Neutrino Oscillation Effects on the Thermodynamic Properties of Hot Electrically Neutral Quark Matter in β-Equilibrium

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INTRODUCTION

Within the framework of the local SU(3) Nambu–Jona-Lasinio (NJL) model, which also takes into account the 't Hooft interaction leading to quark flavor mixing, the influence of neutrino oscillations on the thermodynamic properties of three-flavor hot quark matter in β-equilibrium with trapped neutrinos is studied. For two temperatures T=60 and T=100 MeV, taking into account neutrino oscillations, the thermodynamic characteristics of quark matter are determined.

The end product of the explosion of a supernova star is a star with a central density several times the density in atomic nuclei (a neutron star, hybrid star, quark star), or a black hole.

Explosion time $\tau_G \sim \sqrt{R^3/GM} \sim 1$ sec

Transformation time of {charged baryon + lepton} particle system to uncharged baryon is order of neutron lifetime $\gg \tau_G$.

When the density and temperature in the shrinking core become higher than $10^{12} \div 10^{13}$ g/cm³ and $(6 \div 8)10^{10}K$, then the core matter becomes opaque to the neutrino.

Therefore, during the implosion of the central regions of the pre-supernova star, the relative lepton charge

$$Y_L = \frac{n_e + n_\mu + n_\nu}{n_B}$$

is preserved. Here $n_\nu = n_{\nu_e} + n_{\nu_\mu} + n_{\nu_\tau}$ is the density of the total lepton charge of all the neutrino flavors.

There is every prerequisite to assume that the residual lepton charge $Y_L \approx 0.1 \div 0.45$

S. Wanajo, Y. Sekiguchi, N. Nishimura et al, ApJL **789,** L39, 2014.

A. W. Steiner, M. Prakash, J. M. Lattimer, Phys. Lett. B **509**, 10, 2001, [arXiv:astro-ph/0101566].

E. H. Gudmundson, J. R. Buchler, Astrophys. J., **238**, 717, 1980.

M. Leibendoerfer, Astrophys. J., **633,** 1042, 2005, [arXiv:astro-ph/0504072].

M. Leibendoerfer, S.C. Whitehouse, T. Fischer, Astrophys. J., **698**, 1174, 2009, [arXiv: 0711.2929].

Neutrino oscillations

Experimental observations of neutrino oscillations (Nobel prize in physics, 2015) have initiated many theoretical and experimental studies on neutrino physics. Even as long ago as 1957 Pontecorvo predicted the phenomenon of neutrino oscillations arising from the possible presence of mass in neutrinos.

B. M. Pontecorvo, ZhETF **33**, 549 (1957), (Sov. Phys. JETP **6**, 429 (1958)).

B .M. Pontecorvo, ZhETF **34**, 247 (1957), (Sov. Phys. JETP **7**, 172 (1958)).

In the absence of neutrino oscillations the chemical potentials of the particles of quark matter are related by the formulas

$$\mu_d = \mu_u + \mu_e - \mu_{\nu_e},$$
$$\mu_s = \mu_u + \mu_e - \mu_{\nu_e},$$
$$\mu_e - \mu_{\nu_e} = \mu_\mu - \mu_{\nu_\mu} = \mu_\tau - \mu_{\nu_\tau}.$$

Description of hot quark matter with neutrino confinement in NJL model:

G. S. Hajyan, G. B. Alaverdyan, Astrophysics, ***64****, 370, 2021.*
https://doi.org/10.1007/s10511-021-09696-x

G.S.Hajyan, G.B Alaverdyan, Astrophysics, ***65****, 126, 2022,*
https://doi.org/10.1007/s10511-022-09726-2

Including the phenomenon of neutrino oscillations leads to equality of the chemical potentials of the neutrinos of all flavors, and the relationship between the chemical potentials of the particles takes the form

$$\mu_d = \mu_u + \mu_e - \mu_\nu,$$
$$\mu_s = \mu_u + \mu_e - \mu_\nu,$$
$$\mu_e = \mu_\mu.$$

In fact, from the standpoint of thermodynamics, neutrino oscillations erase all the differences between the different neutrino flavors; formally they all act as neutrinos with a degeneracy factor of g=3.

