

# Pro and con quark matter in Neutron stars

David Blaschke (HZDR/CASUS & IFT UWr)



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**Historic Introduction to Pro and Con**

**Terra Incognita: Agnostic Bayesian Analysis vs. Interpolation**

**Berlin Wall constraint**

**Confining Density Functionals for Quark Matter**

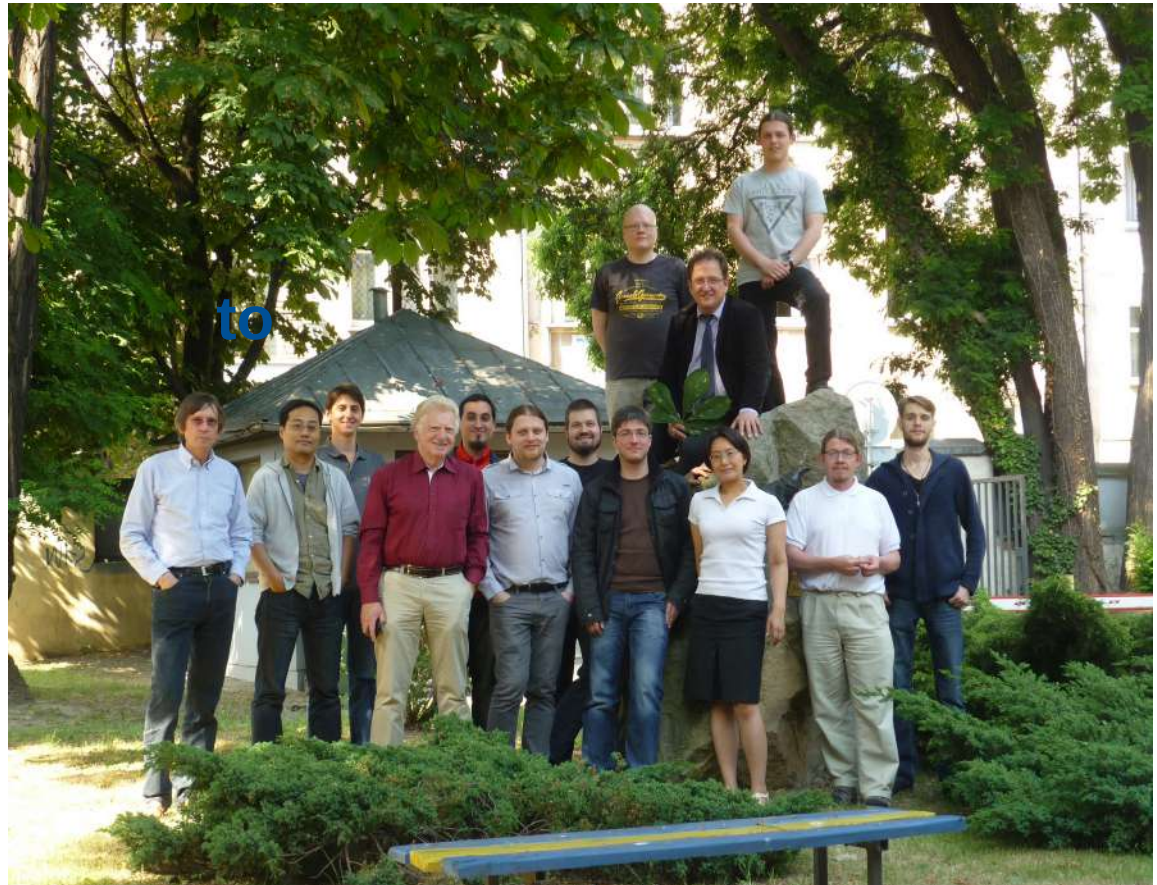
**Special Points**

**Twins and Eccentric Binaries: Discover the 3<sup>rd</sup> Family !**

**Outlook: German Centre for Astrophysics (DZA)**

# Thanks to my collaborators:

T. Fischer, G. Röpke, A. Bauswein, O. Ivanytskyi, N. Bastian,  
M. Cierniak, U. Shukla, S. Liebing, K. Maslov, A. Ayriyan,  
H. Grigorian,  
D.N. Voskresensky,  
M. Kaltenborn,  
G. Grunfeld,  
D. Alvarez-Castillo,  
B. Dönigus, D. Ohse,  
S. Chanlaridis,  
J. Antoniadis ...



**Wroclaw Group ...**



# Pro and con quark matter in neutron stars

## The early days: 1973 – asymptotic freedom of QCD

VOLUME 34, NUMBER 21

PHYSICAL REVIEW LETTERS

26 MAY 1975

### Superdense Matter: Neutrons or Asymptotically Free Quarks?

J. C. Collins and M. J. Perry

*Department of Applied Mathematics and Theoretical Physics, University of Cambridge,  
Cambridge CB3 9EW, England*

(Received 6 January 1975)

We note the following: The quark model implies that superdense matter (found in neutron-star cores, exploding black holes, and the early big-bang universe) consists of quarks rather than of hadrons. Bjorken scaling implies that the quarks interact weakly. An asymptotically free gauge theory allows realistic calculations taking full account of strong interactions.

PRO

$$B/V = \frac{1}{3}N/V = \frac{1}{18}d \sum_i p_{Fi}^3 / \pi^2, \quad (8)$$

$$P = \frac{1}{24}d \sum_i p_{Fi}^4 / \pi^2, \quad (9)$$

$$\rho = E/V = \frac{1}{6}d \sum_i p_{Fi}^4 / \pi^2, \quad (10)$$



$$E/N = BV/N + D(N/V)^{1/3},$$

$$\text{with } D \equiv \frac{3}{4}\pi^2(1 + g_c^2/6\pi^2) \sum_i f_i^{4/3}.$$

# Pro and con quark matter in neutron stars

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Volume 62B, number 2

PHYSICS LETTERS

24 May 1976

### CAN A NEUTRON STAR BE A GIANT MIT BAG?\*

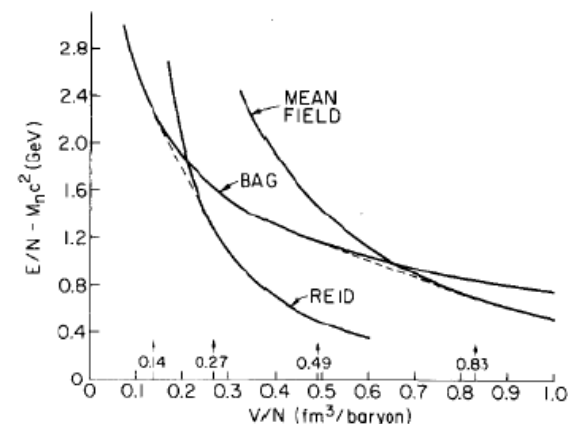
G. BAYM and S.A. CHIN

*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*

Received 30 March 1976

We show, on the basis of the M.I.T. bag model of hadrons, that a neutron matter-quark matter phase transition is energetically favorable at densities around ten to twenty times nuclear matter density. It is unlikely, however, that quark matter can be found within stable neutron stars, or that it may form a third family of dense stellar objects.

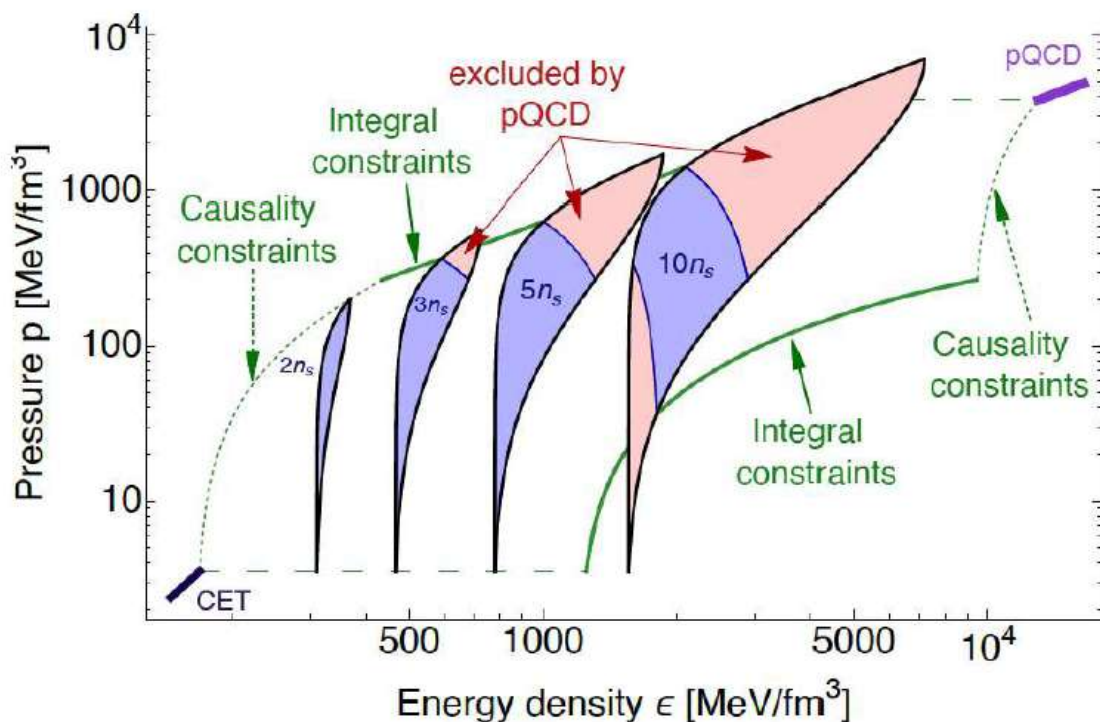
**Con**



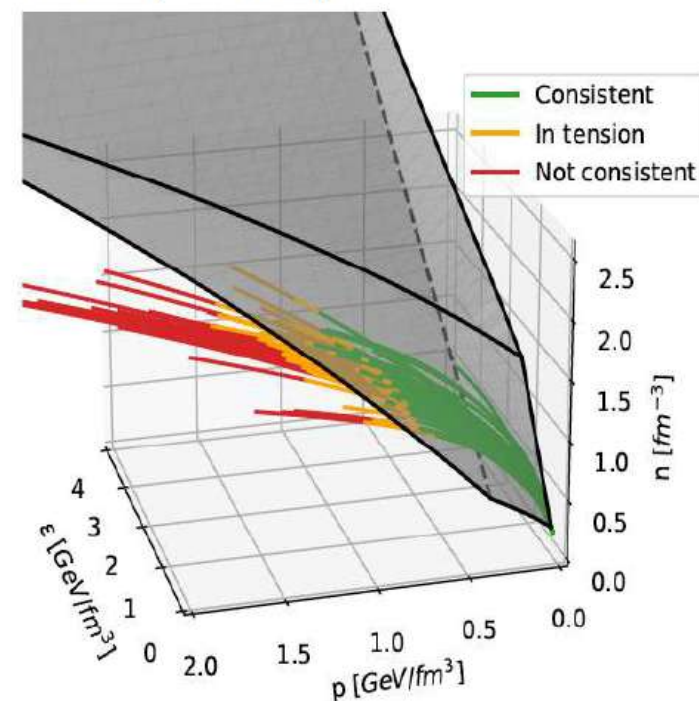
# Pro and con quark matter in neutron stars

Nowadays:

## Neutron star EoS constraint from pQCD



Consistency check for neutron star EoS from the CompOSE library



**Result:** Not all EoS fulfill the consistency check with pQCD asymptotics! pQCD important for NS!

O. Komoltsev and A. Kurkela, Phys. Rev. D 128 (2022) 202701

# QCD Phase Diagram

## Landscape of our investigations

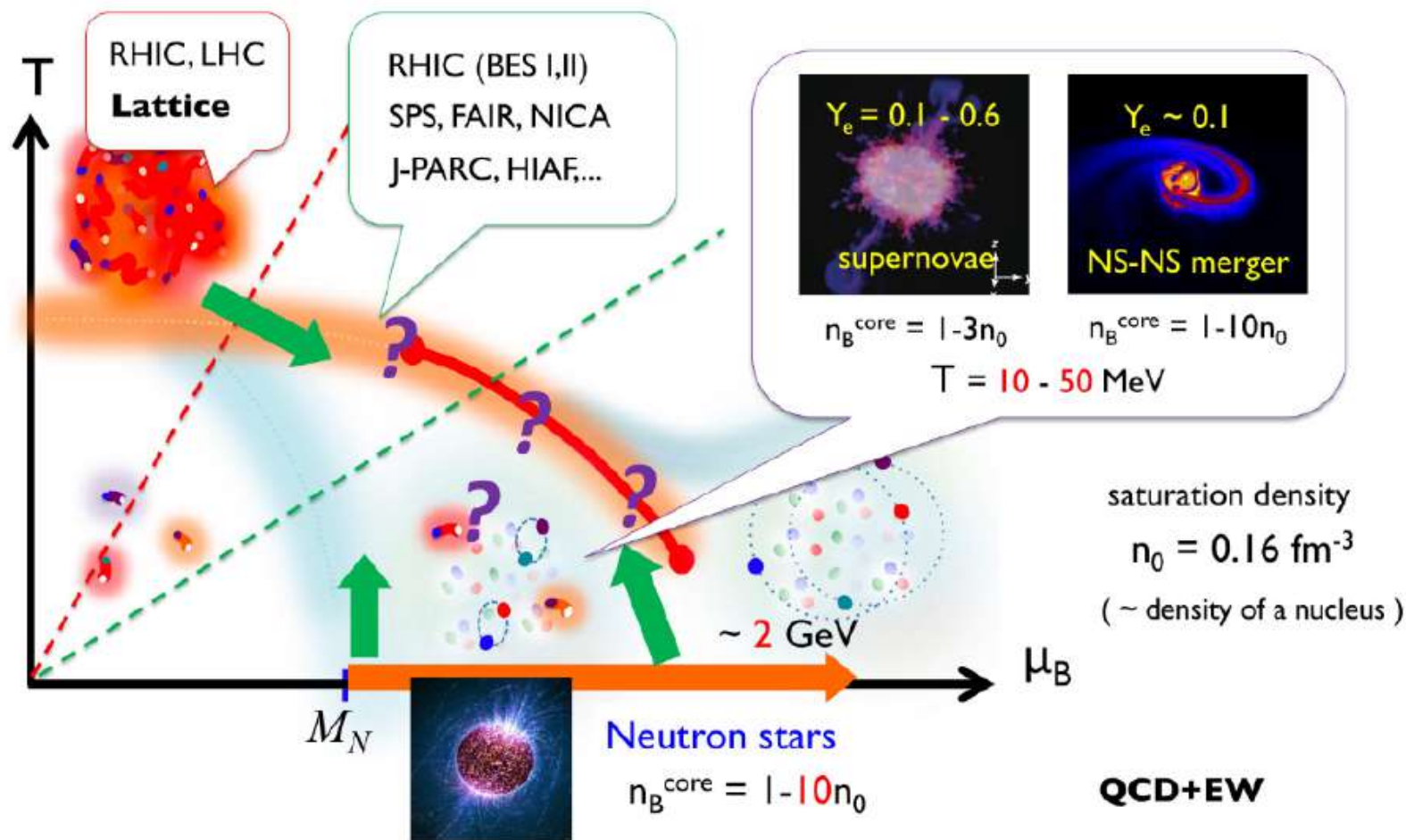
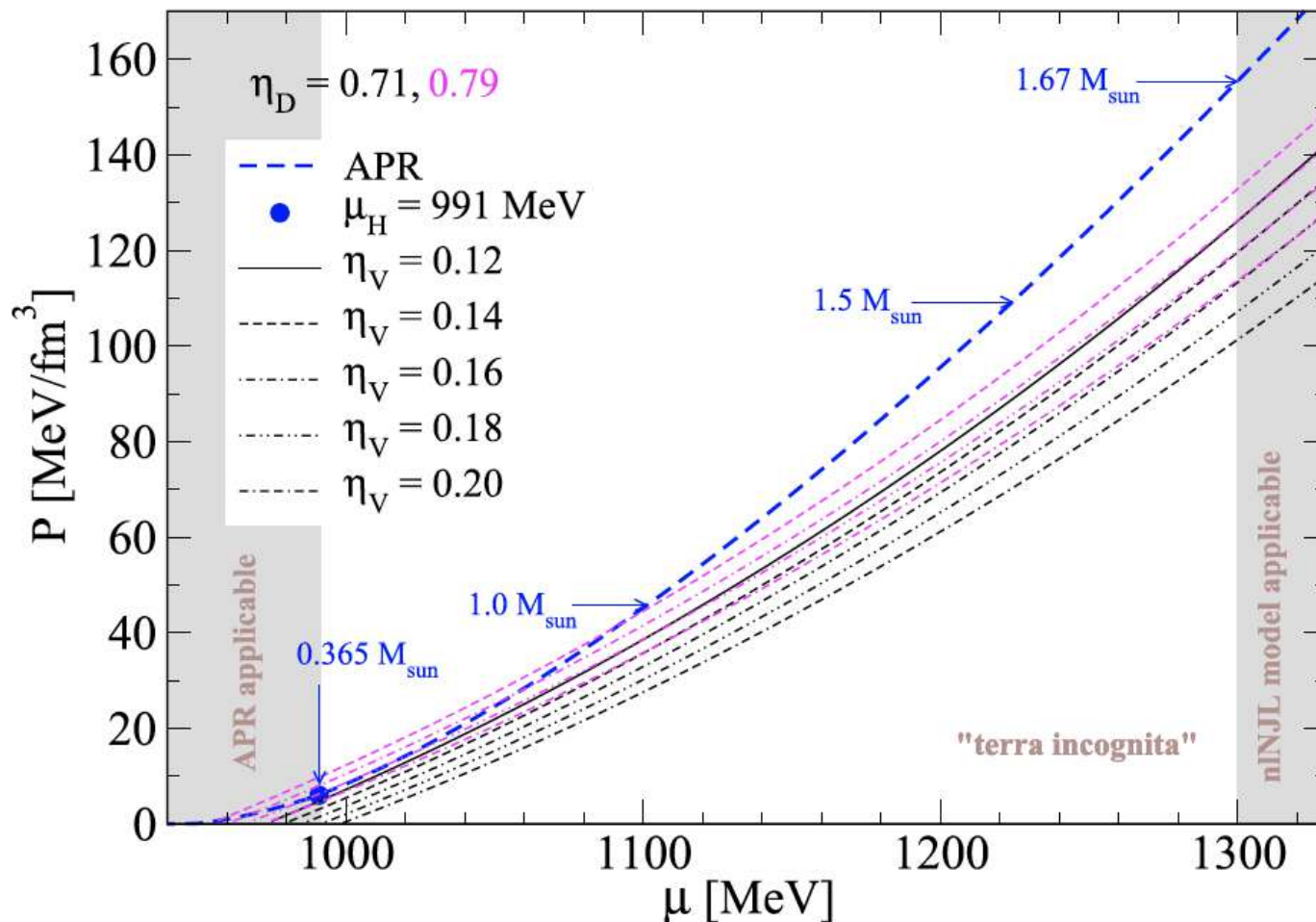


Figure from T. Kojo arXiv:1912.05326 [nucl-th]



# Pro and con quark matter in neutron stars

## Where is deconfinement in “terra incognita” ?



A. Ayriyan, D.B., A.G. Grunfeld, et al.

Eur. Phys. J. A (2021) 57:318

<https://doi.org/10.1140/epja/s10050-021-00619-0>



# Neutron star EOS from multimessenger data

Bayesian analysis, nontrivial  $c_s(p)$ , Gaussian process

D. Mroczek, M.C. Miller, et al., Nontrivial features in  $c_s$  inside neutron stars, arXiv:2309.02345

**The method** of (modified) Gaussian process based on  $c_s(p)$

Auxiliary function:

$$\phi \equiv \ln(d\varepsilon/dp - 1) = \ln(1/c_s^2 - 1)$$

Gaussian process:

$$\phi(\log_{10} p_i) = \mathcal{N}[\mu_i(\log_{10} p_i), \Sigma_{ij}]$$

Quadrature:

$$\varepsilon = \int \frac{dp}{c_s^2(p)} \rightarrow p(\varepsilon)$$

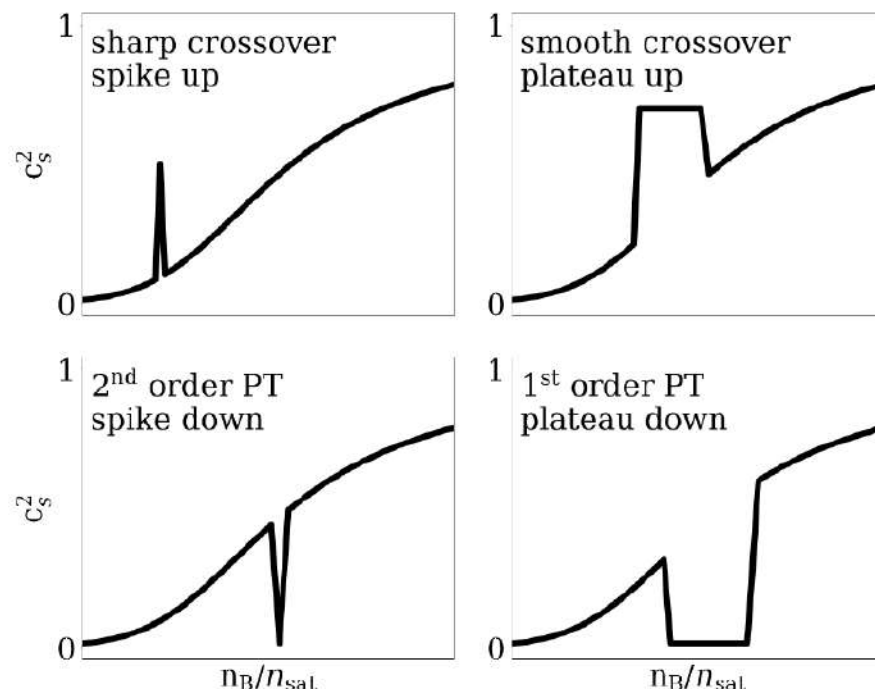
First law of thermodynamics

$$\frac{dn_B}{d\varepsilon} = \frac{n_B}{\varepsilon + p(\varepsilon)}$$

**Result:** EOS sample from GP as a set

$$\{\phi(p), c_s^2(n_B), p(\varepsilon)\}$$

Cartoon of physical features in  $c_s^2(n_B)$



# Neutron star EOS from multimessenger data

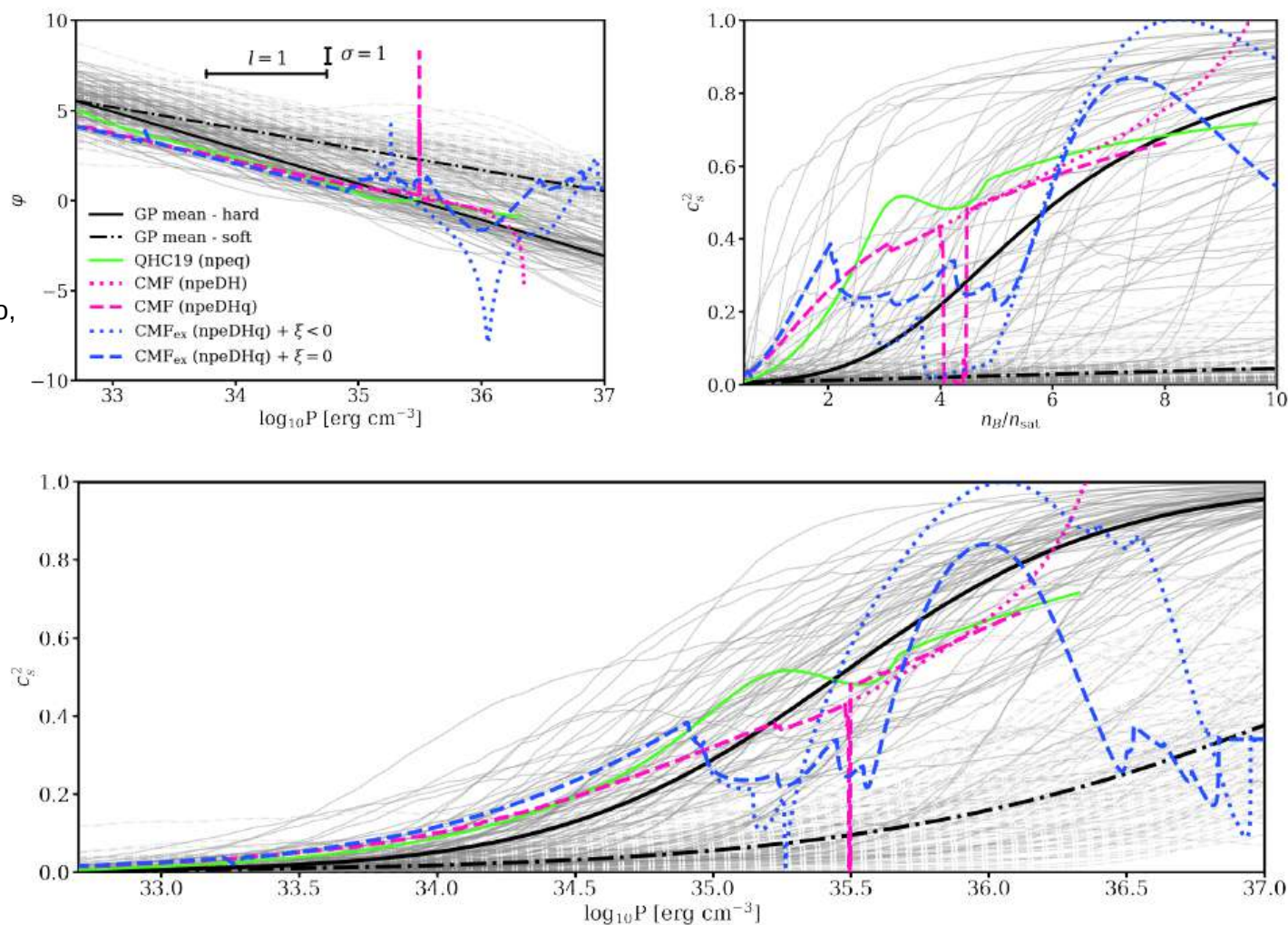
## Bayesian analysis, nontrivial $c_s(p)$ , Gaussian process

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Grey:  
160 forms of  $c_s^2$  from GP

CMF:  
chiral mean field EOS

QHC19:  
Baym, Furusawa, Hatsuda, Kojo,  
Togashi, ApJ 885, 42 (2019)



# Neutron star EOS from multimessenger data

## Bayesian analysis, nontrivial $c_s(p)$ , Gaussian process

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TABLE I. Connection between phase transitions of different orders/crossover to corresponding physical processes in terms of the effect on the speed of sound *in equilibrium* and modifications in the mGP framework. Note that a first-order phase transition has a jump in baryon density across  $\Delta n_B$ .

