Gravitational Waves from Isolated Magnetized Neutron Star

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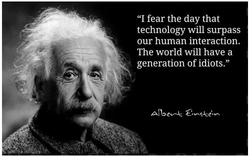
The Long Road Towards Evidence

Albert Einstein (1916)



Joseph Weber (1960)

Hulse and Taylor (1993 Nobel Prize)

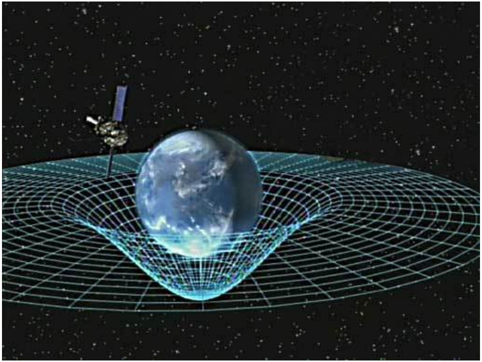
Indirect detection of gravitational waves

Weiss, Thorne and Barish (2017 Nobel Prize)

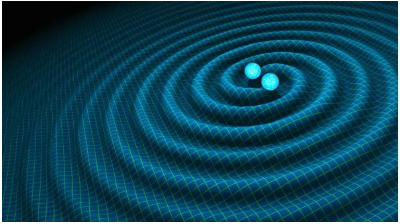
More than four decades of their effort led to gravitational waves finally being observed.

What are Gravitational Waves?

Space-Time is a deformable medium. Mass and Energy deform space-time around them and inversely they follow the deformed paths inside it.



www.ligo.caltech.edu



Only extremely violent phenomena can produce detectable gravitational waves.

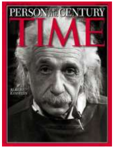
www.livescience.com

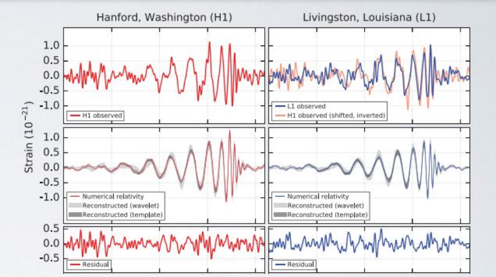
September 14th 2015: first Gravitational Waves detection! (GW150914)

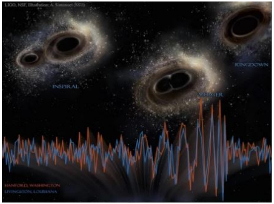
Selected for a Viewpoint in *Physics*
PRL **116**, 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)



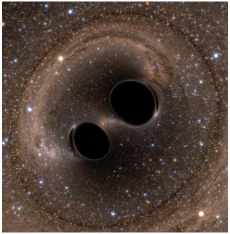




I was right!

