Hot β-equilibrium hadronic matter in the neutrino-trapped regime within the framework of Relativistic Mean-Field theory

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Research Outline

- Hot Hadronic Matter within the $RMF_{\sigma\omega\delta\rho}$ Model
    - Constituent composition and QHD Lagrangian density of many-particle system
    - Mean-field approximation
    - Equations for thermodynamic quantities
    - Computational scheme and numerical results
- Quark Matter within the SU(3) Local NJL Model
    - Constituent composition and Lagrangian density of many-particle system
    - Equations for thermodynamic quantities
    - Computational scheme and numerical results
- Phase transition (Maxwell construction)
    - Parameters of first-order phase transition in neutrino-trapped regime
    - Phase diagram, Critical endpoint

Hadronic Matter Description in Quantum Hadrodynamics (QHD)

Constituents: $n, p, e, \nu_e, \mu, \nu_\mu$

Lagrangian density: $\mathcal{L}_{HM} = \mathcal{L}_{Had} + \mathcal{L}_{Lept}$

$$\mathcal{L}_{Had} = \overline{\psi_N}\left[\gamma^\mu\left(i\partial_\mu - g_\omega \omega_\mu(x) - \frac{1}{2} g_\rho \vec{\tau}_N \vec{\rho}_\mu(x)\right) - \left(m_N - g_\sigma \sigma(x) - g_\delta \vec{\tau}_N \vec{\delta}(x)\right)\right]\psi_N +$$

$$+\frac{1}{2}\left(\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma^2 \sigma(x)^2\right) - U\left(\sigma(x)\right) + \frac{1}{2} m_\omega^2 \omega_\mu(x) \omega^\mu(x) - \frac{1}{4}\Omega_{\mu\nu}(\mathrm{x})\Omega^{\mu\nu}(x) +$$

$$+\frac{1}{2}\left(\partial_\mu \vec{\delta}(x) \partial^\mu \vec{\delta}(x) - m_\delta \vec{\delta}(x)^2\right) + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu(x) \vec{\rho}^\mu(x) - \frac{1}{4}\mathcal{R}_{\mu\nu}(x)\mathcal{R}^{\mu\nu}(x)$$

$$\mathcal{L}_{Lept} = \sum_{i=e,\nu_e,\mu,\nu_\mu} \overline{\psi_i}\left(i\gamma^\mu \partial_\mu - m_i\right)\psi_i$$

$$U(\sigma) = \frac{b}{3}(g_\sigma \sigma)^2 + \frac{c}{4}(g_\sigma \sigma)^3$$

$$\Omega_{\mu\nu}(x) = \partial_\mu \omega_\nu(x) - \partial_\nu \omega_\mu(x),$$

$$\Re_{\mu\nu}(x) = \partial_\mu \rho_\nu(x) - \partial_\nu \rho_\mu(x).$$

Meson fields: $\sigma(x)$, $\omega_\mu(x)$, $\vec{\delta}(x)$, $\vec{\rho}_\mu(x)$

$$x = x_\mu = (t, x, y, z)$$

Mean-Field Approximation

$$g_\sigma \bar{\sigma} \equiv \sigma, \quad g_\omega \bar{\omega}_0 \equiv \omega, \quad g_\delta \bar{\delta}^{(3)} \equiv \delta, \qquad g_\rho \bar{\rho}^{(3)} \equiv \rho,$$

$$\left(\frac{g_\sigma}{m_\sigma}\right)^2 \equiv a_\sigma, \quad \left(\frac{g_\omega}{m_\omega}\right)^2 \equiv a_\omega, \quad \left(\frac{g_\delta}{m_\delta}\right)^2 \equiv a_\delta, \quad \left(\frac{g_\rho}{m_\rho}\right)^2 \equiv a_\rho$$

Euler-Lagrange equations for meson mean fields

$$\sigma = a_\sigma \left(n_{sn} + n_{sp} - b\,\sigma^2 - c\,\sigma^3\right)$$

$$\omega = a_\omega\, n_B$$

$$\delta = -a_\delta \left(n_{sn} - n_{sp}\right)$$

$$\rho = -\frac{1}{2} a_\rho\, n_B\, \alpha$$

Scalar density of nucleons

$$n_{si} = \frac{m_i^{(eff)}}{\pi^2} \int_0^\infty \frac{k^2 dk}{\sqrt{k^2 + m_i^{(eff)\,2}}} \left(f_i^N - f_{ai}^N\right) \qquad (\,i = p, n)$$

Number density of nucleons

$$n_i = \frac{1}{\pi^2} \int_0^\infty k^2 dk \left(f_i^N - f_{ai}^N\right) \qquad (\,i = p, n)$$

Baryon number density: $n_B = n_n + n_p$

Asymmetry parameter: $\alpha = (n_n - n_p)/n_B$

Fermi-Dirac Distribution Functions

Particles

Antiparticles

$$f_i^N = \left[exp\frac{\sqrt{k^2 + m_i^{(eff)^2}} - \mu_i^{(eff)}}{T} + 1 \right]^{-1}$$

$$f_{ai}^N = \left[exp\frac{\sqrt{k^2 + m_i^{(eff)^2}} + \mu_i^{(eff)}}{T} + 1 \right]^{-1}$$

$(i = p, n)$

$$f_i^L = \left[exp\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T} + 1 \right]^{-1}$$

$$f_{ai}^L = \left[exp\frac{\sqrt{k^2 + m_i^2} + \mu_i}{T} + 1 \right]^{-1}$$

$(i = e, \mu, \nu_e, \nu_\mu)$

$$m_n^{(eff)} = m_N - \sigma + \delta \qquad m_p^{(eff)} = m_N - \sigma - \delta$$

$$\mu_n^{(eff)} = \mu_n - \omega + \rho/2 \qquad \mu_p^{(eff)} = \mu_p - \omega - \rho/2$$

Energy density

$$\varepsilon = \varepsilon_H + \varepsilon_L$$

$$\varepsilon_H = \sum_{i=p,n} \frac{1}{\pi^2} \int_0^\infty k^2 \sqrt{k^2 + m_n^{(eff)^2}} \left(f_i^N + f_{ai}^N\right) dk + U(\sigma) + \frac{1}{2}\left(\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} + \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho}\right)$$

$$\varepsilon_L = \sum_{i=e,\mu} \frac{1}{\pi^2} \int_0^\infty k^2 \sqrt{k^2 + m_i{}^2} \left(f_i^L + f_{ai}^L\right) dk + \sum_{i=\nu_e,\nu_\mu} \frac{1}{2\pi^2} \int_0^\infty k^3 \left(f_i^L + f_{ai}^L\right) dk$$

Pressure

$$P = P_H + P_L$$

$$P_H = \sum_{i=p,n} \frac{1}{3\pi^2} \int_0^\infty \frac{k^4}{\sqrt{k^2 + m_n^{(eff)^2}}} \left(f_i^N + f_{ai}^N\right) dk - U(\sigma) + \frac{1}{2}\left(-\frac{\sigma^2}{a_\sigma} + \frac{\omega^2}{a_\omega} - \frac{\delta^2}{a_\delta} + \frac{\rho^2}{a_\rho}\right)$$

$$P_L = \sum_{i=e,\mu} \frac{1}{3\pi^2} \int_0^\infty \frac{k^4}{\sqrt{k^2 + m_i{}^2}} \left(f_i^L + f_{ai}^L\right) dk + \sum_{i=\nu_e,\nu_\mu} \frac{1}{6\pi^2} \int_0^\infty k^3 \left(f_i^L + f_{ai}^L\right) dk$$

Entropy

$$S = S_H + S_L$$

$$S_H = -\sum_{i=p,n} \frac{1}{\pi^2} \int_0^\infty k^2 [f_i^N ln f_i^N + (1 - f_i^N) ln(1 - f_i^N) + f_i^N ln f_{ai}^N + (1 - f_{ai}^N) ln(1 - f_{ai}^N)]\, dk$$

$$S_L = -\sum_{i=e,\mu} \frac{1}{\pi^2} \int_0^\infty k^2 [f_i^L ln f_i^L + (1 - f_i^L) ln(1 - f_i^L) + f_{ai}^L ln f_{ai}^L + (1 - f_{ai}^L) ln(1 - f_{ai}^L)]\, dk$$

$$- \sum_{i=\nu_e,\nu_\mu} \frac{1}{2\pi^2} \int_0^\infty k^2 [f_i^L ln f_i^L + (1 - f_i^L) ln(1 - f_i^L) + f_{ai}^L ln f_{ai}^L + (1 - f_{ai}^L) ln(1 - f_{ai}^L)]\, dk$$

Model Parameters

$$a_\sigma\,,\ a_\omega\,,\ a_\delta\,,\ a_\rho\,,\ b,\ c$$

Symmetric nuclear matter in Saturation density ($\alpha = 0; n = n_0$)

$$m_N^* = \gamma\, m_N\,, \qquad \sigma_0 = (1-\gamma)\, m_N$$

$$\left.\frac{d\varepsilon(n,\alpha)}{dn}\right|_{\substack{n=n_0\\ \alpha=0}} = \frac{\varepsilon(n_0,0)}{n_0} = m_N + f_0\,, \qquad f_0 = \frac{B}{A},$$ Binding energy per baryon

$$\varepsilon_0 = n_0(m_N + f_0) = \frac{2}{\pi^2}\int_0^{k_F(n_0)} \sqrt{k^2+(m_N-\sigma_0)^2}\,k^2dk + \frac{b}{3}m_N\,\sigma_0^{\,3} + \frac{c}{4}\sigma_0^{\,4} + \frac{1}{2}\left(\frac{\sigma_0^{\,2}}{a_\sigma} + n_0^{\,2}a_\omega\right)$$

$$K = 9\, n_0^{\,2}\,\frac{d^2}{dn^2}\left(\frac{\varepsilon(n,\alpha)}{n}\right)\Bigg|_{\substack{n=n_0\\ \alpha=0}}$$ Compressibility module

$$\frac{\varepsilon_{sym}}{n} = E_{sym}(n)\,\alpha^2 \quad \Longrightarrow \quad E_{sym}(n) = \frac{1}{2n}\left.\frac{d^2\varepsilon(n,\alpha)}{d\alpha^2}\right|_{\alpha=0}$$ Symmetry energy

$RMF_{\sigma\omega\delta\rho}$ Model Parameters

Saturation Properties
$m_N = 938,93\ MeV$
$n_0 = 0,153\ fm^{-3}$
$\gamma = \dfrac{m_N^*}{m_N} = 0,78$
$K = 300\ MeV$
$f_0 = -16,3\quad MeV$
$E_{sym}^{(0)} = 32,5\ MeV$

Parameters	σωρ	σωρδ
a_σ , fm^2	9.154	9.154
a_ω , fm^2	4.828	4.828
a_δ , fm^2	0	2.5
a_ρ, fm^2	4.794	13.621
b , fm^{-1}	$1.654\ 10^{-2}$	$1.654\ 10^{-2}$
c	$1.319\ 10^{-2}$	$1.319\ 10^{-2}$

Alaverdyan G.B., *Astrophysics* **52**, 132–150 (2009)

Computational Scheme

10 equations →

$$\sigma = a_\sigma \left(n_{sn} + n_{sp} - b\,\sigma^2 - c\,\sigma^3\right)$$

$$\omega = a_\omega\, n_B$$

$$\delta = -a_\delta \left(n_{sn} - n_{sp}\right)$$

$$\rho = -\frac{1}{2} a_\rho\, n_B\, \alpha$$

$n \rightleftarrows p + e + \tilde{\nu}_e$ ⇒ $$\mu_n = \mu_p + \mu_e - \mu_{\nu_e}$$

$n \rightleftarrows p + \mu + \tilde{\nu}_\mu$ ⇒ $$\mu_n = \mu_p + \mu_\mu - \mu_{\nu_\mu}$$

