]

Two EOS Families

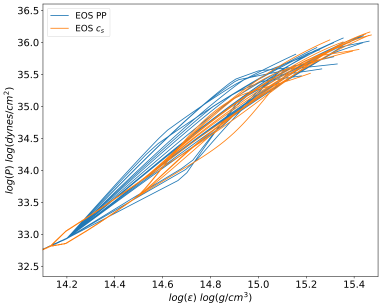


Figure: EOS

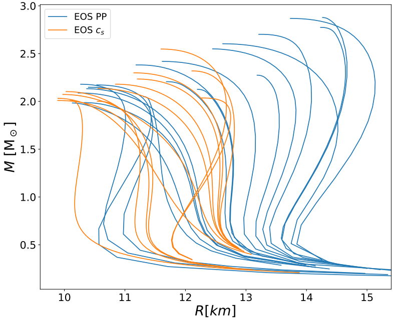


Figure: Mass - Radius curves

Part II : Rotating Neutron Stars And Universal Relations

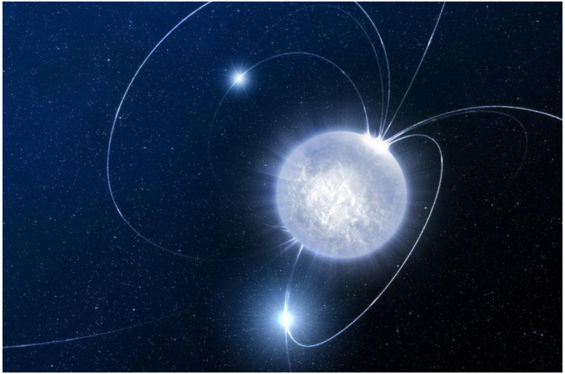
What kind of Neutron Stars?

- **Cold (Temperatures less than the neutrons Fermi energy)**

- Rigidly rotating

- Low magnetic field

- Described by a stationary metric



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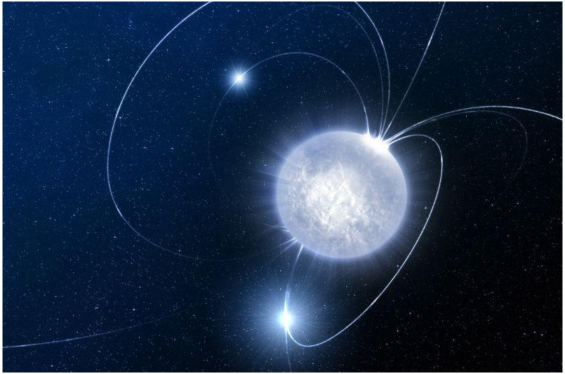

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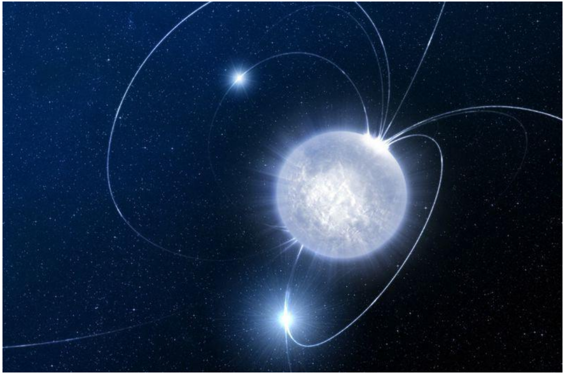

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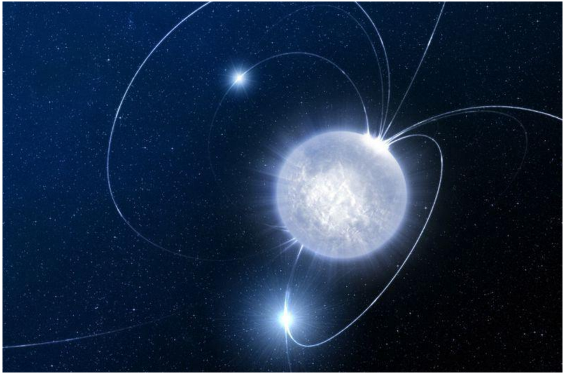

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Rotating Neutron Stars

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- **Axis symmetric metric**

$$ds^2 = -e^{\gamma+\lambda}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\lambda}r^2 sin^2(d\phi - \omega dt)^2$$

λ,γ, α and ω are called metric potentials and depend on r and θ.[Cook et al., 1994]