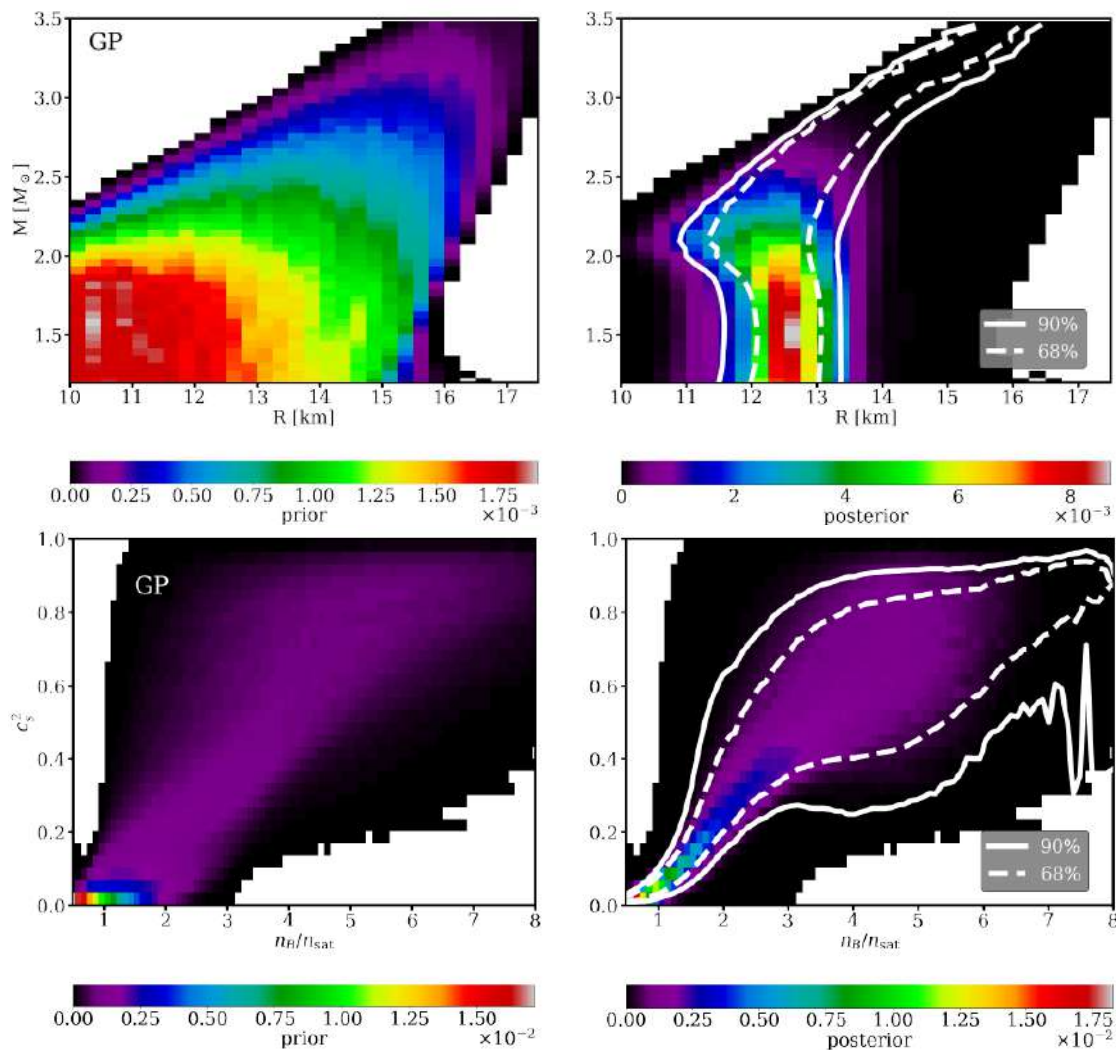
Transition type	Physical Process	Representation in $c_s^2$	Modification
sharp crossover	quarkyonic matter [23, 41–43, 118], percolation to quark matter [38, 44], quark-meson coupling [34, 119] heavy resonances [92–98], hyperons [46, 92], chiral-superfluid transition [120]	for $\delta \ll 1$ , if $n_B = \tilde{n}_B \pm \delta$ , then $(c_s^2)' = \pm \delta^{-1}$	spike up, $c_s^2 \neq 0$
smooth crossover		if $n_B = \tilde{n}_B$ , then $(c_s^2)' = 0$	
$n^{\text{th}}$ -order PT, $n > 2$		for $\delta > 0$ , if $n_B = \tilde{n}_B \pm \delta$ , then $(c_s^2)' = \pm \delta^{-1}$ if $n_B \sim \tilde{n}_B$ , then $(c_s^2)' \sim 0$	plateau up, $c_s^2 = 0$
$2^{\text{nd}}$ order PT	critical point due to exotic quark phases	if $n_B = n_{\text{crit}}$ , then $d^n p / d\mu_B^n \rightarrow \infty$	spike or plateau down, $c_s^2 \neq 0$
$1^{\text{st}}$ order PT	quark deconfinement [92, 94], color-superconductivity, colorflavor-locking [25]	$c_s^2(n_B) = 0$ with $n_B \in [n_B^*, n_B^* + \Delta n_B]$	plateau down at $c_s^2 \approx 0$



# Neutron star EOS from multimessenger data

## Bayesian analysis, nontrivial $c_s(p)$ , Gaussian process

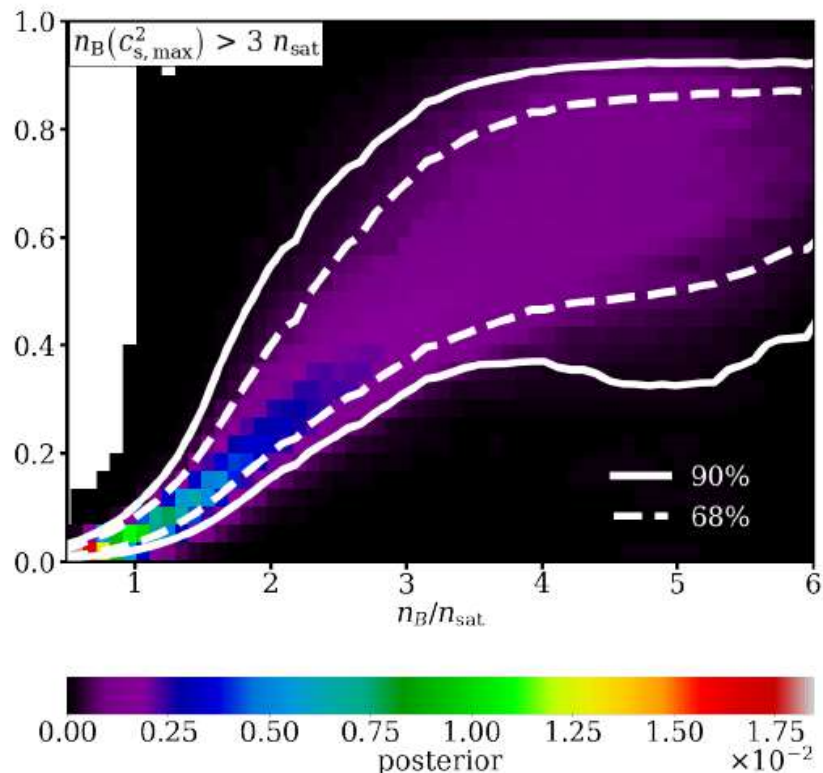
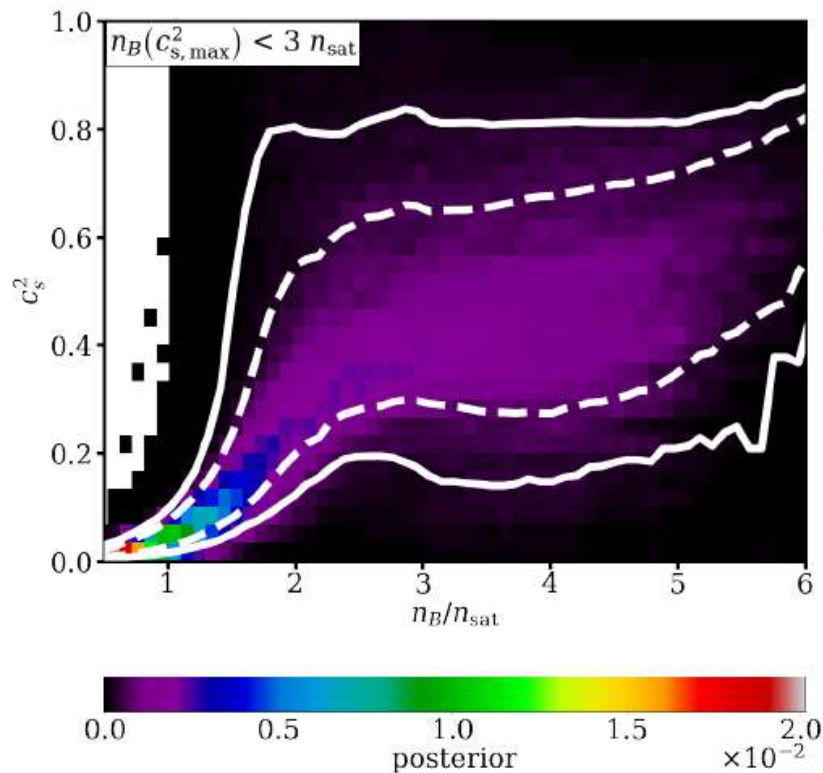
D. Mroczek, M.C. Miller, et al., Nontrivial features in  $c_s$  inside neutron stars, arXiv:2309.02345



# Neutron star EOS from multimessenger data

## Bayesian analysis, nontrivial $c_s(p)$ , Gaussian process

D. Mroczek, M.C. Miller, et al., Nontrivial features in  $c_s$  inside neutron stars, arXiv:2309.02345



- EOS posterior probabilities when a global maximum in  $c_s^2$  is present below (left) and above (right)  $3 n_{\text{sat}}$
- Posterior probability that central density for max. massive star is greater than  $6 n_{\text{sat}}$  is negligible
- Global maximum (indicating softening, perhaps PT) is consistent with, but not required by, current constraints

# Neutron star EOS from multimessenger data

## Bayesian analysis based on $c_s(p)$

L. Brandes, W. Weise, N. Kaiser,  
 Evidence **against** strong 1<sup>st</sup> order PT in NS cores,  
 arXiv:2306.06218

Data and constraints		
Previous	PSR J1614-2230	$M = 1.908 \pm 0.016 M_\odot$ [3]
	PSR J0348+0432	$M = 2.01 \pm 0.04 M_\odot$ [4]
	PSR J0030+0451	$M = 1.34^{+0.15}_{-0.16} M_\odot$
		$R = 12.71^{+1.14}_{-1.19}$ km [7]
	PSR J0740+6620	$M = 2.072^{+0.067}_{-0.066} M_\odot$
		$R = 12.39^{+1.30}_{-0.98}$ km [8]
	GW170817	$\tilde{\Lambda} = 320^{+420}_{-230}$ [12]
	GW190425	$\tilde{\Lambda} \leq 600$ [13]
ChEFT	[34, 35]	
pQCD	[36-38]	
BW	PSR J0952-060	$M = 2.35 \pm 0.17 M_\odot$ [39]
HESS	HESS J1731-347	$M = 0.77^{+0.20}_{-0.17} M_\odot$
		$R = 10.4^{+0.86}_{-0.78}$ km [40]

		Previous		Previous + BW	
		95%	68 %	95%	68 %
1.4 $M_\odot$	$n_c/n_0$	$2.8^{+0.8}_{-0.7}$	$\pm 0.4$	$2.6 \pm 0.7$	$^{+0.3}_{-0.4}$
	$\varepsilon_c$ [MeV fm <sup>-3</sup> ]	$451^{+133}_{-123}$	$^{+62}_{-71}$	$423^{+118}_{-116}$	$^{+56}_{-67}$
	$P_c$ [MeV fm <sup>-3</sup> ]	$64^{+30}_{-23}$	$^{+12}_{-16}$	$60^{+28}_{-20}$	$^{+11}_{-14}$
	$R$ [km]	$12.2^{+0.9}_{-1.0}$	$\pm 0.5$	$12.3^{+0.8}_{-1.0}$	$\pm 0.5$
	$\Lambda$	$396^{+226}_{-197}$	$^{+107}_{-127}$	$421^{+236}_{-200}$	$^{+114}_{-124}$
2.1 $M_\odot$	$n_c/n_0$	$4.1^{+1.9}_{-1.5}$	$^{+0.8}_{-0.9}$	$3.6^{+1.6}_{-1.3}$	$\pm 0.7$
	$\varepsilon_c$ [MeV fm <sup>-3</sup> ]	$716^{+416}_{-326}$	$^{+162}_{-213}$	$628^{+357}_{-251}$	$^{+149}_{-146}$
	$P_c$ [MeV fm <sup>-3</sup> ]	$225^{+239}_{-134}$	$^{+62}_{-110}$	$186^{+184}_{-104}$	$^{+52}_{-80}$
	$R$ [km]	$11.9 \pm 1.3$	$\pm 0.7$	$12.1^{+1.3}_{-1.2}$	$^{+0.6}_{-0.8}$
	$\Lambda$	$21^{+30}_{-15}$	$^{+9}_{-13}$	$26^{+30}_{-20}$	$^{+10}_{-14}$

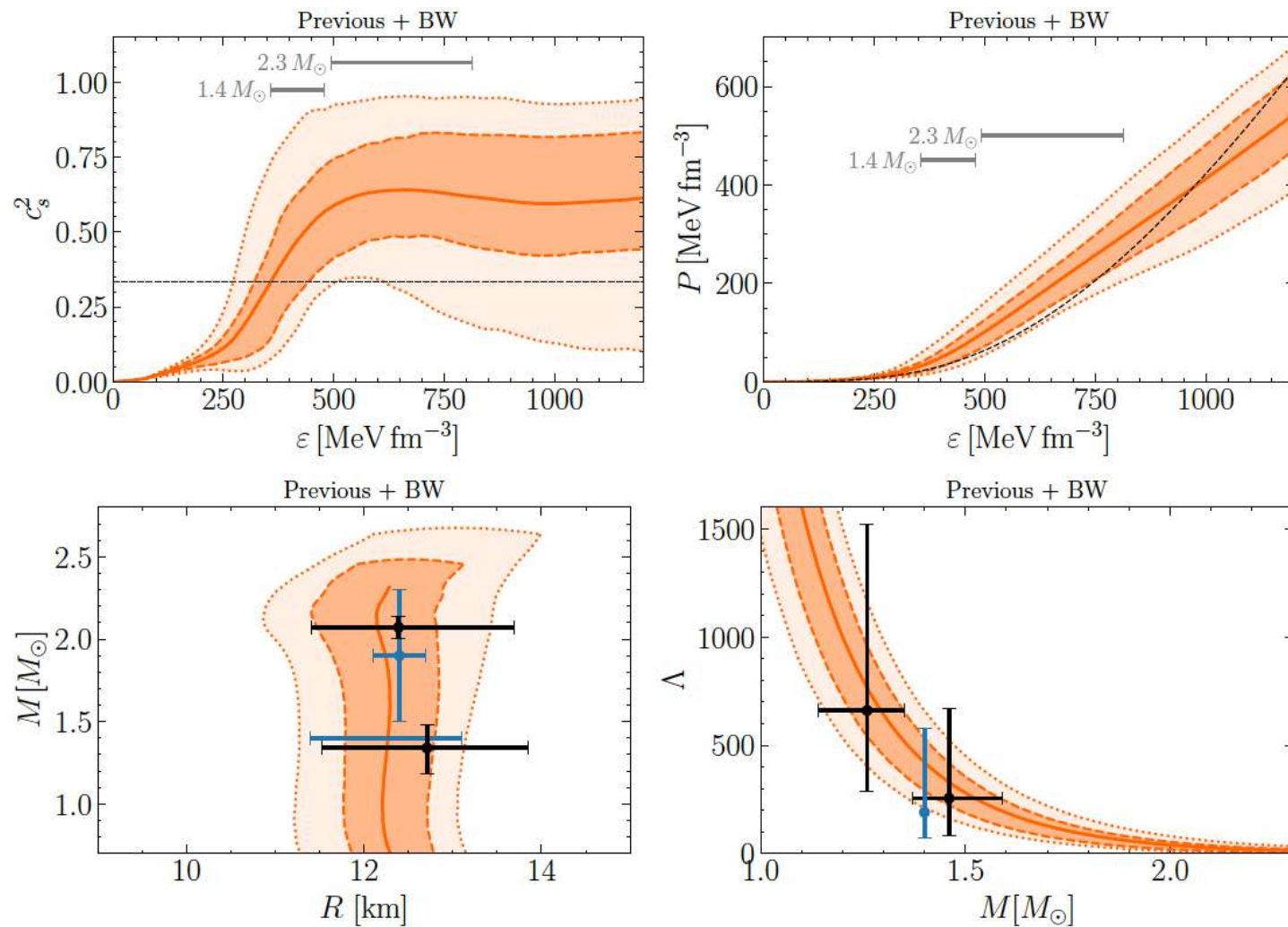
		Previous + BW	
		95%	68 %
2.3 $M_\odot$	$n_c/n_0$	$3.8^{+1.6}_{-1.3}$	$^{+0.7}_{-0.8}$
	$\varepsilon_c$ [MeV fm <sup>-3</sup> ]	$673^{+363}_{-268}$	$^{+140}_{-180}$
	$P_c$ [MeV fm <sup>-3</sup> ]	$237^{+226}_{-134}$	$^{+69}_{-104}$
	$R$ [km]	$12.3 \pm 1.2$	$^{+0.7}_{-0.6}$
	$\Lambda$	$14^{+17}_{-10}$	$^{+4}_{-9}$



# Neutron star EOS from multimessenger data

## Bayesian analysis, nontrivial $c_s(p)$ , Gaussian process

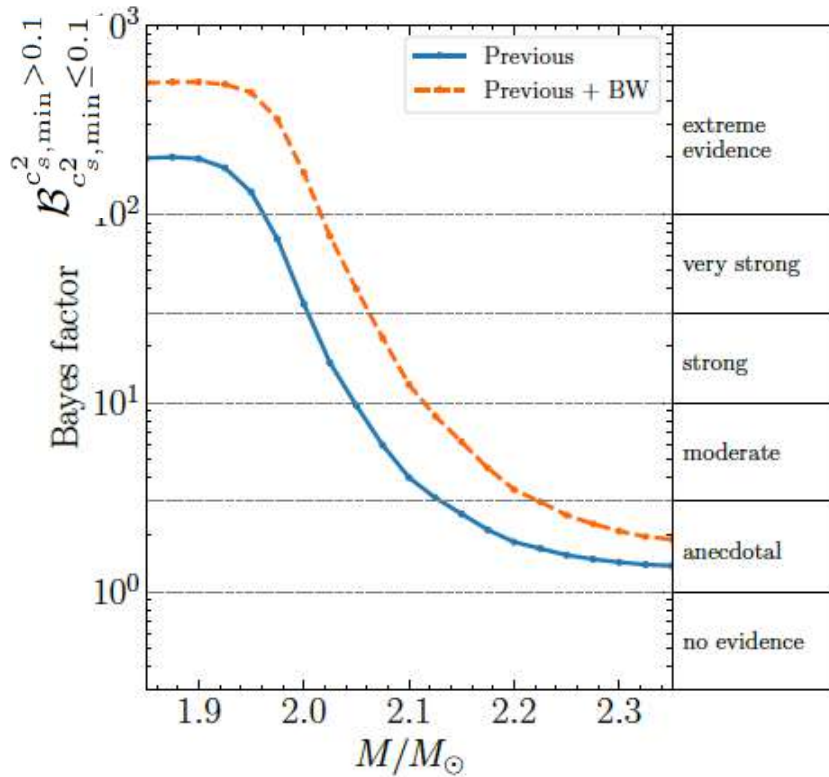
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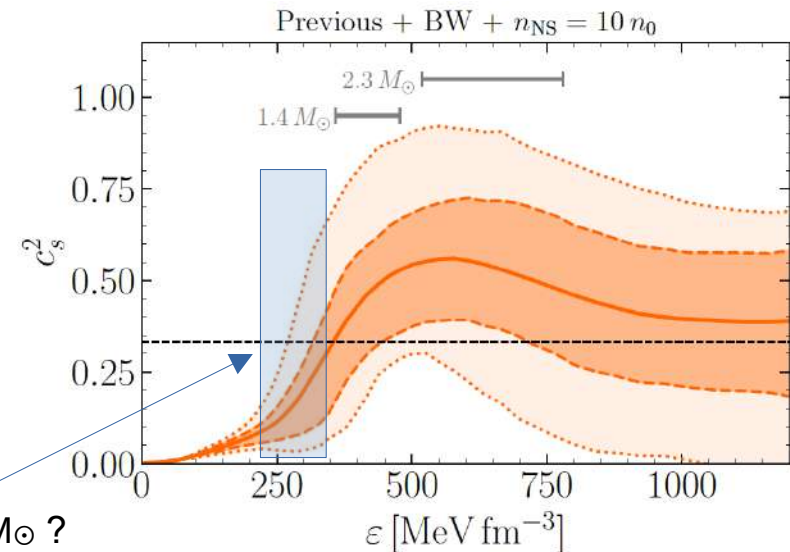
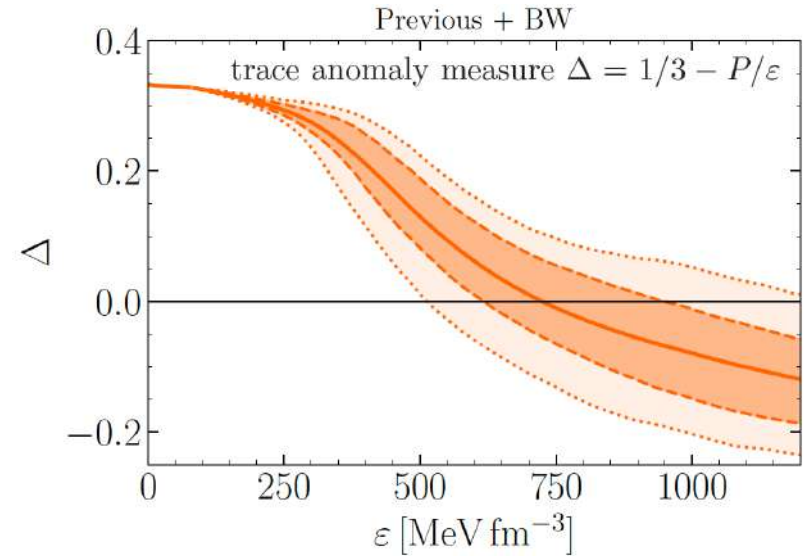
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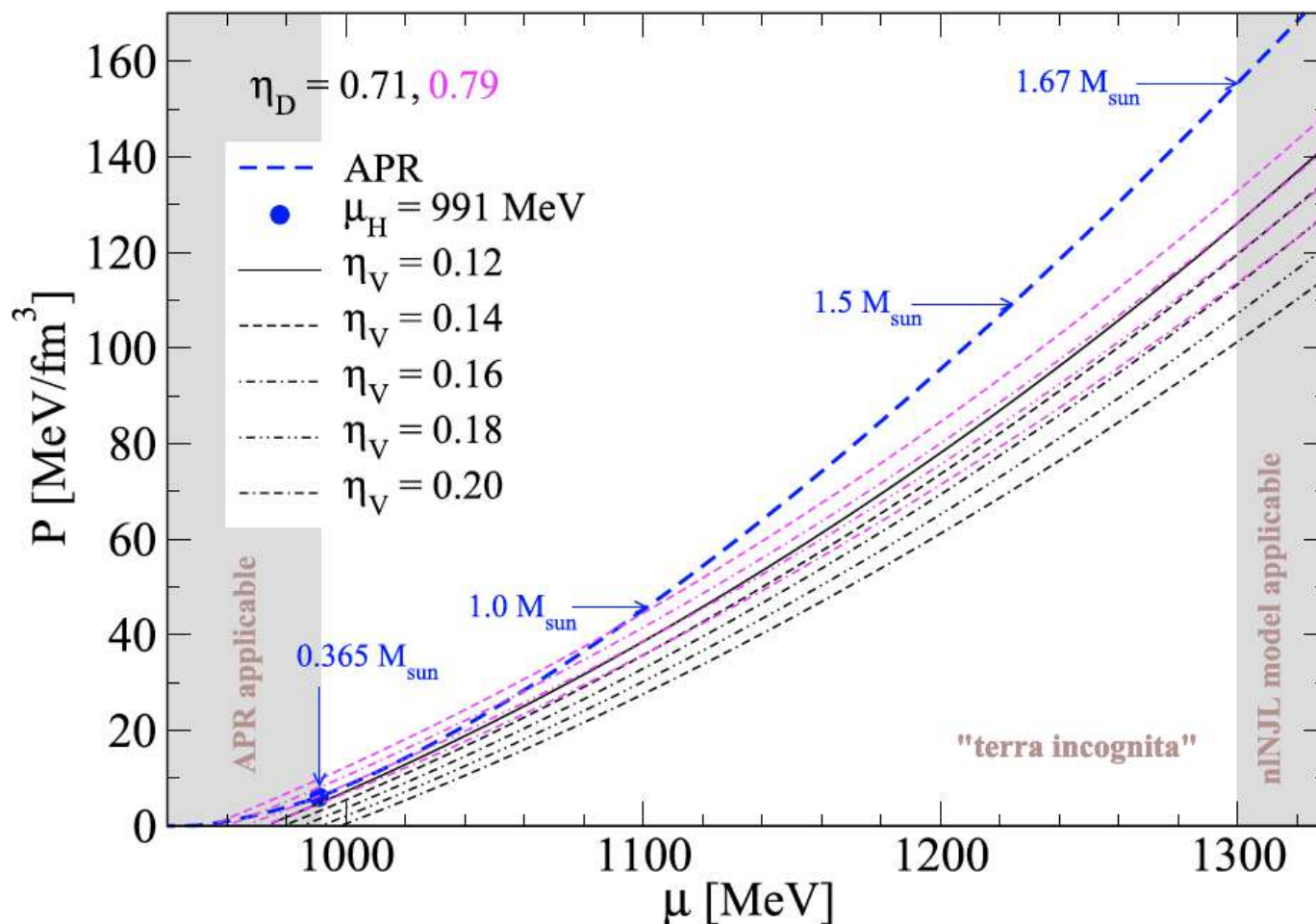
... excluding a strong 1<sup>st</sup> order PT with Maxwell construction