Description of hot quark matter with neutrino confinement in MIT bag model:

*G.S.Hajyan, A.G. Alaverdyan, Astrophysics, **57**, 559, 2014.*

*G. Hajyan, Particles, **4**, 37, 2021.*

NJL Model Lagrangian density

Dirac Lagrangian density

Four-quark interaction term

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - \hat{m}_0)\psi + G\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2]$$

$$-K\{det_f(\bar{\psi}(1+\gamma_5)\psi) + det_f(\bar{\psi}(1-\gamma_5)\psi)\}$$

Six-quark Kobayashi-Maskawa-'t Hooft interaction term

$\psi_f^c \quad f = u, d, s \qquad c = r, g, b$

$\hat{m}_0 = diag(m_{0u}, m_{0d}, m_{0s})$

$\lambda_a (a = 1,2, \ldots 8)$ Gell-Mann matrices, SU(3) generators

$\lambda_0 = \sqrt{2/3}\,\hat{I}$

Hot Quark-Lepton plasma $u, d, s, e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$

$$\Omega_{QP} = \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^\Lambda dk\, k^2 \left(E_f(k, M_{f0}) - E_f(k, M_f)\right)$$

$$- \frac{3T}{\pi^2} \sum_{f=u,d,s} \left\{ \int_0^\Lambda dk\, k^2 \left[ln\left(1 + e^{-\frac{E_f(k,M_f)-\mu_f}{T}}\right) + ln\left(1 + e^{-\frac{E_f(k,M_f)+\mu_f}{T}}\right)\right]\right\}$$

$$+2G({\sigma_u}^2 + {\sigma_d}^2 + {\sigma_s}^2 - {\sigma_{u0}}^2 - {\sigma_{d0}}^2 - {\sigma_{s0}}^2) - 4K\,(\sigma_u\sigma_d\sigma_s - \sigma_{u0}\sigma_{d0}\sigma_{s0})$$

$$- \frac{T}{2\pi^2} \sum_l g_l \int_0^\infty dk\, k^2 \left[ln\left(1 + e^{-\frac{E_l(k)-\mu_l}{T}}\right) + ln\left(1 + e^{-\frac{E_l(k)+\mu_l}{T}}\right)\right]$$

$$E_f(k, M_f) = \sqrt{k^2 + M_f^2} \qquad E_l(k) = \sqrt{k^2 + m_l^2}$$

M_f - Constituent Masses of Quarks

$g_e = g_\mu = 2 \qquad g_{\nu_e} = g_{\nu_\mu} = 1$

Quark Condensates & Constituent Quark Masses

$$\sigma_f(T, M_f, \mu_f) = \langle \bar{\psi}_f \psi_f \rangle =$$

$$= -\frac{3}{\pi^2} M_f \int_0^{\Lambda} dk \, \frac{k^2}{E_f(k, M_f)} \left[1 - \frac{1}{1 + e^{\frac{E_f(k, M_f) - \mu_f}{T}}} - \frac{1}{1 + e^{\frac{E_f(k, M_f) + \mu_f}{T}}} \right].$$

$$M_u = m_{0u} - 4G\,\sigma_u + 2K\sigma_d\sigma_s \,,$$

$$M_d = m_{0d} - 4G\,\sigma_d + 2K\sigma_s\sigma_u \,,$$

$$M_s = m_{0s} - 4G\,\sigma_s + 2K\sigma_u\sigma_d \,.$$

Number Densities of Particles

$$n_f(T, M_f, \mu_f) = \frac{3}{\pi^2} \int_0^\Lambda dk\, k^2 \left[\frac{1}{1 + e^{\frac{E_f(k, M_f) - \mu_f}{T}}} - \frac{1}{1 + e^{\frac{E_f(k, M_f) + \mu_f}{T}}} \right] \qquad f = u, d, s$$

$$n_l(T, \mu_l) = -\frac{\partial \Omega_{QP}}{\partial \mu_l} = \frac{g_l}{2\pi^2} \int_0^\infty dk\, k^2 \left[\frac{1}{1 + e^{\frac{E_l(k) - \mu_l}{T}}} - \frac{1}{1 + e^{\frac{E_l(k) + \mu_l}{T}}} \right] \qquad l = e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$$

