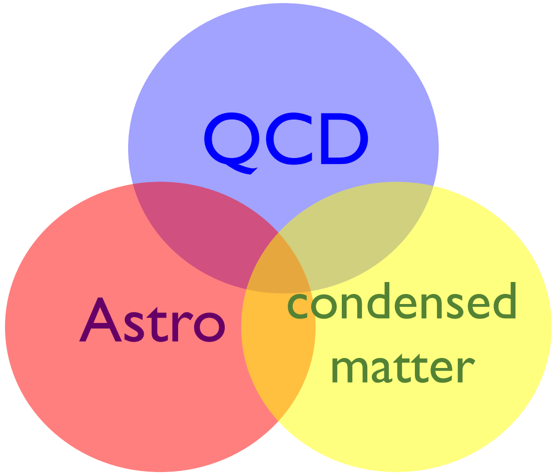
A dual model for neutron star equations of state



Toru Kojo

(**Tohoku Univ.**)

Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

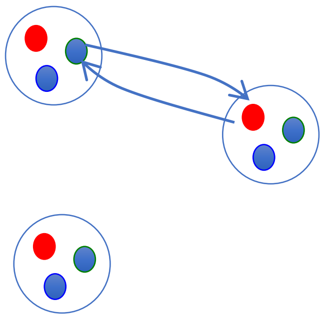
Fujimoto-TK-McLerran, "IdylliQ matter model" arXiv: 2306.04304 [nucl-th]

State of matter: overview

($n_0 = 0.16\ \mathrm{fm}^{-3}$)

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

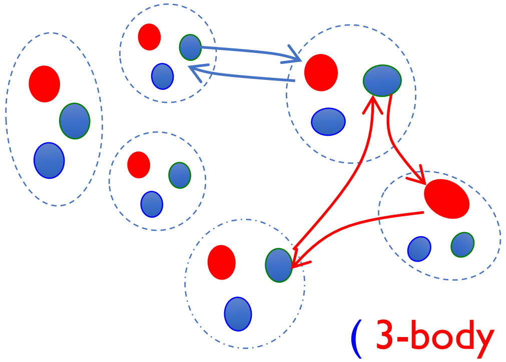


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

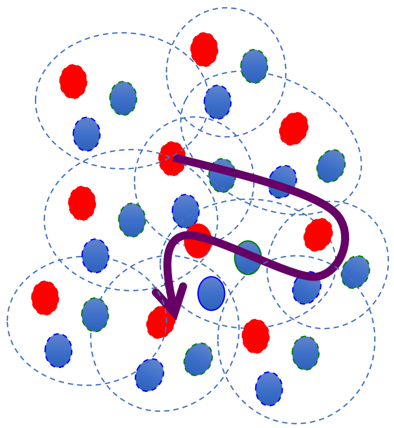


(3-body)

most difficult

(d.o.f ??)

