Three-quark Dynamical System Immersed in a Colored Gluon Thermostat

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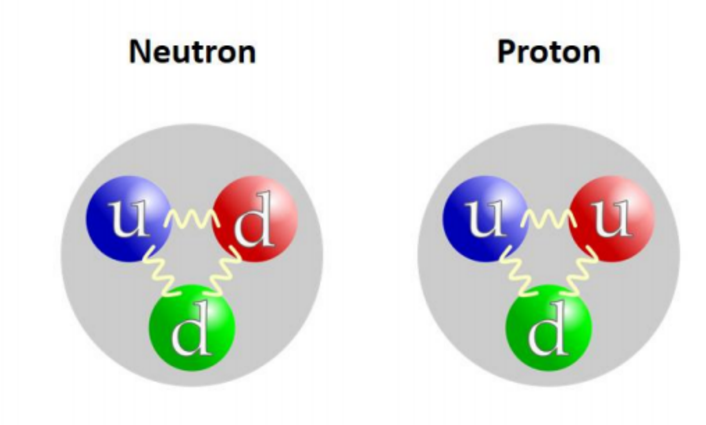
Nucleon as a three-quark dynamical system



Obrázek: *A nucleon in a stable state is white, i.e. blue + red + green = white.*

For the first time, the nucleon as a three-quark system was studied within the framework of a non-relativistic quantum representation in 1968-1969 years in the works;

1. D. Fayman and A. W. Hendry, Phys. Rev. **173**, 1720, (1968); **180**, 1572, (1969).
2. L. A. Copley, G. Karl and E. Obryk, Phys. Letters **29B**, 117, (1969), Nucl. Physics **B13**, 303, (1968).

These works stimulated Feynman and co-authors to develop a relativistic three-quark system in the framework of 4D harmonic oscillator model in 1971;

3. R. P. Feynman, M. Kislinger and F. Ravandal, Phys. Rev, D3, 2706, (1971).

The relativistic quark-model was further explored for study of the electromagnetic excitations of the nucleon

4. R. G. Lipes, Phys. Rev. **D**11, 2849 (1972).

1. Construct a consistent mathematical theory of a relativistic three-quark dynamical system immersed in a colored gluon thermostat, within the framework of a stochastic extension of the Klein-Gordon-Fock equation.

2. Study the geometric and topological features of the three-quark system depending on the spectrum of the color gluon thermostat.

3. Development of mathematical algorithms for calculating various parameters of a quark system during the self-organization of a unified system **three-quark+thermostat**.

Formulation of a regular problem

Lorentz invariant Eq. for the three-quark system can be written as:

$$\left\{\sum_{\zeta=a,b,c}\Box_\zeta+\Omega_0^2\left[(\mathbf{x}_a-\mathbf{x}_b)^2+(\mathbf{x}_a-\mathbf{x}_c)^2+(\mathbf{x}_b-\mathbf{x}_c)^2\right]+m_0^2\right\}\Psi_0=0, \tag{1}$$

where $\mathbf{x}_a, \mathbf{x}_b$ and $\mathbf{x}_c$ are the space-time coordinates of the quarks a, b and c, in addition, Ω_0 is the some constant m_0 denotes the nucleon rest mass, and $\Box_\zeta$ denotes the d'Alembert operator acting on the ζ-th quark, which in natural units $\hbar = c = 1$ is written as:

$$\Box_\zeta = \partial^2_{t_\zeta} - \Delta_\zeta = \partial^2_{t_\zeta} - \partial^2_{x_\zeta} - \partial^2_{y_\zeta} - \partial^2_{z_\zeta}. \tag{2}$$

For further research, let us move on to new coordinates:

$$\mathbf{q}_a = \mathbf{x}_a - \mathbf{x}_b, \qquad \mathbf{q}_b = \mathbf{x}_b - \mathbf{x}_c, \qquad \mathbf{q}_c = \mathbf{x}_c - \mathbf{x}_a, \tag{3}$$

in which the equation (1) can be converted to the form:

$$\left\{\sum_{\zeta=a,b,c}\left[\Box_\zeta + \Omega_0^2\mathbf{q}_\zeta^2\right] + m_0^2\right\}\overline{\Psi}_0 = 0, \tag{4}$$

where $\overline{\Psi}_0(\mathbf{x}_a, \mathbf{x}_b, \mathbf{x}_c) = \overline{\Psi}_0(\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c)$.

