Covariant density functionals for compact star studies

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The Modern Physics of Compact Stars and Relativistic Gravity, 12-16 Sept 2023 Yerevan, Armenia

FIAS Frankfurt Institute for Advanced Studies

Uniwersytet Wrocławski

Talk based on:

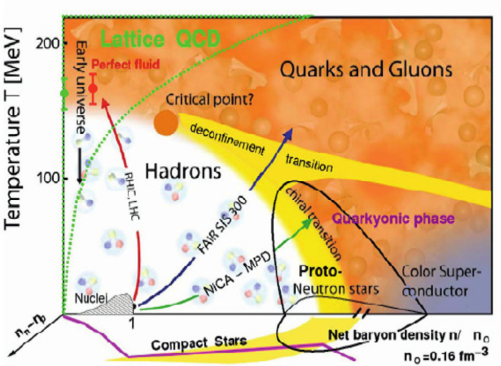
- A. Sedrakian and A. Harutyunyan,
  Delta-resonances and hyperons in proto-neutron stars and merger remnants.
  Eur. Phys. J. A **58** (2022) 137; Universe **7** (2021) 382

- M. Alford, A. Harutyunyan and A. Sedrakian,
  Bulk Viscosity of Relativistic npeμ Matter in Neutron-Star Mergers.
  Phys. Rev. D **104**, (2021) 103027; arXiv:2306.13591; Particles **5** (2022) 361

For a review:

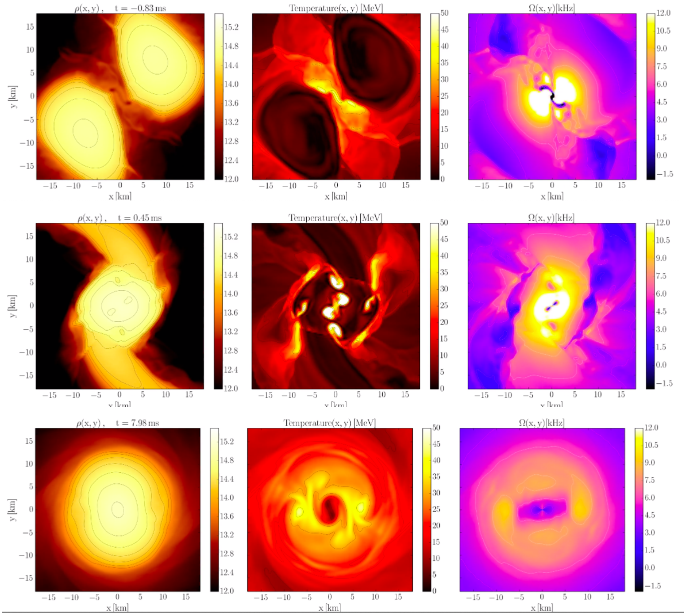
A. Sedrakian, J.-J. Li and F. Weber
Heavy Baryons in Compact Stars.
Prog. Part. Nucl. Phys. 131 (2023) 104041 [arXiv:2212.01086]

Exploration of the strong sector of the Standard Model



The big picture of QCD phase diagram:

1. High-temperature and low-density HIC and lattice QCD simulations
2. High-temperature and high-density - CCSN and BNS mergers
3. Low-temperature and high-density - compact stars
4. Low-temperature and low density - HIC, nuclear structure, compact stars



Numerical simulations of binary neutron star mergers (from L. Rezzolla's group at Goethe-U, Frankfurt-Main). From left to right: density, temperature, angular frequency.

Hyperons and delta-resonances in cold nuclear matter

CDF based equations of states

- Using EoS in the form of density functional: the pressure of dense zero-temperature matter is a functional of energy-density: $P(\varepsilon(r))$.
- The parameters of the functional are adjusted to the available data (astrophysics, laboratory, and ab initio calculations)
- DFT extended to baryon octet and includes hyperons and Delta-resonances
- Fast in implementation to generate quickly families of EoS

- Relativistic models of nuclear matter as DFT:
  (a) relativistic covariance, causality is fulfilled (+)
  (b) The Lorentz structure of interactions is maintained explicitly (+)
  (c) straightforward extension to the strange sector and resonances (+)
  (d) fast implementation (+)
  (e) not a QFT in the QED/QCD sense (-)
- Extended to finite-temperature and iso-entropic case
  The models are studied at S =Const. and Y_e =Const. (early stages of evolution, no significant entropy gradients in the core)
- Mapping of CDF onto the Taylor expansion of energy of nuclear matter
  A family of models is generated with varying symmetry energy, its slope, etc.