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ 1.4 $M_\odot$

~ 2 $M_\odot$

n_B

~ $2n_0$

Hints from NS

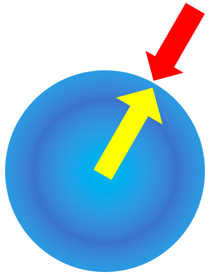
~ $5n_0$

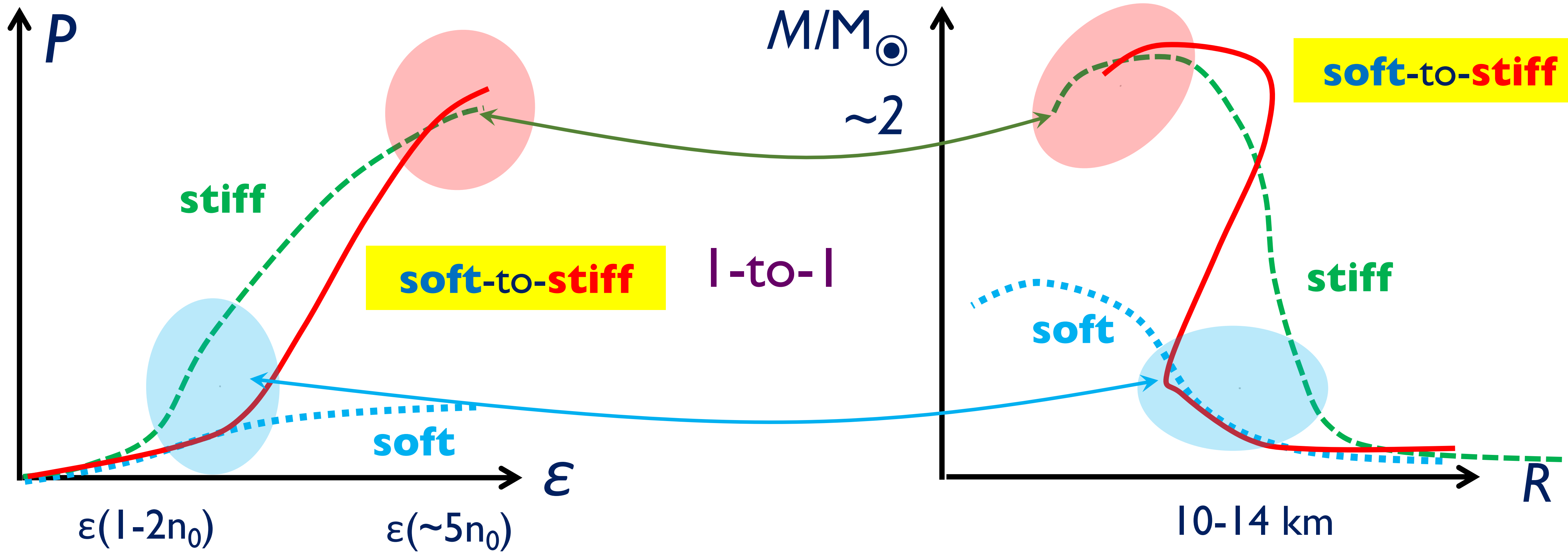
~ $40n_0$

EoS & Neutron Star M-R relation

Einstein eq. : $$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

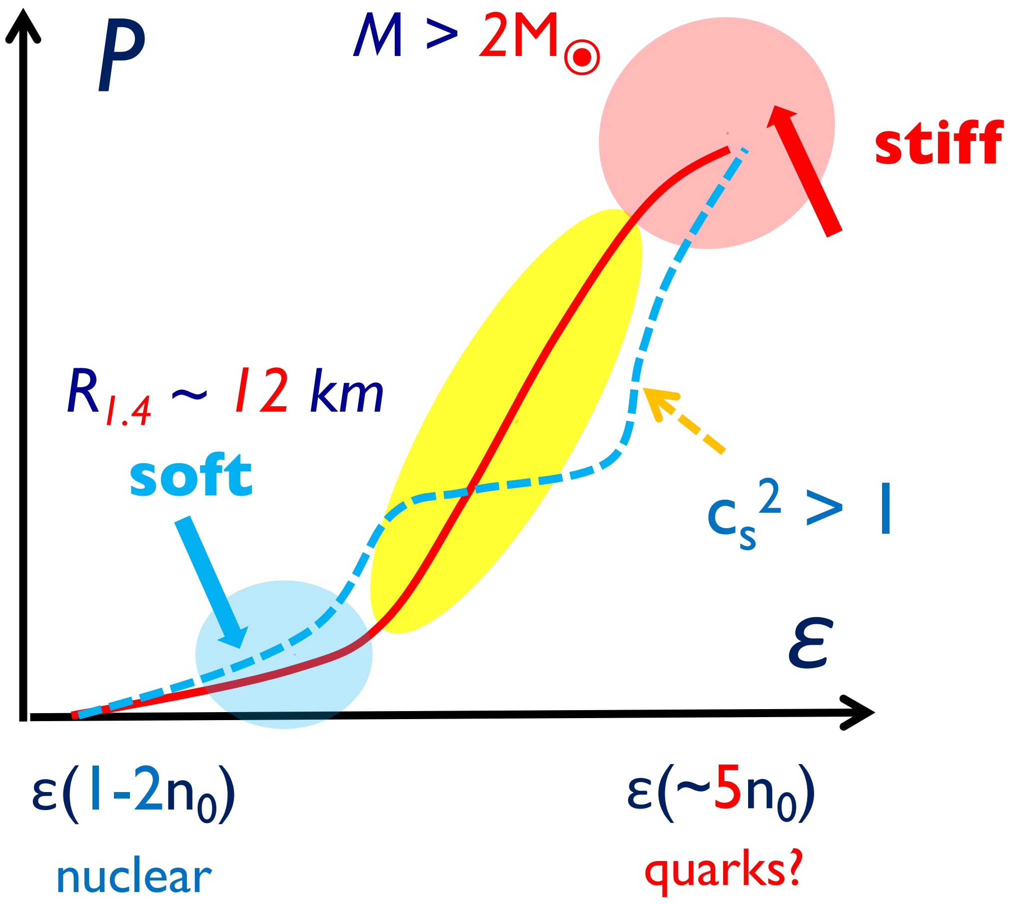
QCD (+EW) EoS

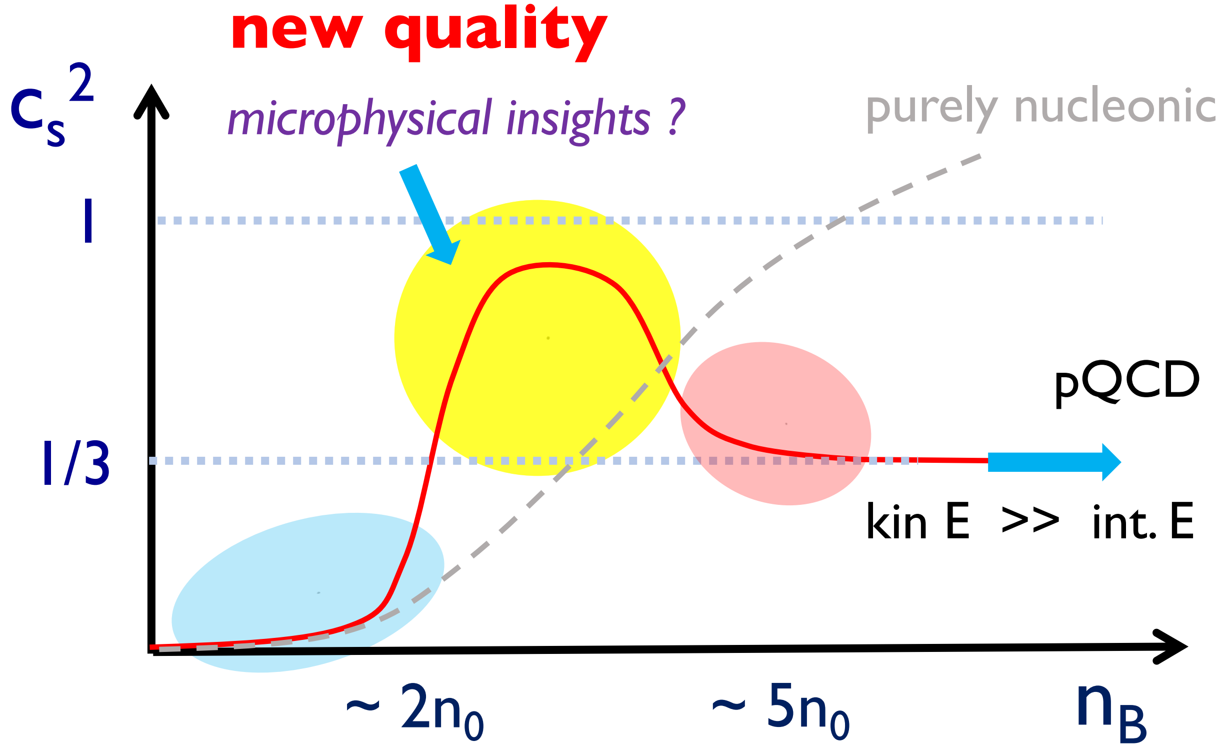




Soft to stiff is challenging:

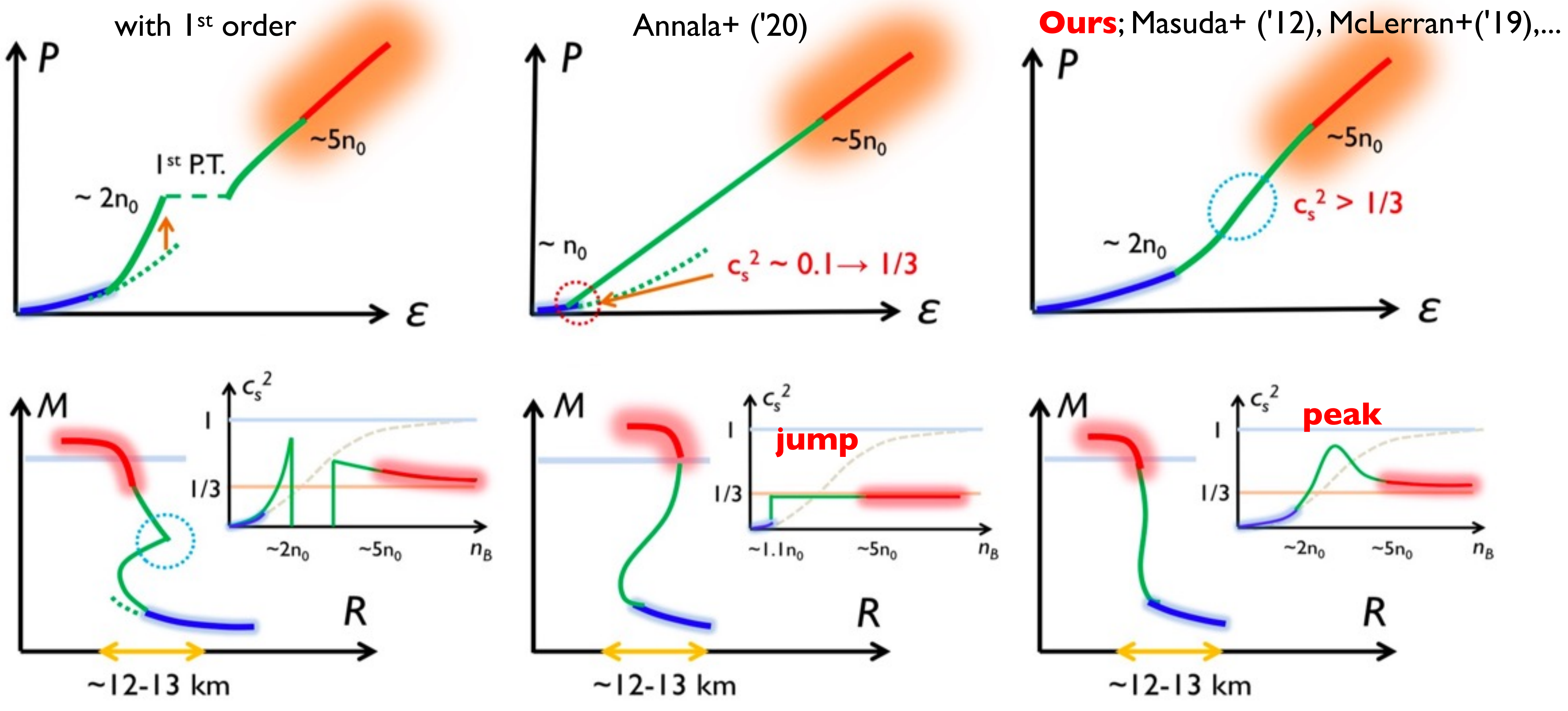
sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*) → *nuclear* & *quark* physics constrain each other





baseline: *quark-hadron continuity* **(QHC)**

Three possible scenarios



→ $R_{1.4}$ and $R_{2.1}$? *→ nuclear physics ?* *→ my favorite*

Contents

pressure from ε(n_B)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle

e.g.)

gas of heavy particles (massive limit)

$$\varepsilon(n_B) = m_N n_B \;\rightarrow\; \varepsilon/n_B = m_N \;\rightarrow\; P = 0$$

gas of relativistic particles (massless limit)

$$\varepsilon(n_B) = a n_B^{4/3} \;\rightarrow\; \varepsilon/n_B = a n_B^{1/3} \;\rightarrow\; P = \frac{\varepsilon}{3}$$

c_s^2 in purely nucleonic models

$$\varepsilon(n_B) = m_N n_B + a\frac{n_B^{5/3}}{m_N} + b n_B^{\alpha} \quad \Rightarrow \quad P = \frac{2}{3}a\frac{n_B^{5/3}}{m_N} + b(\alpha-1)n_B^{\alpha}$$

$m_N n_B$: large (!) $\quad$ $a\frac{n_B^{5/3}}{m_N}$: small (!) $\quad$ $\frac{2}{3}a\frac{n_B^{5/3}}{m_N}$: small (!)

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

⇒ at LO: p << ε (!)

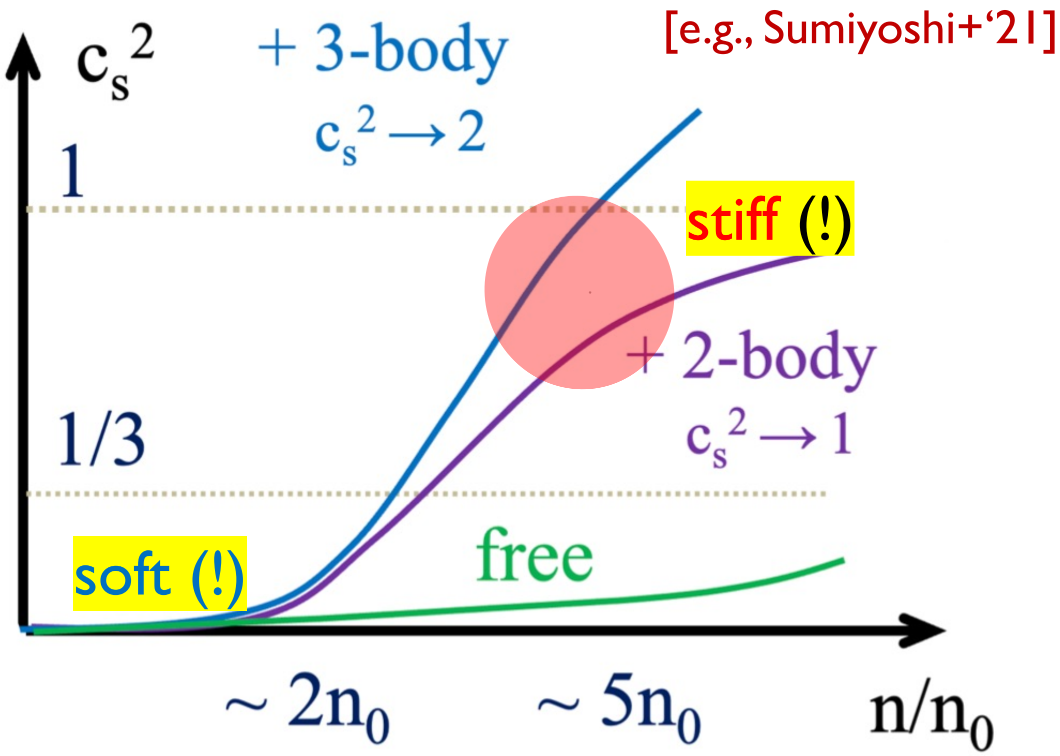
if interactions dominate (at large n_B):

$$P \sim (\alpha - 1)\varepsilon \;\rightarrow\; c_s^2 \sim (\alpha - 1)$$

2-body int. → α = 2 $\qquad$ 3-body int. → α = 3

(contact type)

causality & convergence ??



