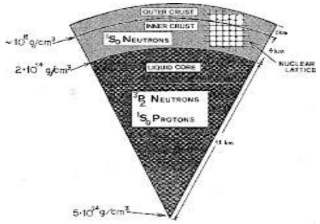
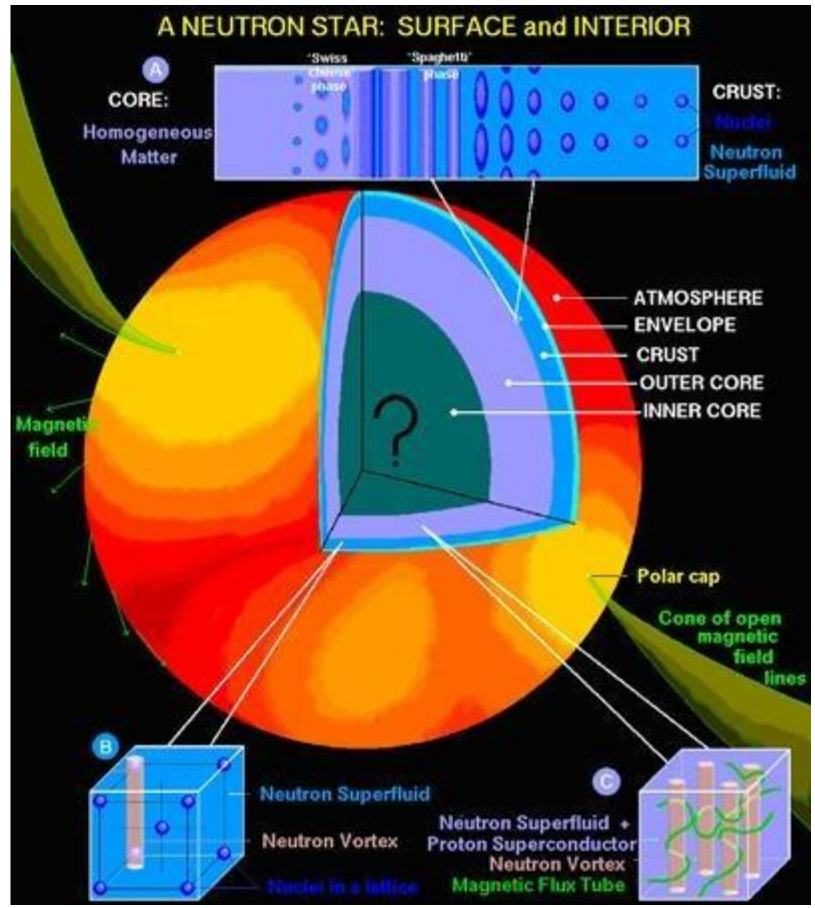
New vortices in two flavor dense quark core of compact star

Karen Shahabasyan

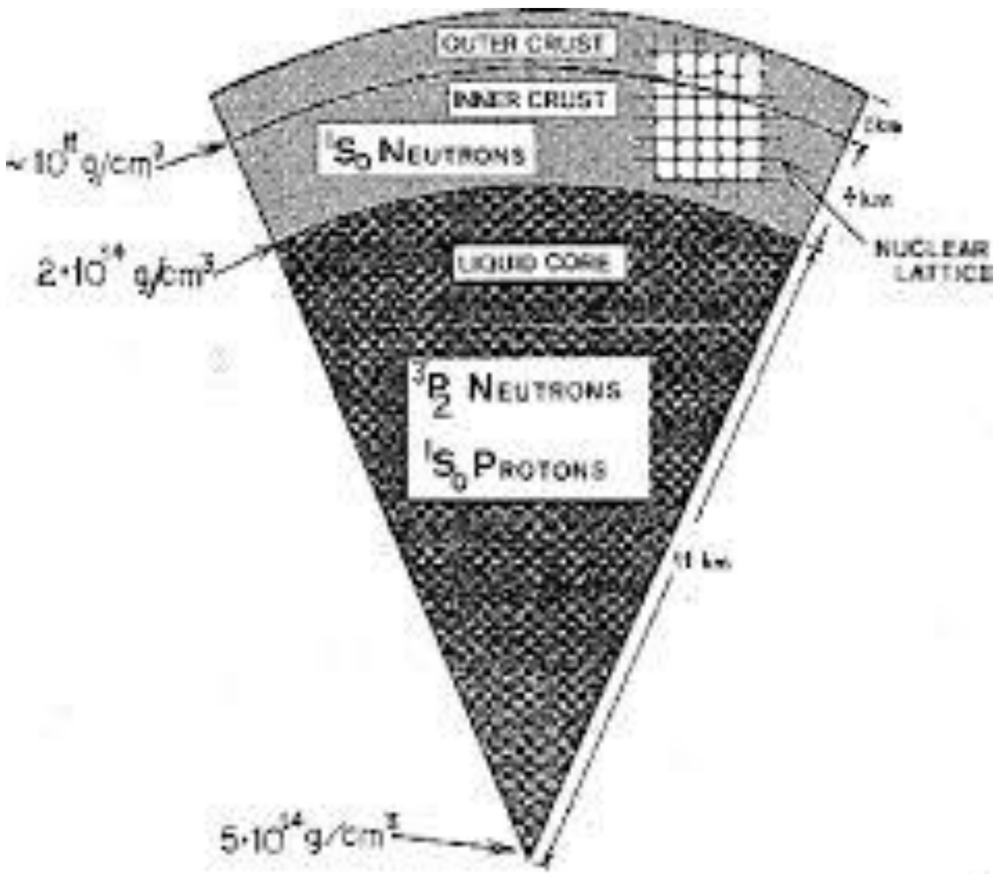
New diquark phase in two flavor dense quark core of compact star

