Probing hybrid stars and the properties of the special points with radial oscillations

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Christoph Gärtlein, Violetta Sagun and David Blaschke

based on CG, OI, CG & DB, 2301.10765 [nucl-th]
additional OI & DB, PRD 2022 and OI & DB, Particles 2022

MPCSRG 2023, Yerevan, 12 September 2023

Phase diagram of strongly interacting matter

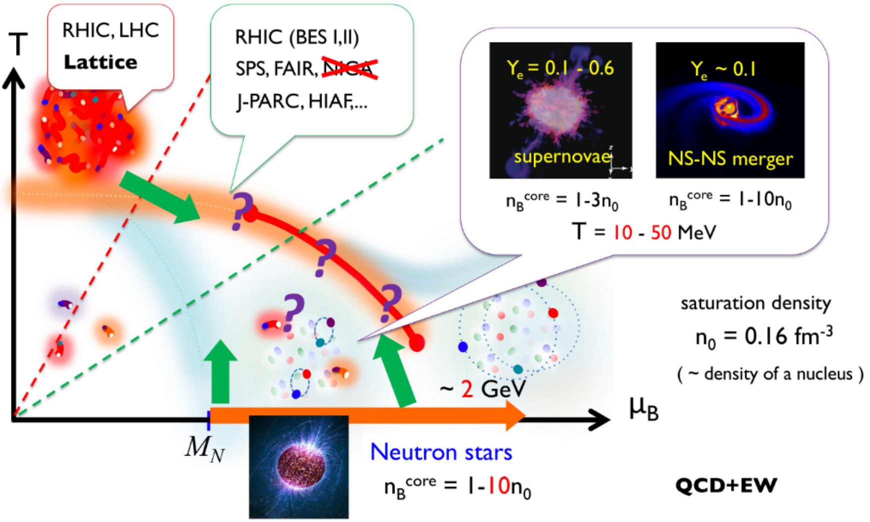
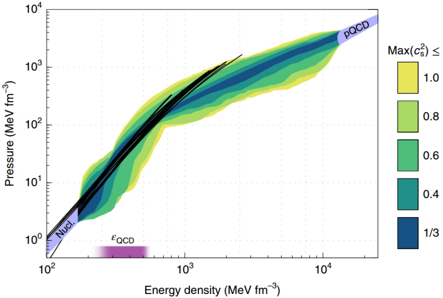
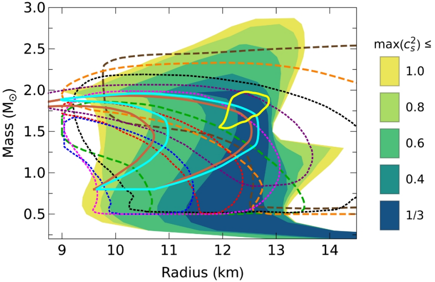


Figure from T. Kojo arXiv:1912.05326 [nucl-th]

pQCD vs $2M_\odot$ compact stars





E. Annala, T. Gorda, A. Kurkela, J. Nättilä, A. Vuorinen, Nature Physics 16, 907 (2020)

Existence of parameterization consistent with pQCD and $2M_\odot$

$\Downarrow$

Argument in favor of quark cores?

Quark matter in supernova explosions

- $2M_\odot$ stars formation? (accretion is too slow)
- Supernovae with progenitor mass $\sim 50\ M_\odot$
- Quark-hadron transition stabilizes collapse

T. Fischer et al., Nature Astronomy 2, 980–986 (2018)

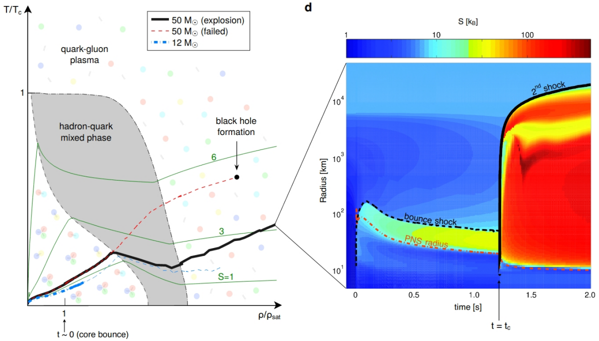


Table 1 | Summary of the supernova simulation results with hadron-quark phase transition

M_{ZAMS} ($M_\odot$)	t_{onset} (s)	$t_{collapse}$ (s)	$\rho\vert_{collapse}$ (ρ_{sat})	$T\vert_{collapse}$ (MeV)	$M_{PNS,collapse}$[a] ($M_\odot$)	t_{final} (s)	$\rho\vert_{final}$ (ρ_{sat})	$T\vert_{final}$ (MeV)	$M_{PNS,final}$[a] ($M_\odot$)	E^*_{expl} (10^{51} erg)
12[12]	3.251	3.489	2.49	28	1.727	3.598	5.5	17	1.732	0.1
18[12]	1.465	1.518	2.53	27	1.958	1.575	5.9	18	1.964	1.6
25[14]	0.905	0.976	2.40	31	2.163	0.983	9.6	19	2.171[b]	-
50[13]	1.110	1.215	2.37	32	2.105	1.224	5.8	31	2.092	2.3

Deconfinement is a supernova engine for massive blue giants

Hybrid quark-hadron EoS

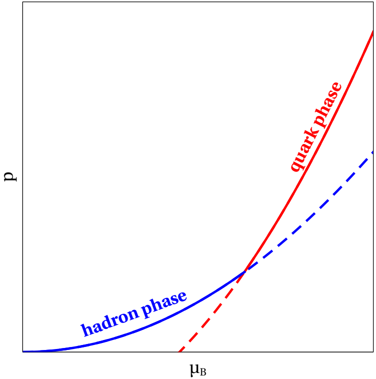
- **Absence of a unified quark-hadron approach**

$$\Downarrow$$

independent modeling of quark, hadron EoSs

- **Phase transition construction** (e.g. Maxwell)

$$p_q(\mu_B) < p_h(\mu_B) \Rightarrow \text{hadron phase}$$
$$p_q(\mu_B) > p_h(\mu_B) \Rightarrow \text{quark phase}$$
$$p_q(\mu_B) = p_h(\mu_B) \Rightarrow \text{mixed phase}$$