alternative **baseline**: quark EOS

e.g.) free massless quarks

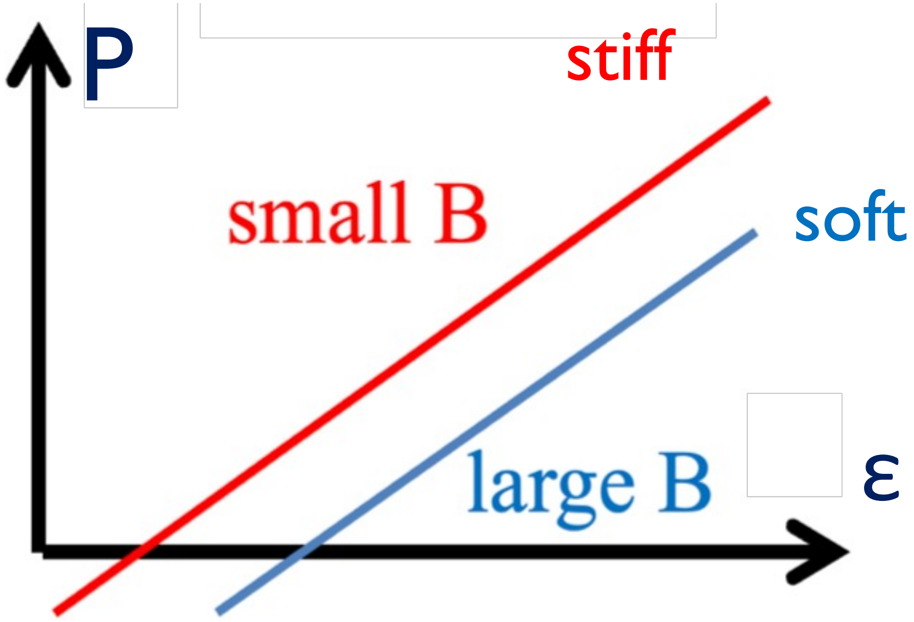
$$P = \frac{\varepsilon}{3} - B'$$

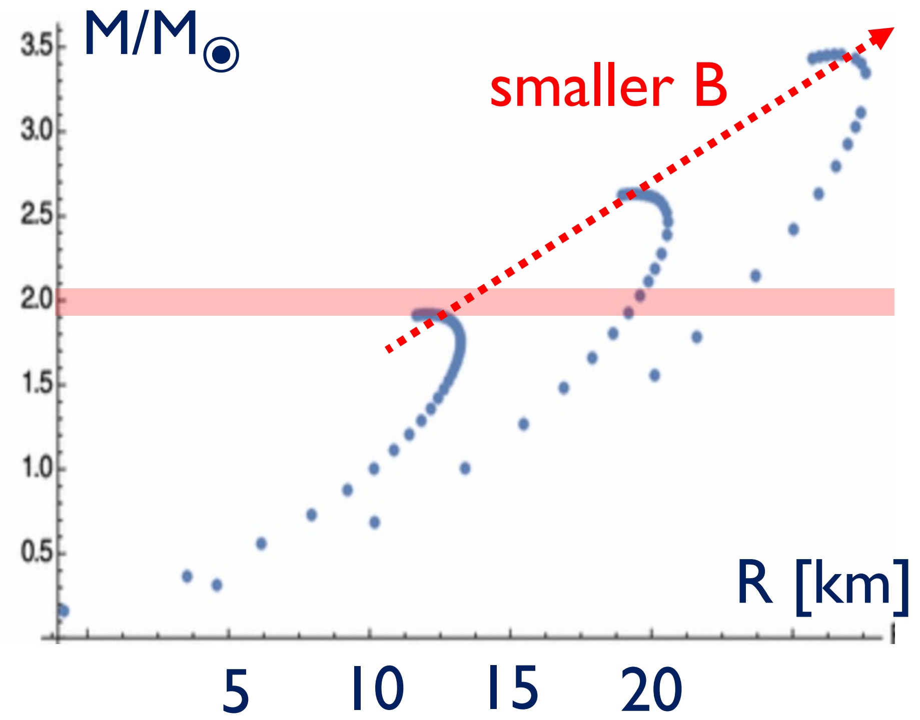
$c_s^2 = 1/3$

normalization

quark kin. E ~ N_c^2 x nucl. kin. E

~ $N_c \times p_F^2/M_q$ ~ $P_F^2/N_c M_q$



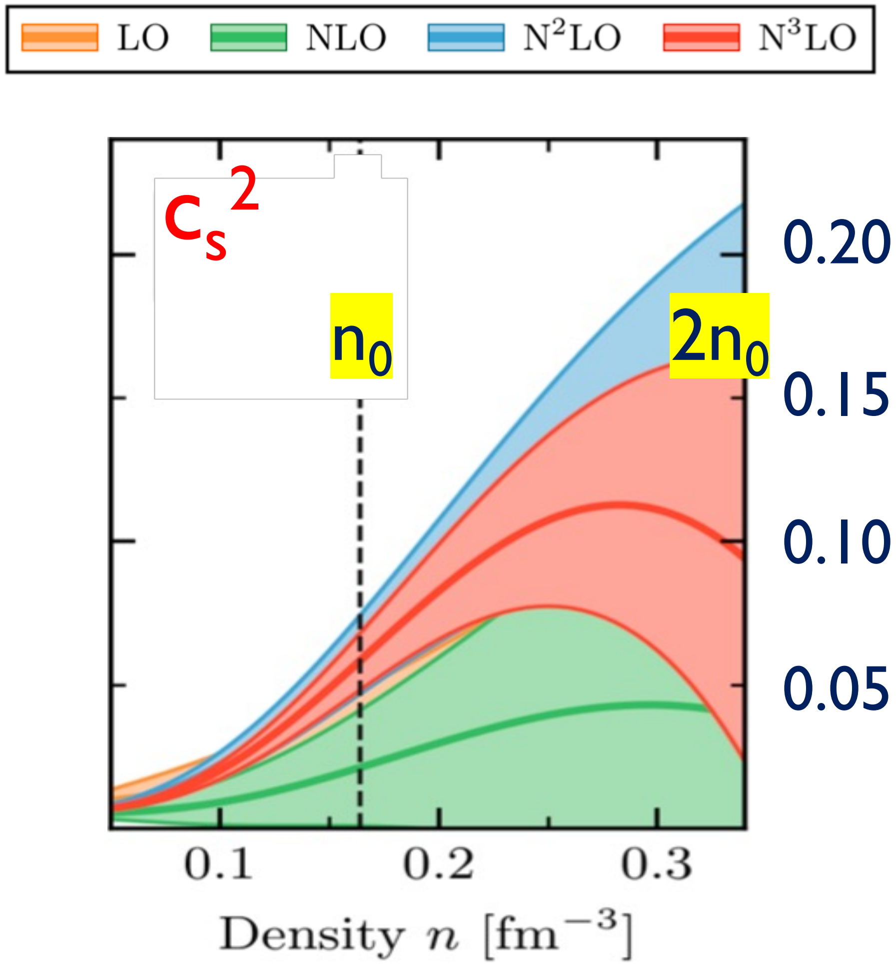


relativistic pressure → stiff EOS

can be a good starting point!?

c_s^2 = 1/3 = 0.33... (at 1-3n_0) is large

[e.g., ChEFT, Drischler+ '21]

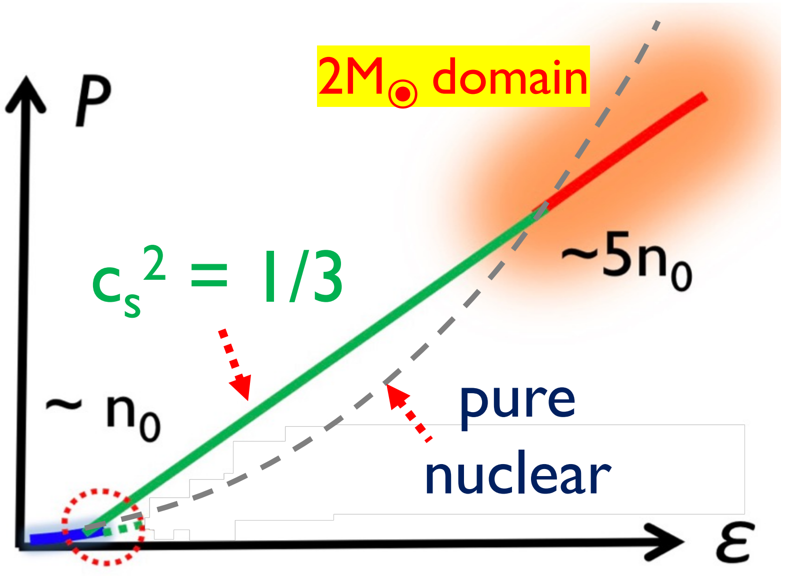


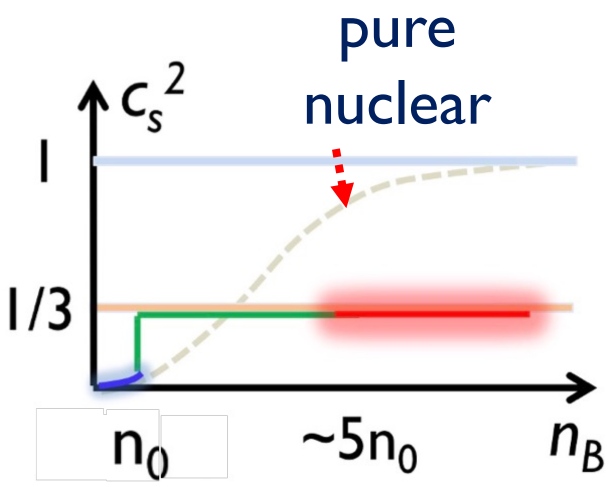
ChEFT (to N³LO)

small..

$c_s^2(n_0)$ ~ 0.05-0.10 $c_s^2(2n_0)$ ~ 0.1-0.2

If we switch to c_s^2 = 1/3 at **low** density...





For systematic analyses, see e.g., Annala+ '20

Contents

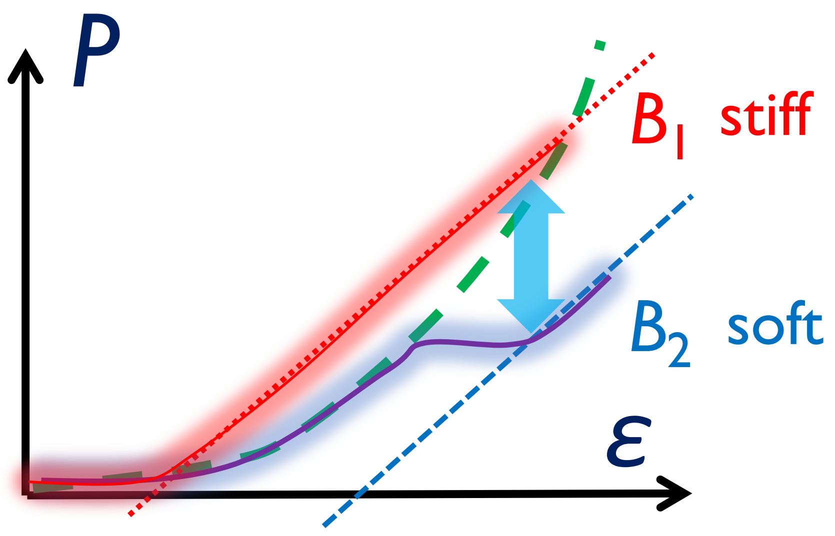
Hadron-to-quark transitions?

