Pions in nuclear and neutron star matter

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Agenda:

- Symmetry energy and pionization of neutron star matter
- Pion-nucleon interaction and the chiral symmetry
- Pion mass in medium at order $O(p_F^3)$ and at order $O(p_F^{5/3})$
- P-wave and new s-wave pion condensation

Symmetry energy. Correlation among parameters

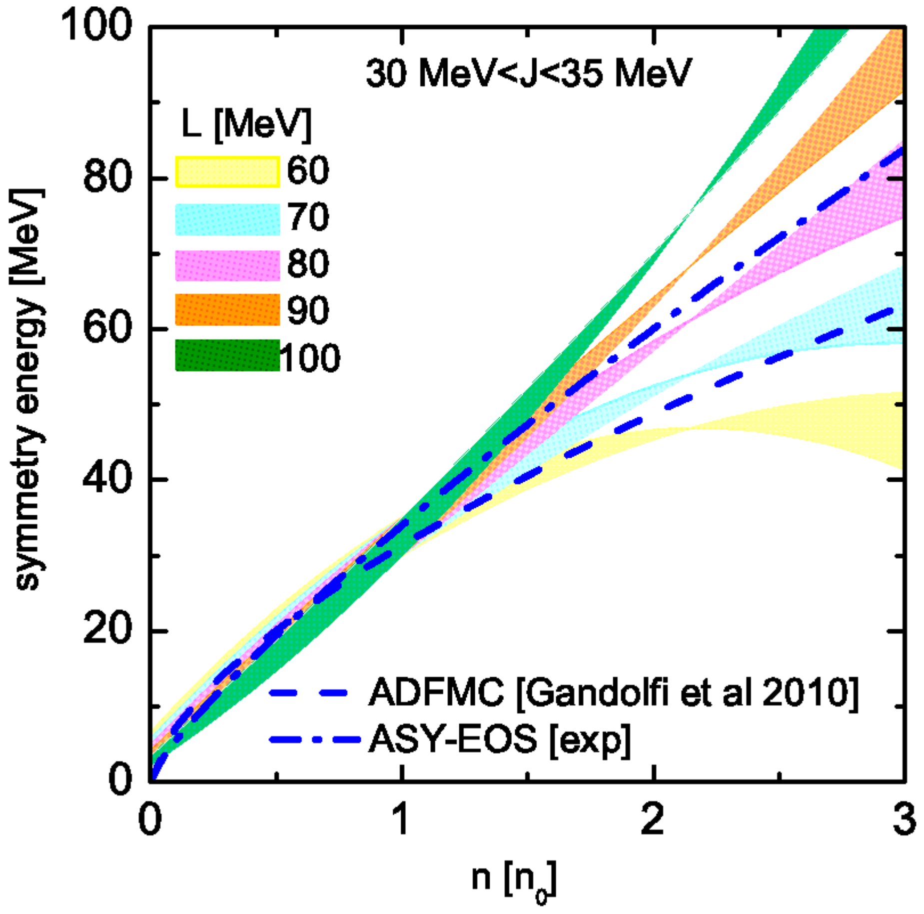
$$\varepsilon_S[n] = J + \frac{L}{3}\frac{n-n_0}{n_0} + \frac{K_{\rm sym}}{18}\frac{(n-n_0)^2}{n_0^2} + \dots$$

If we assume some model for the density dependence of the symmetry energy

$$E_S(n) = C_k\,(n/n_0)^{2/3} + C_1\,n/n_0 + C_2\,(n/n_0)^{\gamma}$$

$$J = C_1 + C_2 + C_k \qquad 3L = C_1 + 2C_k + 3\gamma C_2$$

$$K = -2C_k + 9C_2(\gamma-1)\gamma$$



Eliminate C_1 and C_2

$$L = \left(\frac{2}{3\,\gamma} - 1\right)\,C_k + 3\,J + \frac{K_{\rm sym}}{3\,\gamma}$$

Taking L, J, K_{sym} from models fitted to empirical data we see that this relation works and obtain parameters C_k, γ

RMF: $$L = -11.76\ {\rm MeV} + 3\,J + \frac{K_{\rm sym}}{4.55} \qquad \gamma = 1.5$$

Skyrme $$L = -19.5\ {\rm MeV} + 3\,J + \frac{K_{\rm sym}}{5.50} \qquad \gamma = 1.8$$

$$\frac{4}{3} < \gamma$$

Constraint on the symmetry energy. NS Cooling

Proton concentration

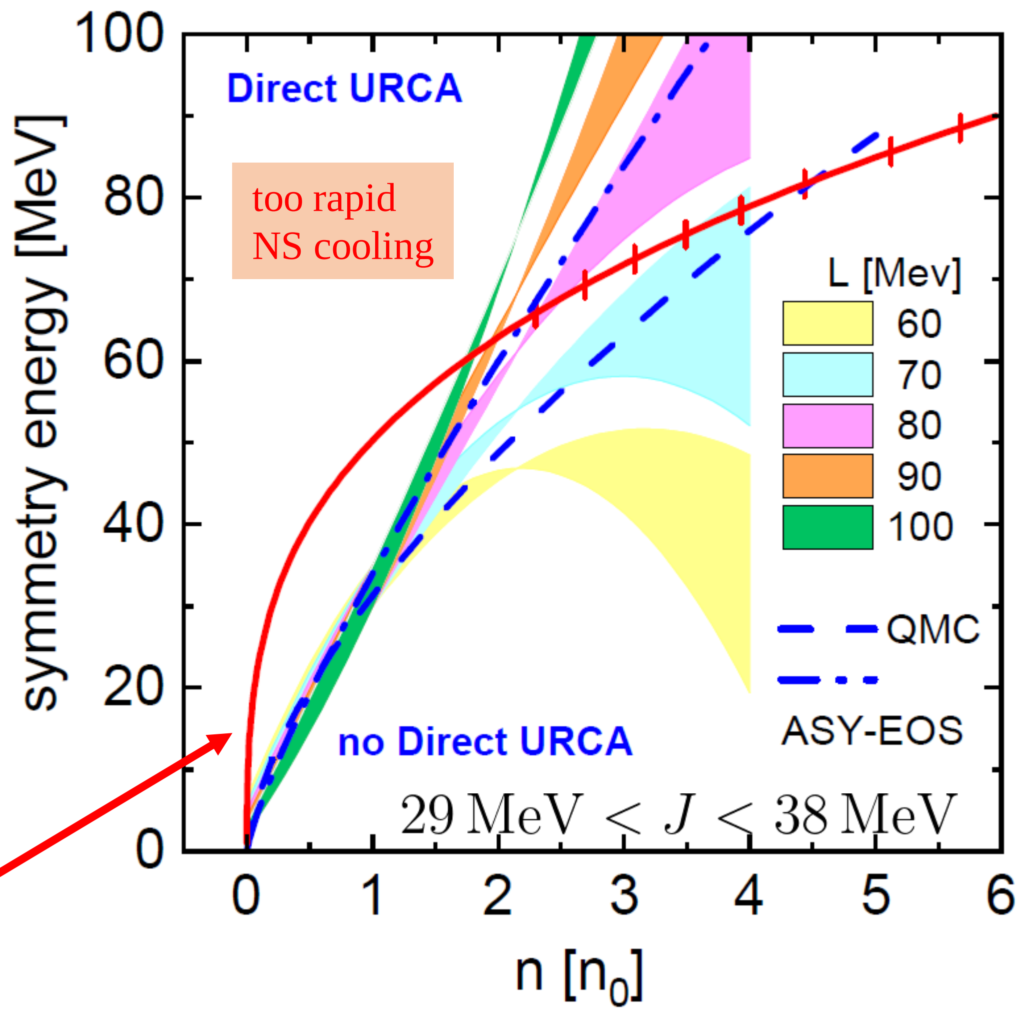
β-equilibrium: $\mu_e = \mu_n - \mu_p = 4\,\varepsilon_S(n)\,(1-2\,x)$

electroneutrality $n_e(\mu_e) + n_\mu(\mu_e) = n\,x$

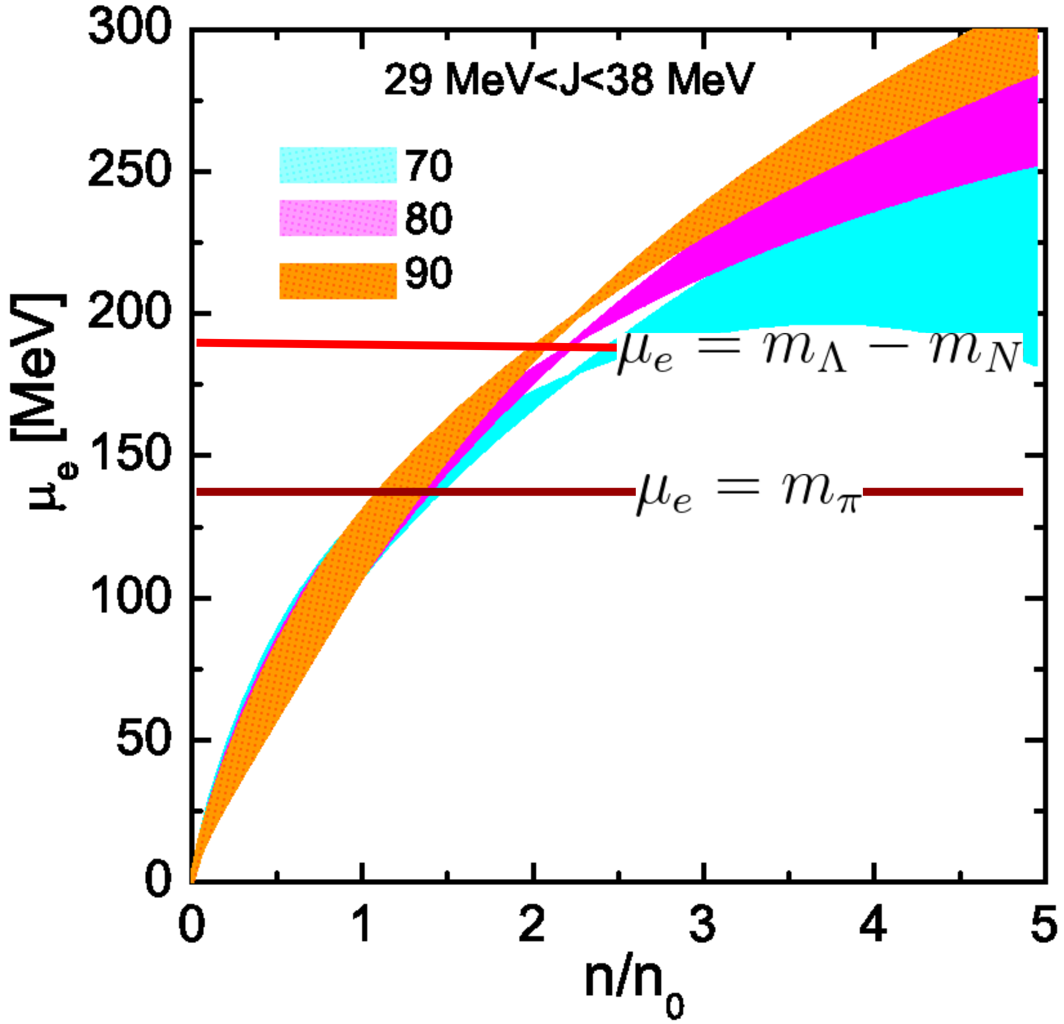
Direct URCE threshold $p_{\mathrm{F},n} \le p_{\mathrm{F},p} + p_{\mathrm{F},e}$