A compact star: surface and interior





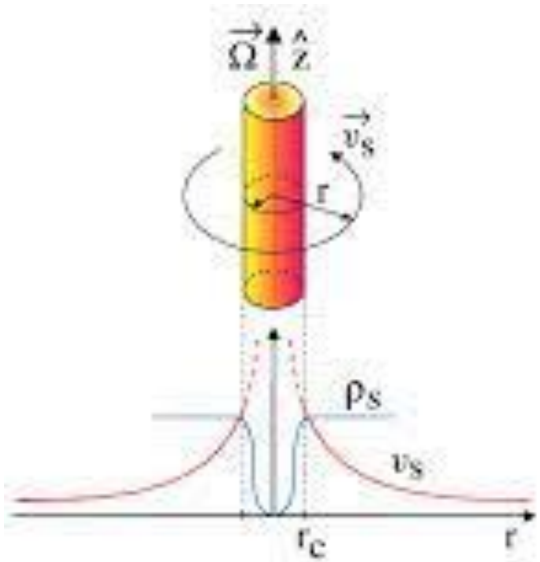
Structure of compact star:
"Ae"-phase, "Aen"-phase, "npe"-phase, quark phase: "2SC"-phase or "CFL"-phase

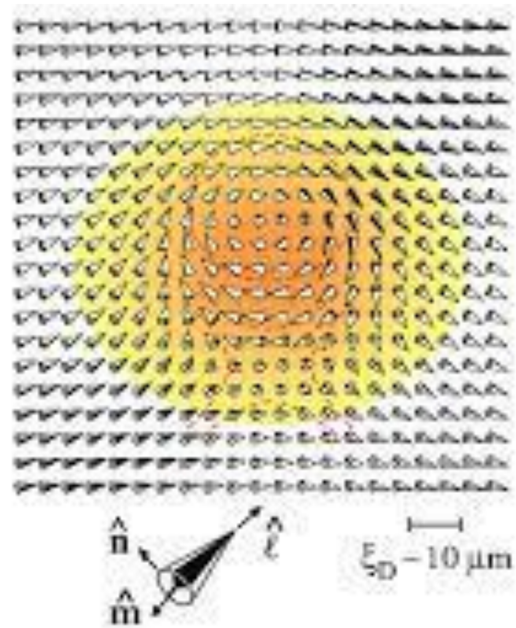


Conceptual framework for a continuous connection between hadronic and quark phases of dense matter described by QCD was suggested in
T.Schaefer,F.Wilczek, Phys. Rev. Lett. 82, 3956,1999.
M.Alford,,J.Berges, K.Rajagopal, Nucl. Phys. B558, 1999.

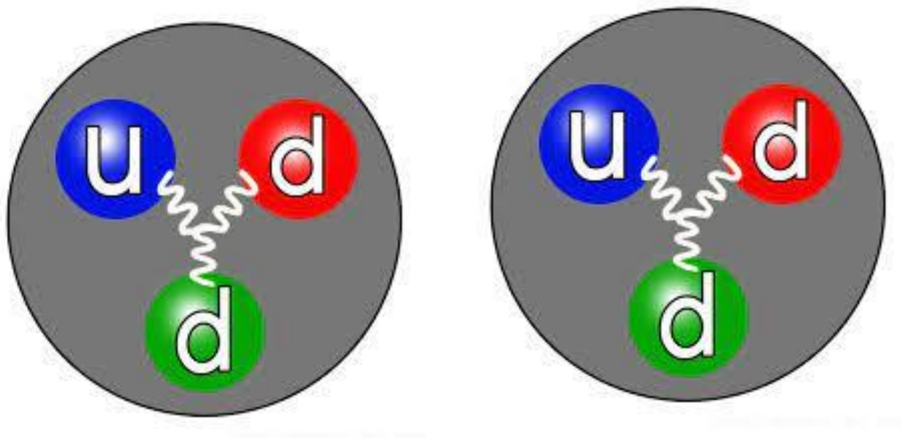
Structure of 1S0 superfluid neutron vortex

$$\vec{v}_s = \frac{\kappa_n}{2\pi r}\hat{\Phi}, \kappa_n = \frac{\pi\hbar}{m_n}, E_k = \frac{1}{2}\rho_s \int_{\xi_n}^{b} v_s^2 2\pi r dr = \rho_s \frac{\kappa_n^2}{4\pi}\ln\frac{b}{\xi_n}$$