Reducing three-quark problem to an oscillator problem 6

Representing the solution of the equation (4) as a product of space-like and time-like wave functions:

$$\overline{\Psi}_0(\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c) = \left[\frac{3\Omega_0}{\pi}\right]^{1/4} e^{-3\Omega_0 t^2/2} \tilde{\Psi}_0(\bar{\mathbf{q}}_a, \bar{\mathbf{q}}_b, \bar{\mathbf{q}}_c), \qquad (5)$$

where $\mathbf{q}_\zeta = (t, \bar{\mathbf{q}}_\zeta)$ and $\bar{\mathbf{q}}_\zeta = (q_{\zeta(x)}, q_{\zeta(y)}, q_{\zeta(z)})$, from (4) we get the following stationary harmonic oscillator equation:

$$\sum_{\zeta=a,b,c} \left\{-\Delta_\zeta + \Omega_0^2 \bar{\mathbf{q}}_\zeta^2\right\} \tilde{\Psi}_0(\bar{\mathbf{q}}_a, \bar{\mathbf{q}}_b, \bar{\mathbf{q}}_c) = 0, \quad \Omega_0 = m_0^2. \qquad (6)$$

Further representing the solution of Eq. (6) in factorized form:

$$\tilde{\Psi}_0(\bar{\mathbf{q}}_a, \bar{\mathbf{q}}_b, \bar{\mathbf{q}}_c) = \psi_{0a}(\bar{\mathbf{q}}_a)\psi_{0b}(\bar{\mathbf{q}}_b)\psi_{0c}(\bar{\mathbf{q}}_c), \qquad (7)$$

from (6) we can get the following Schrödinger-type equation:

$$\left\{-\Delta_\zeta + \Omega_0^2 \bar{\mathbf{q}}_\zeta^2\right\} \psi_{0\zeta}(\bar{\mathbf{q}}_\zeta) = 0. \qquad (8)$$

Wave function of a three-quark system

Using the spherical coordinate system $(q_\zeta, \vartheta_\zeta, \varphi_\zeta)$ as variables, the following solution for equation (8) can be found:

$$\psi_{0\zeta}(\bar{\mathbf{q}}_\zeta) = R_{\lambda_\zeta, l_\zeta}(q_\zeta) Y_{l_\zeta m_\zeta}(\vartheta_\zeta, \varphi_\zeta) e^{-\Omega_0 q_\zeta^2/2}, \qquad (9)$$

where $R_{\lambda_\zeta, l_\zeta}(q_\zeta)$ denotes the normalized radial wave function with $q_\zeta = \sqrt{q^2_{\zeta(x)} + q^2_{\zeta(y)} + q^2_{\zeta(z)}}$, and $Y_{l_\zeta m_\zeta}(\vartheta_\zeta, \varphi_\zeta)$ is the spherical harmonics.

Taking into account the above calculations, we can construct the Lorentz-invariant full wave function of the three-quark dynamical system:

$$\overline{\Psi}_0(\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c) = \left[\frac{3\Omega_0}{\pi}\right]^{1/4} e^{-3\Omega_0 t^2/2} \prod_{\zeta=a,b,c} \psi_{0\zeta}(\bar{\mathbf{q}}_\zeta) =$$

$$\prod_{\zeta=a,b,c} R_{\lambda_\zeta, l_\zeta}(q_\zeta) Y_{l_\zeta m_\zeta}(\vartheta_\zeta, \varphi_\zeta) e^{-\Omega_0 (q_\zeta^2 + 3t^2)/2}, \qquad (10)$$

Three-quark system immersed in a colored gluon TB

Let us consider the following Eq. for the three-quark system:

$$\Big\{\sum_{\zeta=a,b,c}\Box_\zeta+\Omega_a^2(\mathbf{x}_a-\mathbf{x}_b)(\mathbf{x}_a-\mathbf{x}_b)^2+\Omega_b^2(\mathbf{x}_b-\mathbf{x}_c)(\mathbf{x}_b-\mathbf{x}_c)^2$$
$$+\Omega_c^2(\mathbf{x}_c-\mathbf{x}_a)(\mathbf{x}_c-\mathbf{x}_a)^2+m_0^2\Big\}\Psi(\mathbf{x}_a,\mathbf{x}_b,\mathbf{x}_c)=0.\ (11)$$

Since the quarks a, b and c in a nucleon are in three different color states, so that the overall color is always white, then the pair interactions of quarks inside the nucleon will be different, and, accordingly, the space-time functions characterizing these interactions will be denoted by $\Omega_a(\mathbf{x}_a-\mathbf{x}_b)$, $\Omega_b(\mathbf{x}_b-\mathbf{x}_c)$ and $\Omega_c(\mathbf{x}_c-\mathbf{x}_a)$ in general complex random functions. In what follows, we will assume that these functions represent complex random processes that describe elastic and inelastic interactions of quarks through gluons.