Confusing point:

▪ *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)



Strategy:

Keep track of quark states from nuclear to quark matter

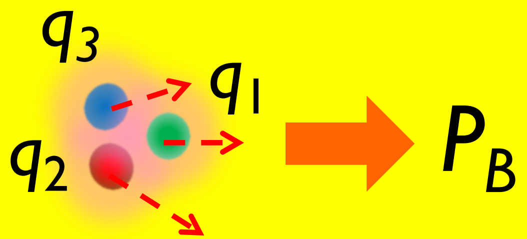
(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

Quarks in a baryon

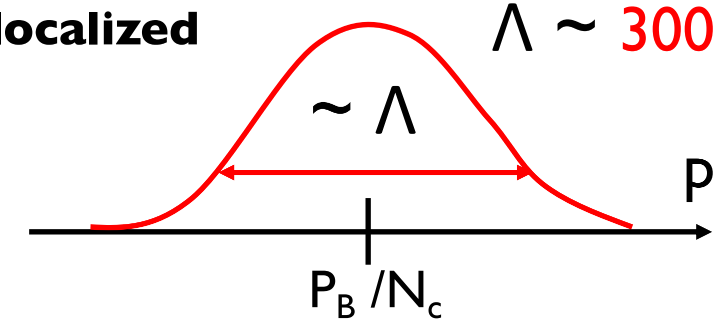
N_c (=3): number of colors

probability density:

$$\varphi(\boldsymbol{q};\boldsymbol{P}_{\rm B}) = \mathcal{N} \mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{q}-\frac{\boldsymbol{P}_{\rm B}}{N_{\rm c}}\right)^2}$$



localized



Λ ~ 300 MeV

variance: $\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$ *energetic !*

→ large "*mechanical*" pressure

$$\langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B} = \mathcal{N}\int_{\boldsymbol{p}} E_q(\boldsymbol{p})\mathrm{e}^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p}-\frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p})\rangle_{\boldsymbol{P}_B=0} + \frac{1}{6}\left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i}\right\rangle_{\boldsymbol{P}_B=0}\left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x N_c x N_c

~ N_c (M_q + E_{kin}) ≫ ~ P_B^2 /($N_c E_q$)

baryon mass | baryon kin. energy

A model of quark-hadron-duality

cf) [TK '21, TK-Suenaga '21]

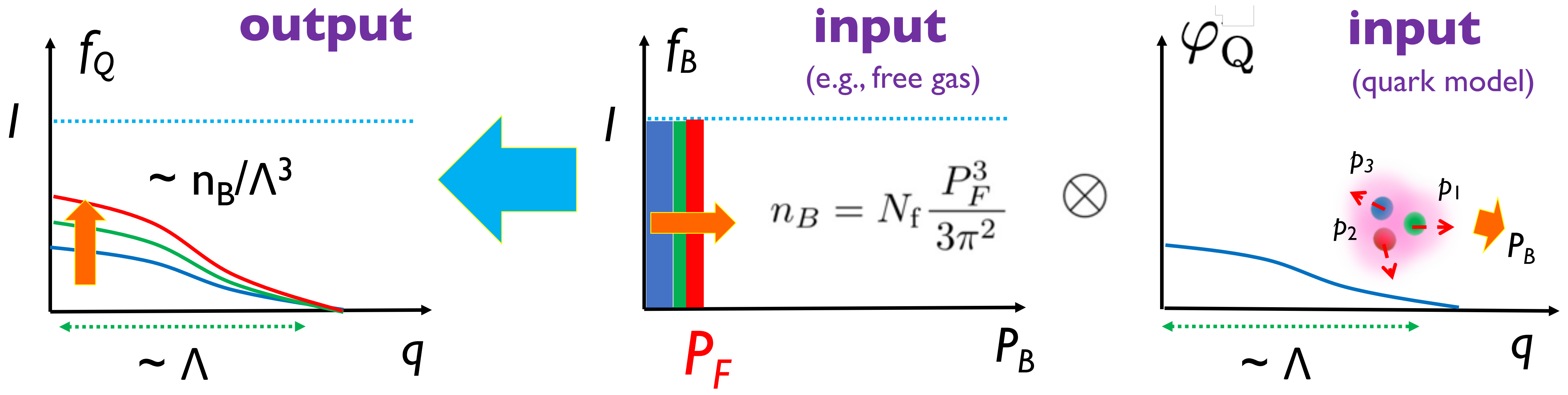
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

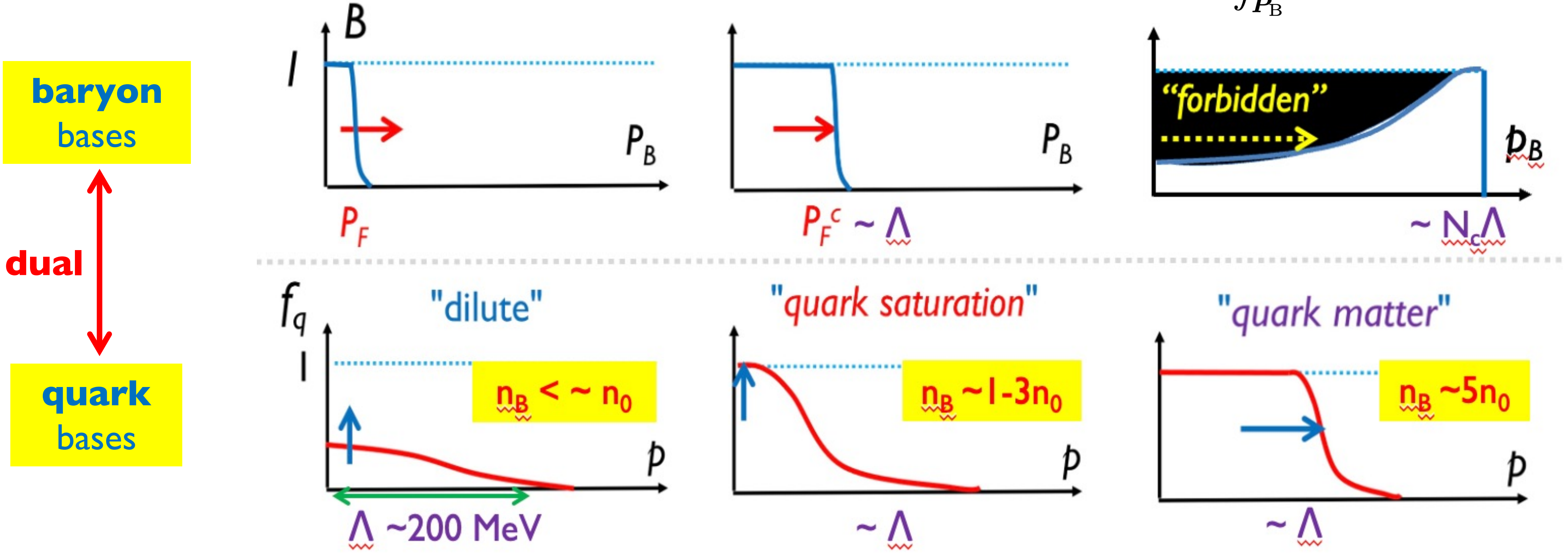
$$f_{\mathrm{Q}}(\boldsymbol{q}; n_{\mathrm{B}}) = \int_{\boldsymbol{P}_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{P}_{\mathrm{B}}; n_{\mathrm{B}}) \varphi^{\mathrm{B}}_{\mathrm{Q}}(\boldsymbol{q}; \boldsymbol{P}_{\mathrm{B}})$$

e.g.) in **ideal** baryonic matter



Evolution of occ. probabilities

$$f_Q(\boldsymbol{q}; n_B) = \int_{\boldsymbol{P}_B} f_B(\boldsymbol{P}_B; n_B)\varphi_Q^B(\boldsymbol{q}; \boldsymbol{P}_B)$$



"**quark saturation**" constraint

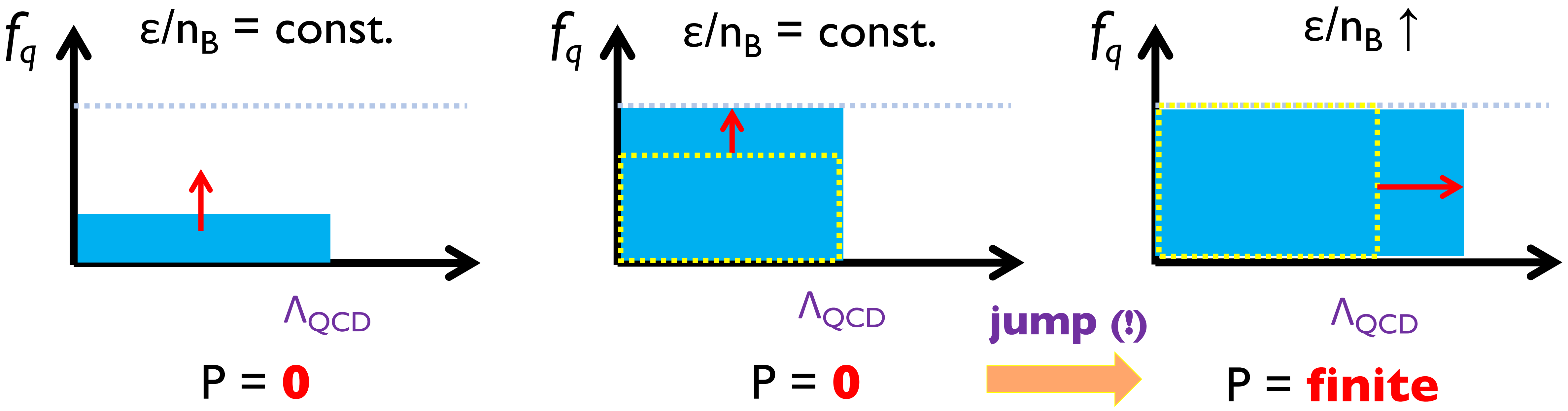
→ **relativistic baryons at low density, n_B ~ 1-3n_0 !**

cf) McLerran-Reddy model (2019); microscopic description, TK (2021)

Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

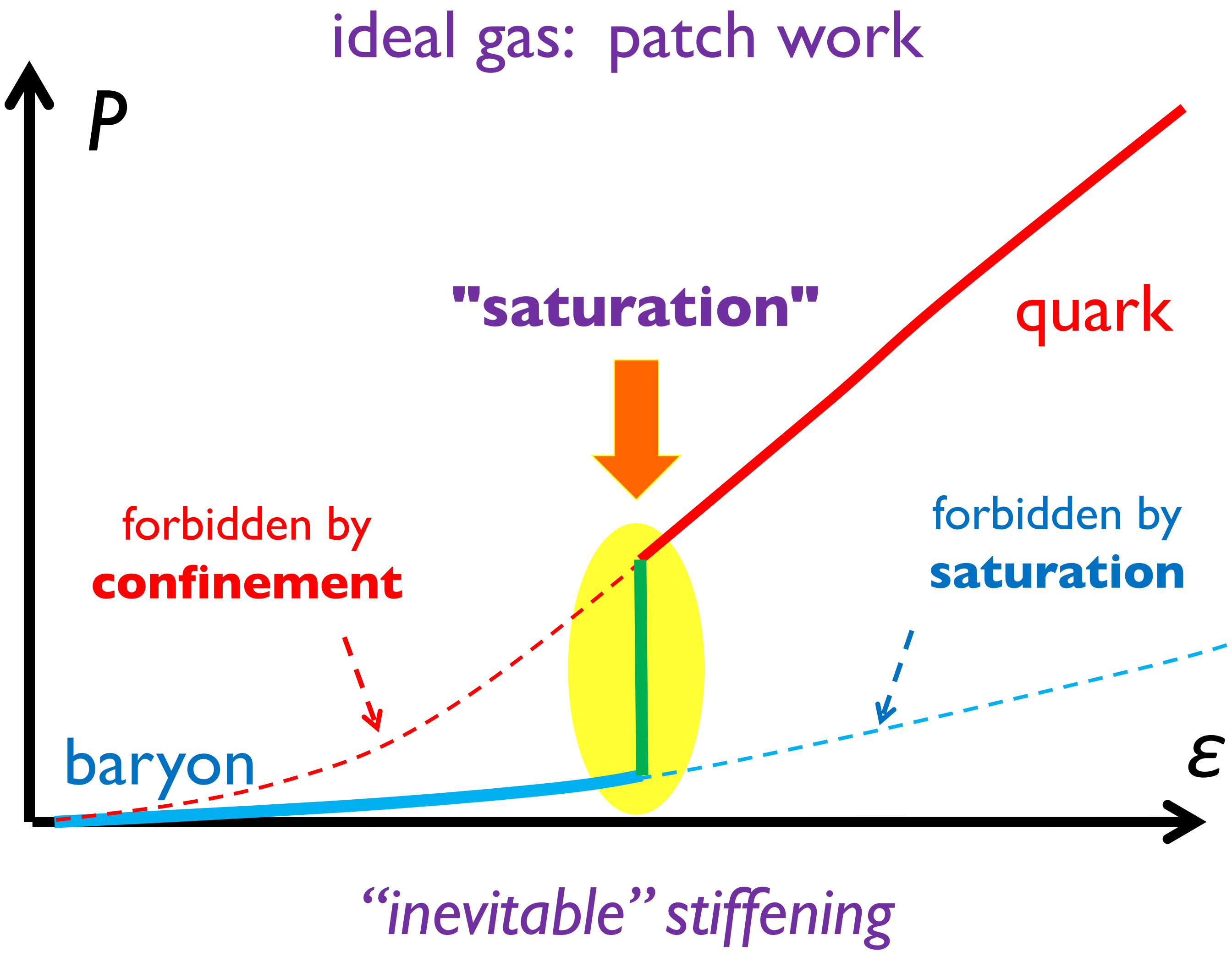
energy per particle

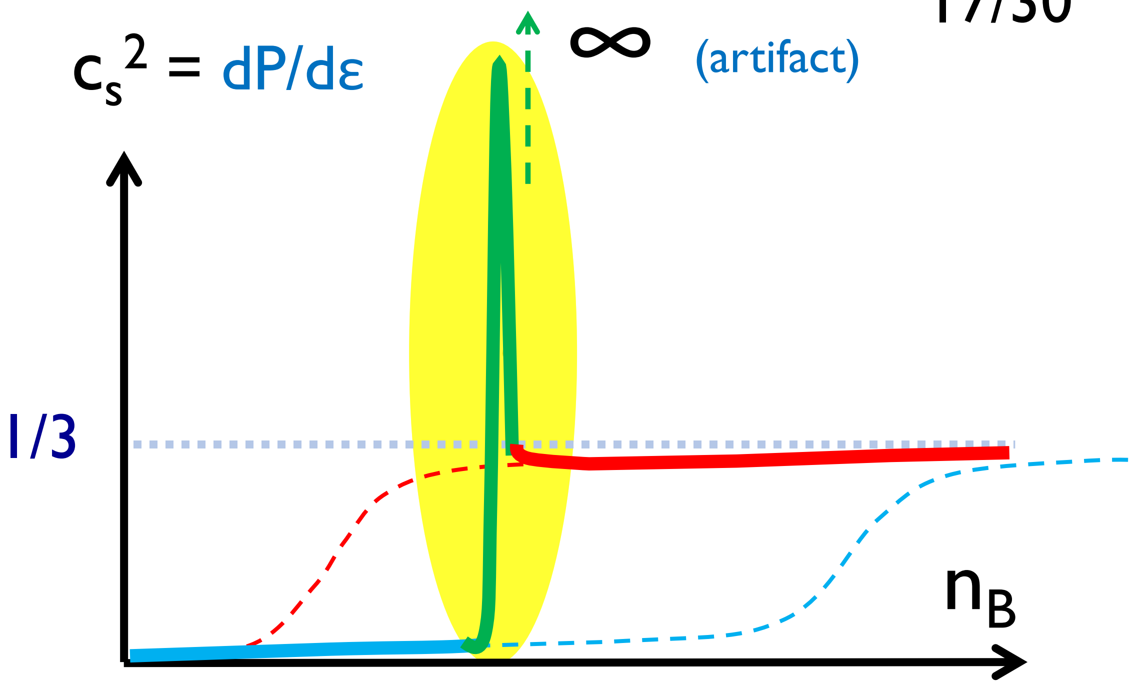


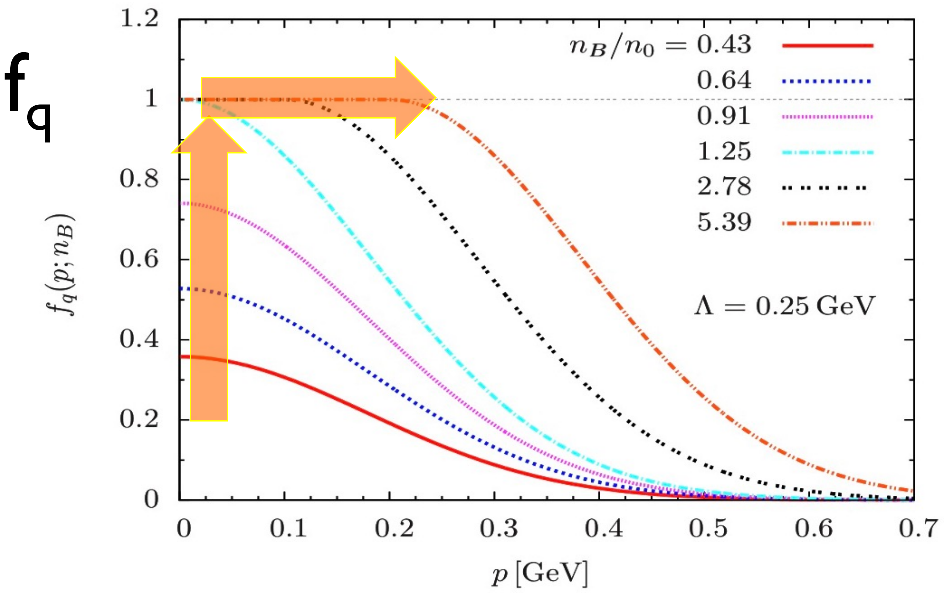
f_q continuous → ε, n_B are continuous

*Quarks do contribute to **ε** even before saturation; but to P only after the saturation!!*