- Use **RNS** code.
[Stergioulas and Friedman, 1995]
Numerically solves the GR equations for one rotating NS

Figure: Curved space-time for rotating systems
https://www.vice.com/da/article/8gmy4a/the-learning-corner-805-v18n5

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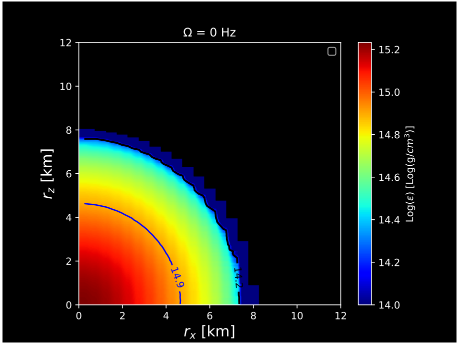
Sequences of Rotating NSs

- Create sequences of rotating NSs with constant ϵ_c in order to see the relation among the rotation and the EOS

- M: Total mass (in $M_\odot$)
- R_e: Equatorial radius of the NS (in km)



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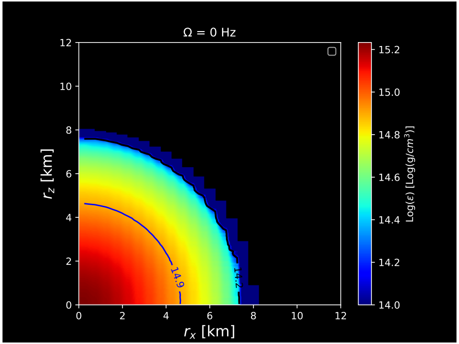

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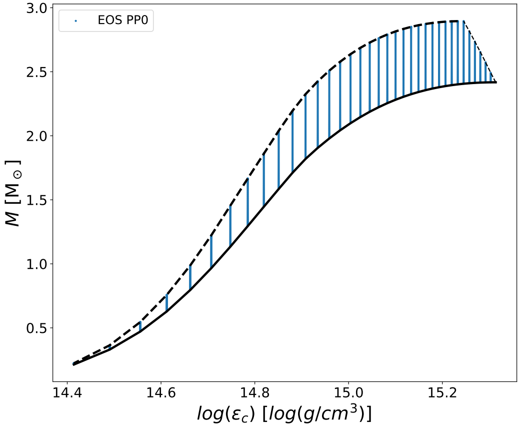


Figure: Total Mass vs central energy density

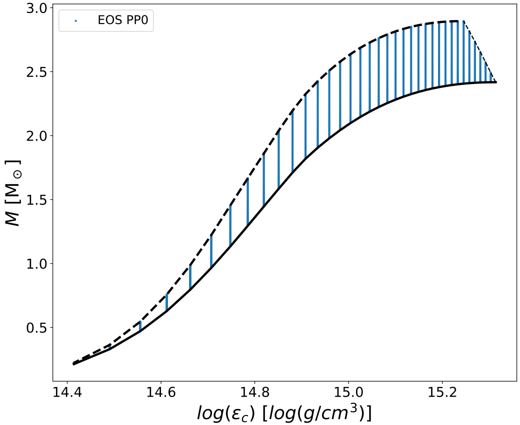
The thesis goal:

- Are there any universal relations?
- If yes,

  Are they useful for modelling the rotating neutron stars?

  Are they useful for our observations?

Rotating neutron stars



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What is not Universality?

- **Non-Universal Relations**:
  Equations that are strongly affected by the EOS structure.

- i.e. Mass-Radius relations.

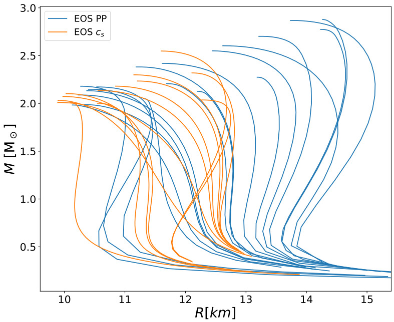


Figure: Mass - Radius curves

What is Universality?

- **Universal Relations**:
  Equations that are **NOT** strongly affected by the EOS structure.

- **Some Examples**:
  BE vs M/R [Lattimer & Prakash (2001)],

  Obleteness [Morsink et al. (2007), Silva et al. (2021)];

  I/M^3 vs M/R [Breu and Rezzolla, 2016],

  I-Love-Q [Yagi and Yunes, 2013].

