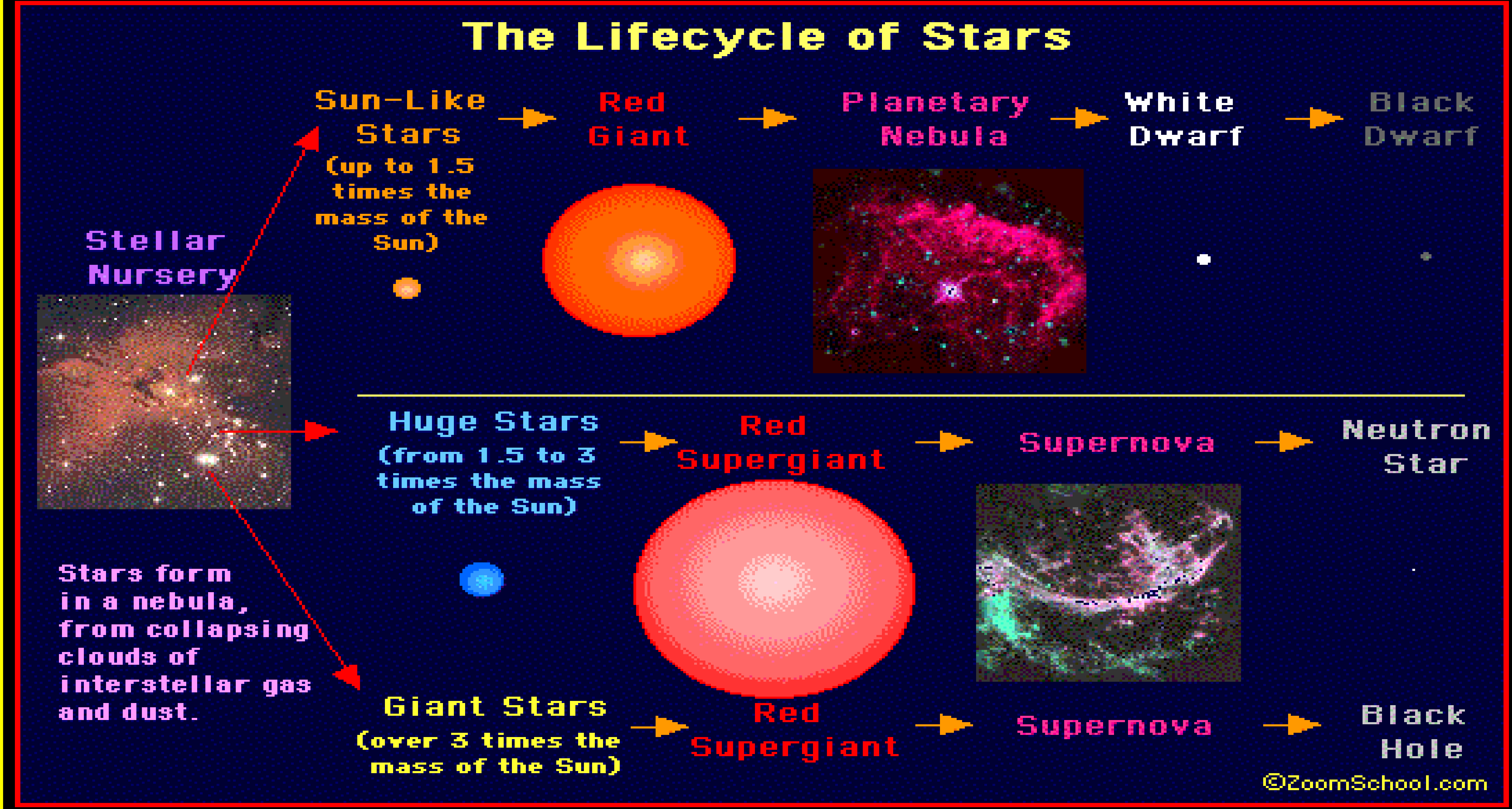
Applications of the QCD Sum Rules (QCDSR) Method to Neutron Stars