**BUT:** What if the PT occurred already in light NS, **below** 1.4  $M_\odot$  ?



# Pro and con quark matter in neutron stars

## Where is deconfinement in “terra incognita” ?



A. Ayriyan, D.B., A.G. Grunfeld, et al.

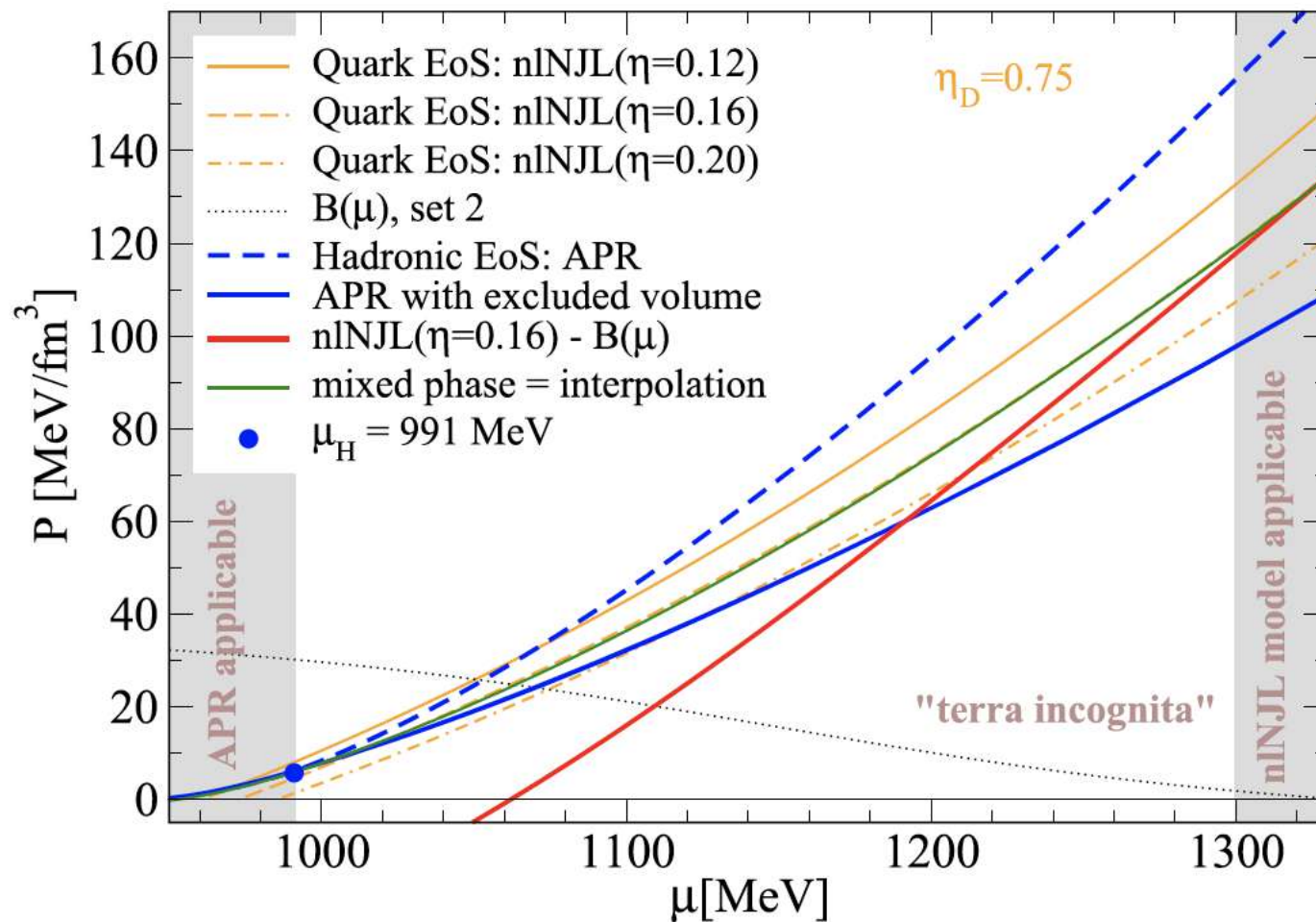
Eur. Phys. J. A (2021) 57:318

<https://doi.org/10.1140/epja/s10050-021-00619-0>



# Pro and con quark matter in neutron stars

Strong 1st order PT masquerades as „crossover” via pasta phases



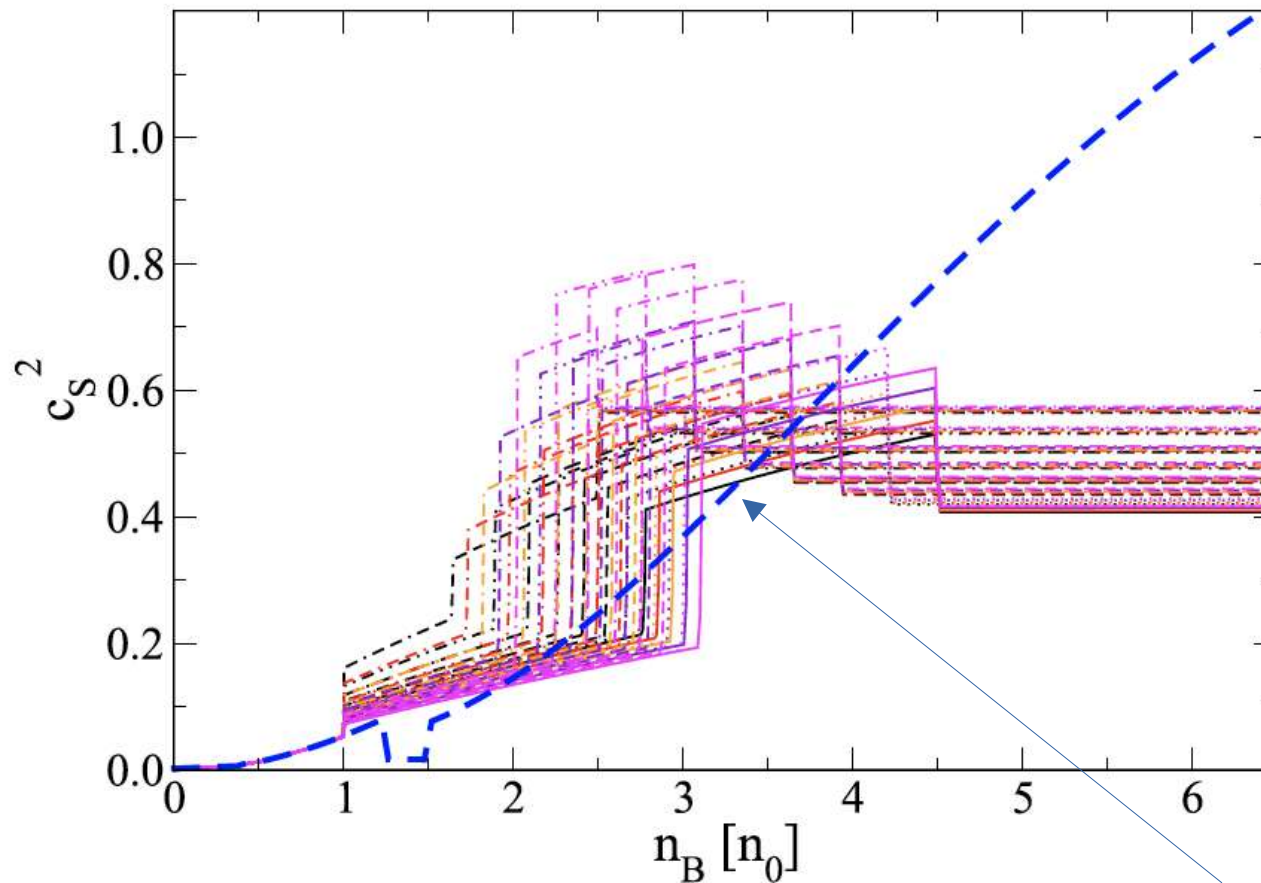
A. Ayriyan, D.B., A.G. Grunfeld, et al.

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# Pro and con quark matter in neutron stars

Strong 1<sup>st</sup> order PT masquerades as „crossover” via pasta phases



Two-zone interpolation scheme (TZIS) with crossover boundary condition → stiffening, analogous to “quarkyonic” matter behavior

A. Ayriyan, D.B., A.G. Grunfeld, et al.

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<https://doi.org/10.1140/epja/s10050-021-00619-0>

# Neutron star phenomenology from TOV eqns.

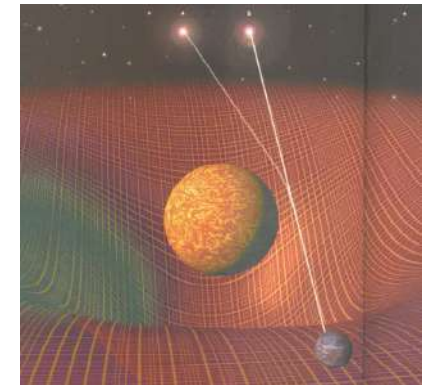
There is a 1:1 correspondence EOS  $\leftrightarrow$  M(R)

## Tolman-Oppenheimer-Volkoff (TOV) equations



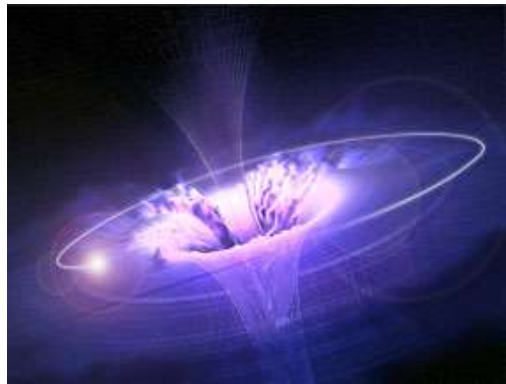
Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Non-rotating, spherical masses  $\rightarrow$  Schwarzschild Metrics

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2d\Omega^2$$



Tolman-Oppenheimer-Volkoff eqs.\*) for structure and stability of spherical compact stars

$$\frac{dP(r)}{dr} = -G \frac{m(r)\epsilon(r)}{r^2} \left(1 + \frac{P(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{m(r)}\right) \left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

Newtonian case    GR corrections from EoS    and metrics

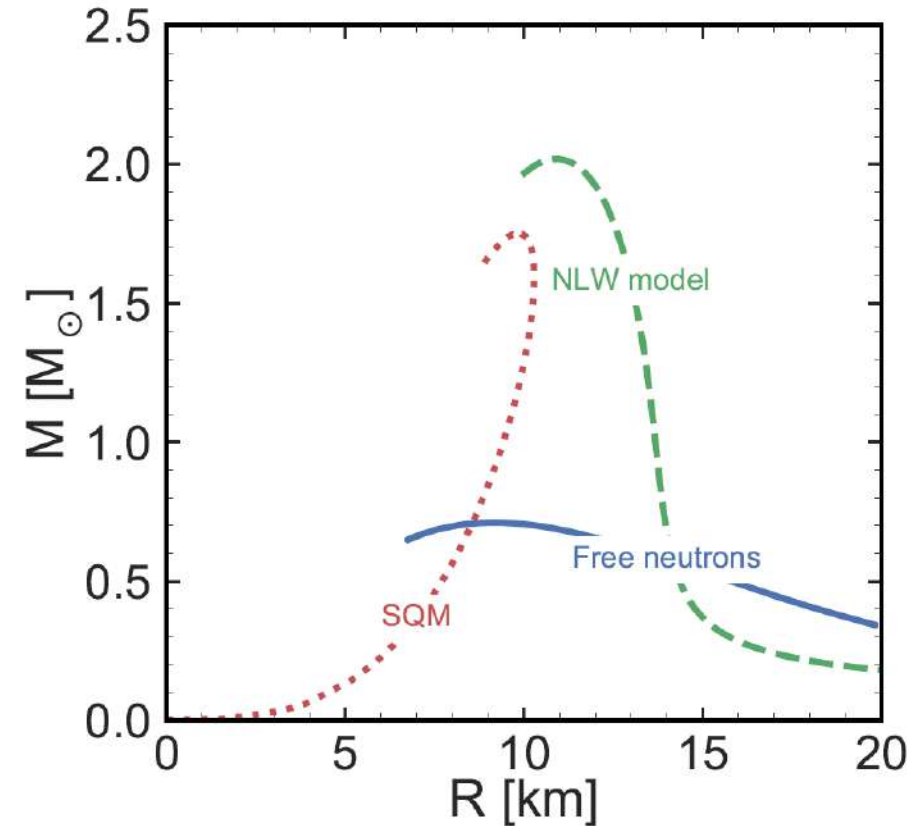
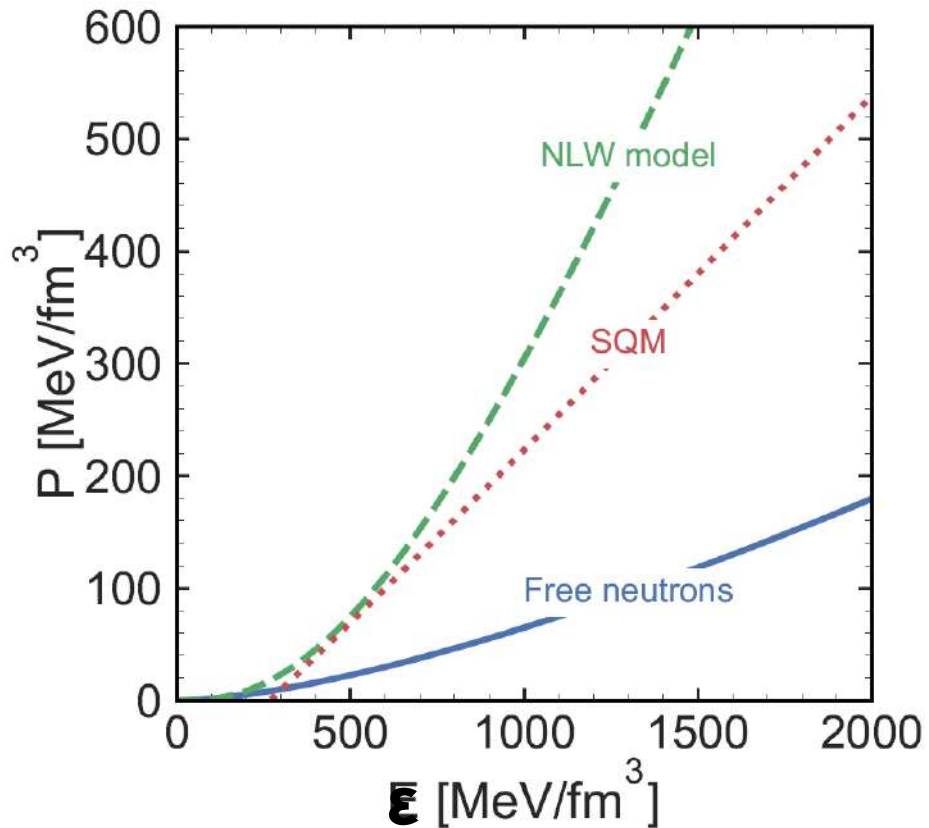
\*)R.C. Tolman, Phys. Rev. 55 (1939) 364; J.R. Oppenheimer, G.M. Volkoff, ibid., 374



# Neutron star phenomenology from TOV eqns.

There is a 1:1 correspondence EOS  $P(\epsilon) \leftrightarrow M(R)$

## Tolman-Oppenheimer-Volkoff (TOV) equations - solutions



**Stiffer equation of state  $\rightarrow$  larger radius and larger maximum mass**

# “Berlin Wall” constraint for neutron stars?

## Mass-radius diagram for purely hadronic EOS

Appearance of hyperons softens the EOS → Limitation for the maximum mass

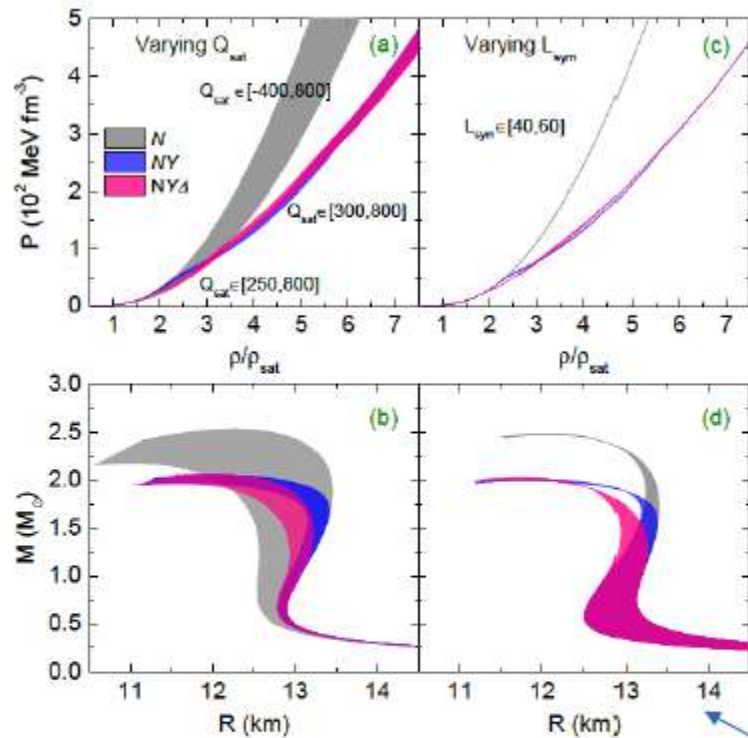


FIG. 4. EoS models and MR relations for  $N$ ,  $NY$ , and  $NY\Delta$  compositions of stellar matter. The bands are generated by varying the parameters  $Q_{\text{sat}}$  [MeV] (a, b) and  $L_{\text{sym}}$  [MeV] (c, d). The ranges of  $Q_{\text{sat}}$  and  $L_{\text{sym}}$  allowed by  $\chi$ EFT and maximum mass constraints are indicated in the figures.

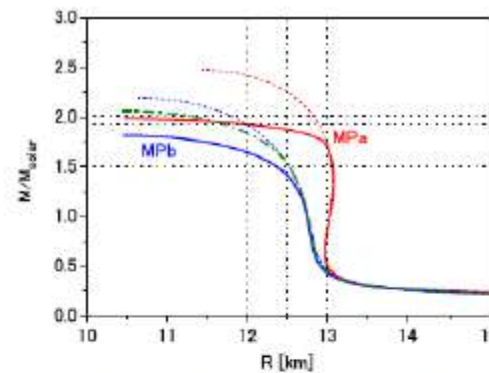


FIG. 7. Neutron-star masses as a function of the radius  $R$ . Solid (dashed) curves are with (without) hyperon ( $\Lambda$  and  $\Sigma^-$ ) mixing for ESC+MPa and ESC+MPb. The dot-dashed curve for MPb is with  $\Lambda$  mixing only. Also see the caption of Fig. 3.

Yamamoto et al., Phys.Rev.C 96 (2017) 06580; arXiv:1708.06163 [nucl-th]

Yamamoto et al., Eur. Phys. J. A 52 (2016) 19; arXiv:1510.06099 [nucl-th]

Ji & Sedrakian, Phys. Rev. C 100 (2019) 015809; arXiv:1903.06057 [astro-ph.HE]

**Examples for realistic hadronic EoS which suggest a Berlin Wall is inferior to the line  $M = 2.0 M_{\text{sun}}$**

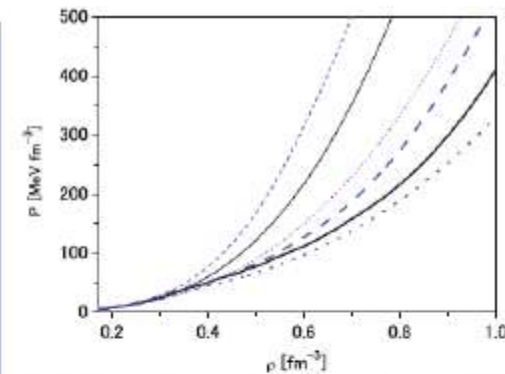


Fig. 8. Pressure  $P$  as a function of baryon density  $\rho$ . Thick (thin) curves are with (without) hyperon mixing. Solid, dashed and dotted curves are for MPa, MPa<sup>+</sup> and MPb.

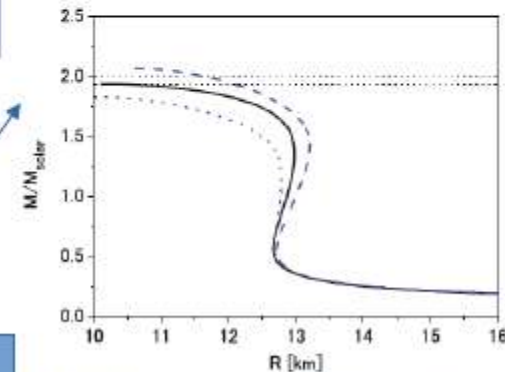
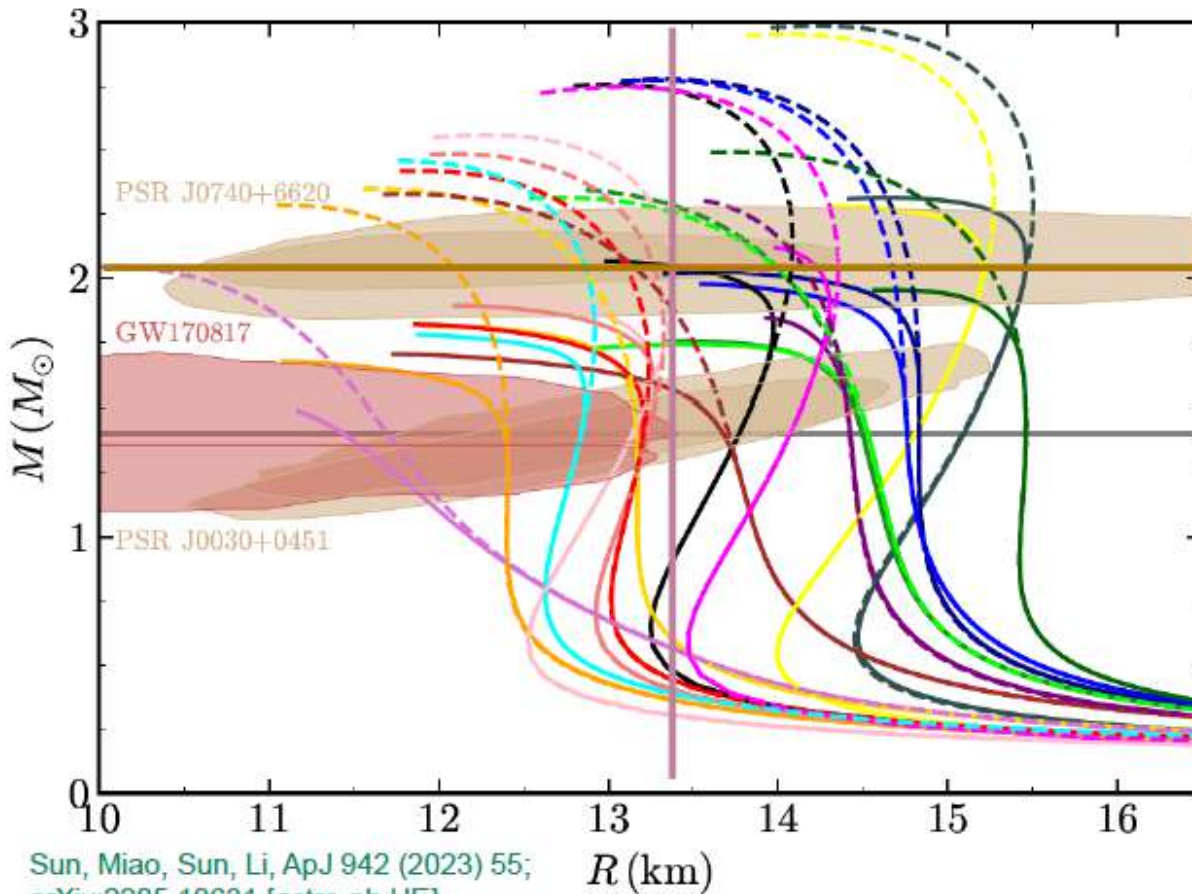


Fig. 9. Neutron-star masses as a function of the radius  $R$ . Solid, dashed and dotted curves are for MPa, MPa<sup>+</sup> and MPb. Two dotted lines show the observed mass  $(1.97 \pm 0.04)M_{\odot}$  of J1614-2230.

