[1]Hebrew University of Jerusalem

[2]Ioffe Institute

Self-similarities of Equations of State and $M-R$ curves of Neutron Stars

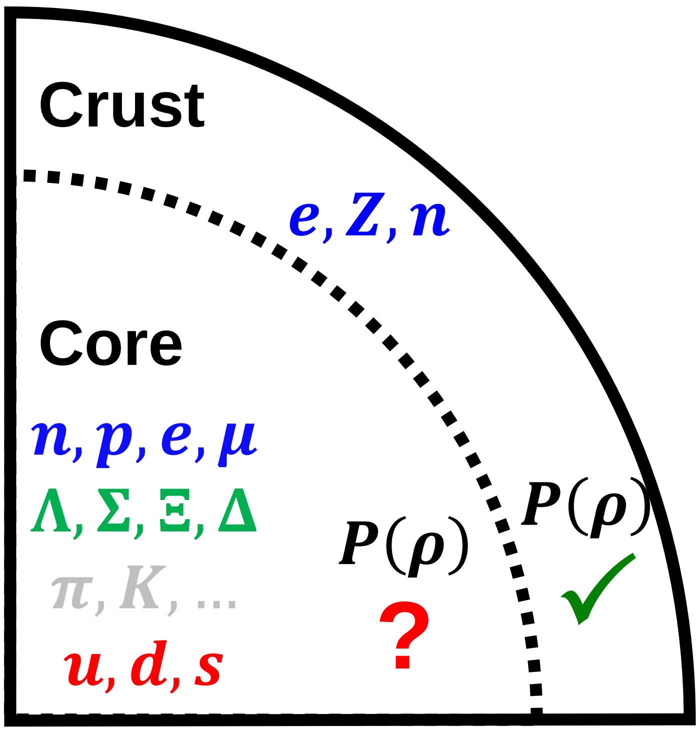
***Dima Ofengeim**[1,2], P. Shternin[2], T. Piran[1]*

The Modern Physics of Compact Stars & Relativistic Gravity – 2023

12 September 2023

NS Equations of State

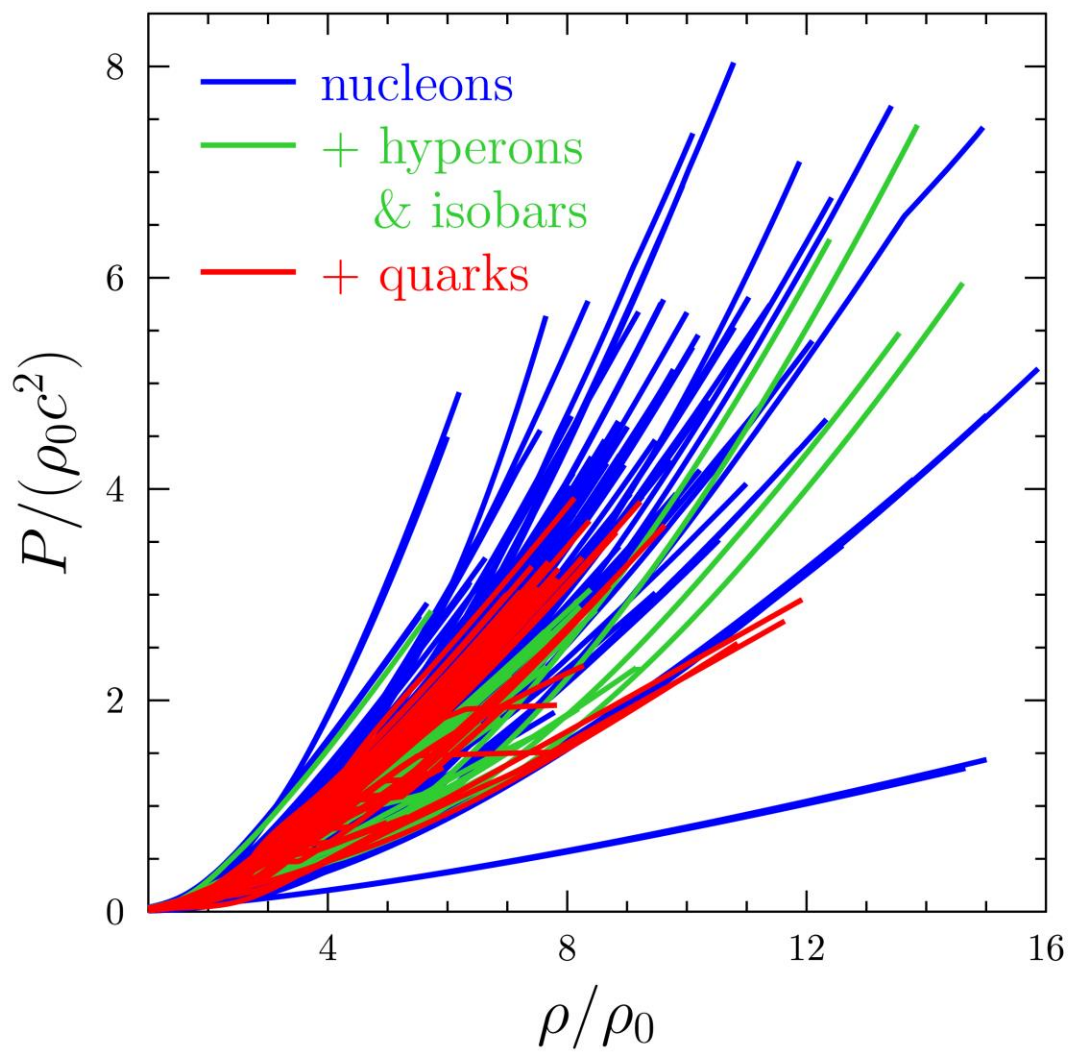
- **Equation of state = EoS**
- ***Cold degenerate matter***
  $T < 10^{10}\mathrm{K},\ T_F \sim 10^{12}\mathrm{K}$



$\sim 0.5\rho_0$

$\rho_0 = 2.8 \times 10^{14}\ \mathrm{g/cm^3}$

- $P(\rho), n(\rho), \mathrm{composition}(\rho), \ldots$



Oppenheimer-Volkoff Mapping

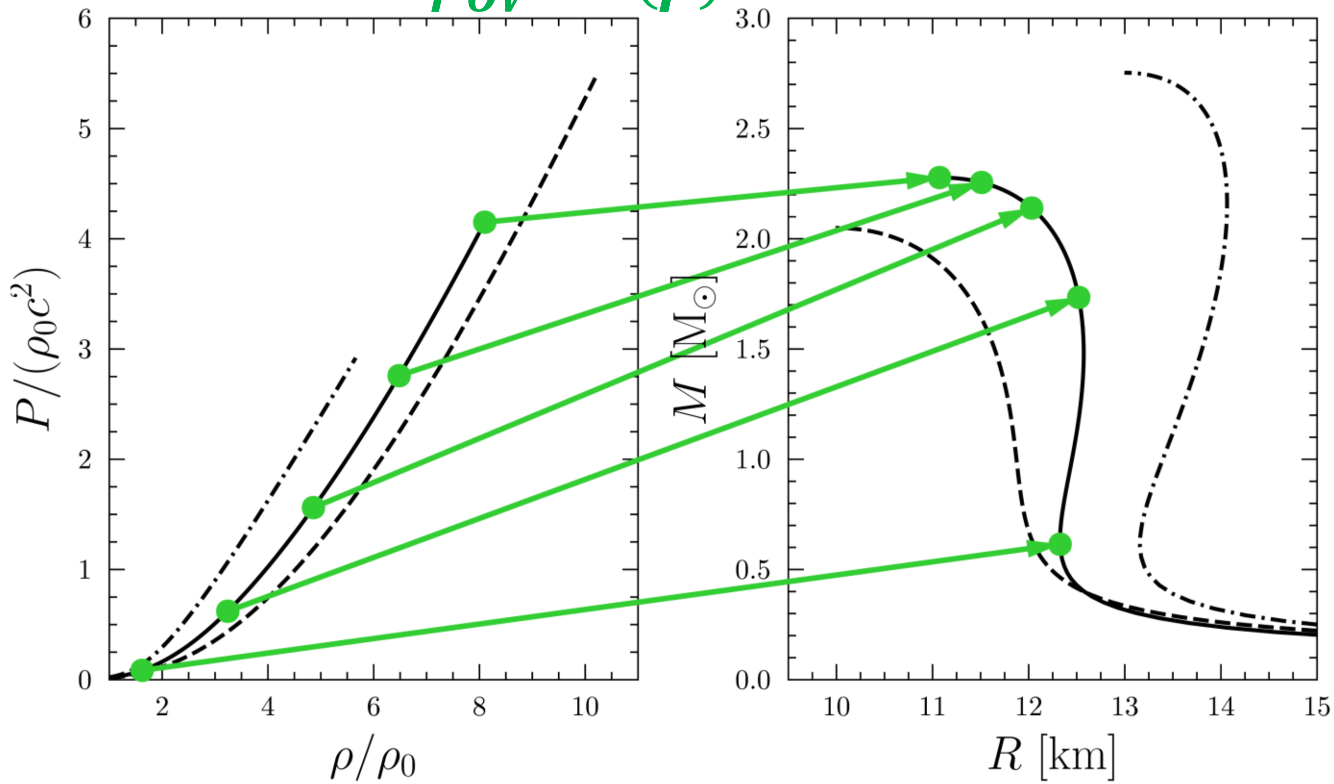
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\frac{\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)}{1-\frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- ***GR hydrostatics***
- ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