Charge neutrality ⇒ $$n_p = n_e + n_\mu$$

Baryon number density $$n_B = n_p + n_n$$

$Y_e = 0.4$ ⇒ $$n_e + n_{\nu_e} = Y_e n_B$$

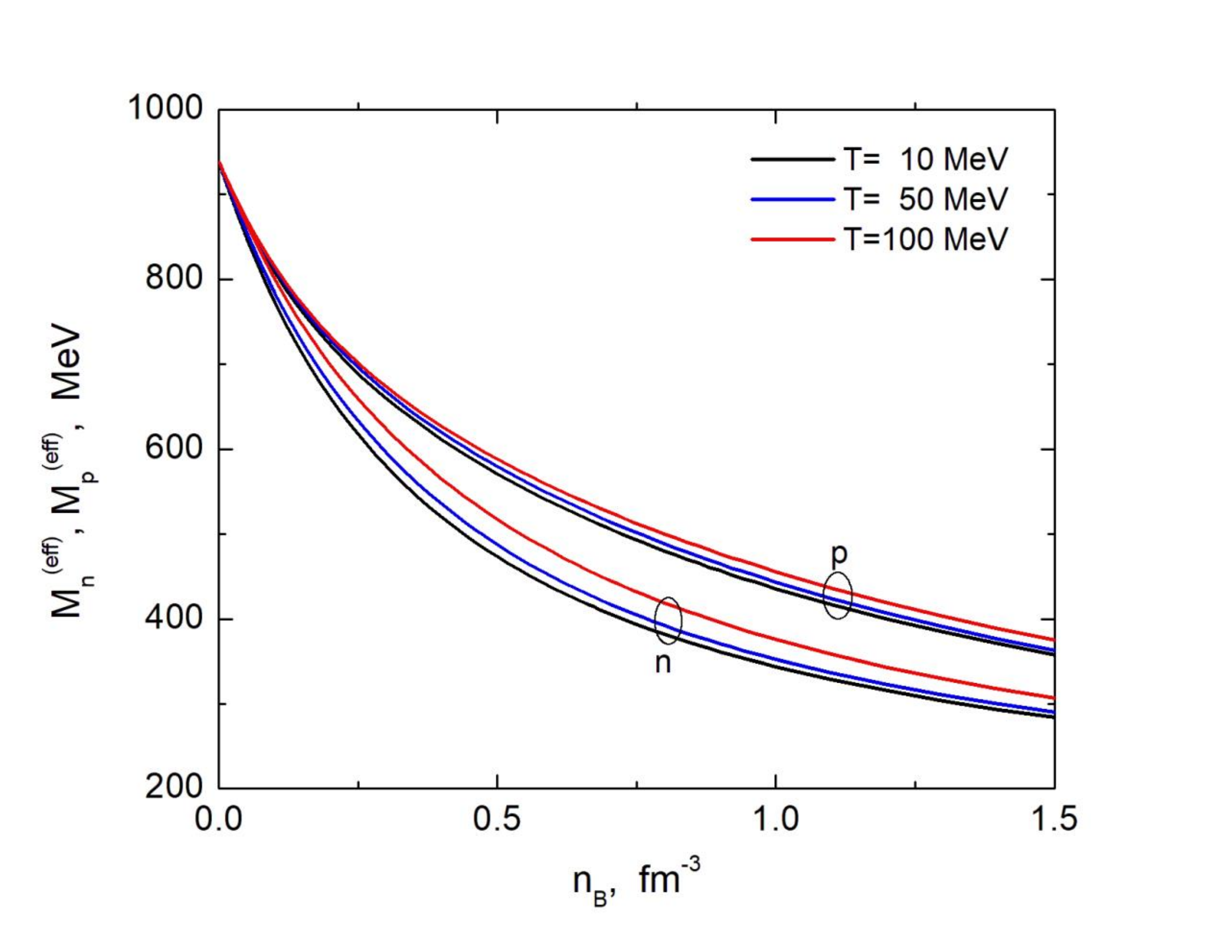
$Y_\mu = 0$ ⇒ $$n_\mu + n_{\nu_\mu} = Y_\mu n_B$$

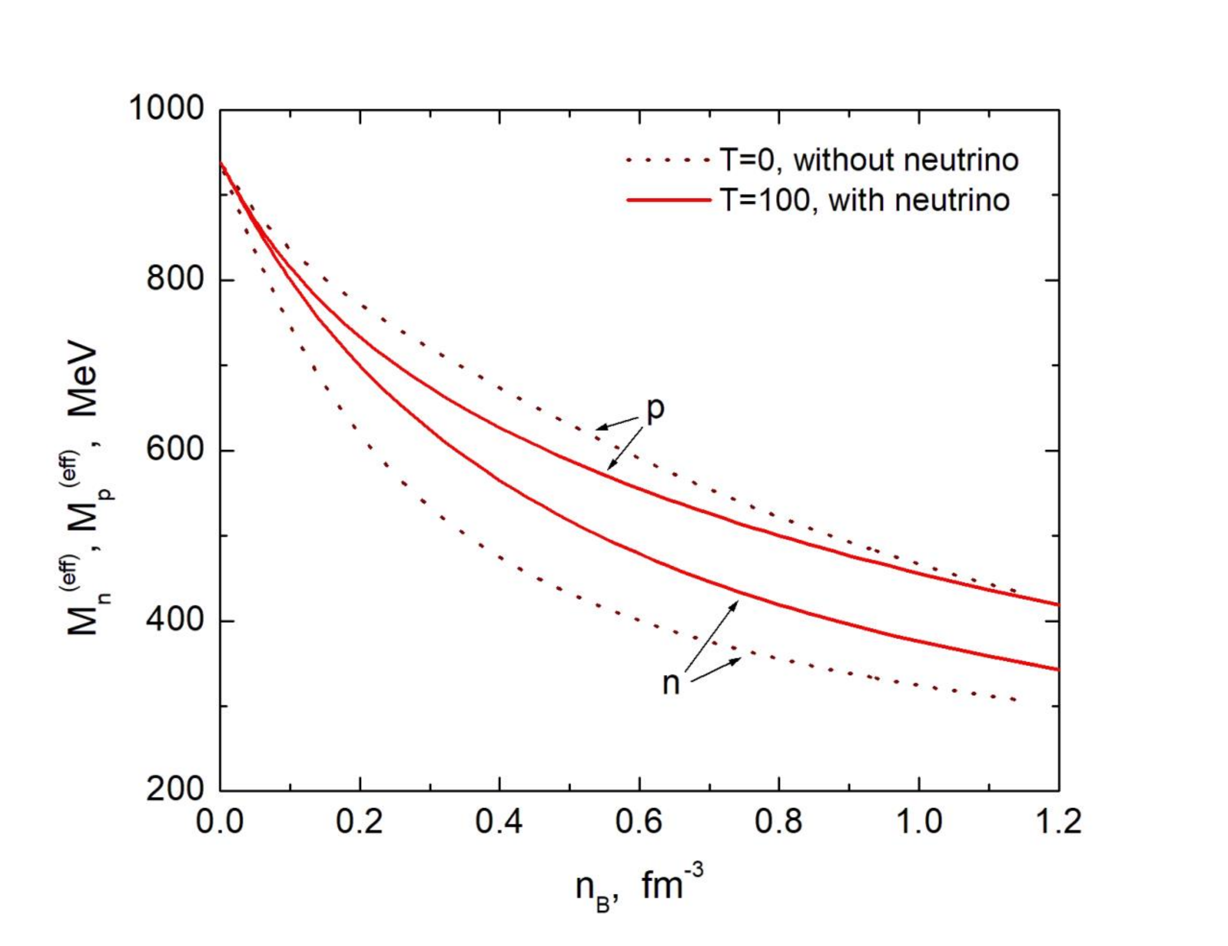
T, n_B

↓

$\sigma, \omega, \delta, \rho, \mu_n, \mu_p, \mu_e, \mu_{\nu_e}, \mu_\mu, \mu_{\nu_\mu}$

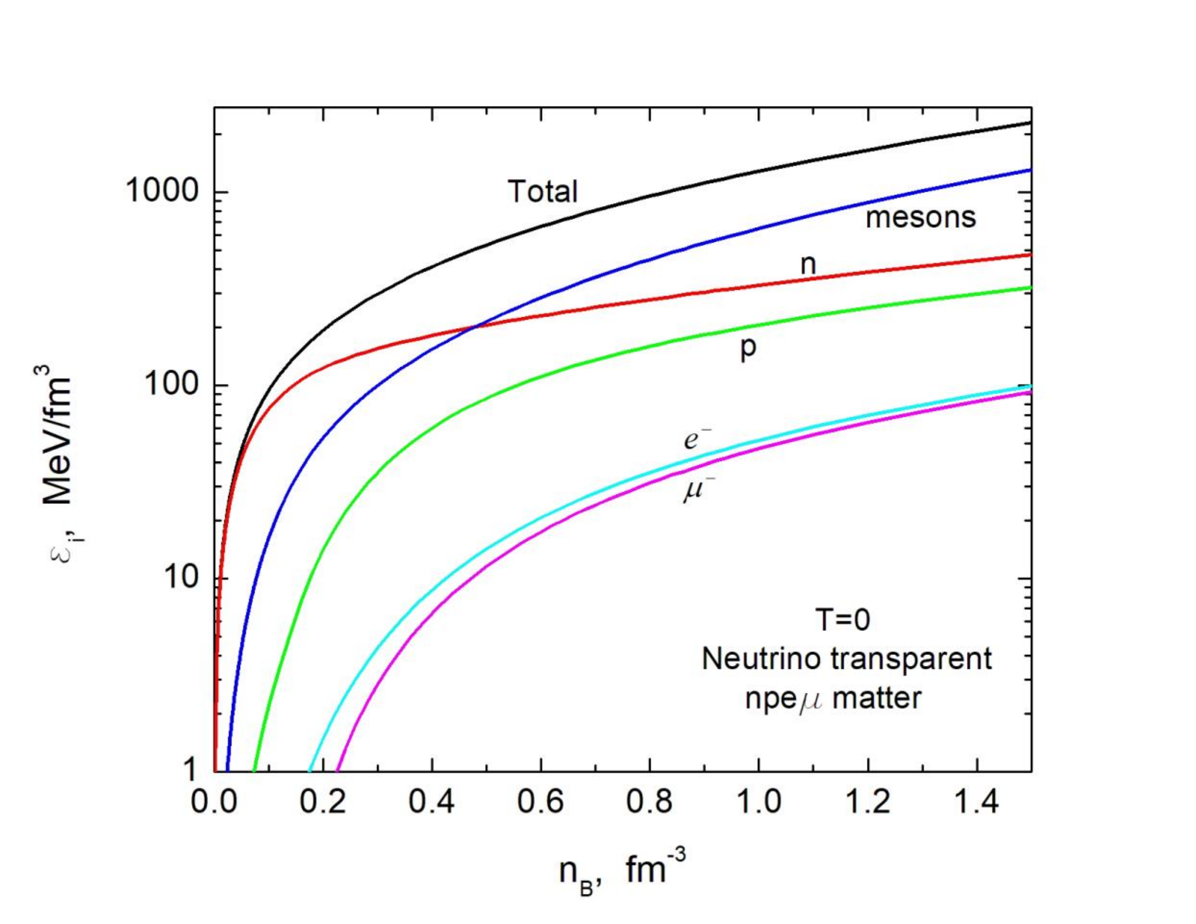
Neutron-proton effective mass splitting due to δ-meson