✓ Introduction
✓ Coupling Constant
✓ Description
✓ Results
✓ Summary
Presentation Main Points


The Lifecycle of Stars
Stellar Nursery
Stars form in a nebula, from collapsing clouds of interstellar gas and dust.
Sun-Like Stars (up to 1.5 times the mass of the Sun)
Red Giant
Planetary Nebula
White Dwarf
Black Dwarf
Huge Stars (from 1.5 to 3 times the mass of the Sun)
Red Supergiant
Supernova
Neutron Star
Giant Stars (over 3 times the mass of the Sun)
Red Supergiant
Supernova
Black Hole
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Neutron Stars:

Neutron stars are born out of core-collapse supernovae and are essential astrophysical events due to their rich physics. For instance, the process in neutron star involves all four known fundamental forces of nature, making it an ideal laboratory for physics.

For the purposes of this research, assume the simple case that the objects we are investigating are static and non-rotating. If the pressure and temperature (hence energy) of the system become great enough, other particles can be formed via weak interactions; for example, hyperons.

In this study we first use a relativistic model (Sigma–omega–rho model) within the mean field approximation to refine our knowledge of the workings of the strong interaction in neutron stars with the presence of hyperons and then we use Baryon-Meson coupling constants that obtained from 2 method:

1. c.c that proposed based on $\sigma - \omega - \rho$ Model
2. c.c that obtained from QCDSR method

to describe the stellar matter. Using both C.Cs, the particle fraction in beta equilibrium is investigated and the neutron star mass-radius profile is obtained in the presence of hyprons.

Lagrangian Density:

The first step to calculating any quantity in Q.F.T is to construct a Lagrangian density, which summarizes the dynamics of the system, and from which the equations of motion can be calculated.

$$\mathscr{L} = \sum_B \mathscr{L}_B + \sum_M \mathscr{L}_M + \sum_\lambda \mathscr{L}_\lambda + \mathscr{L}_{int}.$$

An appropriate generalization of the Lagrangian, based on σ, ω, ρ theory, is

$$\begin{aligned}\mathscr{L} = &\sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{BB\sigma}\sigma - g_{BB\omega}\gamma_\mu\omega^\mu - \frac{1}{2} g_{BB\rho}\gamma_\mu \underline{\tau}.\rho^\mu)\psi_B \\ &+ \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ &- \frac{1}{4}\rho_{\mu\nu}.\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu.\rho^\mu - \frac{1}{3}bm_n(g_{NN\sigma}\sigma)^3 - \frac{1}{4}c(g_{NN\sigma}\sigma)^4 \\ &+ \sum_\lambda \bar{\psi}_\lambda (i\gamma_\mu\partial^\mu - m_\lambda)\psi_\lambda.\end{aligned}$$

$$\psi_N = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix} \quad , \quad \psi_\Lambda = (\psi_\Lambda) \quad , \quad \psi_\Sigma = \begin{pmatrix}\psi_{\Sigma^+} \\ \psi_{\Sigma^0} \\ \psi_{\Sigma^-}\end{pmatrix}.$$

$$\tau_{(N)3} = \frac{1}{2}\begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix} \quad , \quad \tau_{(\Lambda)3} = 0 \quad , \quad \tau_{(\Sigma)3} = \begin{bmatrix}1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1\end{bmatrix}$$

Mean-Field Approximation:

Mean-Field Approximation (MFA) is made on the basis that we can separate the expression for a meson field α into two parts: a constant classical component, and a component due to quantum fluctuations;

$$\alpha = \alpha_{classical} + \alpha_{quantum}.$$

If we then take the vacuum expectation value of these components, the quantum fluctuation term vanishes.

$$\langle \alpha \rangle \equiv \langle \alpha_{classical} \rangle .$$

This component is what we shall use as the meson contribution, and it is constant at any given density. Furthermore, given that the ground-state of matter will contain some proportion of proton and neutron densities, any flavor-changing meson interactions will provide no contribution in the MFA, since the overlap operator between the ground-state $|\Psi\rangle$ and an other state $|\xi\rangle$ is orthogonal.
For this reason, any meson interactions which, say, interact with a proton to form a neutron will produce a state which is not the ground state, and thus provide no contribution to the MFA.