# “Berlin wall” constraint for neutron stars

## Realistic hadronic EOS (with strange baryons)

### Tension with modern multi-messenger observations by LVC and NICER



Sun, Miao, Sun, Li, ApJ 942 (2023) 55; arXiv:2205.10631 [astro-ph.HE]

Examples for hadronic EoS without (dashed lines) and with (solid lines) strange baryons. EoS which fulfill the observational constraints should be left of the vertical line at 1.4 Msun and should cross the horizontal line for the minimal maximum mass at 2.01 Msun. There is no EoS of this sample which fulfills both constraints !!

- LHS
- RMF201
- NL3
- Hybrid
- TM2
- NLSV1
- PK1
- NL3 $\omega\rho$
- S271v6
- HC
- DD-LZ1
- DD-ME2
- DD2
- PKDD
- DD-PC1
- FKVW
- PC-PK1
- OMEG

From Tab. 2 select EoS which fulfill (w. Y)  $70 < \Lambda_{1.4} < 580$  and check their  $M_{\max}$

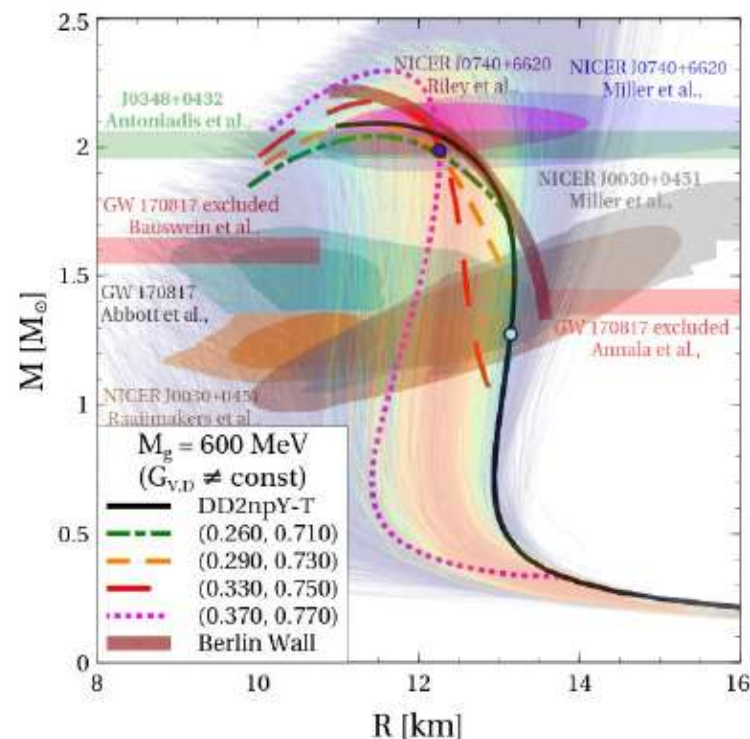
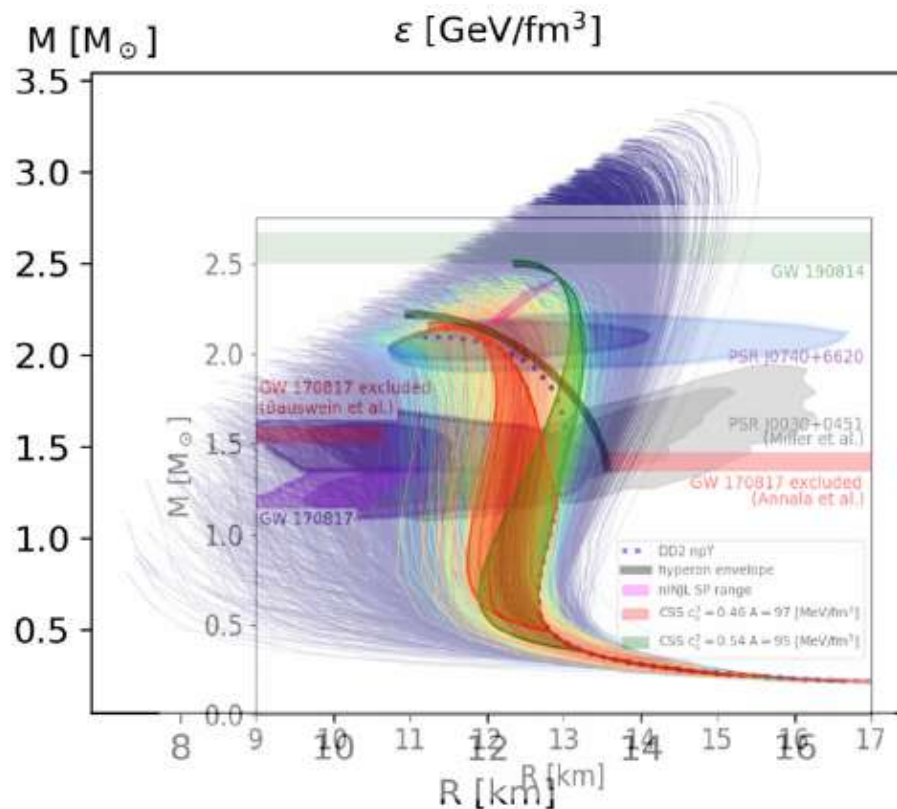
EoS	$M_{\max}$	EoS	$M_{\max}$
NL3 $\omega\rho$	1.974	DD2	1.935
DDLZ1	1.989	PKDD	1.781
DD-ME2	1.971	HC	1.828
OMEG	1.862		



# Breaking the “Berlin wall” constraint

With Bayesian analyses and hybrid EOS

M(R) curves generated by causality, thermodynamic stability and pQCD limit



The conjectured “Berlin Wall” overlaid to the Fig. 2 from Gorda, Komoltsev & Kurkela [2204.11877 [nucl-th]] and hybrid EoS with quark matter described by a CSS model (left) and a confining relativistic density functional (right).



# Relativistic density functionals for QCD

## String-flip model for quark matter confinement



Röpke, Blaschke, Schulz, PRD34 (1986) 3499

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \left\{ \int_0^\beta d\tau \int_V d^3x [\mathcal{L}_{\text{eff}} + \bar{q}\gamma_0\hat{\mu}q] \right\}, \quad q = \begin{pmatrix} q_u \\ q_d \end{pmatrix}, \quad \hat{\mu} = \text{diag}(\mu_u, \mu_d)$$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{free}} - U(\bar{q}q, \bar{q}\gamma_0q), \quad \mathcal{L}_{\text{free}} = \bar{q} \left( -\gamma_0 \frac{\partial}{\partial \tau} + i\vec{\gamma} \cdot \vec{\nabla} - \hat{m} \right) q, \quad \hat{m} = \text{diag}(m_u, m_d)$$

General nonlinear functional of quark density bilinears: scalar, vector, isovector, diquark ...

Expansion around the expectation values:

$$U(\bar{q}q, \bar{q}\gamma_0q) = U(n_s, n_v) + (\bar{q}q - n_s)\Sigma_s + (\bar{q}\gamma_0q - n_v)\Sigma_v + \dots,$$

$$\langle \bar{q}q \rangle = n_s = \sum_{f=u,d} n_{s,f} = - \sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial m_f} \ln \mathcal{Z}, \quad \Sigma_s = \left. \frac{\partial U(\bar{q}q, \bar{q}\gamma_0q)}{\partial (\bar{q}q)} \right|_{\bar{q}q=n_s} = \frac{\partial U(n_s, n_v)}{\partial n_s},$$

$$\langle \bar{q}\gamma_0q \rangle = n_v = \sum_{f=u,d} n_{v,f} = \sum_{f=u,d} \frac{T}{V} \frac{\partial}{\partial \mu_f} \ln \mathcal{Z}, \quad \Sigma_v = \left. \frac{\partial U(\bar{q}q, \bar{q}\gamma_0q)}{\partial (\bar{q}\gamma_0q)} \right|_{\bar{q}\gamma_0q=n_v} = \frac{\partial U(n_s, n_v)}{\partial n_v}$$

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \}, \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v$$

$$\mathcal{S}_{\text{quasi}}[\bar{q}, q] = \beta \sum_n \sum_{\vec{p}} \bar{q} G^{-1}(\omega_n, \vec{p}) q, \quad G^{-1}(\omega_n, \vec{p}) = \gamma_0(-i\omega_n + \hat{\mu}^*) - \vec{\gamma} \cdot \vec{p} - \hat{m}^*$$

# Relativistic density functionals for QCD

$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] - \beta V \Theta[n_s, n_v] \}, \quad \Theta[n_s, n_v] = U(n_s, n_v) - \Sigma_s n_s - \Sigma_v n_v$$

$$\mathcal{Z}_{\text{quasi}} = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \{ \mathcal{S}_{\text{quasi}}[\bar{q}, q] \} = \det[\beta G^{-1}], \quad \ln \det A = \text{Tr} \ln A$$

$$P_{\text{quasi}} = \frac{T}{V} \ln \mathcal{Z}_{\text{quasi}} = \frac{T}{V} \text{Tr} \ln[\beta G^{-1}] \quad \text{"no sea" approximation ...}$$

$$= 2N_c \sum_{f=u,d} \int \frac{d^3p}{(2\pi)^3} \left\{ T \ln \left[ 1 + e^{-\beta(E_f^* - \mu_f^*)} \right] + T \ln \left[ 1 + e^{-\beta(E_f^* + \mu_f^*)} \right] \right\}$$

$$P_{\text{quasi}} = \sum_{f=u,d} \int \frac{dp}{\pi^2} \frac{p^4}{E_f^*} [f(E_f^* - \mu_f^*) + f(E_f^* + \mu_f^*)] \quad \begin{aligned} E_f^* &= \sqrt{p^2 + m_f^{*2}} \\ f(E) &= 1/[1 + \exp(\beta E)] \end{aligned}$$

$$P = \sum_{f=u,d} \int_0^{p_{F,f}} \frac{dp}{\pi^2} \frac{p^4}{E_f^*} - \Theta[n_s, n_v], \quad p_{F,f} = \sqrt{\mu_f^{*2} - m_f^{*2}}$$

$$\begin{aligned} \hat{m}^* &= \hat{m} + \Sigma_s \\ \hat{\mu}^* &= \hat{\mu} - \Sigma_v \end{aligned}$$

Selfconsistent densities

$$n_s = - \sum_{f=u,d} \frac{\partial P}{\partial m_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{F,f}} dp p^2 \frac{m_f^*}{E_f^*}, \quad n_v = \sum_{f=u,d} \frac{\partial P}{\partial \mu_f} = \frac{3}{\pi^2} \sum_{f=u,d} \int_0^{p_{F,f}} dp p^2 = \frac{p_{F,u}^3 + p_{F,d}^3}{\pi^2}.$$



# Relativistic density functionals for QCD

## String-flip model for quark matter

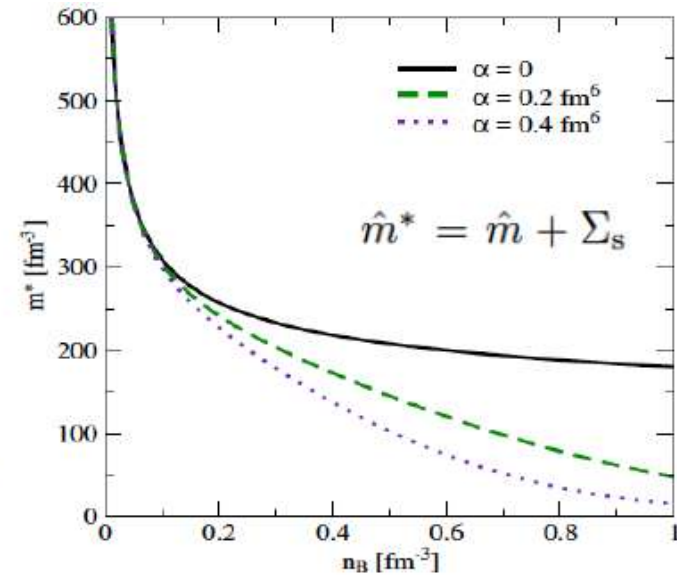
Density functional for the SFM

$$U(n_s, n_v) = D(n_v)n_s^{2/3} + an_v^2 + \frac{bn_v^4}{1 + cn_v^2},$$

Quark selfenergies

$$\Sigma_s = \frac{2}{3}D(n_v)n_s^{-1/3}, \quad \text{Quark "confinement"}$$

$$\Sigma_v = 2an_v + \frac{4bn_v^3}{1 + cn_v^2} - \frac{2bcn_v^5}{(1 + cn_v^2)^2} + \frac{\partial D(n_v)}{\partial n_v}n_s^{2/3}$$

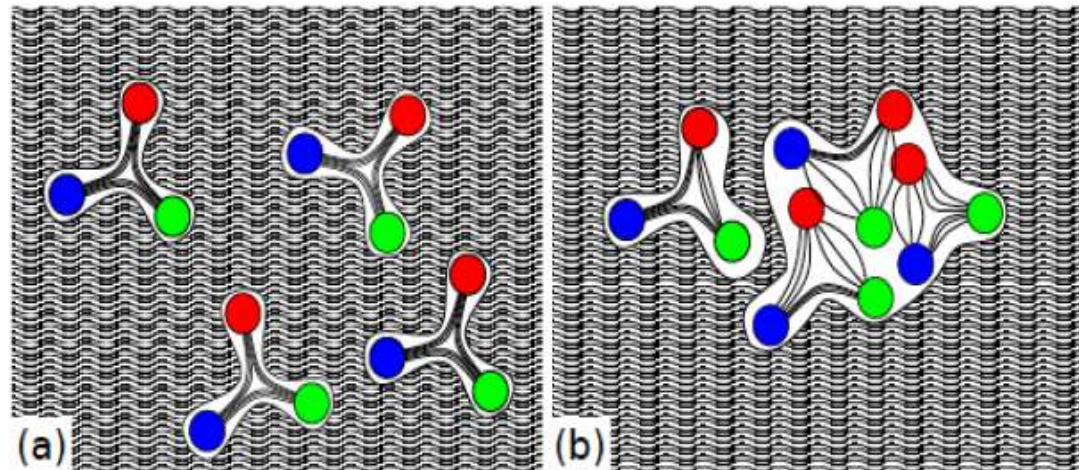


String tension & confinement due to dual Meissner effect (dual superconductor model)

$$D(n_v) = D_0\Phi(n_v)$$

Effective screening of the string tension in dense matter by a reduction of the available volume  $\alpha = v|v|/2$

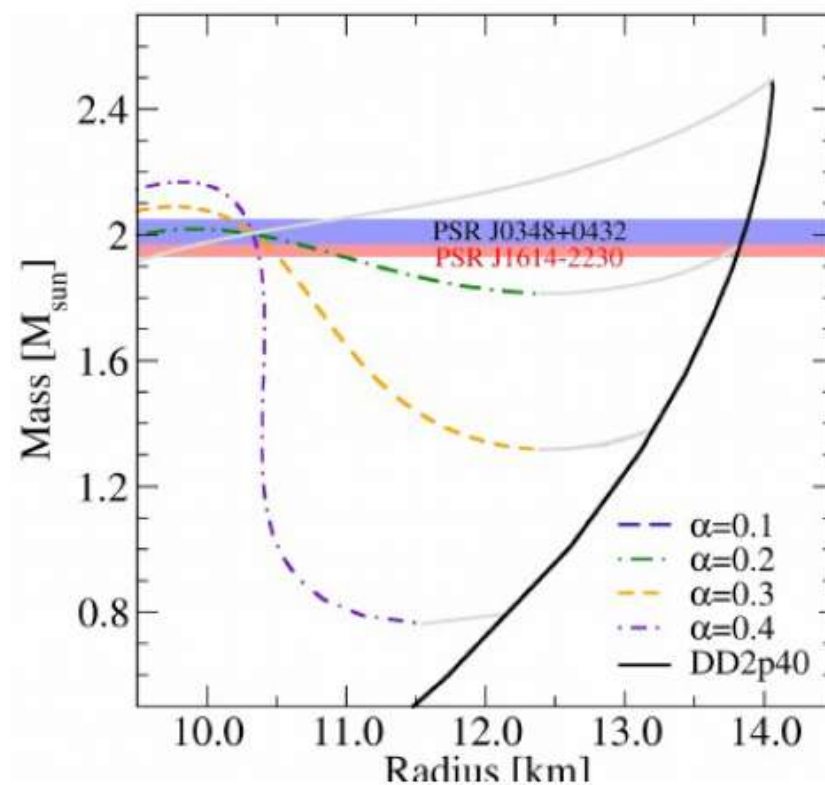
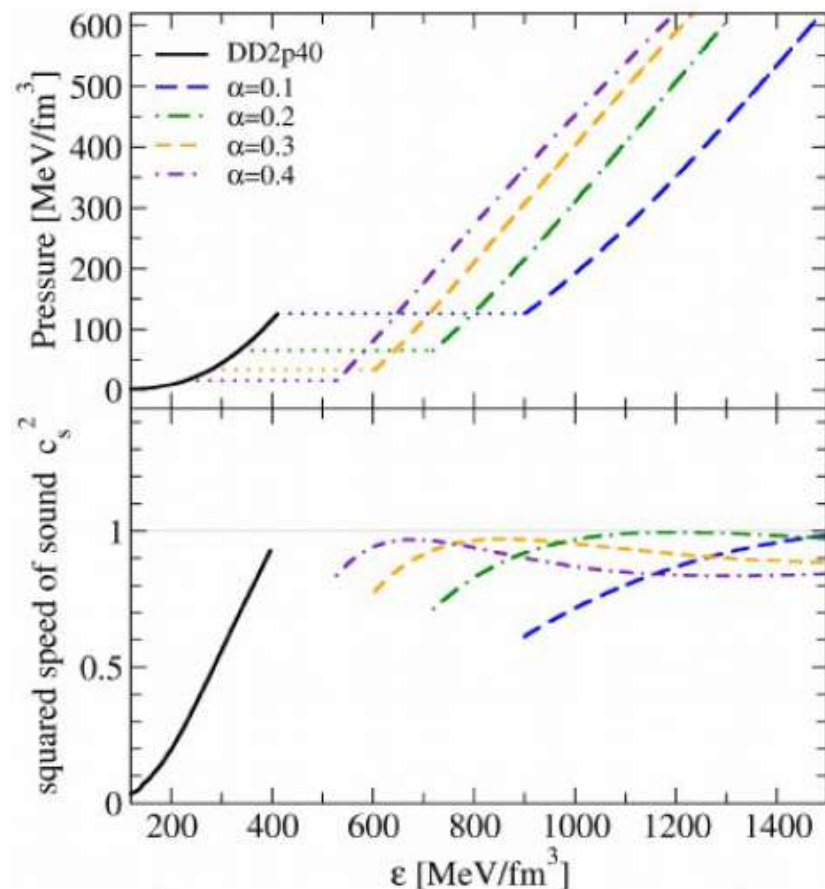
$$\Phi(n_B) = \begin{cases} 1, & \text{if } n_B < n_0 \\ e^{-\alpha(n_B - n_0)^2}, & \text{if } n_B > n_0 \end{cases}$$



# Relativistic density functionals for QCD

## String-flip model for quark matter

### Results for 1st order phase transition by Maxwell construction with DD2p40



Kaltenborn, Bastian, Blaschke, arXiv:1701.04400



Phys. Rev. D 96, 056024 (2017)



# QCD Phase Diagram

## Landscape of our investigations

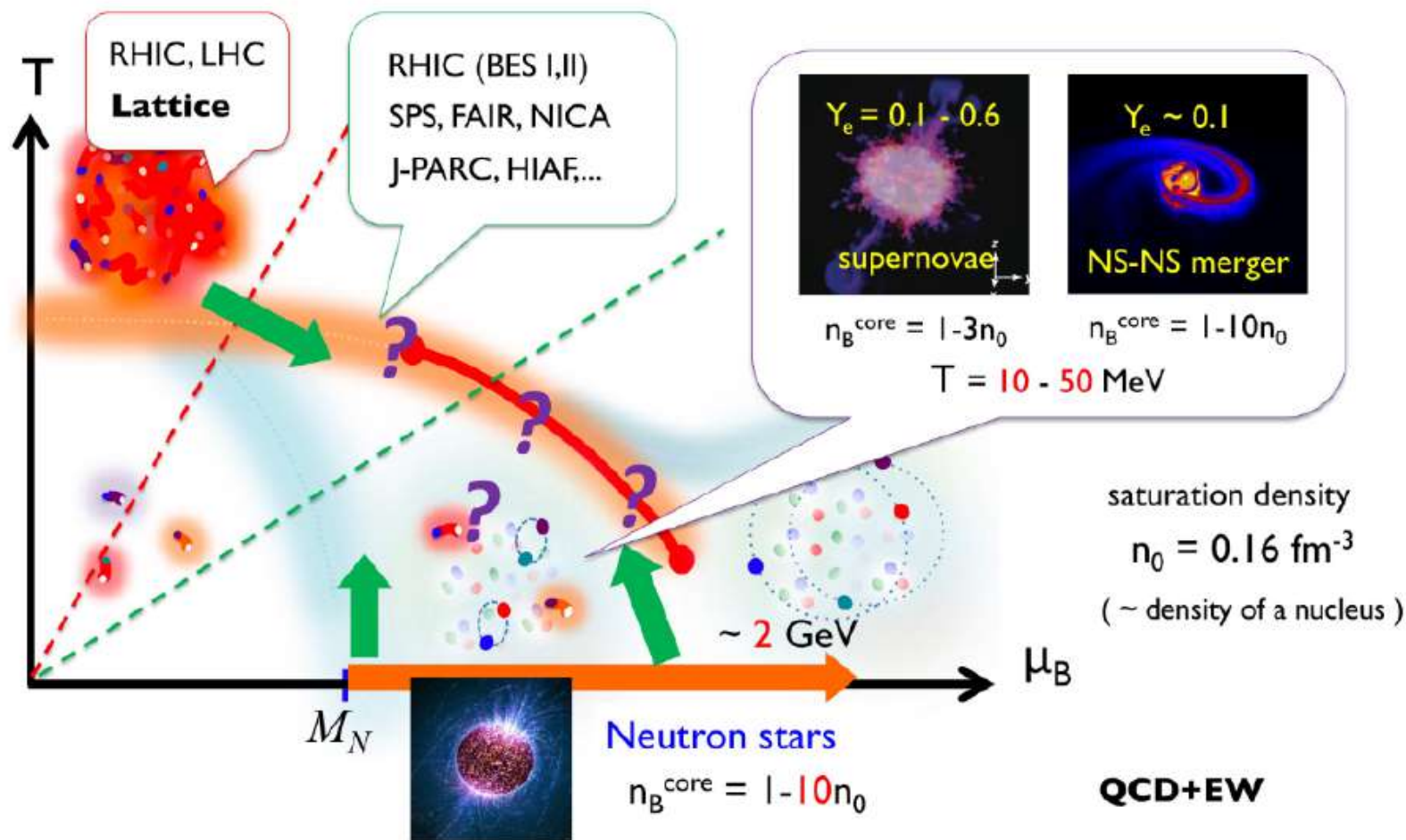
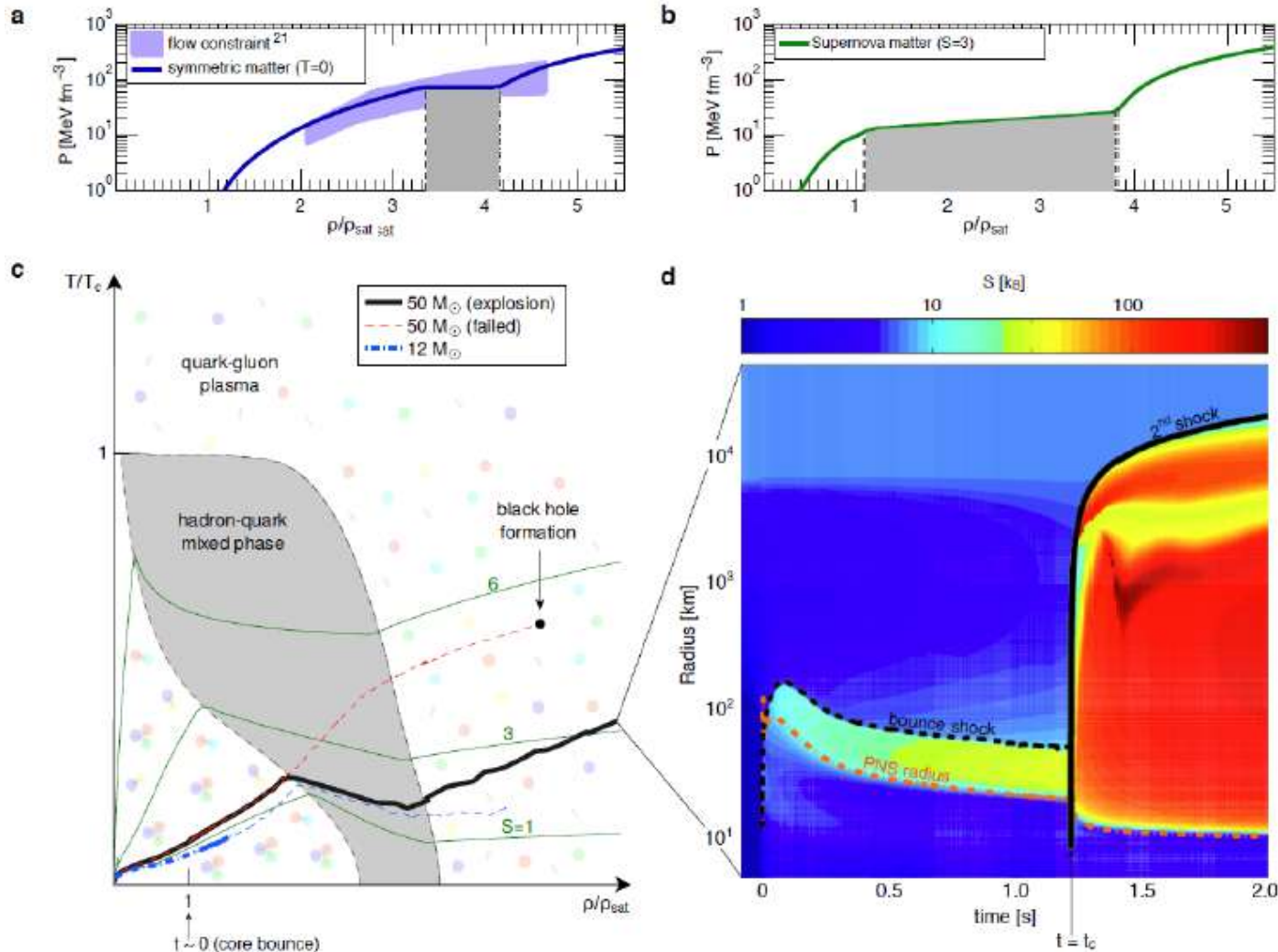