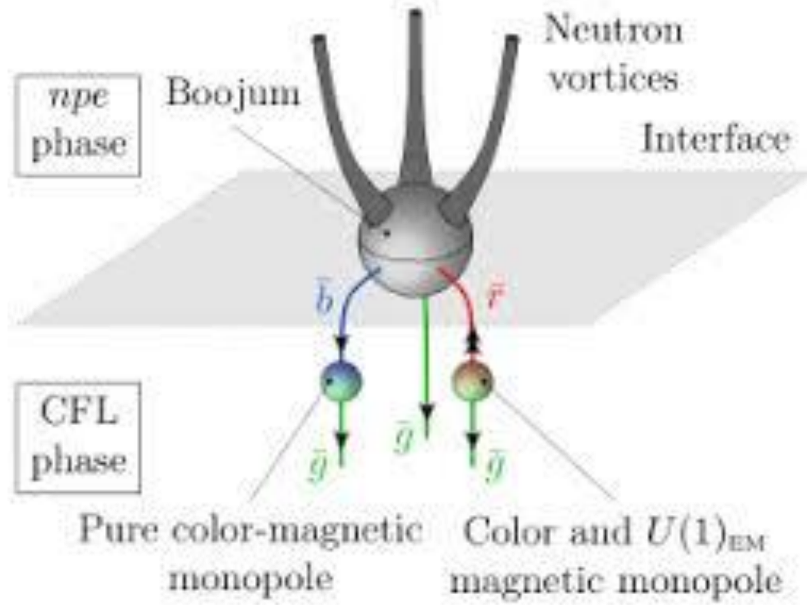


The Quark Structure of the Neutron Pair



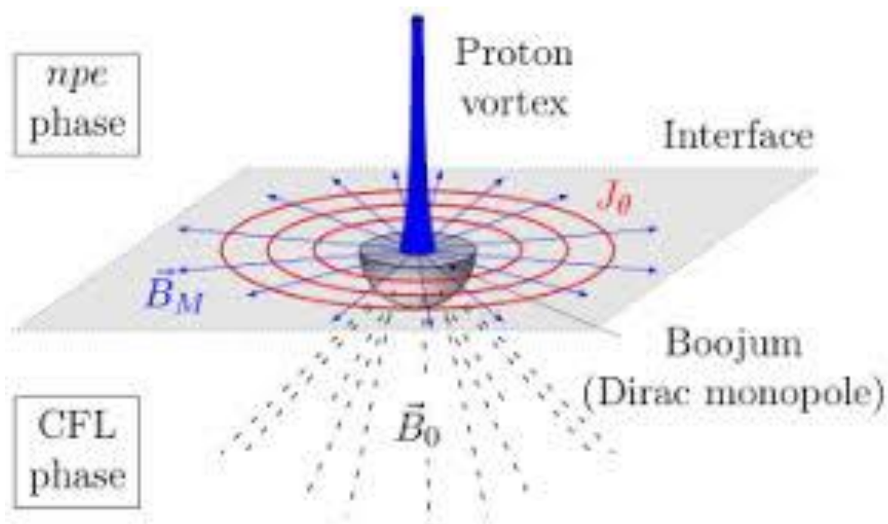
Three neutron vortices ending in a boojum at the interface

M. Cipriani, W. Vinci, M. Nitta, Phys. Rev. D 86, 121704,2012



A superconducting proton vortex ending on interface

M. Cipriani, W. Vinci, M. Nitta, Phys. Rev. D 86, 121704, 2012.



The neutron-neutron spin-orbit interactions is generated by Lorentz scalar and vector couplings of neutrons. Phenomenological boson exchange models associate these interactions with scalar and vector boson fields, σ(x) and $v^{\mu}(x)$

$$\mathcal{L}_S = -\, g_S\, \bar{\psi}(x)\psi(x)\,\sigma(x)\,,$$

$$\mathcal{L}_V = -\, g_V\, \bar{\psi}(x)\gamma_\mu\psi(x)\, v^\mu(x) + \frac{g_T}{2m_N}\bar{\psi}(x)\sigma_{\mu\nu}\psi(x)\,\partial^\nu v^\mu(x)\,,$$

$$P = \frac{1}{2}(p+p')\,, \quad q = p' - p\,. \qquad S = s_1 + s_2,$$

$$\langle p'|V_{LS}|p\rangle = -\frac{i}{2m_N^2}\left[\frac{g_S^2}{q^2+m_S^2} + \frac{3g_V^2 + 4g_V g_T}{q^2+m_V^2}\right] S\cdot(P\times q)$$