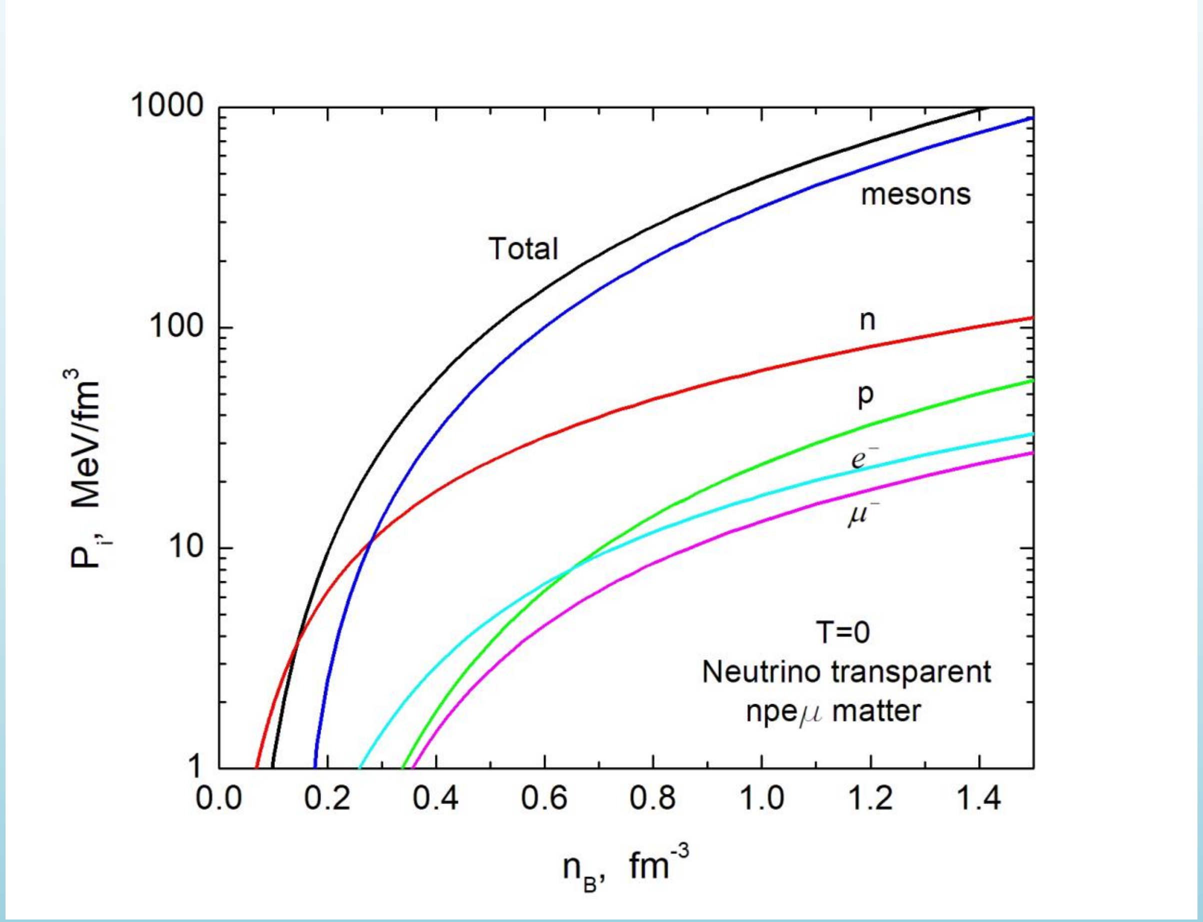


Cold neutrino-transparent hadronic matter

Constituents contribution to the energy density

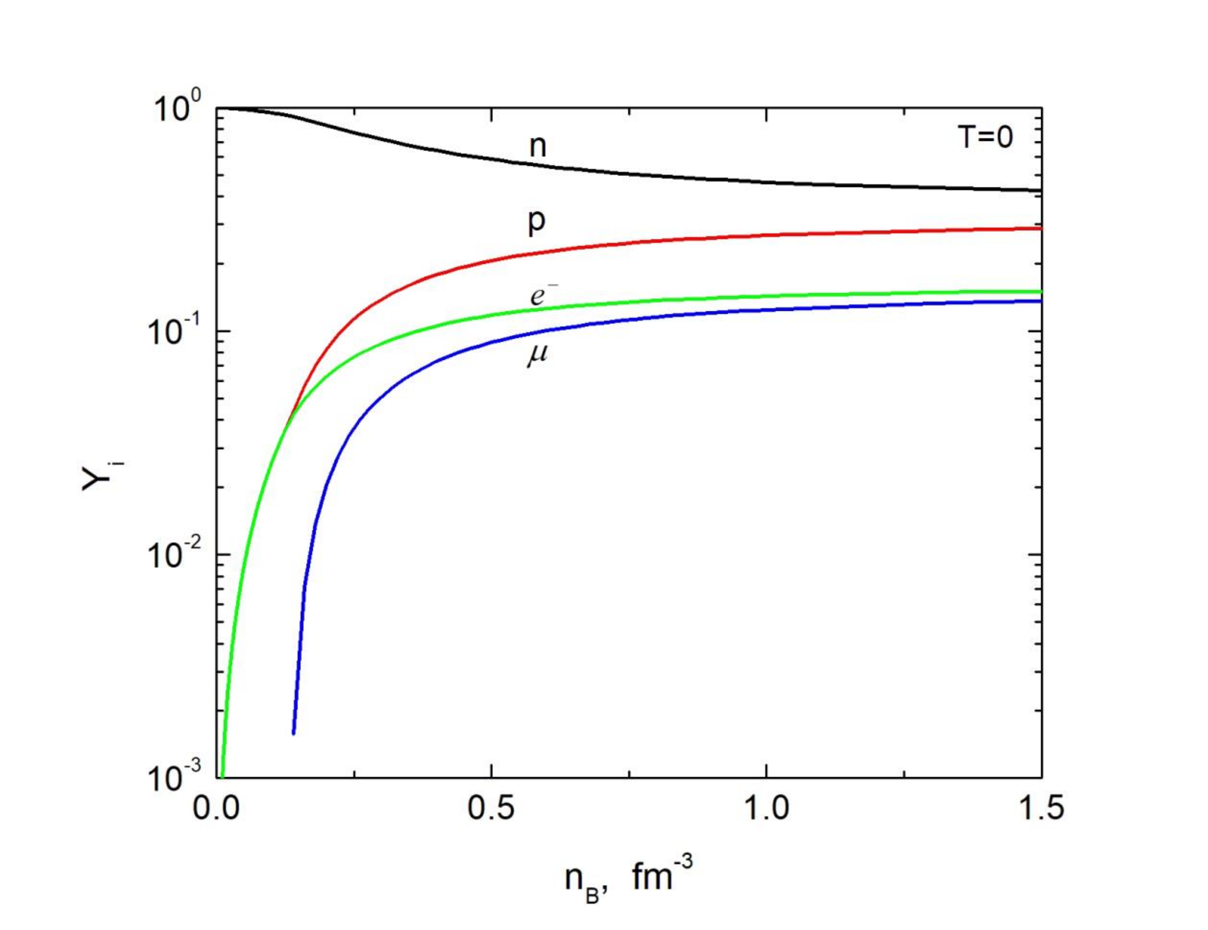


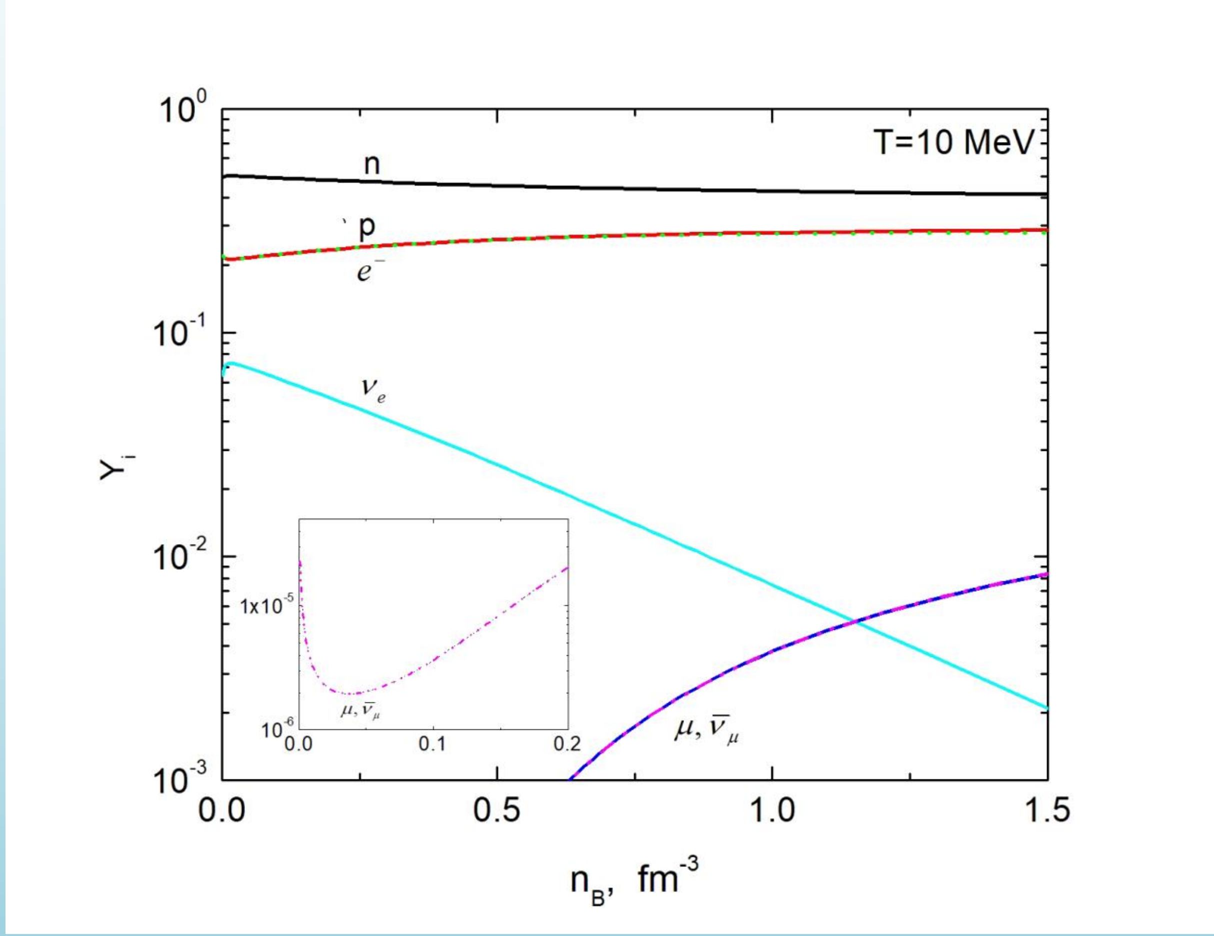
Constituents contribution to the pressure



Particle composition

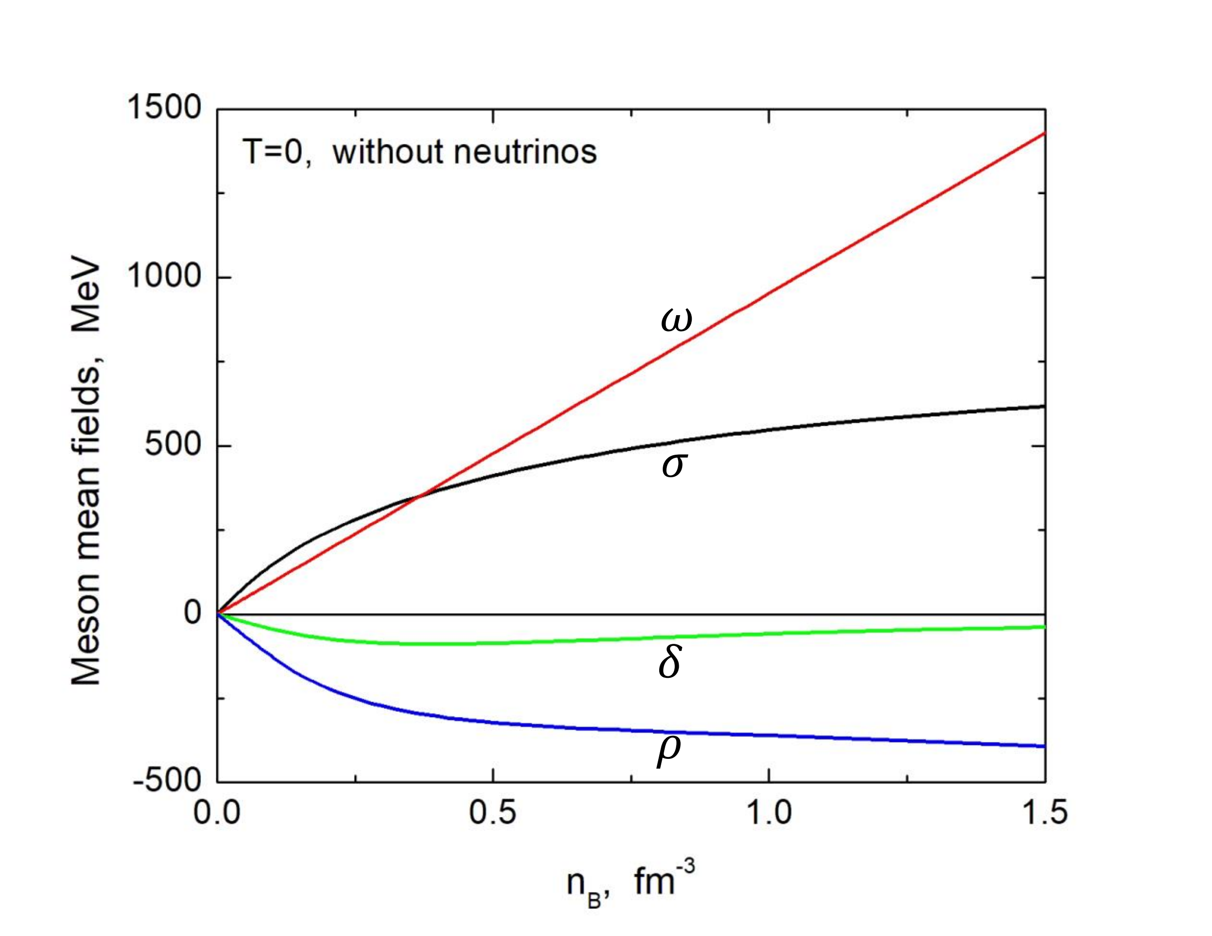
$$Y_i = \frac{n_i}{n_B}, \quad \text{i} = \text{n,p,e,μ}, \nu_e, \nu_\mu$$