$\psi_{OV}: \ P(\rho) \mapsto M - R$



Oppenheimer-Volkoff Mapping

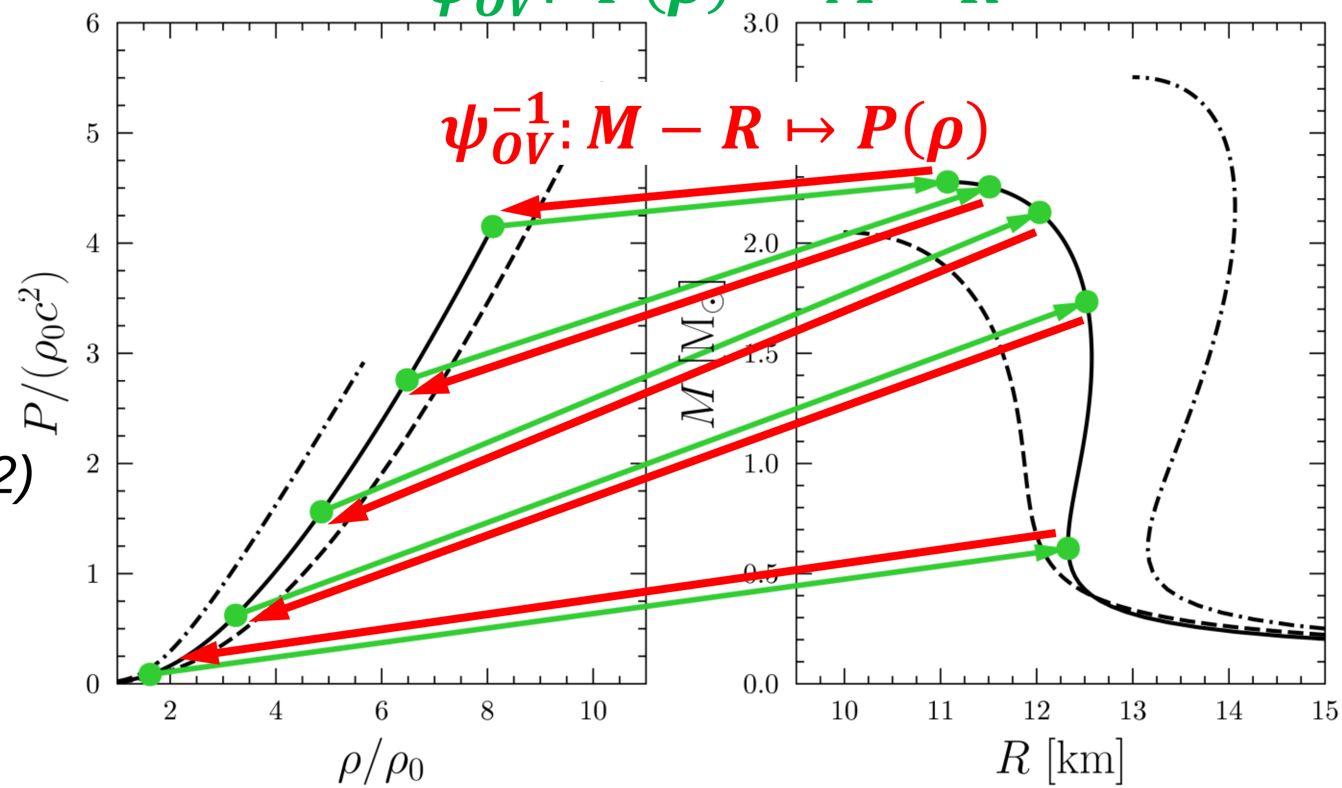
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\frac{\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)}{1-\frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- ***GR hydrostatics***
- ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

ψ_{OV}: $P(\rho) \mapsto M - R$



- **Lindblom (1992):** $\exists\psi_{OV}^{-1}$
  - numerics
  - neural networks *(Soma+ JCAP 2022)*

Oppenheimer-Volkoff Mapping

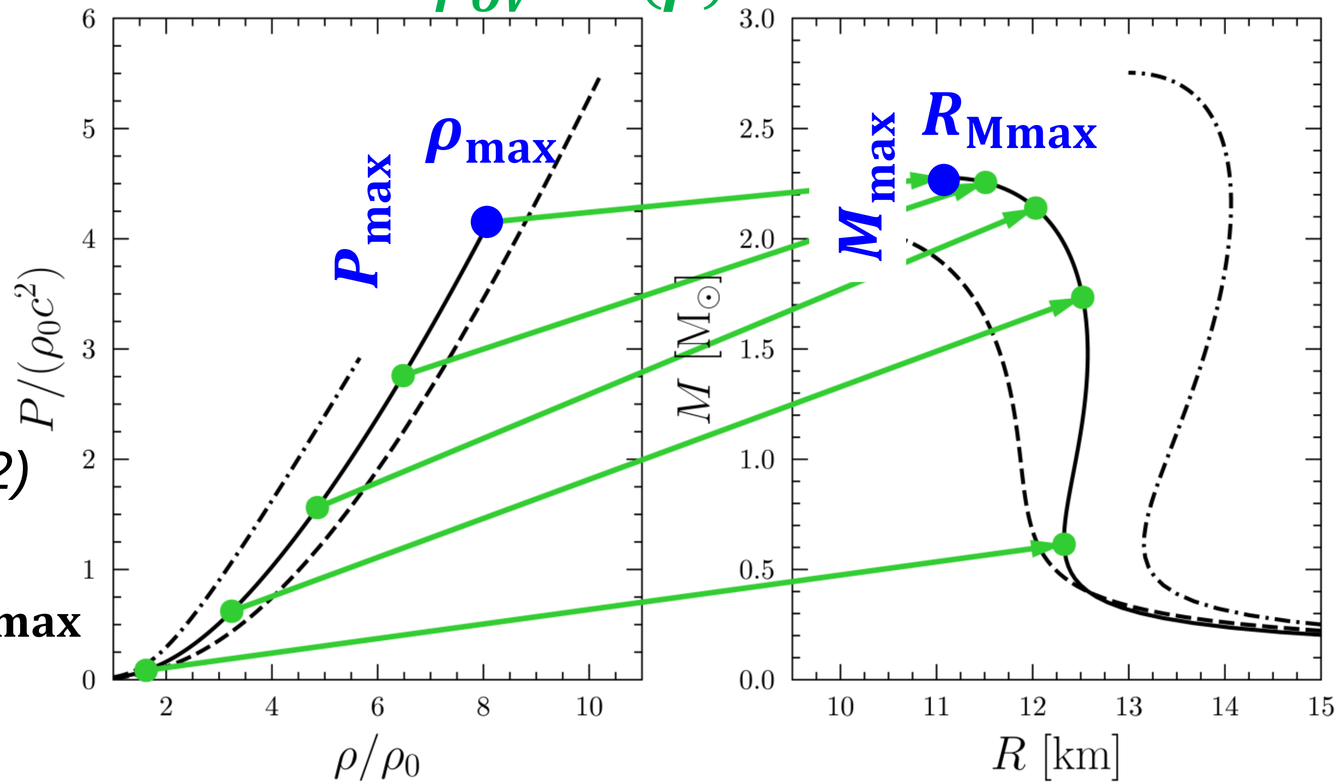
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\rho}{r^2}\frac{\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)}{1-\frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- ***GR hydrostatics***
- ~~***Rotation***~~

Tolman (1939); Oppenheimer & Volkoff (1939)

ψ_{OV}: $P(\rho) \mapsto M - R$



- **Lindblom (1992): $\exists \psi_{OV}^{-1}$**
  - numerics
  - neural networks *(Soma+ JCAP 2022)*
- $\exists$ $\boldsymbol{M_{\max} \leftrightarrow \rho_{\max}, P_{\max}}$

Dimension of EoS Manifold

Lindblom (2010): spectral representation

- 34 «realistic» EoS ← $npe\mu$ ← $npe\mu\Lambda\Sigma$ ← $npe\mu\Lambda\Sigma uds$
- universal fit

$$\ln\Gamma(P) = \gamma_0 + \gamma_1 \ln P + \gamma_2 (\ln P)^2 + \gamma_3 (\ln P)^3 + \gamma_4 (\ln P)^4 + \cdots$$

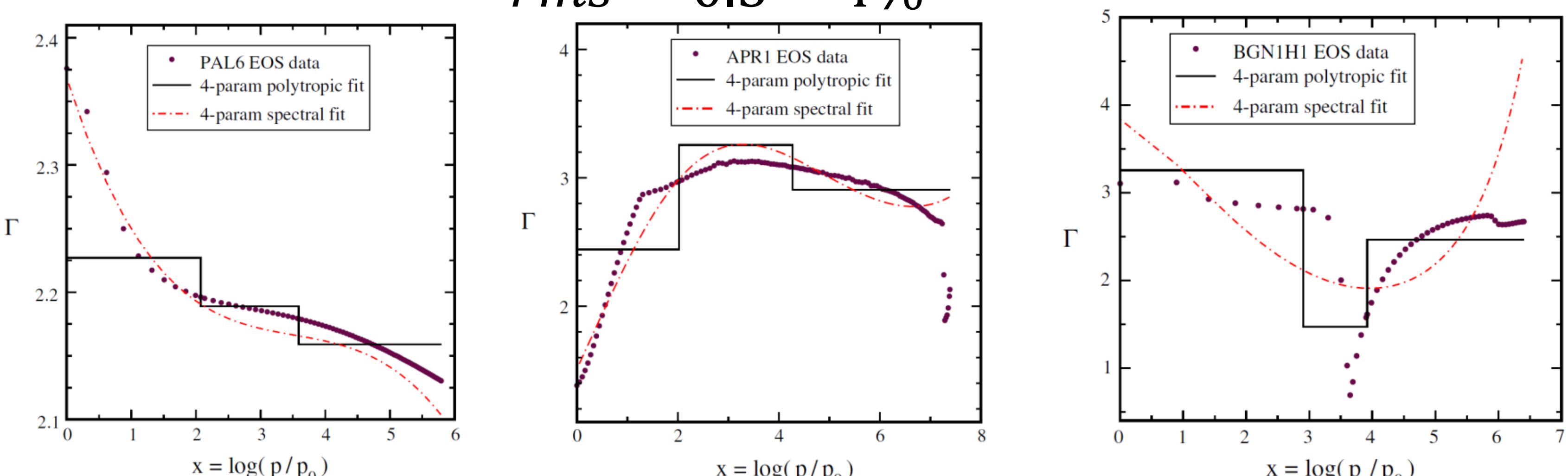
$P - \rho$ ↕ $M - R$

$\gamma_0 + \gamma_1 \ln P$: $rms \sim 3-8\%$

up to $\gamma_2 (\ln P)^2$: $rms \sim 1-5\%$

up to $\gamma_3 (\ln P)^3$: $rms \sim 0.5-4\%$

$$\Gamma = \frac{\rho c^2 + P}{\rho c^2} \frac{\mathrm{d}P}{\mathrm{d}(\rho c^2)}$$



Lindblom (2010)

Dimension of EoS Manifold

Lindblom (2010): spectral representation

- 34 «realistic» EoS ← $npe\mu$ ← $npe\mu\Lambda\Sigma$ ← $npe\mu\Lambda\Sigma uds$
- universal fit

$$\ln\Gamma(P) = \gamma_0 + \gamma_1 \ln P + \gamma_2 (\ln P)^2 + \gamma_3 (\ln P)^3 + \gamma_4 (\ln P)^4 + \cdots$$

$\Updownarrow$

$P-\rho$

$\Updownarrow$

$M-R$

$rms \sim 3-8\%$

$rms \sim 1-5\%$

$rms \sim 0.5-4\%$

$$\Gamma = \frac{\rho c^2 + P}{\rho c^2}\frac{\mathrm{d}P}{\mathrm{d}(\rho c^2)}$$

Idea of this work

handful parametrization:

$M_{\max}, R_{\mathrm{Mmax}}$

$\Updownarrow$

$P_{\max}, \rho_{\max}$

⇒ **self-similarity of $M-R$ & $P-\rho$ curves**

⇒ **explicit ψ_{OV}^{-1}**

Plan of the Further Talk

1. **Describe the zoo of used EoSs**

2. **Build the Inverse OV mapping**
   - **Step I:** $P_{\mathrm{max}}, \rho_{\mathrm{max}} \leftrightarrow M_{\mathrm{max}}, R_{\mathrm{max}}$
   - **Step II: self-similar curves** $P/P_{\mathrm{max}} - \rho/\rho_{\mathrm{max}}$
   - **Step III: self-similar curves** $M/M_{\mathrm{max}} - R/R_{\mathrm{max}}$

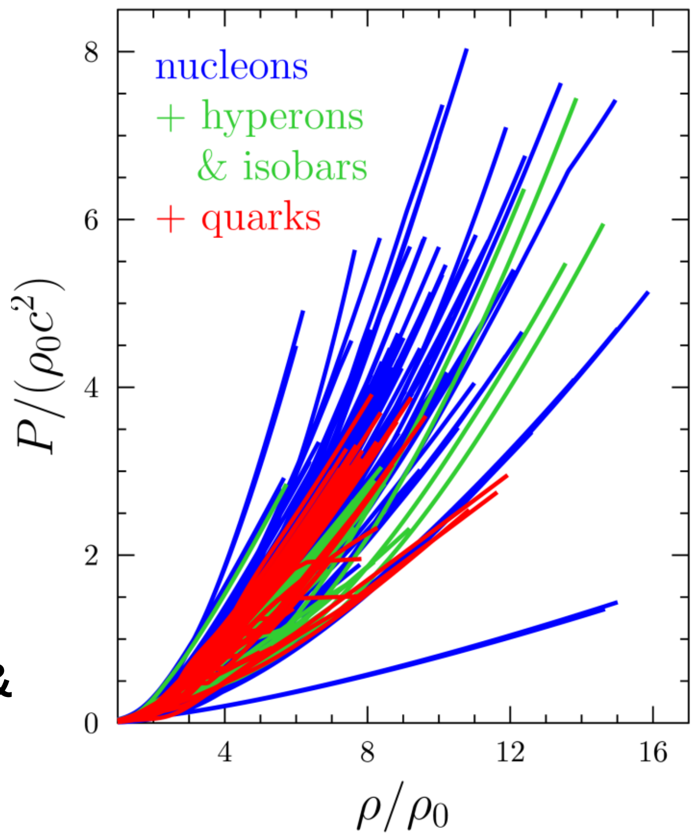
   } **Universal fits**

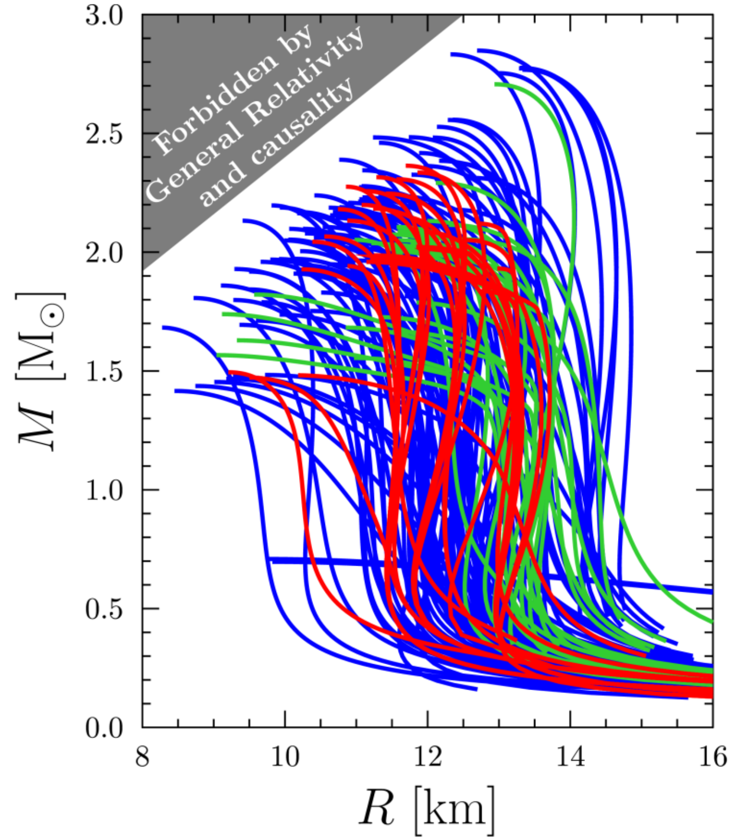
3. **Apply the Inverse OV to observations**

EoS Zoo

162 models → **97 $npe\mu$** → **32 $+\Lambda\Sigma\Xi\Delta$** → **33 $+uds$**

- **CompOSE**
  https://compose.obspm.fr/
- **Read+2009**
  ↔Lindblom (2010)
- **Ozel & Freira 2016**
- **Gusakov, Kantor & Haensel 2014, Fortin+2017, Ofengeim+2019,…**



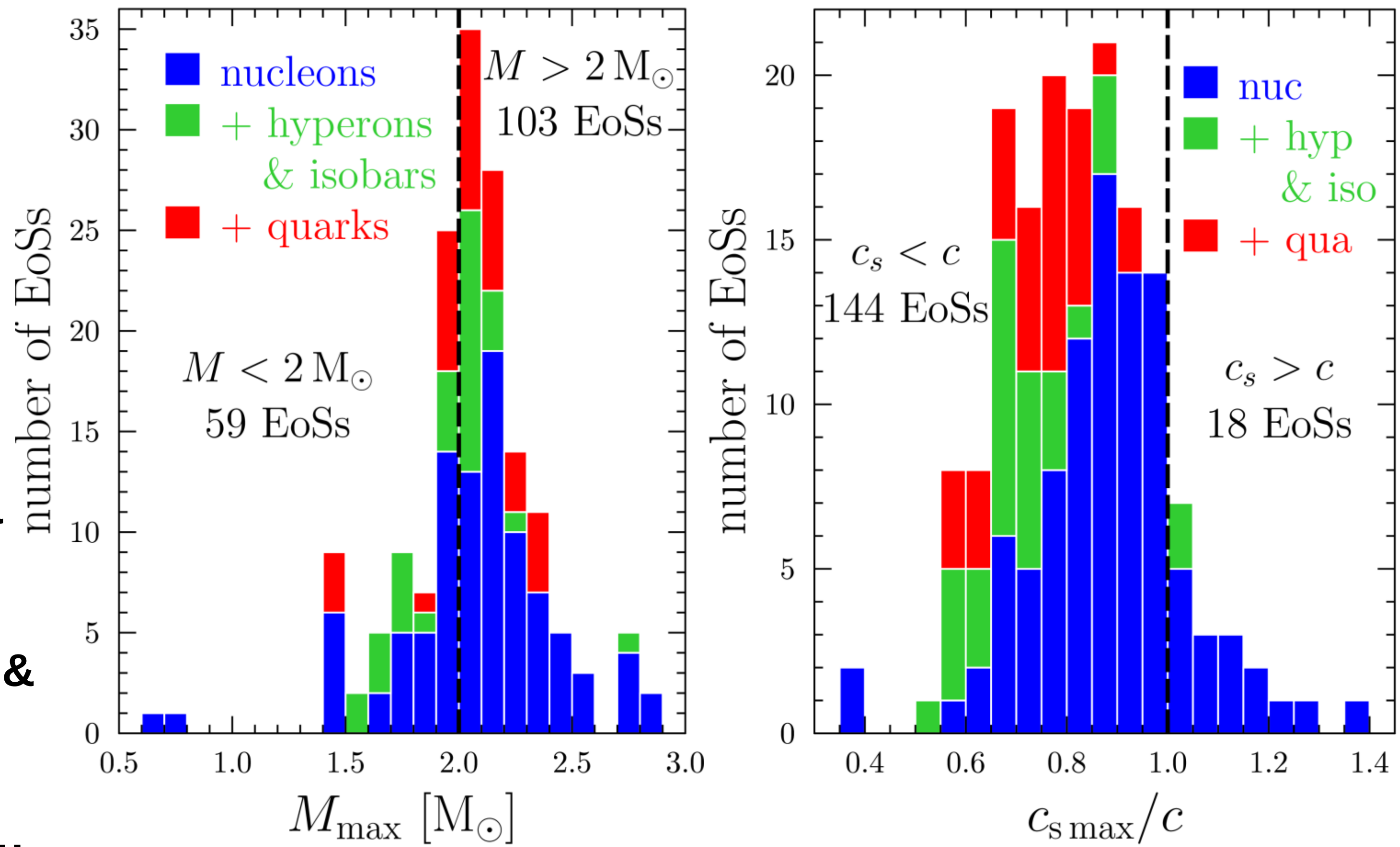


…free npe, PAL, HHJ, variational, Skyrme, RMF, QMC, QHC,…

EoS Zoo

162 models → **97 $npe\mu$** → **32 $+\Lambda\Sigma\Xi\Delta$** → **33 $+uds$**

- **CompOSE**
  https://compose.obspm.fr/
- **Read+2009**
  ↔Lindblom (2010)
- **Ozel & Freira 2016**
- **Gusakov, Kantor & Haensel 2014, Fortin+2017, Ofengeim+2019,…**

