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Intense Magnetic Field Effect On Mass-Radius Relation Of Neutron Stars

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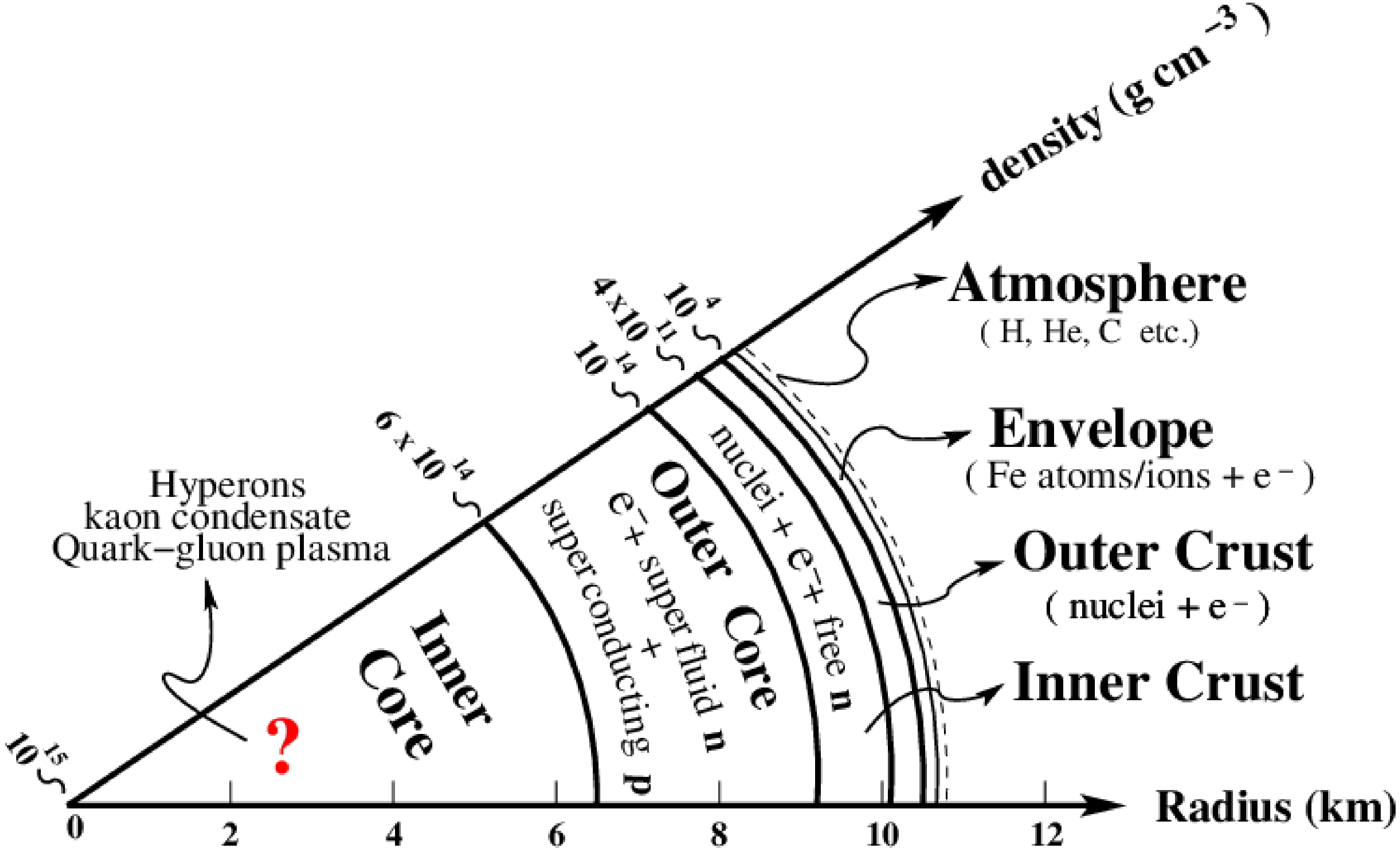
Introduction

Neutron stars are one of the densest known objects in the universe. These stars have a mass about twice that of the Sun and a radius of approximately 11 kilometers. The structure of these stars is predominantly composed of neutrons, with a small amount of protons and electrons in beta equilibrium. Many research have been accomplished on neutron stars and their various properties. One of the unique characteristics of these stars is the presence of an extremely strong magnetic field. We want to investigate the effect of this magnetic field on the Landau levels in neutron star matter and, consequently, its impact on the radius of these stars. we study the magnetic field's effect on the equation of state for two different models in the crust. For the equation of state of the stellar core, we will consider the lowest order constrained variational (LOCV) method with the three-body force [1].