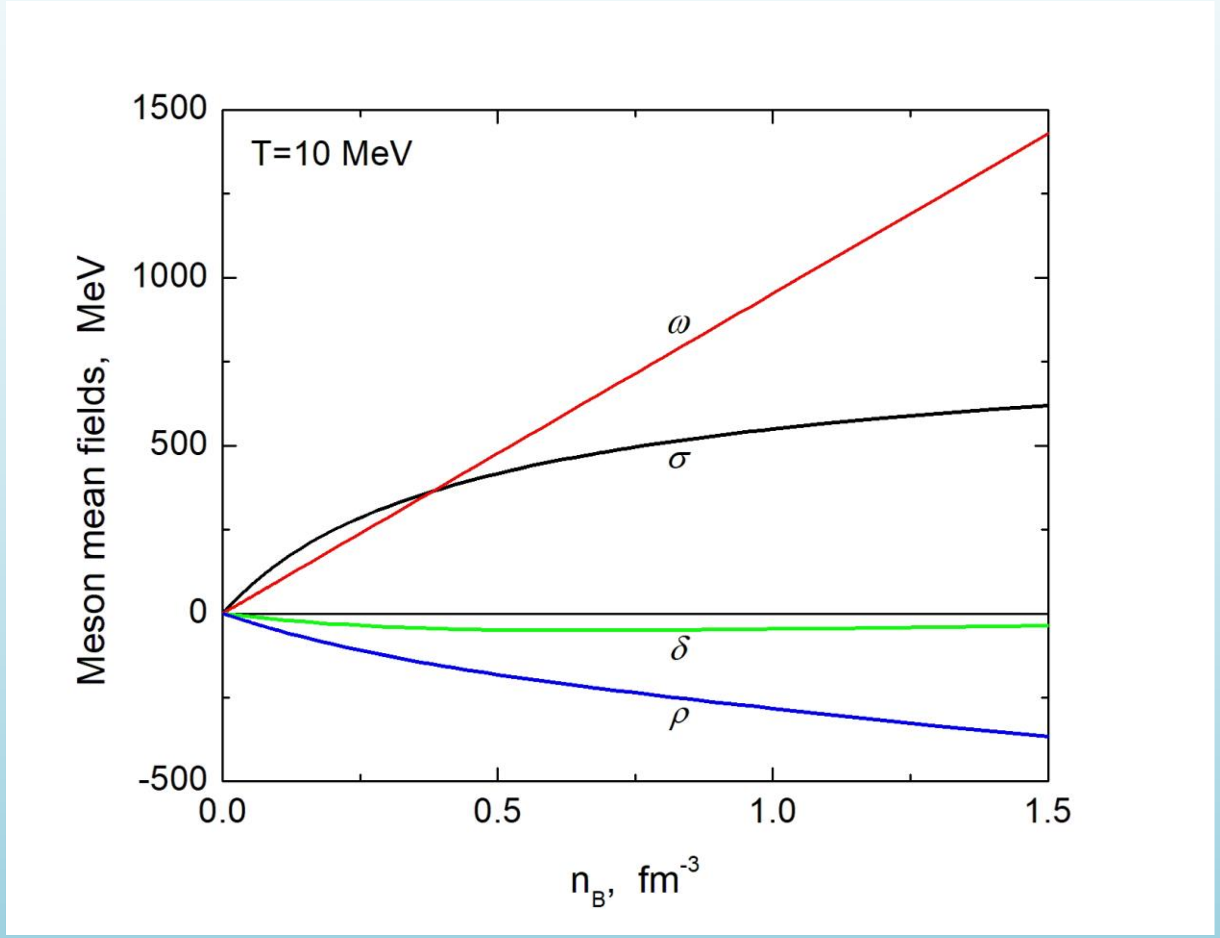


Meson mean-fields

$T = 0$, without neutrinos

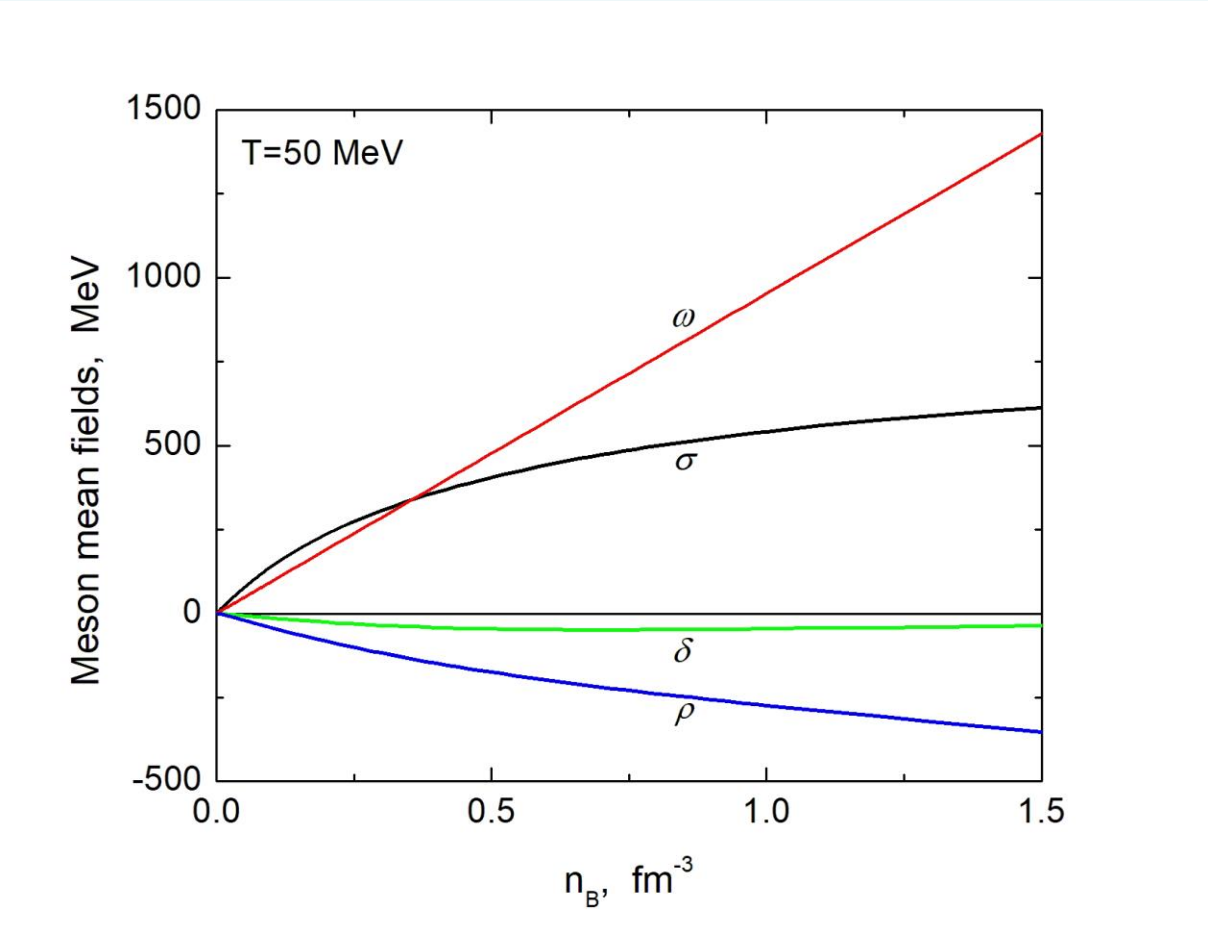


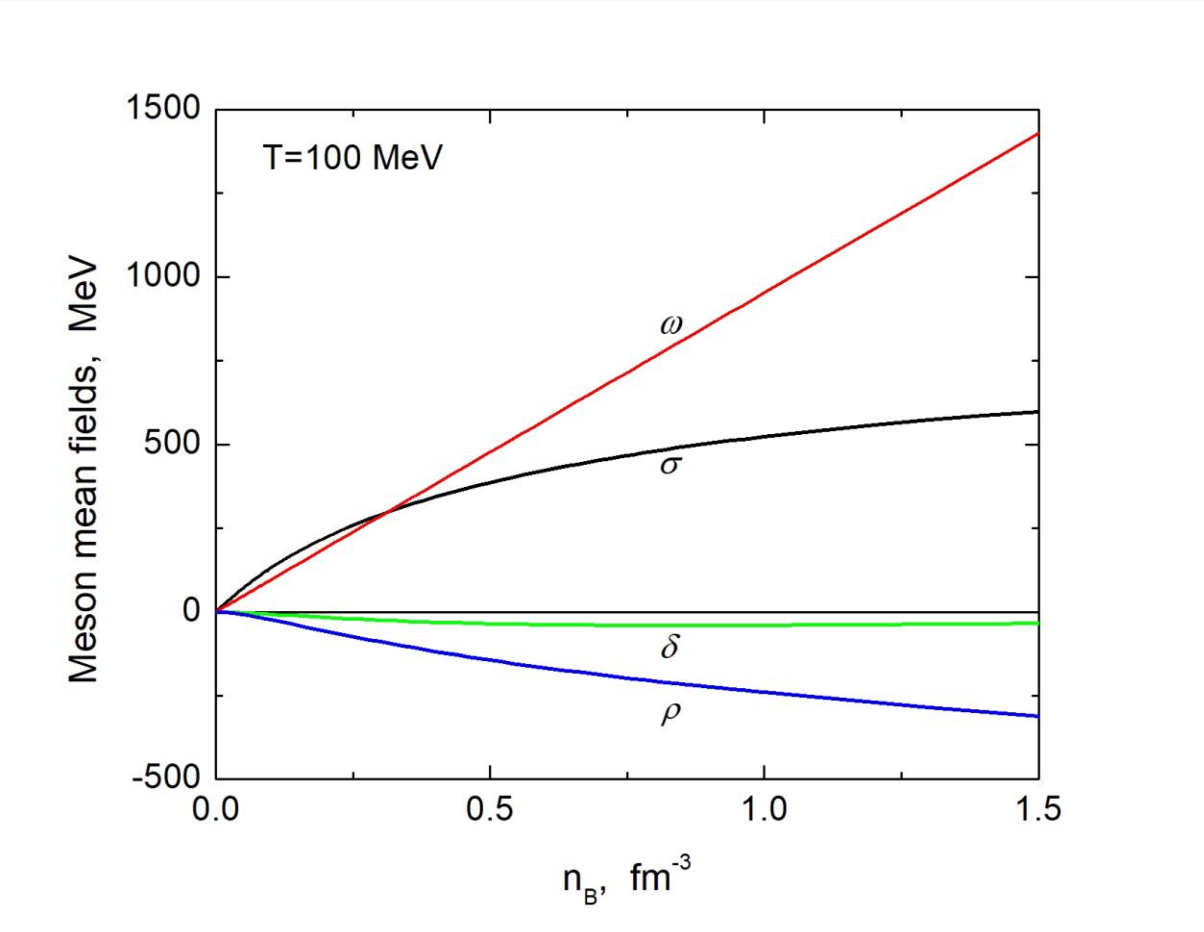
$T = 10\ MeV$, with neutrinos



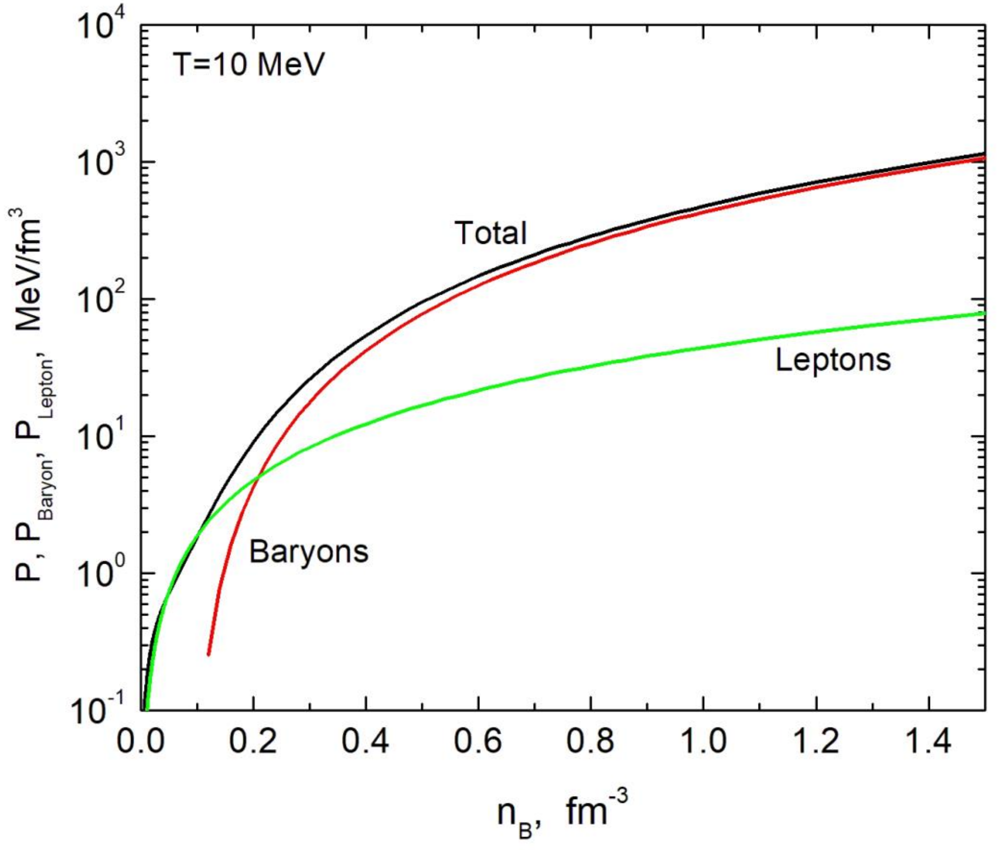
Meson mean-fields

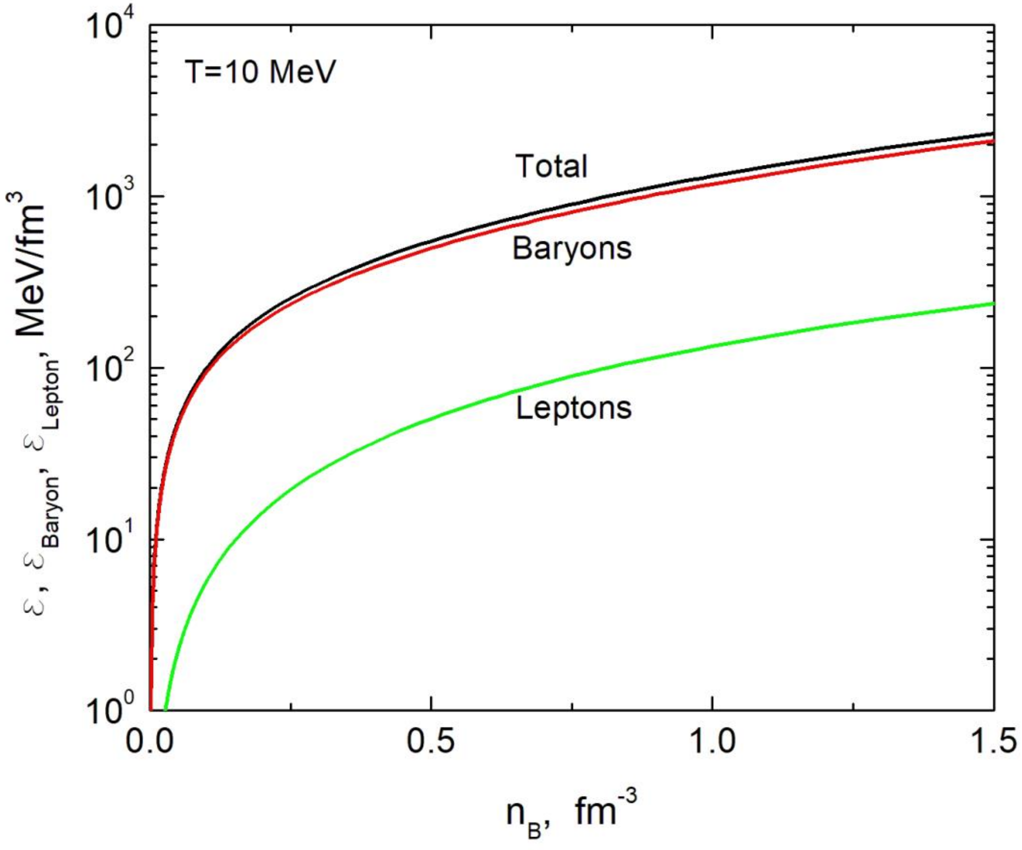
With neutrinos



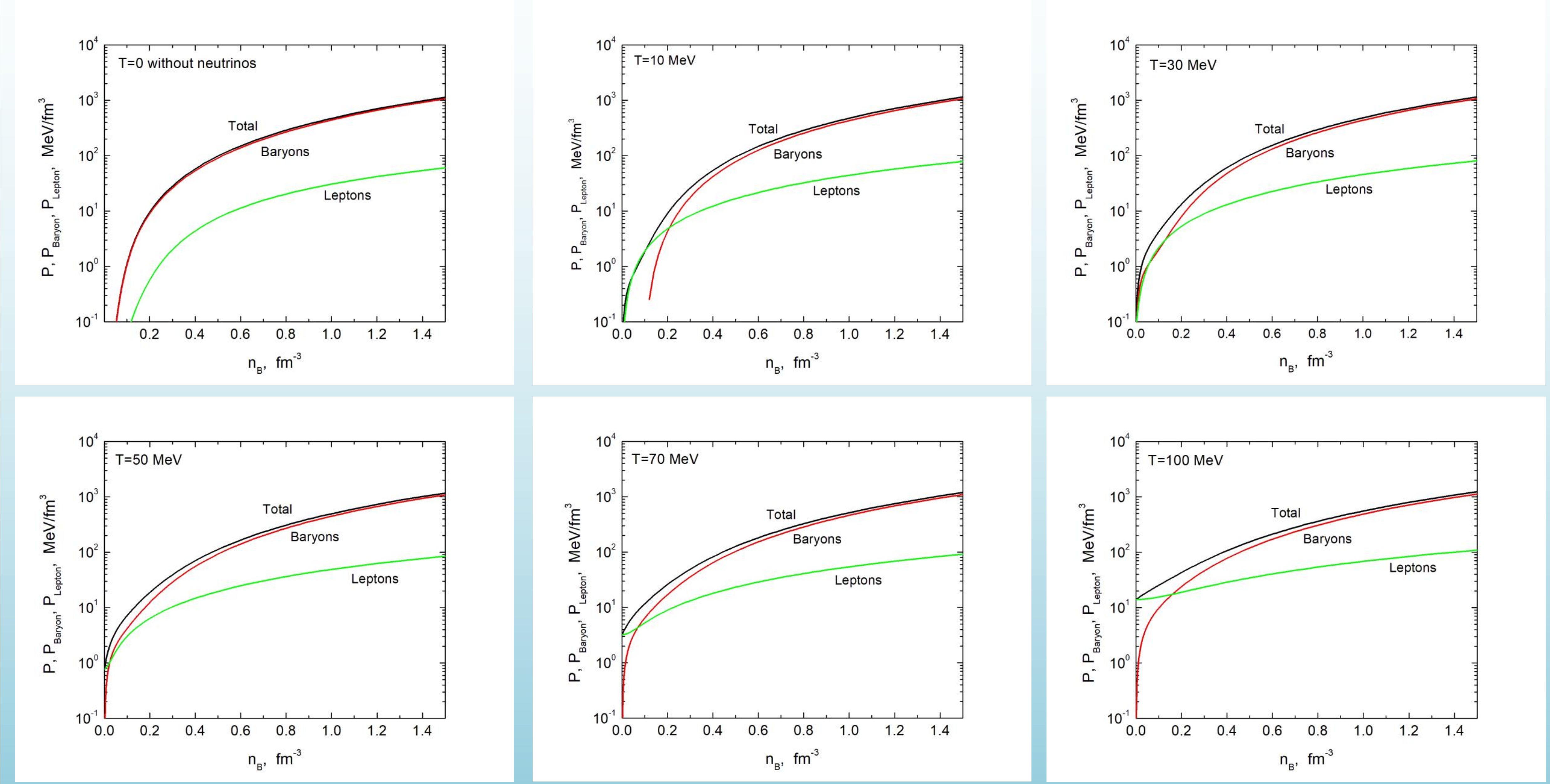


Baryon and lepton contributions

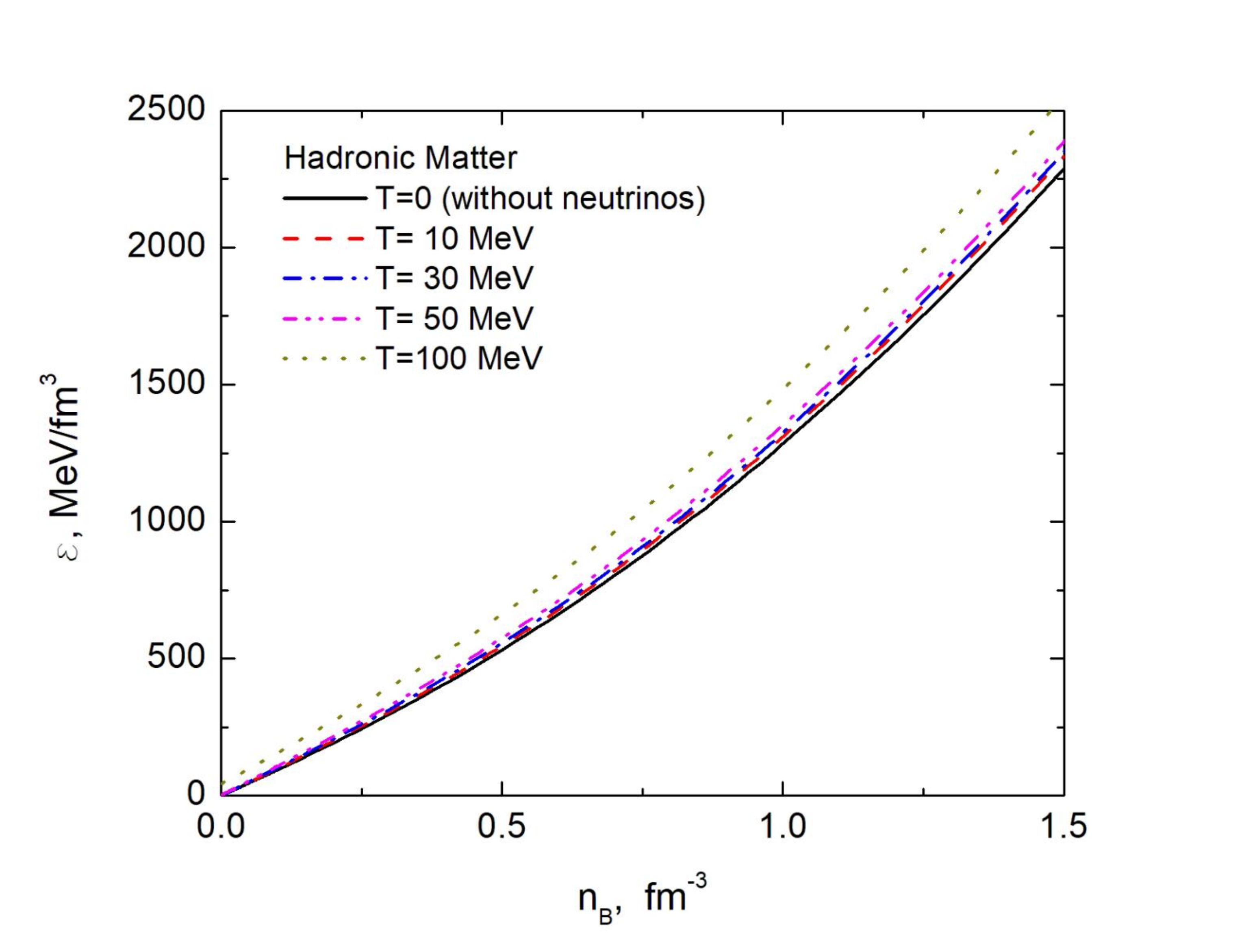




Baryon and lepton contributions to the pressure



Energy Density



Quark Matter Description within NJL model

Constituents: $u, d, s, e, \nu_e, \mu, \nu_\mu, \tau, \nu_\tau$

$$\mathcal{L}_{QM} = \mathcal{L}_{NJL} + \mathcal{L}_{Lept}$$

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \hat{m}_0\right)\psi + G\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2] - K\left\{det_f(\bar{\psi}(1+\gamma_5)\psi) + det_f(\bar{\psi}(1-\gamma_5)\psi)\right\}$$

$$\mathcal{L}_{Lept} = \sum_{i=e,\nu_e,\mu,\nu_\mu} \overline{\psi_i}\left(i\gamma^\mu\partial_\mu - m_i\right)\psi_i$$

ψ_f^c $\quad f = u, d, s$ $\quad c = r, g, b$ $\quad \hat{m}_0 = diag(m_{0u}, m_{0d}, m_{0s})$

$\lambda_a (a = 1,2, \dots 8)$ Gell-Mann matrices, SU(3) generators

$$\lambda_0 = \sqrt{2/_3}\,\hat{I}$$

Constituent Quark Masses & Quark Condensates

$$M_u = m_{0u} - 4G\,\sigma_u + 2K\sigma_d\sigma_s\,,$$

$$M_d = m_{0d} - 4G\,\sigma_d + 2K\sigma_s\sigma_u\,,$$