$$x = \frac{1}{1+\left(1+x_e^{1/3}\right)^3}\,, \quad x_e = \frac{n_e}{x\,n}$$

From these 3 equations we express $\varepsilon_{\mathrm{sym}}^{(\max)}[n]$



- [*Pionization of the NS matter*]() $n+p+e+\mu$ matter



$$e^- + p \longrightarrow \Lambda + \nu_e$$

$$e^- \longrightarrow \pi^- + \nu_e$$

Pion-nucleon interaction

isospin even and odd amplitudes $$T^{(\pm)} = \frac{1}{2}\left[T^{(\pi^- p)} \pm T^{(\pi^- n)}\right]$$

At the threshold: scattering amplitues $$T^{(\pm)}(\sqrt{s} = m_\pi + m_N) = 4\pi\left(1 + \frac{m_\pi}{m_N}\right) a_S^{\pm}$$

$T^{(+)}[m_\pi]$ –0.122[KA86]; 0 [SP98]; 0.06 [EM98]; –0.003 [PSI]

$T^{(-)}[m_\pi]$ 1.32 [KA86]; 1.27 [SP98]; 1.11 [EM98]; 1.26 [PSI]

Current algebra prediction
Soft-pion theorem

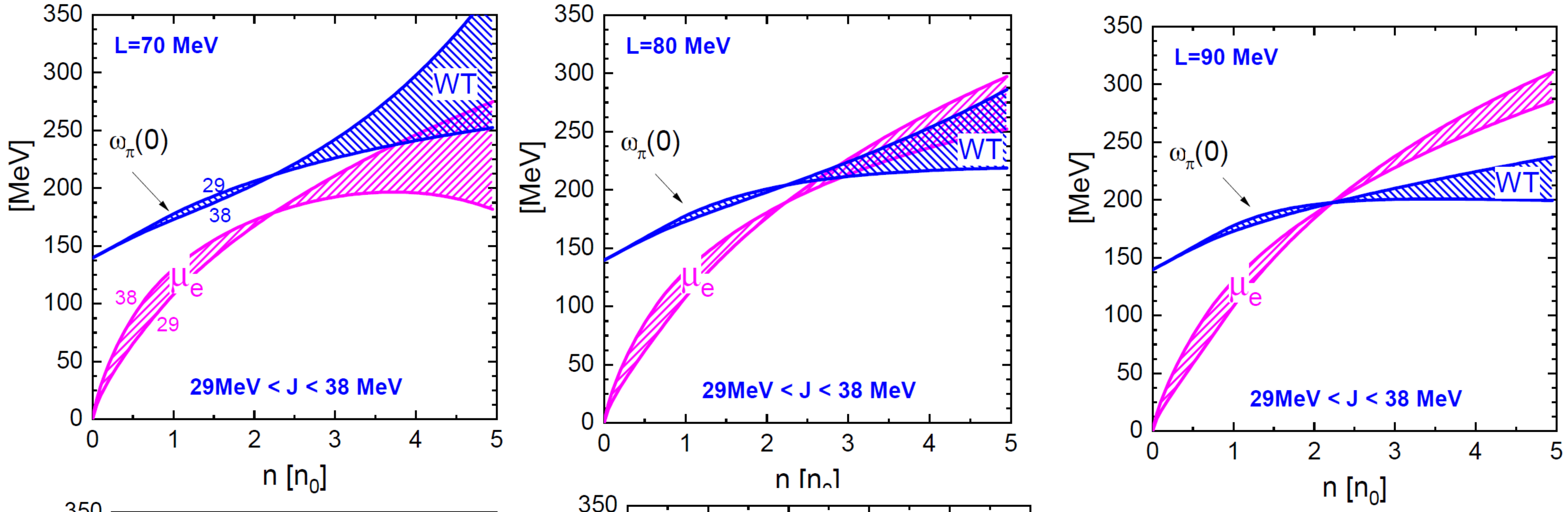
$$T^{(-)}(\omega) \approx \frac{\omega}{2 f_\pi^2}$$

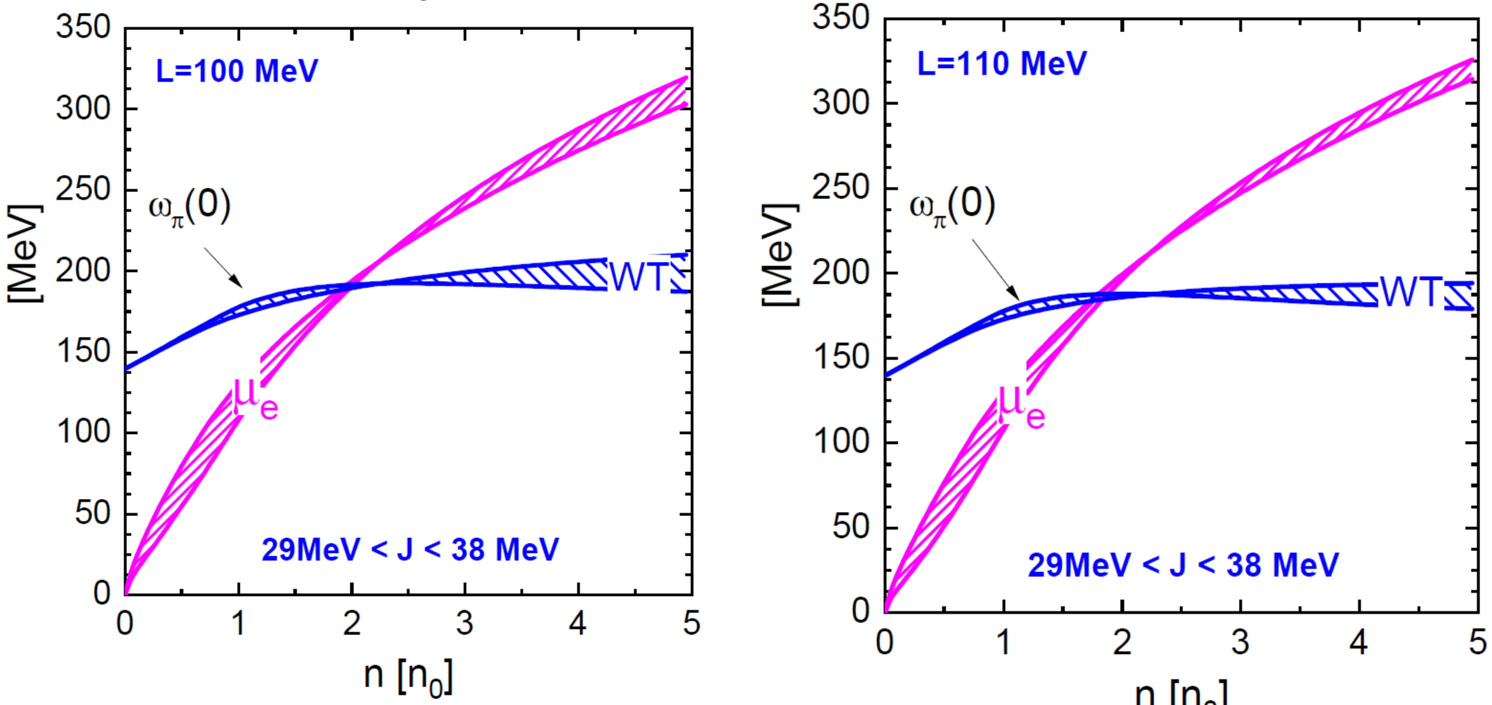
- **polarization operator**

$$\Pi_S(\omega) = -T^{(-)}(\omega)\,(n_p - n_n)$$

repulsive in neutron reach matter

- **spectrum** $$D^{-1}(\omega, k = 0) = \omega^2 - m_\pi^2 - \Pi_S(\omega) = 0$$





Weinberg-Tomazawa term does not protect against pionization for too stiff symmetry energy, L>90 MeV