Figure from T. Kojo arXiv:1912.05326 [nucl-th]

# Deconfinement as supernova engine

## Of massive blue supergiant star explosions

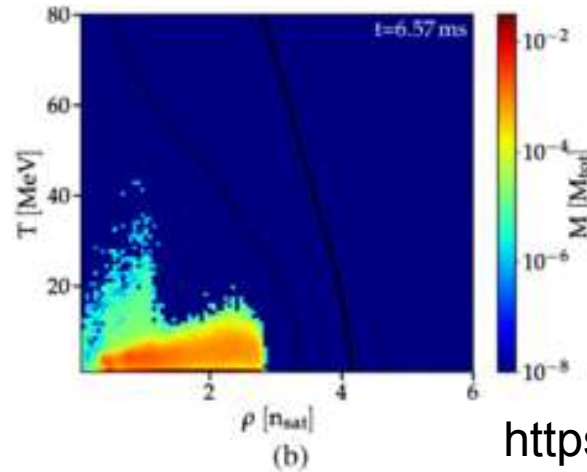
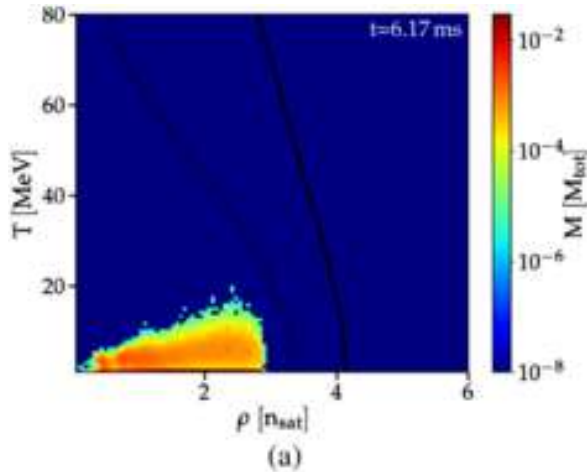


Progenitor:  
 $M = 50 M_{\odot}$

T. Fischer et al., Nature Astronomy 2, 960 (2018)

# Ultra-heavy Nucleus-Nucleus Collisions !

## Population of the QCD phase diagram in a merger



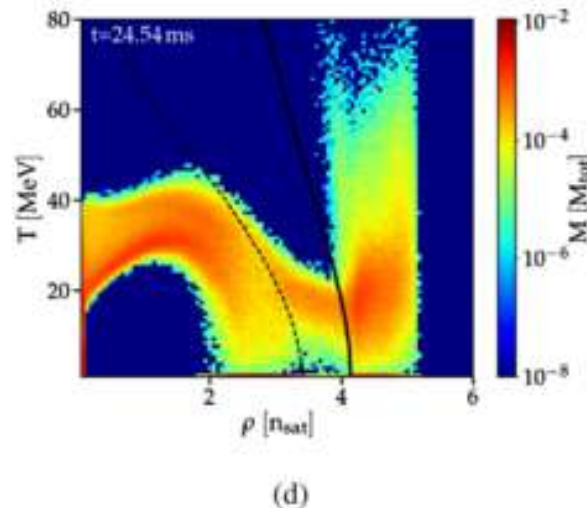
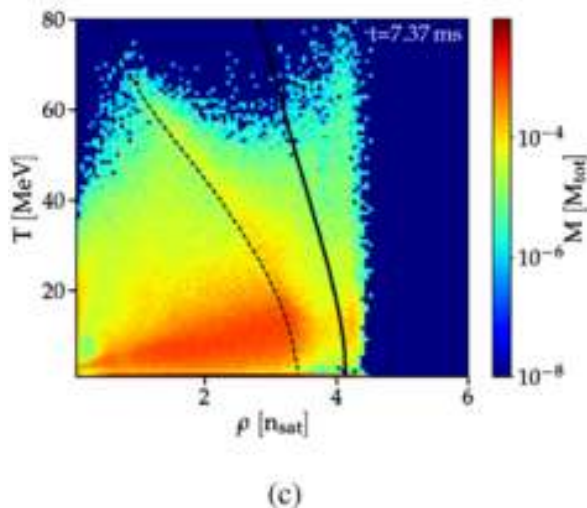
1.35  $M_{\text{sun}}$  + 1.35  $M_{\text{sun}}$

EoS for supernova and merger simulations:

**CompOSE**

With deconfinement:

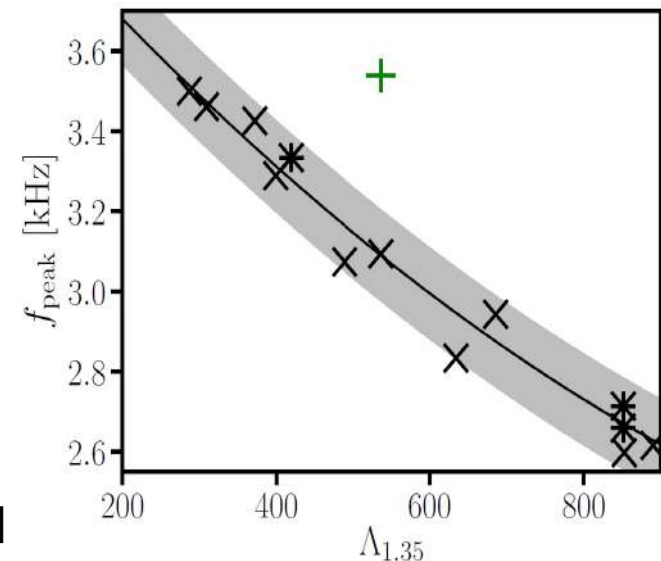
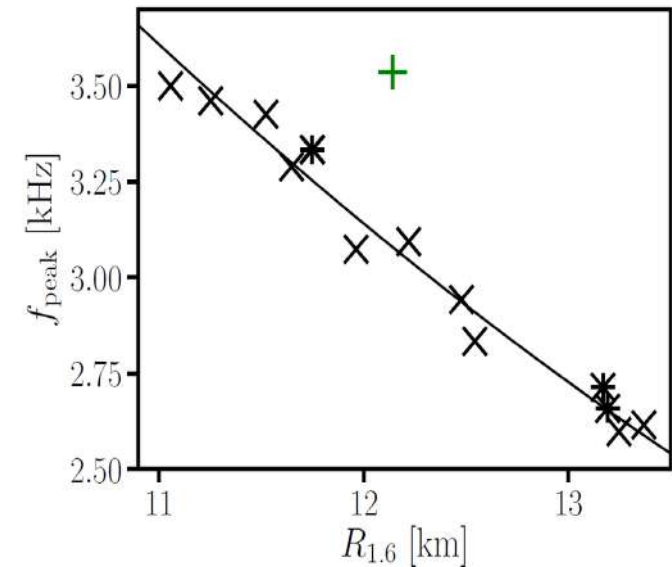
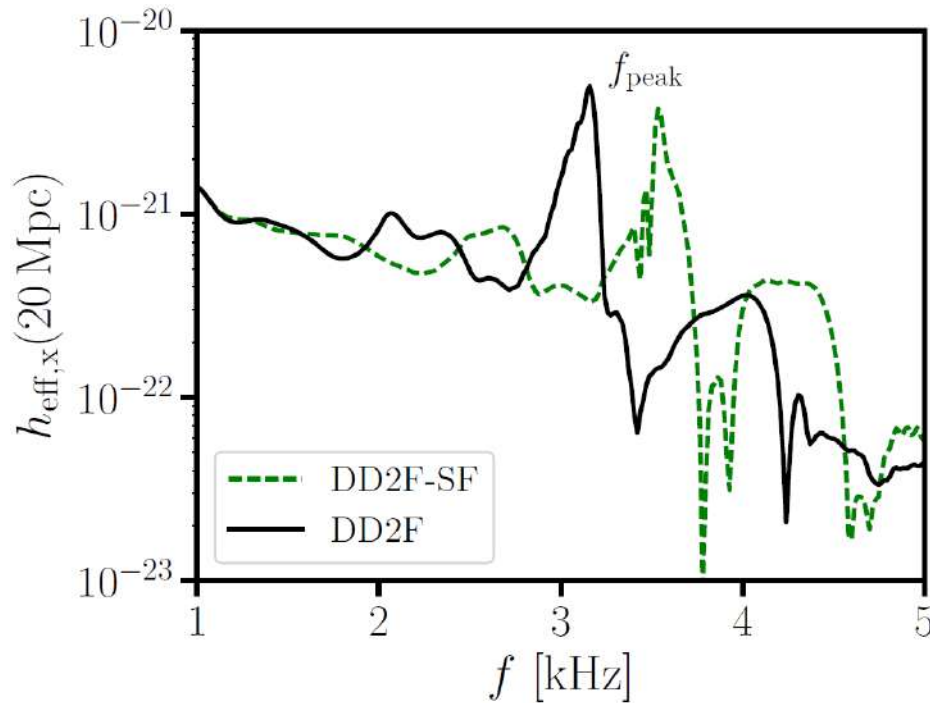
<https://compose.obspm.fr/eos/166>



S. Blacker, A. Bauswein, et al.,  
Phys. Rev. D 102 (2020) 123023

# Ultra-heavy Nucleus-Nucleus Collisions !

## Signal of a deconfinement transition



**Strong deviation** from  $f_{\text{peak}} - R_{1.6}$  relation signals **strong phase transition in NS merger!**

**Complementarity of  $f_{\text{peak}}$  from postmerger with tidal deformability  $\Lambda_{1.35}$  from inspiral phase.**

A. Bauswein et al., PRL 122 (2019) 061102; [arxiv:1809.01116]



# Relativistic density functional for quark matter

## With chiral symmetry, color SC & confinement

**Lagrangian**  $\mathcal{L} = \bar{q}(i\cancel{\partial} - \hat{m})q - \mathcal{U} + \mathcal{L}_V + \mathcal{L}_I + \mathcal{L}_D$

- **Scalar & pseudoscalar interaction channels**

$$\mathcal{U} = G_0 [(1 + \alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5q)^2]^{\frac{1}{3}}$$

(motivated by String Flip Model,  $\chi$ -dynamics, quark "confinement")

- **Vector-isoscalar interaction channel**

$$\mathcal{L}_V = -G_V(\bar{q}\gamma_\mu q)^2$$

(motivated by gluon exchange, stiff EoS needed to reach  $2M_\odot$ )

- **Vector-isovector interaction channel**

$$\mathcal{L}_I = -G_I(\bar{q}\gamma_\mu\vec{\tau}q)^2$$

(motivated by gluon exchange, isospin sensitive interaction)

- **Diquark interaction channel**

$$\mathcal{L}_D = G_D \sum_{A=2,5,7} (\bar{q}i\gamma_5\tau_2\lambda_A q^c)(\bar{q}^c i\gamma_5\tau_2\lambda_A q)$$

(motivated by Cooper theorem, color superconductivity)

## What is new?

O. Ivanytskyi & D.B., Phys. Rev. D 105 (2022) 114042

**Interaction** 
$$\mathcal{U} = D_0 [(1 + \alpha)\langle\bar{q}q\rangle_0^2 - (\bar{q}q)^2 - (\bar{q}i\vec{\tau}\gamma_5q)^2]^\kappa$$

- Parameters**

$D_0$  - dimensionfull coupling, controls interaction strength

$\alpha$  - dimensionless constant, controls vacuum quark mass

$\langle\bar{q}q\rangle_0$  -  $\chi$ -condensate in vacuum (introduced for the sake of convenience)

$$\kappa = 1/3$$



motivated by String Flip model

$$\mathcal{U}_{SFM} \propto \langle q^+q \rangle^{2/3}$$

$$\Sigma_{SFM} = \frac{\partial \mathcal{U}_{SFM}}{\partial \langle q^+q \rangle} \propto \langle q^+q \rangle^{-1/3} \propto \text{separation}$$

$$\kappa = 1$$



Nambu–Jona-Lasinio model



- Dimensionality**

$$\begin{aligned} [U] &= \text{energy}^4 \\ [\bar{q}q] &= \text{energy}^3 \end{aligned} \Rightarrow [D_0]_{\kappa=1/3} = \text{energy}^2 = [\text{string tension}]$$

**self energy = string tension × separation ⇒ confinement**

## Expansion around mean fields

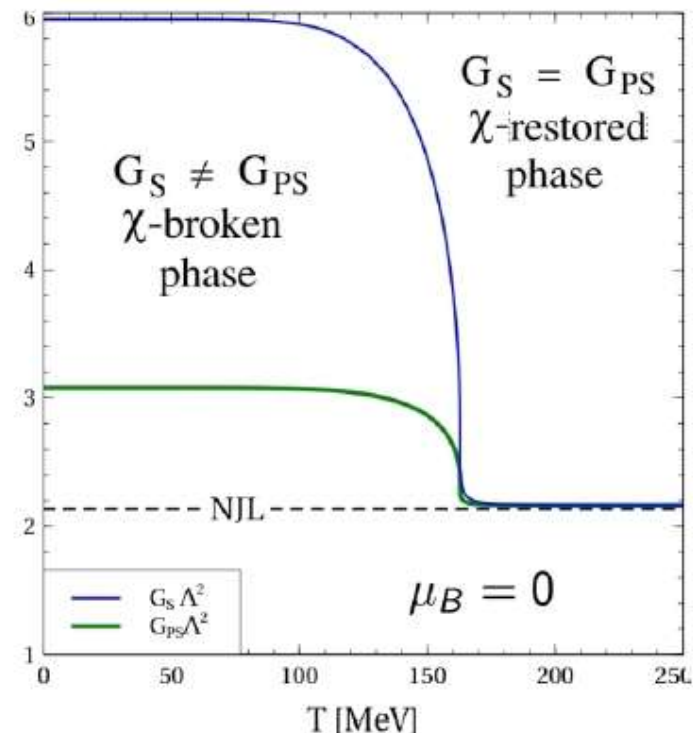
$$\mathcal{U} = \underbrace{\mathcal{U}_{MF}}_{0^{\text{th}} \text{ order}} + \underbrace{(\bar{q}q - \langle \bar{q}q \rangle) \Sigma_S}_{1^{\text{st}} \text{ order}} - \underbrace{G_S (\bar{q}q - \langle \bar{q}q \rangle)^2 - G_{PS} (\bar{q}i\vec{\tau}\gamma_5 q)^2}_{2^{\text{nd}} \text{ order}} + \dots$$

- Mean-field scalar self-energy

$$\Sigma_S = \frac{\partial \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle}$$

- Effective medium dependent couplings

$$G_S = -\frac{1}{2} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}q \rangle^2}, \quad G_{PS} = -\frac{1}{6} \frac{\partial^2 \mathcal{U}_{MF}}{\partial \langle \bar{q}i\vec{\tau}\gamma_5 q \rangle^2}$$



## Comparison to Nambu—Jona-Lasinio model

$$\mathcal{L} = \bar{q}(i\not{\partial} - \underbrace{(m + \Sigma_S)}_{\text{effective mass } m^*})q + G_S(\bar{q}q)^2 + G_{PS}(\bar{q}i\vec{\tau}\gamma_5q)^2 + \dots + \mathcal{L}_V + \mathcal{L}_D$$

### • Similarities:

- current-current interaction
- (pseudo)scalar, vector, diquark, ... channels

### • Differences:

- high  $m^*$  at low  $T, \mu \Rightarrow$  “confinement”

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_0 \Rightarrow m^* = m - \frac{2G_0}{3\alpha^{2/3}\langle \bar{q}q \rangle_0^{1/3}}$$

$\Downarrow$

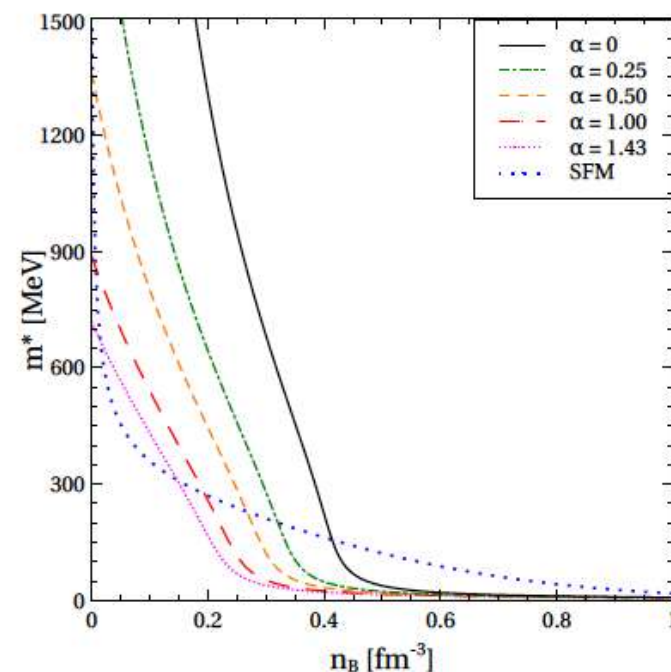
$$m^* \rightarrow \infty \text{ at } \alpha \rightarrow 0$$

- medium dependent couplings:

$$\text{low } T, \mu, \Rightarrow G_S \neq G_{PS} \Rightarrow \chi\text{-broken}$$

$$\text{high } T, \mu, \Rightarrow G_S = G_{PS} \Rightarrow \chi\text{-symmetric}$$

$T = 0$





# Relativistic density functional for quark matter

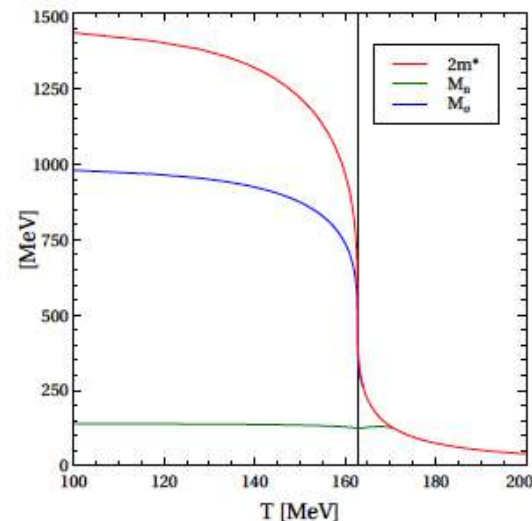
## Model setup – parameter fixing with observables

- (Pseudo)scalar interaction channels  
(chiral condensate &  $\pi$ ,  $\sigma$  mesons)

$m$ [MeV]	$\Lambda$ [MeV]	$\alpha$	$D_0\Lambda^{-2}$
4.2	573	1.43	1.39
$M_\pi$ [MeV]	$F_\pi$ [MeV]	$M_\sigma$ [MeV]	$\langle \bar{l}l \rangle_0^{1/3}$ [MeV]
140	92	980	-267

### Pseudocritical temperature

$$T_c = 163 \text{ MeV}$$



- low T:  $2m_{quark} > M_\pi, M_\sigma$   
(stable mesons, confined quarks)
- high T:  $2m_{quark} < M_\pi, M_\sigma$   
(unstable mesons, deconfined quarks)

- Vector-isoscalar & vector-isovector channels ( $\omega$ ,  $\rho$  mesons)

$$M_\omega = 783 \text{ MeV} \Rightarrow \eta_V \equiv \frac{G_{V0}}{G_{S0}} = 0.452, \quad M_\rho = 775 \text{ MeV} \Rightarrow \eta_I \equiv \frac{G_{I0}}{G_{S0}} = 0.454$$

- Diquark pairing channel (Fierz transformation)  $\eta_D \equiv \frac{G_{D0}}{G_{S0}} = 1.5\eta_V = 0.678$

## Onset of color superconductivity

- **Single quark energy and distribution**

$$E_f^\pm = \text{sgn}(E_f \mp \mu_f) \sqrt{(E_f \mp \mu_f)^2 + \Delta^2}$$

$$f_f^\pm = [\exp(E_f^\pm / T) + 1]^{-1}$$

- **Gap equation**

$$\frac{\partial \Omega}{\partial \Delta} = \frac{\Delta}{2G_D} - 2\Delta \sum_{f,a=\pm} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1 - 2f_f^a}{E_f^a} = 0$$

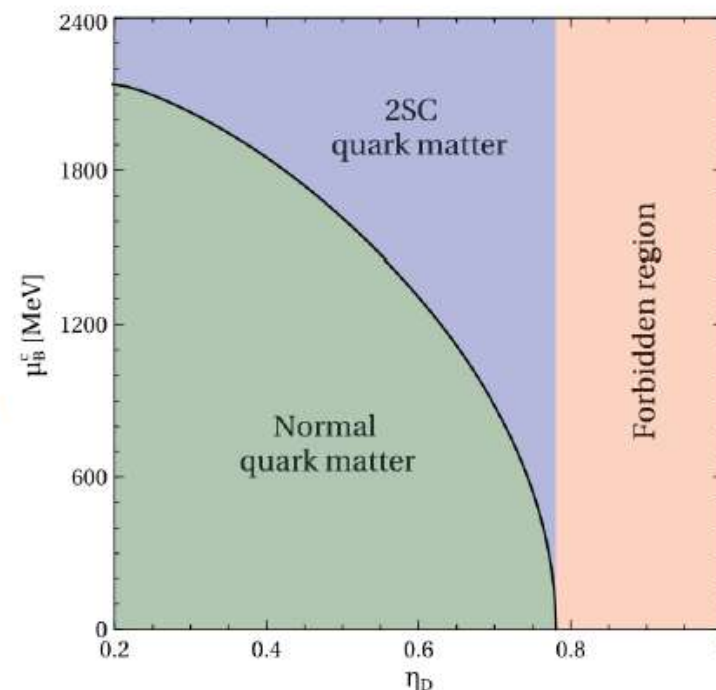
⇓

two solutions :  $\Delta = 0$  or  $\Delta \neq 0$

- **Two solutions coincide  $\Rightarrow$  SC onset**

$$\left. \frac{\partial^2 \Omega}{\partial \Delta^2} \right|_{\Delta=0} = 0 \quad \Rightarrow \quad \mu_B = \mu_B(G_D)$$