Chemical Potential and Meson fields :

For uniform static matter, the chemical potential μ, also known as the Fermi energy ϵ_F , is the energy of a particle at the top of the Fermi sea. By solving Euler-Lagrange equations ,and use mean field approximation for meson field, we able to calculate the field and their eigenvalues. thus, the baryon field in momentum representation is:

$$[\gamma_\mu(k^\mu - g_{BB\omega}\omega^\mu - \frac{1}{2}g_{BB\rho}\tau.\rho^\mu) - (m_B - g_{BB\sigma}\sigma)]\psi_B(k) = 0,$$

And their eigenvalues for particle and antiparticle are:

$$e_B(k) = g_{BB\omega}\omega_0 + g_{BB\rho}\rho_{03}\underbrace{I_{3B}}_{\text{isospin projection}} + \sqrt{\underbrace{k^2}_{\text{Fermi momentum}} + (m_B - g_{BB\sigma}\sigma)^2},$$

$$\bar{e}_B(k) = -g_{BB\omega}\omega_0 - g_{BB\rho}\rho_{03}\bar{I}_{3B} + \sqrt{k^2 + (m_B - g_{BB\sigma}\sigma)^2}.$$

Meson fields :

σ, ω0 and ρ03 are meson fields in the uniform static matter and we can write them as:

$$\omega_0 = \sum_B \frac{g_{BB\omega}}{m_\omega^2} \rho_B,$$

$$\rho_{03} = \sum_B \frac{g_{BB\rho}}{m_\rho^2} I_{3B} \rho_B,$$

Spin

$$m_\sigma^2 \sigma = -b m_n g_{NN\sigma} (g_{NN\sigma}\sigma)^3 + \sum_B \frac{2J_B + 1}{2\pi^2} g_{BB\sigma} \int_0^{k_B} \frac{m_B - g_{BB\sigma}\sigma}{\sqrt{k^2 + (m_B - g_{BB\sigma}\sigma)^2}} k^2 dk.$$

Note:

Although the total baryon density is a useful control parameter, many of the parameters of the models are dependent on the density via K_F . The relation between the Fermi momentum and the total density is found as below,

$$\rho_{total} = \sum_i \rho_i = \sum_i \frac{k_{F_i}^3}{3\pi^2},$$

Equation of State :

By solving the Euler-Lagrange equations, using MFA and finally apply Energy-Momentum Tensor to the Lagrangian density, Energy and Pressure obtained as below :

$$\epsilon = \frac{1}{3} b m_n (g_{NN\sigma}\sigma)^3 + \frac{1}{4} c (g_{NN\sigma}\sigma)^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \sum_B \frac{2J_B+1}{2\pi^2} \int_0^{k_B} \sqrt{k^2 + (m_B - g_{BB\sigma}\sigma)^2} k^2 dk + \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \sqrt{k^2 + m_\lambda^2} k^2 dk$$

$$p = -\frac{1}{3} b m_n (g_{NN\sigma}\sigma)^3 - \frac{1}{4} c (g_{NN\sigma}\sigma)^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{1}{3} \sum_B \frac{2J_B+1}{2\pi^2} \int_0^{k_B} \frac{k^4 dk}{\sqrt{k^2 + (m_B - g_{BB\sigma}\sigma)^2}} + \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \frac{k^2 dk}{\sqrt{k^2 + m_\lambda^2}}.$$

Calculation of Coupling Constants

Note:

- Coupling Constant in $\sigma - \omega - \rho$ Model
- QCD Sum Rule (QCDSR) Method

Coupling Constant in σ − ω − ρ Model

The nuclear properties that define C.C are the saturation values of the binding, baryon density, symmetry energy coefficient, compression modulus, and nucleon effective mass, which are reported in Table 1. Using these properties, the nucleon coupling constants at saturation density are determined, which are shown in Table 2.

1.