● Deeply bound pionic atoms

Missing repulsion problem

energy independent potential ○

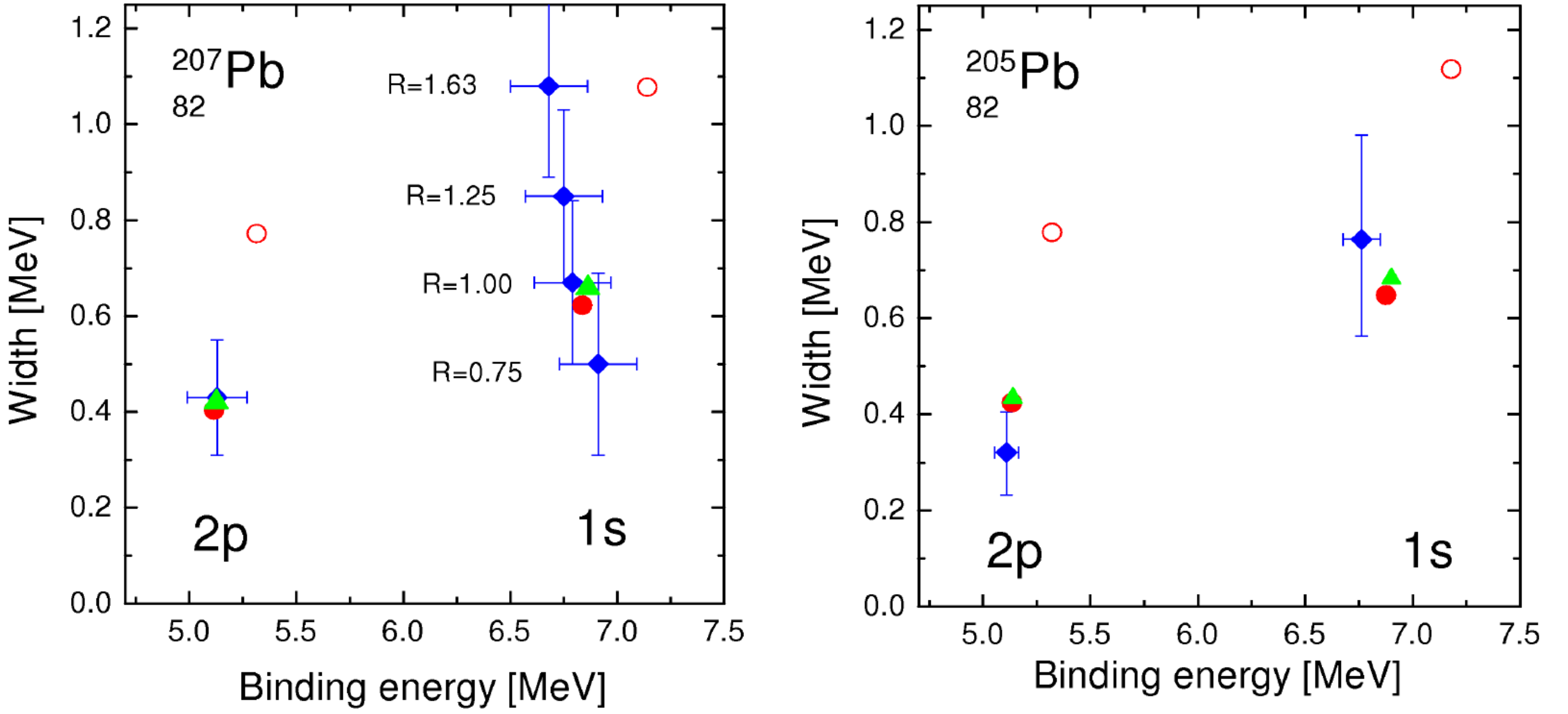
energy dependent potential

● phenomenological model [Weise 2001] $f_\pi \to f_\pi^*(n) < f_\pi$

▲ in-medium chiral perturbation theory [EEK,Kaiser,Weise 2003]

$T^{(+)}(\omega - V_C)$

Reveals a new scale of πN interaction



Chiral symmetry

- *QCD with light quarks*

✔ **quarks** $q_L(x) = \frac{1+\gamma_5}{2} \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}$

✔ **gluons** $G^a(x) \longleftarrow \mathrm{SU}(N_c)$ gauge bosons

$D_\mu(G) = \partial_\mu - i\,\frac{g}{2}\,G^a_\mu(x)\,\lambda_a$

$$\mathcal{L}_{\mathrm{QCD}}(x) = \bar{q}_L(x)\, i\,\gamma^\mu D_\mu(G) q_L(x) + \bar{q}_R(x)\, i\,\gamma^\mu D_\mu(G) q_R(x) - \frac{1}{4} \sum_{a=1}^{N_c^2-1} G^{\mu\nu}_a(x)\, G_{\mu\nu,a}(x)$$

$$- \bar{q}_L(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_R(x) - \bar{q}_R(x) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q_L(x)$$

Accidental symmetries

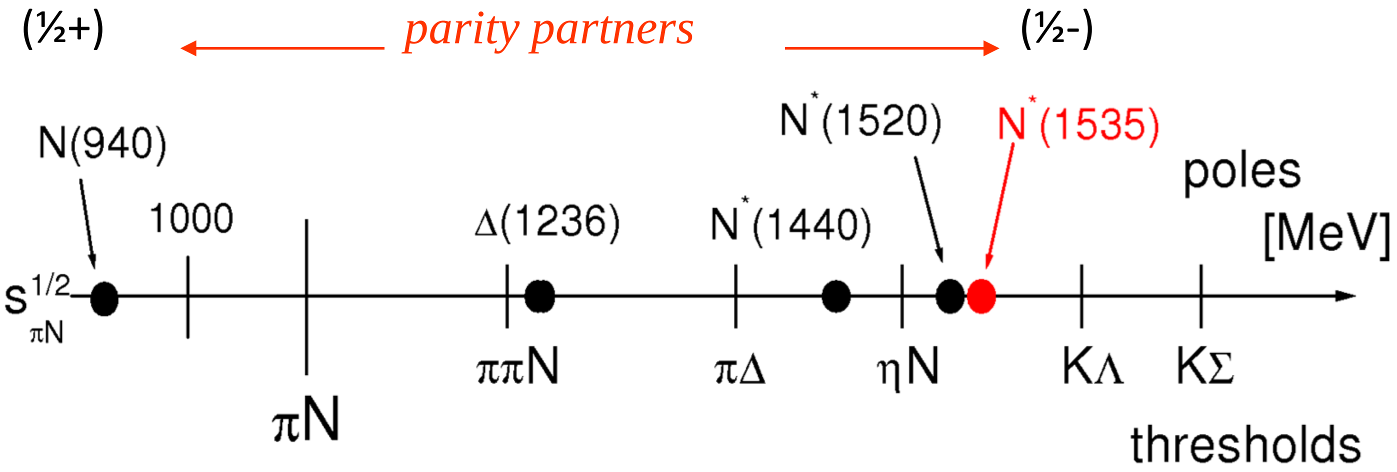
✔ consider $m_{u,d,s}$ to be small

- approximate $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ chiral symmetry
- parity doublets in hadron spectrum (if not broken spontaneously !)

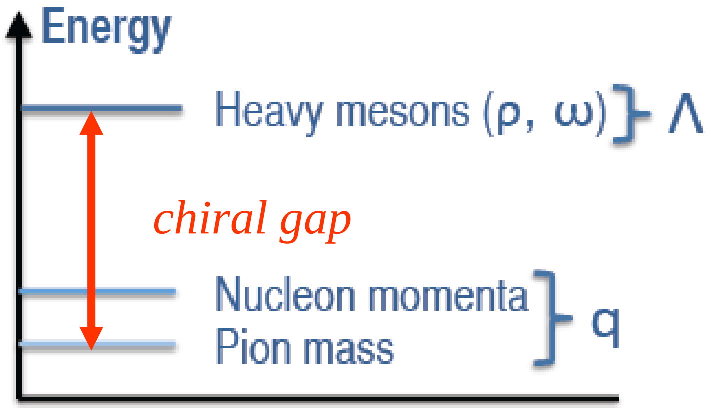
✔ consider number of colors $N_c = 3$ to be large

- contracted spin-flavor symmetry SU(6)

world of pion-nucleon interaction



Pions – Goldstone bosons of chiral symmetry breaking



- gradient couplings
- expansion in small pion momenta and masses

effective chiral Lagrangian

Building blocks: B baryon field matrix

$$U_\mu = \frac{1}{2} e^{-i\frac{\Phi}{2f}} \left(\partial_\mu e^{i\frac{\Phi}{2f}} \right) e^{+i\frac{\Phi}{2f}} = \partial_\mu \Phi + \ldots$$

Φ meson field matrix

Meson mass terms

Chiral Lagrangian at Q^2 order

$$\mathcal{L}_{\rm int} = -\frac{1}{4f^2}\bar{N}\gamma_\mu(\boldsymbol{\tau}\cdot[\boldsymbol{\phi}\times(\partial^\mu\boldsymbol{\phi})])N + \frac{g_A}{2f}\bar{N}\gamma_5\gamma_\mu(\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\phi})N$$

LO: Q^1 Weinberg-Tomazawa and πNN

$$-\frac{2c_1}{f^2}m_\pi^2\bar{N}(\boldsymbol{\phi}\cdot\boldsymbol{\phi})N$$

NLO: Q^2 σ-term

$$-\frac{c_2}{2f^2m_N^2}\{\bar{N}(\partial_\mu\boldsymbol{\phi})\cdot(\partial_\nu\boldsymbol{\phi})\partial^\mu\partial^\nu N + \text{h.c.}\} + \frac{c_3}{f^2}\bar{N}(\partial_\mu\boldsymbol{\phi})\cdot(\partial^\mu\boldsymbol{\phi})N - \frac{c_4}{2f^2}\bar{N}\sigma^{\mu\nu}(\boldsymbol{\tau}\cdot[(\partial_\mu\boldsymbol{\phi})\times(\partial_\nu\boldsymbol{\phi})])N$$