Special points of the mass-radius diagram

- **Quark EoS**

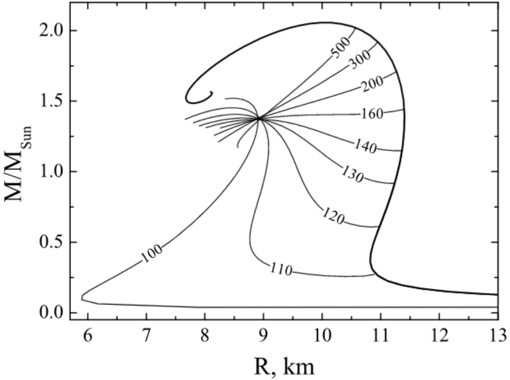
$$p = \frac{\varepsilon}{3} - \frac{4B}{3}$$

B - bag constant

- **Variation of B**

$$\Downarrow$$

family of hybrid quark-hadron Eo



- **Special point** - narrow range of intersection of M-R curves

A. V. Yudin et al., Astron. Lett. 40, 201 (2014)

Special points vs properties of hybrid EoS

- **Weak sensitivity to hadron EoS**

  M. Cierniak and D. Blaschke, Eur. Phys. J. ST 229, 3663 (2020)

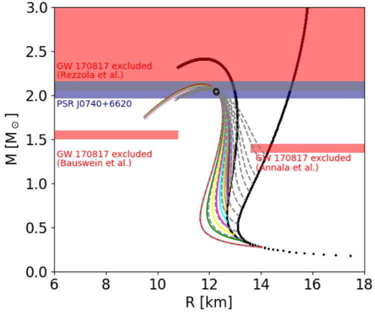
- **Weak sensitivity to details of quark-to-hadron transition**

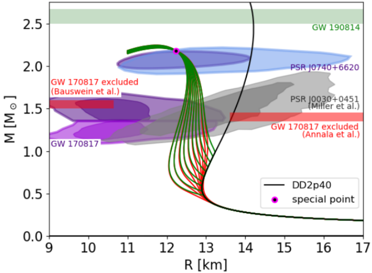
  M. Cierniak and D. Blaschke, Astron. Nachr. 342, 819-825 (2021)

- **Sensitivity to quark EoS only**

$$\Downarrow$$

SP can be used in order to test quark EoS





Goal:

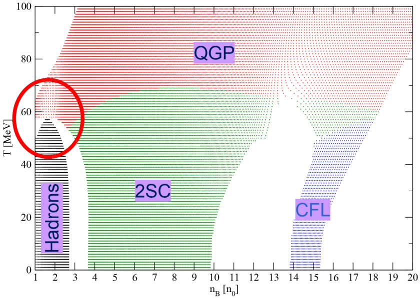
To probe properties of quark matter with SPs based on a microscopic model

False quark dominance in hybrid quark-hadron EoS

- **Hadronic EoS consistent with astro (DDf4) + NJL model**

$\Downarrow$

False quark onset already @ T $\simeq$ 60 MeV



- **Hadron decays are energetically favorable**

$$M_q \simeq 330 MeV \qquad M_{meson} > 2M_q$$
$$M_\omega = 783 MeV \quad \Rightarrow \quad quarks\ are\ too\ light$$
$$M_\rho = 775 MeV \qquad to\ be\ confined$$

Effective quark "confinement" is needed

Model

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q - \mathcal{U} + \mathcal{L}_V + \mathcal{L}_D$$

- **Scalar & pseudoscalar interaction channels**

  $\mathcal{U}$ – χ-symmetric density functional (details below)

- **Vector-isoscalar interaction channel**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2$$

  (motivated by gluon exchange, stiff EoS needed to reach $2M_\odot$)

- **Diquark interaction channel**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q}i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

  (motivated by Cooper theorem, color superconductivity)

Relativistic density functional

$$\mathcal{U} = D_0\left[(1+\alpha)\langle\overline{q}q\rangle_0^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\varkappa}$$

- **Parameters**

  D_0 - dimensionfull coupling, controls interaction strength

  α - dimensionless constant, controls vacuum quark mass

  $\langle\overline{q}q\rangle_0$ - χ-condensate in vacuum (introduced for the sake of convenience)

$\varkappa = 1/3$
$\Downarrow$
motivated by String Flip model

$\varkappa = 1$
$\Downarrow$
Nambu–Jona-Lasinio model

$$\mathcal{U}_{SFM} \propto \langle q^+q\rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial\mathcal{U}_{SFM}}{\partial\langle q^+q\rangle} \propto \langle q^+q\rangle^{-1/3} \propto \textit{separation}$$

- **Dimensionality**

$$\begin{matrix}[\mathcal{U}] = \textit{energy}^4 \\ [\overline{q}q] = \textit{energy}^3\end{matrix} \quad \Rightarrow \quad [D_0]_{\varkappa=1/3} = \textit{energy}^2 = [\textit{string tension}]$$

self energy = string tension × separation ⇒ confinement

Expansion around $\langle\bar{q}q\rangle$ and $\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle=0$

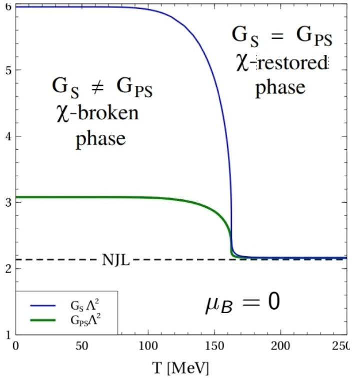
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}}\ \text{order}} + \underbrace{\left(\bar{q}q - \langle\bar{q}q\rangle\right)\Sigma_S}_{1^{\text{st}}\ \text{order}} - \underbrace{G_S\left(\bar{q}q - \langle\bar{q}q\rangle\right)^2 - G_{PS}\left(\bar{q}i\vec{\tau}\gamma_5 q\right)^2}_{2^{\text{nd}}\ \text{order}} + \ldots$$

- **Mean-field scalar self-energy**

$$\Sigma_S = \frac{\partial\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle}$$