$\rho_0(fm^{-3})$	$B/A(MeV)$	$a_{sym}(MeV)$	$K(MeV)$	M^*/M
0.153	-16.3	32.5	300	0.78

2.

$(g_{NN\sigma}/m_\sigma)^2 fm^2$	$(g_{NN\omega}/m_\omega)^2 fm^2$	$(g_{NN\rho}/m_\rho)^2 fm^2$	b	c
9.031	4.733	4.825	0.003305	0.01529

The coupling constants of the mesons’ interaction with the nucleons were proposed as a function of the mass of the nucleons and baryons by R. Karimi and H. R. Moshfegh:

$$\chi_\sigma = g_{BB\sigma}/g_{NN\sigma}, \qquad \chi_\omega = g_{BB\omega}/g_{NN\omega}, \qquad \chi_\rho = g_{BB\rho}/g_{NN\rho}.$$

$$\chi_\sigma = \chi_\omega = \chi_\rho = \left(\frac{m_{baryon}}{m_{nucleon}}\right)^\zeta, \qquad \zeta \in \mathscr{R}.$$

QCD Sum Rule (QCDSR) Method

Window1:

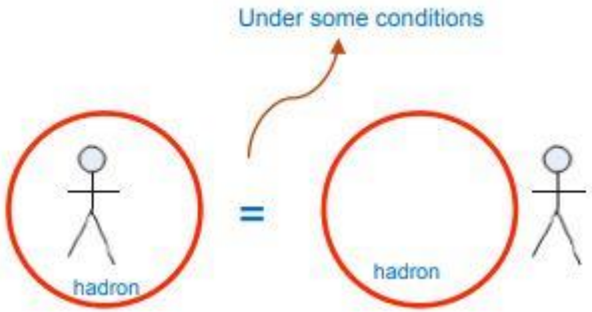
We see a hadron which has a mass, Decay constant, lifetime and other ha-Dronic parameters

ote:

Window2:

We see quarks and gluons and their interaction

QCDSR:



QCD sum rules for physical quantities.

Coupling Constant in QCDSR Method

To calculate the C.Cs in QCDSR, we need to perform these steps:

1. Start with the two-point correlation function of the baryon and meson currents.

2. Calculate the correlation function using the operator product expansion (OPE) to obtain the QCD side.

3. Calculate the phenomenological or hadronic side of the correlation function by inserting intermediate hadron states.

4. Apply the quark-hadron duality assumption and equate the QCD side and phenomenological side of the correlation function in some energy range.

5. Use input values for the QCD parameters from other sources to extract the numerical value of the coupling constant from the sum rule.

6. Perform various checks like Borel stability analysis, continuum contribution analysis, and variation of threshold parameter to estimate the uncertainty in the calculated coupling constant.

Meson-Baryon coupling constants

σ − ω − ρ Model

QCDSR Method

c.c	Ref. [1] $\zeta=-0.5$	Ref. [1] $\zeta=-1$	Ref. [1] $\zeta=-2$	Ref. [2]	Ref. [3]	Ref. [4]	Ref. [5]	Ref. [6]	Ref. [7]
$g_{NN\sigma}$	8.375	8.375	8.375	-	-	-	14.4 ± 3.7	-	-
$g_{NN\omega}$	8.621	8.621	8.621	-8.9 ± 1.1	-	7.2 ± 1.8	-	6.5 ± 1.1	10.34
$g_{NN\rho}$	8.571	8.571	8.571	-2.5 ± 1.1	3.2 ± 0.9	2.4 ± 0.6	-	3.0 ± 1.0	2.96
$g_{\Lambda\Lambda\sigma}$	7.683	7.048	5.932	-	-	-	7.0 ± 1.9	-	-
$g_{\Lambda\Lambda\omega}$	7.908	7.255	6.106	-7.1 ± 1.1	-	4.8 ± 1.2	-	2.8 ± 0.4	4.95
$g_{\Lambda\Lambda\rho}$	7.862	7.212	6.070	-	-	0	-	-	-
$g_{\Sigma^-\Sigma^-\sigma}$	7.415	6.566	5.149	-	-	-	-	-	-
$g_{\Sigma^-\Sigma^-\omega}$	7.633	6.759	5.300	-	-	-	-	-	-
$g_{\Sigma^-\Sigma^-\rho}$	7.589	6.720	5.269	-	-	-	-	-	-
$g_{\Sigma^+\Sigma^+\sigma}$	7.441	6.611	5.219	-	-	-	14.1 ± 3.7	-	-
$g_{\Sigma^+\Sigma^+\omega}$	7.659	6.805	5.372	6.6 ± 1.0	-	4.8 ± 1.2	-	4.6 ± 1.0	7.97
$g_{\Sigma^+\Sigma^+\rho}$	7.615	6.766	5.341	7.2 ± 1.2	4.0 ± 1.0	4.8 ± 1.2	-	4.6 ± 1.0	5.92