Spin-orbit interaction favoring J=2 neutron pairing in “npe-phase
Y. Fujimoto,K.Fukushima, W. Weise, Phys. Rev. D 101, 094009,2020

$$\vec{\mathrm{L}}\cdot\vec{\mathrm{S}} = \frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right] \qquad \vec{\mathrm{L}}\cdot\vec{\mathrm{S}} : -2, -1, +1, \mathrm{in} : {}^3P_o,\ {}^3P_1,\ {}^3P_2$$

$$V_{LS}(\boldsymbol{r}) = \frac{1}{2m_N^2 r}\frac{df(r)}{dr}\boldsymbol{L}\cdot\boldsymbol{S},$$

$$f(r) = \frac{g_S^2}{4\pi}\frac{e^{-m_S r}}{r} + \frac{g_V^2}{4\pi}\left(3 + \frac{4g_T}{g_V}\right)\frac{e^{-m_V r}}{r},$$

$$\frac{g_s^2}{4\pi}\frac{d}{dr}\left(\frac{e^{-mr}}{r}\right) = -\frac{g_s^2}{4\pi}\frac{(1+mr)e^{-mr}}{r^2} < 0, \qquad \mathrm{V}_{LS}(r) < 0, \left\langle \vec{\mathrm{L}}\cdot\vec{\mathrm{S}} \right\rangle = +1,\ {}^3P_2,$$

The spin-orbit interaction between quarks can be produced by one-gluon exchange

$$\langle \boldsymbol{p}' | V_{LS} | \boldsymbol{p} \rangle = \frac{-i}{2m_q^2} \left(\sum_A T_1^A T_2^A \right) \frac{12\pi\alpha_s}{\boldsymbol{q}^2} \boldsymbol{S} \cdot (\boldsymbol{P} \times \boldsymbol{q}).$$

$$V_{LS}^{\mathrm{OGE}}(\boldsymbol{r}) = -\frac{\alpha_s}{2m_q^2 r^3} \boldsymbol{L} \cdot \boldsymbol{S},$$

$\sum_A T_1^A T_2^A = 1/3.$

Spin-orbit interactions quark quasiparticles

$$\mathcal{L}_S = -\tilde{g}_S\,\bar{q}(x)q(x)\,\sigma(x)\,,$$
$$\mathcal{L}_V = -\tilde{g}_V\,\bar{q}(x)\gamma_\mu q(x)\,v^\mu(x)\,.$$

$$\sigma = -\left(\tilde{g}_S / \Lambda^2\right)\bar{q}q, \mathrm{G} = \tilde{g}_S / \Lambda^2$$

$$\langle\sigma\rangle = -\left(\tilde{g}_S / \Lambda^2\right)\langle\bar{q}q\rangle$$

$$(\boldsymbol{\nabla}^2 - \Lambda^2)\sigma(x) = \tilde{g}_S\,\bar{q}(x)q(x)\,,$$

$$m_q = -2G\langle\bar{q}q\rangle,$$

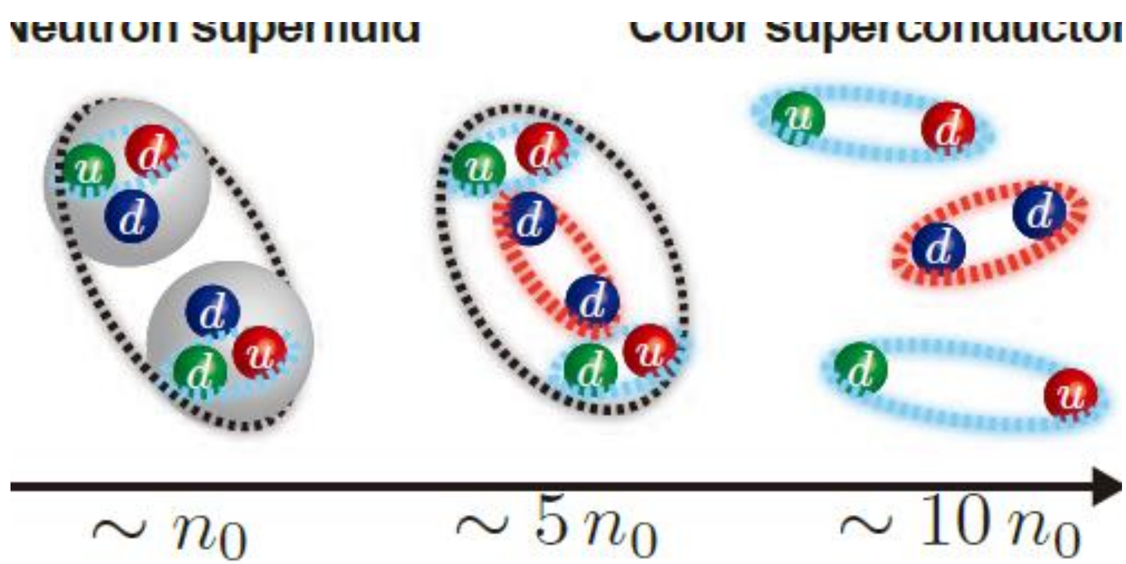
$$\langle \boldsymbol{p}'|V_{LS}|\boldsymbol{p}\rangle = -\frac{i}{2m_q^2}\left[\frac{\tilde{g}_S^2 + 3\tilde{g}_V^2}{\boldsymbol{q}^2 + \Lambda^2}\right]\boldsymbol{S}\cdot(\boldsymbol{P}\times\boldsymbol{q})\,.$$

$$V_{LS}^{qq}(\boldsymbol{r}) = \frac{1}{2m_q^2\,r}\frac{df(r)}{dr}\,\boldsymbol{L}\cdot\boldsymbol{S}\,,$$