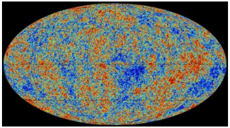
Abbott et al. '2016

GW astrophysical sources



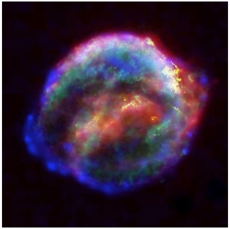
Coalescing Binary Systems

1. BH – BH
2. NS – NS
3. BH-NS



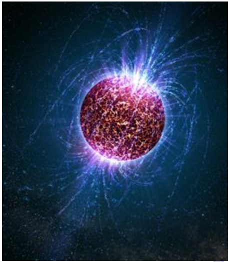
Cosmic GW Background

1. residue of the Big Bang,
2. stochastic, incoherent background



Transient Burst Sources

1. Core Collapse Supernovae
2. cosmic strings

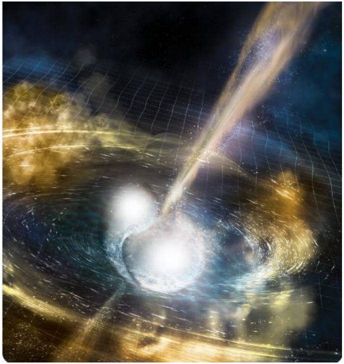


Continuous Sources

1. Spinning neutron stars

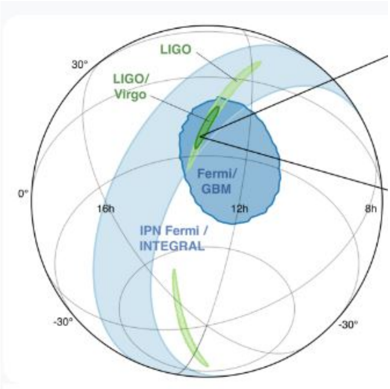
www.ligo.caltech.edu

Multimessenger Astronomy with Gravitational Waves



Artist's illustration of two merging neutron stars. The rippling spacetime grid represents gravitational waves that travel out from the collision, while the narrow beams show the bursts of gamma rays that are shot out just seconds after the gravitational waves.

Credit: NSF, LIGO, Sonoma State University and A. Simonnet



The localization of GW170817 by LIGO, Virgo and the Fermi/Integral satellites

Fundamental Questions Addressed through GW Observations

1. What is the physics of stellar core collapse?

2. What is the equation of state of neutron stars?

3. What are the multi-messenger emission mechanisms of high-energy transients (gamma-ray bursts)?

3. How do BBHs of tens of solar masses form and evolve?

4. How did super-massive black holes at the cores of galactic nuclei form?

5. Are black holes, neutron stars and white dwarfs the only compact objects in our Universe, or are there even more exotic objects?

6. How does gravity behave in the strong/highly dynamical regime?

7. Are black hole spacetimes as predicted by general relativity?

8. Are there any signatures of horizon structure or other manifestations of quantum gravity accessible to gravitational-wave observations?

9. Is dark matter composed, in part, of primordial black holes, or must it be composed solely from exotic matter such as axions or dark fermions?

10. What is the expansion rate of the Universe?

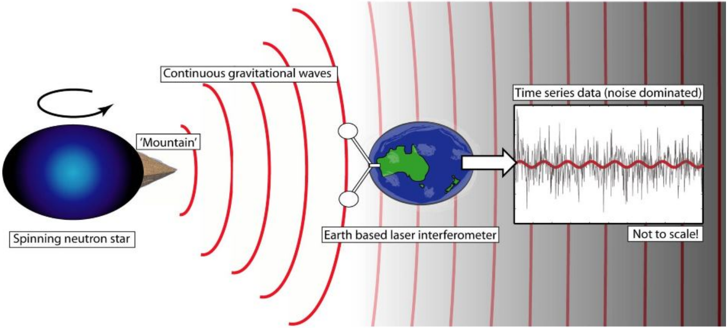
11. What is the nature of dark energy?

12. Do we live in a Universe with large extra dimensions?

13. Is there a measurable gravitational-wave stochastic background due to phase transitions in the early Universe? If so, what were its properties?

Continuous Gravitational Waves

Continuous gravitational waves are produced by systems that have a constant and well-defined frequency.

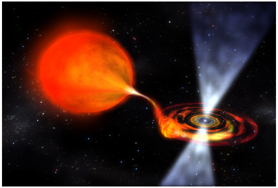


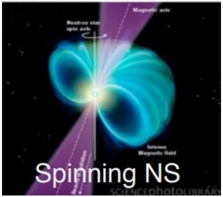
The sound these gravitational waves would produce is a continuous tone since their frequency is nearly constant.

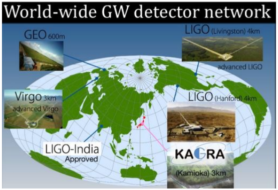
These sources are expected to produce comparatively weak gravitational waves since they evolve over longer periods of time and are usually less catastrophic than sources producing inspiral or burst gravitational waves.

Why CGWs detection is important?

Among the uppermost priorities of the gravitational-wave community, undetected continuous gravitational waves stand out, the detection of which is crucial for the comprehension of matter at extreme supranuclear densities and highly relativistic regimes. These signals are typically emitted by non-axisymmetric and rapidly rotating neutron stars.



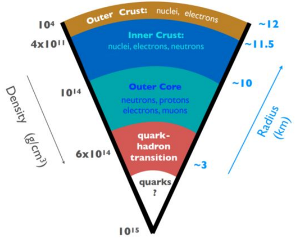




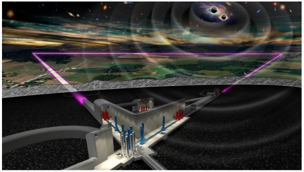
In practice, the most important LIGO search targets for continuous gravitational waves are neutron stars in our own Galaxy. Once we reach sufficient detector sensitivity to find such a signal, we expect we can observe it continuously for many months or years.