Equilibrium Conditions:

For a system such as that which consider here, each conserved quantity is associated with a chemical potential. In this system, we consider two conserved quantities: total baron number and total charge, and so we have a chemical potential related to each of these.
We can construct the chemical potential for each particle species by multiplying each conserved charge by its associated chemical potential to obtain a general relation. Thus

$$\mu_i = B_i\mu_n - Q_i\mu_e.$$

baryon charge electric charge

Also, charge neutrality is established in the system as:

$$\rho_p = \rho_e + \rho_\mu + \rho_{\Sigma^-}.$$

And the total baryon density (ρb) is defined as the sum of the densities of each baryon:

$$\rho_b = \rho_n + \rho_p + \rho_{\Sigma^-} + \rho_\Lambda.$$

processes in these system are as fallow:

$$n \longrightarrow p + e + \bar{\nu}_e,$$

$$p + e \longrightarrow n + \nu_e.$$

$$e \longrightarrow \mu + \bar{\nu}_\mu + \nu_e.$$

$$n + e \longrightarrow \Sigma^- + \nu_e,$$

$$n + n \longrightarrow n + \Lambda.$$

$$\mu_p = \mu_n - \mu_e,$$

$$\mu_e = \mu_\mu,$$

$$\mu_{\Sigma^-} = \mu_n + \mu_e,$$

$$\mu_\Lambda = \mu_n.$$

TOV Equations:

The Tolman-Oppenheimer-Volkoff (TOV) equations describe the conditions of stability against gravitational collapse for an EOS. These equations relate the change in pressure with radius to various state variables from the EOS

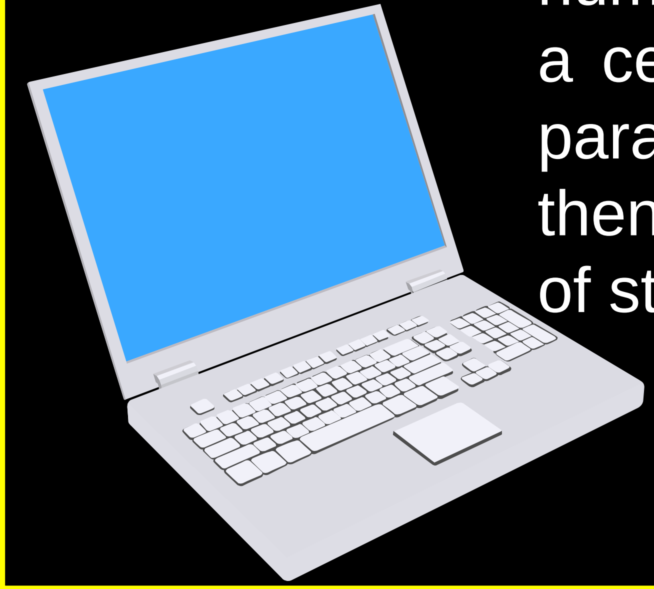
$$\frac{dP(r)}{dr} = -\frac{GM(r)\epsilon(r)}{c^2 r^2}\left(1 + \frac{P(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{rc^2}\right)^{-1},$$

$$\frac{dM(r)}{dr} = \frac{4\pi\epsilon(r)r^2}{c^2}.$$

Mass-Radius relation of Neutron Stars:

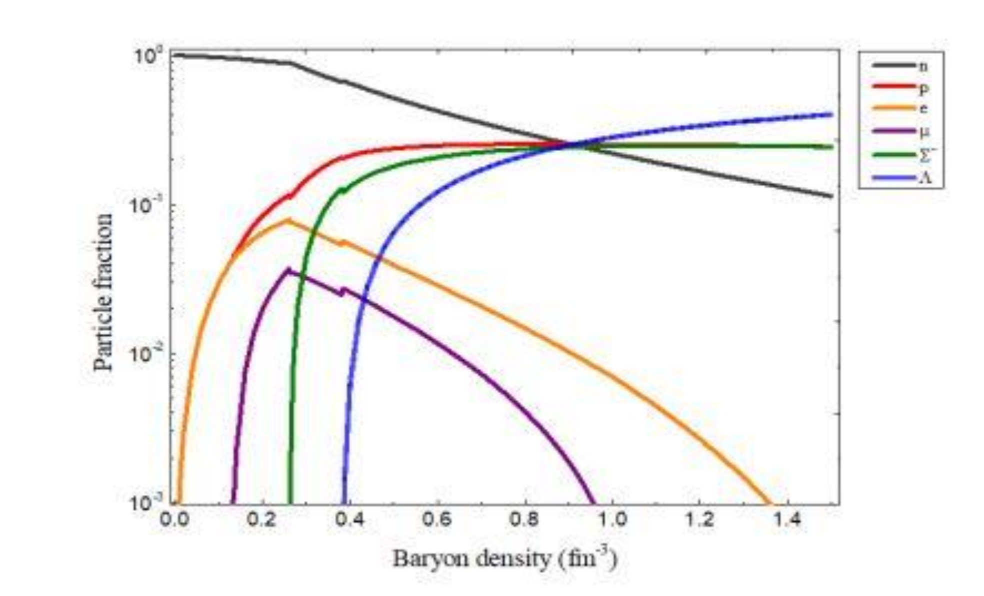
In order to calculate the properties of compact stellar objects, we need to solve TOV equations. These equations provide a connection between the infinite matter equation of state and the properties of macroscopic stellar objects.

We use fourth-order Runge-Kutta integration, which provides a useful scheme for integrating since it only requires a small number of function evaluations. For our purposes, we select a central density of a stellar object pcentral as the control parameter. The EOS data calculated for a particular model then provides the energy density and pressure at the center of star.



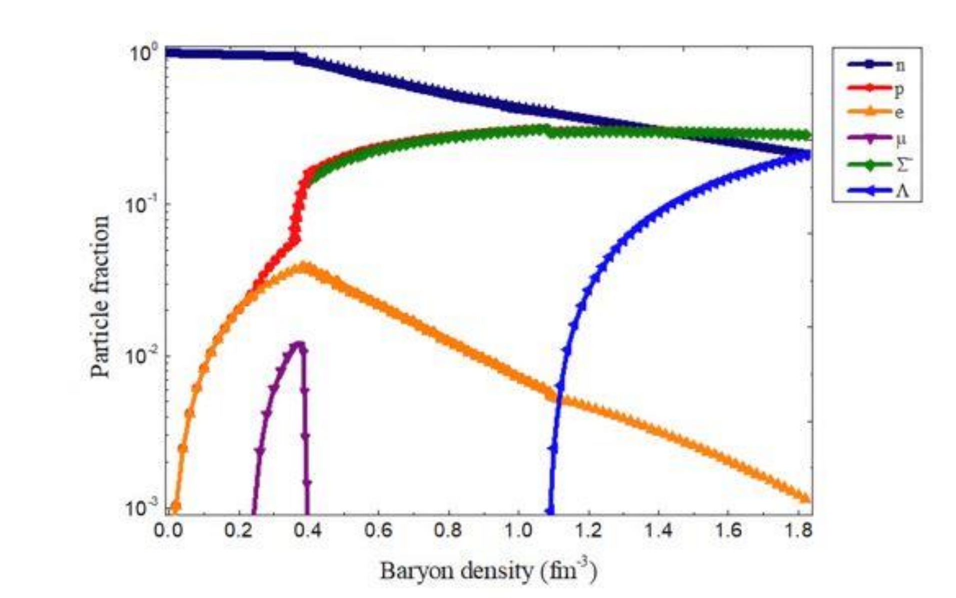
Particle Fraction

$\sigma - \omega - \rho$



Particle fraction Yi for neutron, proton, Sigma and Lambda with electron and muon leptons using coupling constants of Ref. [1] with $\zeta = -2$.