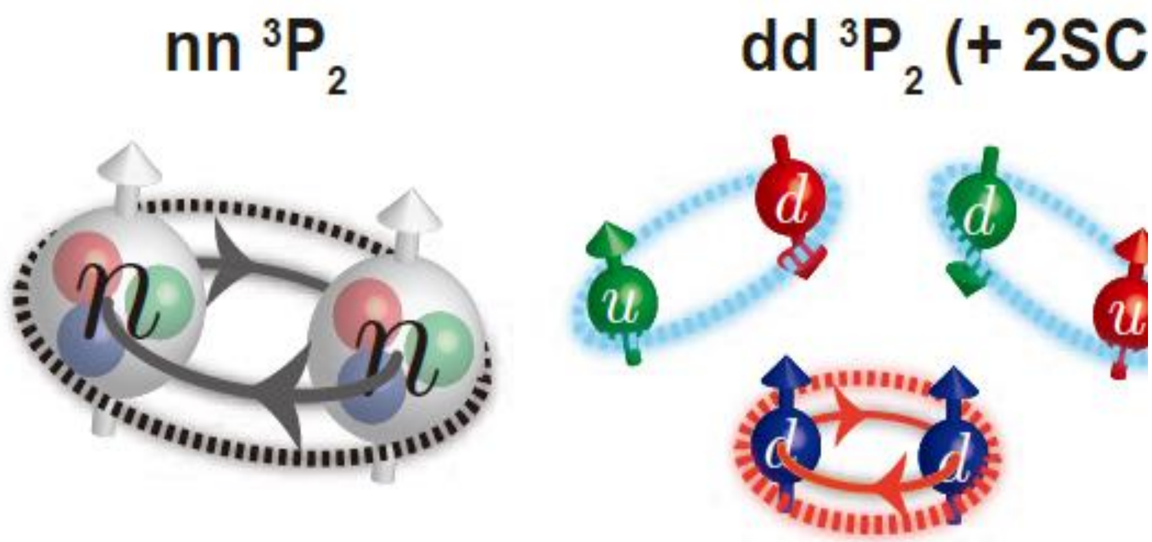
$$f(r) = \frac{\left(\tilde{g}_S^2 + 3\tilde{g}_V^2\right)e^{-\Lambda r}}{4\pi\,r}\,.$$

$$\frac{df}{dr} = -\left(\Lambda + \frac{1}{r}\right)\frac{\left(\tilde{g}_S^2 + 3\tilde{g}_V^2\right)e^{-\Lambda r}}{4\pi r} < 0$$

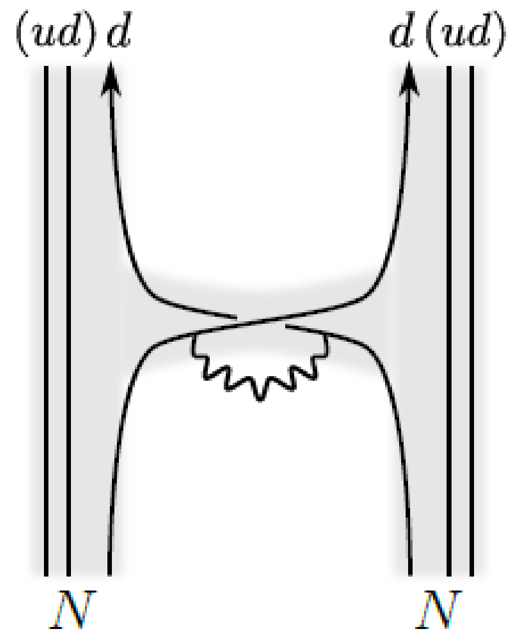
Schematic picture of quark-hadron continuity between neutron superfluid and color superconductor.
Y. Fujimoto, K. Fukushima, W. Weise,Phys. Rev. D 101,094009, 2020.