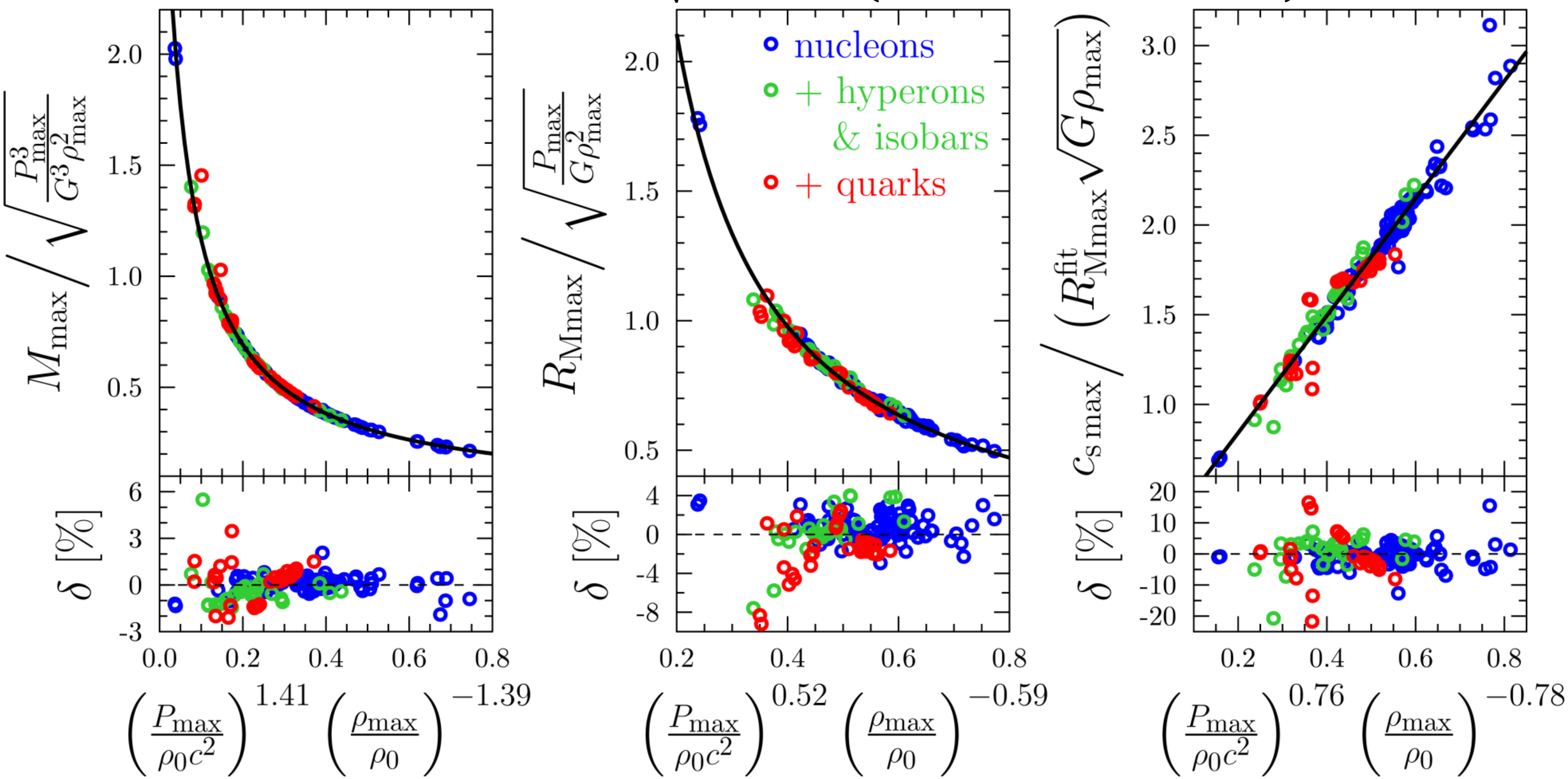


…free npe, PAL, HHJ, variational, Skyrme, RMF, QMC, QHC,…

Step I: $M_{\max}, R_{\max} \leftrightarrow P_{\max}, \rho_{\max}$

$$M_{\max} = \sqrt{\frac{P_{\max}^3}{G^3\rho_{\max}^2}}\, f_M\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right) \qquad R_{\max} = \sqrt{\frac{P_{\max}}{G\rho_{\max}^2}}\, f_R\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right)$$

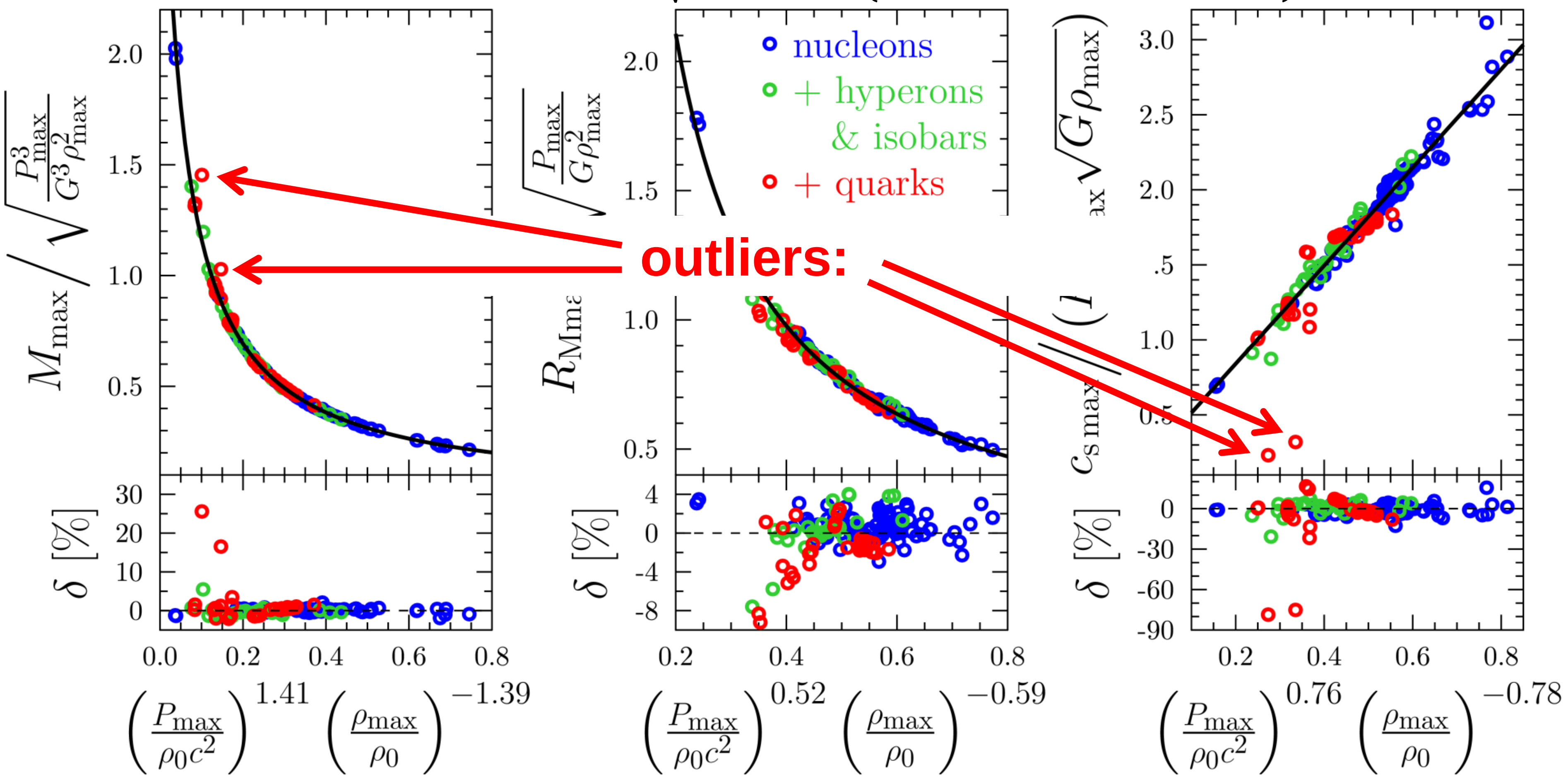
$$c_{s\,\max} = R_{\mathrm{Mmax}}^{\mathrm{fit}}\sqrt{G\rho_{\max}}\, f_c\left(\rho_{\max}/\rho_0\,, P_{\max}/\rho_0 c^2\right)$$



Step I: $M_{\max}, R_{\max} \leftrightarrow P_{\max}, \rho_{\max}$

$$M_{\max} = \sqrt{\frac{P_{\max}^3}{G^3\rho_{\max}^2}}\, f_M\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right) \qquad R_{\max} = \sqrt{\frac{P_{\max}}{G\rho_{\max}^2}}\, f_R\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right)$$

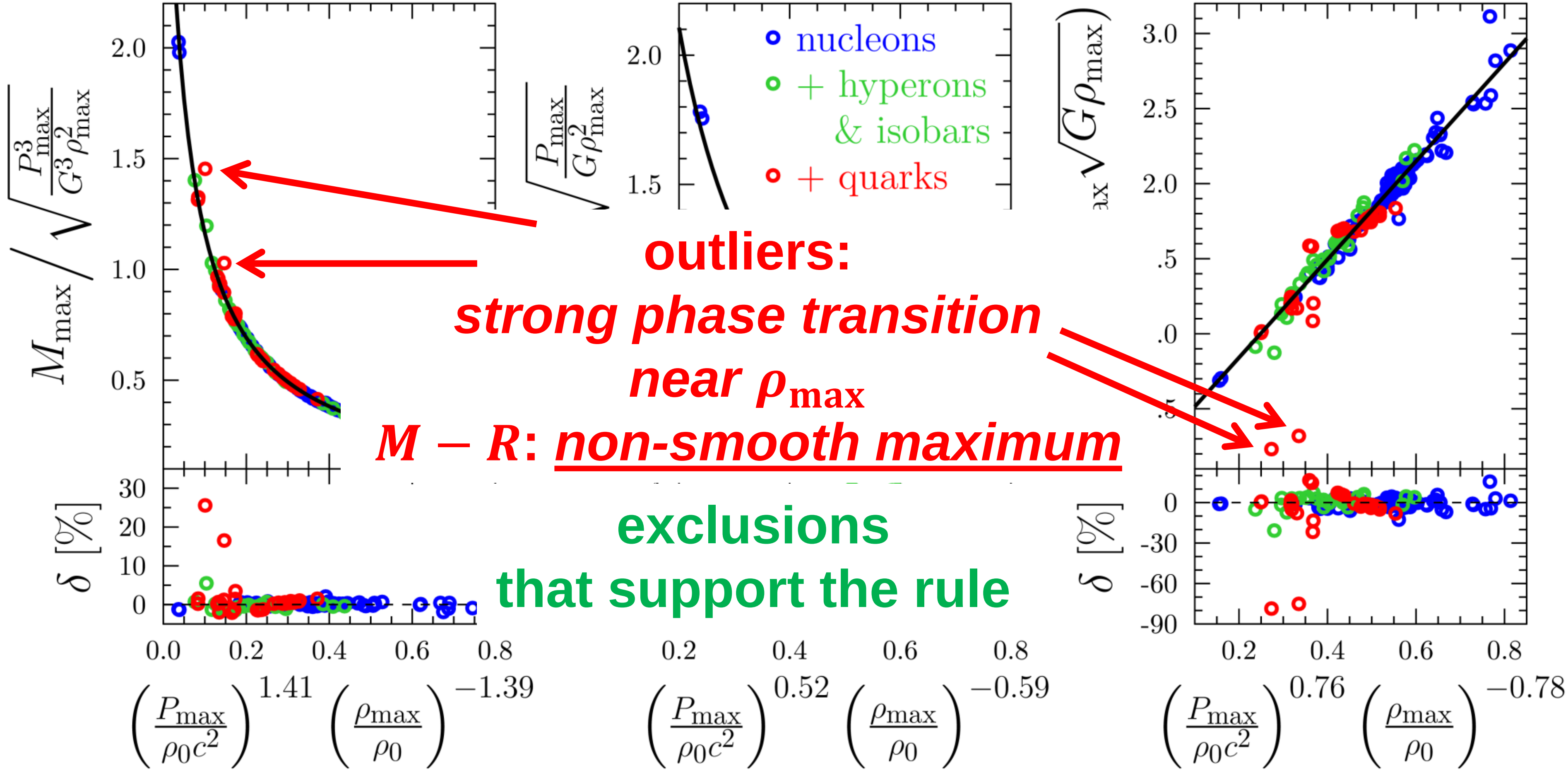
$$c_{s\,\max} = R_{\mathrm{Mmax}}^{\mathrm{fit}}\sqrt{G\rho_{\max}}\, f_c\left(\rho_{\max}/\rho_0\,, P_{\max}/\rho_0 c^2\right)$$