CGW signals: What will a Detections Tell Us?

1. NS internal structure (EoS, viscosity, superfluidity)
2. Maximum spin allowed for a NS
3. Intensity and configuration of interior magnetic field
4. Accretion physics
5. Possible distance estimation
6. moment of inertia estimation
7. Testing GR



Neutron star structure

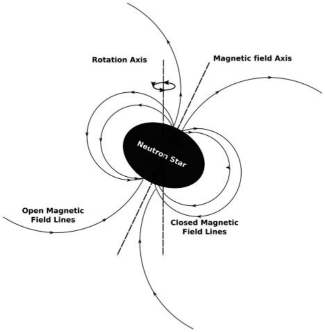


The proposed Einstein Telescope gravitational wave observatory.

www.aei.mpg.de/einsteintelescope

CGWs constrain the NS parameters, like its ellipticity, the internal magnetic field and its EOS.

Deformatoin of Magnetic Rotating NSs



A representation of a neutron star, with its rotation and magnetic field axes misaligned with respect to each other.

Chandra et al. '2020

total quadrupole moment

Magnetic fields are known to cause a star to become oblate or prolate, depending on the field configuration (Chandrasekhar and Fermi '1953; Ferraro '1954). This generates a quadrupole moment and associated quadrupole ellipticity. In cases where the rotation and magnetic axes do not coincide, this opens up the possibility of generating continuous gravitational waves.

As far as the non-axisymmetric deformation of neutron stars are very tiny, the total quadrupole moment can be linearly decomposed into the sum of two pieces(Gourgoulhon et al. '1996):

$$Q_{ij} = Q_{ij}^{rot} + Q_{ij}^{dist}$$

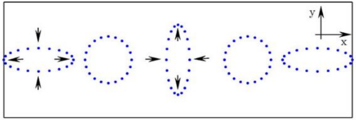
Q_{ij}^{rot} is the quadrupole moment due to rotation and Q_{ij}^{dist} is the quadrupole moment due to the process that distorts the star, for example an internal magnetic field.

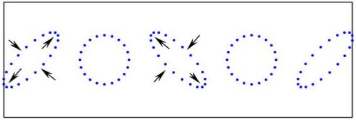
A General Formula for Gravitatinal Emission by a Magnetic Rotating Star

quadrupole formula

$$\left[h_{ij}^{TT}(t,\vec{x})\right]_{quad} = \frac{4G}{rc^4}\ddot{Q}_{ij}(t-\frac{r}{c})$$

Q is the mass quadrupole moment of the object.





gravitational wave polarizations

The two modes h_+ and $h_\times$ of the gravitational radiation field in a TT gauge are,

$$h_+ = h_0 \sin\alpha \left[\frac{1}{2}\cos\alpha \sin i \cos i \cos\Omega t - \sin\alpha \frac{1+\cos^2 i}{2}\cos 2\Omega t\right]$$

$$h_\times = h_0 \sin\alpha \left[\frac{1}{2}\cos\alpha \sin i \sin\Omega t - \sin\alpha \sin i \cos 2\Omega t\right]$$

where i is the inclination angle of the "line of sight" with respect to the rotation axis and,

$$h_0 = \frac{16\pi^2 G}{rc^4}\frac{I\epsilon}{p^2}$$

where r is the distance of the star, P is the rotation period of the star, I is its moment of inertia with respect of the rotation axis and ϵ is the ellipticity resulting from the distortion process.

Relativistic Calculation of Deformation

The deformation of magnetized rotating neutron stars can be expressed by the second-order quantities with respect to the rotation rate or the magnetic field strength. The line element can be written in the form,

$$ds^2 = -e^{\nu(r)}\{1 + 2[h_0(r) + h_2(r)P_2(cos\theta)]\}dt^2 + e^{\lambda(r)}\left\{1 + \frac{2e^{\lambda(r)}}{r}[m_0(r) + m_2(r)P_2(cos\theta)]\right\}dr^2$$
$$+ r^2[1 + 2k_2(r)P_2(cos\theta)](d\theta^2 + sin^2\theta(d\phi - \omega(r)dt)^2)$$

where $P_2(cos\theta)$ is the Legendre polynomial of order 2 and h_0, h_2, m_0, m_2 and k_2 are corrections which correspond to deviation from spherical shape. The quantity ω is the angular velocity of the local inertial frame acquired by an observer falling freely from infinity to a point r, and is equal to zero for the static magnetic field deformation. (Hartle and Thorne '1968)