- **Effective medium dependent couplings**

$$G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}q\rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{MF}}{\partial\langle\bar{q}i\vec{\tau}\gamma_5 q\rangle^2}$$



Expansion around $\langle \bar{q}q \rangle$ and $\langle \bar{q}i\vec{\tau}\gamma_5 q \rangle = 0$

$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{th}\ \text{order}} + \underbrace{(\bar{q}q - \langle \bar{q}q \rangle)\, \Sigma_{MF}}_{1^{st}\ \text{order}} - \underbrace{G_S\, (\bar{q}q - \langle \bar{q}q \rangle)^2 - G_{PS}\, (\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{nd}\ \text{order}} + \dots$$

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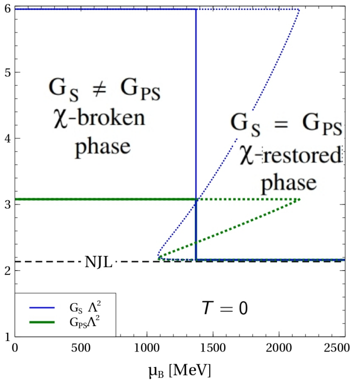


Comparison to NJL model

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \underbrace{(m + \Sigma_S)}_{\textbf{effective mass } \mathbf{m}^*})q + G_S(\bar{q}q)^2 + G_{PS}\,(\bar{q}i\vec{\tau}\gamma_5 q)^2 + \cdots + \mathcal{L}_V + \mathcal{L}_D$$

- **Similarities:**
  - current-current interaction
  - (pseudo)scalar, vector, diquark, ... channels
- **Differences:**
  - high m^* at low T, μ $\Rightarrow$ **"confinement"**

$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2G_0}{3\alpha^{2/3}\langle\bar{q}q\rangle_0^{1/3}}$$

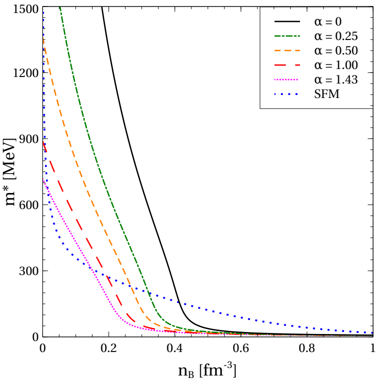
$$\Downarrow$$

$$m^* \to \infty \; at \; \alpha \to 0$$

  - medium dependent couplings:

$$\begin{array}{llll} \text{low } T,\ \mu, & \Rightarrow\ G_S \neq G_{PS} & \Rightarrow & \chi\text{-broken} \\ \text{high } T,\ \mu, & \Rightarrow\ G_S = G_{PS} & \Rightarrow & \chi\text{-symmetric} \end{array}$$

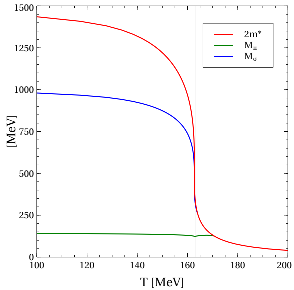
T = 0



Model setup

- **(Pseudo)scalar interaction channels (chiral condensate & π, σ mesons)**

m [MeV]	Λ [MeV]	α	$D_0\Lambda^{-2}$
4.2	573	1.43	1.39
M_π [MeV]	F_π [MeV]	M_σ [MeV]	$\langle \bar{l}l \rangle_0^{1/3}$ [MeV]
140	92	980	-267



Pseudocritical temperature

$$T_c = 163 \text{ MeV}$$

- low T: $2m_{quark} > M_\pi, M_\sigma$ **(stable mesons, confined quarks)**
- high T: $2m_{quark} < M_\pi, M_\sigma$ **(unstable mesons, deconfined quarks)**

- **Vector repulsion channel (ω-meson)**

$$M_\omega = 783 \text{ MeV} \Rightarrow \eta_V \equiv \frac{G_V}{G_S} = 0.452$$

- **Diquark pairing channel (Fierz transformation)** $\eta_D \equiv \frac{G_D}{G_S} = 1.5\eta_V = 0.678$

High density asymptotic at constant G_V and G_D

- $\mathbf{G_V \neq 0}$

$$p \to G_V \langle q^+ q \rangle^2 \propto \mu_B^2, \quad c_S^2 \to 1$$

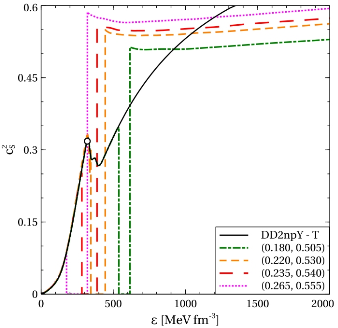
- $\mathbf{G_V = 0, \; G_D \neq 0}$

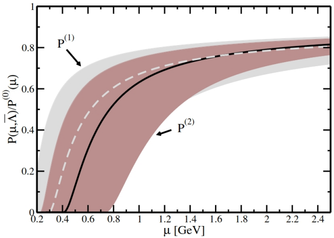
$$p \to G_D |\langle \overline{q}^c i\gamma_5 \tau_2 \lambda_2 q \rangle^2| \propto \mu_B^6, \quad c_S^2 \to \frac{1}{5}$$

- **Perturbative QCD**

$$p \to 0.8 p_{SB} \propto \mu_B^4, \quad c_S^2 \to \frac{1}{3}$$

A. Kurkela, P. Romatschke, A. Vuorinen, Phys. Rev. D 81, (2010)





Medium dependent couplings?