NLO: Q^2 range-term

Scattering amplitude $\pi^+(q) + n(p) \to \pi^+(\bar{q}) + n(\bar{p})$

$$\hat{T}^{(\pi^+n)} = +\frac{\not\bar{q}+\not q}{4f^2} - \frac{4c_1}{f^2}m_\pi^2 + \frac{c_2}{f^2m_N^2}[(p\cdot\bar{q})(p\cdot q)+(\bar{p}\cdot\bar{q})(\bar{p}\cdot q)] + 2\frac{c_3}{f^2}(\bar{q}\cdot q) + \frac{c_4}{2f^2}[\not\bar{q}\not q - \not q\not\bar{q}] - \frac{g_A^2}{2f^2}\gamma_5\not\bar{q}\hat{G}_p^{(0)}(p+q)\gamma_5\not q$$

S-wave amplitude (tree level)

$$\sigma_{\pi N} = -4c_1m_\pi^2 \qquad \beta = -2(c_3+c_2)m_\pi^2 + \frac{g_A^2m_\pi^2}{4m_N} \qquad f = 90\,\text{MeV}$$

$$T_{\pi N,s}^{(1/2)}(\sqrt{s}=m_N+\omega) = \frac{\omega}{f^2} + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T^+ = \frac{1}{3}T^{(1/2)} + \frac{2}{3}T^{(3/2)} = \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T_{\pi N,s}^{(3/2)}(\sqrt{s}=m_N+\omega) = -\frac{\omega}{2f^2} + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\frac{\omega^2}{m_\pi^2}$$

$$T^- = \frac{1}{3}\left(T^{(1/2)} - T^{(3/2)}\right) = \frac{\omega}{2f^2}$$

- *Pion polarization operator from the scattering amplitude*

Polarization operator of the $\pi^{\pm}$ meson is equal to

Vector mean fields

$$\Pi^{(+)}(q) = \Pi_n^{(+)}(q) + \Pi_p^{(+)}(q)$$

$$\Pi_n^{(+)}(q) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\mathrm{Tr}\{\hat{G}_n^{(\mathrm{m})}(p - v_n u; m_N^*)\hat{T}_{\mathrm{forw}}^{(\pi^+ n)}\},$$

$$\Pi_p^{(+)}(q) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} i\mathrm{Tr}\{\hat{G}_p^{(\mathrm{m})}(p - v_p u; m_N^*)\hat{T}_{\mathrm{forw}}^{(\pi^+ p)}\}$$

Effective mass

In-medium nucleon propagator

$$\hat{G}_a^*(p) = \hat{G}^{(0)}(p - v_a u; m_N^*) + \hat{G}_a^{(\mathrm{m})}(p - v_a u; m_N^*)$$

$$\hat{G}_a^{(\mathrm{m})}(p; m) = 2\pi i n_a(p)\hat{S}(p; m)\delta(p^2 - m^2)\theta(p_0)$$

$$n_a(p) = \theta(p_{\mathrm{F},a}^2 + p^2 - (p \cdot u)^2)$$

$$u = (1, 0, 0, 0)$$

$$n_a = p_{\mathrm{F},a}^3/(3\pi^2)$$

- *S-wave pion polarization operator at chiral Q^2 order* (approximated) $O(p_F^3)$

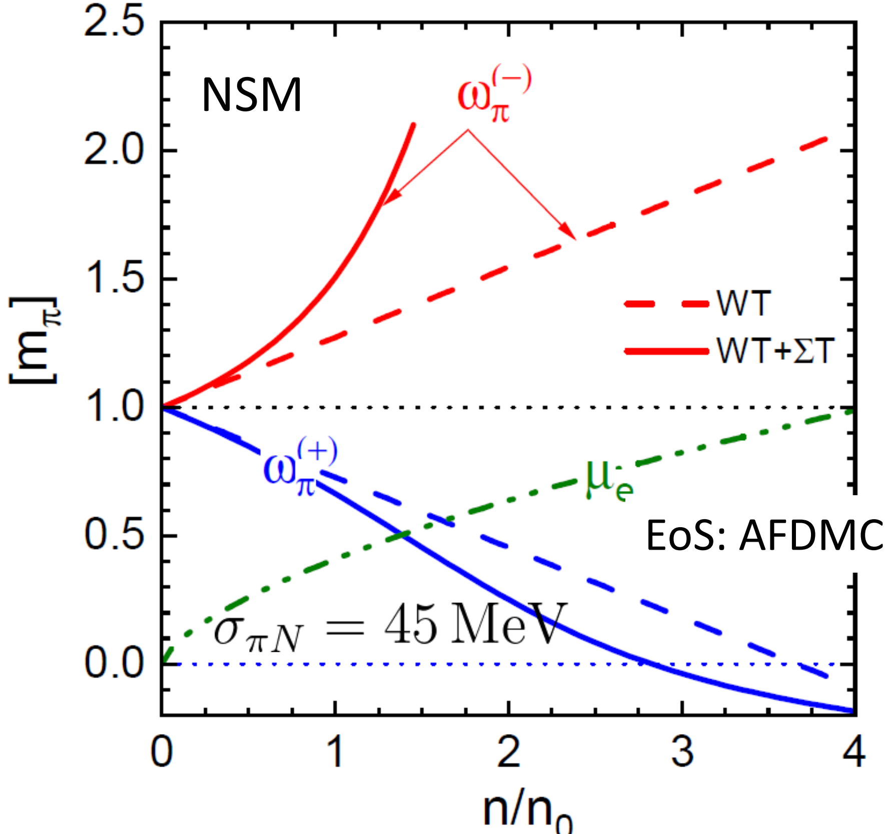
$$\Pi^{(\pm)}_{1,S}(\omega) = \mp\frac{\omega}{2f^2}(n_n - n_p) - \frac{\sigma_{\pi N}}{f^2}(n_p + n_n) + \frac{\beta\omega^2}{m_\pi^2 f^2}(n_p + n_n)$$

$$\omega_\pi^{(\pm)} = \frac{1}{1 - \frac{n}{n_{c,\beta}}}\left\{\sqrt{m_\pi^2\left(1 - \frac{n}{n_{c,\sigma}}\right)^2 + \frac{(n_n - n_p)^2}{16 f^4}} \mp \frac{n_n - n_p}{4f^2}\right\}, \quad n = n_p + n_n$$

$$n_{c,\beta} = \frac{f^2 m_\pi^2}{\beta}$$

$$n_{c,\sigma} = \frac{f^2 m_\pi^2}{\sigma_{\pi N}}$$

From experiment we know: $\beta \approx \sigma_{\pi N}$ → $n_{c,\beta} \approx n_{c,\sigma}$



EoS: AFDMC [Gandolfi]

Small densities $\omega_\pi^{(\pm)} = m_\pi \mp \frac{n_n - n_p}{4f^2} + O(n^2)$

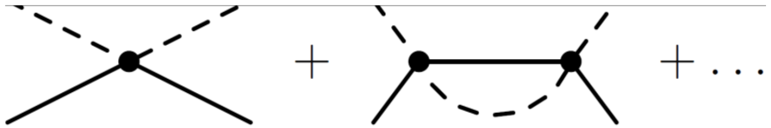
In isospin symmetric matter $\omega^\pm = m_\pi$

At $n \to n_{c,\beta}$, the pion masses behave as

$$\omega_\pi^{(+)} \to 0\,, \quad \text{and} \quad \omega_\pi^{(-)} \to \infty\,.$$

See also Onishi, Jido et al, PRC 80, 038202 (2009)

- *Pion–Nucleon scattering amplitude*



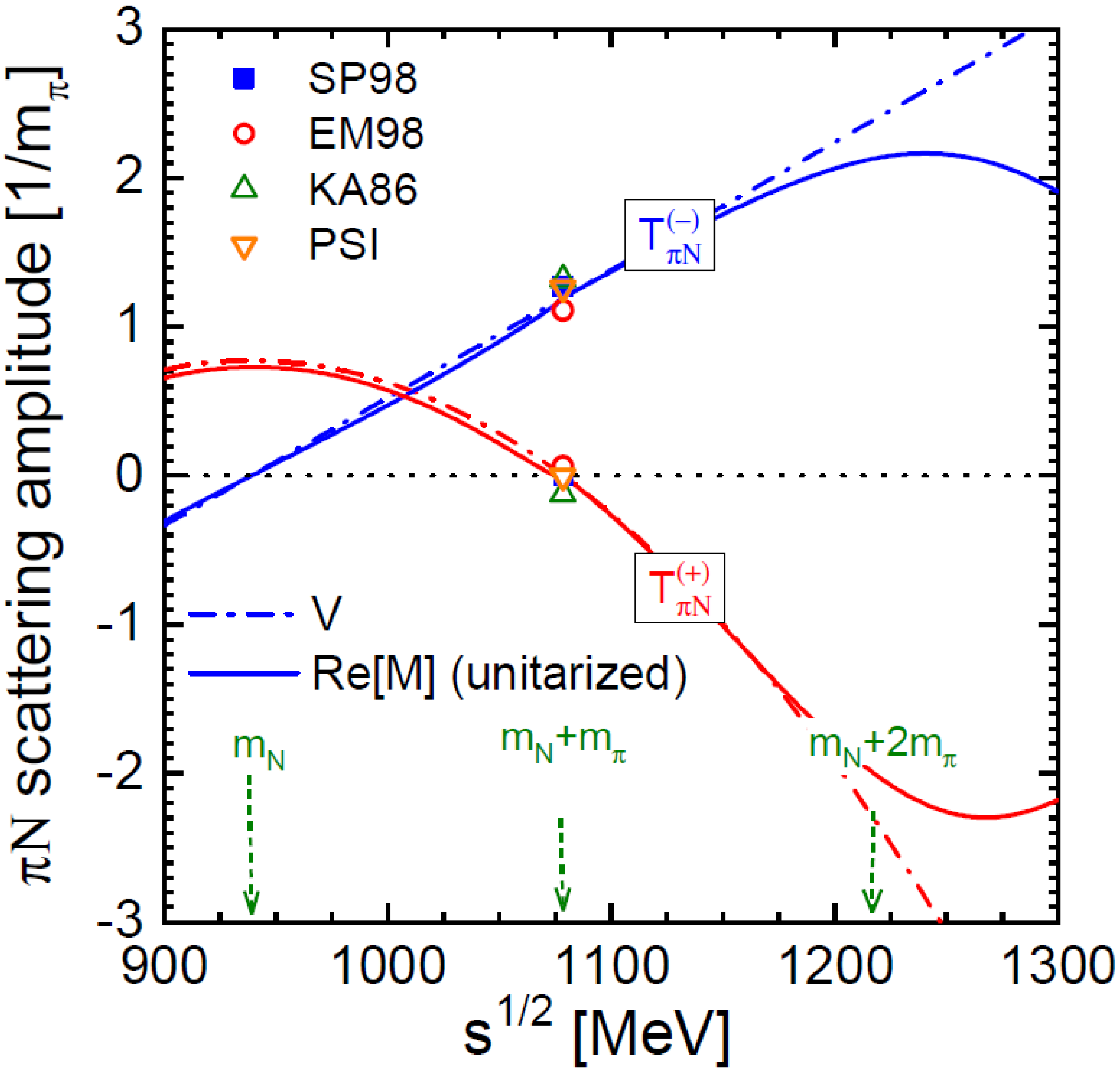
$$T^{(I)}(\sqrt{s}) = \frac{1}{[V^{(I)}(\sqrt{s})]^{-1} - J(\sqrt{s})} \qquad I = \frac{1}{2}, \frac{3}{2}$$