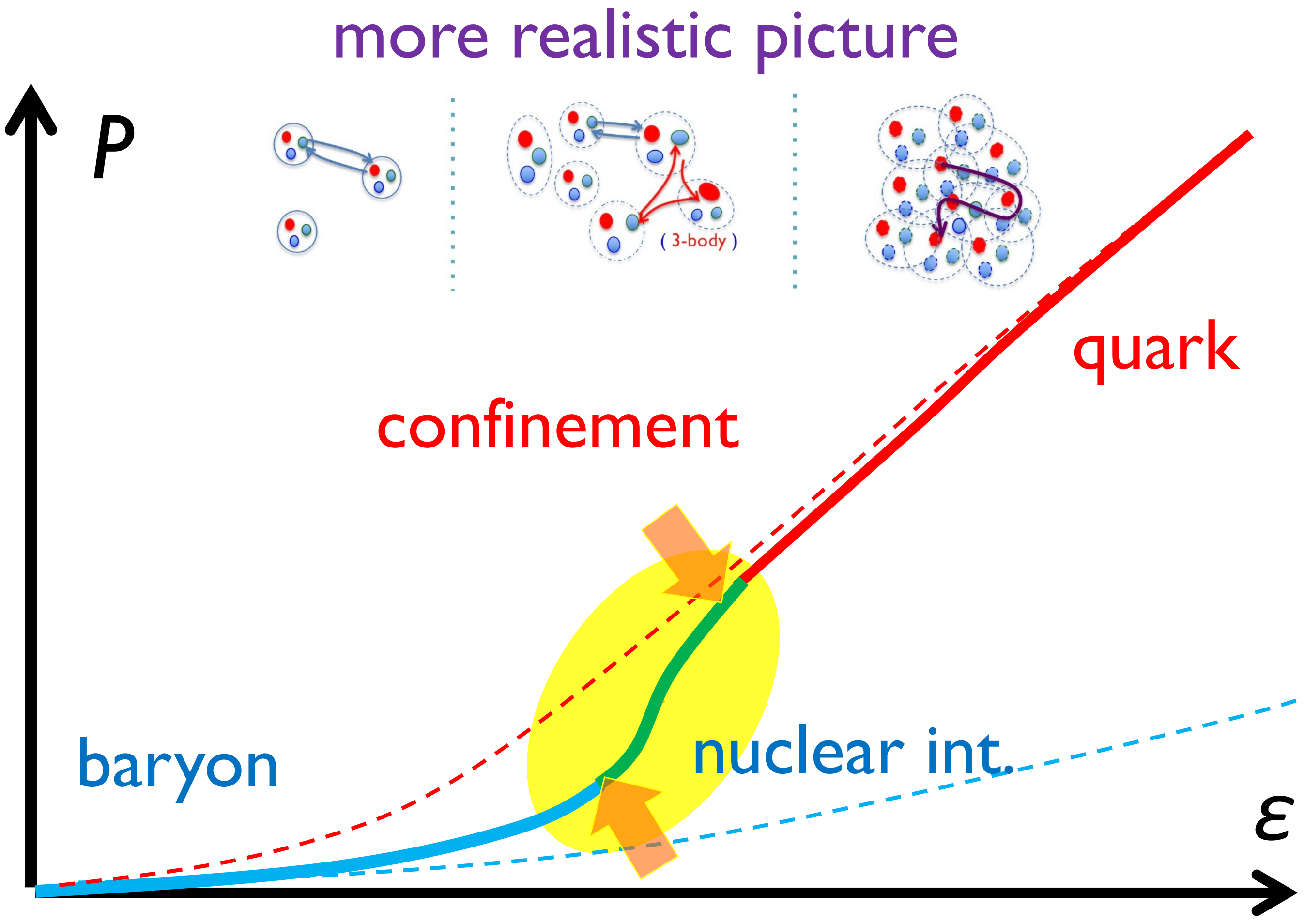
Peak in sound velocity

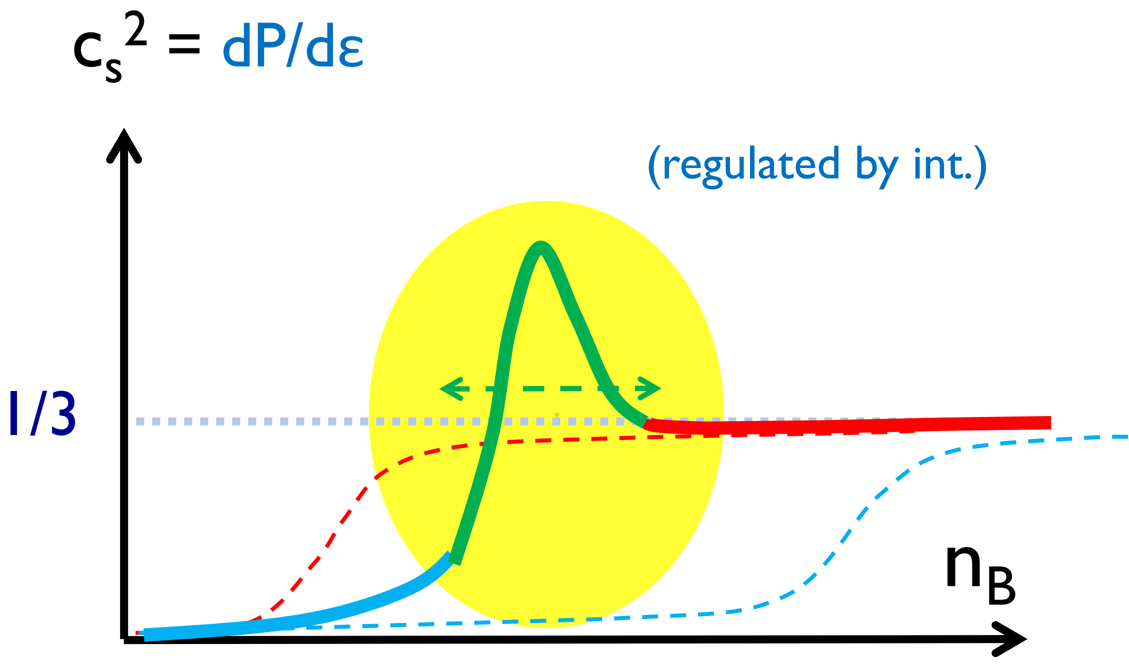


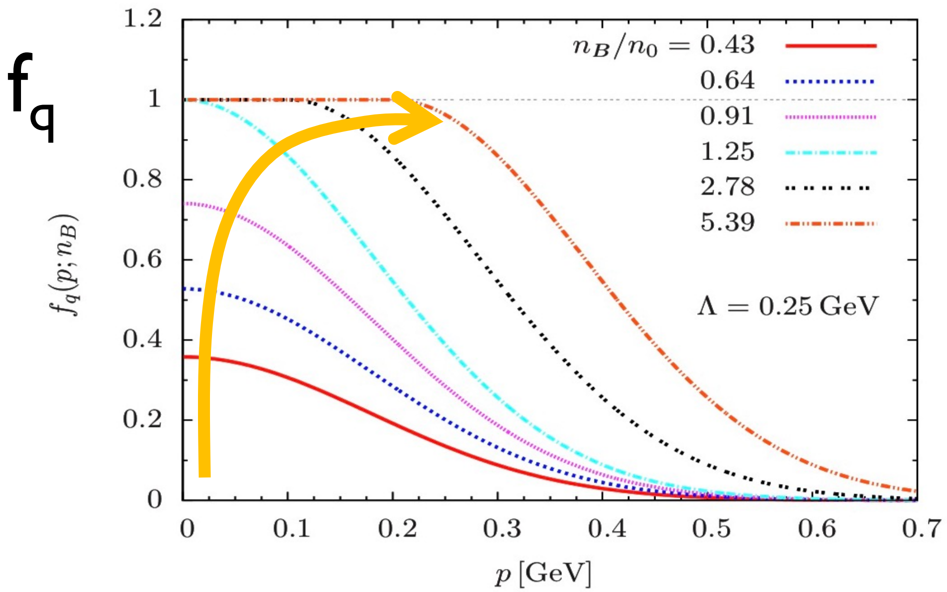




Peak in sound velocity







A solvable model

[Fujimoto-TK-McLerran, ‘23]

duality:

$$f_{\rm Q}(\boldsymbol{q}; n_{\rm B}) = \int_{\boldsymbol{P}_{\rm B}} f_{\rm B}(\boldsymbol{P}_{\rm B}; n_{\rm B}) \varphi_{\rm Q}^{\rm B}(\boldsymbol{q}; \boldsymbol{P}_{\rm B})$$

$f_B \rightarrow f_Q$ always doable, how about $f_Q \rightarrow f_B$??

global problem!

a useful model: with a specific quark distribution $\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3} \frac{e^{-q/\Lambda}}{q/\Lambda}$

$$\hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2} \delta(\boldsymbol{p}-\boldsymbol{q})$$

local f_B from local f_Q: $f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3} \hat{L}\left[f_{\rm Q}(\boldsymbol{q})\right]$

A solvable model [Fujimoto-TK-McLerran, '23]

isospin, spin

2-flavor model: $$\varepsilon_{\rm B}[f_{\rm B}] = 4 \int_k E_{\rm B}(k) f_{\rm B}(k)$$

IdylliQ matter (Ideal dual-lyllic Quarkyonic)

Ideal: *except* the confining forces that trap quarks, all interactions are neglected.

dual: *explicit* dual relations between baryons and quarks.

Quarkyonic: quark matter with non-perturbative (*confining*) gluons.

Variational problem

[Fujimoto-TK-McLerran, '23]

$$E_{\mathrm{B}}(k) = \sqrt{M_{\mathrm{B}}^2 + k^2}$$

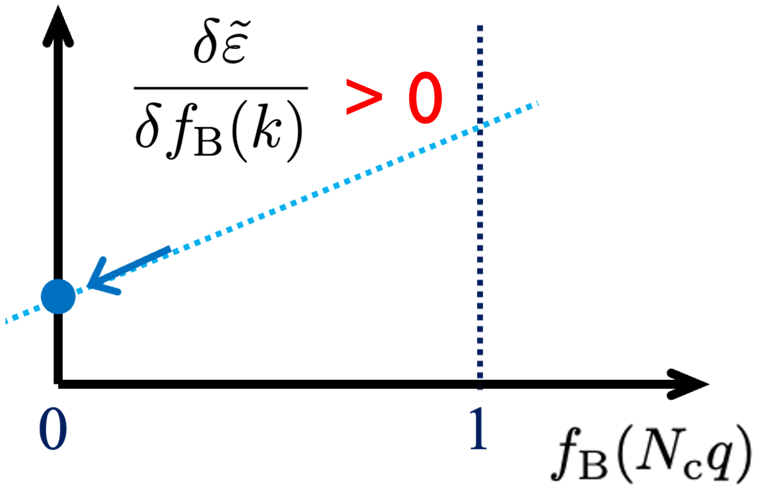
$$n_{\mathrm{B}} = 4\int_k f_{\mathrm{B}}(k)$$

Lagrange multiplier

$$\tilde{\varepsilon} = \varepsilon_{\mathrm{B}}[f_{\mathrm{B}}] - \lambda_{\mathrm{B}} n_{\mathrm{B}}$$

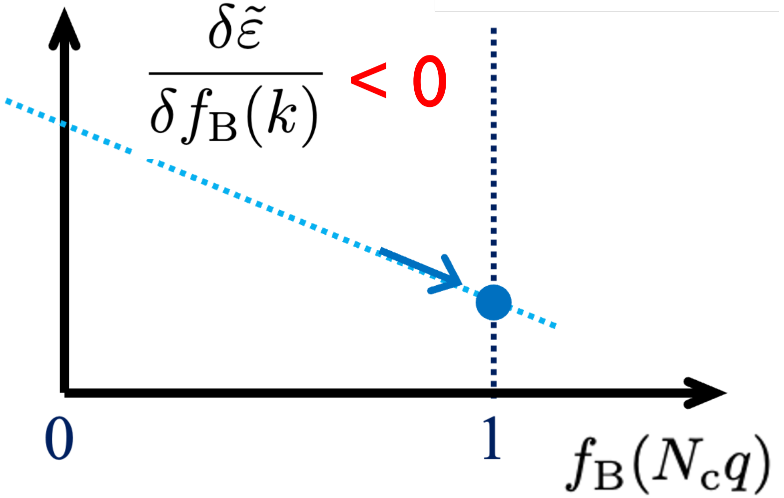
optimization: $$\frac{\delta \tilde{\varepsilon}}{\delta f_{\mathrm{B}}(k)} = E_{\mathrm{B}}(k) - \lambda_{\mathrm{B}}$$ *at a given k*