$T = 0$



No vacuum superconductivity

⇓

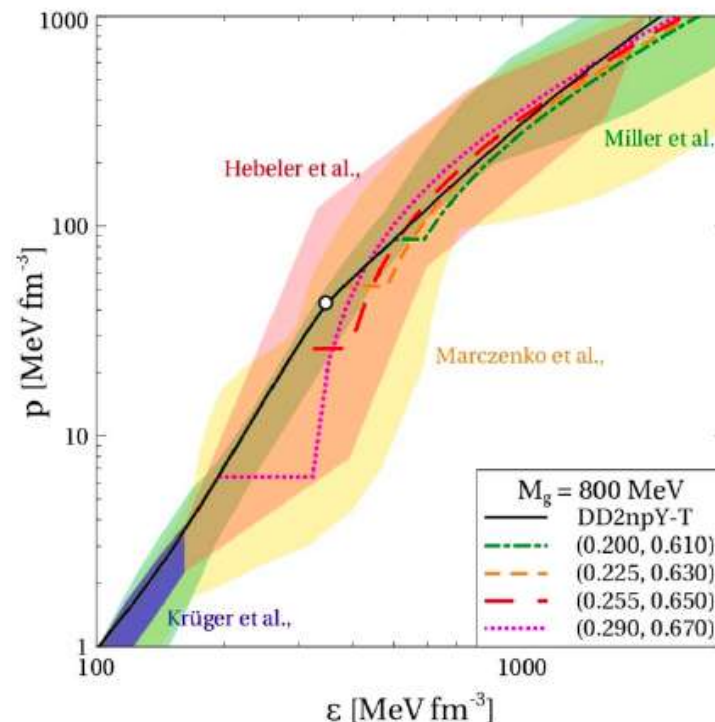
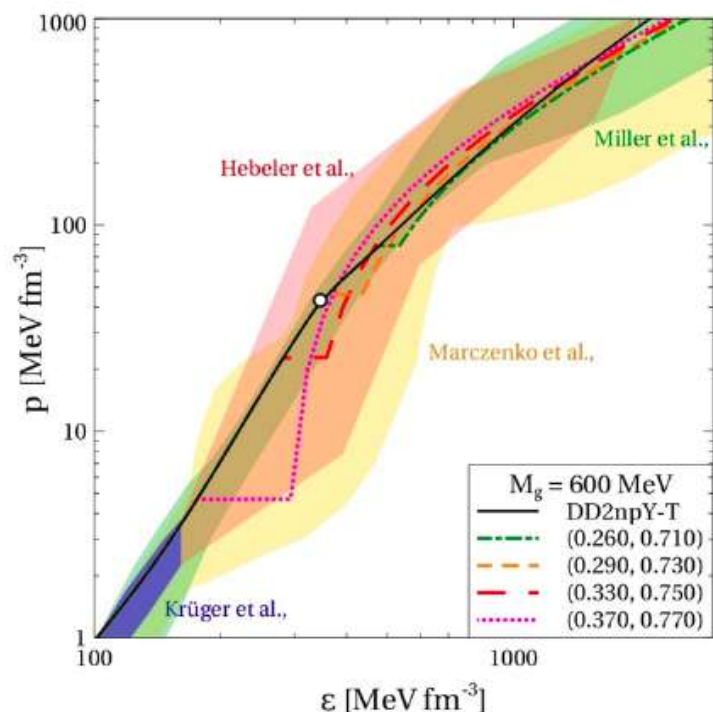
$$\eta_D \lesssim 0.78$$

(agrees with the Fierz value)

# Relativistic density functional for quark matter

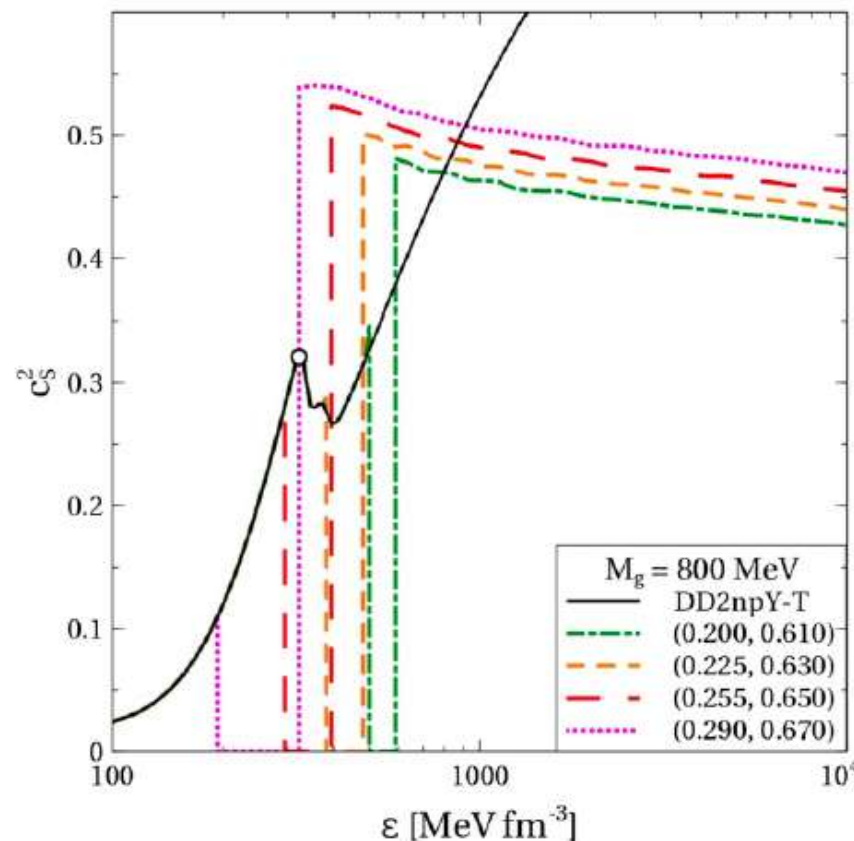
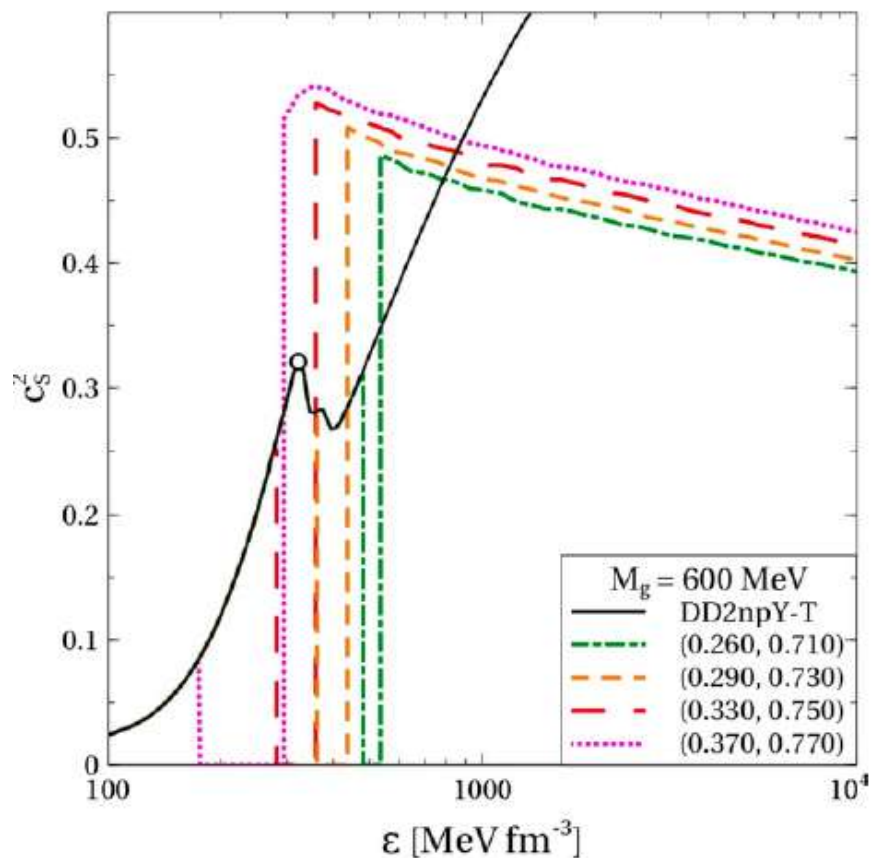
## Asymptotically conformal EOS for neutron stars

- **Setup:** electric neutrality,  $\beta$ -equilibrium, Maxwell construction with DD2 EoS
- **Scanning over  $\eta_V$  and  $\eta_D$  at  $M_{gD} = M_{gV}$**



The  $\omega$ -meson value of  $\eta_V$  and the Fierz value of  $\eta_D$   
prefer early deconfinement?

## Speed of sound

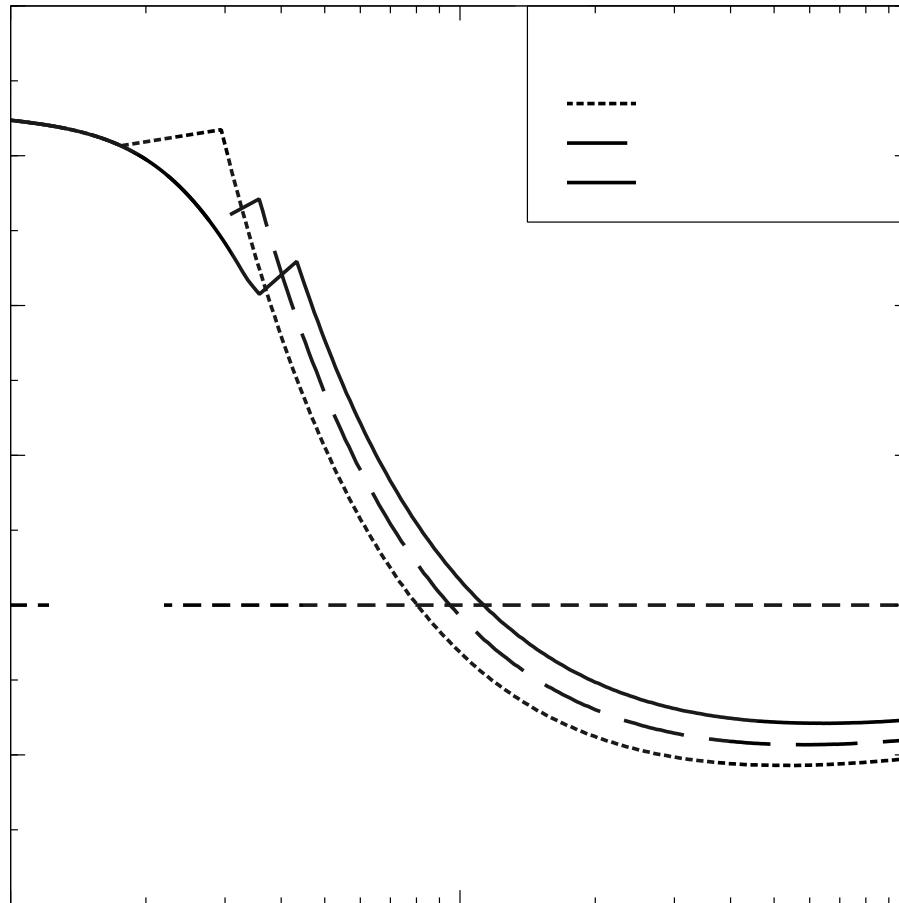


O. Ivanytskyi and D. Blaschke, *Particles* 5 (2022) 514 - 534



# Relativistic density functional for quark matter

Conformality measure  $\Delta = 1/3 - P/\epsilon$



Courtesy:

O. Ivanytskyi, derived from

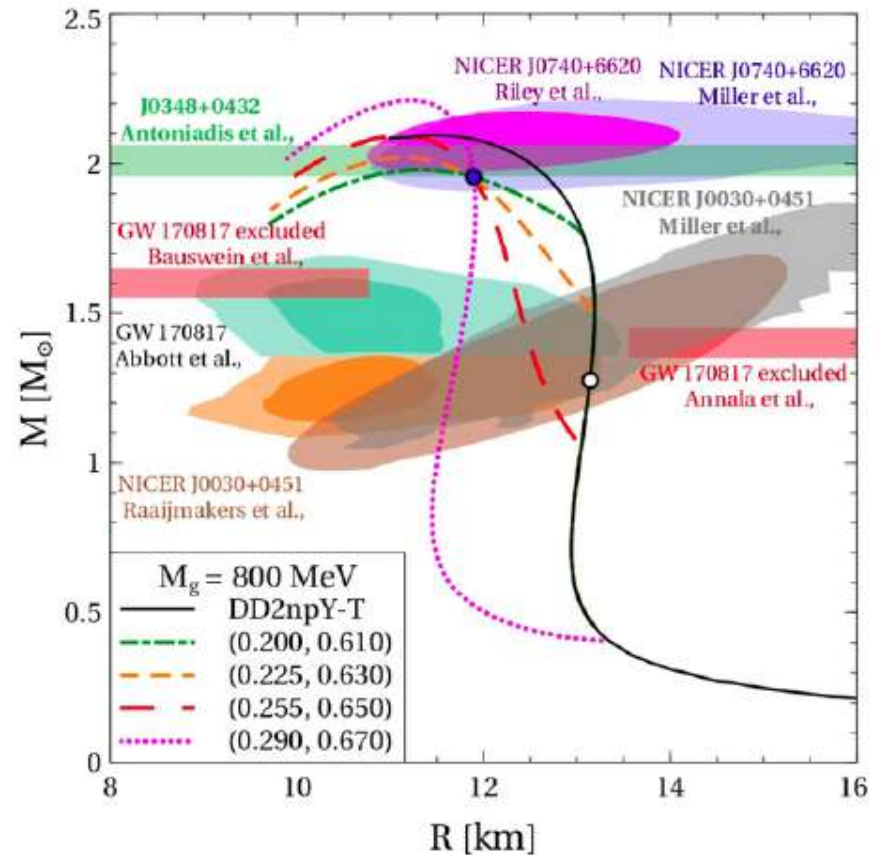
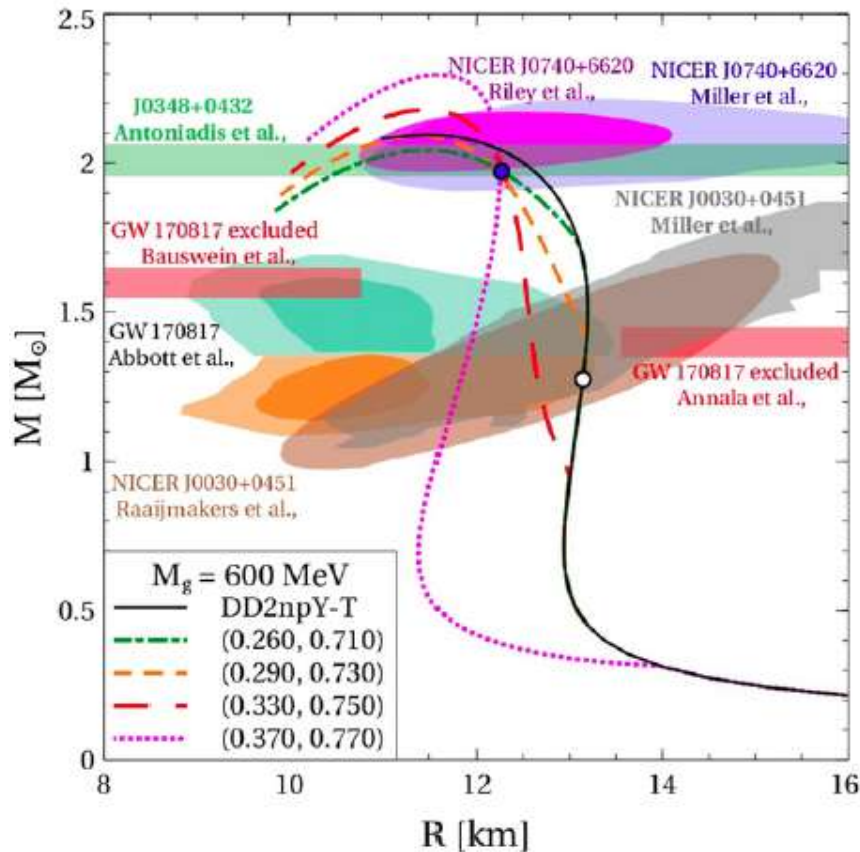
Particles 5 (2022) 514 and

M. Marczenko et al.,

Phys. Rev. C 107 (2023) 025802

# Relativistic density functional for quark matter

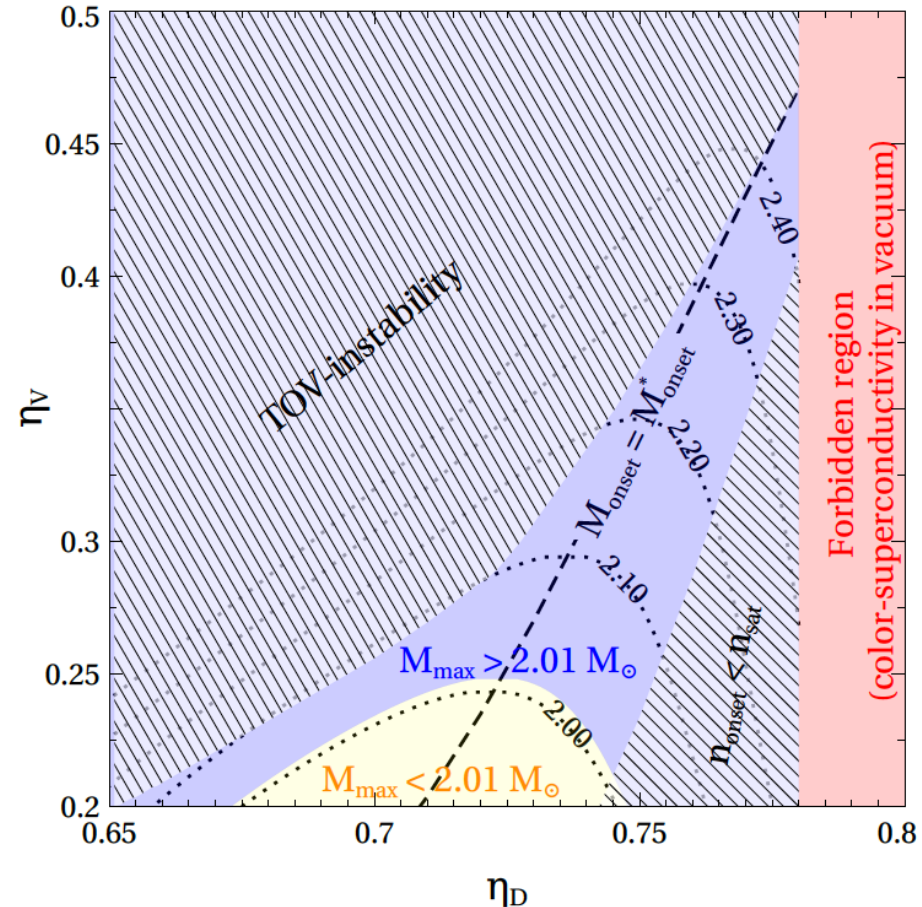
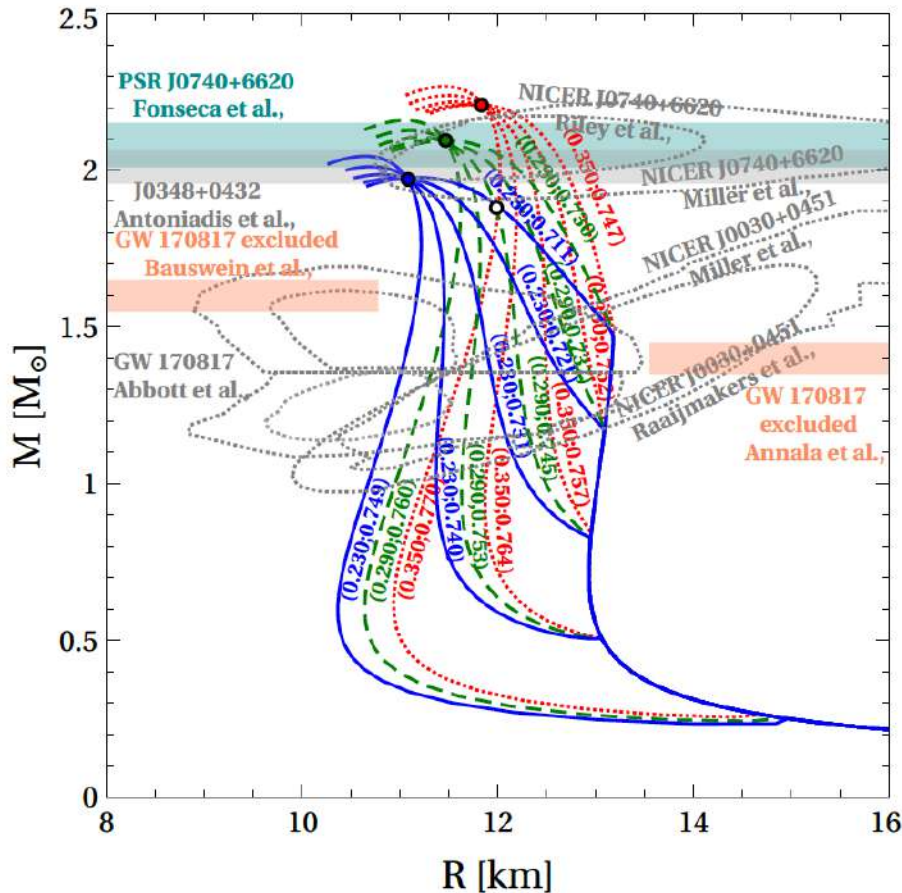
## Mass-radius diagram for hybrid neutron stars



Observational data prefer early deconfinement?