Goals:

- Construct an EoS in the form of density functional: the pressure of dense zero-temperature matter is a functional of energy-density: $P(\varepsilon(r))$
- The parameters of the functional are adjusted to the available data; in our case astrophysics and laboratory data.
- *Ab initio* calculations are data $\rightarrow$ check compatibility and adjust if required.
- DFT must be versatile enough to accommodate the baryon spin-1/2 octet and spin-3/2 decouplet.
- Fast in implementation to generate quickly families of EoS

DFT's :

- Relativistic mean-field models of nuclear matter reinterpreted as DFT:
  (a) relativistic covariance, causality is fulfilled automatically (+)
  (b) The Lorentz structure of interactions is maintained explicitly (+)
  (c) straightforward extension to the strange sector and resonances (+)
  (d) fast implementation (+)
  (e) the microscopic counterpart is unknown [not a QFT in the QED/QCD sense] (-)
  (f) uncertainties can be quantified in terms of Taylor expansion coefficients
- Non-relativistic DFTs (e.g. Skyrme or Gogny classes):
  (a) high accuracy at low-densities (+)
  (b) extensive tests on laboratory nuclei (+)
  (c) relativistic covariance is lost and high-density extrapolation is not obvious (-)
  (d) extensions to heavy baryons not straightforward (-)

Nuclear matter Lagrangian:

$$\mathcal{L}_{NM} = \underbrace{\sum_B \bar\psi_B\left[\gamma^\mu\left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2}g_{\rho BB}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\right) - (m_B - g_{\sigma BB}\sigma)\right]\psi_B}_{\text{baryons}}$$

$$+ \underbrace{\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu}_{\text{mesons}}$$

$$- \underbrace{\frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu}_{\text{mesons}} + \underbrace{\sum_\lambda \bar\psi_\lambda(i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda}_{\text{leptons}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{electromagnetism}},$$

- B-sum is over the baryonic octet
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Two types of relativistic density functionals based on relativistic Lagrangians

- linear mesonic fields, density-dependent couplings (DDME2, DD2, etc.)
- non-linear mesonic fields; coupling constants are just numbers (NL3, GM1-3, etc.)

Fixing the couplings: nucleonic sector

$$g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \qquad h_i(x) = a_i\frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2} \quad i=\sigma,\omega,$$
$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0)\exp[-a_\rho(x-1)], \quad i=\rho,(\pi-HF)$$

Meson (i)	m_i (MeV)	a_i	b_i	c_i	d_i	g_{iN}
σ	550.1238	1.3881	1.0943	1.7057	0.4421	10.5396
ω	783	1.3892	0.9240	1.4620	0.4775	13.0189
ρ	763	0.5647				7.3672

$h_i(1)=1$, $h_i''(0)=0$ and $h_\sigma''(1)=h_\omega''(1)$, which reduce the number of free parameters to three in this sector.

– DD-ME2 parametrization, G. Lalazissis, et al., Phys. Rev. **C71**, 024312 (2005)
– DD2 parametrizations, S. Typel, Eur. Phys. J. **A52**, 16 (2016)
– DD-ME2+LQ parametrizations, J. J. Li, Sedrakian, Phys. Rev. **C100**, 015809 (2019)

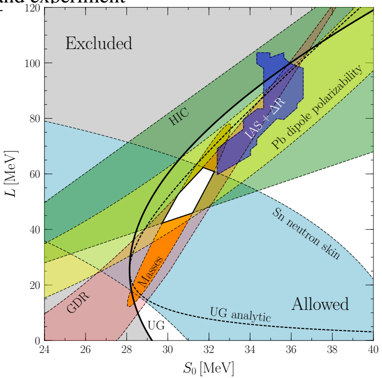
Taylor expansion of nuclear energy

$$E(\chi,\delta) \simeq E_0 + \frac{1}{2!}K_0\chi^2 + \frac{1}{3!}Q_{sym}\chi^3 + E_{sym}\delta^2 + L\delta^2\chi + \mathcal{O}(\chi^4,\chi^2\delta^2), \quad (1)$$

where $\delta = (n_n - n_p)/(n_n + n_p)$ and $\chi = (\rho - \rho_0)/3\rho_0$.