Energy Density

$$\varepsilon_{QP} = \frac{3}{\pi^2}\sum_{f=u,d,s}\left\{\int_0^{\Lambda} dk\, k^2\left[E_f(k,M_{f0}) - E_f(k,M_f)\left(1 - \frac{1}{1+e^{\frac{E_f(k,M_f)-\mu_f}{T}}} - \frac{1}{1+e^{\frac{E_f(k,M_f)+\mu_f}{T}}}\right)\right]\right\}$$

$$+2G({\sigma_u}^2 + {\sigma_d}^2 + {\sigma_s}^2 - {\sigma_{u0}}^2 - {\sigma_{d0}}^2 - {\sigma_{s0}}^2) - 4K\,(\sigma_u\sigma_d\sigma_s - \sigma_{u0}\sigma_{d0}\sigma_{s0})$$

$$+\frac{1}{2\pi^2}\sum_l g_l \int_0^{\infty} dk\, k^2\, E_l(k)\left[\frac{1}{1+e^{\frac{E_l(k)-\mu_l}{T}}} + \frac{1}{1+e^{\frac{E_l(k)+\mu_l}{T}}}\right]$$

Pressure

$$P_{QP} = \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^{\Lambda} dk\, k^2 \left(E_f(k, M_f) - E_f(k, M_{f0}) \right)$$

$$+ \frac{3T}{\pi^2} \sum_{f=u,d,s} \left\{ \int_0^{\Lambda} dk\, k^2 \left[ln\left(1 + e^{-\frac{E_f(k,M_f)-\mu_f}{T}}\right) + ln\left(1 + e^{-\frac{E_f(k,M_f)+\mu_f}{T}}\right) \right] \right\}$$

$$-2G({\sigma_u}^2 + {\sigma_d}^2 + {\sigma_s}^2 - {\sigma_{u0}}^2 - {\sigma_{d0}}^2 - {\sigma_{s0}}^2) + 4K\, (\sigma_u \sigma_d \sigma_s - \sigma_{u0}\sigma_{d0}\sigma_{s0})$$

$$+ \frac{T}{2\pi^2} \sum_l g_l \int_0^{\infty} dk\, k^2 \left[ln\left(1 + e^{-\frac{E_l(k)-\mu_l}{T}}\right) + ln\left(1 + e^{-\frac{E_l(k)+\mu_l}{T}}\right) \right]$$

Entropy Density

$$S_{QP}=\frac{3}{\pi^2}\sum_{f=u,d,s}\left\{\int_0^{\Lambda}dk\,k^2\left[ln\left(1+e^{-\frac{E_f(k,M_f)-\mu_f}{T}}\right)+ln\left(1+e^{-\frac{E_f(k,M_f)+\mu_f}{T}}\right)\right]\right\}$$

$$+\frac{3}{\pi^2 T}\sum_{f=u,d,s}\int_0^{\Lambda}dk\,k^2\,E_f(k,M_f)\left[\frac{1}{1+e^{\frac{E_f(k,M_f)-\mu_f}{T}}}+\frac{1}{1+e^{\frac{E_f(k,M_f)+\mu_f}{T}}}\right]-\frac{1}{T}\sum_{f=u,d,s}\mu_f n_f$$

$$+\sum_l\left\{\frac{g_l}{2\pi^2}\int_0^{\infty}dk\,k^2\left[ln\left(1+e^{-\frac{E_l(k)-\mu_l}{T}}\right)+ln\left(1+e^{-\frac{E_l(k)+\mu_l}{T}}\right)\right]-\frac{1}{T}\mu_l n_l\right\}$$

β – Equilibrium

$$\mu_d = \mu_u + \mu_e - \mu_\nu$$

$$\mu_s = \mu_u + \mu_e - \mu_\nu$$

$$\mu_e = \mu_\mu$$

Lepton Charge

$$Y_L = \frac{n_e + n_\mu + n_\nu}{n_B}$$

Electrical Neutrality

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu - n_\tau = 0$$

Baryonic Number Density

$$n_B = \frac{1}{3}(n_u + n_d + n_s)$$

Numerical Calculation Results

$$m_{0u} = m_{0d} = 5.5 \text{ МэВ}, \ m_{0s} = 140.7 \text{ МэВ},$$

$$\Lambda = 602.3 \text{ МэВ}, \ G = 1.835/\Lambda^2, \ K = 12.36/\Lambda^5.$$

P. Rehberg, S.P. Klevansky, J. Hüfner, Phys. Rev. C, **53**, 410, 1996.

Numerical calculations are carried out for the two values of temperature T = (60; 100) Mev and relative lepton charge $Y_L = (0.1; 0.4)$.