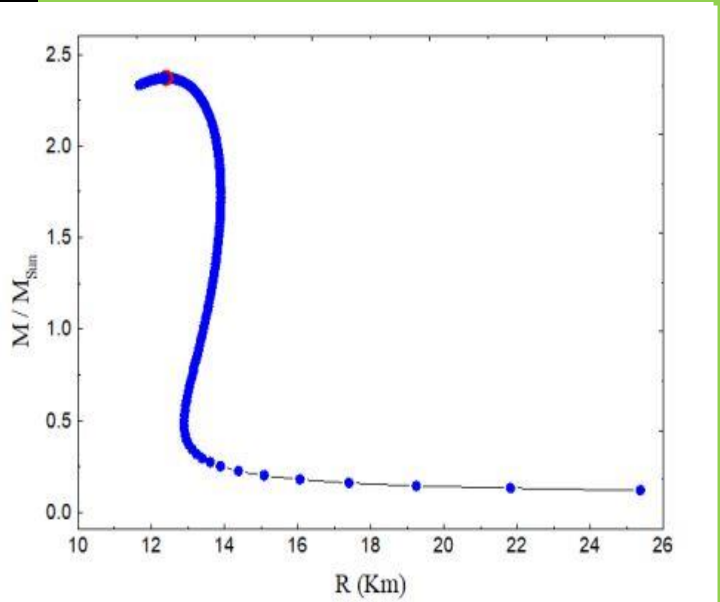
QCDSR



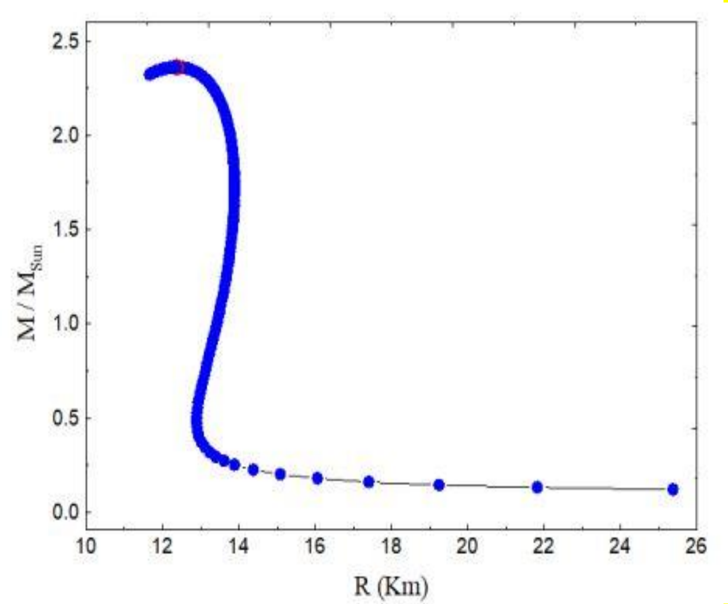
Particle fraction Yi for neutron, proton, Sigma and Lambda with electron and muon leptons in the neutron star using coupling constants of Ref. [2, 5].