$$M_s = m_{0s} - 4G\,\sigma_s + 2K\sigma_u\sigma_d\,.$$

$$\sigma_f\left(T, M_f, \mu_f\right) = \langle\bar{\psi}_f\psi_f\rangle =$$

$$= -\frac{3}{\pi^2}M_f\int_0^\Lambda dk\,\frac{k^2}{E_f\left(k, M_f\right)}\left[1 - \frac{1}{1+e^{\frac{E_f(k,M_f)-\mu_f}{T}}} - \frac{1}{1+e^{\frac{E_f(k,M_f)+\mu_f}{T}}}\right]. \qquad (f = u, d, s)$$

$$E_f\left(k, M_f\right) = \sqrt{k^2 + M_f^2}$$

Grand Potential for Quark Matter

$$\Omega_{QP} = \frac{3}{\pi^2} \sum_{f=u,d,s} \int_0^{\Lambda} dk\, k^2 \left(E_f(k, M_{f0}) - E_f(k, M_f)\right)$$

$$- \frac{3T}{\pi^2} \sum_{f=u,d,s} \left\{ \int_0^{\Lambda} dk\, k^2 \left[ln\left(1 + e^{-\frac{E_f(k,M_f)-\mu_f}{T}}\right) + ln\left(1 + e^{-\frac{E_f(k,M_f)+\mu_f}{T}}\right)\right]\right\}$$

$$+2G({\sigma_u}^2 + {\sigma_d}^2 + {\sigma_s}^2 - {\sigma_{u0}}^2 - {\sigma_{d0}}^2 - {\sigma_{s0}}^2) - 4K\,(\sigma_u\sigma_d\sigma_s - \sigma_{u0}\sigma_{d0}\sigma_{s0})$$

$$- \frac{T}{2\pi^2} \sum_{l} g_l \int_0^{\infty} dk\, k^2 \left[ln\left(1 + e^{-\frac{E_l(k)-\mu_l}{T}}\right) + ln\left(1 + e^{-\frac{E_l(k)+\mu_l}{T}}\right)\right]$$

$$E_l(k) = \sqrt{k^2 + m_l^2}$$

Lepton spin degeneracy:

$g_e = g_\mu = 2 \qquad g_{\nu_e} = g_{\nu_\mu} = 1$

Computation Scheme of Thermodynamic Quantities

$m_{0u} = m_{0d} = 5.5$ MeV, $m_{0s} = 140.7$ MeV,

$\Lambda = 602.3$ MeV, $G = 1.835/\Lambda^2$, $K = 12.36/\Lambda^5$.