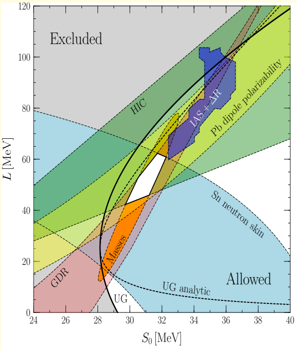
Consistency between the density functional and experiment

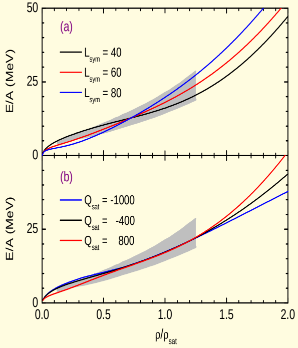
- saturation density $\rho_0 = 0.152\ \mathrm{fm}^{-3}$
- binding energy per nucleon $E/A = -16.14$ MeV,
- incompressibility $K_{sat} = 251.15$ MeV,
- skweness $Q_{sat} = 479$
- symmetry energy $E_{sym} = 32.30$ MeV,
- symmetry energy slope $L_{sym} = 51.27$ MeV,
- symmetry incompressibility $K_{sym} = -87.19$ MeV



Credit: Tews, et al ApJ, 2017

Consistency between the density functional with experiment and ab initio theory

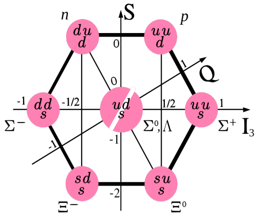


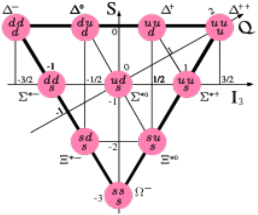


– Uncertainties will be quantified in terms of variation of higher-order characteristics around the central fit values.
– Low density physics depends strongly on the value of L_{sym} with a strong correlation to the radius of the star and tidal deformability
– High-density physics strongly depends on the value of Q_{sym} with strong correlations to the mass of the star.

Beyond nucleons: Baryon octet $J^p = 1/2^+$ and baryon decuplet $J^p = 3/2^+$

Strangeness carrying baryons + resonances (nucleon excitations)





$R_{\alpha Y} = g_{\alpha Y}/g_{\alpha N}$ and $\kappa_{\alpha Y} = f_{\alpha Y}/g_{\alpha Y}$ for hyperons in SU(6) spin-flavor model

$R \backslash Y$	Λ	Σ	Ξ
$R_{\sigma Y}$	2/3	2/3	1/3
$R_{\sigma^* Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$R_{\omega Y}$	2/3	2/3	1/3
$\kappa_{\omega Y}$	-1	$1 + 2\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$
$R_{\phi Y}$	$-\sqrt{2}/3$	$-\sqrt{2}/3$	$-2\sqrt{2}/3$
$\kappa_{\phi Y}$	$2 + 3\kappa_{\omega N}$	$-2 - \kappa_{\omega N}$	$1 + 2\kappa_{\omega N}$
$R_{\rho Y}$	0	2	1
$\kappa_{\rho Y}$	0	$-3/5 + (2/5)\kappa_{\rho N}$	$-6/5 - (1/5)\kappa_{\rho N}$
$f_{\pi Y}$	0	$2\alpha_{ps}$	$-(1/2)\alpha_{ps}$

$\alpha_{ps} = 0.40$. κ is the ratio of the tensor to vector couplings of the vector mesons.

The depth of hyperonic potentials in the symmetric nuclear matter are used as a guide the range of hyperonic couplings:

- Λ particle: $V_{\Lambda}^{(N)}(\rho_0) \simeq -30$ MeV
- Ξ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq -14$ MeV
- Σ particle: $V_{\Xi}^{(N)}(\rho_0) \simeq +30$ MeV

These ranges capture the most interesting regions of the parameter space of masses and radii.

The depth of Δ-potentials in the symmetric nuclear matter is used as a guide for the range of the couplings:

- Electron and pion scattering: $-30 \text{ MeV} + V_{\Delta}^{(N)}(\rho_0) \leq V_{\Delta}(\rho_0) \leq V_N(\rho_0)$
- Use instead $R_{m\Delta} = g_{m\Delta}/g_{mN}$ for which the the typical range used is

$$R_{\rho\Delta} = 1, \quad 0.8 \leq R_{\omega\Delta} \leq 1.6, \quad R_{\sigma\Delta} = R_{\omega\Delta} \pm 0.2.$$

Hyperons and delta-resonances and in proto-neutron stars and merger remnants

The equation of state (EoS) and composition of dense and hot Δ-resonance admixed hypernuclear matter is studied under conditions that are characteristic of neutron star binary merger remnants and supernovas.