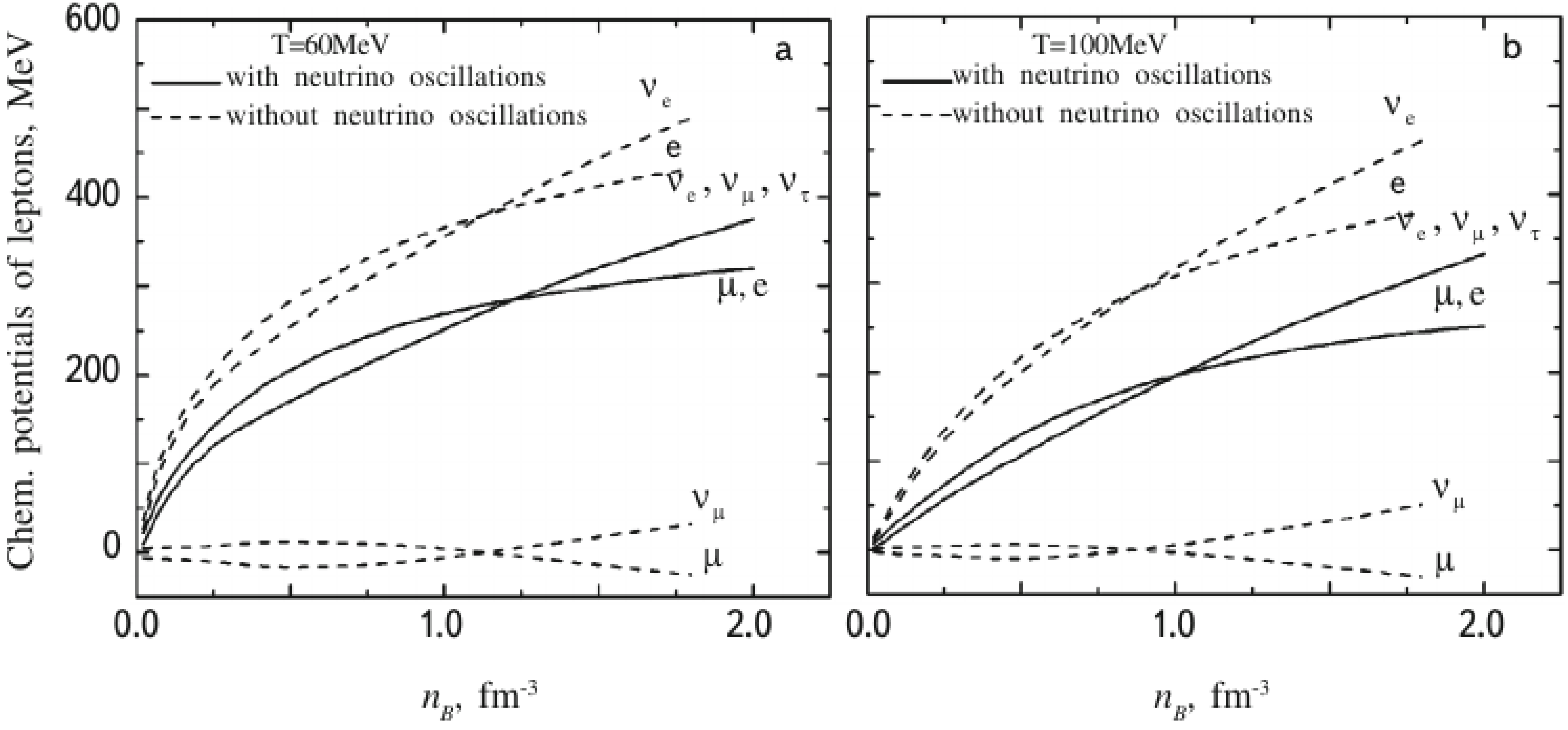

Fig. 1. Chemical potentials of leptons $\mu_e, \mu_\mu, \mu_{\nu_e}, \mu_{\nu_\mu}, \mu_{\nu_\tau}$ as functions of the baryon number density n_B for two values of the temperature: (a) $T=60$MeV; (b) $T=100$MeV. The continuous curves correspond to the case when mixing (oscillations) of neutrinos occur, the dashed curves , to the absence of neutrino oscillations.