# Relativistic density functional for quark matter

## Mass-radius diagram for hybrid neutron stars



$$M_{\max} = M_{\text{SP}} + \delta |M_{\text{onset}}^* - M_{\text{onset}}|^{\kappa}$$

$$M_{\text{SP}} = k_{M_{\text{SP}}} \eta_V + b_{M_{\text{SP}}}; \quad \delta = k_{\delta} \eta_V + b_{\delta}; \quad M_{\text{onset}}^* = 1.254 M_{\odot}$$

C. Gärtlein et al., arXiv:2301.10765v2 ; For more details, see talk by **Oleksii Ivanytskyi**

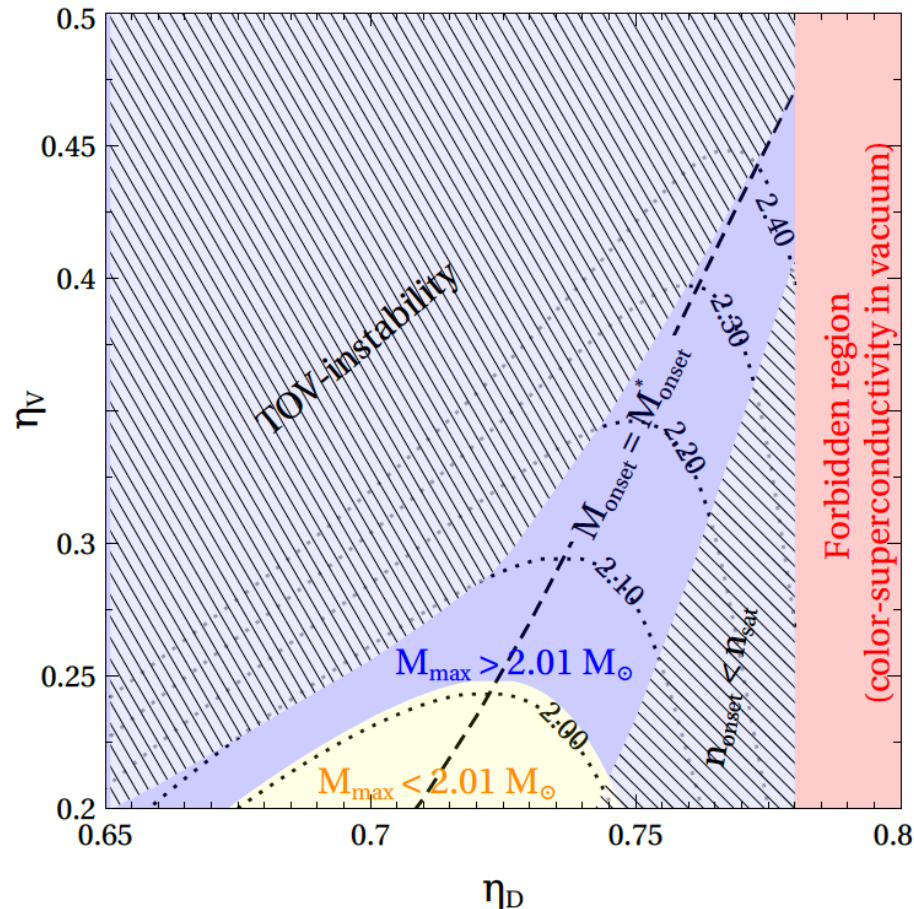
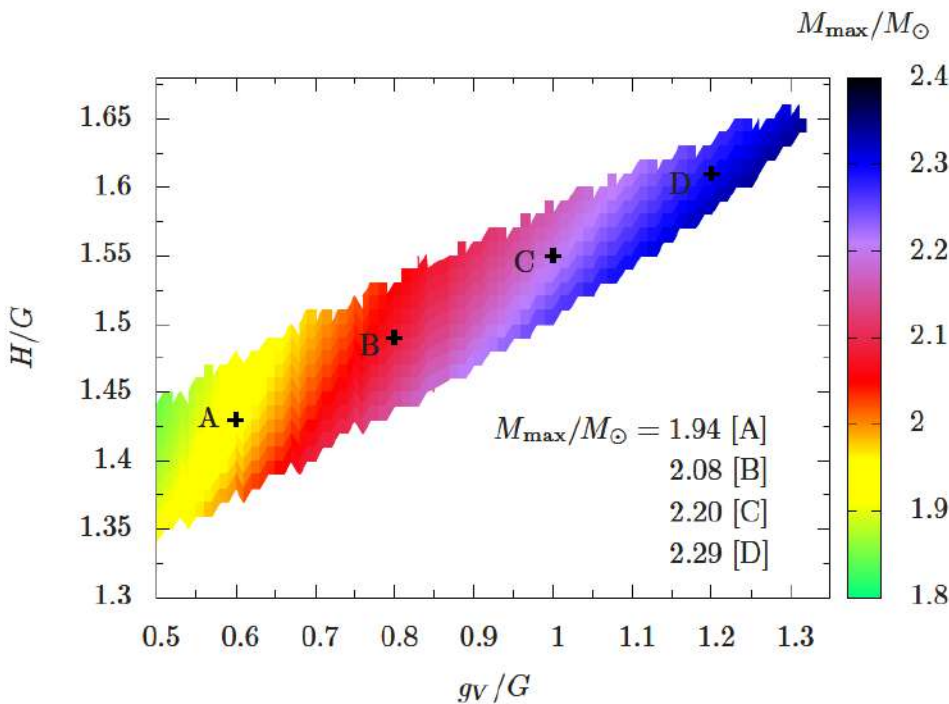


# Relativistic density functional for quark matter

## Mass-radius diagram for hybrid neutron stars

Compare to:

G. Baym, S. Furusawa, T. Hatsuda, T. Kojo, H. Togashi, ApJ 885, 42 (2019)



$$M_{\max} = M_{\text{SP}} + \delta |M_{\text{onset}}^* - M_{\text{onset}}|^{\kappa} \quad M_{\text{SP}} = k_{M_{\text{SP}}} \eta_V + b_{M_{\text{SP}}}; \quad \delta = k_{\delta} \eta_V + b_{\delta}; \quad M_{\text{onset}}^* = 1.254 M_{\odot}$$

C. Gärtlein et al., arXiv:2301.10765v2 ; For more details, see talk by **Oleksii Ivanytskyi**



# Relativistic density functional for quark matter

## Phase diagram with two-zone interpolation

- **Normal quark matter**

2 spin  $\times$  2 flavor  $\times$  3 color = 12

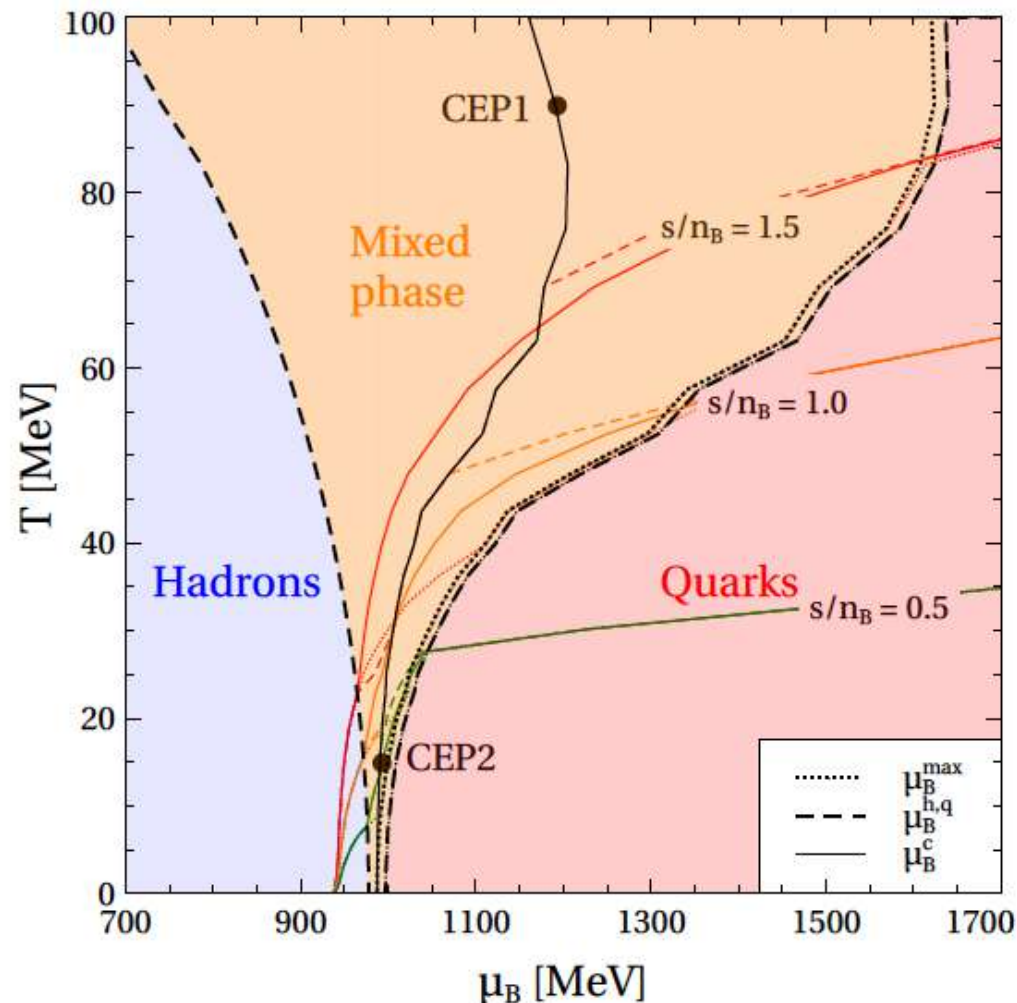
- **2SC quark matter**

2 spin  $\times$  2 flavor  $\times$  1 color + 1 = 5

Quark pairing reduces  
number of quark states



requires higher  $T$   
along adiabat



→ EOS tables are prepared for simulation of supernovae and NS mergers

# Relativistic density functional for quark matter

## Phase diagram with two-zone interpolation

- **Normal quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 3 \text{ color} = 12$$

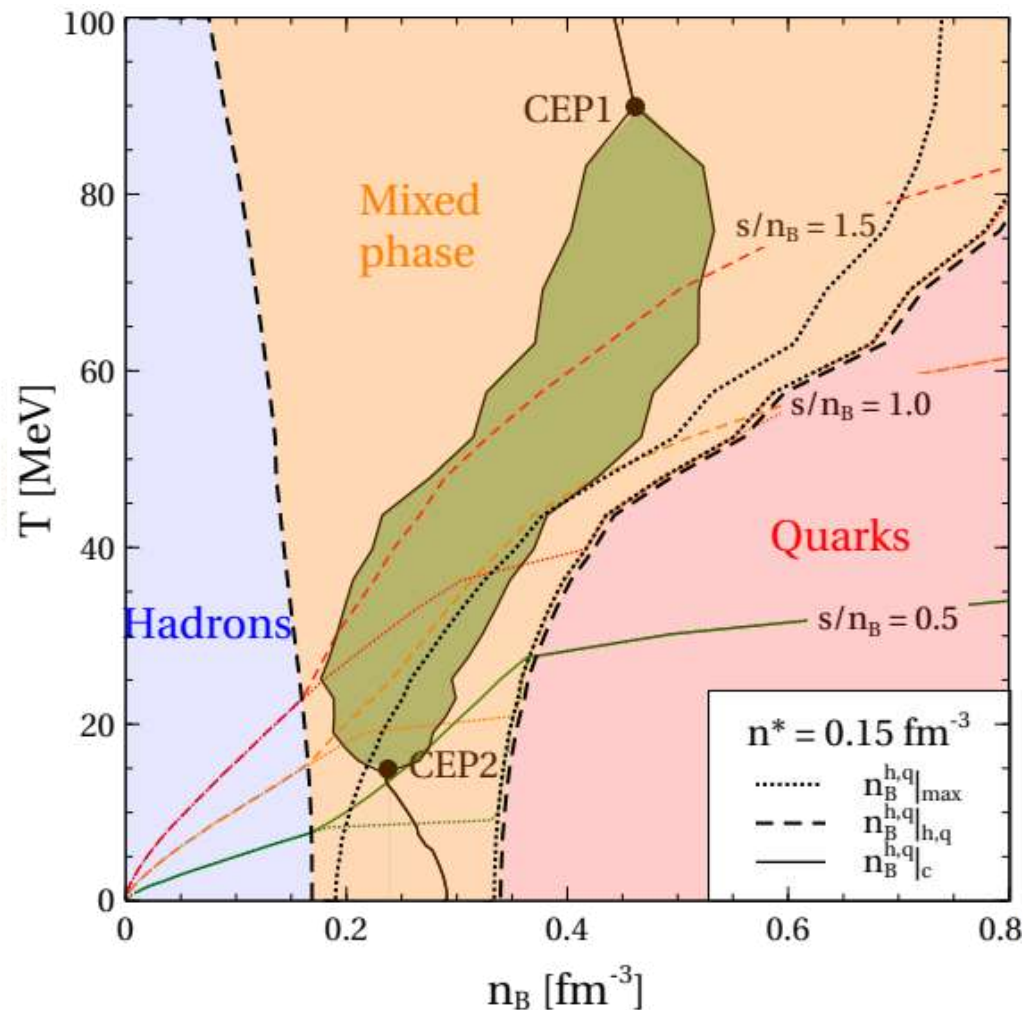
- **2SC quark matter**

$$2 \text{ spin} \times 2 \text{ flavor} \times 1 \text{ color} + 1 = 5$$

Quark pairing reduces number of quark states

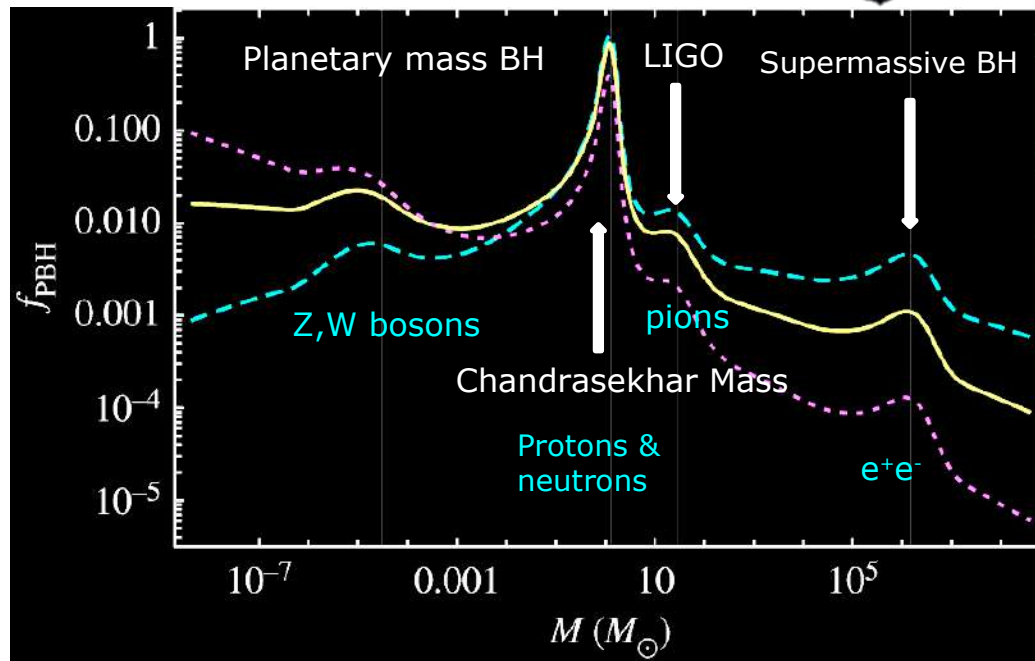
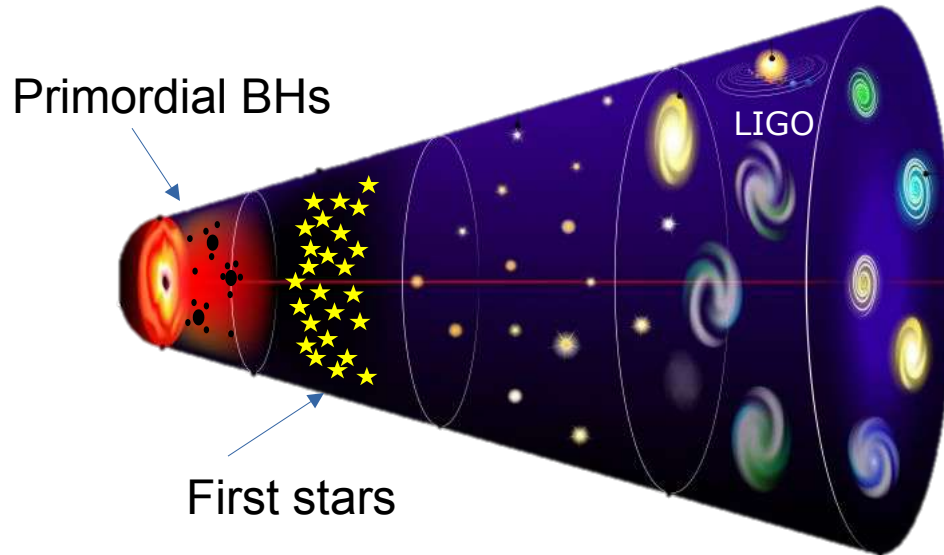


requires higher  $T$  along adiabat



→ EOS tables are prepared for simulation of supernovae and NS mergers

# JWST results – primordial black holes !



Talk at University of Wroclaw  
by Günther Hasinger,  
Founding director of the  
German Centre for Astrophysics  
In Görlitz:



**Key role plays the QCD  
hadronization transition !**

Different peaks correspond to  
different particles created at the early  
universe phase transitions and the  
corresponding reduction in the sound  
velocity.

BH mass corresponds to the horizon  
size at each time.

Only requirement is enough  
fluctuation power in a volume fraction  
of  $10^{-9}$  of the early Universe.

**Carr, Clesse, García-Bellido 2019**



# Let us discover the 3<sup>rd</sup> family of compact stars!

(Title of a talk I gave 5 years ago at MPIfR Bonn)



# Let us discover the 3<sup>rd</sup> family of compact stars!

Prehistory ... 55 years ago: Gerlach coined „3rd family“

PHYSICAL REVIEW

VOLUME 172, NUMBER 5

25 AUGUST 1968

## Equation of State at Supranuclear Densities and the Existence of a Third Family of Superdense Stars\*†

ULRICH H. GERLACH‡§

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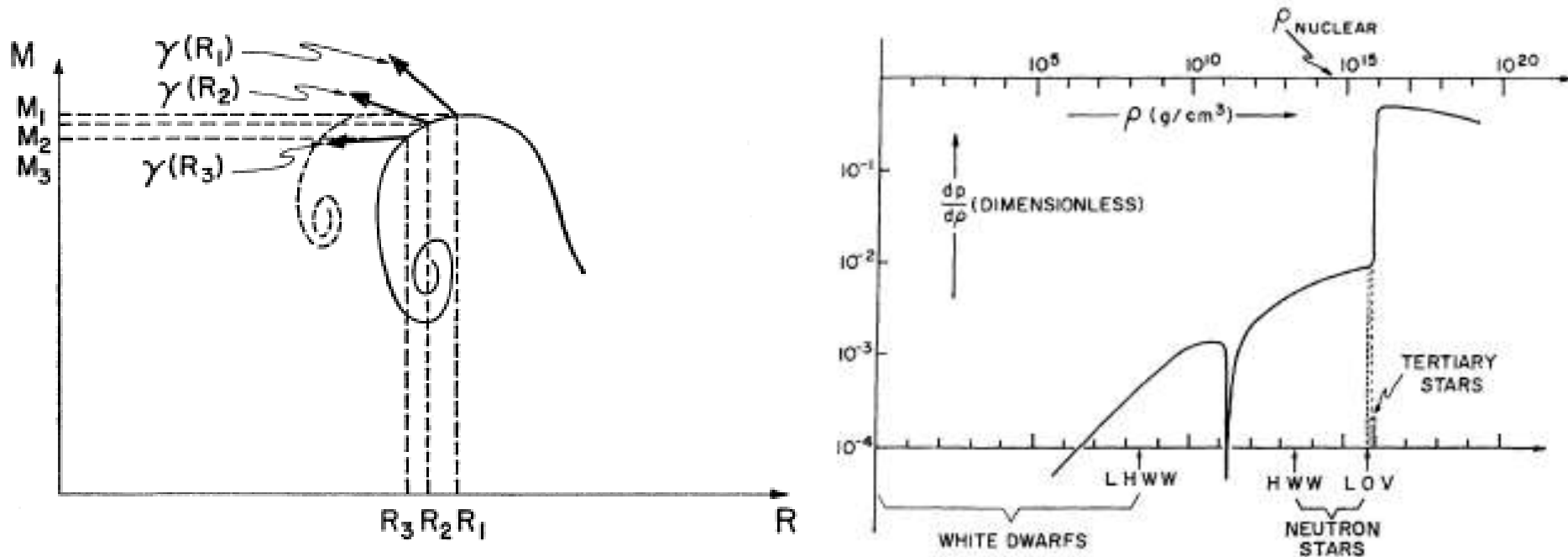
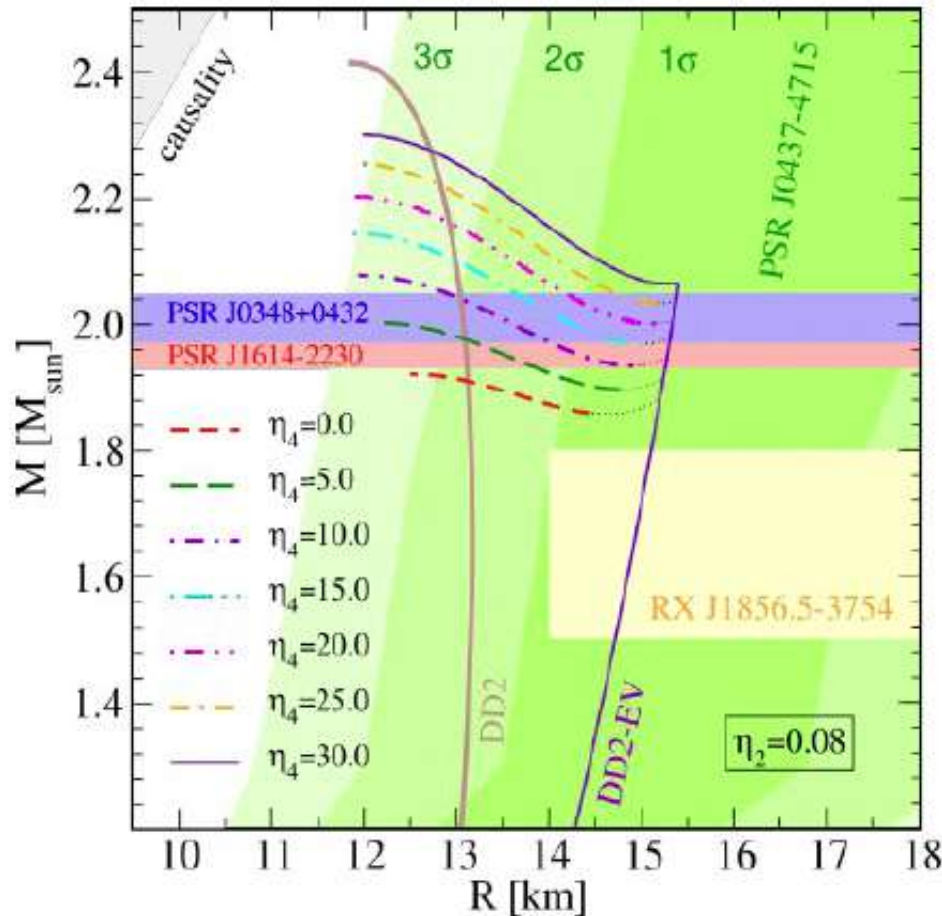


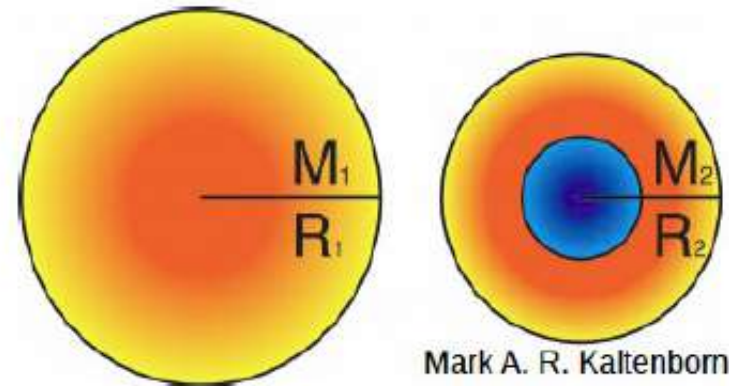
FIG. 3. A blow-up of the LOV maximum. The central densities

# Let us discover the 3<sup>rd</sup> family of compact stars!

Prehistory ... 10 years ago (2013) we revived the idea



- Star configurations with same masses, but different radii

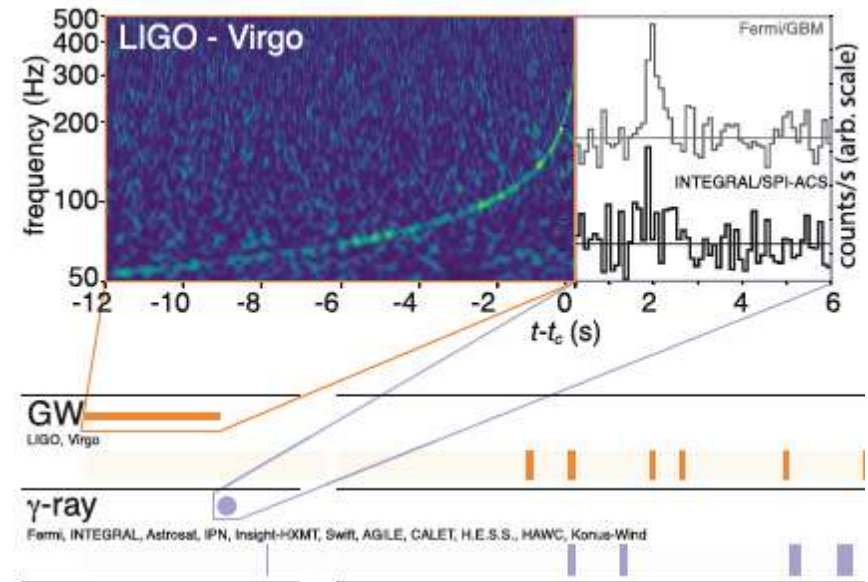
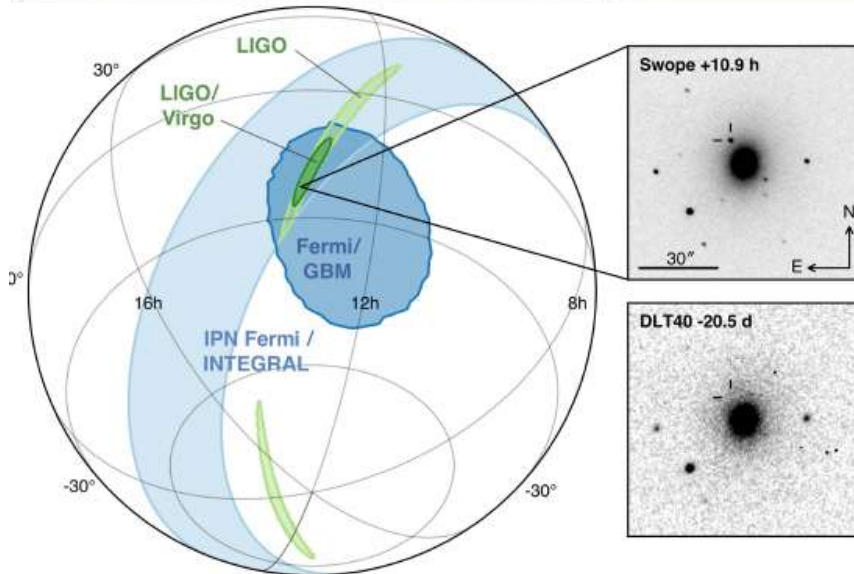


- **New class of EOS, that features high mass twins**
- NASA NICER mission: radii measurements  $\sim 0.5$  km
- Existence of twins implies 1<sup>st</sup> order phase-transition and hence a critical point



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Prehistory ... GW170817 happened!