$E_B(k) > \lambda_B$

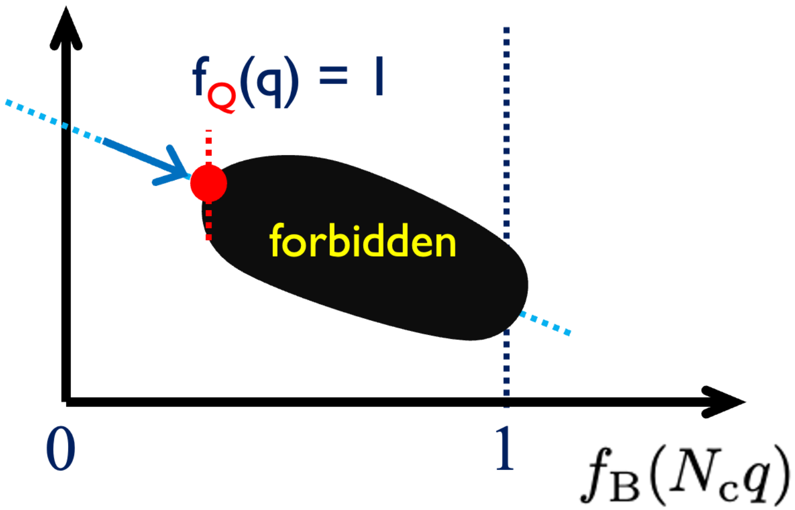


$E_B(k) < \lambda_B$

before saturation

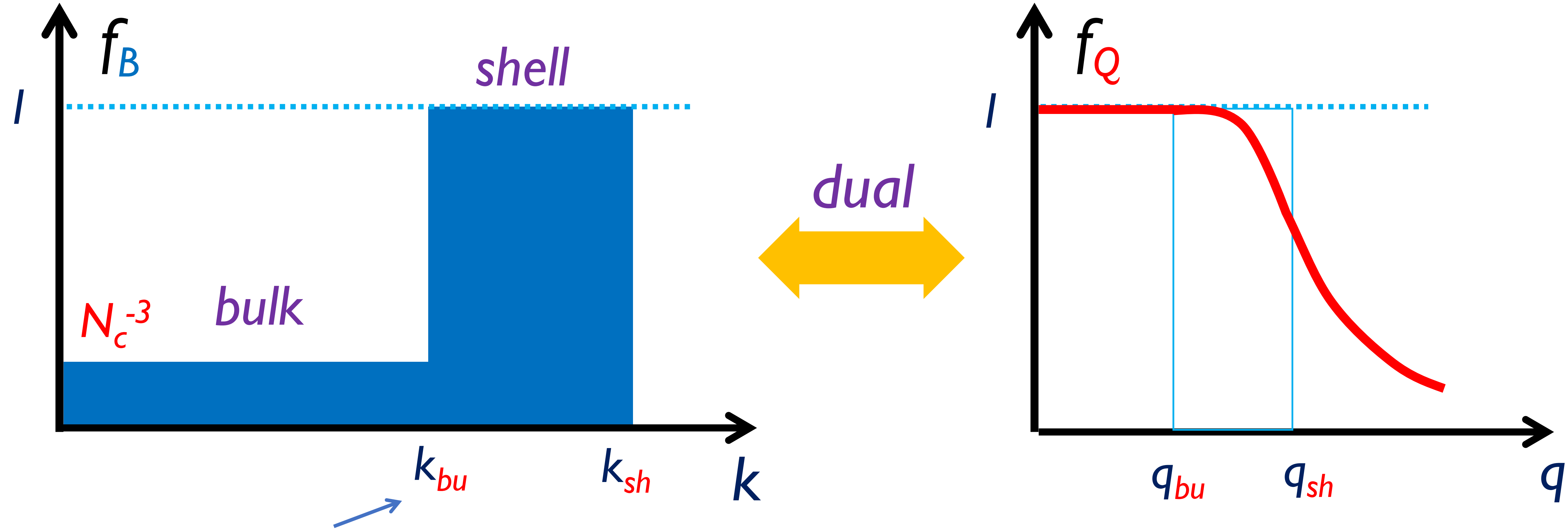


after saturation



Solution (post saturation)

[Fujimoto-TK-McLerran, '23]



determined by saturation condition

$$k_{\rm sh} = \sqrt{\lambda_{\rm B}^2 - M_{\rm B}^2}$$

$q = k/N_c$

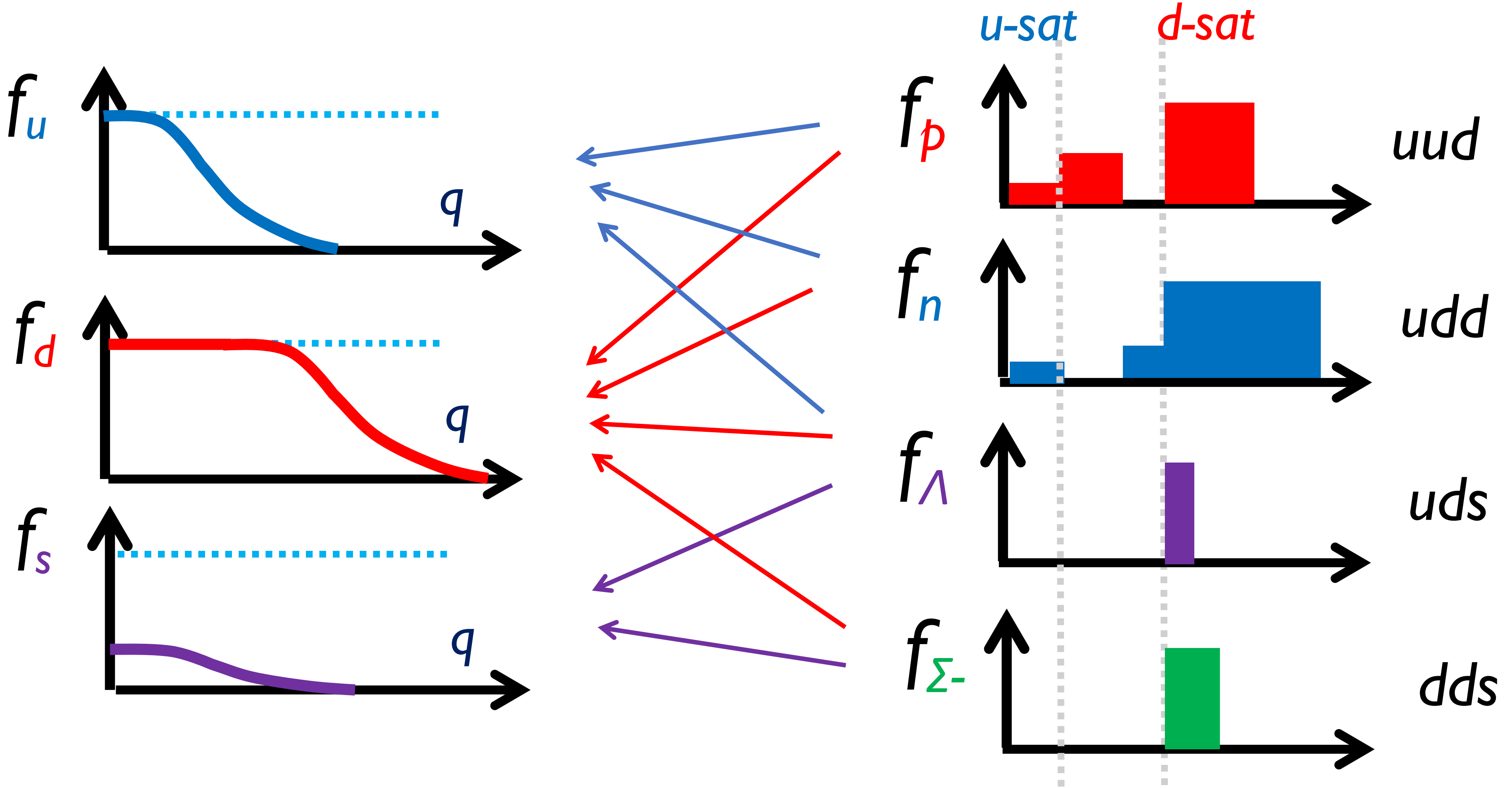
The ***most compact*** *distributions* ***compatible with*** *the saturation*

Multi-flavor extension

[Fujimoto-TK-McLerran, '23, to appear soon]

$$f_{\rm Q}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\rm Q}^{\rm B} \int_{\boldsymbol{k}} f_{\rm B}(\boldsymbol{k}) \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\rm c}}\right)$$

Q = u, d, s



and so on...

saturation constraints

flavor correlations

Contents

Stiff quark matter

The appearance of c_s^2 peak is characteristic in the QHC scenarios, but is not sufficient condition for ~ 2.1-2.3$M_\odot$ NS.

Just after the crossover, quarks are not fully relativistic.

Can the chiral restoration makes quarks massless and stiffens EOS?

Unlikely: it adds "*the bag constant*" to the energy density! (look at Dirac sea!)

→ ε increases & P decreases: **significant** softening!

Now, we consider interactions on top of IdylliQ models.