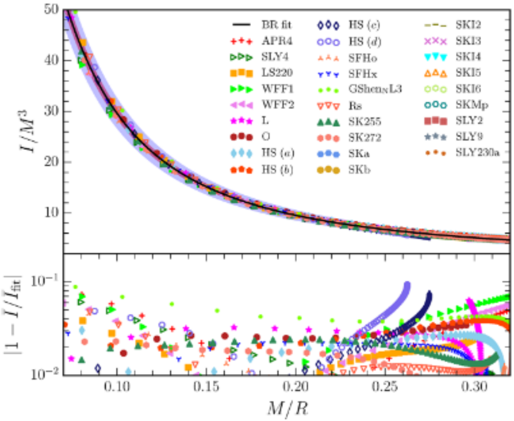


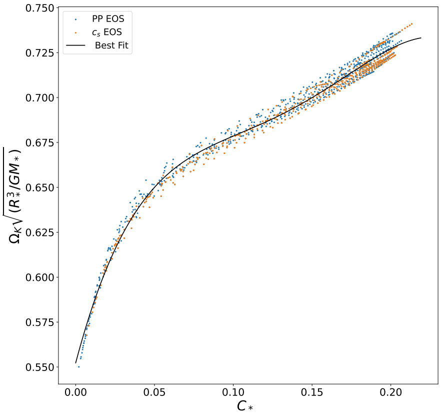
Figure: I/M^3 vs M/R [Breu and Rezzolla, 2016]

Part III : Results & Applications

Kepler Frequency

- $\Omega_n = \frac{\Omega}{\Omega_K(empirical)}$
- **Empirical approximation**:
  Previus work: Haensel & Zdunik (1989) ,Friedman et al. (1989), Haensel et al. (2009), Haskell et al. (2018), and Koliogiannis & Moustakidis (2020)
  Our work:
  $\Omega_K(empirical) = N_\Omega(C_*)\sqrt{\frac{GM_*}{R_*^3}}$
- $N_\Omega(C_*) = (a_1C_*^4 + a_2C_*^3 + a_3C_*^2 + a_4C_* + a_5)$

  $\text{Dev}(\Omega_K)_{max} = 1.60\%$
- $\text{Dev(Z)} = |Z_{data} - Z_{bestfit}|/Z_{data}$



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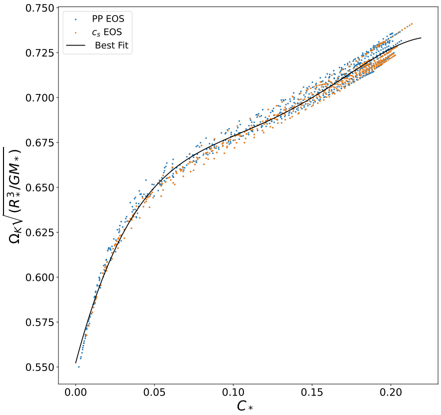

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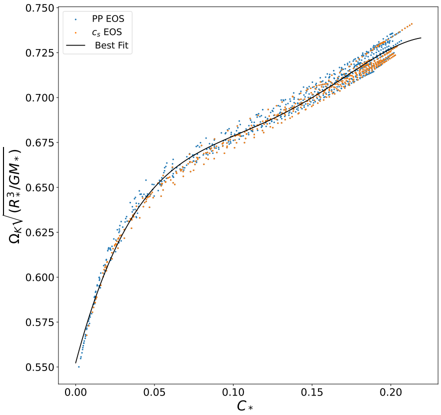

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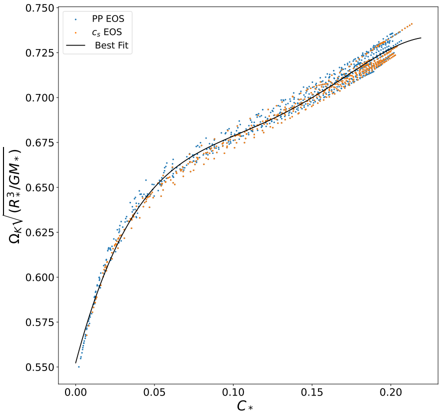

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Compactness

- C_e: Equatorial compactness $\frac{M}{R_e}\frac{km}{M_\odot}$
- Compactness is strongly related to the central energy density
- C_*: Initial compactness $\frac{M_*}{R_*}\frac{km}{M_\odot}$