GW170817, announced on 16.10.2017

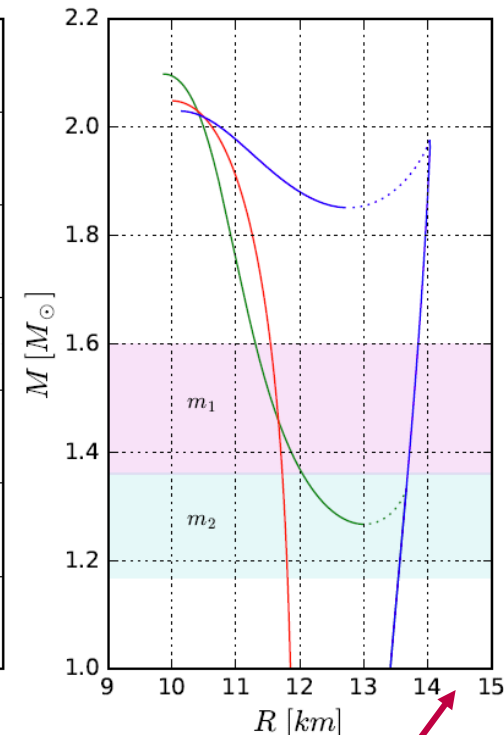
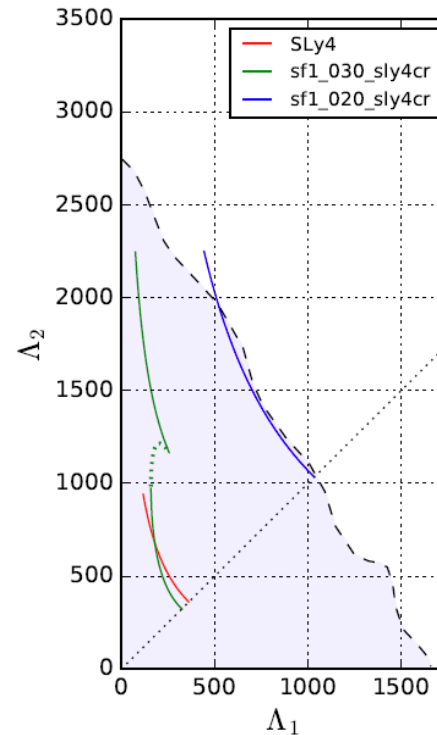
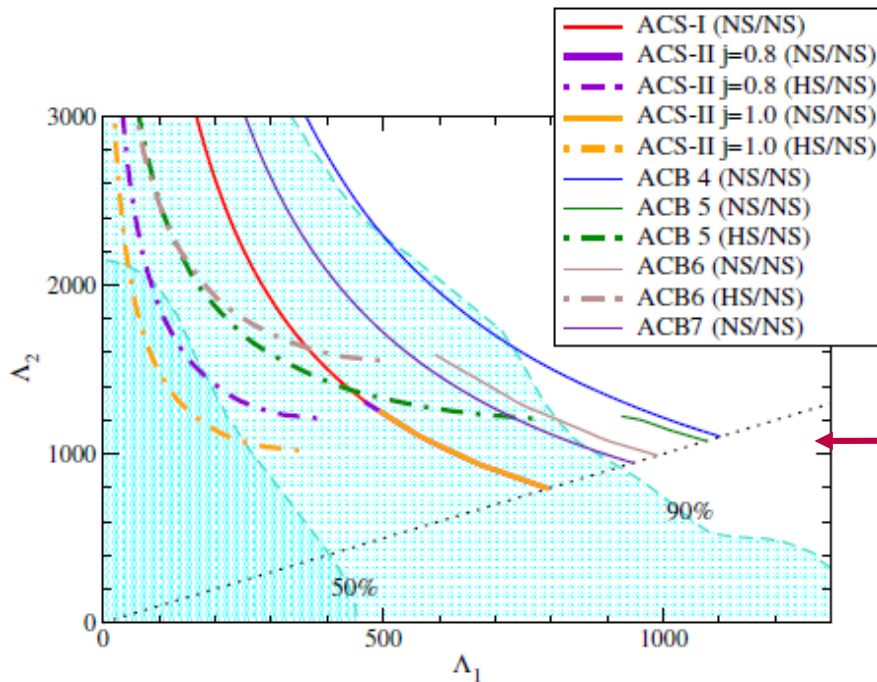
B.P. Abbott et al. [LIGO/Virgo Collab.], PRL 119, 161101 (2017); ApJLett 848, L12 (2017)

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Prehistory ...

Low-spin priors ( $|\chi| \leq 0.05$ )

Primary mass $m_1$	1.36–1.60 $M_\odot$
Secondary mass $m_2$	1.17–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio $m_2/m_1$	0.7–1.0
Total mass $m_{\text{tot}}$	$2.74^{+0.04}_{-0.01} M_\odot$
Radiated energy $E_{\text{rad}}$	$> 0.025 M_\odot c^2$
Luminosity distance $D_L$	$40^{+8}_{-14}$ Mpc



M. Bejger, D.B., et al., A&A 600 (2017) A39

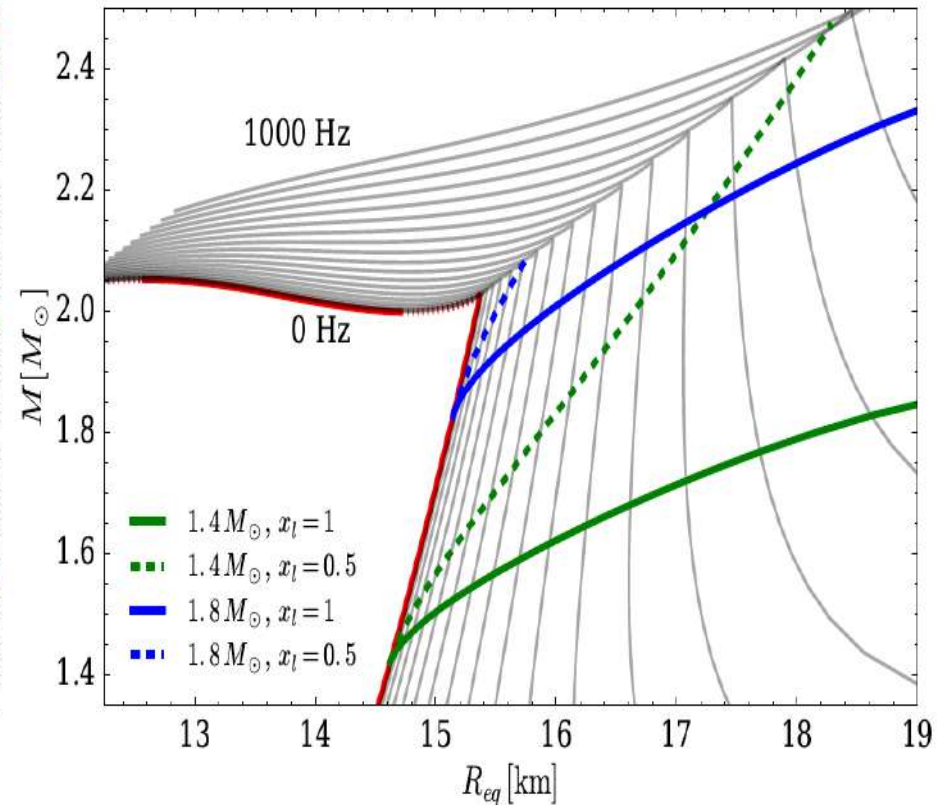
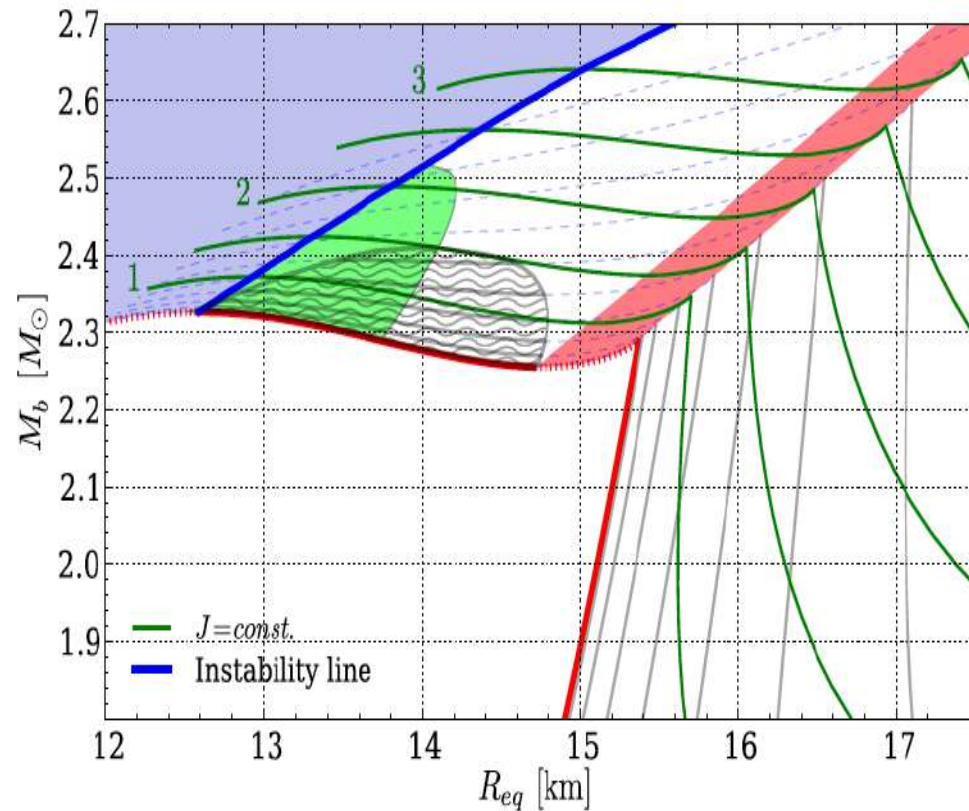
V. Paschalidis, K. Yagi, D. Alvarez-Castillo, D.B., A. Sedrakian, arxiv:1712.00451  
Phys. Rev. D97 (2018) 084038

**Suggestion:** The heavier NS be a hybrid star (HS) with a quark core, evtl. member of a “third family”!

# Let us discover the 3<sup>rd</sup> family of compact stars!

## Observation:

With a strong PT (mass twins), a sudden transition NS  $\rightarrow$  HS is possible,  
Triggered by accretion, under simultaneous conservation of  $M_b$  and  $J$



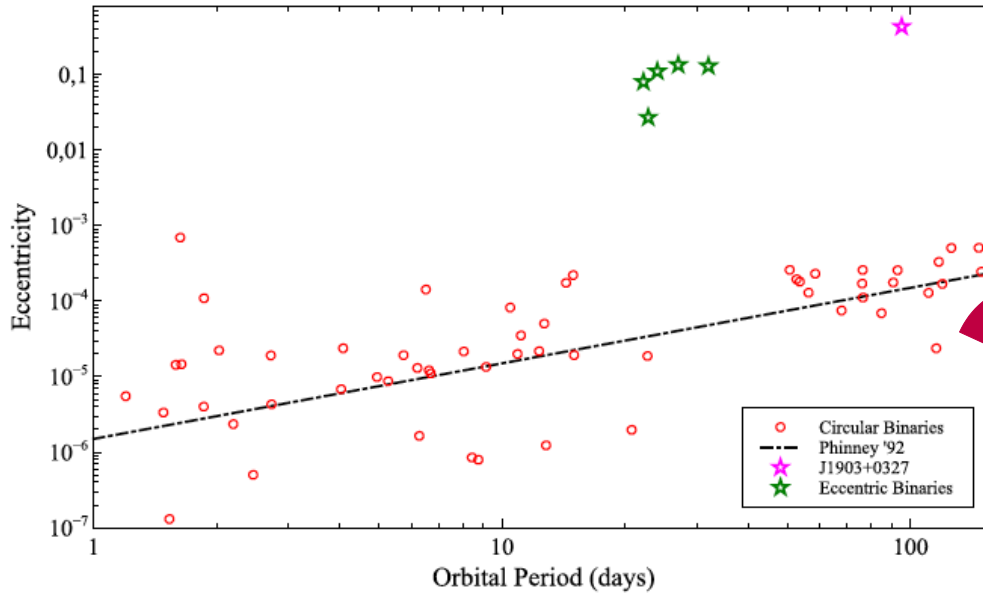
M. Bejger, D.B., et al., A&A 600 (2017) A39

Evolutionary tracks for disc accretion of mass with Efficiency  $x_l=1(0.5)$  of angular momentum transfer



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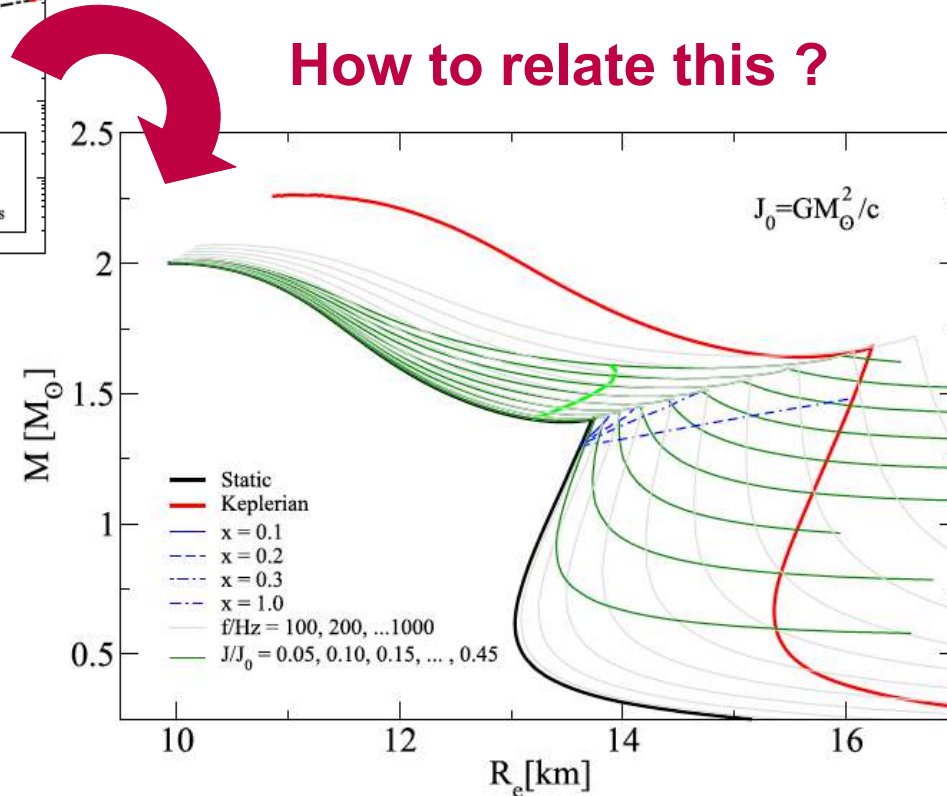
## Antoniadis-puzzle



J. Antoniadis, ApJ Lett. 797, L24 (2014)

K. Stovall, P.C.C. Freire, J. Antoniadis, ApJ 870(2), 74 (2019)

**How to relate this ?**



D.E. Alvarez.Castillo, J. Antoniadis, A. Ayriyan, D. B., V. Danchev, H. Grigorian, N. Khosravi Largani, F. Weber,

Accretion-induced collapse to third family compact stars as trigger for eccentric orbits of Millisecond pulsars in binaries,

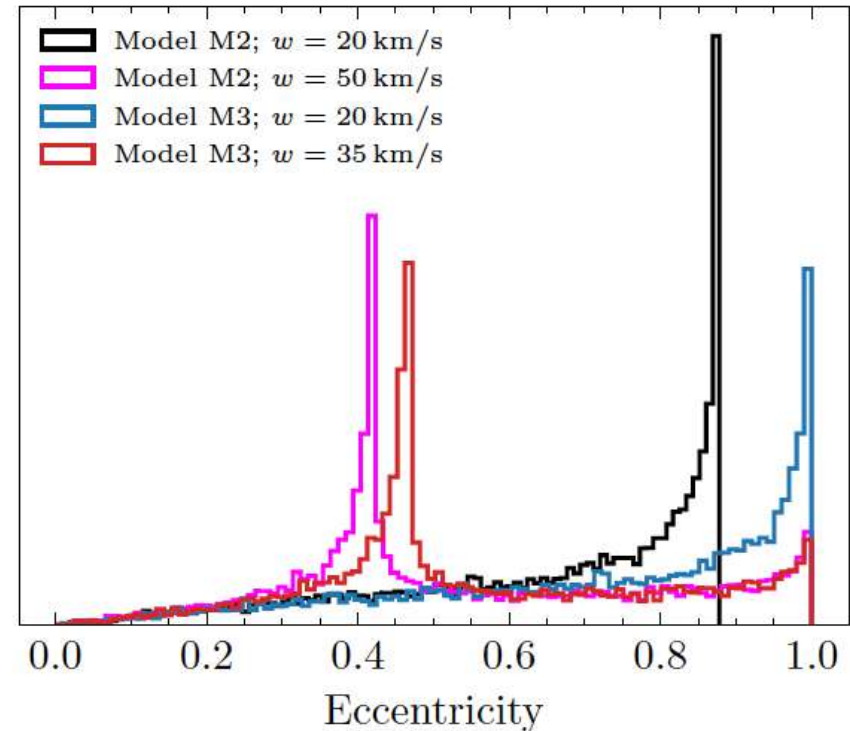
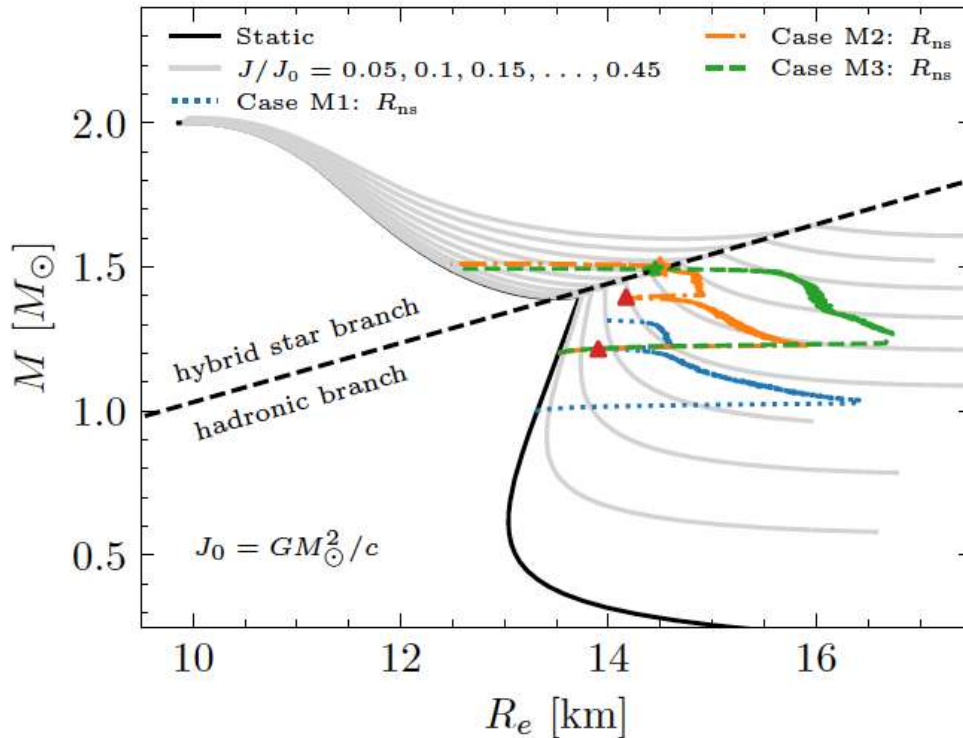
Astron. Nachr. 340 (2019) 878;  
 ArXiv:1912.08782 [astro-ph.HE]



# Let us discover the 3<sup>rd</sup> family of compact stars!

Work in preparation ...

S. Chanlaridis, D. Ohse, A. Aspradakis, J. Antoniadis, D. Blaschke, D.E. Alvarez-Castillo, V. Danchev and N. Langer

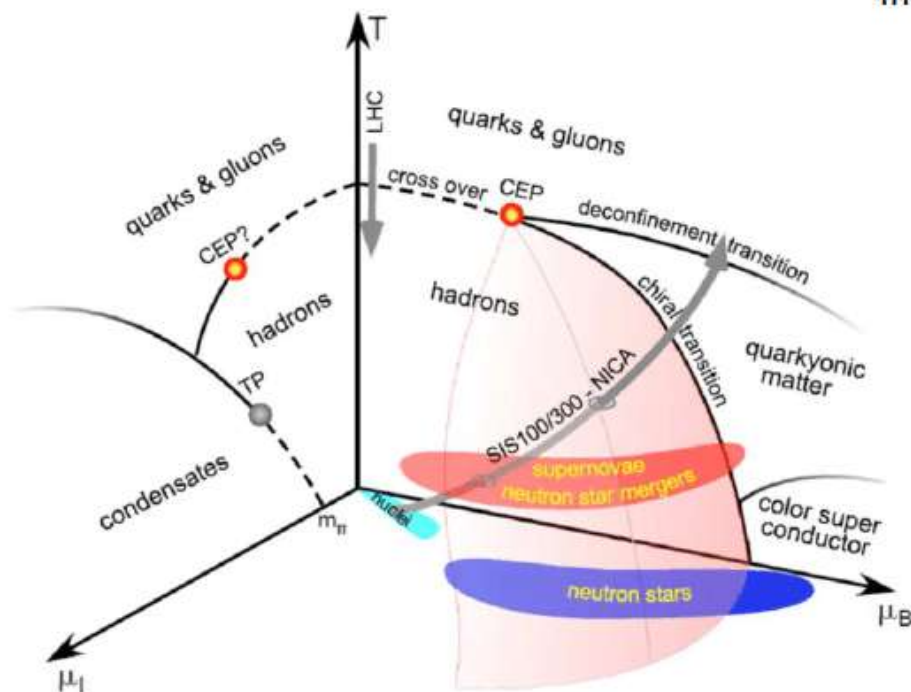


Prescription for transition-triggered kicks adapted from supernova case following:  
J.G. Hills, ApJ 267, 322 (1983) and T.M. Tauris et al., ApJ 846, 170 (2017)

# Conclusion

## Density functional methods solve obstacles in neutron star astrophysics

Prominent contributions to deconfinement in modern multimessenger Astrophysics:



From: NuPECC Long Range Plan 2017

- Quark deconfinement transition triggers the **supernova explosion** of a very massive ( $M = 50M_{\odot}$ ) blue supergiant progenitor star  
T. Fischer et al., Nature Astron. 2 (2018) 960
- Unambiguous signal of a strong phase transition in the postmerger GW from a binary **NS merger** predicted  
A. Bauswein et al., Phys. Rev. Lett. 122 (2019) 061102
- Strong deconfinement phase transition in NS can be detected by observing the **mass twin star** phenomenon  
D. B. et al., Universe 6 (2020) 81

See also: Agnieszka Sorensen et al., Dense nuclear matter EOS from HIC, arXiv:2301.13253

## Research Technology Digitization

„Science Creating Prospects  
for the Region!“



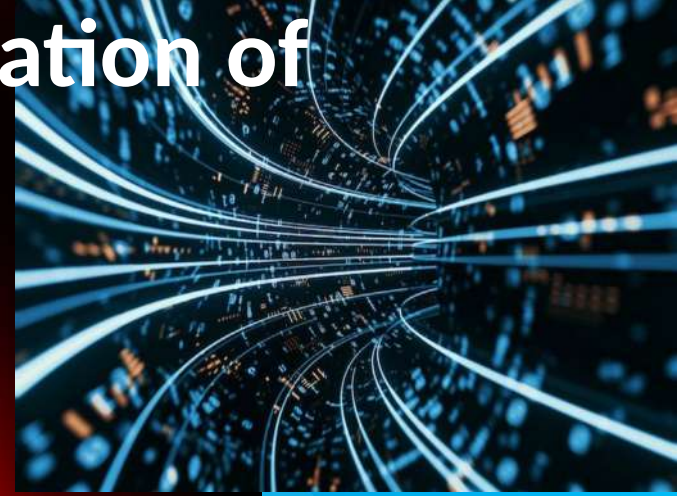
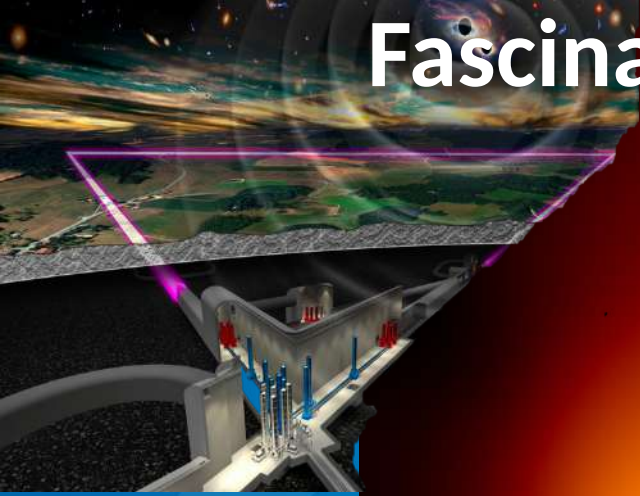
Scientific Commission: 13. July 2022

Structural and Transfer-Commission: 30. August 2022

Final decision (Approval): 29. September 2022



# Fascination and inspiration of astrophysics

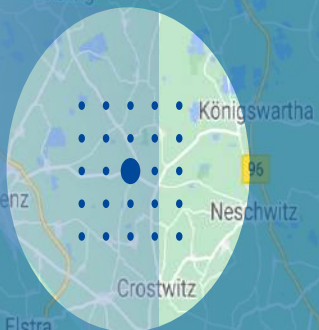




# Why in Saxony? Lusatia is a unique region for Astrophysics, Technology and Digitization



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