High density asymptotic at constant G_V and G_D

- $\mathbf{G_V \neq 0}$

$$p \to G_V \langle q^+ q\rangle^2 \propto \mu_B^2, \quad c_S^2 \to 1$$

- $\mathbf{G_V = G_I = 0,\ G_D \neq 0}$

$$p \to G_D |\langle \overline{q}^c i\gamma_5 \tau_2 \lambda_2 q\rangle^2| \propto \mu_B^6, \quad c_S^2 \to \frac{1}{5}$$

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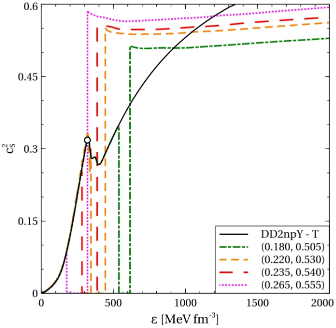
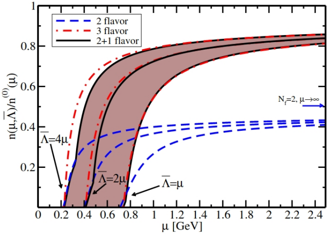




Medium dependent couplings?

Effective repulsion from non-perturbative gluon exchange

- **Fock energy with** $D_{gluon} \propto \frac{1}{k^2 - M_{gluon}^2}$

$$\varepsilon_{repulsion} = G_V \langle q^+ q \rangle^2 \quad with \quad G_V = \frac{4\pi\alpha_s/3}{9M_{gluon}^2 + 8k_F^2}, \quad \alpha_s \; - \; frozen$$

Y. Song, G. Baym, T. Hatsuda, and T. Kojo Phys. Rev. D 100, 034018 (2019)

- **Density dependent coupling**

$$k_F = \left(\frac{6\pi^2 \langle q^+ q \rangle}{2 \cdot 2 \cdot 3} \right)^{1/3} \quad \Rightarrow \quad G_V = \frac{G_V^{vacuum}}{1 + \frac{8}{9M_{gluon}^2} \left(\frac{\pi^2 \langle q^+ q \rangle}{2} \right)^{2/3}}$$

- **High density asymptotic**

$$\varepsilon_{repulsion} \propto \langle q^+ q \rangle^{4/3} \quad \Rightarrow \quad c_S^2 \rightarrow \frac{1}{3}$$

Medium dependent couplings

$$G_V(n_V) = G_V^{vacuum} \cdot f(n_V), \qquad G_D(n_D) = G_D^{vacuum} \cdot f(n_D)$$

$$n_V = \langle q^+ q \rangle, \qquad n_D = |\langle \overline{q}^c i\tau_2\gamma_5\lambda_2 q \rangle|$$

- **Medium dependence**

$$f(n) = \left[1 + \frac{8}{9M_{gluon}^2} \left(\frac{\pi^2 n}{2} \right)^{2/3} \right]^{-1}$$

- **Rearrangement terms** (needed for thermodynamic consistency)

$$\langle f^+ f \rangle = -\frac{\partial \Omega}{\partial \mu_f}$$

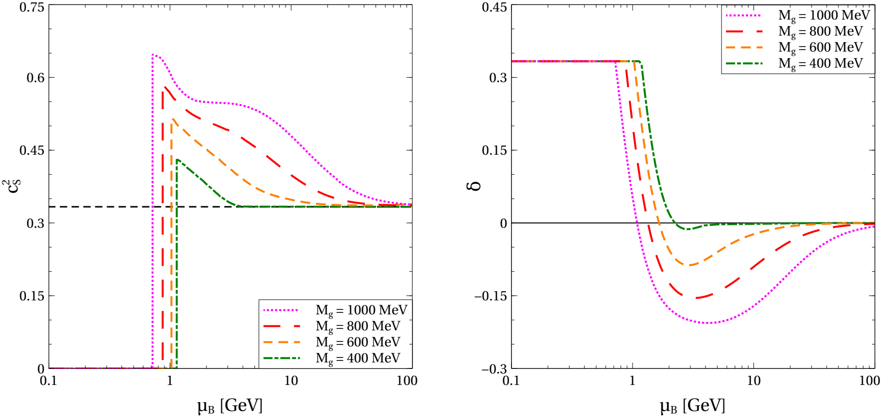
$$\Downarrow$$

$$\Omega \to \Omega - \Theta_V + \Theta_D \quad with \quad \Theta_i = \int_0^{n_i} dn \ n^2 \frac{\partial G_i(n)}{\partial n}$$

Asymptotically conformal EoS (symmetric matter @ T=0)

- **Conformal matter**

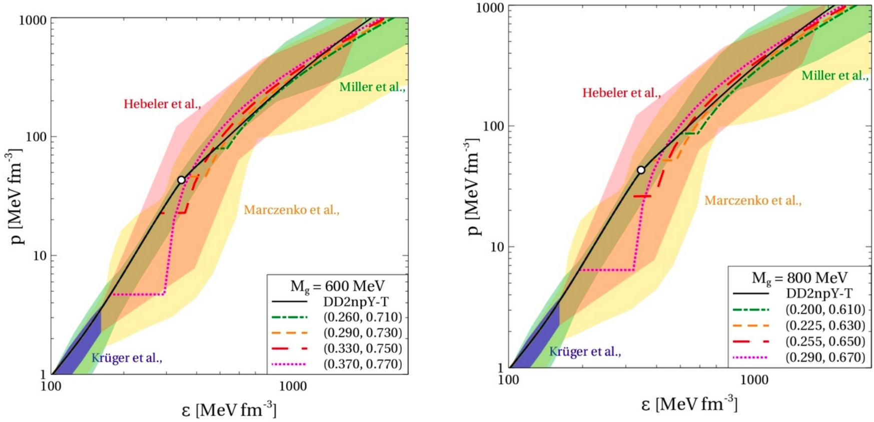
$$\varepsilon \propto \mu^{d+1} \quad \Rightarrow \quad c_S^2 \to \frac{1}{3}, \quad \delta \equiv \frac{T^\mu_\mu}{3\varepsilon} = \frac{1}{3} - \frac{p}{\varepsilon} \to 0$$



Conformality is reached at $\mu_B/3 \gg M_{gluon}$

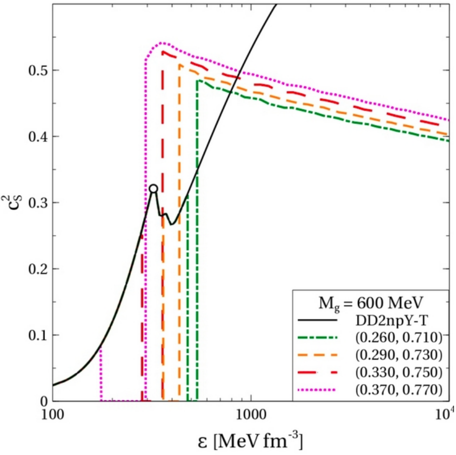
Asymptotically conformal EoS of neutron stars