The stress-energy tensor of the perfect fluid body is described by,

$$T_m^{\mu\nu} = (\varepsilon_m + p_m)u^\mu u^\nu + p_m g^{\mu\nu}$$

When we consider the magnetic field deformation, we further take into account the stress-energy tensor arising from the magnetic field,

$$T_f^{\mu\nu} = \frac{1}{4\pi}F^{\mu\lambda}F_\lambda^\nu - \frac{1}{16\pi}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}$$

Magnetic Effect on Star Deformation

Einstein-Maxwell equations

$$\frac{dm_0}{dr} = 4\pi r^2 p_0,$$

$$\frac{dh_0}{dr} = 4\pi r e^{\lambda} p_0 + \left(\frac{\frac{d\nu}{dr}}{r} + \frac{1}{r^2} \right) e^{\lambda} m_0,$$

$$h_2 + \frac{e^{\lambda}}{r} m_2 = 0,$$

$$\frac{dk_2}{dr} = \frac{2p_2 \frac{d\nu}{dr} + \frac{dp_2}{dr}}{\varepsilon_m + p_m},$$

$$\frac{dh_2}{dr} = \frac{-p_2 \frac{d\nu}{dr} - \frac{dp_2}{dr}}{\varepsilon_m + p_m}.$$

magnetic ellipticity

Here, the total pressure, p, can be expanded in spherical harmonics as,

$$p = p_m + [p_0 + p_2 P_2(cos\theta)]$$

where $p_0 = \frac{1}{3}(\frac{B^2}{8\pi})$ and $p_2 = -\frac{4}{3}(\frac{B^2}{8\pi})$ are the monopole and quadrupole contribution of the magnetic pressure,respectively.

The ellipticity of the relativistic star can be calculated from the definition,

$$\epsilon_B = -\frac{3}{2}\left(\frac{\xi_2}{r} + k_2 \right)$$

where ξ_2 represents the displacement of quadrupole deformation.

$$\xi_2(r) = \frac{r^2 \left[1 - \frac{2m(r)}{r}\right]}{\left[4\pi r^3 p_m + m(r)\right]} h_2.$$

(Zamani and Bigdeli '2019)

Magnetic Field Distribution

magnetic field profile

we assume that the magnetic field is density-dependent, (Bandyopadhyay et al. 1997)

$$B = B_{surf} + B_{cent}\left[1 - exp(-\beta(\frac{\rho}{\rho_0})^{\gamma})\right]$$

where the parameters are chosen to be $\beta = 0.01$ and $\gamma = 3$. The surface and central magnetic field is considered to be $B_{surf} = \alpha B_c^e$ and $B_{cent} = \alpha \times 10^4 B_c^e$, where α is a free parameter to obtain the effects of different magnetic fields.

maximum magnetic field strength of stable neutron star

Based on different equations of state, we an find a critical value for α. These values corresponds to maximum magnetic field strength (B_{max}) in the interior of stable neutron star based on our results for magnetized neutron matter.

We take two differet critical values for α:

1. $\alpha = 7.3 \Longrightarrow \mathrm{B}_{max} \simeq 3.2 \times 10^{18} G$
2. $\alpha = 11.06 \Longrightarrow \mathrm{B}_{max} \simeq 4.8 \times 10^{18} G$

(Zamani and Bigdeli '2019)