Neutron star structure



Neutron Star Core Equation of State

The lowest order constrained variational method is an efficient and well-established approach for investigating nuclear properties. In this method, the single-particle trial wave function of particle $\psi = \mathcal{F}\varphi$ is considered as the basis for the calculations. φ is Slater determinant of single-particle wave functions for A-body non-interaction nucleon system and $\mathcal{F}$ is the A-body correlation operator which will be assumed in terms of two-body correlation operators in Jastrow form, the nonrelativistic many-body Hamiltonian is given by [2] :

$$H_{NR} = \sum_{i=1}^{A} \frac{P_i^2}{2m} + \sum_{i<j} v(ij) + \sum_{i<j<k} v(ijk)$$

Where $v(ij)$ and $v(ijk)$ are two and three body potential respectively[3].

Neutron star crust equation of state and the presence of the magnetic field

Fermi Gas Model

The presence of a magnetic field in neutron stars has been investigated from various perspectives. However, one of the most important aspects is the effect of the magnetic field on the Landau levels of electrons. Landau levels are defined for charged particles, so in a neutron star, only electrons and protons are involved in the final pressure due to changes in Landau levels. The energy of an electron is obtained by solving the Dirac relativistic equation as follows[4]:

$$E_e^2 = P_z^2 c^2 + m^2 c^4 \left(1 + (2\nu + 1 + \sigma)\frac{e\hbar B}{m^2 c^3}\right)$$

The critical magnetic field is defined as $B_{cr} = \frac{m^2 c^3}{e\hbar}$ After counting the number of states per unit volume in phase space, we obtain the energy and pressure resulting from the degenerate electron gas in the presence of a magnetic field[5].

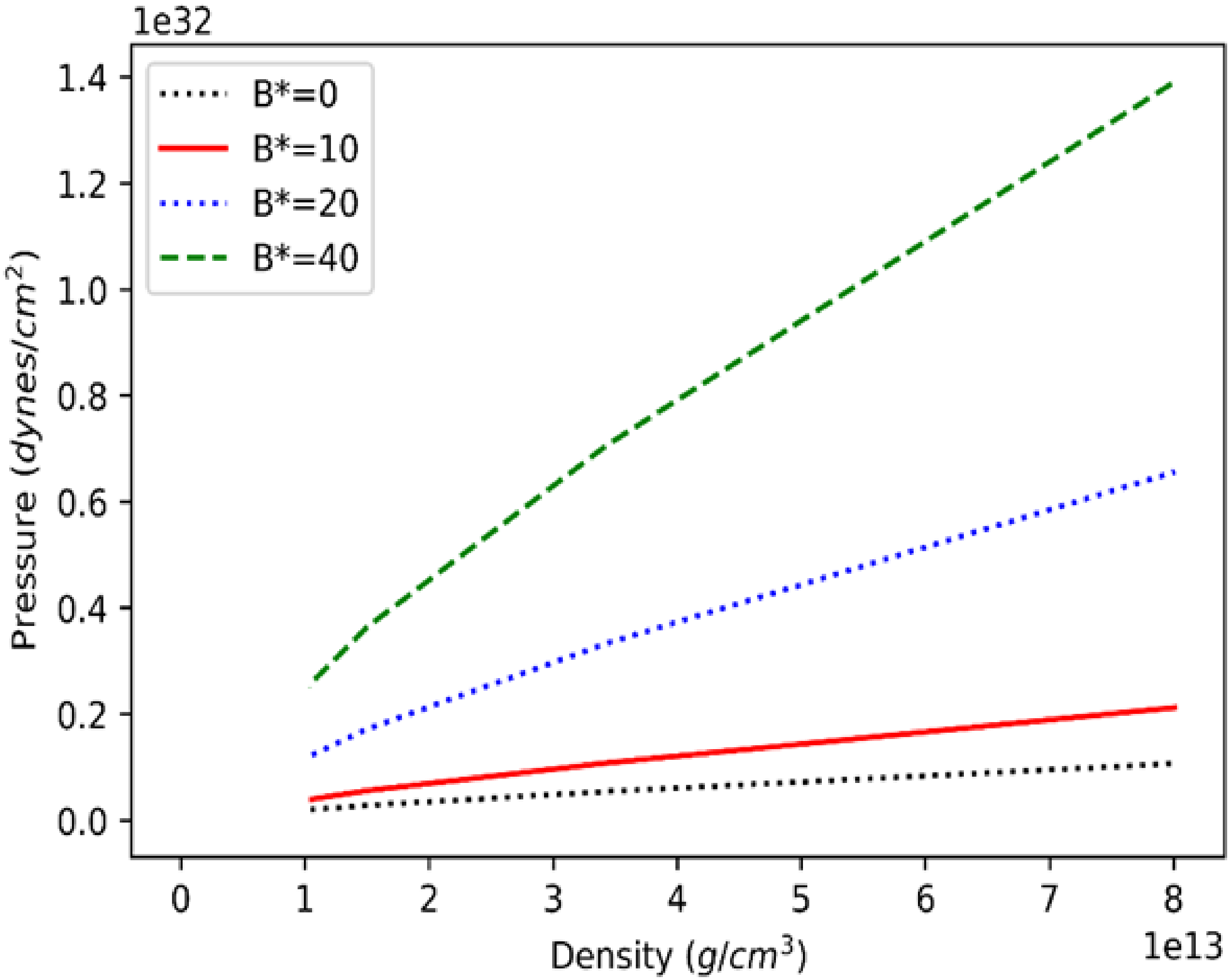
$$\varepsilon_e = \frac{2B^*}{(2\pi)^2\lambda_e^3}mc^2\sum_{\nu=0}^{\nu_m} g_\nu \times (1+2\nu B^*)\Psi\left[\frac{\chi_e(\nu)}{(1+2\nu B^*)^{\frac{1}{2}}}\right]$$