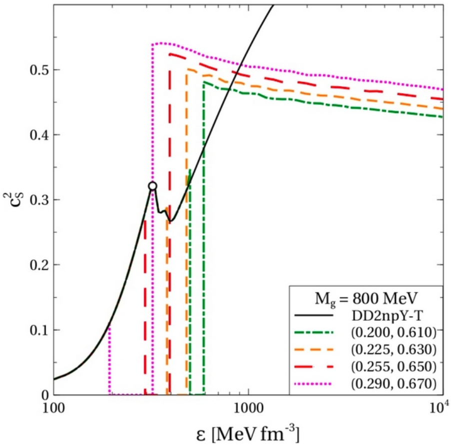
- **Setup:** electric neutrality, β-equilibrium, Maxwell construction with DD2 EoS
- **Scanning over** $\eta_V = \frac{G_V}{G_S}\Big|_{vacuum}$ **and** $\eta_D = \frac{G_D}{G_S}\Big|_{vacuum}$



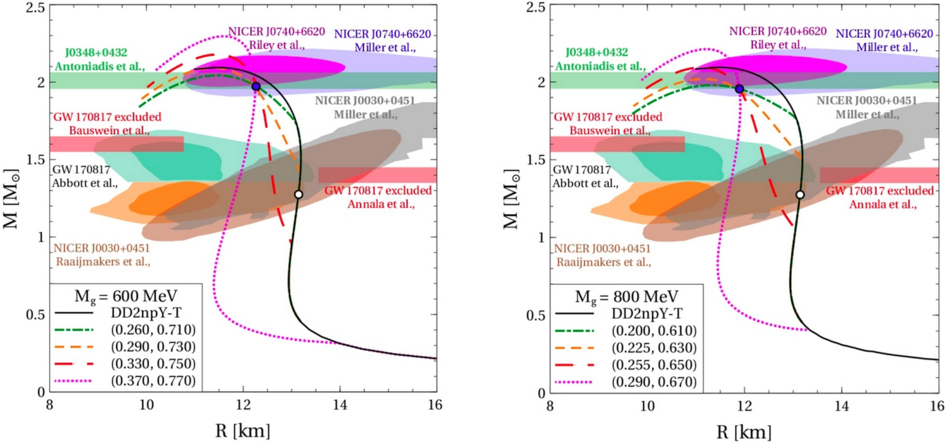
The ω-meson value of η_V and the Fierz value of η_D prefer early deconfinement?

Speed of sound



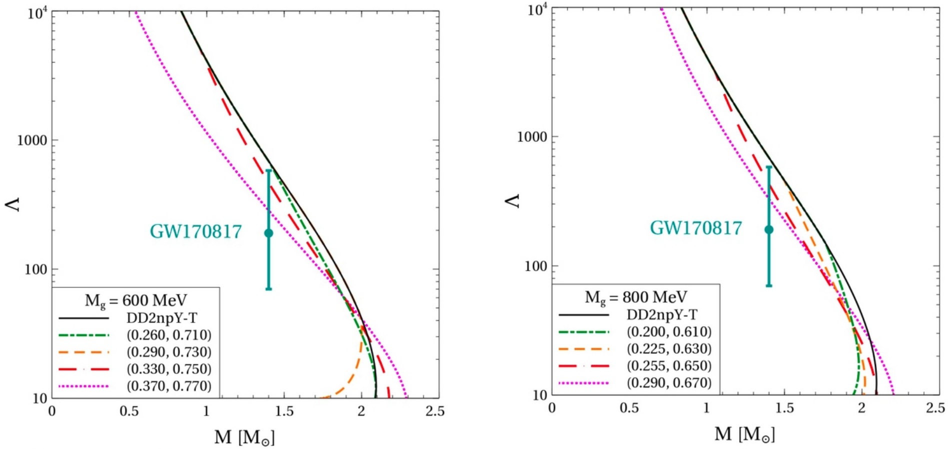


Mass-radius diagram



Observational data prefer early deconfinement?

Tidal deformability



Observational data prefer early deconfinement?

Conformality in neutrons stars?

- **Speed of sound**

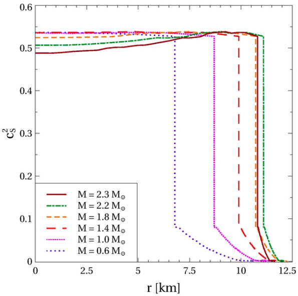
  $c_S^2 = \frac{1}{3}$ in conformal matter