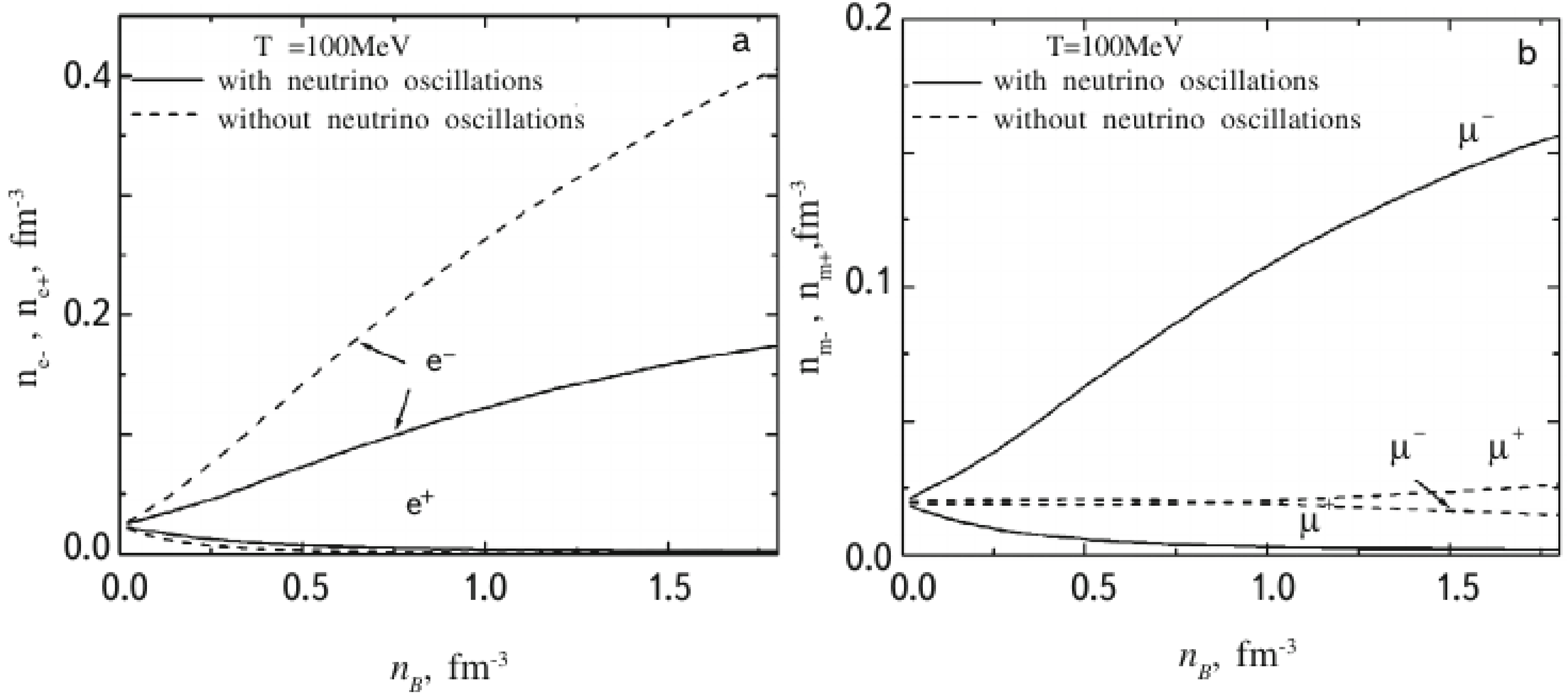

Fig. 2. The concentration of charged leptons: (a) electrons and positrons, (b) muons and antimuons as functions of baryon number density n_B for temperature $T = 100$ MeV. The smooth curves correspond to the case when neutrino mixing (oscillations) occurs and the dotted curves, to the case of a lack of neutrino oscillations.

