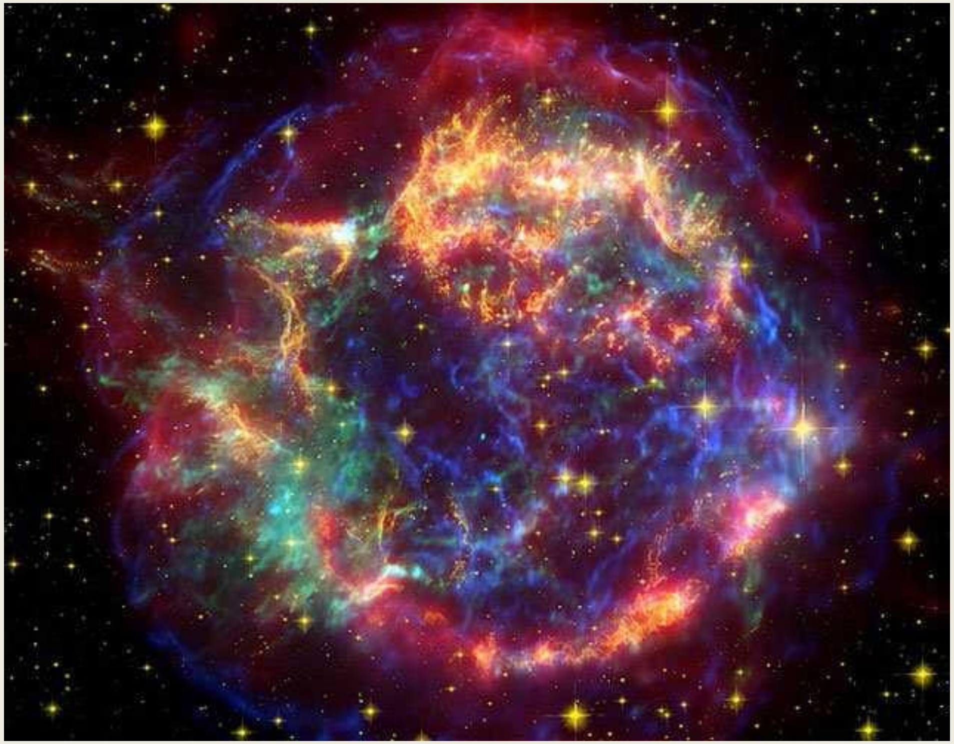
Neutron Stars Mass estimations from Cooling Evolution



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(Yeravan, Armenia)

MPCS – 2023
12-16 September
Yerevan, Armenia

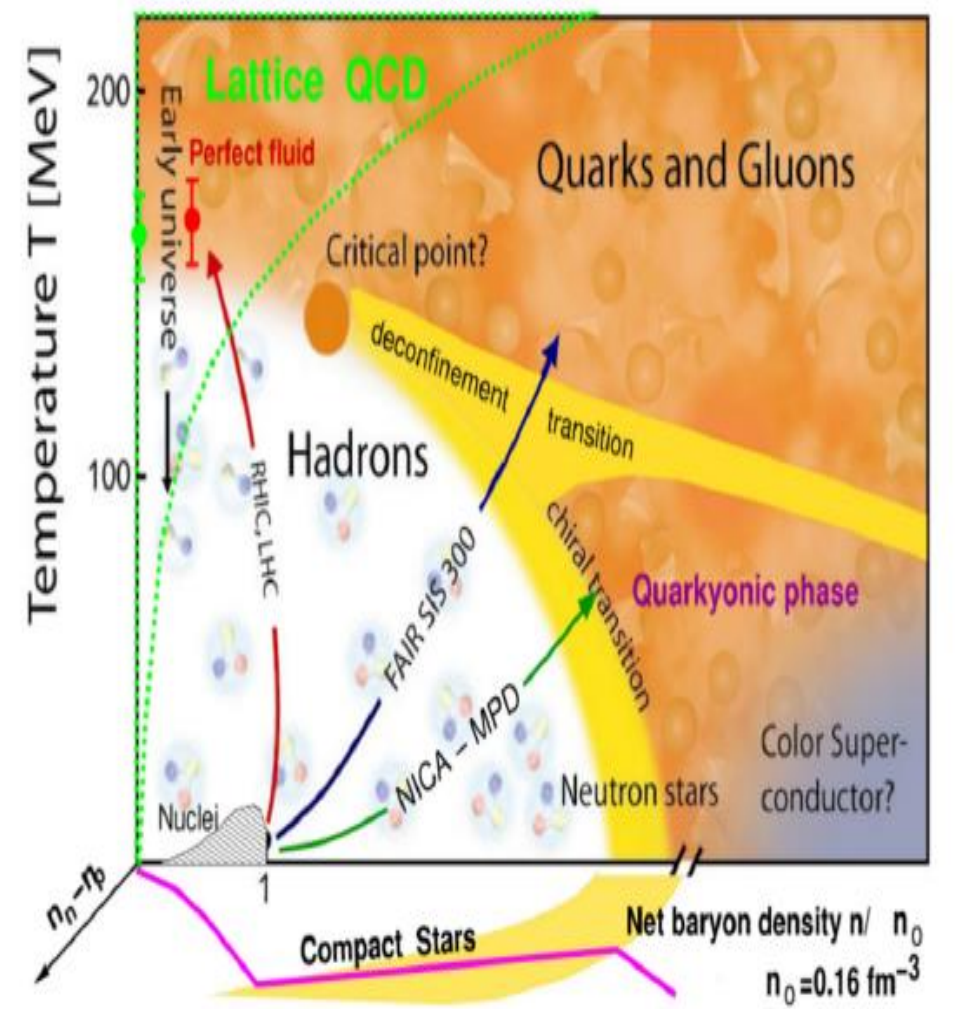
my co-authors: D.Blaschke, D.Voskresensky,
A. Ayriyan E. Kolomeitsev, K. Maslov,