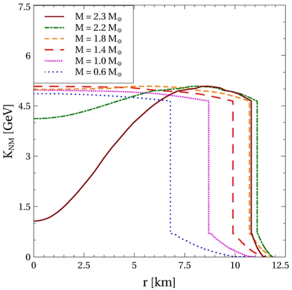
- **Compression modulus**

  $$K_{NM} = 9 n_B^2 \frac{\partial^2}{\partial n_B^2} \frac{E}{A}$$

  $$K_{NM} = -\frac{3\mu_B}{2} < 0 \text{ in conformal matter}$$

Both c_S^2 and K_{NM} contradict conformality in neutron stars





Summary of quark EoS

- **Phenomenological "confinement"**

  $p \simeq -B$ at small densities

- **Asymptotically conformal**

  $p \propto \mu_B^4$ at high densities

- **Color superconductivity**

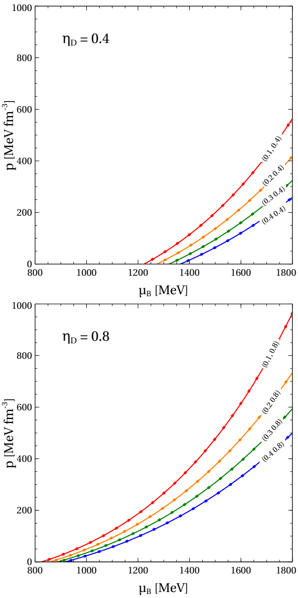
  correction $\propto \mu_B^2 \Delta^2$

- **ABPR-like parameterization** $(M_{gluon} = 600 \text{ MeV})$

  $$p = A_4 \mu_B^4 + \Delta^2 \mu_B^2 - B$$

  $A_4, \ \Delta, \ B$ depend on $\eta_V, \ \eta_D$

  C. Gärtlein et al., 2301.10765 [nucl-th]



Special points of the mass-radius diagram

- **Variation of η_D at fixed η_V**

  $\Downarrow$

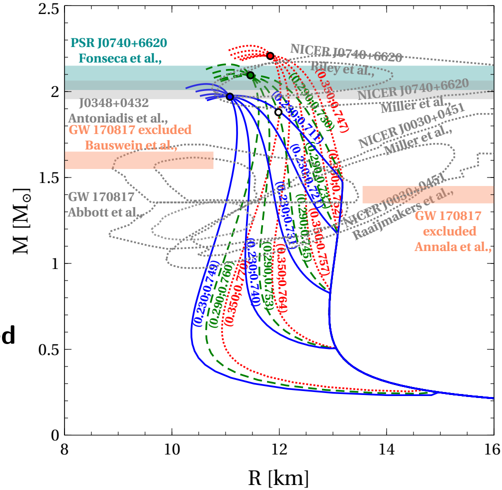
  Special point

- **SPs are equidistant**

- **M_{max} and M_{onset} are anticorrelated**

  M_{max} – observationally constrained

  M_{onset} – controlled by η_V, η_D



Is it possible to constrain η_V and η_D?

M_{max}, M_{onset} and special point

- **Onset mass**

  M_{onset} is controlled by η_V, η_D

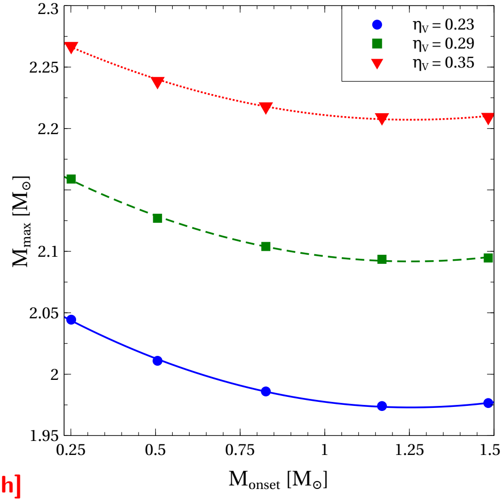
- **Maximum mass**

  $$\mathbf{M_{max}} = \mathbf{M_{SP}} + \delta \left| \mathbf{M^*_{onset}} - \mathbf{M_{onset}} \right|^2$$

  δ depends on η_V and η_D

  $M^*_{onset} = 1.245\ M_{\odot}$ - universal

C. Gärtlein et al., 2301.10765 [nucl-th]



Constraining vector and diquark couplings

- **No vacuum color-superconductivity**

$$\eta_D < 0.78$$

O. Ivanytskyi, D. Blaschke, PRD (2022)

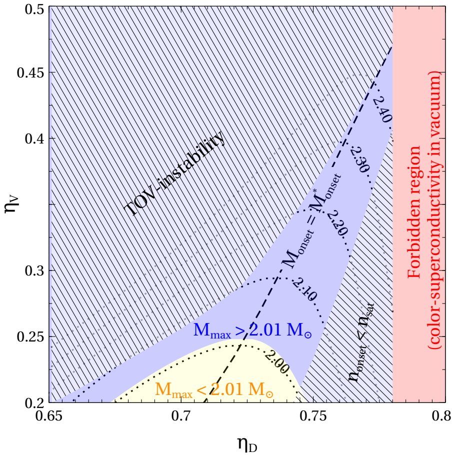
- $\mathbf{M_{max} = 2.08^{+0.07}_{-0.07} M_\odot}$

E. Fonseca et al., Astrophys. J. Lett. 915, L12 (2021)

- **Not too early deconfinement**

$$n_{onset} > n_{saturation}$$

- **Stability of the quark branch**



$$M_\omega = 783\ \mathrm{MeV} \Rightarrow \eta_V = 0.452$$

Are the couplings constrained to the small region suggesting $M_{onset} < 0.5 M_\odot$ and $M_{max} > 2.4 M_\odot$?

Conclusions

- Effective "confining" chiral model with color superconductivity is derived based on the χ-symmetric density functiobal
- Medium dependent quark-meson couplings provide conformal limit
- Neutron star matter is unlikely to be conformal
- Simple analytical parameterization of the model
- Constraint on the parameters of quark matter, suggesting an early deconfinement and heavy compact stars

Hyperon puzzle

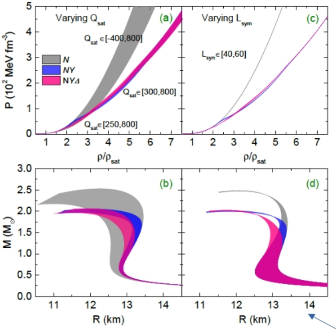


FIG. 4. EoS models and MR relations for N, NY, and $NY\Delta$ compositions of stellar matter. The bands are generated by varying the parameters Q_{sat} [MeV] (a, b) and L_{sym} [MeV] (c, d). The ranges of Q_{sat} and L_{sym} allowed by χEFT and maximum mass constraints are indicated in the figures.

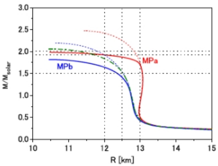


FIG. 7. Neutron-star masses as a function of the radius R. Solid (dashed) curves are with (without) hyperon (Λ and Σ^-) mixing for ESC+MPa and ESC+MPb. The dot-dashed curve for MPb is with Λ mixing only. Also see the caption of Fig. 3.