$$P_e = \frac{2B^*}{(2\pi)^2\lambda_e^3}mc^2\sum_{\nu=0}^{\nu_m} g_\nu \times (1+2\nu B^*)\eta\left[\frac{\chi_e(\nu)}{(1+2\nu B^*)^{\frac{1}{2}}}\right]$$

$B^* = \frac{B}{B_{cr}}$, $\chi_e(v) = \frac{P_F^e(v)}{mc}$ and

$$\Psi(x) = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}ln\left(x+\sqrt{1+x^2}\right)$$

$$\eta(x) = \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}ln\left(x+\sqrt{1+x^2}\right)$$



1e32
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Pressure (dynes/cm²)
B*=0
B*=10
B*=20
B*=40
0
1
2
3
4
5
6
7
8
1e13
Density (g/cm³)

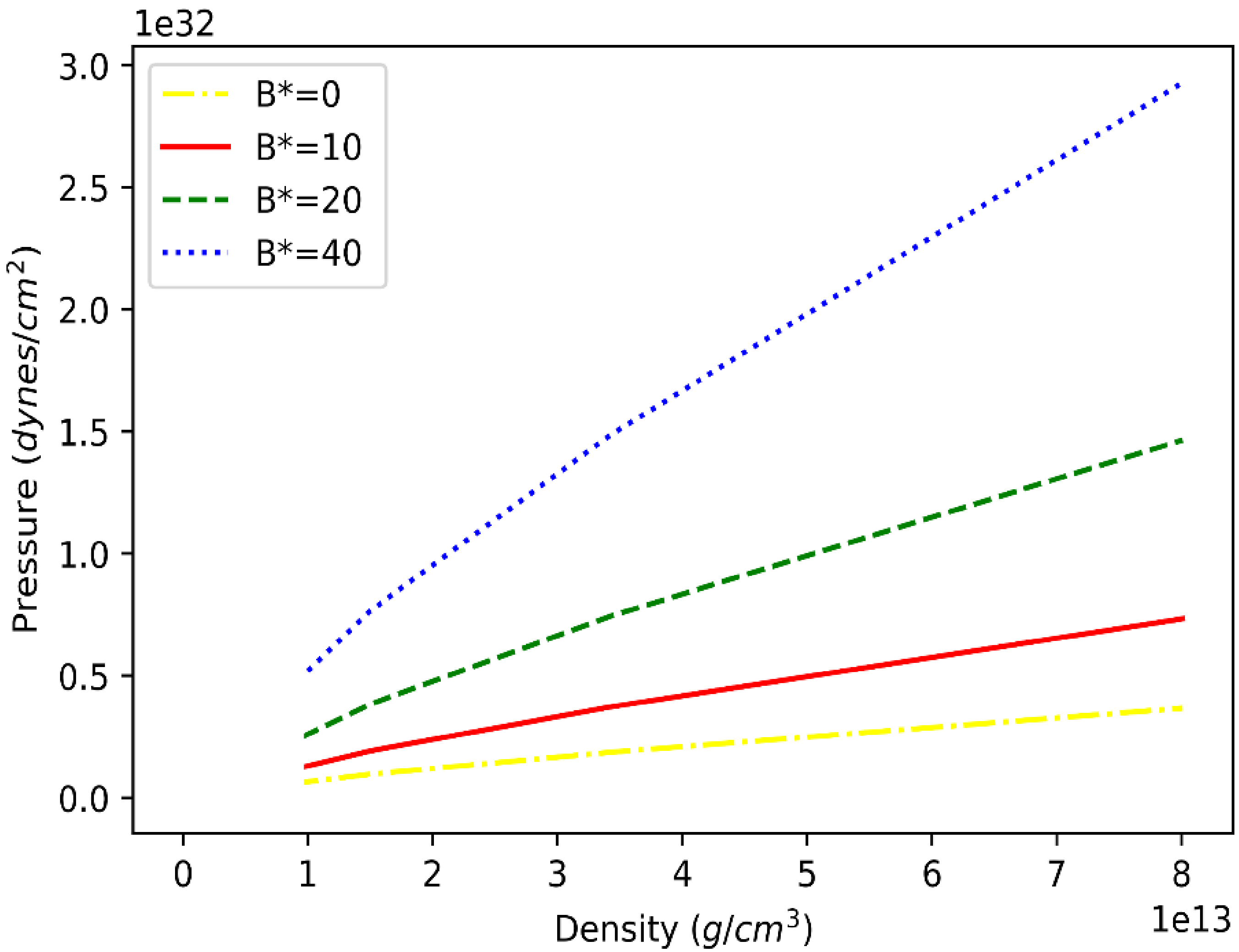
BPS Model

The basis of the BPS model is to consider the coulomb’s energy. In the crust region, we have a lattice of nuclei where electrons have formed an electron gas. Therefore, we should also consider the interactions of e-e and e-p. the lattice pressure is calculated as follows [6]

$$P_L = n_e^2 \frac{d}{dn_e}\left(\frac{E_c}{Z}\right) = -\frac{3}{10}\left(\frac{4\pi}{3}\right)^{\frac{1}{3}} Z^{\frac{2}{3}} e^2 n_e^{\frac{4}{3}}$$

and finally we obtain degenerate electron gas pressure [7] [8]

$$P_e = 6.266 \times 10^{30}\left(\frac{Y_e}{0.05}\frac{\rho}{\rho_0}\frac{B}{B_{cr}}\right)$$


1e32
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Pressure $(dynes/cm^2)$
B*=0
B*=10
B*=20
B*=40
0
1
2
3
4
5
6
7
8
1e13
Density (g/cm^3)

Stellar structure equations

In the end, we use the following equations, which are known as the Tolman-Oppenheimer-Volkoff (TOV) equations, to determine the radius and mass of a neutron star, taking into account the equations of state for both the core and the crust of the star [9].

$$\frac{dP(r)}{dr} = -\frac{G\rho(r)m(r)}{r^2}\left(1 + \frac{P(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 P(r)}{m(r)c^2}\right)\left(1 - \frac{Gm(r)}{rc^2}\right)^{-1}$$