$$V^{(1/2)}(\sqrt{s}) = \frac{1}{f^2}\,(\sqrt{s} - m_N) + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f2}\,\frac{(\sqrt{s} - m_N)^2}{m_\pi^2}$$

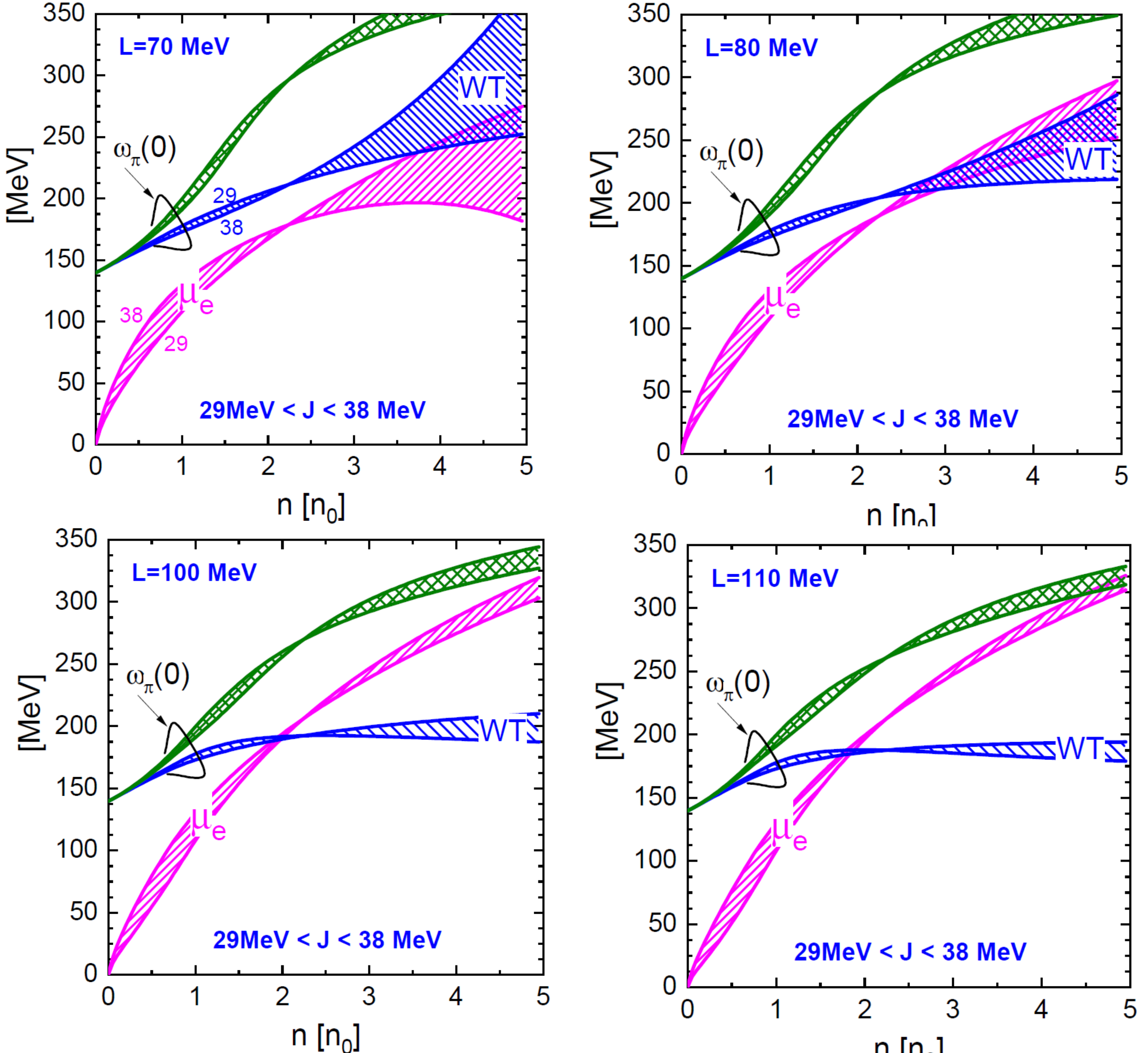
$$V^{(3/2)}(\sqrt{s}) = -\frac{1}{2f^2}\,(\sqrt{s} - m_N) + \frac{\sigma_{\pi N}}{f^2} - \frac{\beta}{f^2}\,\frac{(\sqrt{s} - m_N)^2}{m_\pi^2}$$

$$J(\sqrt{s}) = (E_{\rm cm} + m_N)\,\big(I(\sqrt{s}) - I(\mu_M)\big)$$

[Lutz, EEK, NPA700]



$$I(\sqrt{s}) = \frac{1}{16\,\pi^2}\left[\frac{p_{\rm cm}}{\sqrt{s}}\left(\ln\left(1 - \frac{s - 2\,p_{\rm cm}\sqrt{s}}{m_\pi^2 + m_N^2}\right) - \ln\left(1 - \frac{s + 2\,p_{\rm cm}\sqrt{s}}{m_\pi^2 + m_N^2}\right)\right) + \left(\frac{1}{2}\,\frac{m_\pi^2 + m_N^2}{m_\pi^2 - m_N^2} - \frac{m_\pi^2 - m_N^2}{2\,s}\right)\ln\left(\frac{m_\pi^2}{m_N^2}\right) + 1\right] + I(0)$$



Iterated pN amplitude, including Sigma-and range terms, could provide a shield against pionization

● *New s-wave pion condensation*

We are interested now in $\omega<m_\pi$

Iteration is not important

$$\Pi^{(\pm)}_{1,S}(\omega) = -\frac{\sigma_{\pi N}}{f^2}(n_p+n_n) \mp \frac{\omega}{2f^2}(n_n-n_p) + \frac{\beta\omega^2}{m_\pi^2 f^2}(n_p+n_n)$$

Pion propagator $$D^{(-)}_\pi(\omega,\boldsymbol{q}) = \omega^2 - \boldsymbol{q}^2 - m_\pi^2 - \Pi^{(-)}_{1,S}(\omega)$$

Effective pion gap $$\widetilde{\omega}_\pi^2 = -D^{(-)}_\pi(0,0) = m_\pi^2 + \Pi^{(-)}_{1,S}(0) = m_\pi^2 - \frac{\sigma_{\pi N}}{f^2}(n_p+n_n) = m_\pi\Big(1-\frac{n}{n_{c,\sigma}}\Big)$$

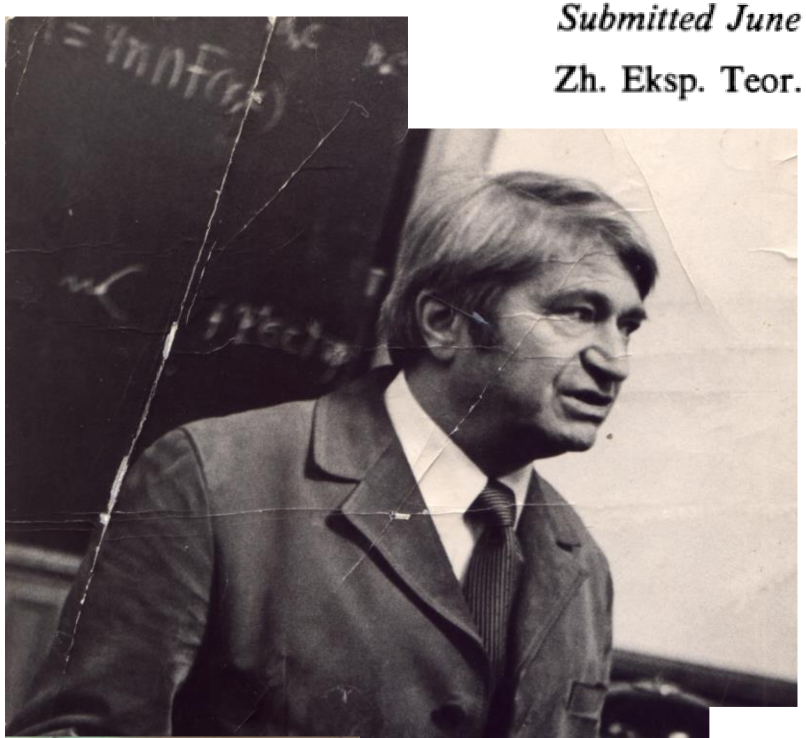
It vanishes at $$n_{c,\sigma} = \frac{f^2 m_\pi^2}{\sigma_{\pi N}} = 2.83\, n_0$$ for $$\sigma_{\pi N} = 45\,\mathrm{MeV}$$