- Baryon and lepton charges:

$$Y_Q = n_Q/n_B, \quad Y_{e,\mu} = (n_{e,\mu} - n_{e^+,\mu^+})/n_B$$
$$n_Q = n_p + n_{\Sigma^+} + 2n_{\Delta^{++}} + n_{\Delta^+} - (n_{\Sigma^-} + n_{\Xi^-} + n_{\Delta^-}).$$

- Trapped regime - fixed lepton numbers

$$Y_{L,e} = Y_e + Y_{\nu_e} \quad Y_{L,\mu} = Y_\mu + Y_{\nu_\mu},$$
$$\text{BNS} : Y_{L,e} = Y_{L,\mu} = 0.1 \quad \text{Supernova} : Y_{L,e} = 0.4 \quad Y_{L,\mu} = 0.$$

- Transparent regime (neutrino chemical potentials vanish) - equilibrium with respect to the weak processes imply

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_{\Delta^0} = \mu_n = \mu_B, \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_{\Delta^-} = \mu_B - \mu_Q,$$
$$\mu_{\Sigma^+} = \mu_{\Delta^+} = \mu_B + \mu_Q, \quad \mu_{\Delta^{++}} = \mu_B + 2\mu_Q,$$

where the baryon μ_B and charge $\mu_Q = \mu_p - \mu_n$ chemical potentials are associated with conservations of these quantities.

- Thus the conditions are

$$\mu_e = \mu_\mu = -\mu_Q = \mu_n - \mu_p, \quad \text{(free streaming)}$$
$$\mu_e = \mu_{L,e} - \mu_Q, \quad \mu_\mu = \mu_{L,\mu} - \mu_Q. \quad \text{(trapped)}$$

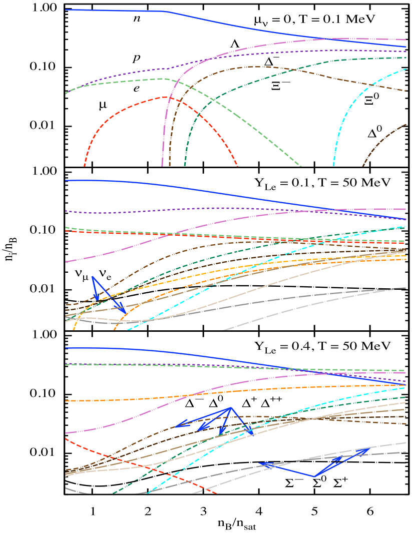
- BNS mergers, the initial conditions correspond to two cold neutron stars,

$$Y_{L,e} = Y_{L,\mu} = 0.1,$$

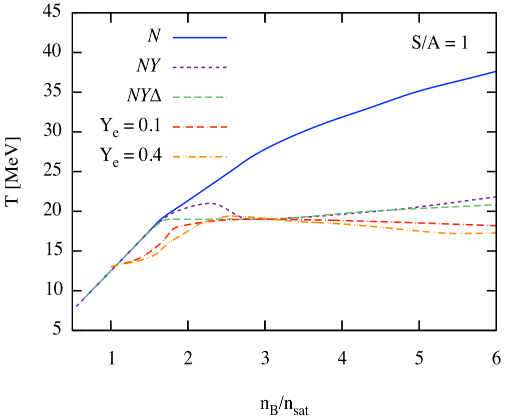
- For supernova matter the predicted electron and μ-on lepton numbers are typically

$$Y_{L,e} = 0.4, \quad Y_{L,\mu} = 0.$$

Dependence of composition on baryon density for fixed T.

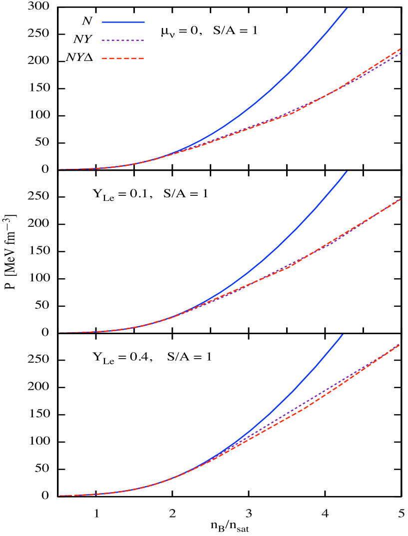


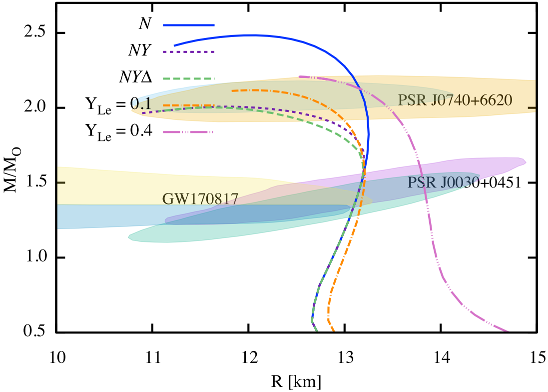
Dependence of temperature on density for fixed $S/A = 1$.



No significant changes in the composition compared to fixed T.