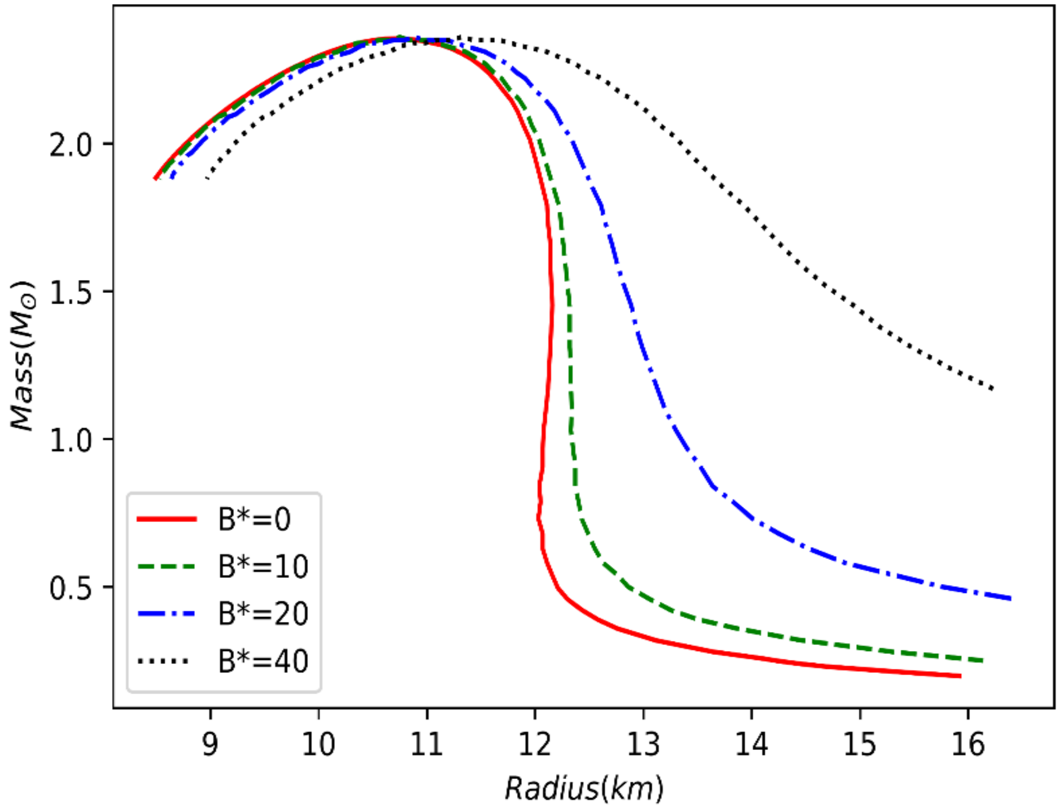
$$\frac{dm(r)}{dr} = \rho(r)4\pi r^2$$

These equations are solved under a set of initial conditions for mass and pressure.

Conclusion

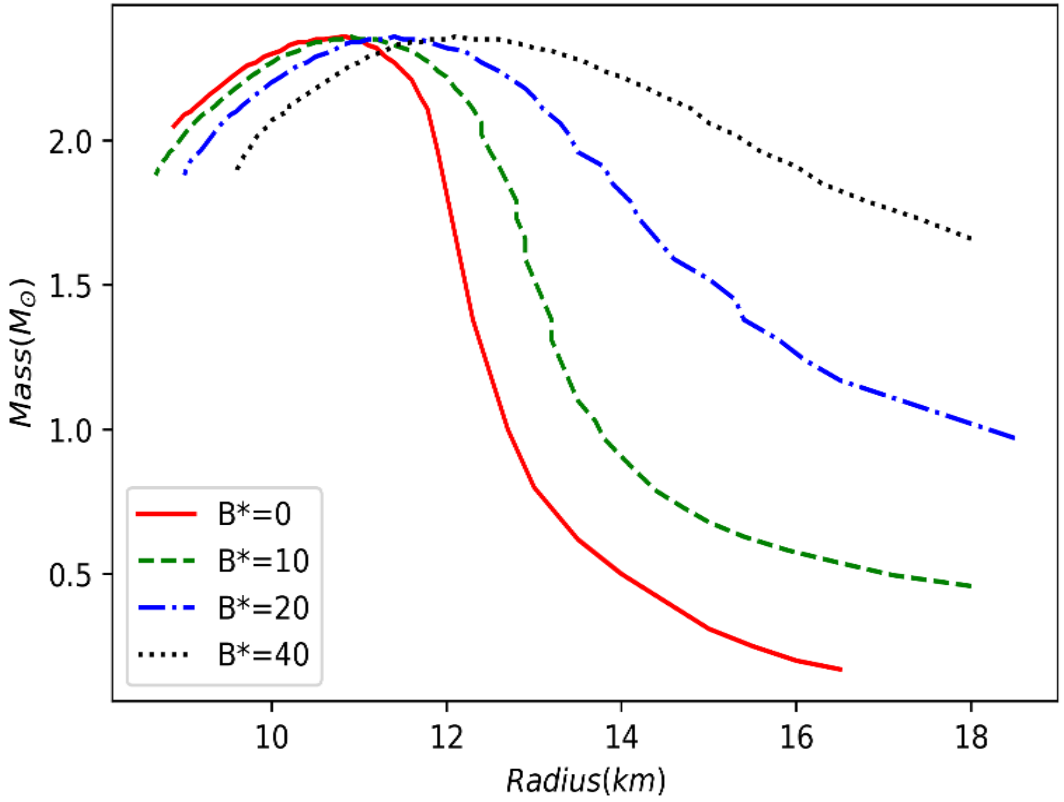
Results for fermi gas model

B^*	Max Mass $(M_\odot)$	Radius (km)	$R_{M=1.4M_\odot}$ (km)
0	2.35	10.71	12.16
10	2.35	10.75	12.32
20	2.35	10.90	12.89
40	2.35	11.32	14.95



Results for fermi gas model

B^*	Max Mass ($M_\odot$)	Radius (km)	$R_{M=1.4M_\odot}$ (km)
0	2.35	10.81	12.37
10	2.35	10.94	13.10
20	2.35	11.38	15.25
40	2.35	12.11	20.21



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Thank
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