Eq. of motion for a “3-quark+TB ” joint system

Using (3) the equation (11) can be written as:

$$\Big\{\sum_{\zeta=a,b,c}\big[\Box_\zeta+\Omega_\zeta^2(\mathbf{q}_\zeta)\mathbf{q}_\zeta^2\big]+m_0^2\Big\}\overline{\Psi}(\mathbf{q}_a,\mathbf{q}_b,\mathbf{q}_c)=0. \qquad (12)$$

Further, representing the total wave function as a product:

$$\overline{\Psi}(\mathbf{q}_a,\mathbf{q}_b,\mathbf{q}_c)=\left[\frac{3\Omega_0}{\pi}\right]^{1/4}e^{-3\Omega_0 t^2/2}\tilde{\Psi}(\mathbf{q}_a,\mathbf{q}_b,\mathbf{q}_c), \qquad (13)$$

the equation (12) can be reduced to the form:

$$\Big\{\sum_{\zeta=a,b,c}\big[\Box_\zeta+\Omega_\zeta^2(\mathbf{q}_\zeta)\mathbf{q}_\zeta^2\big]-3\Omega_0^2t^2\Big\}\tilde{\Psi}(\mathbf{q}_a,\mathbf{q}_b,\mathbf{q}_c)=0. \qquad (14)$$

Now let us present the solution to equation (14) in factorized form:

$$\tilde{\Psi}(\mathbf{q}_a,\mathbf{q}_b,\mathbf{q}_c)=\psi_a(\mathbf{q}_a)\psi_b(\mathbf{q}_b)\psi_c(\mathbf{q}_c), \qquad (15)$$

Using (14) from equation (13), we can obtain three new equations:

$$\big\{\Box_\zeta+\Omega_\zeta^2(\mathbf{q}_\zeta)\mathbf{q}_\zeta^2-\Omega_0^2t^2\big\}\psi_\zeta(\mathbf{q}_\zeta)=0, \quad \zeta=a,b,c, \qquad (16)$$

Note that these three equations are related by one source- by GB.

Since we are considering a relativistic problem, it is natural to use a 4-vector defining a point or event in space-time as a parameter describing the evolution of a dynamical system:

$$s_\zeta^2 = t^2 - q_{\zeta(x)}^2 - q_{\zeta(y)}^2 - q_{\zeta(z)}^2, \qquad \zeta = a, b, c. \tag{17}$$

In view of the foregoing, the solution of each of the equations (16) can be represented as:

$$\psi_\zeta(\mathbf{q}_\zeta) = \psi_{0\zeta}(\bar{\mathbf{q}}_\zeta) \exp\left(\int_0^{s_\zeta} \Lambda_\zeta(s'; \bar{\mathbf{q}}_\zeta, t) ds'\right). \tag{18}$$

Recall that the wave function (18) is a complex probabilistic process, which should be further averaged to obtain the mathematical expectation of the wave function of the relative motion of two quarks, taking into account the influence of the gluon thermostat.

Now we define the function $\Omega_\zeta(\mathbf{q}_\zeta)$, representing it as a sum:

$$\Omega_\zeta^2(\mathbf{q}_\zeta) = \Omega_0^2 + F_\zeta(s_\zeta), \qquad F_\zeta(s_\zeta) = f_\zeta^{(r)}(s_\zeta) + i f_\zeta^{(i)}(s_\zeta).$$

For definiteness, we will assume that random number generators satisfy Markov-Gaussian random processes or *white noise* correlation relations:

$$\mathbb{E}\big[f_\zeta^{(\upsilon)}(s)\big] = 0, \quad \mathbb{E}\big[f_\zeta^{(\upsilon)}(s) f_\zeta^{(\upsilon)}(s')\big] = 2\varepsilon_\zeta^{(\upsilon)}\,\delta(s - s'), \qquad (19)$$

where $\mathbb{E}\big[...\big]$ is mean value of the random variable and $\upsilon = (i, r)$. Substituting (18) into Eq. (16) taking into account that $\Lambda_\zeta(s_\zeta) = u_{\zeta 1}(s_\zeta) + u_{\zeta 2}(s_\zeta)$, we get:

$$\begin{cases} \dot{u}_{\zeta 1} + u_{\zeta 1}^2 - u_{\zeta 2}^2 - k u_{\zeta 1} + \Omega_0^2 h \bar{\mathbf{q}}_\zeta^2 + h \bar{\mathbf{q}}_\zeta^2 f^{(r)}(s_\zeta) = 0, \\ \dot{u}_{\zeta 2} + 2 u_{\zeta 1} u_{\zeta 2} - k u_{\zeta 2} + \bar{\mathbf{q}}_\zeta^2 h f_\zeta^{(i)}(s_\zeta) = 0. \end{cases} \qquad (20)$$

where

$$k(\bar{\mathbf{q}}_\zeta, t) = \frac{2 - \bar{\mathbf{q}}_\zeta \nabla \ln \psi_{0\zeta}(\bar{\mathbf{q}}_\zeta)}{t - x_\zeta - y_\zeta - z_\zeta}, \quad h(\bar{\mathbf{q}}_\zeta, t) = \frac{s_\zeta}{t - x_\zeta - y_\zeta - z_\zeta}.$$

Using the equations (20) taking into account the synchronization of $4D$ event intervals $s=\min\{s_a,\,s_b,\,s_c\}$, we can write:

$$\begin{cases} \dot{u}_{a1}+u_{a1}^2-u_{a2}^2-k_a u_{a1}+\Omega_0^2\bar{\mathbf{q}}_a^2 h_a+\bar{\mathbf{q}}_a^2 h_a f_a^{(r)}(s)=0, \\ \qquad \dot{u}_{a2}+2u_{a1}u_{a2}-k_a u_{a2}+\bar{\mathbf{q}}_a^2 h_a f_a^{(i)}(s)=0, \\ \dot{u}_{b1}+u_{b1}^2-u_{b2}^2-k_b u_{b1}+\Omega_0^2\bar{\mathbf{q}}_b^2 h_b+\bar{\mathbf{q}}_b^2 h_b f_b^{(r)}(s)=0, \\ \qquad \dot{u}_{b2}+2u_{b1}u_{b2}-k_b u_{b2}+\bar{\mathbf{q}}_b^2 h_b f_b^{(i)}(s)=0, \\ \dot{u}_{c1}+u_{c1}^2-u_{c2}^2-k_c u_{c1}+\Omega_0^2\bar{\mathbf{q}}_c^2 h_c+\bar{\mathbf{q}}_c^2 h_c f_c^{(r)}(s)=0, \\ \qquad \dot{u}_{c2}+2u_{c1}u_{c2}-k_c u_{a2}+\bar{\mathbf{q}}_c^2 h_c f_c^{(i)}(s)=0. \end{cases} \tag{21}$$

Let us represent the distribution of colored gluon fields in the form:

$$\mathcal{P}(\mathbf{u},s|\mathbf{u},s_0)=\Big\langle \prod_{\zeta=a,b,c}\ \prod_{i=1,2}\delta\big(u_{\zeta i}(s)-u_{\zeta i}^0\big)\Big\rangle, \tag{22}$$

$\mathbf{u}(s)=\big\{\big(u_{a1}(s),u_{a2}(s)\big);\big(u_{b1}(s),u_{b2}(s)\big);\big(u_{c1}(s),u_{c2}(s)\big)\big\}\in$ $\Xi_{\{\mathbf{u}(s)\}}$ denotes the six-component colored gluon field of the event "s" and $u_{\zeta i}^0$- component of the gluon field of the event $s=0$.

Using (21) taking into account (20), for the distribution of gluon fields (22) we find the following Fokker-Planck type equation:

$$\frac{\partial \mathcal{P}}{\partial s} = \widehat{\mathcal{L}}(\mathbf{u}|\bar{\mathbf{q}}, t)\mathcal{P}, \qquad \bar{\mathbf{q}} = (\mathbf{q}_a, \mathbf{q}_b, \mathbf{q}_c), \tag{23}$$

where

$$\widehat{\mathcal{L}}(\mathbf{u}|\bar{\mathbf{q}}, t) = \sum_{\zeta=a,b,c} \Bigg\{ h_\zeta(\bar{\mathbf{q}}_\zeta, t)\bar{\mathbf{q}}_\zeta^2 \left(\varepsilon_\zeta^{(r)} \frac{\partial}{\partial u_{\zeta 1}^2} + \varepsilon_\zeta^{(i)} \frac{\partial}{\partial u_{\zeta 2}^2} \right) +$$