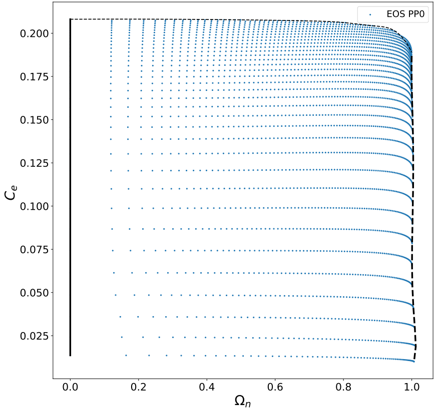
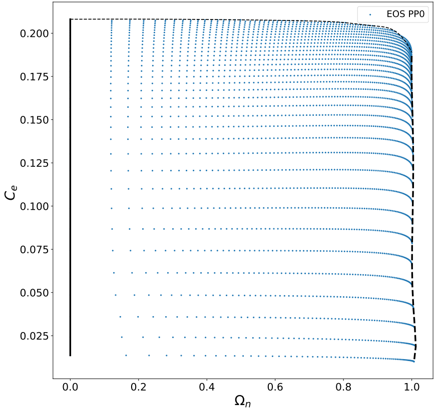


Figure: C_e vs Ω_n for PP EOS 0

Compactness

- C_e: Equatorial compactness $\frac{M}{R_e}\frac{km}{M_\odot}$
- Compactness is strongly related to the central energy density
- C_*: Initial compactness $\frac{M_*}{R_*}\frac{km}{M_\odot}$



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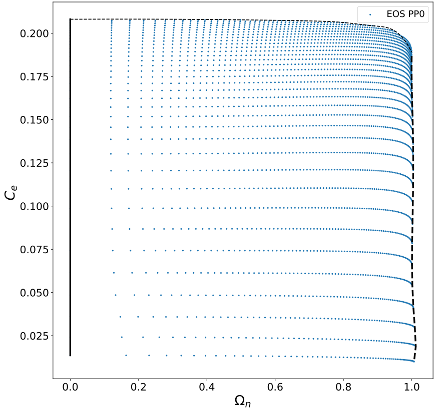

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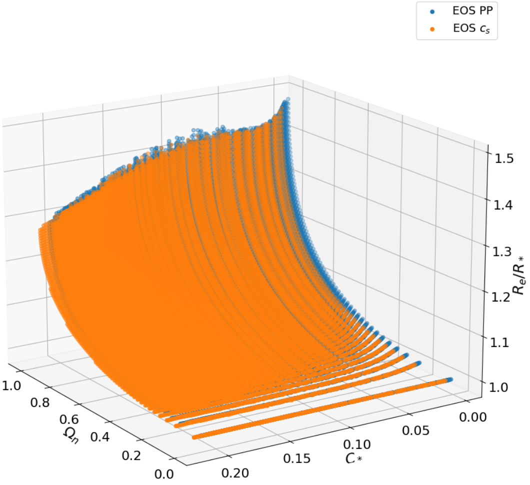
R_e/R_* Spin corrections



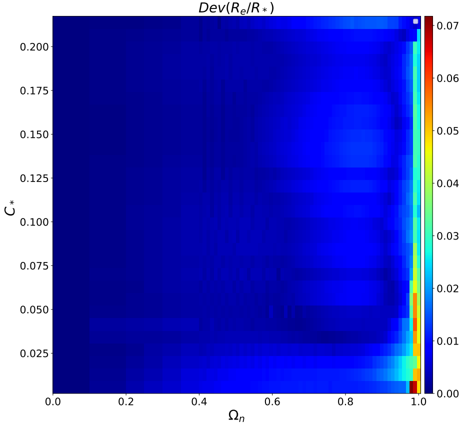
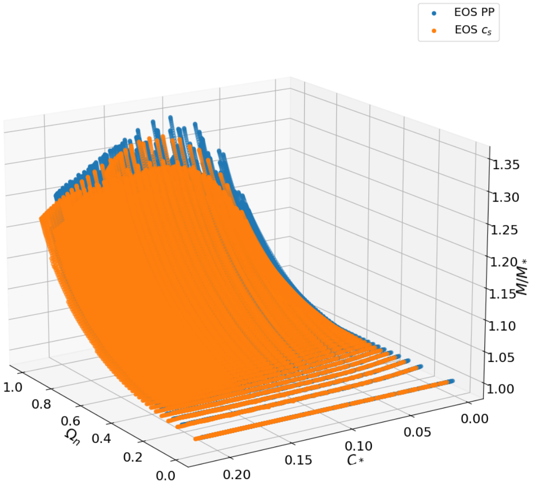