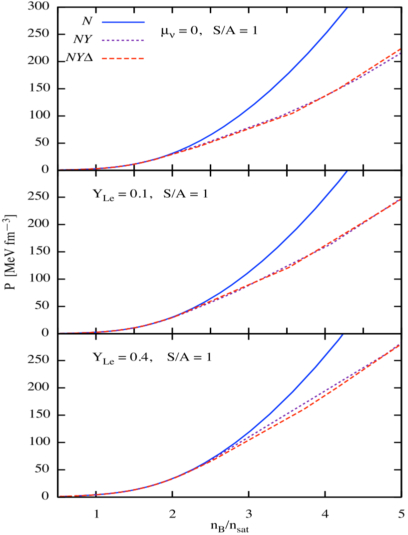
Dependence of pressure on baryon density for $S/A = 1$.





Gravitational mass versus radius for non-rotating spherically-symmetric stars. Three sequences are shown for β-equilibrated, neutrino-transparent stars with nucleonic (*N*), hypernuclear (*NY*) and Δ-admixed hypernuclear ($NY\Delta$) composition for $T = 0.1$ MeV. In addition, we show sequences of fixed $S/A = 1$ neutrino-trapped, isentropic stars composed of $NY\Delta$ matter in two cases of constant lepton fractions $Y_{Le} = Y_{L\mu} = 0.1$ and $Y_{Le} = 0.4,\ Y_{L\mu} = 0$. The ellipses show 90% CI regions for PSR J0030+0451, PSR J0740+6620 and gravitational wave event GW170817.

Dependence of pressure on baryon density for $S/A = 1$.



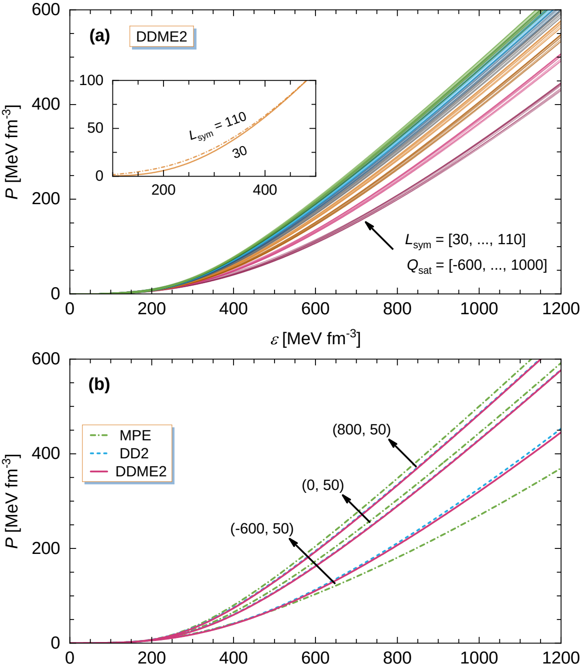
Generated a large number of EoS based on DDME2, DD2 and MPE functionals ($9 \times 9 = 81$ for each)

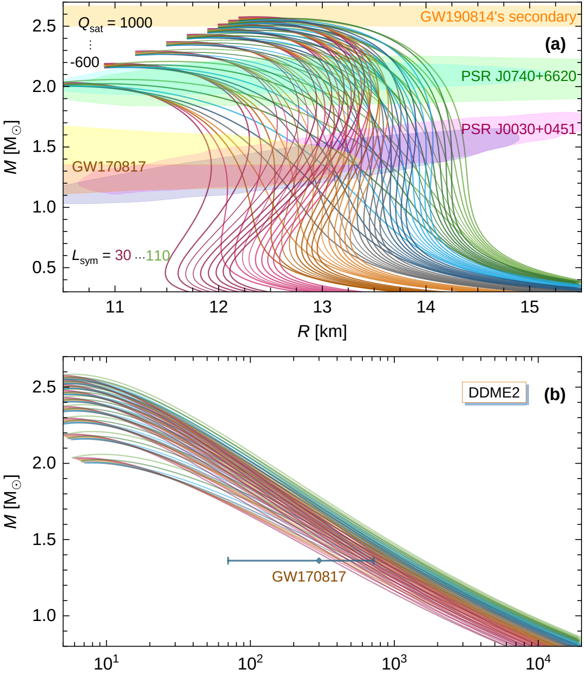
- skweness $-600 \leq Q \leq 1000$ MeV
- symmetry energy slope $30 \leq L_{\rm sym} \leq 110$ MeV