Schematic picture of quark-hadron continuity between the 3P2 neutron superfluid and the 2SC+(dd) color superconductor



Short –range interaction between neutrons mediated by quark-gluon exchange



2SC+(dd) phase from quark-hadron continuity

Y. Fujimoto, M. Nitta, Phys. Rev. D103,114003, 2021

order parameter 3P2 neutron superfluid is

$$\hat{A}^{ij} = \hat{n}^T C\gamma^i \nabla^j \hat{n} \,,$$

$$\hat{n} = \epsilon^{\alpha\beta\gamma}(\hat{u}_\alpha^T C\gamma^5 d_\beta) d_\gamma$$

$$\begin{aligned} \hat{A}^{ij} &\propto \epsilon^{\alpha\beta\gamma}\epsilon^{\alpha'\beta'\gamma'}(\hat{u}_\alpha^T \mathcal{C}\gamma^5 \hat{d}_\beta)(\hat{u}_{\alpha'}^T \mathcal{C}\gamma^5 \hat{d}_{\beta'})(\hat{d}_\gamma^T \mathcal{C}\gamma^i \nabla^j \hat{d}_{\gamma'}) \\ &= (\hat{\Phi}_{2\mathrm{SC}})^\gamma (\hat{\Phi}_{2\mathrm{SC}})^{\gamma'} (\hat{\Phi}_{dd})^{ij}_{\gamma\gamma'} \qquad (\end{aligned}$$

$$(\hat{\Phi}_{2\mathrm{SC}})^\alpha \equiv \epsilon^{\alpha\beta\gamma} \hat{u}_\beta^T \mathcal{C}\gamma^5 \hat{d}_\gamma \,,$$

$$(\hat{\Phi}_{dd})^{ij}_{\alpha\beta} \equiv \hat{d}_\alpha^T \mathcal{C}\gamma^i \nabla^j \hat{d}_\beta \,.$$

Expectations values of the order parameters in the hadronic and in the quark phases are

$$\langle \hat{A}^{ij} \rangle = \langle \hat{n}^T C \gamma^i \nabla^j \hat{n} \rangle \, .$$

$$\langle \hat{A}^{ij} \rangle \simeq (\Phi_{2SC})^{\alpha} (\Phi_{2SC})^{\beta} (\Phi_{dd})^{ij}_{\alpha\beta} \, ,$$

$$\Phi_{2SC} \equiv \langle \ddot{\Phi}_{2SC} \rangle \, ,$$
$$\Phi_{dd} \equiv \langle \hat{\Phi}_{dd} \rangle \, .$$

Minimal topological vortex in the "dd" phase is non abelian semi-superfluid Alice string

- A. Abelian vortex. It carries a unit quantized circulation in U(1)B.

$$\Phi_{dd}(\varphi) = f_0(r) e^{i\varphi} \Delta_{dd} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$f_0(0) = 0,\ f_0(\infty) = 1$$

$$\pi_1\left[U(1)_B\right] = Z$$

B. Pure color magnetic flux tubes for the case of 2SC and CFL phases:
K. Iida, Phys. Rev. D 71, 054011, 2005.
M. G. Alford, A. Sedrakian, J. Phys. G 37, 075202, 2010.

$$\Phi_{dd}(\varphi) = \Delta_{dd} \begin{pmatrix} f(r)e^{-2i\varphi} & 0 & 0 \\ 0 & f(r)e^{-2i\varphi} & 0 \\ 0 & 0 & f(r)e^{4i\varphi} \end{pmatrix},$$

$$A_i = -\frac{a(r)}{g}\frac{\epsilon_{ij}x^j}{r^2}\mathrm{diag}(-1,-1,2),$$

$$f(0) = a(0) = 0, \quad f(\infty) = a(\infty) = 1.$$

$$\pi_1[SU(3)_C] = 0.$$

$$\Phi_{dd}(\varphi = 0) = \Delta_{dd} f(r) 1_3 \sim \Delta_{dd} 1_3.$$

$$\mathcal{F}_0 \equiv \frac{2\pi}{g}.$$

$$\int d^2x F_{12} = \frac{2\pi}{g}\mathrm{diag}(-1,-1,2) = \mathcal{F}_0 \mathrm{diag}(-1,-1,2),$$

Semi-superfluid vortices in the CFL phase

A.P. Balachandran,S.Digal,T. Matsuura, Phys. Rev. D73,074009,2006.
E. Nakano,M.Nitta, T. Matsuura, Phys. Phys. Rev. D78,045002,2008.
D.M.Sedrakian,K.M.Shahabasyan,D.Blaschke,M.K.Shahabasyan,Astrophys.51,544,2008

$$\Phi_{ur}(\varphi) = \Delta_{CFL} \begin{pmatrix} f(r)e^{i\varphi} & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}$$

$$A_i^{ur} = \frac{1}{3g_s} \frac{\varepsilon_{ij} x_j}{r^2} \left(1 - h(r)\right) diag(2,-1,-1),$$

$$f(0) = 0, \partial_r g(r)_{r=0} = 0, h(0) = 0,$$

$$f(\infty) = g(\infty) = 1, \mathrm{h}(\infty) = 0.$$

$$\Phi \sim \Delta_{CFL} e^{i\varphi/3} \exp\left(-ig_s \int_0^{\varphi} \vec{A} d\overleftarrow{l} \right) diag(1,1,1), r \to \infty$$

Quark hadron continuity connects hadronic matter with neutron 3P2 super-fluidity and 2-flavor dense quark matter [1]. This 2-flavor quark phase consists of the coexistense of the 2SC condensate of the ud quarks and P-wave diquark condensate of d-quarks, which gives rise to color superconductivity, as well as superfluidity. Stable vortices are non-Abelian Alice strings which are superfluid vortices with fractional circulation and non-Abelian color magnetic fluxes [2].