$$\sigma_{\zeta 1}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t)\frac{\partial}{\partial u_{\zeta 1}} + \sigma_{\zeta 2}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t)\frac{\partial}{\partial u_{\zeta 2}} + \sigma_{\zeta 0}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t) \Bigg\}, \tag{24}$$

where the following notations are made:

$$\sigma_{\zeta 0}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t) = 4u_{\zeta 1} - 2k_\zeta, \quad \sigma_{\zeta 1}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t) = \left(u_{\zeta 1}^2 - u_2^2 - k_\zeta u_{\zeta 1} + \Omega_0^2 h_\zeta \bar{\mathbf{q}}_\zeta^2\right),$$

$$\sigma_{\zeta 2}(\mathbf{u}_\zeta|\bar{\mathbf{q}}_\zeta, t) = \left(2u_{\zeta 1} - k_\zeta\right)u_{\zeta 2}, \qquad \mathbf{u}_\zeta = (u_{\zeta 1}, u_{\zeta 2}).$$

Let us the probability distribution at each point of Minkowski space-time $(\bar{\mathbf{q}}, t) \in \mathbb{R}^4$ satisfy the following limit condition:

$$\lim_{s \to s'} \mathcal{P}(\mathbf{u}, s; \bar{\mathbf{q}}, t|\mathbf{u}', s') = \delta(\mathbf{u} - \mathbf{u}'), \qquad s = s' + \Delta s. \tag{25}$$

Taking into account (25) for small intervals of events, that is, for $\Delta s = s - s' \ll 1$, we can present the solution to Eq. (24) as:

$$P(\mathbf{u}, s; \bar{\mathbf{q}}, t|\mathbf{u}', s') = \exp\left\{ -\frac{\left[\mathbf{u} - \mathbf{u}' - \boldsymbol{\sigma}(\mathbf{u}, s; \bar{\mathbf{q}}, t)\Delta s\right]^T}{\sqrt{2\Delta s}} \times \frac{\bar{\varepsilon}^{-1}\left[\mathbf{u} - \mathbf{u}' - \boldsymbol{\sigma}(\mathbf{u}, s; \bar{\mathbf{q}}, t)\Delta s\right]}{\sqrt{2\Delta s}} \right\} \frac{1}{2\pi\sqrt{|\det \bar{\varepsilon}|\,\Delta s}}, \tag{26}$$

where $\bar{\varepsilon} = \varepsilon h \bar{\mathbf{q}}^2$ is the second-rank matrix with elements; $\varepsilon_{11} = \bar{\varepsilon}^{(r)}$, $\varepsilon_{22} = \bar{\varepsilon}^{(i)}$ and $\varepsilon_{12} = \varepsilon_{21} = 0$, while $[\cdots]^T$ denotes a vector transposition. Additionally, in (26) vector $\boldsymbol{\sigma}$ is defined as:

$$\boldsymbol{\sigma}(\mathbf{u}, s; \bar{\mathbf{q}}, t) = \begin{cases} \sigma_1 = u_1^2 - u_2^2 - k(\bar{\mathbf{q}}, t)u_1 + \Omega_0^2 h(\bar{\mathbf{q}}, t)\bar{\mathbf{q}}^2, \\ \sigma_2 = [2u_1 - k(\bar{\mathbf{q}}, t)]u_2. \end{cases} \tag{27}$$

Finally we can write down the mathematical expectation of the nucleon internal motion:

$$\mathbb{E}[\Psi(\mathbf{q})] = \frac{\Psi_0(\bar{\mathbf{q}})}{\alpha(s)} \int \cdots \int_{\Xi_{\{\mathbf{u}(s)\}}} D\mu(\mathbf{u})\, e^{\sum_{\zeta=a,b,c} \int_0^s \Lambda_\zeta(s';\bar{\mathbf{q}}_\zeta,t)ds'}, \tag{28}$$

where $\alpha(s)$ is the normalization constant:

$$\alpha(s) = \int \cdots \int_{\Xi_{\{\mathbf{u}(s)\}}} D\mu(\mathbf{u}) = \int \cdots \int_{\Xi^6_{\mathbf{u}}} \mathcal{P}(\mathbf{u}, s; \bar{\mathbf{q}}) \prod_{\zeta=a,b,c} du_{\zeta 1} du_{\zeta 2}.$$