Rotational Effect on Star Deformation

Hydrostatic Equilibrium Condition

$$\frac{dm_0}{dr} = 4\pi r^2 \frac{dE}{dP}(E+P)p_0^* + \frac{1}{12}j^2 r^4 \left(\frac{d\varpi}{dr}\right)^2 - \frac{1}{3}r^3 \frac{dj^2}{dr}\varpi^2$$

$$\frac{dp_0^*}{dr} = -\frac{m_0(1+8\pi r^2 P)}{(r-2M)^2} - \frac{4\pi(E+P)r^2}{(r-2M)}p_0^* + \frac{1}{12}\frac{r^4 j^2}{(r-2M)}\left(\frac{d\varpi}{dr}\right)^2 + \frac{1}{3}\frac{d}{dr}\left(\frac{r^3 j^2 \varpi^2}{r-2M}\right)$$

$$\frac{dv_2}{dr} = -\frac{d\nu}{dr}h_2 + \left(\frac{1}{r} + \frac{1}{2}\frac{d\nu}{dr}\right)\left[-\frac{1}{3}r^3\frac{dj^2}{dr}\varpi^2 + \frac{1}{6}j^2 r^4\left(\frac{d\varpi}{dr}\right)^2\right]$$

$$\frac{dv_2}{dr} = \left\{-\frac{d\nu}{dr} + \frac{r}{r-2M}\left(\frac{d\nu}{dr}\right)^{-1}\left[8\pi(E+P) - \frac{4M}{r^3}\right]\right\}$$

where $\varpi \equiv \Omega - \omega$ is the angular velocity of the fluid relative to the local inertial frame and $k_2 = v_2 - h_2$.

Rotational Ellipticity

In equilibrium, a rotating star attains a balance between pressure forces, gravitational forces, and centrifugal forces. The magnitude of the centrifugal force is determined by its angular velocity relative to the local inertial frame, $\varpi(r)$. This quantity is of first order in Ω .

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j \frac{d\varpi}{dr}\right) + \frac{4}{r}\frac{dj}{dr}\varpi = 0$$

And the ellipticity of the slowly rotating star is expressed as,

$$\epsilon_\Omega = \frac{3}{r\nu\prime} + \frac{r}{\nu\prime}e^{-\nu}(\Omega-\omega)^2 - \frac{3}{2}k_2$$

where $\frac{d\nu}{dr} = -\frac{2}{\varepsilon_m + p_m}\frac{dp}{dr}$.

How about the EoS?

LOCV method

Maximum allowed asymmetry of a neuton star depends on its equation of state.
Here, we employ Lowest Order Constraint Variational (LOCV) method in addition to the modern AV_{18} two-body potential to compute the EoS of magnetized neutron matter.
Then we add Three Nucleon Interactions (TNI) to the AV_{18} potenial.

1. $AV_{18} \Longrightarrow \alpha = 7.3$
2. $AV_{18} + TNI \Longrightarrow \alpha = 11.06$

(Zamani and Bigdeli '2019)

LOCV method

In the LOCV method, we consider a trial many-body wave function of the form $\psi = \mathcal{F}\phi$, in which ϕ is Slater determinant of the plane waves and $\mathcal{F} = \mathcal{F}(1 \cdots N)$ is a proper N-body correlation operator which can be replaced by a Jastrow form i.e.,

$$\mathcal{F} = \mathcal{S} \prod_{i>j} f(ij),$$

E is the total energy per particle of spin polarized neutron matter:

$$E = \frac{1}{N} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2 \cdot$$

We consider the cluster expansion of the energy functional up to the two-body term.

LOCV method

$$E_1 = \sum_{\sigma=\uparrow,\downarrow} \sum_{k \leq k_F{}^{\sigma}} \frac{\hbar^2 k^2}{2m_n}$$

Fermi momentum: $k_F{}^{\sigma} = (6\pi^2 \rho^{\sigma})^{1/3}$

$$E_2 = \frac{1}{2N} \sum_{ij} \langle ij \,| \nu(12) |\, ij - ji \rangle$$

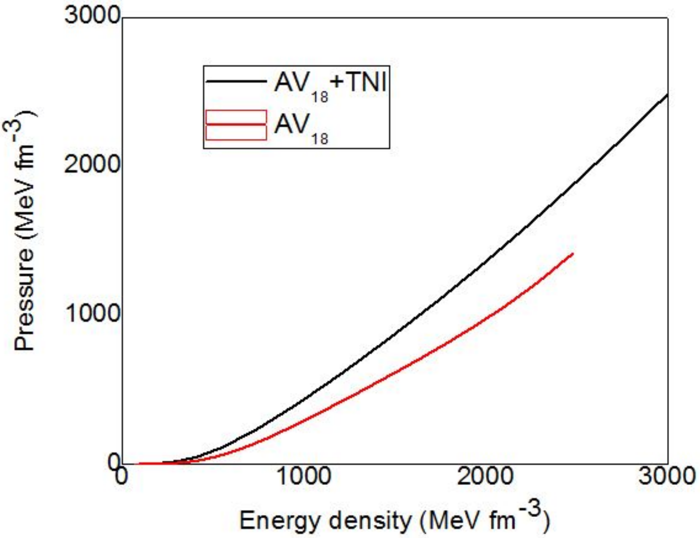
$$\nu(12) = -\frac{\hbar^2}{2m_n(B)} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12)$$