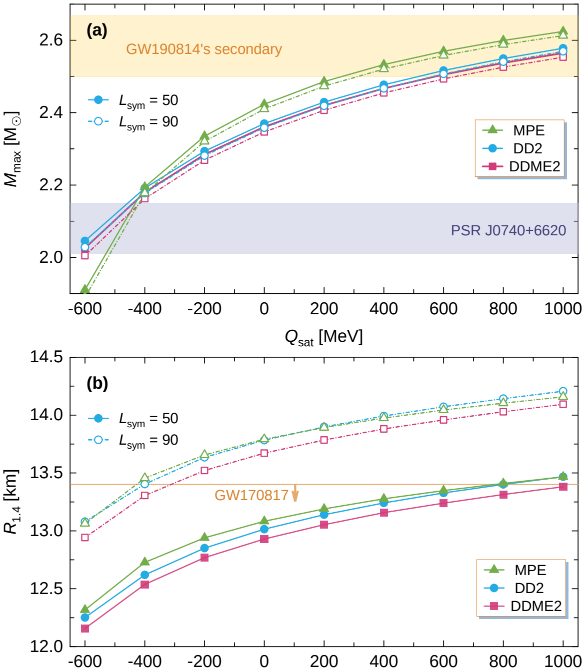
Cold nuclear matter equation of state

$$E(\chi,\delta) \simeq E_0 + \frac{1}{2!}K_0\chi^2 + \frac{1}{3!}Q_{\rm sym}\chi^3 + E_{\rm sym}\delta^2 + L\delta^2\chi + \mathcal{O}(\chi^4,\chi^2\delta^2), \qquad (2)$$

where $\delta = (n_n - n_p)/(n_n + n_p)$ and $\chi = (\rho - \rho_0)/3\rho_0$.


(a)
DDME2
L_{sym} = 110
30
L_{sym} = [30, ..., 110]
Q_{sat} = [-600, ..., 1000]
P [MeV fm^{-3}]
ε [MeV fm^{-3}]
(b)
MPE
DD2
DDME2
(800, 50)
(0, 50)
(-600, 50)
P [MeV fm^{-3}]


Q_{sat} = 1000
⋮
-600
GW190814's secondary
(a)
PSR J0740+6620
PSR J0030+0451
GW170817
L_{sym} = 30 ...110
M [M$_\odot$]
R [km]
DDME2
(b)
GW170817

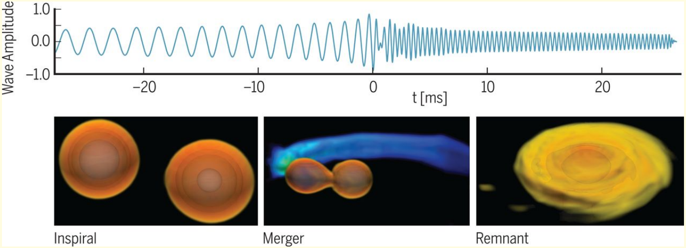

(a)
GW190814's secondary
L_{sym} = 50
L_{sym} = 90
MPE
DD2
DDME2
PSR J0740+6620
M_{max} [$M_\odot$]
2.0
2.2
2.4
2.6
-600
-400
-200
0
200
400
600
800
1000
Q_{sat} [MeV]
(b)
L_{sym} = 50
L_{sym} = 90
GW170817
MPE
DD2
DDME2
$R_{1.4}$ [km]
12.0
12.5
13.0
13.5
14.0
14.5

Physics output:

- Large number of stellar models for injection studies of the Einstein Telescope (mass, radius, tidal deformabilities, variation of characteristics L and Q of the EoS).
- 3D tables for numerical simulations (in progress)
- More on properties of hot compact stars: rotation, universal relation, arXiv:2306.14190, arXiv:2102.00988, arXiv:2008.00213
- At a more fundamental level - improved DFs and, in particular, CDFs...
- 2D EoS tables can be downloaded from `https://github.com/asedrakian/DD_CDFs/` repository.

Dissipation and bulk viscosity

Sketch of gravitational wave amplitude of BNS merger



- the gravitational wave signal carries information about the equation of state and eventually composition of hot and dense matter
- current modeling of the emitted gravitational wave is based on numerical relativity which uses ideal (non-dissipative) hydrodynamics
- Our motivation is *assessment of the effects of dissipation and effects of the equation of state and composition on these processes*

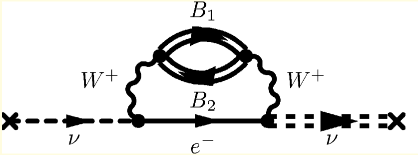
Urca rates including muons and leptonic processes

Urca reactions included (μ-ons as a new factor)

$$
\begin{aligned}
n &\rightleftarrows p + e^- + \bar{\nu}_e \quad &(\text{neutron e} - \text{decay}),\\
p + e^- &\rightleftarrows n + \nu_e \quad &(\text{electron capture}),\\
n &\rightleftarrows p + \mu^- + \bar{\nu}_\mu \quad &(\text{neutron } \mu - \text{decay}),\\
p + \mu^- &\rightleftarrows n + \nu_\mu \quad &(\text{muon capture}).
\end{aligned}
$$