Step I: $M_{\max}, R_{\max} \leftrightarrow P_{\max}, \rho_{\max}$

$$M_{\max} = \sqrt{\frac{P_{\max}^3}{G^3 \rho_{\max}^2}} f_M\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right) \qquad R_{\max} = \sqrt{\frac{P_{\max}}{G \rho_{\max}^2}} f_R\left(\frac{\rho_{\max}}{\rho_0}, \frac{P_{\max}}{\rho_0 c^2}\right)$$

$$c_{s\,\max} = R_{\mathrm{Mmax}}^{\mathrm{fit}} \sqrt{G\rho_{\max}} f_c(\rho_{\max}/\rho_0, P_{\max}/\rho_0 c^2)$$



outliers:

strong phase transition

near $\rho_{\max}$

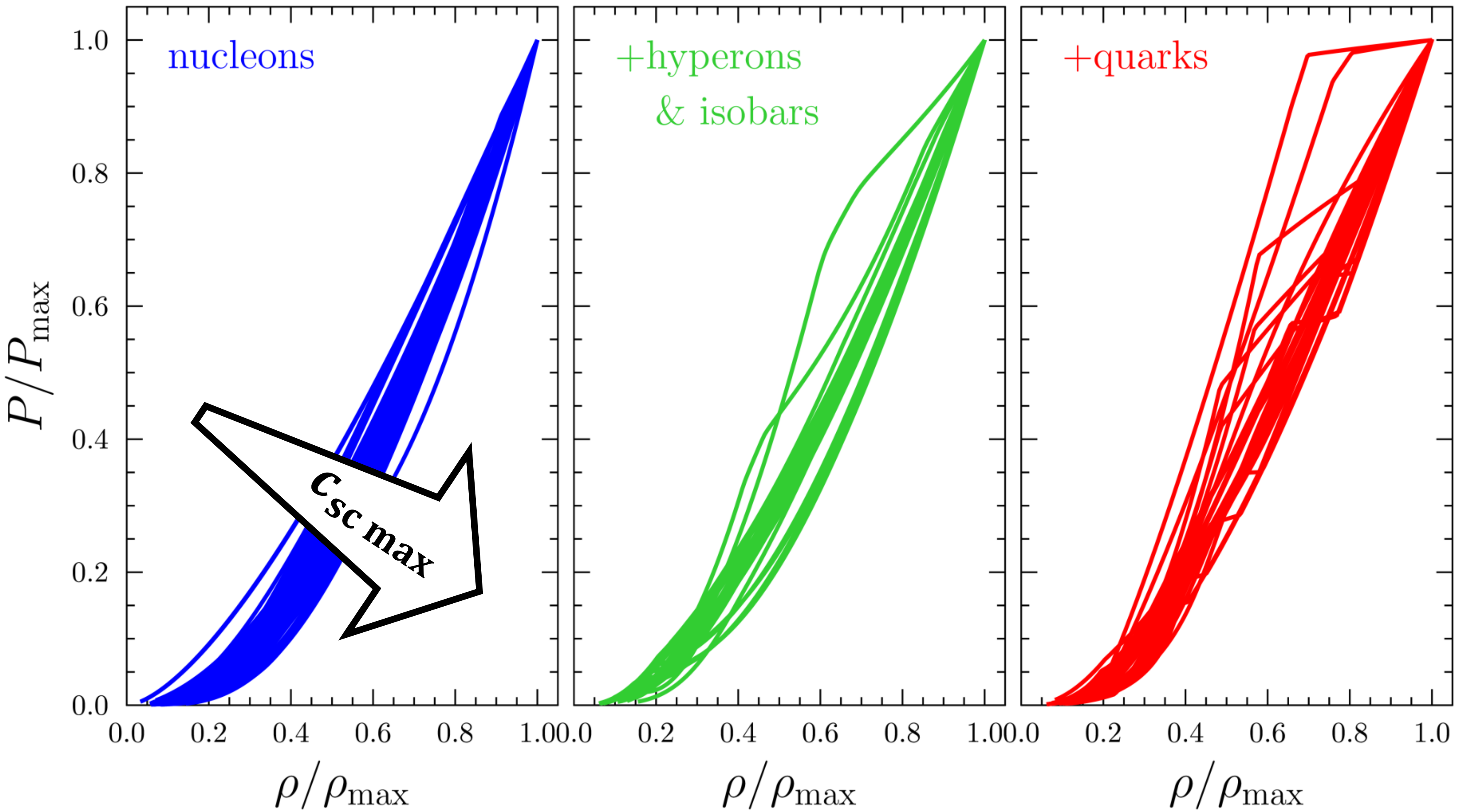
$M - R$: non-smooth maximum

exclusions

that support the rule

Step II: Self-similar $P(\rho)$

$$\frac{P}{P_{\max}} \approx g\left(\frac{\rho}{\rho_{\max}}\right)$$



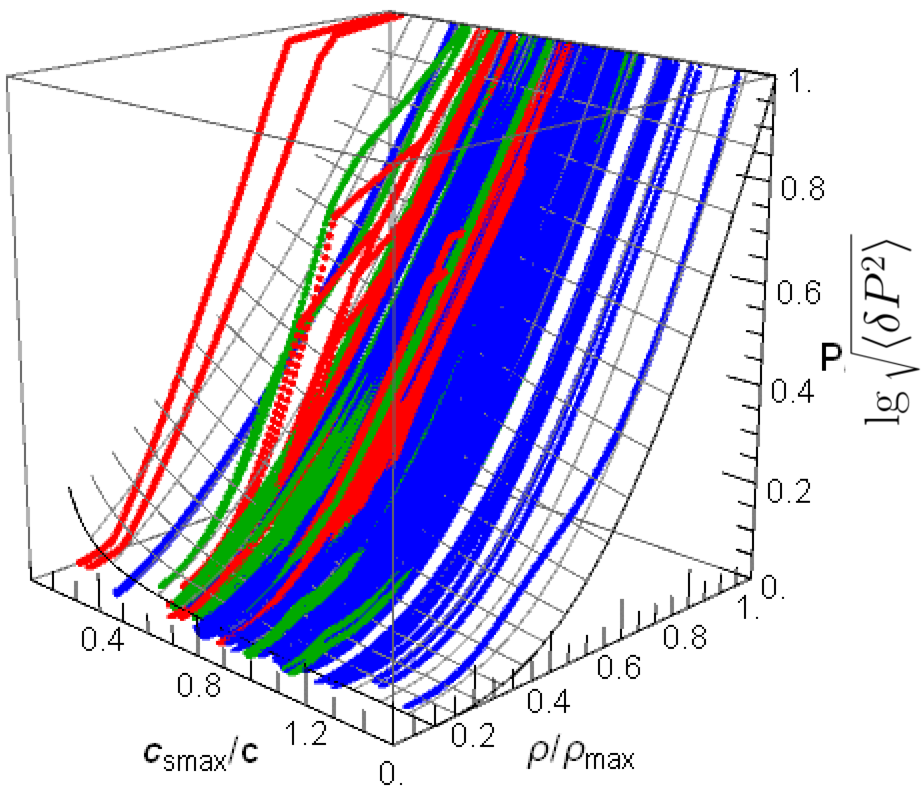
Step II: Universal Fit $P(\rho)$

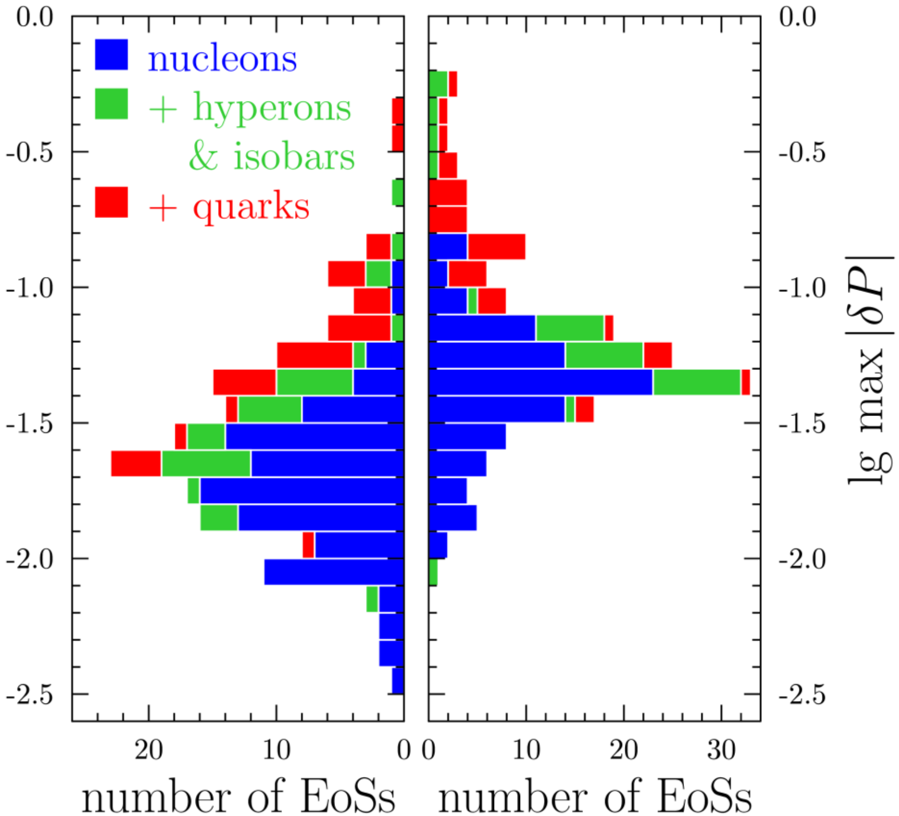
$$\frac{P}{P_{\max}} = g\left(\frac{\rho}{\rho_{\max}}; c_{s\,\max}, \gamma_{\max}\right)$$

$c_{s\,\max}(P_{\max}, \rho_{\max})$

$$\gamma_{\max} = \frac{\rho_{\max}}{P_{\max}} c_{s\,\max}^2(P_{\max}, \rho_{\max})$$