P. Rehberg, S.P. Klevansky, J. Hüfner,
Phys. Rev. C, **53**, 410, 1996.

$$M_u = m_{0u} - 4G\,\sigma_u + 2K\sigma_d\sigma_s\,,$$
$$M_d = m_{0d} - 4G\,\sigma_d + 2K\sigma_s\sigma_u\,,$$
$$M_s = m_{0s} - 4G\,\sigma_s + 2K\sigma_u\sigma_d\,.$$
$$n_f(T, M_f, \mu_f) = \frac{3}{\pi^2}\int_0^\Lambda dk\, k^2 \left[\frac{1}{1 + e^{\frac{E_f(k,M_f)-\mu_f}{T}}} - \frac{1}{1 + e^{\frac{E_f(k,M_f)+\mu_f}{T}}}\right] \qquad (f = u, d, s)$$

13 equations →

$$n_l(T, \mu_l) = \frac{g_l}{2\pi^2}\int_0^\infty dk\, k^2 \left[\frac{1}{1 + e^{\frac{E_l(k)-\mu_l}{T}}} - \frac{1}{1 + e^{\frac{E_l(k)+\mu_l}{T}}}\right] \qquad (l = e, \nu_e, \mu)$$
$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e - n_\mu = 0$$
$$n_\mu + n_{\nu_\mu} = Y_\mu n_B = 0$$
$$n_e + n_{\nu_e} = Y_e n_B$$
$$n_u + n_d + n_s = 3n_B$$

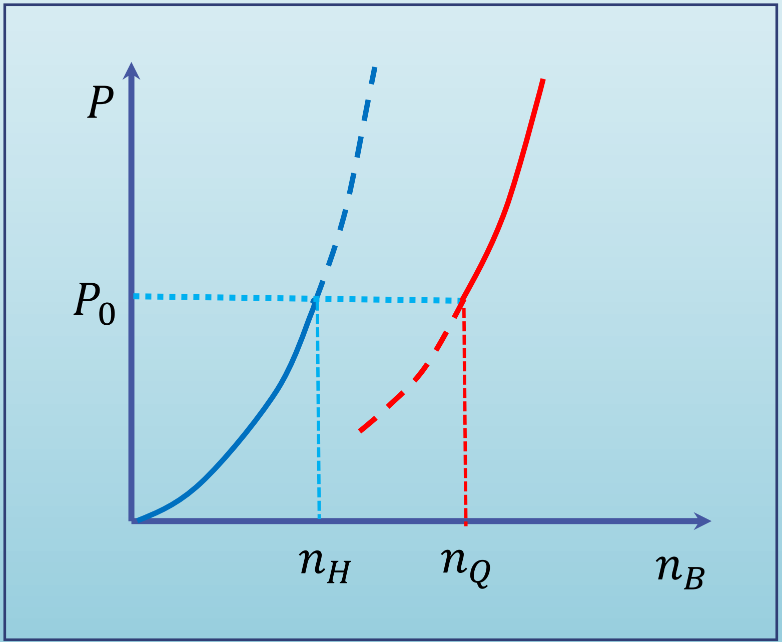
T, n_B → $M_u, M_d, M_s, n_u, n_d, n_s, n_e, n_\mu, n_{\nu_e}, \mu_u, \mu_e, \mu_\mu, \mu_{\nu_e}$

Hadron-Quark Phase Transition Scenario

Models

- Hadronic matter ⟹ $RMF_{\sigma\omega\delta\rho}$ model
- Quark matter ⟹ $SU(3)\ Local\ NJL$ model

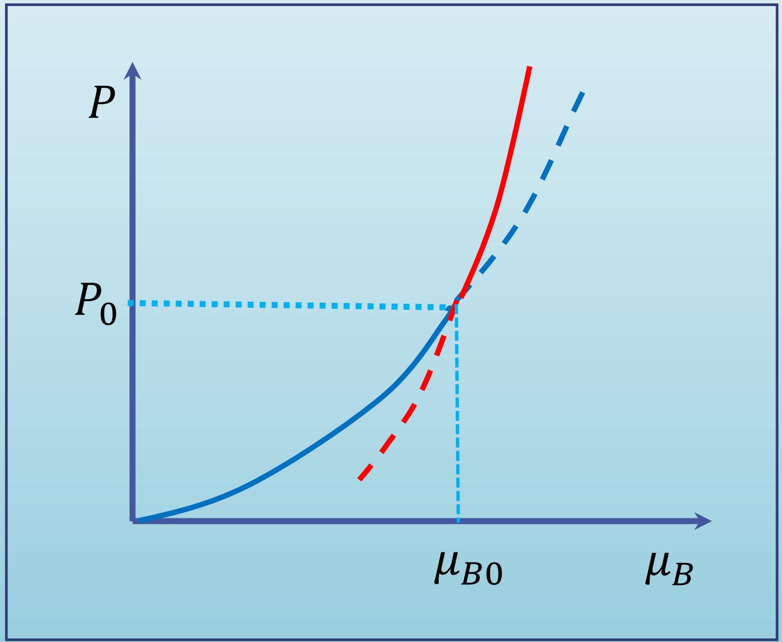
It was assumed that the surface tension is so high that the phase transition proceeds according to the Maxwell scenario.