Underlying picture (guess)

- *Gluons remain non-perturbative at 5-10n_0*

  (see, e.g., lattice results for 2-color & isospin QCD)

- *Chiral restoration occurs mildly*

} implicitly included in IdylliQ type models

- *Continuity*: interactions in quark matter should have *natural counterpart* in hadron physics

Short range correlations in a baryon:

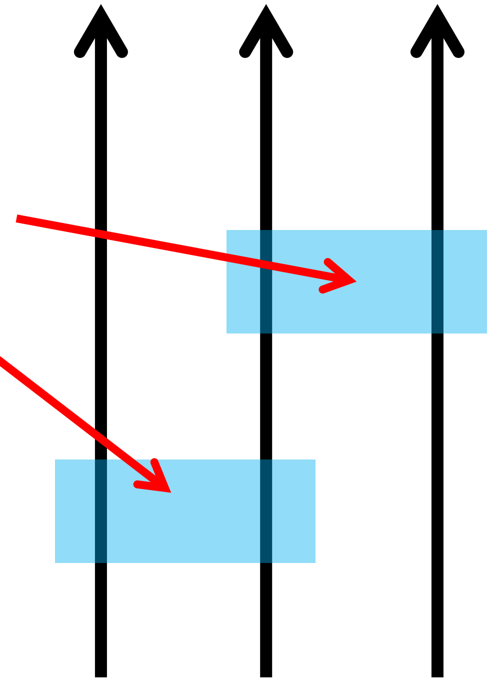
my favorite: *color-electric & magnetic interactions*

a baryon in dilute regime

(color-singlet)

(always) color-
antisymmetric

(attractive
electric int.)



e.g., **nucleons**

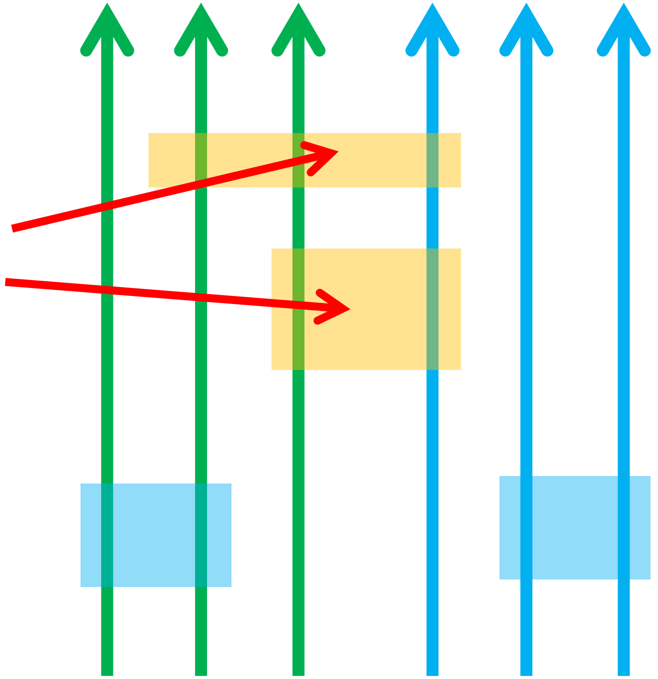
$$M_N \sim \underbrace{3M_{q.} + \text{kin.}} + \text{color-EM}$$

~ 940MeV ~ 1100MeV ~ −150-200MeV

in dense regime

sometimes
color-
symmetric

(repulsive)



more chances to feel repulsion

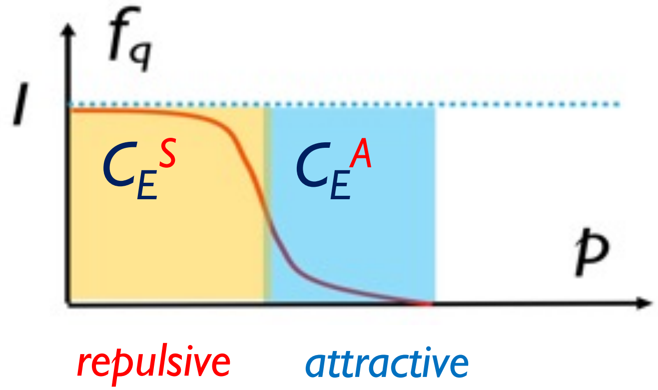
EoS with interactions

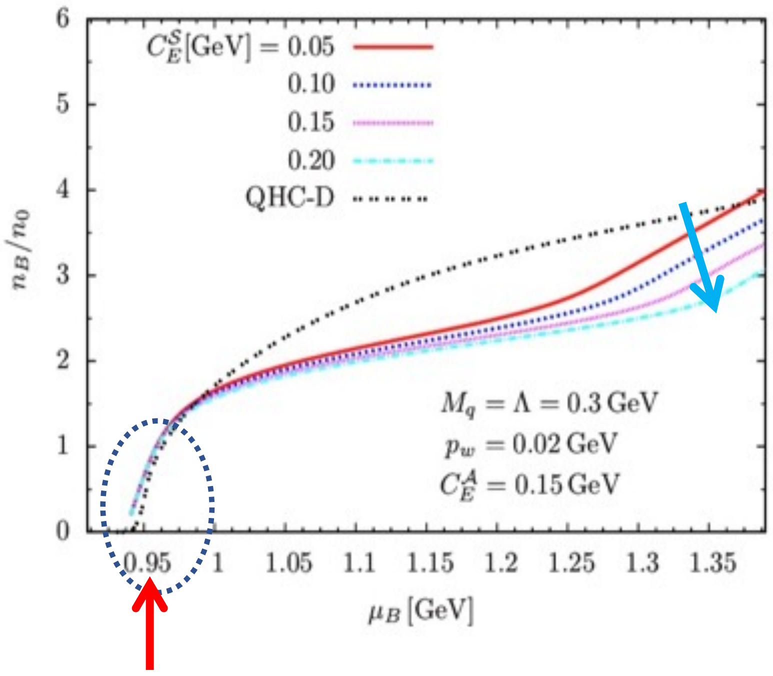
cf) [TK '21, TK-Suenaga '21]

e.g., $$\mathcal{V}[f_Q] = -C_E^{\mathcal{A}}\left[1-(f_Q)^n\right] + C_E^{\mathcal{S}}(f_Q)^n$$

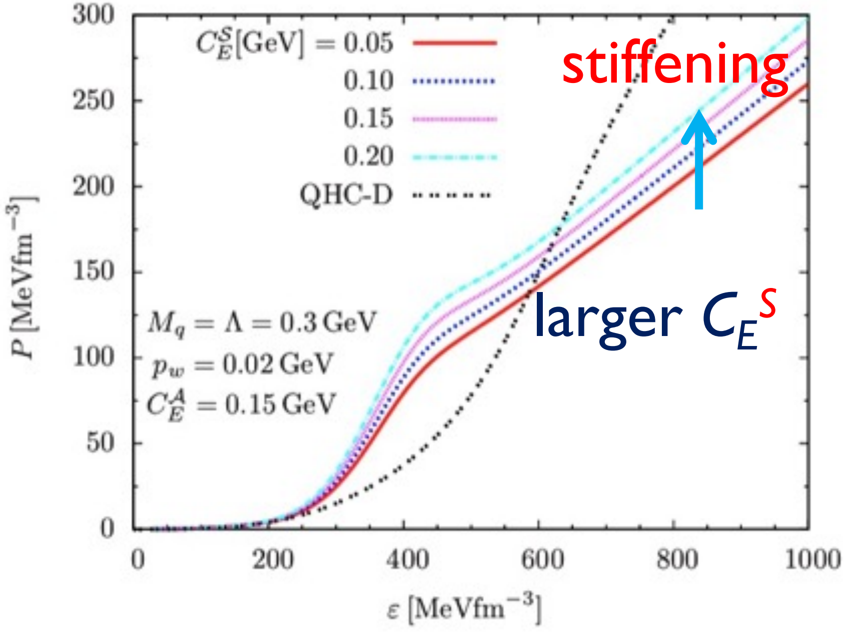
$\left[1-(f_Q)^n\right]$: → 1 (dilute), → 0 (dense)

$(f_Q)^n$: → 0 (dilute), → 1 (dense)

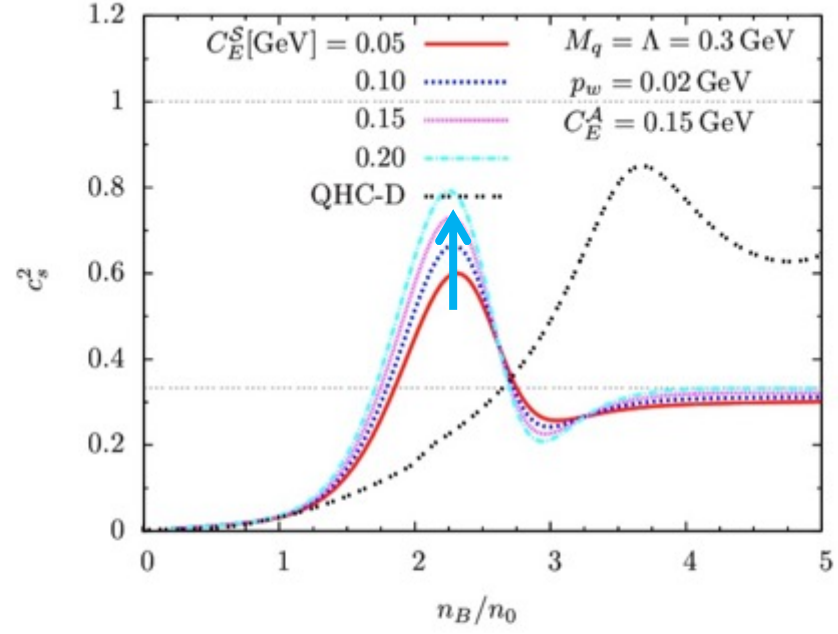




adjust $C_E{}^A$ (fit M_B = 939 MeV)



high density stiffening



stronger peak in c_s

Simple parametric analyses

[TK-Powell-Song-Baym, '14]

rela. kin. energy — interactions

$$\varepsilon(n) = an^{4/3} + bn^{\alpha}$$

(n: quark density)

ideal — interactions

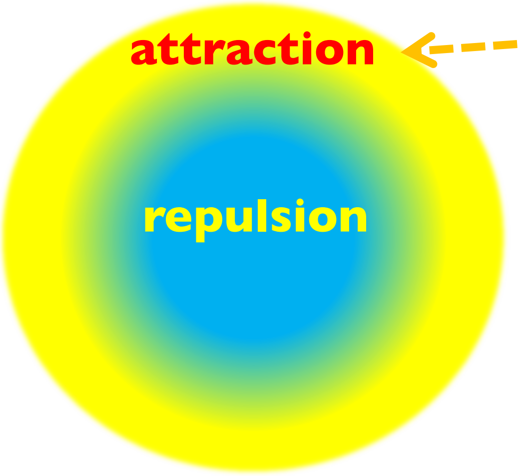
$$P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right)n^{\alpha}$$

For stiff EOS:
(for large P)

for $\alpha > 4/3$: $b > 0$ (e.g. bulk repulsion, ~ $+ n_B^2/\Lambda^2$)

for $\alpha < 4/3$: $b < 0$ (e.g. surface pairings, ~ $- \Lambda^2 n_B^{2/3}$)

quark
Fermi sea
(ideal combo)



2- or 3-quark correlations

Stiff EOS from attractive forces

Summary

- For soft-to-stiff EOS: QHC is a good baseline
- Quark saturation effects: likely occur at ~1-3n_0
- Saturation triggers rapid stiffening (sound velocity peak)
- Hyperons are not independent; highly suppressed by saturation

  [Fujimoto-TK-McLerran, to appear]
- Bulk repulsion and Fermi surface attraction → stiffening of EOS

Quarks are important for NS physics in multiple contexts