Yamamoto et al., Phys.Rev.C 96 (2017) 06580; arXiv:1708.06163 [nucl-th]
Yamamoto et al., Eur. Phys. J. A 52 (2016) 19; arXiv:1510.06099 [nucl-th]
Ji & Sedrakian, Phys. Rev. C 100 (2019) 015809; arXiv:1903.06057 [astro-ph.HE]

Examples for realistic hadronic EoS which suggest a Berlin Wall is inferior to the line M = 2.0 M_sun

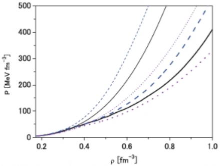


Fig. 8. Pressure P as a function of baryon density ρ. Thick (thin) curves are with (without) hyperon mixing. Solid, dashed and dotted curves are for MPa, MPa$^+$ and MPb.

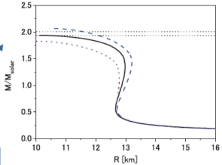
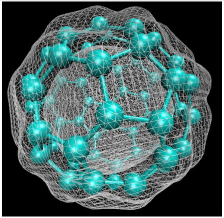
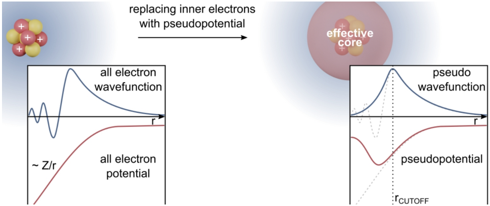


Fig. 9. Neutron-star masses as a function of the radius R. Solid, dashed and dotted curves are for MPa, MPa$^+$ and MPb. Two dotted lines show the observed mass $(1.97 \pm 0.04)M_\odot$ of J1614-2230.

Hyperons soften EoS, prevent neutron stars from reaching $2M_\odot$

Context: Density functional theory





(Dirac)Brueckner-Hartree-Fock T-, G-matrix based theories

$$\Updownarrow$$

Density functional theory

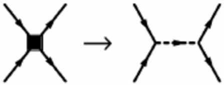
- **Many body problems**
- **Quantum chemistry**
- **Skyrme-type models for nuclear physics**
- **String Flip model for quark matter**
- **...**

Bosonization

- **Hubbard-Stratonovich transformation**

$$\exp\left[\int dx\ G(\bar{q}\hat{\Gamma}q)^2\right] = \int [D\phi]\exp\left[-\int dx\left(\frac{\phi^2}{4G}+\phi\bar{q}\hat{\Gamma}q\right)\right]$$

- **Vertexes:** $\hat{\Gamma}_S = 1 \ \Rightarrow$ scalar-isoscalar σ-field
  $\hat{\Gamma}_{PS} = i\gamma^5\vec{\tau} \ \Rightarrow$ pseuscalar-isoscalar $\vec{\pi}$-field
  $\hat{\Gamma}^{\mu}_{V} = \gamma^{\mu} \ \Rightarrow$ vector-isoscalar ω^{μ}-field
  $\hat{\Gamma}^{\mu}_{I} = \gamma^{\mu}\vec{\tau} \ \Rightarrow$ vector-isovector $\vec{\rho}^{\mu}$-field
  $\hat{\Gamma}^{A}_{D} = i\gamma^5\lambda_A\tau_2 \ \Rightarrow$ scalar diquark Δ_A-field



- **Bosonized Lagrangian ($m^* = m + \Sigma_S$ - effective mass, $Q^T = (q\ q^c)/\sqrt{2}$)**

$$\mathcal{L} + q^{+}\hat{\mu}q = \overline{Q}\hat{S}^{-1}Q - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \frac{\omega^2}{4G_V} + \frac{\vec{\rho}^2}{4G_I} - \frac{\Delta_A\Delta_A^*}{4G_D} - \mathcal{U}_{MF} + \langle\bar{q}q\rangle(\Sigma_S+\sigma)$$

$$\hat{S}^{-1} = \begin{pmatrix} \hat{\mathcal{S}}_{+}^{-1} & i\Delta_A\gamma_5\tau_2\lambda_A \\ i\Delta_A^*\gamma_5\tau_2\lambda_A & \hat{\mathcal{S}}_{-}^{-1} \end{pmatrix}, \quad \hat{\mathcal{S}}_{\pm}^{-1} = i\partial\!\!\!/ - m^* - \sigma - i\gamma^5\vec{\pi}\cdot\vec{\tau} \pm (\gamma_0\hat{\mu} + \omega\!\!\!/ + \vec{\rho\!\!\!/}\cdot\vec{\tau})$$

Mean field

- **Field equations for σ and $\vec{\pi}$**

$$\begin{cases} \sigma = 2G_S(\langle \overline{q}q \rangle - \overline{q}q) \\ \vec{\pi} = -2G_{PS}\overline{q}i\vec{\tau}\gamma_5 q \end{cases} \Rightarrow \langle \sigma \rangle = \langle \vec{\pi} \rangle = 0 \Rightarrow \sigma, \vec{\pi} - \textbf{beyond MF}$$

comment: $\langle \sigma \rangle = 0$ does not assume χ-symmetry since $\langle \overline{q}q \rangle \neq 0$

- **Thermodynamic potential**

$$\langle \omega_\mu \rangle = \delta_{\mu 0}\omega, \quad \langle \rho^a_\mu \rangle = \delta_{\mu 0}\delta_{a3}\rho, \quad |\langle \Delta_A \rangle| = \delta_{A2}\Delta$$

$$\Downarrow$$

$$\Omega = -\frac{1}{2\beta V} Tr \ln(\beta \hat{S}^{-1}) - \frac{\omega^2}{4G_V} - \frac{\rho^2}{4G_I} + \frac{\Delta^2}{4G_D} + \mathcal{U}_{MF} - \langle \overline{q}q \rangle \Sigma_S$$

- **Vector fields, diquark gap, χ-condensate**

$$\frac{\partial \Omega}{\partial \omega} = 0, \quad \frac{\partial \Omega}{\partial \rho} = 0, \quad \frac{\partial \Omega}{\partial \Delta} = 0, \quad \langle \overline{q}q \rangle = \sum \frac{\partial \Omega}{\partial m_f}$$