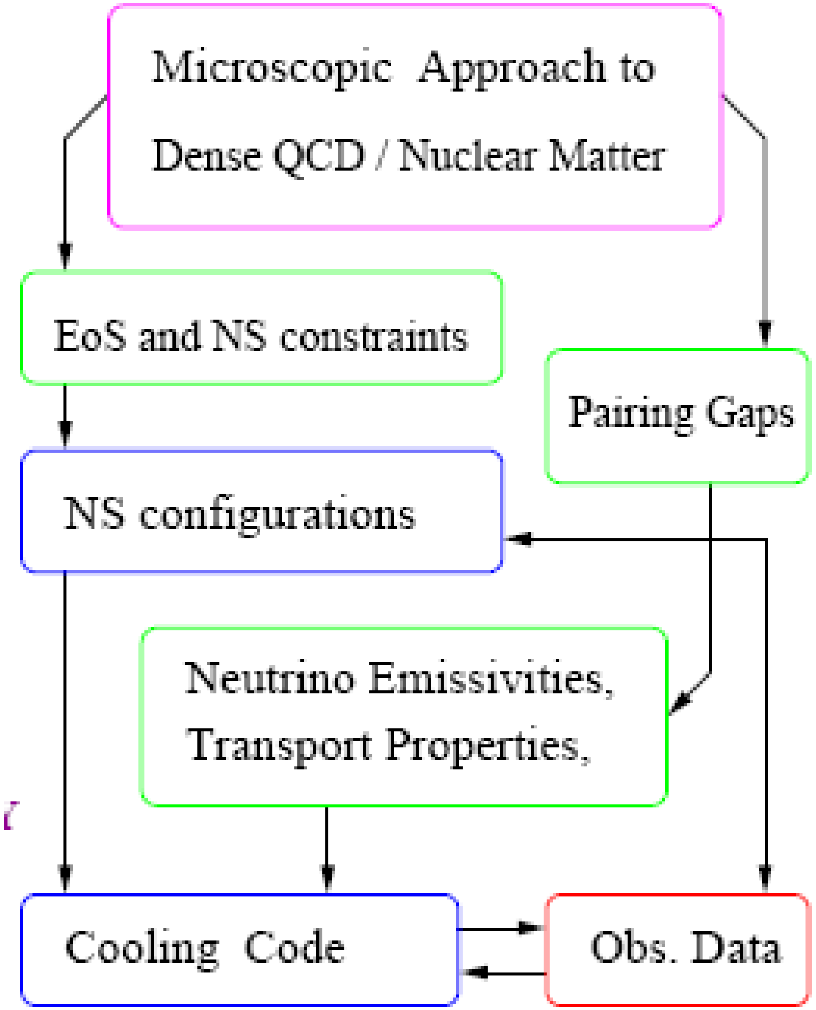
Simulation of Cooling Evolution of Neutron Stars