$\rho \gtrsim 3\rho_0 \leftrightarrow$ center of $1M_\odot$





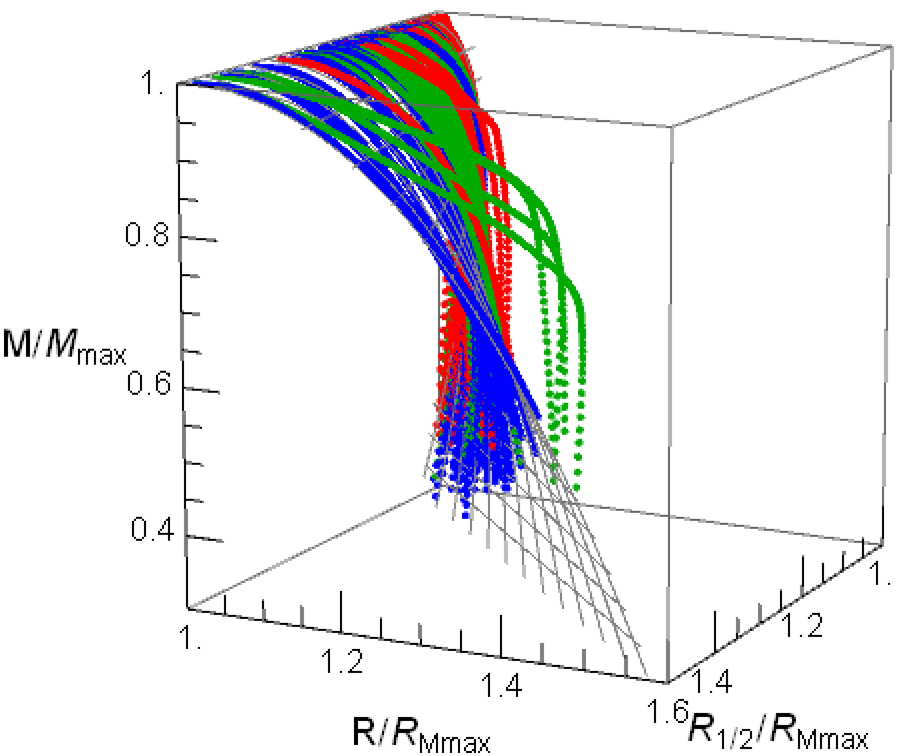
$$\frac{R}{R_{\mathrm{Mmax}}} = 1 + \left[2\left(\sqrt{2}-1\right)\frac{R_{1/2}}{R_{\mathrm{Mmax}}} - a\right]\sqrt{1-\frac{M}{M_{\max}}} + \left[2\left(\sqrt{2}-1\right)\frac{R_{1/2}}{R_{\mathrm{Mmax}}} - 2 + a\sqrt{2}\right]\left(1-\frac{M}{M_{\max}}\right)$$

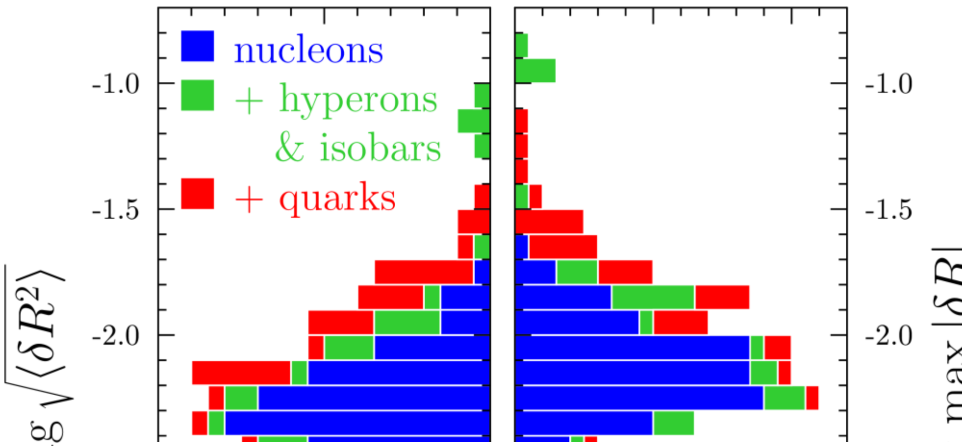
$a = 0.478$

1 fitting parameter

$R_{1/2} = R(M_{\max}/2)$

$M \gtrsim 1 M_\odot$



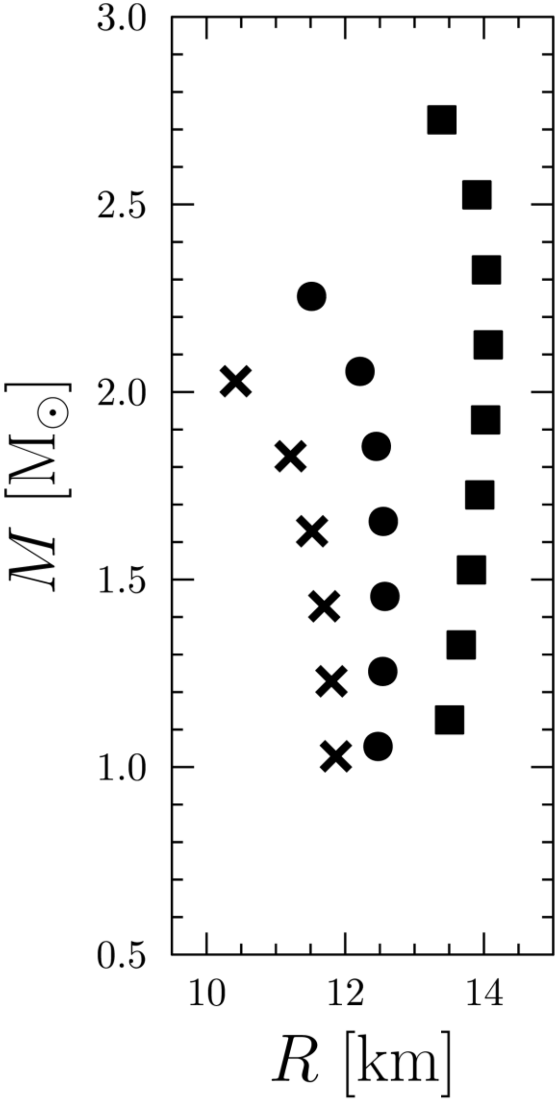


$R_{1/2} \neq f(M_{\max}, R_{\mathrm{Mmax}})$

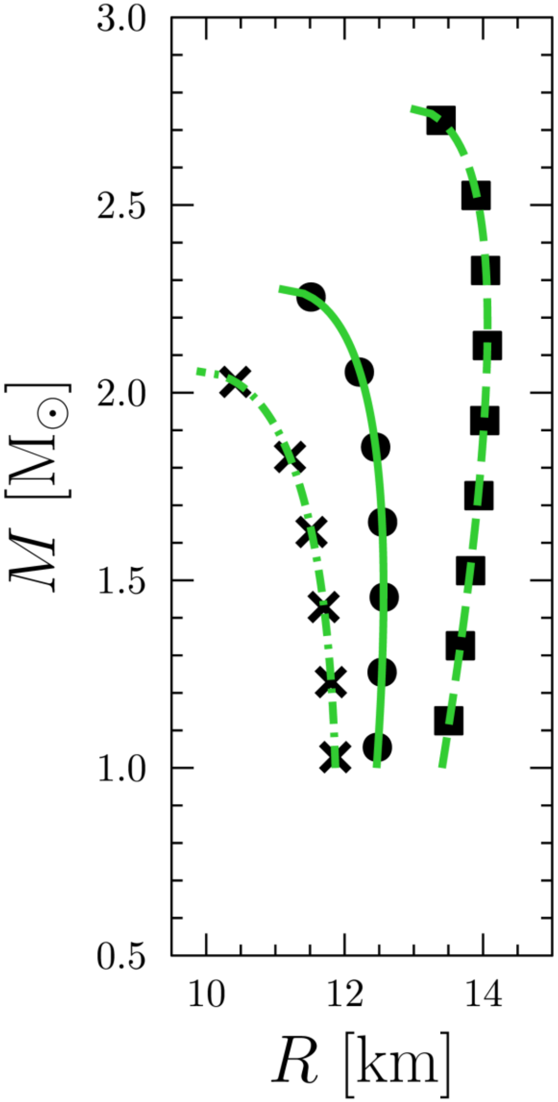
3 parameters: $M_{\max}, R_{\mathrm{Mmax}}, R_{1/2}$