Leptonic reactions:

$$
\begin{aligned}
\mu &\rightleftarrows e^- + \bar{\nu}_e + \nu_\mu \quad &(\text{muon decay}),\\
\mu + \nu_e &\rightleftarrows e^- + \nu_\mu \quad &(\text{neutrino scattering}),\\
\mu + \bar{\nu}_\mu &\rightleftarrows e^- + \bar{\nu}_e \quad &(\text{antineutrino scattering}).
\end{aligned}
$$



Density oscillations in neutron-star matter

Consider now small-amplitude density oscillations in baryonic matter with frequency ω

$$n_j(t) = n_{j0} + \delta n_j(t), \quad \delta n_j(t) = \delta n_j^{\rm eq}(t) + \delta n_j'(t), \quad j = \{n, p, e, \nu\},$$

The oscillations cause perturbations in particle densities due to which the chemical equilibrium of matter is disturbed leading to a small shift which can be written as

$$\mu_\Delta(t) = A_n \delta n_n(t) + A_\nu \delta n_\nu(t) - A_p \delta n_p(t) - A_e \delta n_e(t), \qquad A_{ij} = \frac{\partial \mu_i}{\partial n_j}.$$

Out of equilibrium the chemical equilibration rate to linear order in $\mu_\Delta(t)$ is given by

$$\Gamma_\Delta \equiv \Gamma_p - \Gamma_n = \lambda \mu_\Delta, \quad \lambda > 0,$$

Definition of bulk viscosity

The rate equations which take into account the loss and gain of particles read as

$$\frac{\partial}{\partial t}\delta n_n(t) = -\theta n_{n0} - \lambda \mu_\Delta(t), \quad etc.$$

The non-equilibrium density perturbations can be found according

$$\delta n'_p = \delta n'_e = -\delta n'_n = -\delta n'_\nu = \frac{C}{A(i\omega + \gamma)}\theta,$$

$$C = n_{n0}A_n + n_{\nu 0}A_\nu - n_{p0}A_p - n_{e0}A_e = n_B\left(\frac{\partial \mu_\Delta}{\partial n_B}\right)_{Y_n}$$

The non-equilibrium part of the pressure:

$$\Pi = \sum_j \frac{\partial p}{\partial n_j}\delta n'_j = \sum_{lj} n_{l0}A_{lj}\delta n'_j = -\zeta\theta \qquad \boxed{\zeta = \frac{C^2}{A}\frac{\gamma}{\omega^2+\gamma^2}}$$

$$\zeta_{\rm max} = \frac{C^2}{2A} \qquad \zeta_{\rm slow} = \frac{C^2}{A}\frac{\gamma}{\omega^2} \qquad \zeta_{\rm fast} = \frac{C^2}{A\gamma}$$

Damping time-scale

The energy dissipation rate by the bulk viscosity per unit volume is

$$\frac{d\epsilon}{dt} = \frac{\omega^2\zeta}{2}\left(\frac{\delta n_B}{n_B}\right)^2.$$

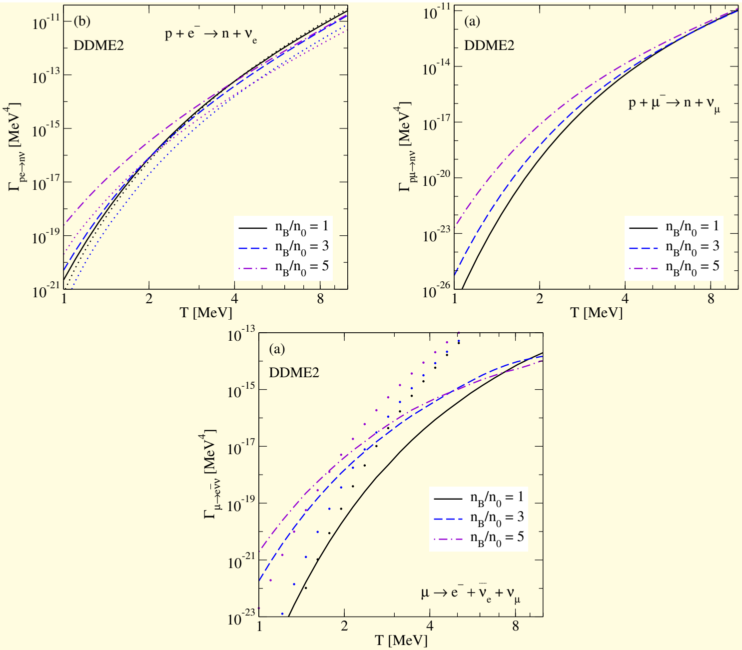
The characteristic timescale required for damping of oscillations