Figure: R_e/R_* vs C_* vs Ω_n

$$\frac{R-R_*}{R_*} = (e^{\alpha_r \Omega_n^2} - 1 - \beta_r \ln(1-(\frac{\Omega_n}{1.1})^4)^2) \times \left(1 + \Sigma_{i=1}^{5} r_i C_*^i\right)$$

M/M_* Spin corrections



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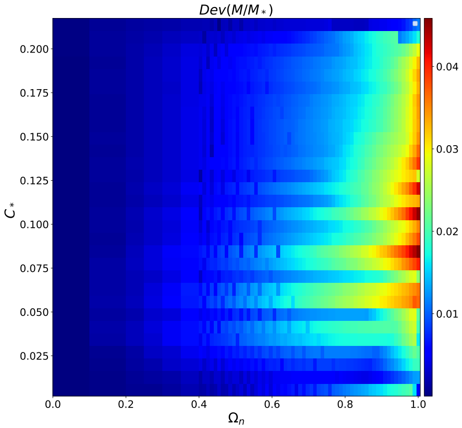

$$\frac{M-M_*}{M_*} = (e^{\alpha_m \Omega_n^2} - 1) \times \left(\Sigma_{i=0}^{4} m_i C_*^i\right)$$

Applications

- Solve the TOV Equations for a specific EOS (Black solid line)
  This means that we know M_*, R_* and C_*
- Use the **best fit surfaces** for M/M_* and R_e/R_* for $\Omega_n = 0.95$ (Orange solid line)

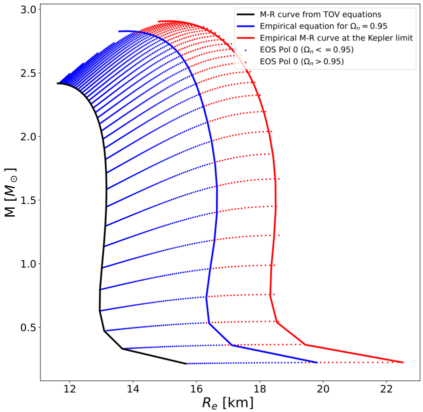

Figure: Mass - Radius curves for rotating NSs

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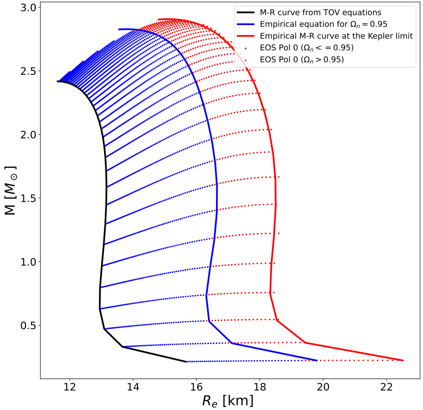


Figure: Mass - Radius curves for rotating NSs

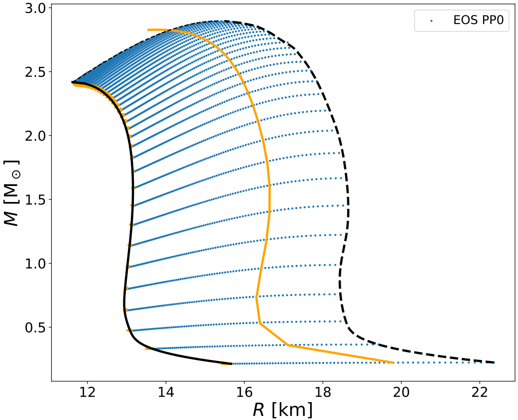
Applications

- $\Omega_n(\Omega, R_*, M_*) \rightarrow \Omega_n(\Omega, R_e, M)$ (Max Dev $\approx$ 1.5%)

  $\frac{M}{M_*}(\Omega, R_*, M_*) \rightarrow \frac{M}{M_*}(\Omega, R_e, M)$(Max Dev $\approx$ 5%)

  $\frac{R_e}{R_*}(\Omega, R_*, M_*) \rightarrow \frac{R_e}{R_*}(\Omega, R_e, M)$ (Max Dev $\approx$ 1.8%)

- We can find the non rotating NS with the same central energy density (Orange points)



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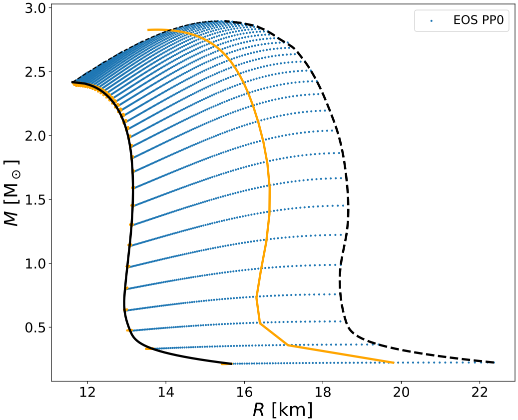