It was argued by D.N. Voskresensky that at the density $n = n_{c,\sigma}$ there appears the spatially constant pion field varying with time as

$$\phi(t) = e^{i\alpha}\theta\big(n_{c,\beta}-n\big)\frac{m_\pi}{\sqrt{\Lambda}}\big(n/n_{c,\beta}-1\big)^{1/2}\tanh\frac{m_\pi t}{\sqrt{2}}$$

[D.N. Voskresensky, S-wave pion condensation in symmetric nuclear matter, Phys. Rev. D 105 (2022) 116007]

- p-wave pion condensation



Stability of Vacuum and Limiting Fields

A. B. MIGDAL

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

Submitted June 21, 1971

Zh. Eksp. Teor. Fiz. **61**, 2209–2224 (December, 1972)

Condensed π^- Phase in Neutron-Star Matter*

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(Received 29 March 1972)

π^- Condensate in Dense Nuclear Matter*

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(Received 17 April 1972)

Pion Condensation in Nuclear and Neutron Star Matter*

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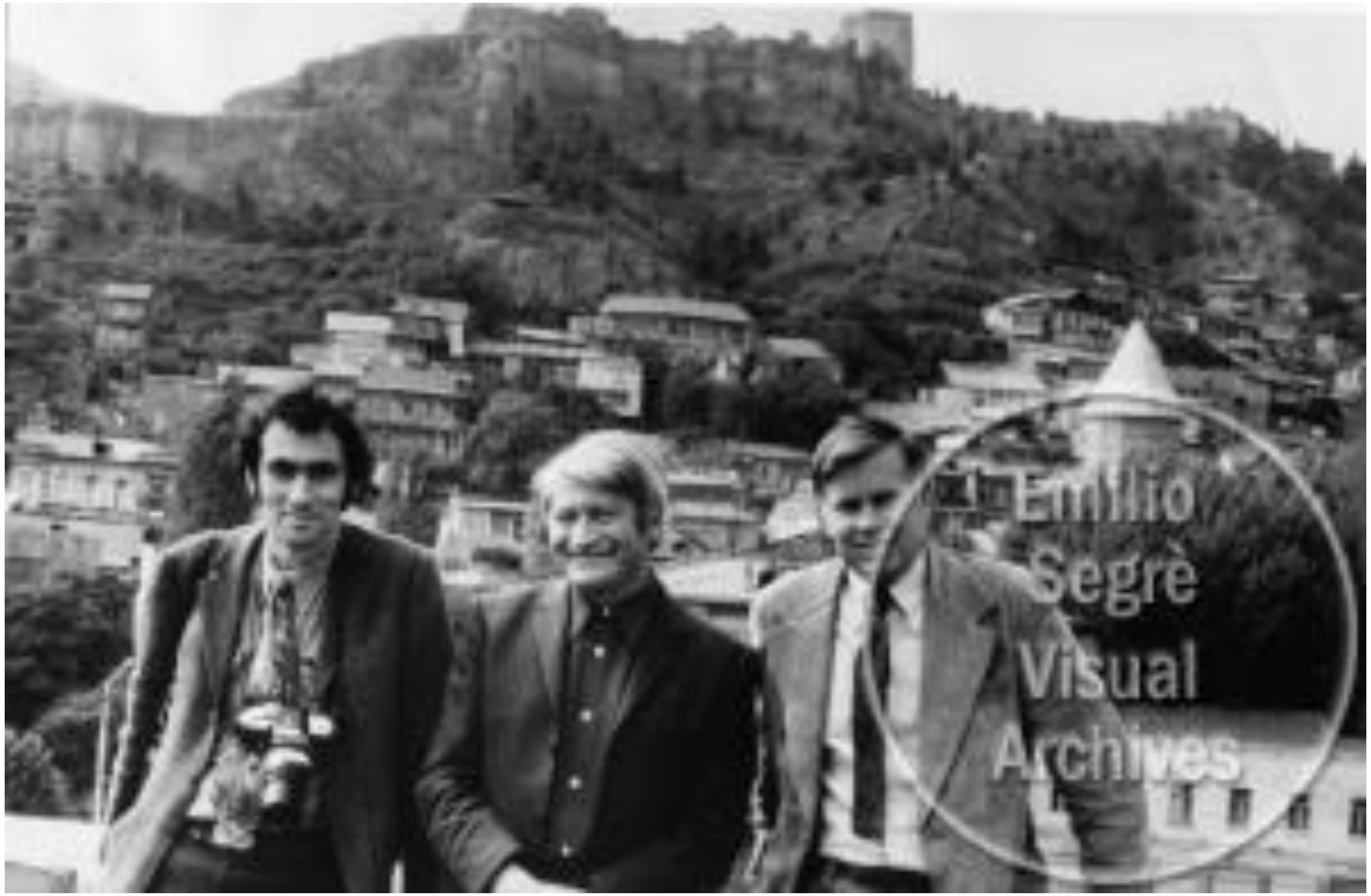
(Received 13 April 1973)

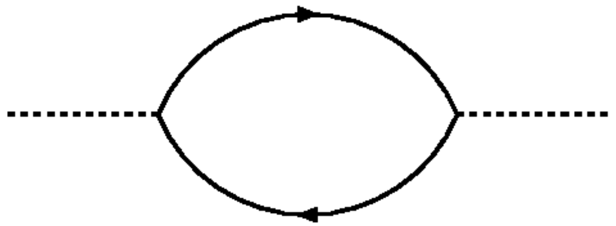


Baym Migdal Scalapino

1974 Tbilisi

- *Nucleon particle-hole polarization operator*

$$\mathcal{L} = \frac{g_A}{2f}\bar{N}\gamma_5\gamma_\mu(\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\phi})N$$



π^+ polarization operator

$$\Pi^{(+)}_{\mathrm{ph},a}(q) = 2f^2_{\pi NN}\Big\{(\omega-\lambda_a\Delta v)n_a + (\boldsymbol{q}^{\,2}+(\Delta v)^2-\omega^2)A(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a}) + (\Delta v)^2 B(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a})\Big\}$$

$$+\frac{(\Delta v)^2}{2m^*_N}2f^2_{\pi NN}\Big\{-\chi_{p^2}(\xi_a)n_a + (\omega+2\lambda_a\Delta v)C(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a}) + \frac{(\omega+\lambda_a\Delta v)^2-\boldsymbol{q}^{\,2}}{2m^*_N}A(\omega+\lambda_a\Delta v,\boldsymbol{q},p_{\mathrm{F},a})\Big\},$$

$$f_{\pi NN} = \frac{g_A}{2f}, \quad \Delta v = v_n - v_p, \quad \lambda_n = +1,\ \lambda_p = -1, \quad a = n,p$$

$$A(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{m^{*2}_N}{E^2_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}},$$

$$B(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{\boldsymbol{p}^{\,2}}{E^2_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}}.$$

$$C(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \int_0^{p_{\mathrm{F}}}\frac{2\mathrm{d}^3p}{(2\pi)^3}\frac{m^*_N}{E_{\boldsymbol{p}}}\frac{1}{\omega-\frac{\boldsymbol{p}\boldsymbol{q}}{E_{\boldsymbol{p}}}+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2E_{\boldsymbol{p}}}},$$

$$E_{\boldsymbol{p}} = \sqrt{m^{*2}_N+\boldsymbol{p}^{\,2}}.$$

$$A(\omega,\boldsymbol{q},p_{\mathrm{F}}) \approx -\frac{m_N p_{\mathrm{F}}}{\pi^2}\phi_1\Big(\omega+\frac{\omega^2-\boldsymbol{q}^{\,2}}{2m_N},\boldsymbol{q},p_{\mathrm{F}}\Big)$$

$$\phi_1(\omega,\boldsymbol{q},p_{\mathrm{F}}) = \frac{m_N}{2|\boldsymbol{q}|^3 v_{\mathrm{F}}}\Big(\frac{\omega^2-\boldsymbol{q}^{\,2}v^2_{\mathrm{F}}}{2}\log\frac{\omega+|\boldsymbol{q}|v_{\mathrm{F}}}{\omega-|\boldsymbol{q}|v_{\mathrm{F}}}-\omega|\boldsymbol{q}|v_{\mathrm{F}}\Big)$$

$$v_{\mathrm{F}} = \frac{p_{\mathrm{F}}}{m_N}$$

Migdal function

Nucleon particle-hole polarization operator

No vector potentials $\Pi^{(+)}_{\mathrm{ph},a}(q) = 2f^2_{\pi NN}\Big\{\omega n_a + (\boldsymbol{q}^{\,2} - \omega^2)A(\omega, \boldsymbol{q}, p_{\mathrm{F},a})\Big\}$

$$A(\omega, 0, p_{\mathrm{F},a}) \approx \frac{n_a}{\omega + \frac{\omega^2}{2m^*_N}}$$

Migdal model

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π Condensation in Nuclear Matter