$$P_{QM} = P_{HM} = P_0$$

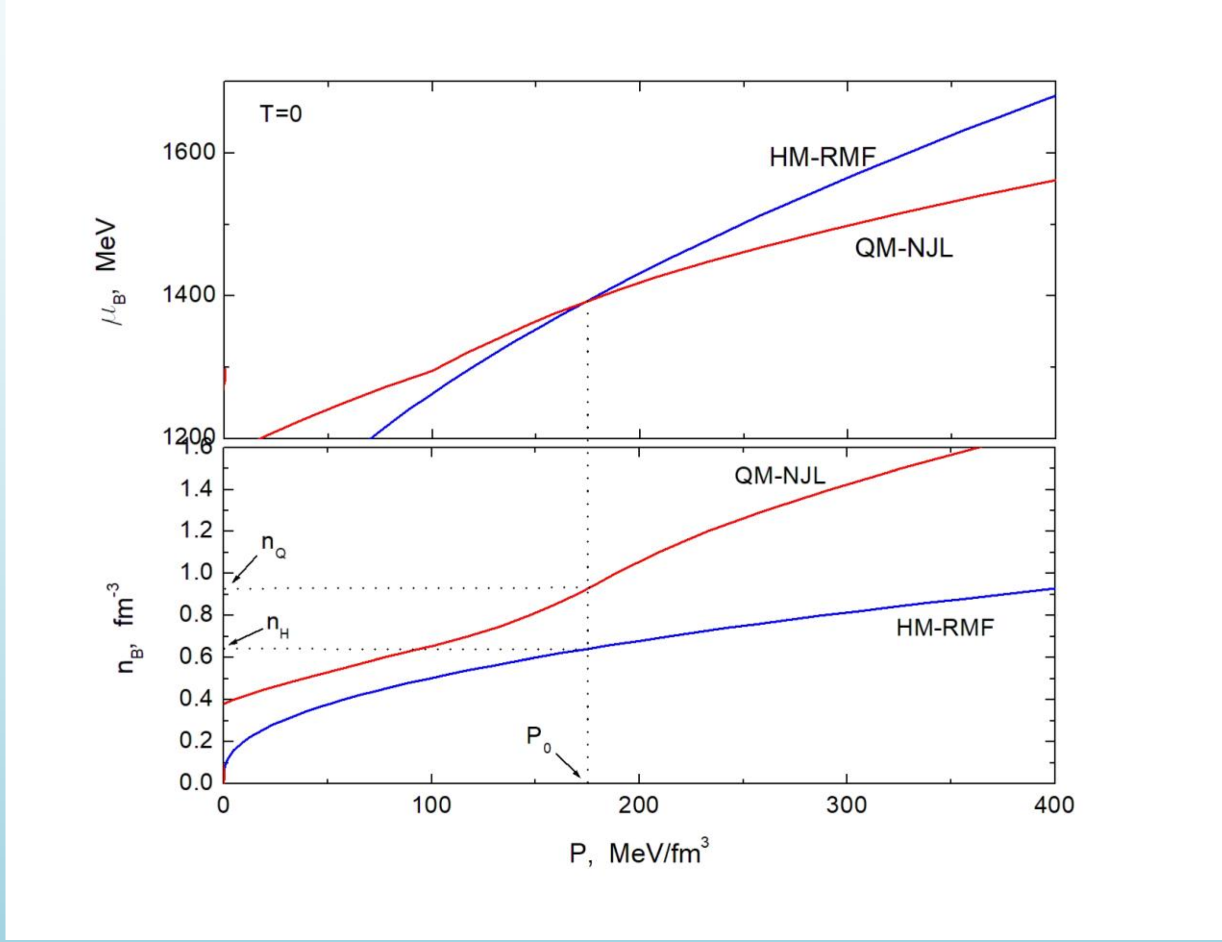
$$\mu_B^{QM} = \mu_B^{HM} = \mu_{B0}$$

$$T_{QM} = T_{HM} = T$$

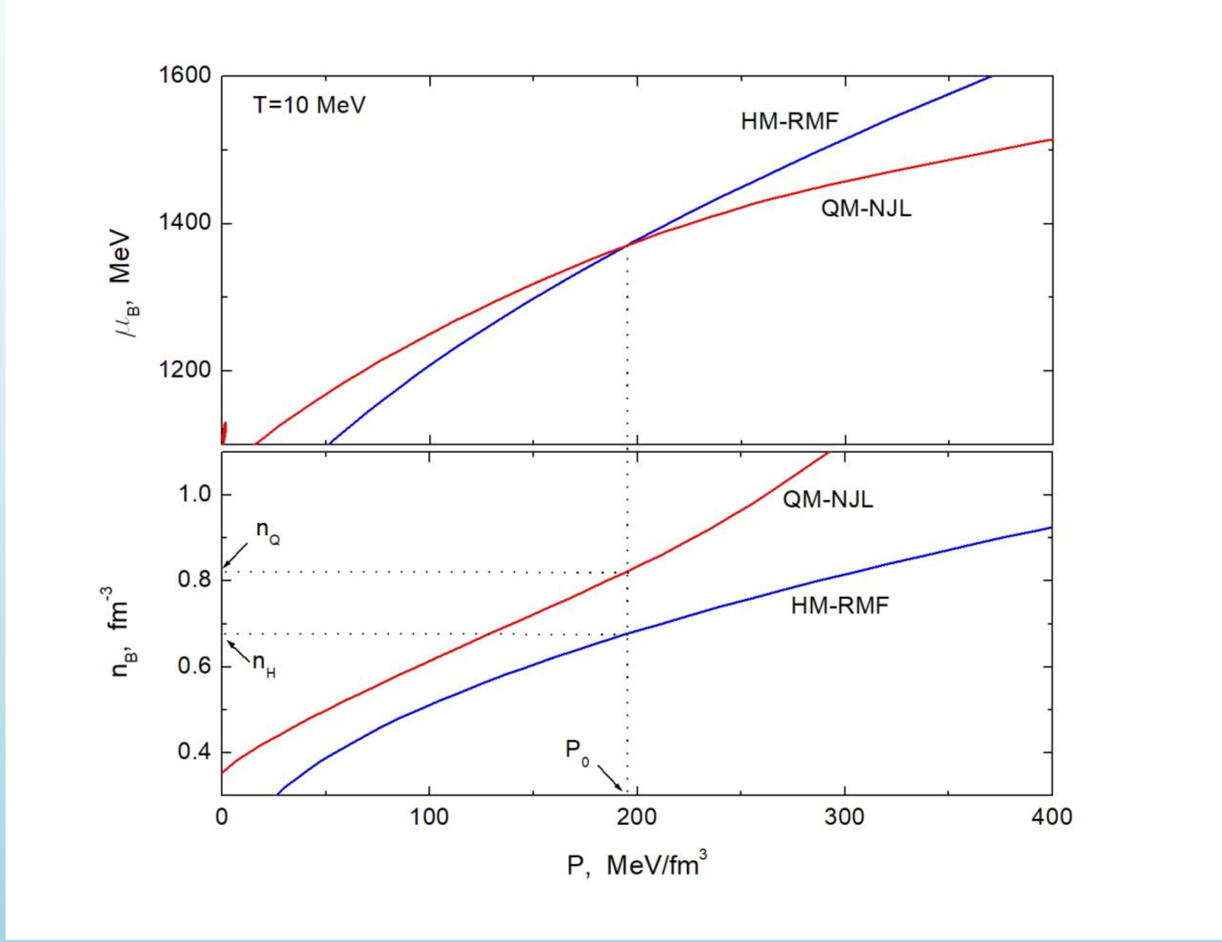


Phase Transition Parameters

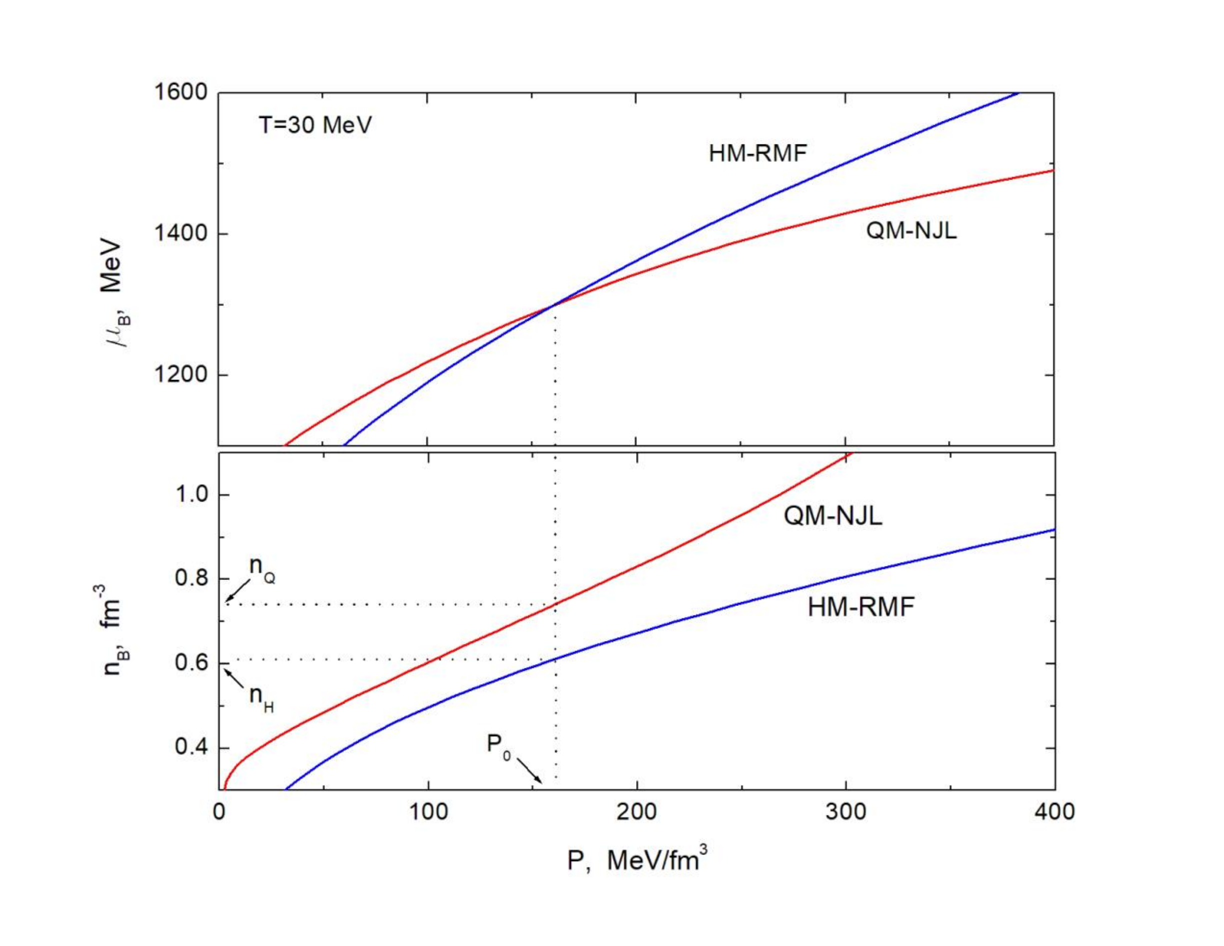
$T = 0,$ without neutrinos

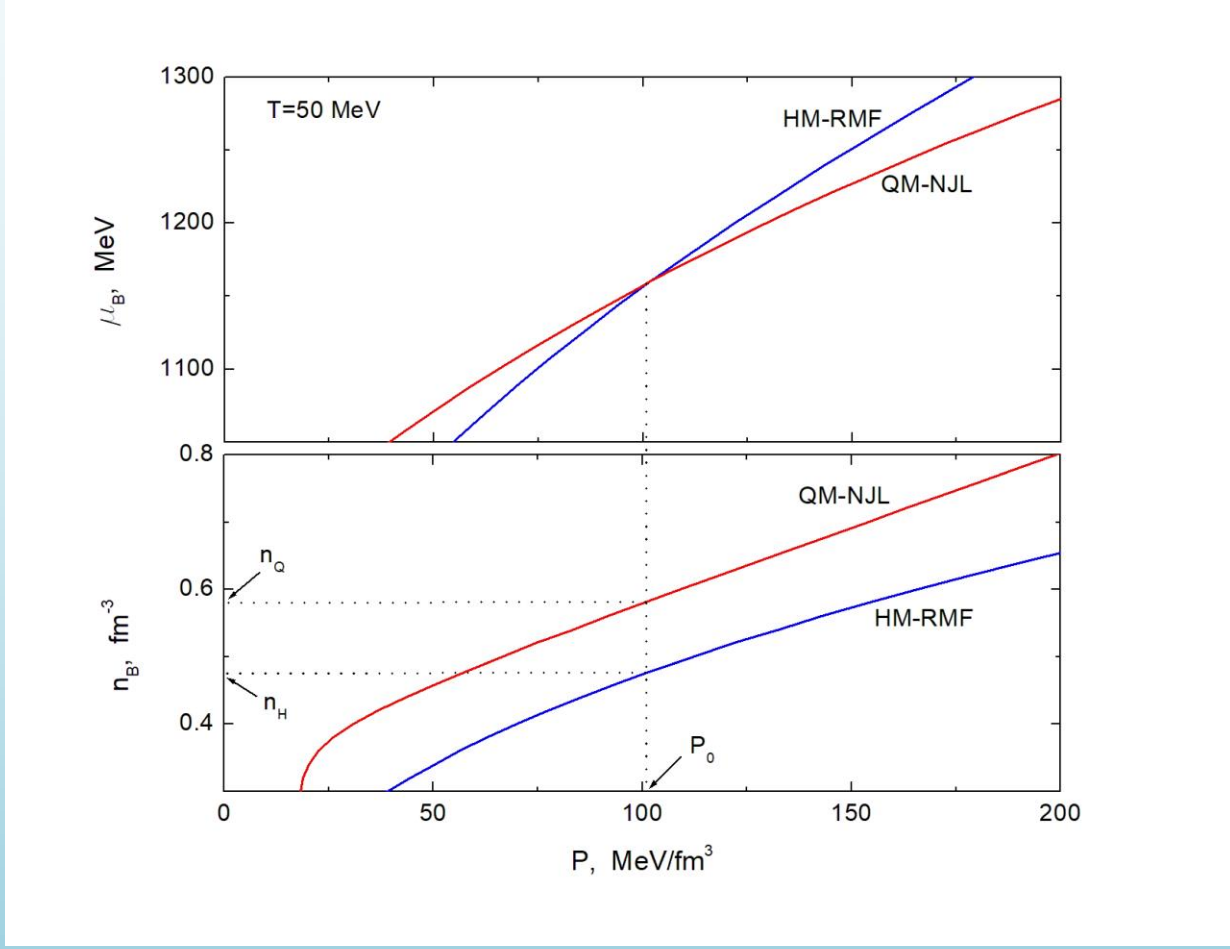


$T = 10\ MeV,$ with neutrinos



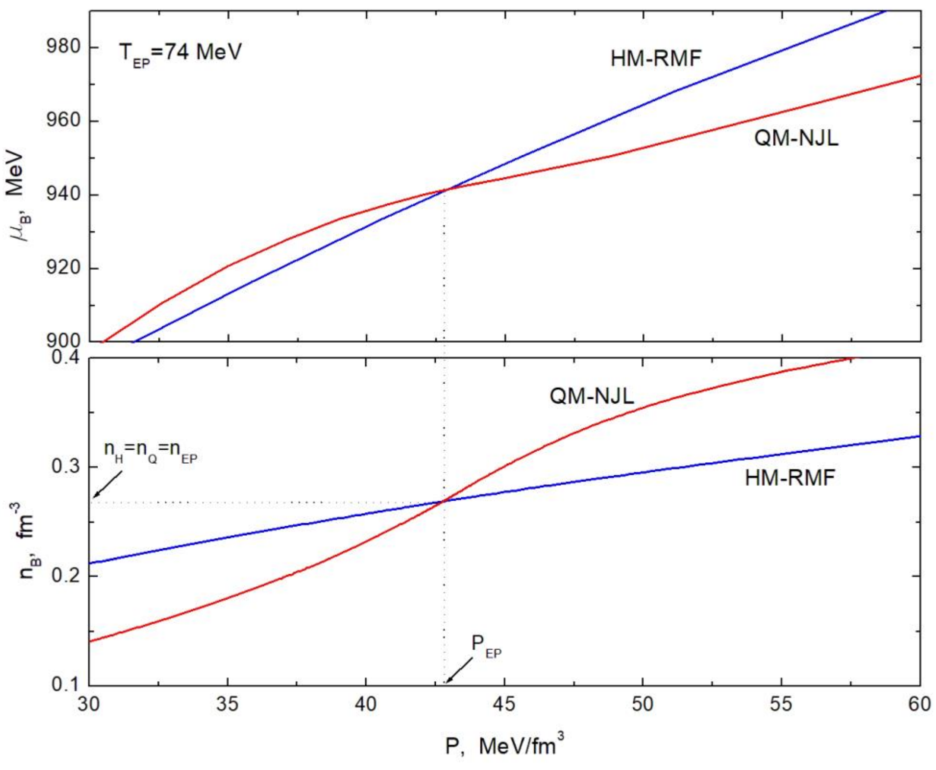
Phase Transition Parameters



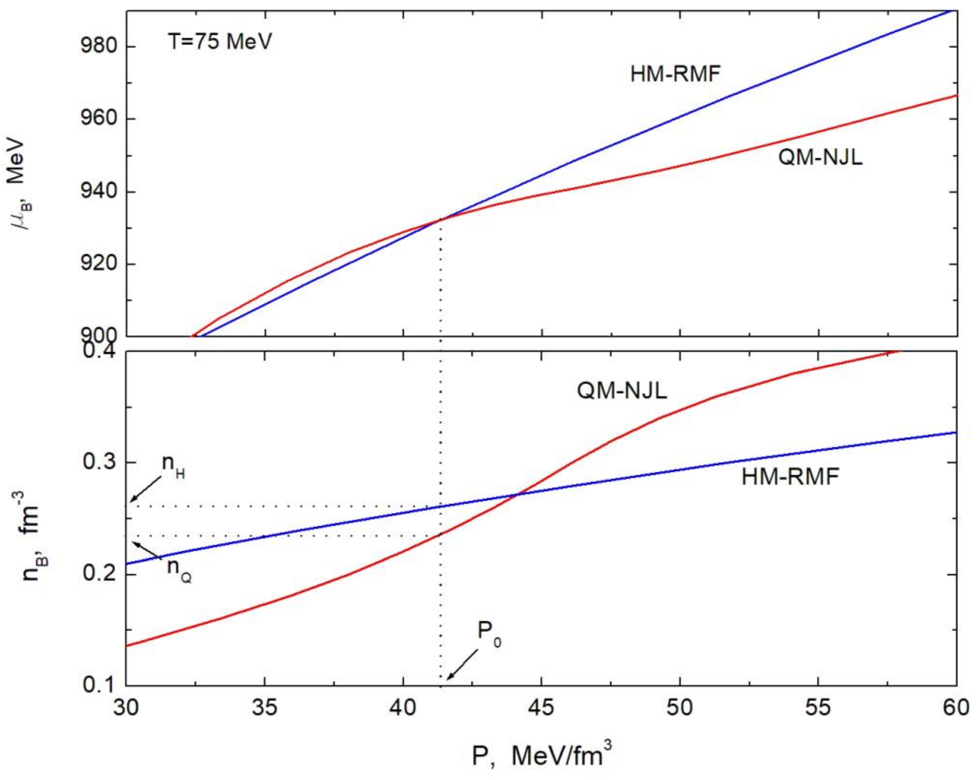


Critical End Point

Critical End Point



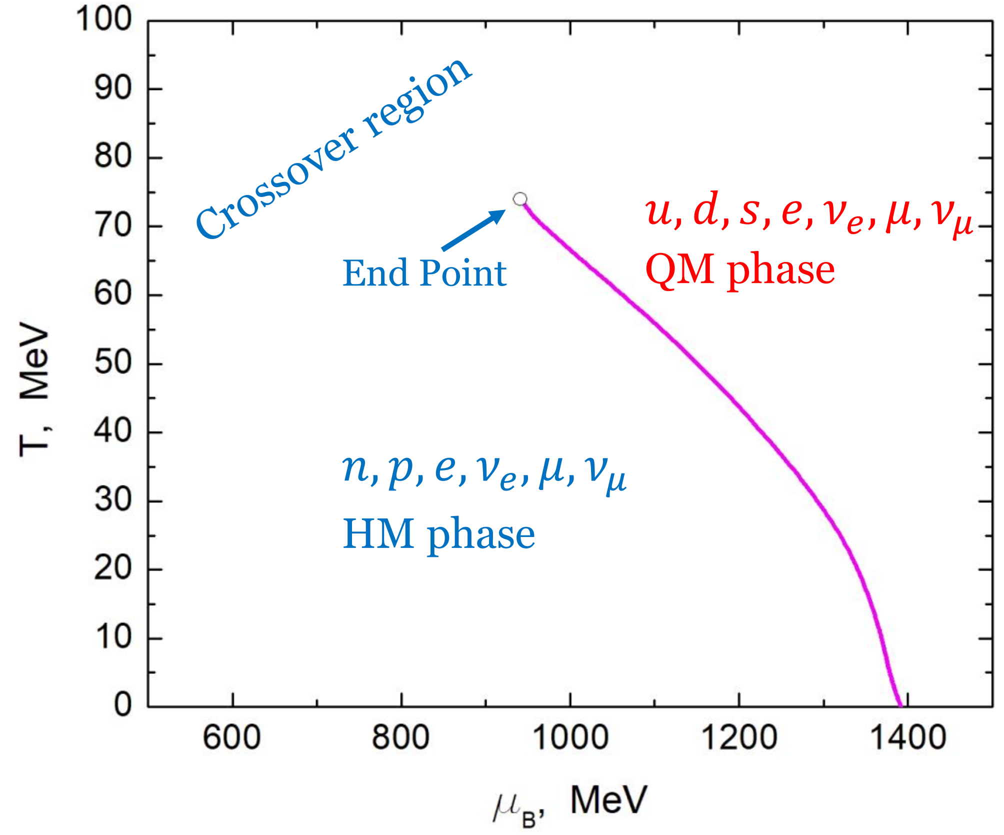
Phase Equilibrium in Crossover Region

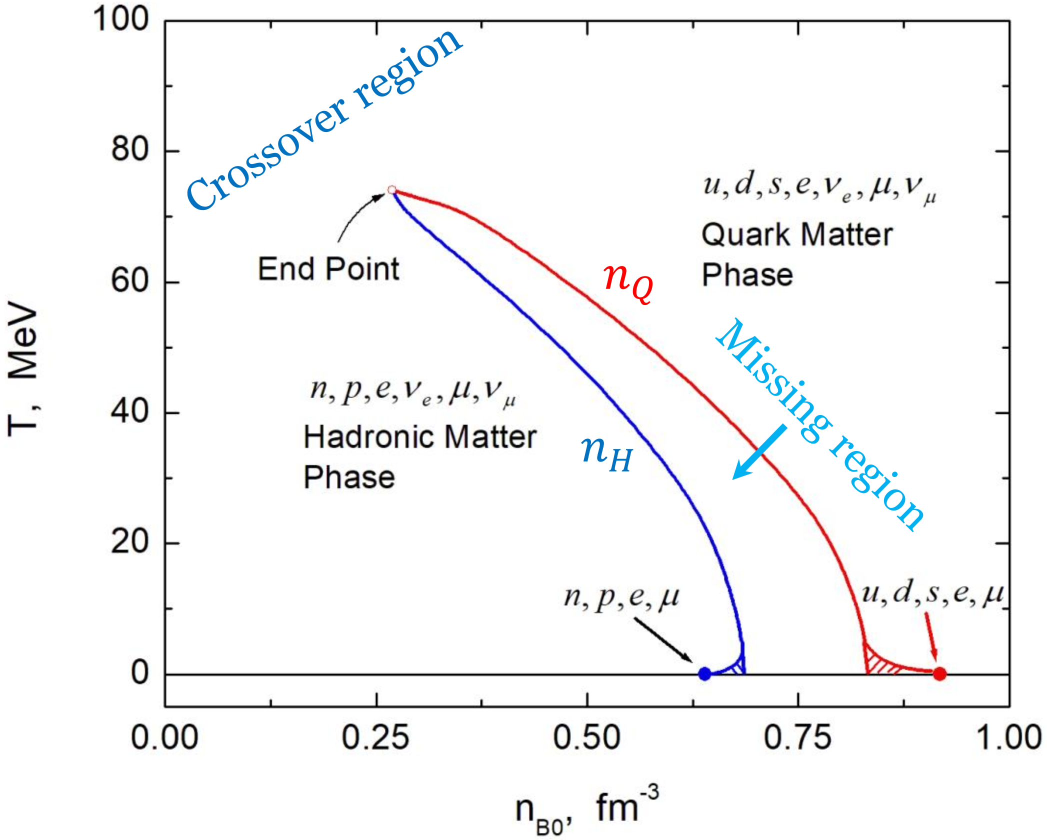


Two-Phase Coexistence Parameters

T MeV	P_0 MeV/fm³	μ_B MeV	n_H fm⁻³	n_Q fm⁻³	ε_H MeV/fm³	ε_Q MeV/fm³	S_H fm⁻³	S_Q fm⁻³
0	173.6	1391.8	0.639	0.918	716.3	1109.3	0	0
5	198.7	1377.9	0.683	0.829	795.9	1058.7	0.154	0.346
10	194.8	1370.5	0.676	0.821	787.2	1049.8	0.307	0.683
20	181.3	1343.8	0.652	0.972	756.1	1015.0	0.601	1.322
30	159.2	1298.4	0.609	0.738	703.4	950.1	0.865	1.858
50	101.9	1160.2	0.478	0.584	547.0	766.4	1.232	2.509
70	46.2	966.9	0.290	0.372	336.4	522.3	1.253	2.541
72	44.1	953.1	0.278	0.326	324.6	467.4	1.255	2.443
74	42.8	941.2	0.269	0.269	315.6	397.3	1.261	2.275

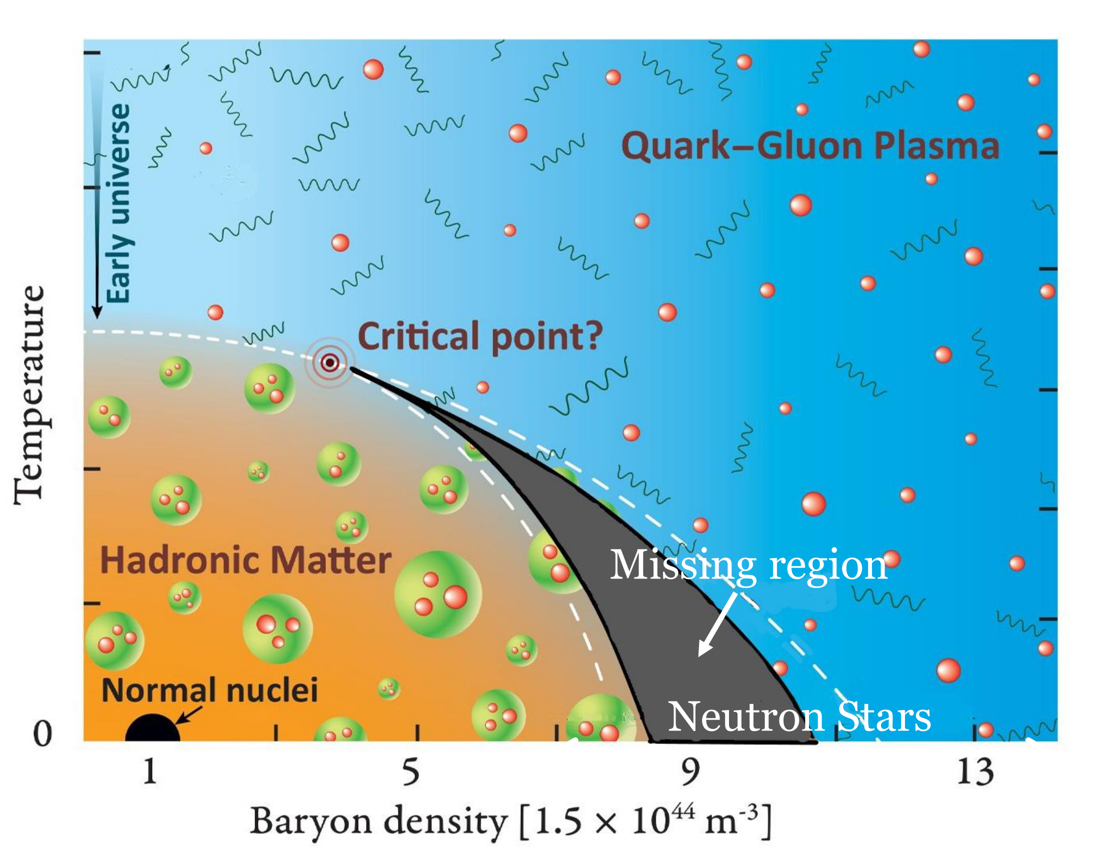
Phase Diagram





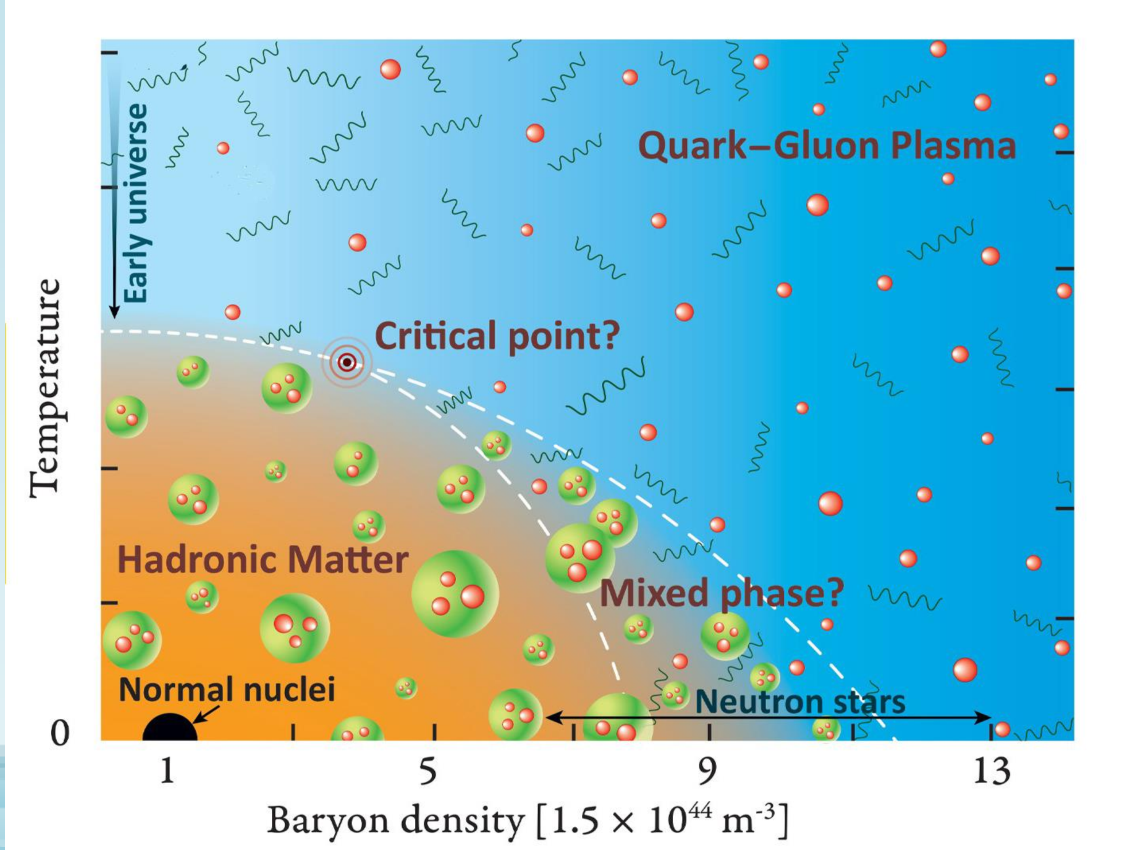
Strong quark-hadron interface tension

Ordinary first-order phase transition at constant pressure (Maxwell Construction)



Weak quark-hadron interface tension

Phase transition with the formation of a Quark-Hadron mixed phase (Gibbs Construction)



Conclusions

- We investigate the thermodynamic properties of the hot β-equilibrated hadronic matter in a neutrino-trapped regime. To describe such matter, we use an improved version of RMF model at a finite temperature, where, in addition to the effective fields of σ-, ω-, and ρ-mesons, the scalar-isovector δ-meson effective field is also taken into account.

- For different values of temperature T in the range of 0-100 MeV, the dependences of pressure P, energy density ε, entropy density S, and baryon chemical potential μ_B on the baryon number density n_B have been determined.

- We show that splitting of the proton and neutron effective masses slightly decreases with increasing temperature.

- The temperature dependencies of the parameters of the first-order phase transition from hadronic matter to strange quark matter are studied by using the quark phase described in the SU(3) local NJL model.

- A phase diagram is obtained corresponding to the equilibrium coexistence of hadron and quark phases in $(T-\mu_B)$ and $(T-n_B)$ planes. The thermodynamic parameters of the critical endpoint in the phase coexistence curve are found.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