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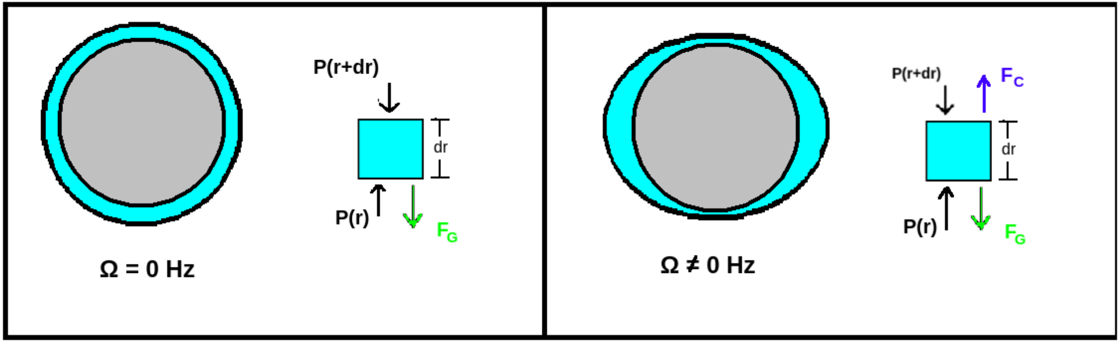
Part IV : Why Universality?

R_e/R_* Universality



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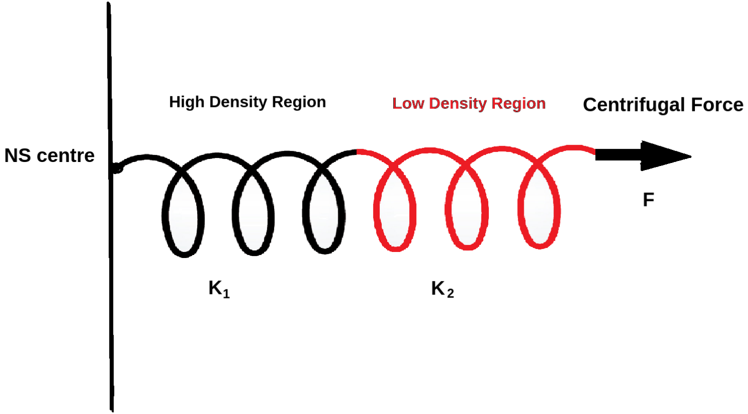

- $K_{eff} = \frac{K1K2}{K1+K2}$
- $K_{eff} \rightarrow K2$, when $K2 << K1$

M/M_* Universality

- A star with a uniform constant density ρ_0
- M $= \frac{4\pi\rho_0}{3} R_e^2 R_p$
- $R_e = R_* + \delta R_e(\Omega)$
  $R_p = R_* - \delta R_p(\Omega)$
- $V_i \sim V_f$, and $\frac{\delta R_e(\Omega)}{R_*} << 1$
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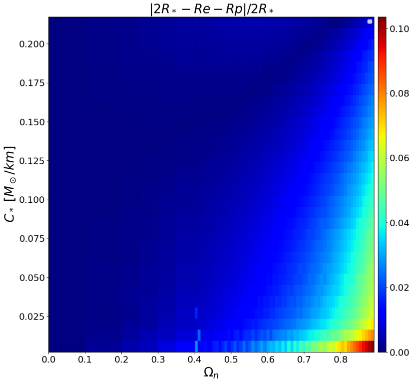
  $R_p \times R_e = R_*(1 + \alpha_e(\Omega)) \times R_*(1 - \alpha_p(\Omega)) \approx R_*^2$
- $M \approx \frac{4\pi\rho_0}{3} R_*^2 R_e \Rightarrow \frac{M}{M_*} \approx \frac{R_e}{R_*}$



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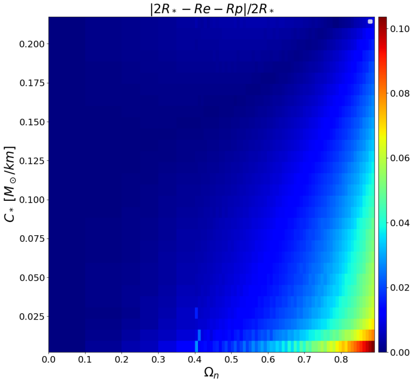