Superconductivity onset

- **Single quark energy and distribution**

$$E_f^{\pm} = sgn(E_f \mp \mu_f)\sqrt{(E_f \mp \mu_f)^2 + \Delta^2}$$

$$f_f^{\pm} = \left[\exp(E_f^{\pm}/T) + 1\right]^{-1}$$

- **Gap equation**

$$\frac{\partial \Omega}{\partial \Delta} = \frac{\Delta}{2G_D} - 2\Delta \sum_{f,a=\pm} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1-2f_f^a}{E_f^a} = 0$$

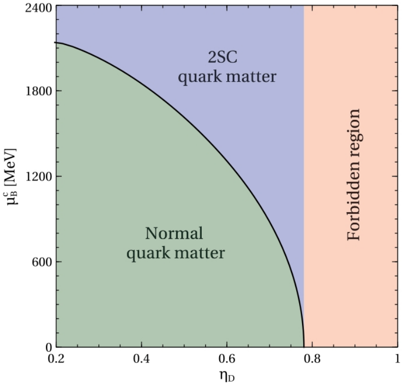
$$\Downarrow$$

two solutions : $\Delta = 0$ **or** $\Delta \neq 0$

- **Two solutions coincide** $\Rightarrow$ **SC onset**

$$\left.\frac{\partial^2 \Omega}{\partial \Delta^2}\right|_{\Delta=0} = 0 \quad \Rightarrow \quad \mu_B = \mu_B(G_D)$$

$T = 0$



No vacuum superconductivity

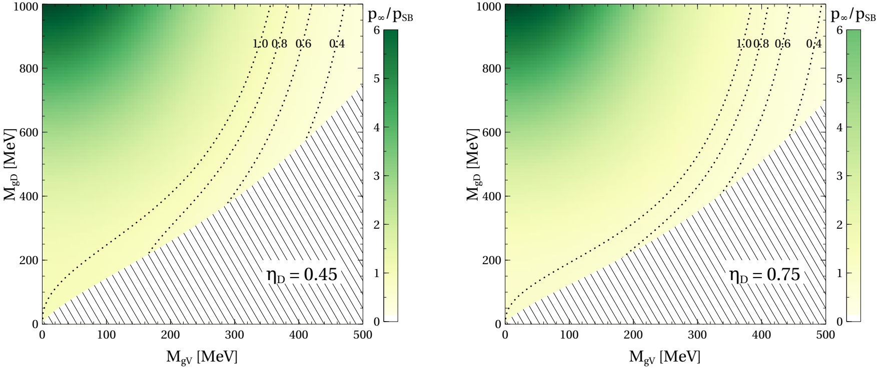
$$\Downarrow$$

$$\eta_D \lesssim 0.78$$

(agrees with the Fierz value)

Asymptotic of pressure

- **Perturbative QCD:** $p/p_{SB} \simeq 0.8$, $\delta \to +0$ in symmetric matter
- **Scanning over M_{gD} vs M_{gV} at different η_D**



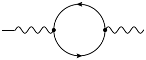
hatchet region: $\delta \to -0$ - contradiction with pQCD

$M_{gV} \simeq 400$ MeV and $M_{gV} \simeq 800$ MeV?

How to define G_V, G_I and G_D?

- **Mesonic correlations**

$$\mathcal{L} = \ldots + \bar{q}(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau} + \not{\omega} + \vec{\not{\rho}}\cdot\tau)q - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \frac{\omega^2}{4G_V} + \frac{\vec{\rho}^2}{4G_I}$$

$D_i^{-1}(p^2) = \frac{1}{2G_i}$  - one-loop mesonic propagator

$$D_i^{-1}(M_i^2) = 0 \;\Rightarrow\; \text{mesonic masses}$$

- **Fierz transformation** - rearrangement of Dirac, color and flavor indexes

$$(\gamma^\mu)_{mn}(\gamma_\mu)_{m'n'} = \mathbf{1}_{mn'}\mathbf{1}_{m'n} + (i\gamma_5)_{mn'}(i\gamma_5)_{mn'} - \frac{1}{2}(\gamma^\mu)_{mn'}(\gamma_\mu)_{m'n} - \frac{1}{2}(\gamma^\mu\gamma_5)_{mn'}(\gamma_\mu\gamma_5)_{m'n}$$

$$\mathbf{1}_{ij}\mathbf{1}_{kl} = \frac{1}{3}\mathbf{1}_{il}\mathbf{1}_{kj} + \frac{1}{2}(\tau_a)_{il}(\tau_a)_{kj}$$

$$\lambda_\alpha^{ab}\lambda_\alpha^{a'b'} = \frac{16}{9}\mathbf{1}_{ab'}\mathbf{1}_{a'b} - \frac{1}{3}\lambda_\alpha^{ab'}\lambda_\alpha^{a'b}$$

coefficients - proportional to couplings

$$\mathbf{G_S : G_V : G_I : G_D = 1 : 0.5 : 0.5 : 0.75}$$

Phase diagram (Q-neutral, β-equilibrium, $M_{gluon} \to \infty$)

- **Normal quark matter**

  2 spin $\times$ 2 flavor $\times$ 3 color $=12$

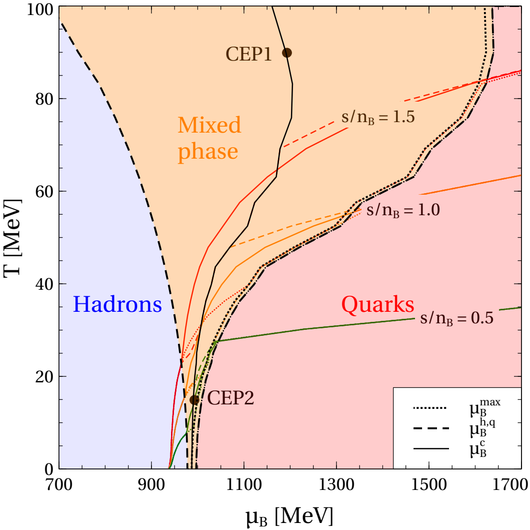
- **2SC quark matter**

  2 spin $\times$ 2 flavor $\times$ 1 color$+1=5$

Quark pairing reduces number of quark states

$\Downarrow$

requires higher T along adiabat



OI & David Blaschke, EPJ A, 2022