$f(12)$: two-body correlation function, $V(12)$: two-body potential

Energy Density

$$\varepsilon_m = \rho(E + m_n)$$

EoS of Neutron Matter



Gravitational Waves Amplitude (h_0)

Table: The values of magnetic field strength, mass and radius of neutron stars, magnetic and rotational ellipticity and the amplitude of gravitational waves. The amplitude of gravitational waves is obtained for r=1 kpc and P=0.01 s.

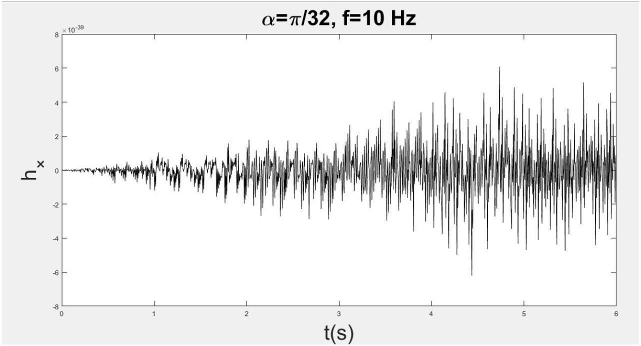
EoS	α	$M_{NS}(M_\odot)$	$R(km)$	$\epsilon_B(\times 10^{-5})$	$\epsilon_\Omega(\times 10^{-5})$	$h_0(\times 10^{-25})$
AV_{18}	7.3	1.87	9.35	0.4048	0.0185	2.32
$AV_{18}+TNI$	11.06	2.13	10.23	0.6622	0.6329	9.67

The GW amplitude (h_0) may be directly related to the strain of the GW detector arms. The reach in h_0 for Advanced LIGO and the proposed Einstein telescope are around $10^{-24}-10^{-26}$ and $10^{-26}-10^{-27}$, respectively (Glampedakis et al. '2017; Abbott et al. '2019) in the 10–100 Hz frequency range of interest.

	Known waveform	Unknown waveform
Long-lived (continuous)	Rotating neutron stars $h_0 \sim 10^{-25}$	Stochastic background $h_0 \sim 10^{-28}$
Short-lived ($T \sim 0.1$ s)	Compact binaries coalescences $h_0 \sim 10^{-21}$	Supernovæ $h_0 \sim 10^{-21}$

General taxonomy of GW sources and their expected GW amplitude h_0 (Sieniawska and Bejger '2019)

Gravitational Waveforms of a Rotating Magnetized Neutron Stars ($\alpha = 7.3$)

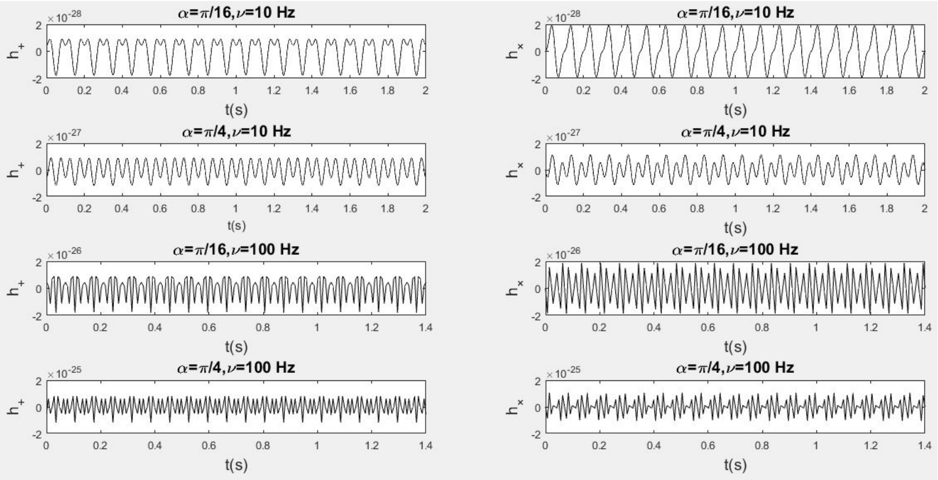


Schematic of a continuous gravitational wave buried in detector noise.