Mass-Radius relation of Neutron Stars at :

QCDSR



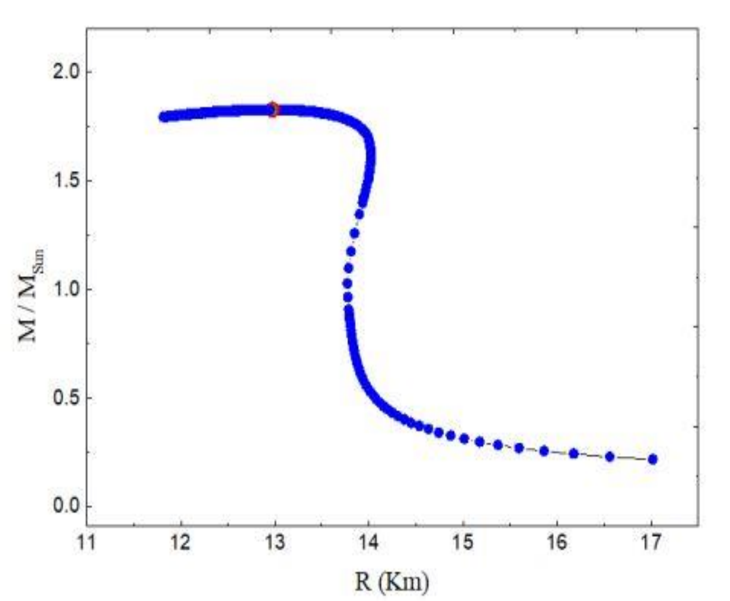
The mass-radius relation of neutron star for neutron, proton and electron in the neutron star using coupling constants of **QCDSR**.



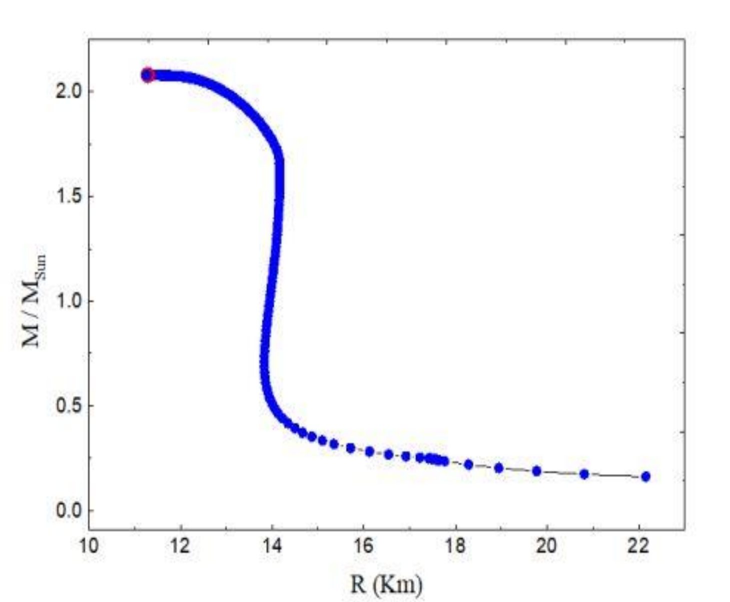
The mass-radius relation of neutron star for neutron, proton with electron and muon leptons in the neutron star using coupling constants of **QCDSR**.

Matter content	Pcentral (fm−3)	Mmax/M⊙	Radius (Km)
nep	0.89	2.37	12.40
nepµ	0.89	2.36	12.38

Mass-Radius relation of Hybrid Stars:



The mass-radius relation of neutron star for neutron, proton, Sigma and Lambda with electron and muon leptons using coupling constants of **$\sigma - \omega - \rho$ Model** with $\zeta = -2$.



The mass-radius relation of neutron star for neutron, proton, Sigma and Lambda with electron and muon leptons in the neutron star using coupling constants of **QCDSR.**

Matter content	Pcentral (fm^{-3})	$M_{max}/M_\odot$	Radius (Km)
$\sigma - \omega - \rho$ Model	0.78	1.82	12.95
QCDSR Method	1.12	2.08	11.29

Summary

In this study we first use a relativistic model within the mean-field approximation and then we use Baryon-Meson coupling constants from two methods to describe the stellar matter. Using both methods for the coupling constants, the particlle fraction in beta equilibrium is investigated and the neutron star mass-radius profile is obtained in the presence of hyprons.

The results Show that the use of QCDSR-based C.Cs in addition to having a physical basis, also produces a better mass-radius for NS, which is within the range predicted by observations.

References

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Thank You!