$M - R$ feels $P(\rho < 3\rho_0)$ at $\forall M$

Inverse Oppenheimer-Volkoff Mapping



Inverse Oppenheimer-Volkoff Mapping



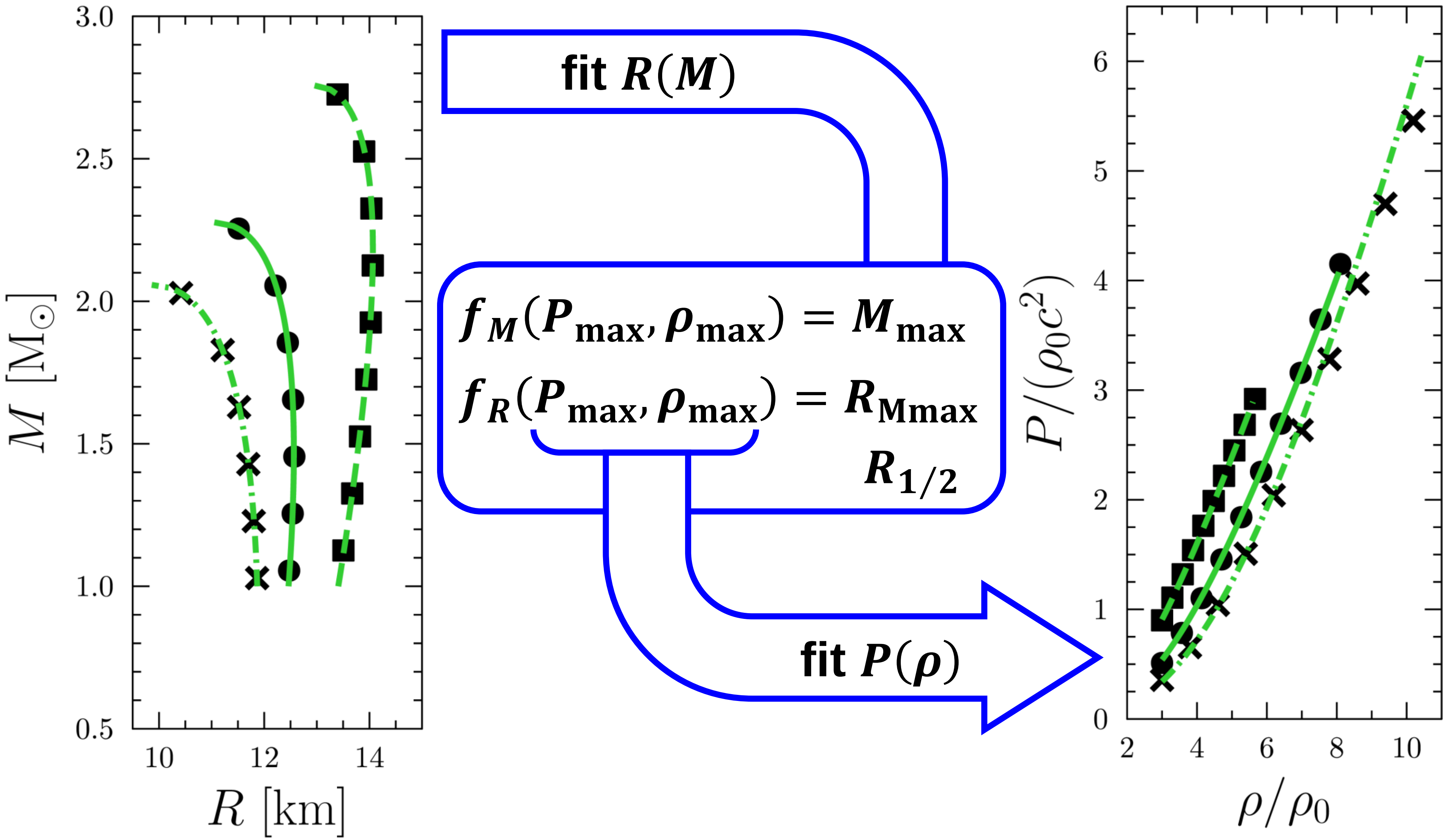
fit $R(M)$

$f_M(P_{\max}, \rho_{\max}) = M_{\max}$

$f_R(P_{\max}, \rho_{\max}) = R_{\mathrm{Mmax}}$

$R_{1/2}$

Inverse Oppenheimer-Volkoff Mapping



Analysis of Observations: Method

$M_{\max} >$ **radiopulsars**

PSR J1614-2230

PSR J0348+0432

Arzoumanian+'18; Fonseca+'21

GWs: $\Lambda = f\left(\frac{2GM}{Rc^2}\right)$

Yagi & Younes'13,'16

GW170817

Abbot+'18

$M_i, R_i \in \left(\text{fit of NS spectrum}\right)_i$

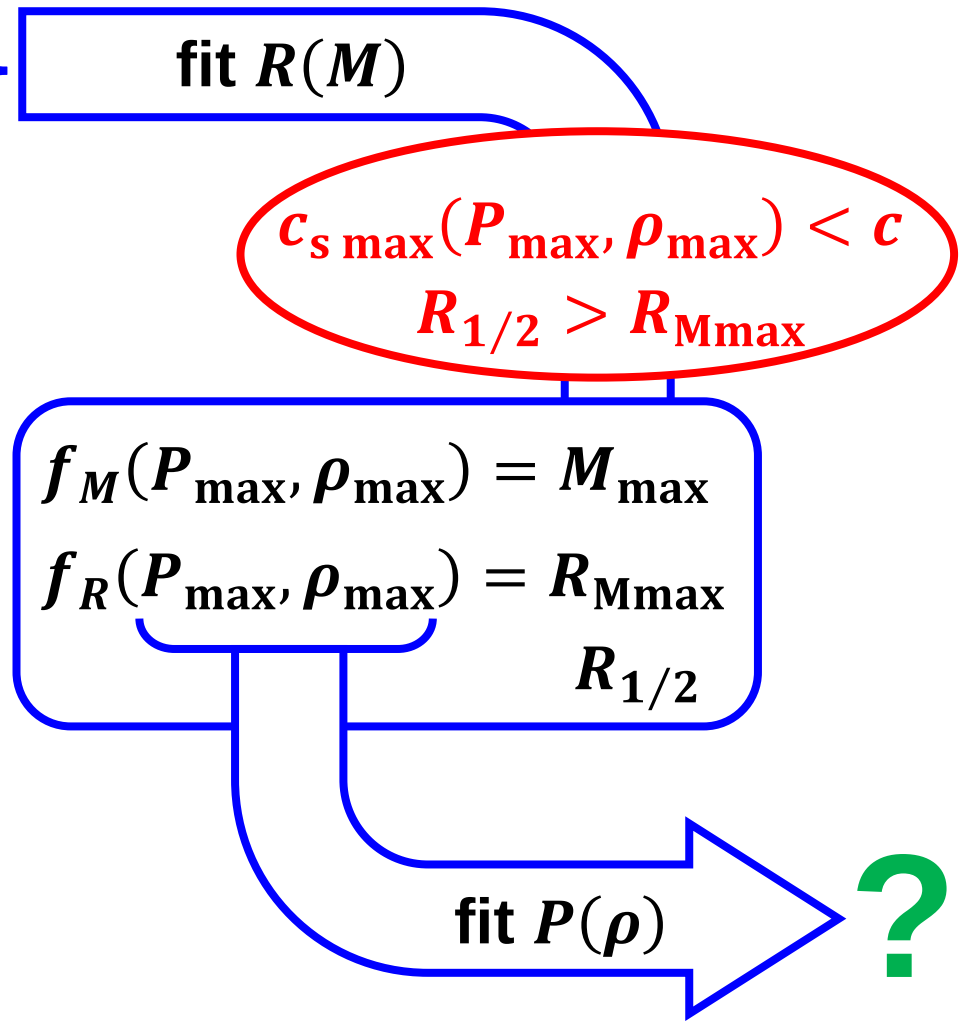
PSR J0740+6620

Miller+'19; Riley+'19

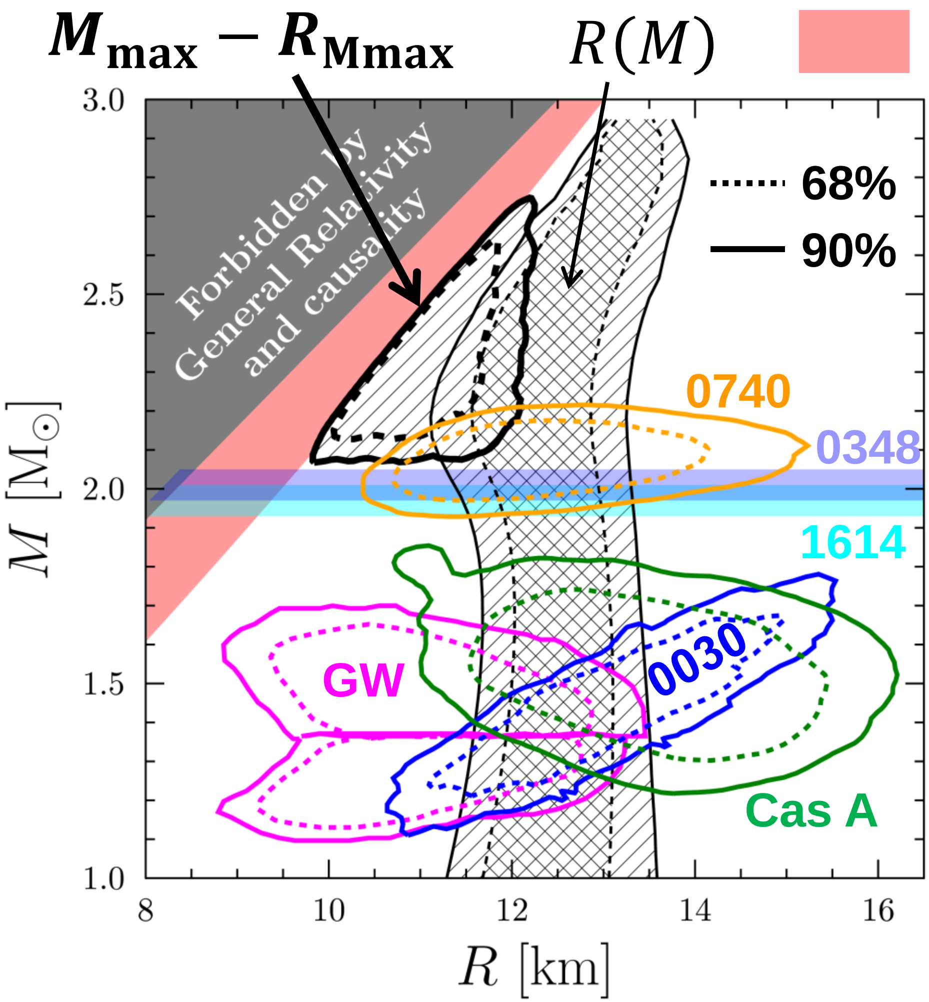
PSR J0030+0451

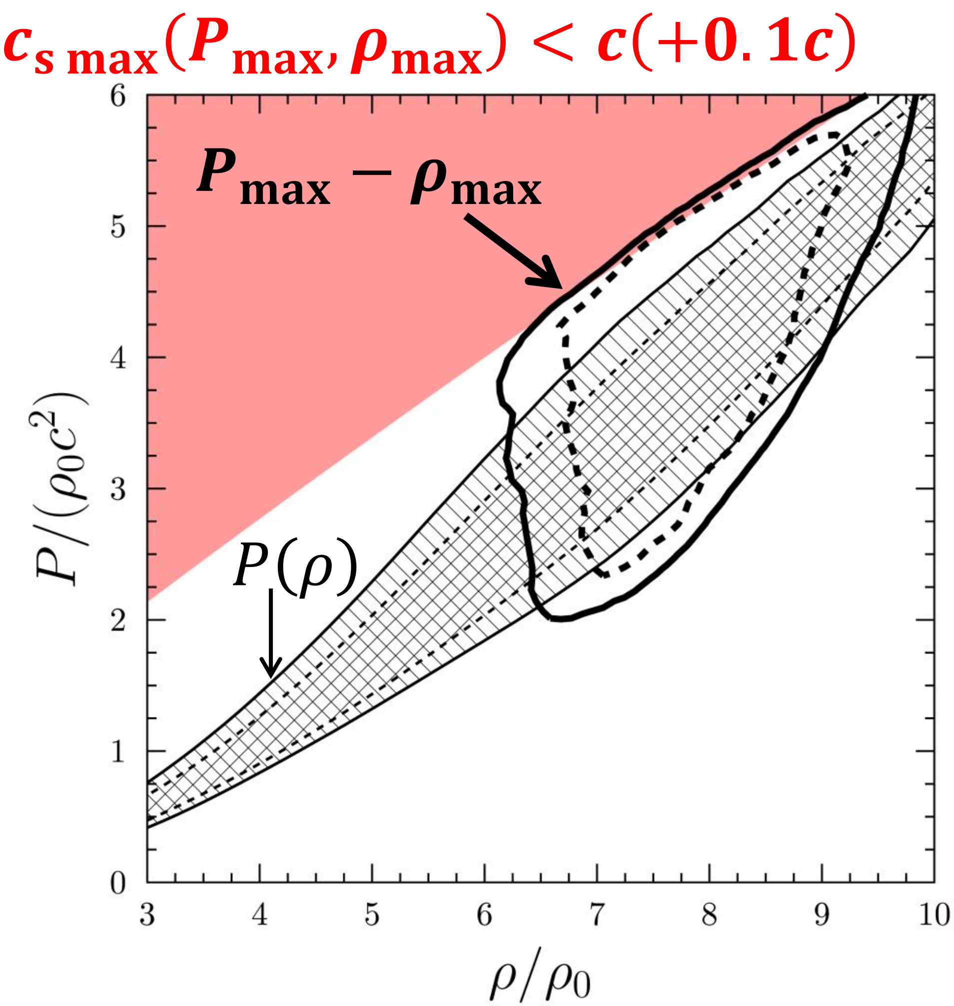
Miller+'21; Riley+'21

Cas A Shternin+'23



Analysis of Observations: Results





$$M_{max} = 2.29^{+0.15}_{-0.15}\, M_\odot$$
$$R_{max} = 11.3^{+0.5}_{-0.6}\ \text{km}$$

$$R_{1/2} = 12.4^{+0.7}_{-0.6}\text{km}$$

$$P_{max} = 4.1^{+1.0}_{-1.0}\, \rho_0 c^2$$
$$\rho_{max} = 7.7^{+1.0}_{-0.8}\, \rho_0$$