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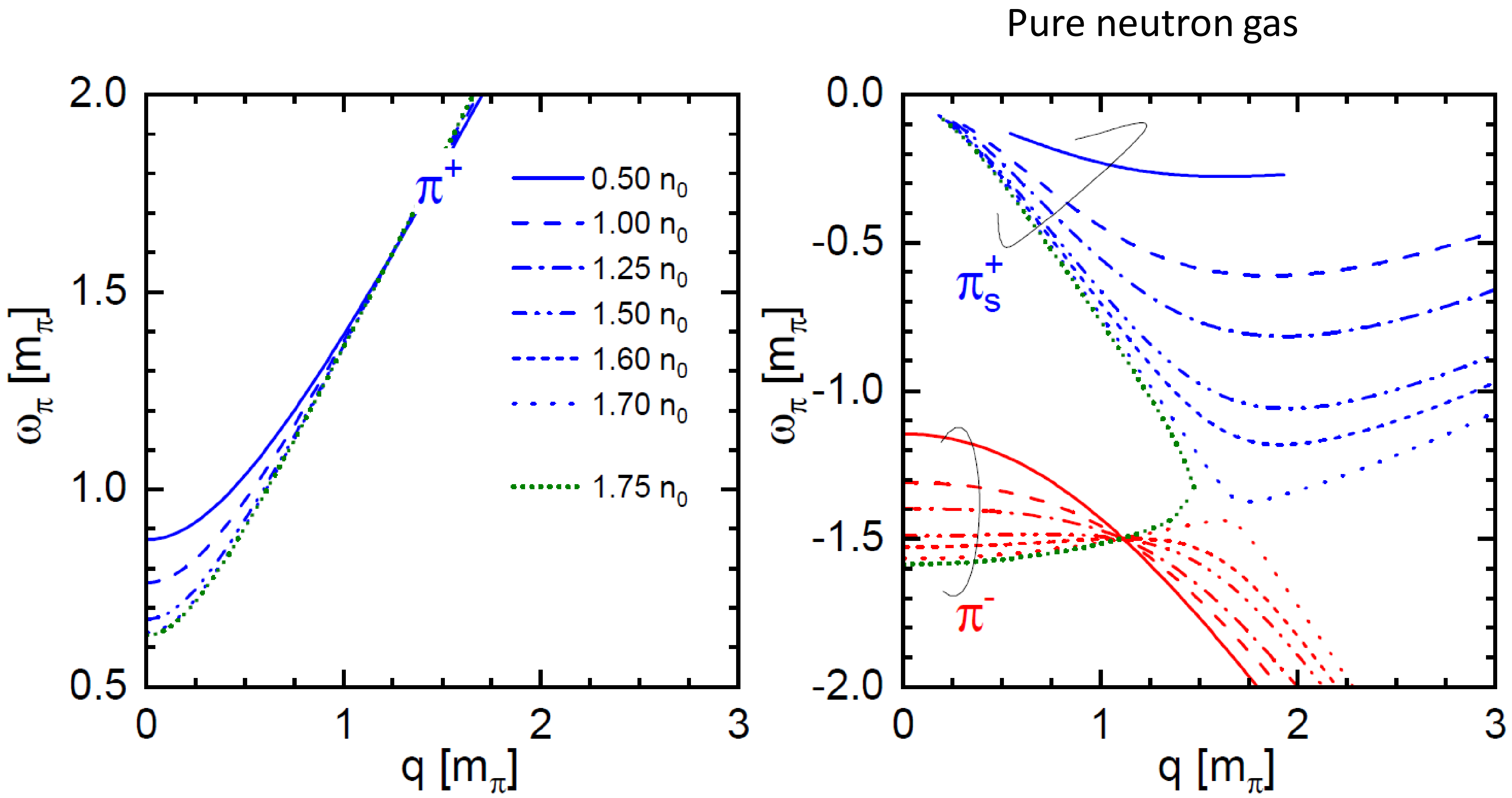
(Received 17 April 1973)

It is shown that in nuclear matter at $Z=0$ (neutron star) at a density $n_1 < n_{\mathrm{nucl}}$ a π^0 condensate appears. Nearly at the same density an electrically neutral π^+, π^- condensate arises. The π^- condensate assumed by other workers apparently does not arise even at very high densities.

$$\Pi^{(+)}_{\mathrm{n.g.,Mig}}(\omega, \boldsymbol{q},) = -\frac{\omega}{2f^2}n_n + 2f^2_{\pi NN}\boldsymbol{q}^{\,2}A(\omega, \boldsymbol{q}, p_{\mathrm{F},n})$$

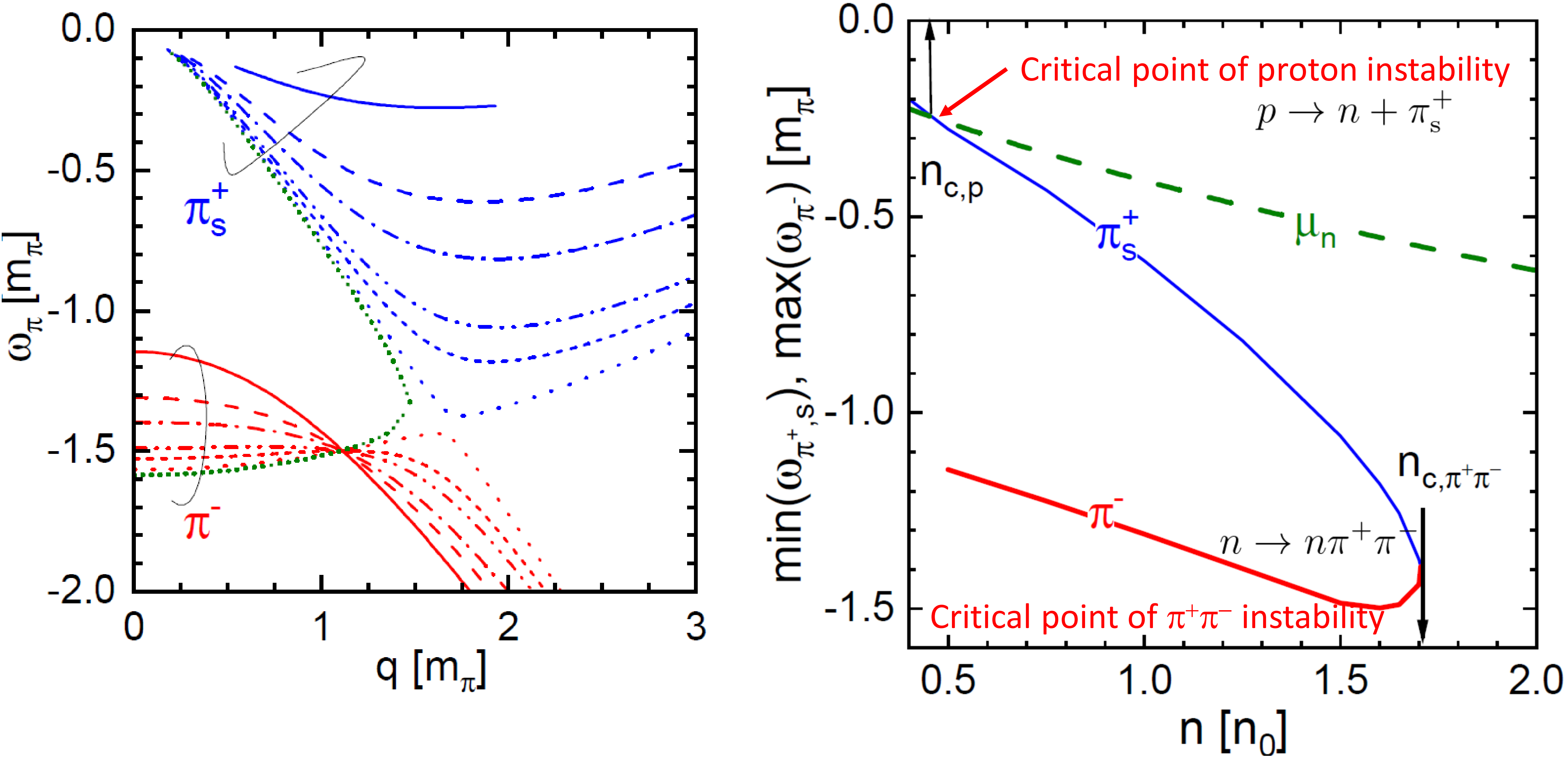
$$\approx -\frac{\omega}{2f^2}n_n - 2f^2_{\pi NN}\boldsymbol{q}^{\,2}\frac{m_N p_{\mathrm{F},n}}{\pi^2}\phi_1\left(\omega - \frac{\boldsymbol{q}^{\,2} - \omega^2}{2m_N}, \boldsymbol{q}, p_{\mathrm{F},n}\right)$$