(www.aei.mpg.de)

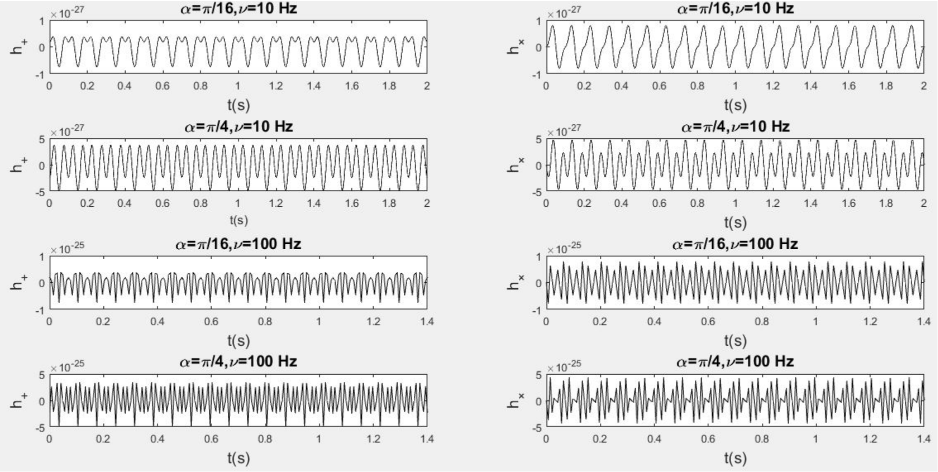
Gravitational Waveforms of a Rotating Magnetized NSs

$(\alpha = 7.3)$

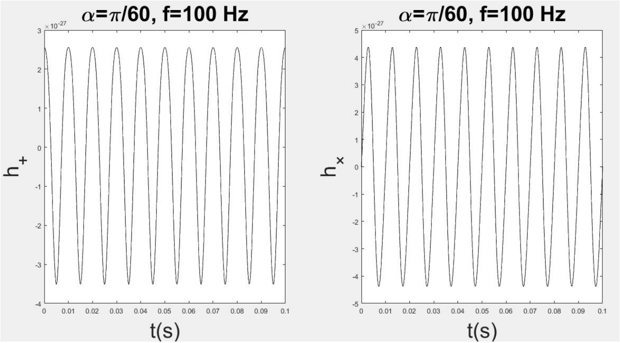


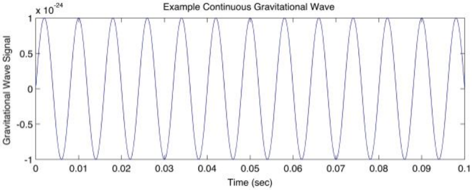
Gravitational Waveforms of a Rotating Magnetized NSs

($\alpha = 11.06$)



A short snippet of GW strain from a CGW source ($\alpha = 7.3$)





[Image: A. Stuver/LIGO] www.ligo.org/science/GW-Continuous.php

Summary

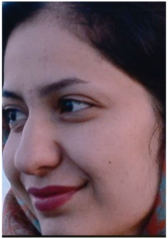
1. Here we target a different type of gravitational wave signal: the long continuous waveform expected from a magnetized spinning neutron star.
2. We investigate the effets of different EoSs and magnetic field strengths on ellipticities of relativistic star.
3. We find out for stiffer EoS the rotational and magnetic elliptity and hence the amplitude of GW is greater.
4. Because the star's sky location, spin rate, and deformation from axisymmetry are unknown, there is a large ambiguity to search the CGW.
5. Over short times, a continuous gravitational-wave signal from neutron star will look almost perfectly constant in both frequency and amplitude. However, over longer durations, the frequency of the signal will slowly change.

Our Neutron Star Group in University of Zanjan

Mozhgan Shahbazi (R-mode instability of NSs)

Mohsen Bigdeli (Head)

Mina Zamani (Magnetized NSs)



Maryam Asadnezhad (Minimal Dilatonic Gravity model in NSs)

Morteza Taghilo (NS crust)

Thank you