Add More Observations

$M_{\max} >$ **radiopulsars**

PSRs J1614-2230, J0348+0432

+ spider PSRs *J0952−0607, J1311−3430, J1653−0158, J1810+1744*

Kandel&Romani'23

GWs: $\Lambda = f\left(\frac{2GM}{Rc^2}\right)$ Yagi & Younes '13,'16

GW170817

$M_i, R_i \in \begin{pmatrix}\text{fit of NS}\\ \text{spectrum}\end{pmatrix}_i$

PSRs J0740+6620, J0030+0451

Cas A

+ X-ray bursters

4U 1702-429, 4U1724-307, SAX J1810.8-2609

fit $R(M)$

$c_{s\,\max}(P_{\max}, \rho_{\max}) < c$

$R_{1/2} > R_{M\max}$

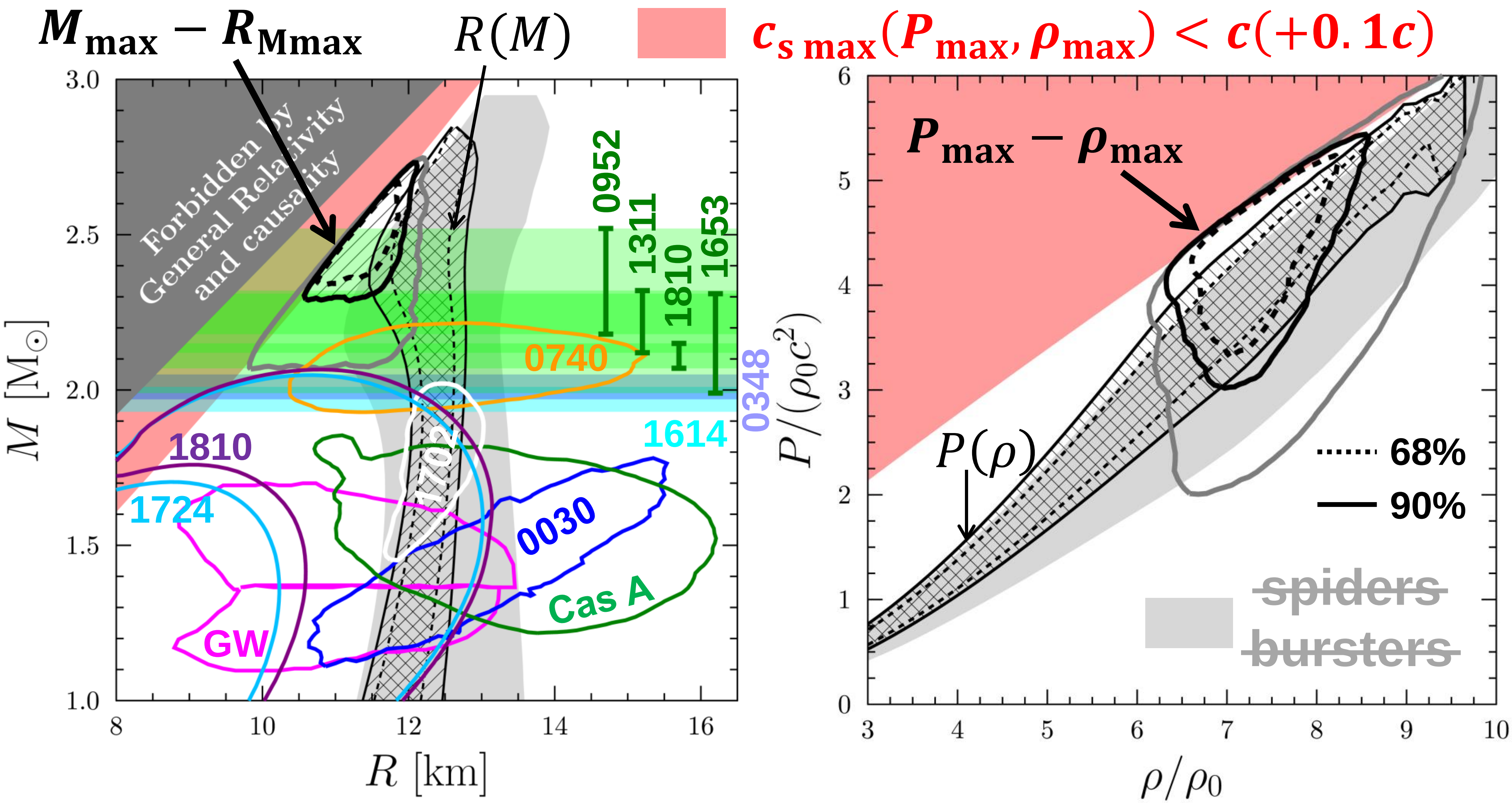
$f_M(P_{\max}, \rho_{\max}) = M_{\max}$

$f_R(P_{\max}, \rho_{\max}) = R_{M\max}$

$R_{1/2}$

fit $P(\rho)$ **?**

Results for More Observations



$M_{max} = 2.47^{+0.09}_{-0.12} M_\odot$

$R_{max} = 11.5^{+0.3}_{-0.3}$ km

$R_{1/2} = 12.0^{+0.3}_{-0.3}$ km

$P_{max} = 4.5^{+0.3}_{-0.8} \rho_0 c^2$

$\rho_{max} = 7.3^{+0.5}_{-0.5} \rho_0$

Discussion

- **Physics or antropology?**
- **Extend to $\boldsymbol{\rho < 3\rho_0}$?**
  - *Lattimer & Prakash 2001:* $R_{1.4} \propto P^{1/4}(2n_0)$
- **Extend to $\boldsymbol{M < 1M_\odot}$?**
  - *Sufficient dimension of* $M - R$ *curves family?*
- **Account for rotation?**
  - *Talk by A. Konstantinou*

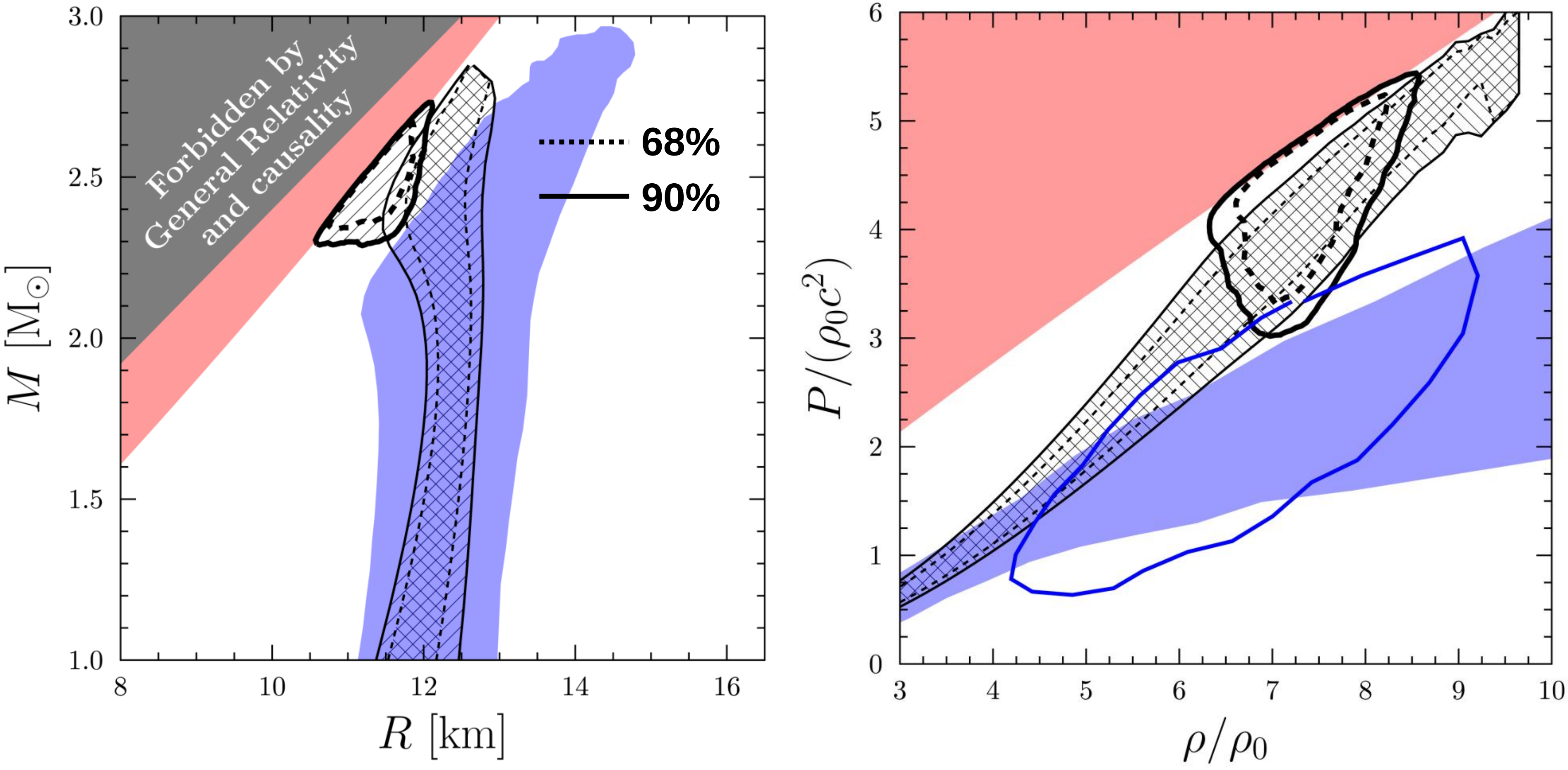
Conclusion

- **Maximum-mass NS – handful universal scale of hydrostatic properties of NSs**
- **Using this scale we**
  - **provide universal fits for $P-\rho$ and $M-R$**
  - **derived explicit (semi)analytic inverse Oppenheimer-Volkoff mapping**
- **This inverse OV mapping – new handful tool to gain properties of superdense matter from NS onservations**

Thank you!

More equations of state are welcome

Comparison with Other Works



90% Jiang, Ecker & Rezzolla 2023