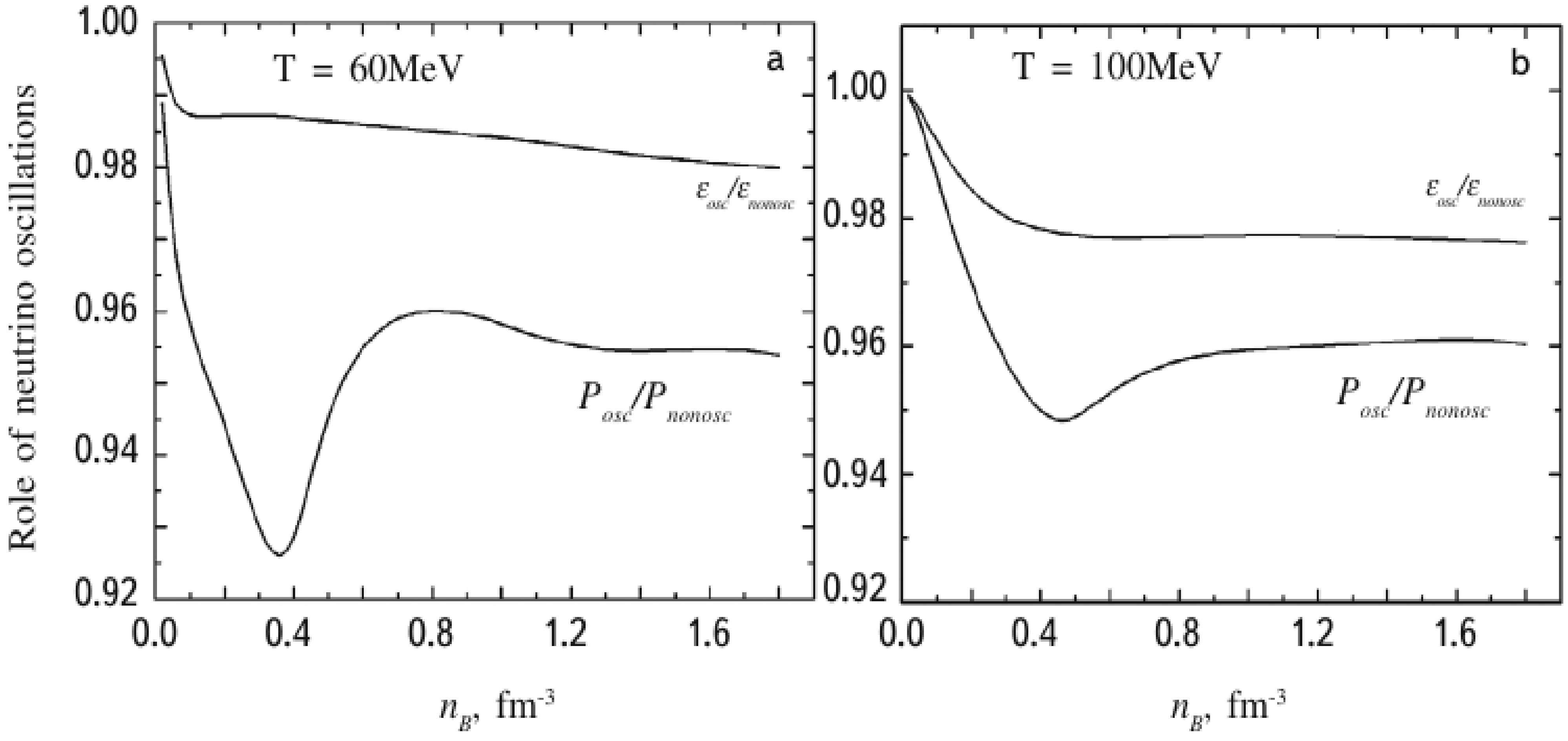

Fig. 3. The ratios of the pressures P_{osc}/P_{nonosc} and energy densities $\varepsilon_{osc}/\varepsilon_{nonosc}$ as functions of the baryon number density n_B for two temperatures: (a) T = 60 MeV, (b) T=100 MeV.