1. Y. Fujimoto, Nucl. Phys. A 1005, 121757,2021.
2. Y. Fujimoto, M. Nitta, Phys. Rev. D 103, 054002, 2021.

There are also green and red Alice strings respectively:

$$\Phi_{dd}(\varphi) = \Delta_{dd} \begin{pmatrix} g(r) & 0 & 0 \\ 0 & f(r)e^{i\varphi} & 0 \\ 0 & 0 & g(r) \end{pmatrix},$$

$$A_i = -\frac{a(r)}{6g}\frac{\epsilon_{ij}x^j}{r^2}\mathrm{diag}(-1, 2, -1),$$

$$\Phi_{dd}(\varphi) = \Delta_{dd} \begin{pmatrix} f(r)e^{i\varphi} & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}$$

$$A_i = -\frac{a(r)}{6g}\frac{\epsilon_{ij}x^j}{r^2}\mathrm{diag}(2, -1, -1).$$

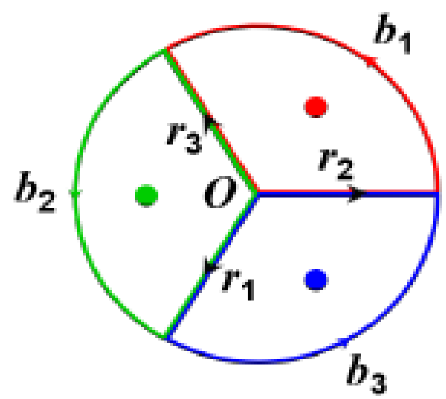
Decay of an Abelian U(1)B vortex into three non-abelian Alice strings, denoted by the red, green, blue blobs. The b_1, b_2, b_3 are the paths with angles 2 π/3 at the boundary at the spatial infinity, and r_1, r_2, r_3 denote the paths from the origin O to the spatial infinities.

- Divergent tension of superfluid vortices: $E \sim n^2 \log\Lambda, n=1, U(1)_B, n=1/3, Alice,$

$$E(n=1) = 9E(n=1/3) > 3E(n=1/3)\,.$$

$$\Phi_{dd} = \Phi_{dd}^{\mathrm{U(1)_B}} \;\to\; \Phi_{dd} = \Phi_{dd}^{r} + \Phi_{dd}^{g} + \Phi_{dd}^{b}\,,$$

$$A_i = A_i^{\mathrm{U(1)_B}} = 0 \;\to\; A_i = A_i^{r} + A_i^{g} + A_j^{b}\,,$$



Three 3P2 neutron vortices in the “npe”-phase are joined to three non-Abelian semi-superfluid vortices in “2SC”+”dd” phase.