Calculating the functional integral (28) we find:

$$\mathbb{E}[\Psi(\mathbf{q})] = \frac{\Psi_0(\bar{\mathbf{q}})}{\alpha(s)} \int \cdots \int_{\Xi^6_{\mathbf{u}}} Q(\mathbf{u}, t, s; \bar{\mathbf{q}}) \prod_{\zeta=a,b,c} du_{\zeta 1} du_{\zeta 2}, \tag{29}$$

where $Q(\mathbf{u}, s; \mathbf{q})$ is the solution of the following partial differential equation:

$$\partial_s Q = \Big\{\widehat{\mathcal{L}}(\mathbf{u}|\bar{\mathbf{q}}t, t) + \sum_{\zeta=a,b,c}(u_{\zeta 1} + iu_{\zeta 2})\Big\} Q. \tag{30}$$

Representing the solution $Q(\mathbf{u}, s; \mathbf{q})$ in the form of following sum:

$$Q(\mathbf{u}, s; \mathbf{q}) = Q^{(r)}(\mathbf{u}, s; \mathbf{q}) + iQ^{(i)}(\mathbf{u}, s; \mathbf{q}),$$

from Eq. (28) can find a system of two real coupled PDEs:

$$\begin{cases} \partial_s Q^{(r)} = \big\{\widehat{\mathcal{L}}(\mathbf{u}|\bar{\mathbf{q}}, t) + \sum_{\zeta=a,b,c} u_{\zeta 1}\big\} Q^{(r)} - \sum_{\zeta=a,b,c} u_{\zeta 2} Q^{(i)}, \\ \partial_s Q^{(i)} = \big\{\widehat{\mathcal{L}}(\mathbf{u}|\bar{\mathbf{q}}, t) + \sum_{\zeta=a,b,c} u_{\zeta 1}\big\} Q^{(i)} + \sum_{\zeta=a,b,c} u_{\zeta 2} Q^{(r)}. \end{cases}$$

Having given the functions $Q^{(r)}$ and $Q^{(i)}$ a probabilistic meaning, we write:

$$\bar{Q}^{(\upsilon)}(\mathbf{u}, s; \bar{\mathbf{q}}) = \lambda^{-1}(\bar{\mathbf{q}}, s) Q^{(\upsilon)}(\mathbf{u}, s; \bar{\mathbf{q}}),$$

$$\lambda(\bar{\mathbf{q}}, s) = \int \cdots \int_{\Xi^6_{\mathbf{u}}} \sum_{\upsilon=r,i} Q^{(\upsilon)}(\mathbf{u}, s; \bar{\mathbf{q}}) \prod_{\zeta=a,b,c} du_{\zeta 1} du_{\zeta 2}, \tag{31}$$

and, correspondingly; $\int \cdots \int_{\Xi^6_{\mathbf{u}}} \sum_{\upsilon=r,i} \bar{Q}^{(\upsilon)}(\mathbf{u}, s; \bar{\mathbf{q}}) du_1 du_2 = 1.$

Theorem. *If the motion of a dynamical system is described by a Langevin-type stochastic differential equations (21), then in the limit of statistical equilibrium, the functional space* $\Xi_{\{\mathbf{u}(s)\}}$ *is compactified into the 6D subspace* $\Xi^6_{\mathbf{u}}$*, which, in general, has an antisymmetric metric tensor and, accordingly, its geometry is a non-commutative.*

It is proved that antisymmetric elements obey algebraic equations of the fourth degree, which can generate two-dimensional manifolds with topological Betti singularities of order $n \leq 4$. In this case, the additional submanifold is represented as a decomposition; $\Xi^6_{\{\mathbf{u}\}} = \Xi^2_{\{\mathbf{u}\}} \bigotimes \Xi^2_{\{\mathbf{u}\}} \bigotimes \Xi^2_{\{\mathbf{u}\}}$.

Within the framework of the stochastic extension of the Klein-Gordon-Fock equation, the nucleon problem is considered as a problem of self-organization of a three-quark system in a color gluon thermostat.

1. The mathematical expectation of the total wave function of the internal motion of a three-quark system is constructed, taking into account colored gluon exchanges between quarks and conservation of nucleon color in the form of a six-fold integral representation.

2. Within the framework of the proposed approach, the influence of only six out of eight color quarks on the evolution of the quark system is taken into account, and this is due to the neglect of quarks spin.

3. It is shown that the additional six-dimensional subspace $\Xi^6_{\mathbf{u}}$, depending on the magnitude of the fluctuation powers, can have the character of a non-commutative geometry with topological features.

THANK YOU FOR ATTENTION!