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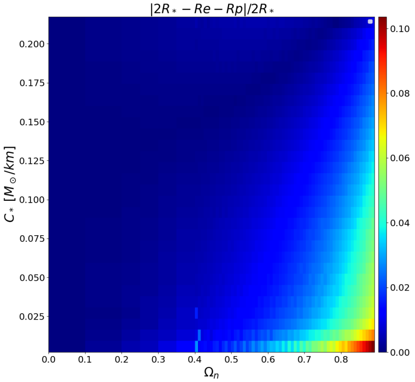

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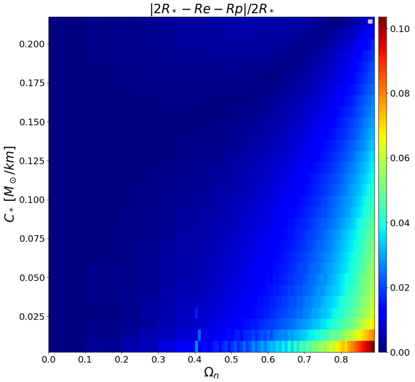

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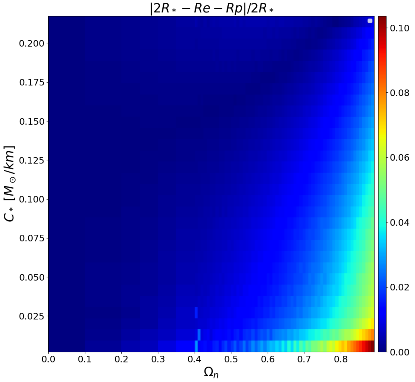

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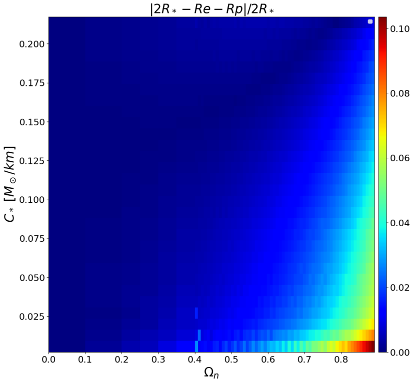

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- $\rho = \rho_{center}[1 - G(\sqrt{x^2/R_e^2 + y^2/R_e^2 + z^2/R_p^2})]$
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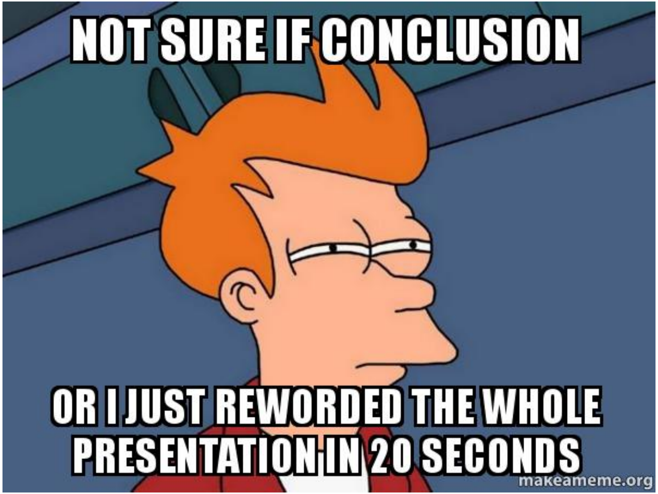
Part V : Conclusions



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- We used rns code to create sequences of rapidly rotating NSs with constant ϵ_c.
- We provide a new empirical equation for the Kepler frequency.
- M/M_* and R_e/R_* are universal.
- We provide best fit equations for them, so we can recreate the Mass-Radius curves for the rotating NSs.
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Figure: Universal Relations for the Increase in the Mass and Radius of a Rotating Neutron Star (Konstantinou Morsink 2022)

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Thank you for your attention!