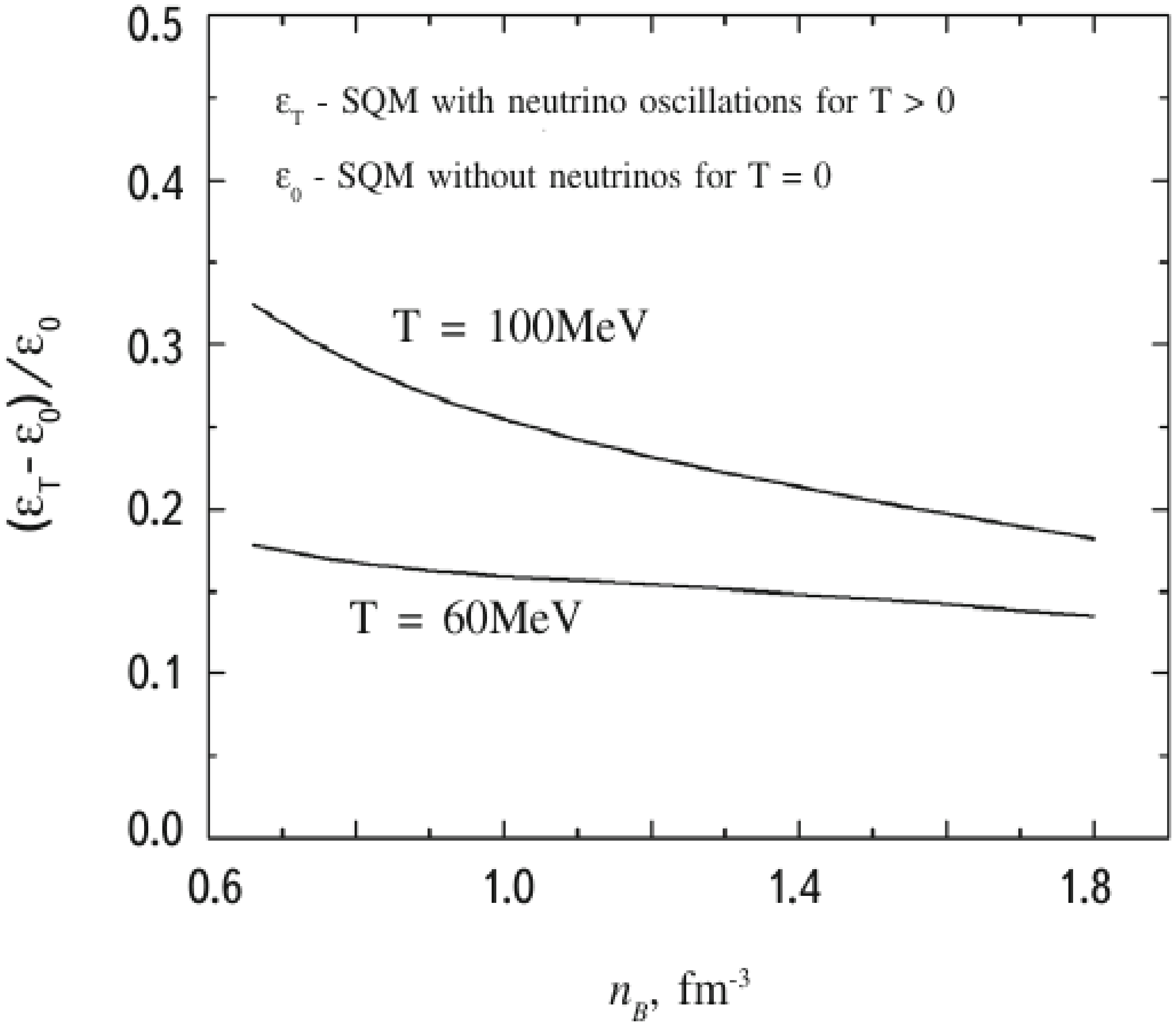

Fig. 4. Normalized difference of the energies of HSQM that is opaque to neutrinos with mixing (oscillations) of neutrinos and transparent for neutrinos in cold quark matter, after emergence of all the neutrinos depending on the baryonic charge density n_B for two values of the temperature T = 60MeV and T = 100MeV.