Y. Fujimoto, M. Nitta, Phys. Rev. D 103, 114003,2021.

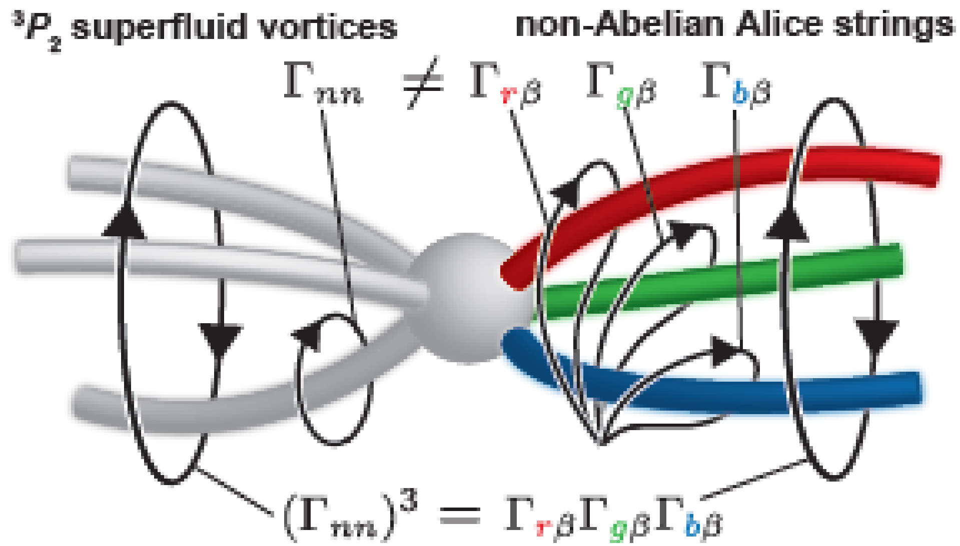
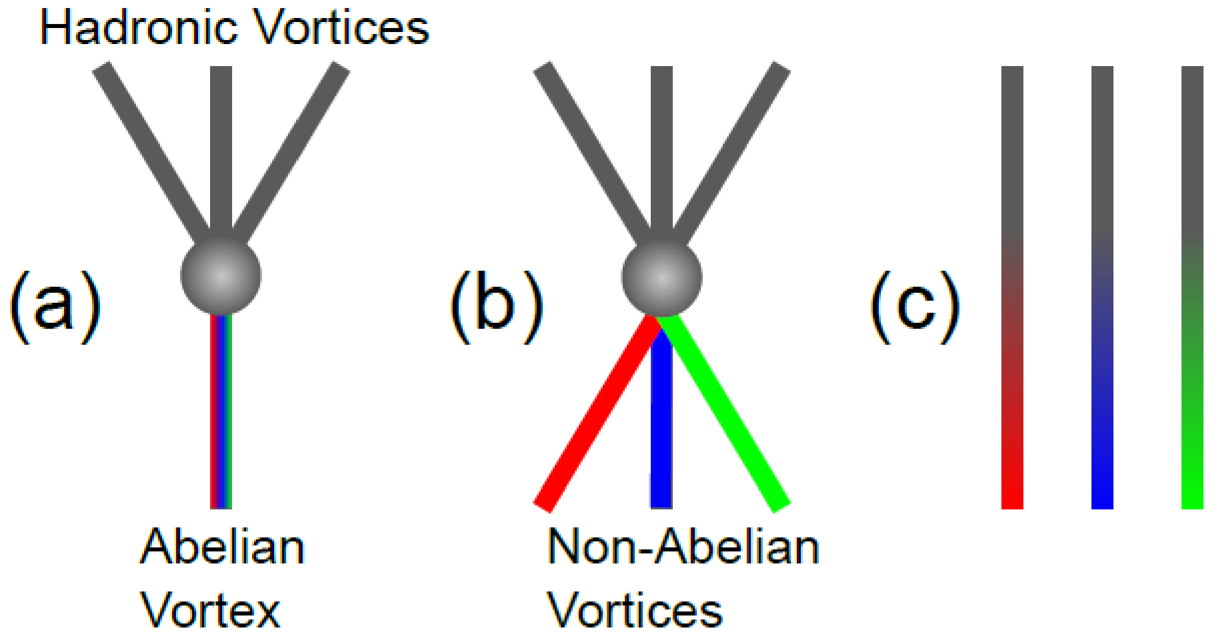
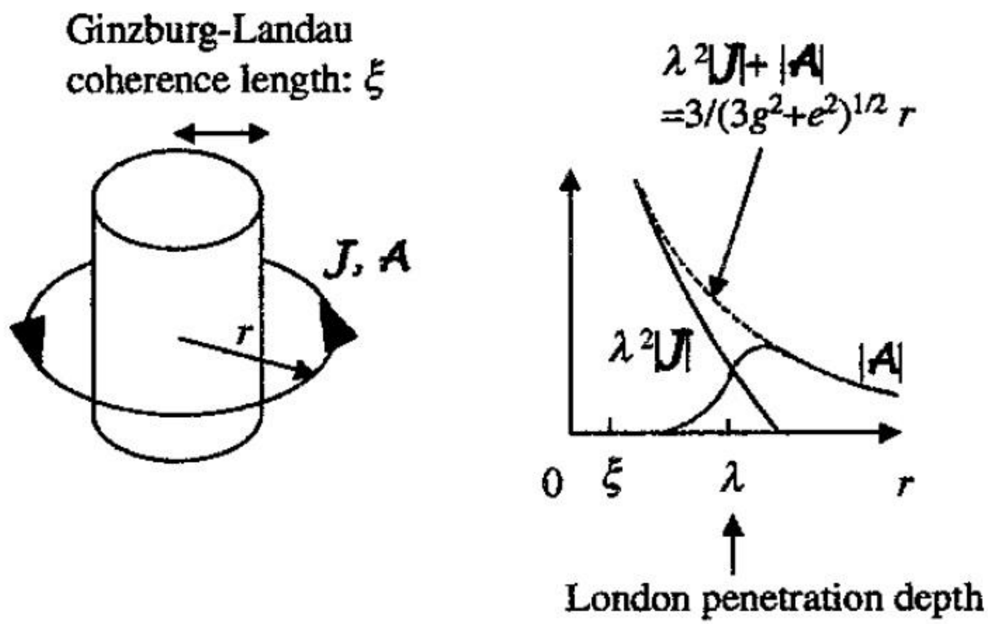


FIG. 1. A schematic figure of the Boojum. Three $^3P_2$ neutron vortices in the hadronic phase are joined to three non-Abelian Alice strings in the color-superconducting phase. We also show the Aharonov-Bohm phase around each vortices.

Schematic illustrations for connecting vortices.
M.G. Alford ,G. Baym,K. Fukusima,T. Hatsuda,M. Tachibana, Phys. Rev. D 99, 036004, 2019.



D.M. Sedrakian,D. Blaschke, nucl-th/ 0006038,
K. Iida,G. Baym, Nuclear Physics A , 718, 697, 2003.
2SC condensate is type II superconductor and magnetic field penetrates into quark core in the form of quark magnetic vortices.



a) Magnetic vortex in the 2SC phase

Thank You!