Pion condensation in the simplified Migdal model



- *Pion condensation in the simplified Migdal model*

Pure neutron gas

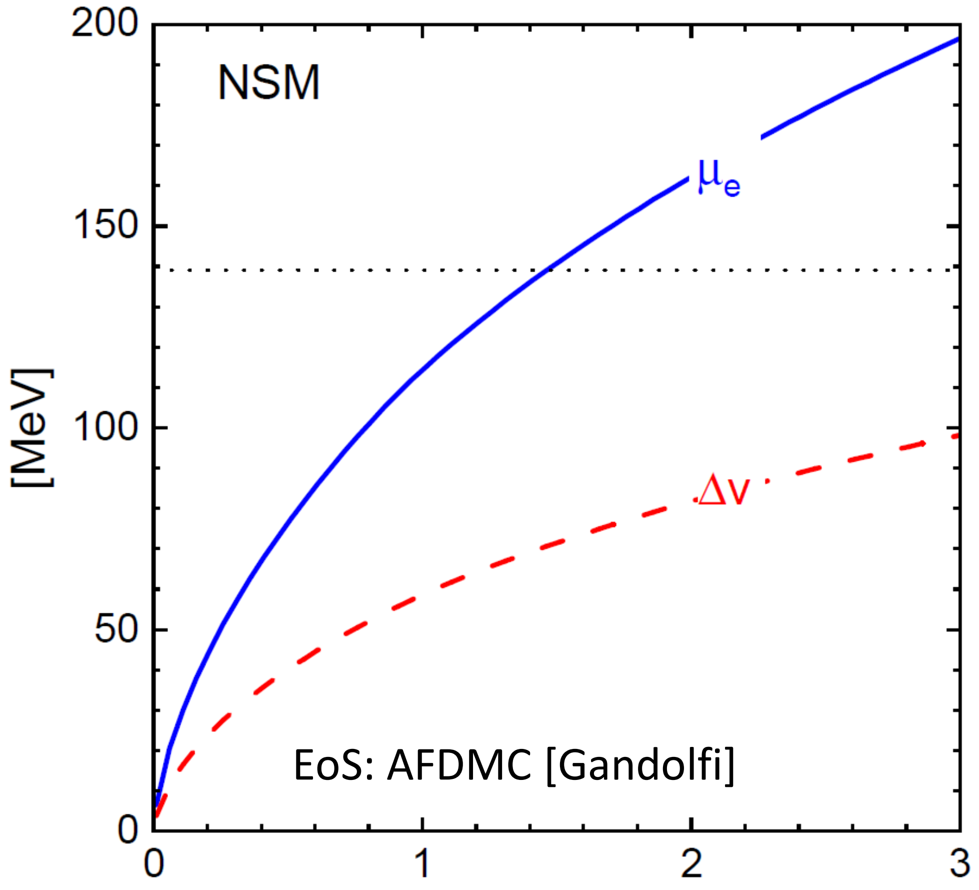


New branches in pion spectrum

B. Fore, N. Kaiser, S. Reddy, N.C. Warrington, The mass of charged pions in neutron star matter, arXiv:2301.07226.

Vector potentials (Vector part of the nucleon self-energy) are important

EoS: $E(n,x)$ $\quad \mu_n = \frac{\partial E(n,x)}{\partial n} - \frac{x}{n}\frac{\partial E(n,x)}{\partial x}$ $\quad \mu_p = \frac{\partial E(n,x)}{\partial n} + \frac{1-x}{n}\frac{\partial E(n,x)}{\partial x}$



$$\mu_a = \sqrt{m_N^{*2} + p_{\mathrm{F},a}^2} + v_a$$

$$\Delta v = v_n - v_p = \mu_n - \mu_p - \sqrt{m_N^{*2} + p_{\mathrm{F},n}^2} + \sqrt{m_N^{*2} + p_{\mathrm{F},p}^2}$$

assumed $\quad m_N^*(n) = \frac{m_N}{1 + \left(\frac{1}{0.85} - 1\right)\sqrt{\frac{n}{n_0}}}$

S-wave pion polarization operator at chiral order Q^2 including $p_F^{5/3}$ corrections

Pure neutron matter

We will keep the correction terms of the order $\xi_n^2 = p_{F,n}^2/m_N^{*2}$ and also will assume $\omega/m_N^* \ll 1$, and $\Delta v/m_N^* \ll 1$. So we will drop terms proportional to $\xi_n^2\omega/m_N^*$ and $\xi_n \Delta v/m_N^*$.

$$\Pi^{(+)}_{\mathrm{n.m.},S}(\omega) = -\frac{\omega}{2f^2}n_n - \left(\frac{\sigma_{\pi N}}{f^2} - \frac{\beta\omega^2}{m_\pi^2 f^2}\right)\left(n_n - \frac{3\xi_n^2}{10}n_n\right) - \frac{2c_2}{f^2}\omega^2\frac{3}{5}\xi_n^2 n_n$$
$$+ 2f_{\pi NN}^2\left\{(\omega-\Delta v)n_n + \left((\Delta v)^2-\omega^2\right)A(\omega+\Delta v,0,p_{F,n}) + (\Delta v)^2 B(\omega+\Delta v,0,p_{F,n}) - \omega^2\frac{n_n}{2m_N}\right\}$$

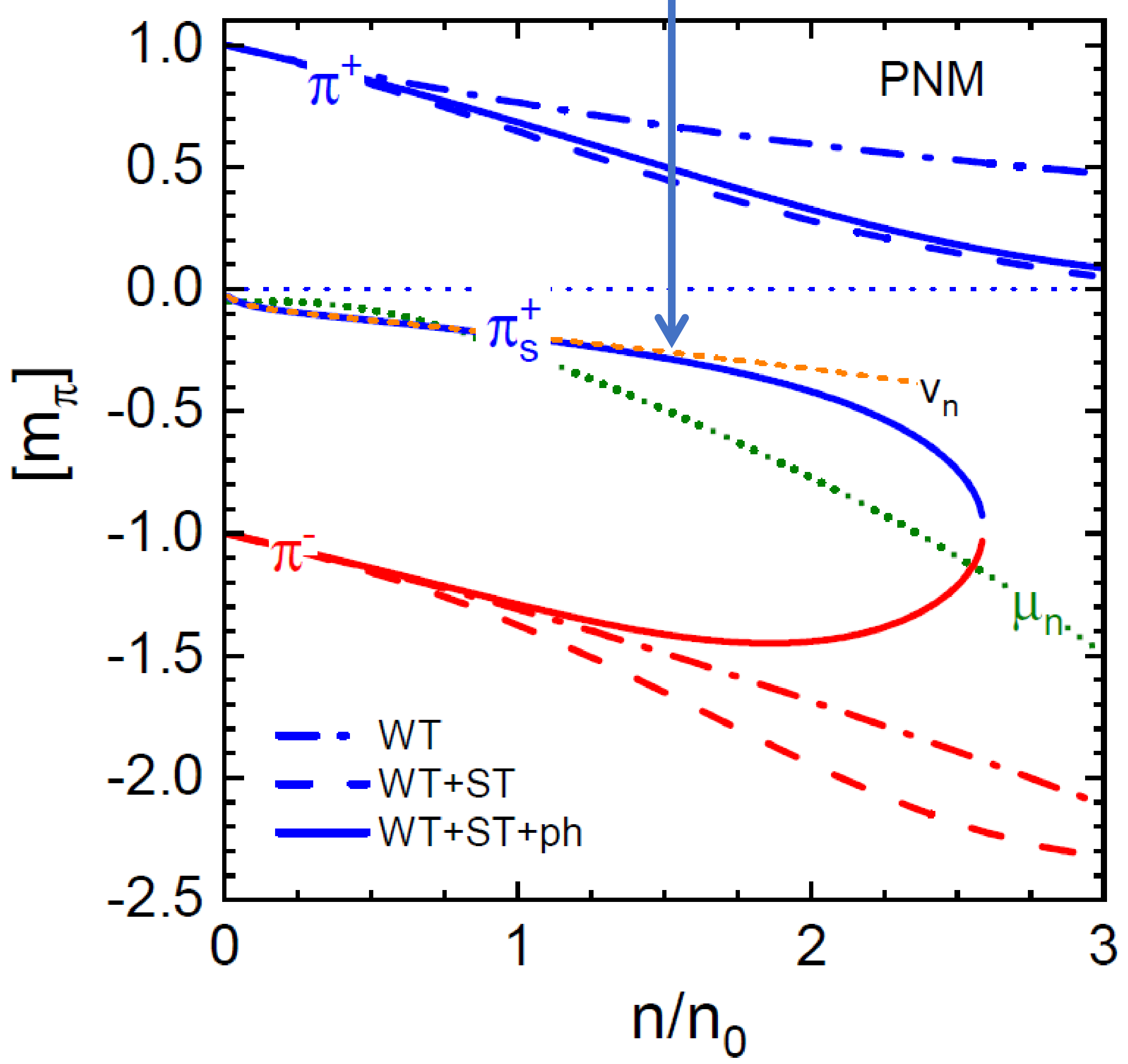
$$A(\omega,0,p_{F,a}) \approx \frac{n_a - \frac{3}{5}\xi_a^2 n_a}{\omega + \frac{\omega^2}{2m_N^*}}, \quad B(\omega,0,p_{F,a}) \approx \frac{\frac{3}{5}\xi_a^2 n_a}{\omega + \frac{\omega^2}{2m_N^*}}$$

$$\Pi^{(+)}_{\mathrm{n.m.},S}(\omega) = -m_\pi^2\frac{n_n}{n_{c,\sigma}}\left(1-\frac{3\xi_n^2}{10}\right) - \frac{\omega}{2f^2}n_n + \omega^2\frac{n}{n_{c,\beta}}\left(1 - \frac{3\xi_n^2}{10} - \frac{2c_2 m_\pi^2}{\beta}\frac{3}{5}\xi_n^2\right)$$
$$+ 2f_{\pi NN}^2\left\{\frac{(\Delta v)^2 n_n\left(\frac{3}{5}\xi_n^2 + \frac{\Delta v}{2m_N^*}\right)}{\omega+\Delta v} + n_n(\omega-\Delta v)\frac{3}{5}\xi_n^2 + \omega^2 n_n\frac{m_N - m_N^*}{2m_N^* m_N}\right\}$$

Pole term! → new branch
[Fore et all.]

No term of the order n, only higher orders in density

New branch at vanishing momentum



Sigma-term instability is shifted to higher densities

$$\widetilde{\omega}_\pi^2 = -D_\pi(0,0)$$
$$= m_\pi^2 - m_\pi^2 \frac{n_n}{n_{c,\sigma}}\Big(1 - \frac{3\xi_n^2}{10}\Big) + 2f_{\pi NN}^2 \frac{v_n^2 n_n}{2m_N^*}$$

Conclusion

- For non-interacting pions, the pionization of the NS matter occurs at very low density ~1.3n_0
- The Weinber-Tomazawa term in the πN interaction alone does not protect against pionization for the stiff symmetry energy
- Sigma-term and range term (Next-to-leading chiral order) has to be taken into account
- π^- mass diverges at $n_{c,\beta} = \frac{f^2 m_\pi^2}{\beta} \approx 2.8 n_0$ and π^+ mass vanishes
- One has to use the unitarized amplitudes. Pionization could be prevented.
- New type of the s-wave condensation occurs at density ~2.8n_0
- In the particle-hole term one has to take into account the nucleon vector potential. New spectral branch at the vanishing momentum which connects to the Migdal branch at finite moments.
- New instabilities!