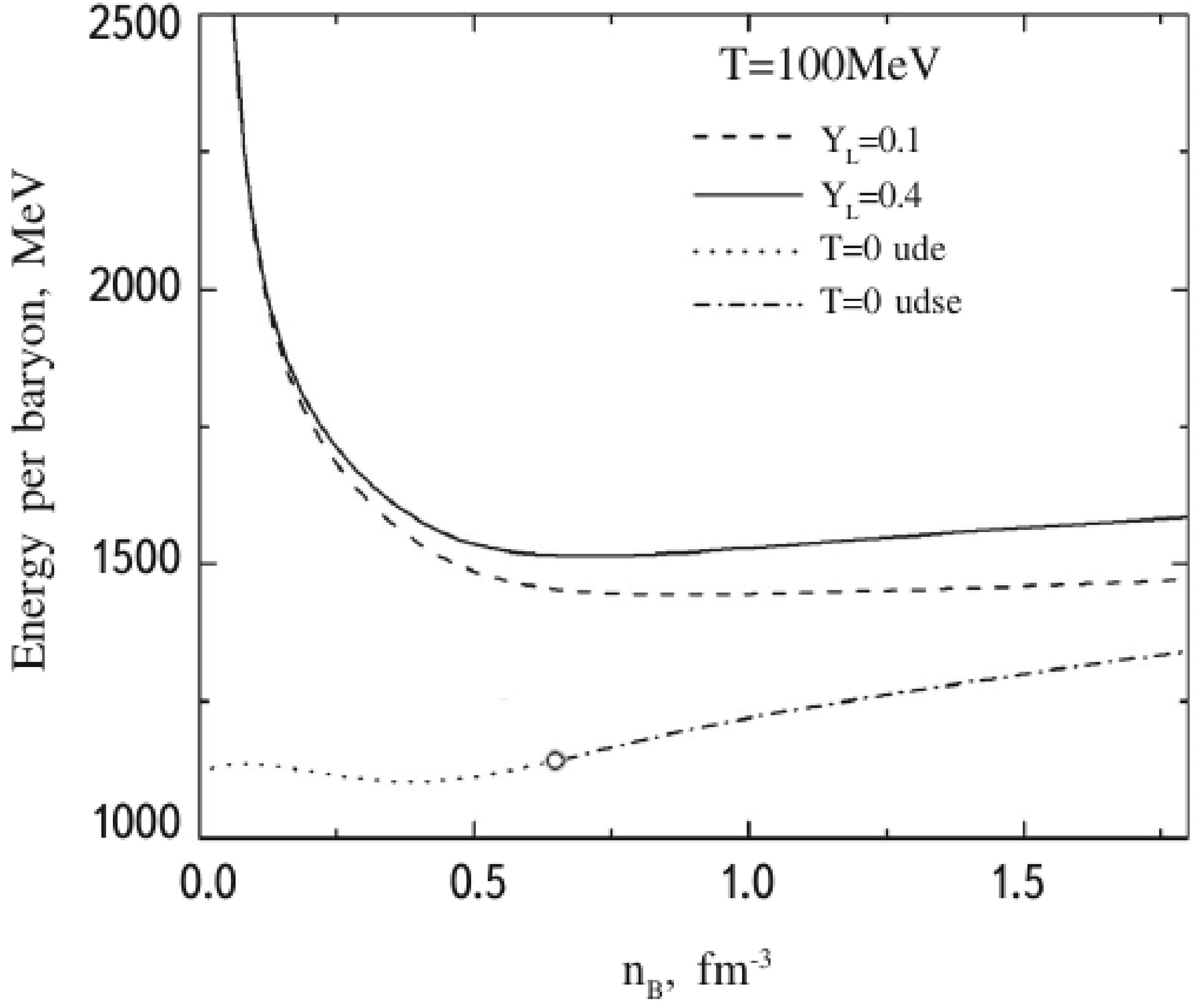

Fig. 5. The energy $E_1 = \varepsilon / n_B$ per unit baryon charge in HSQM that is opaque for neutrinos with neutrino oscillations at a temperature of T=100MeV as a function of the baryon number density n_B for two values of the specific lepton charge Y_L=0.1 and Y_L=0.4. For comparison the analogous curve is also shown for quark matter at zero temperature, after which, all the neutrinos will have already left the system. The circle denotes the threshold for creation of a quark in β equilibrium cold quark matter.

CONCLUSIONS

1. Confinement of neutrinos in the matter of the quark core of a proto-neutron star can provide stored energy up to 250 MeV per baryon.

2. Due to neutrino oscillations, the energy and pressure of the hot quark matter will decrease by about 2 and 4 percent, respectively.

3. The neutrinos leaving the star will have an energy of 200-350 MeV energy. The interaction of such high-energy neutrinos with external non-degenerate layers will be much stronger than at an energy of 10-50 MeV, when the neutrino is not retained by matter.


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Thank you