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C_e Spin corrections

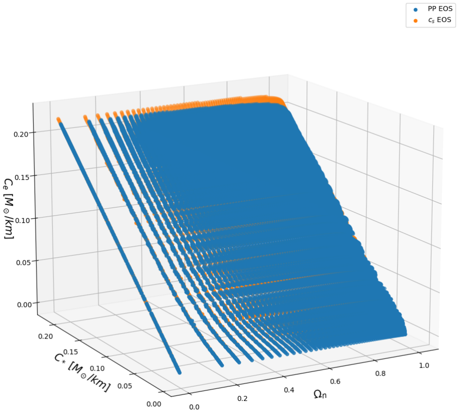
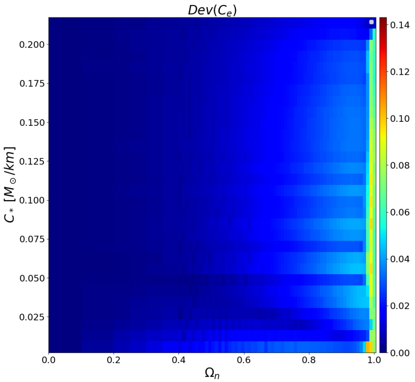
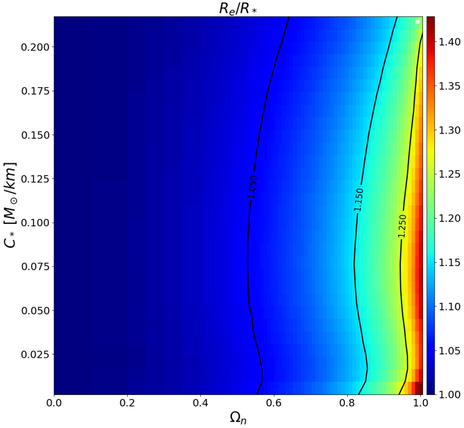


Figure: C_e vs C_* vs Ω_n



$$C_e \approx C_* + D_0 \times ln(1 - (\frac{\Omega_n}{1.1})^3) \times (1 + D_1 \times C_* + D_2 \times C_*^2 + D_3 \times C_*^4 + D_5 \times C_*^6)$$

R_e/R_* Spin corrections



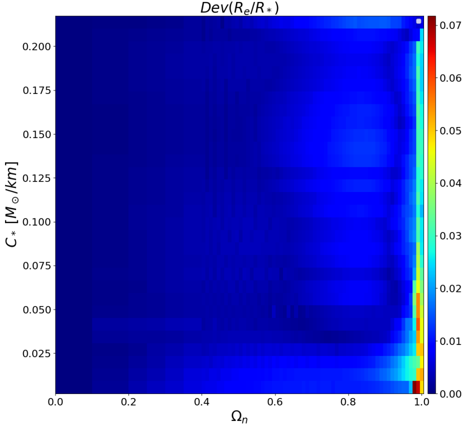


Figure: R_e/R_* vs C_* vs Ω_n

$$\frac{R-R_*}{R_*} = (e^{\alpha_r \Omega_n^2} - 1 - \beta_r \ln(1-(\frac{\Omega_n}{1.1})^4)^2) \times \left(1 + \Sigma_{i=1}^5 r_i C_*^i\right)$$

M/M_* Spin corrections

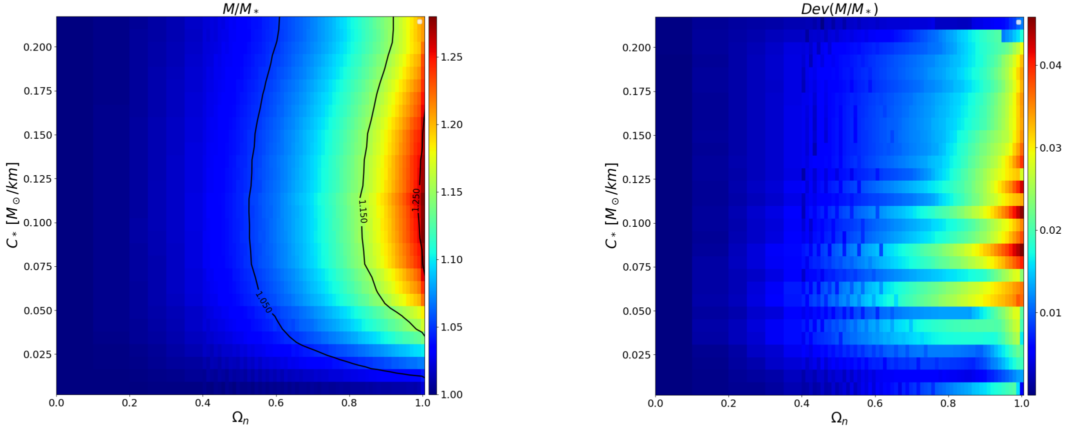


Figure: M/M_* vs C_* vs Ω_n

$$\frac{M - M_*}{M_*} = \left(e^{\alpha_m \Omega_n^2} - 1\right) \times \left(\Sigma_{i=0}^{4} m_i C_*^i\right)$$