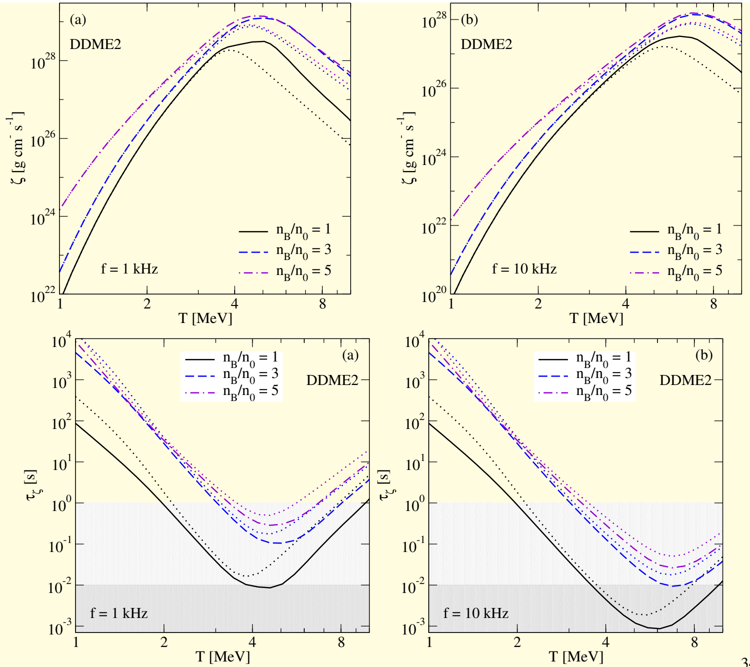
$$\tau_\zeta = \epsilon\left(\frac{d\epsilon}{dt}\right)^{-1} = \frac{1}{9}\frac{Kn_B}{\omega^2\zeta}.$$

The minimal/maximal value of the damping timescale is

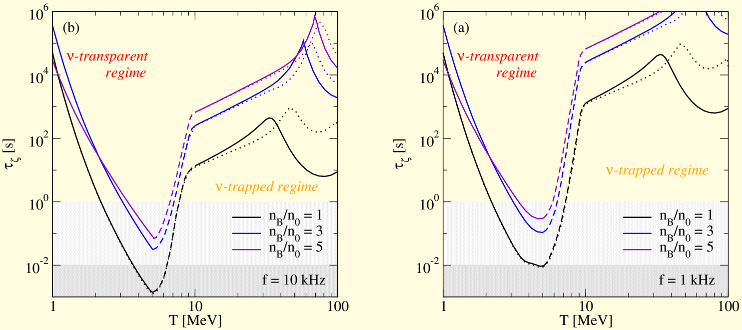
$$\tau_\zeta^{\rm min} = \frac{2}{9\omega}\frac{Kn_B}{C^2/A}. \qquad \tau_\zeta^{\rm slow} = \frac{1}{9\gamma}\frac{Kn_B}{C^2/A}, \qquad \tau_\zeta^{\rm fast} = \frac{\gamma}{9\omega^2}\frac{Kn_B}{C^2/A}.$$

In the limits of slow and fast equilibration the damping timescale is given by


(b)
DDME2
p + e⁻ → n + ν_e
Γ_pe→nν [MeV⁴]
T [MeV]
n_B/n_0 = 1
n_B/n_0 = 3
n_B/n_0 = 5
(a)
DDME2
p + μ⁻ → n + ν_μ
Γ_pμ→nν [MeV⁴]
T [MeV]
n_B/n_0 = 1
n_B/n_0 = 3
n_B/n_0 = 5
(a)
DDME2
μ → e⁻ + ν̄_e + ν_μ
Γ_μ→eνν̄ [MeV⁴]
T [MeV]
n_B/n_0 = 1
n_B/n_0 = 3
n_B/n_0 = 5


(a)
DDME2
ζ [g cm^{-} s^{-1}]
10^{28}
10^{26}
10^{24}
10^{22}
f = 1 kHz
n_B/n_0 = 1
n_B/n_0 = 3
n_B/n_0 = 5
1
2
4
8
T [MeV]
(b)
DDME2
10^{28}
10^{26}
10^{24}
10^{22}
10^{20}
f = 10 kHz
τ_ζ [s]
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}

Combined neutrino transparent and trapped regimes:



Physics output:

- Combined EoS and viscosity tables for input in numerical simulations (in progress)
- Including other compositions – heavy baryons and quark matter
- More fundamental level – many-body effects on neutrinos and in nucleonic matter....