- Motivation
- Neutron Stars structure
- Neutron Stars cooling problem
- ***Results for NS cooling***
- ***Mass extraction***

H. Grigorian, D. N. Voskresensky and D. Blaschke
Eur. Phys. J. A **52: 67 (2016).**

Phase Diagramm & Cooling Simulation





Microscopic Approach to
Dense QCD / Nuclear Matter

EoS and NS constraints

Pairing Gaps

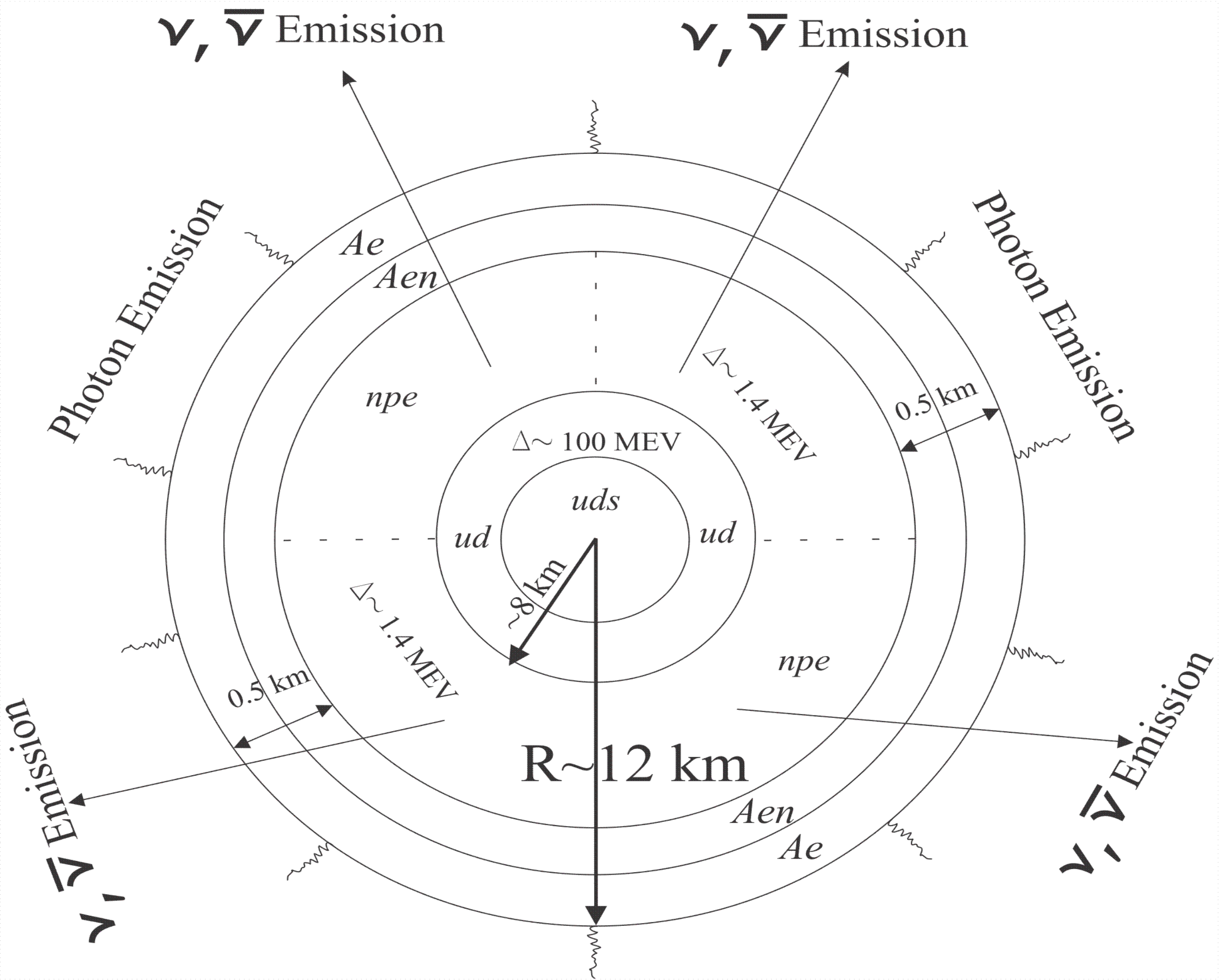
NS configurations

Neutrino Emissivities,
Transport Properties,

Cooling Code

Obs. Data

Structure Of Hybrid Star



Static neutron star mass and radius

The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2]:

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2} \dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)}; \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\ \dfrac{dN_B(r)}{dr} = 4\pi r^2 \left(1-\dfrac{2GM(r)}{r}\right)^{-1/2} n(r). \end{cases}$$

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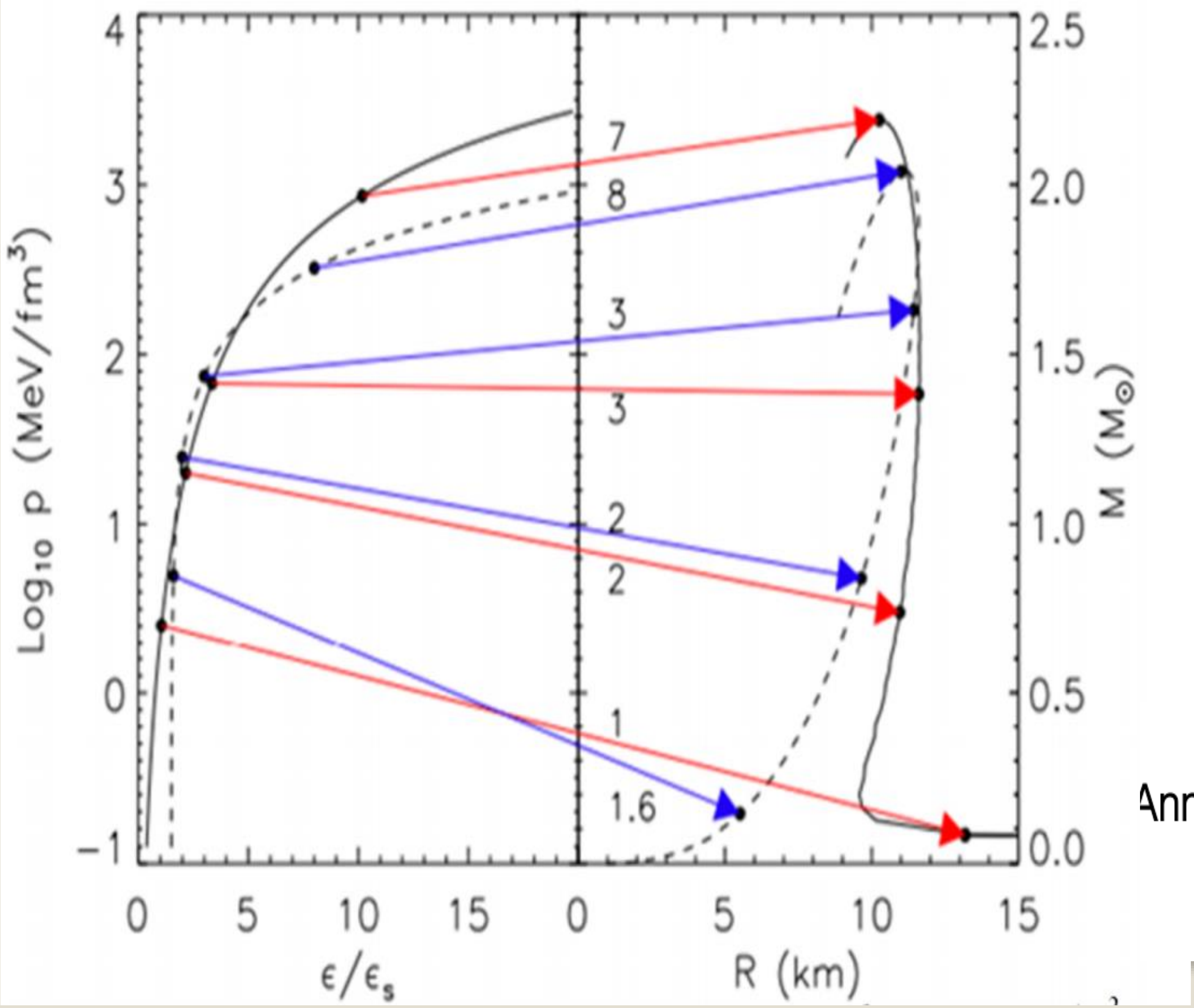
[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939).

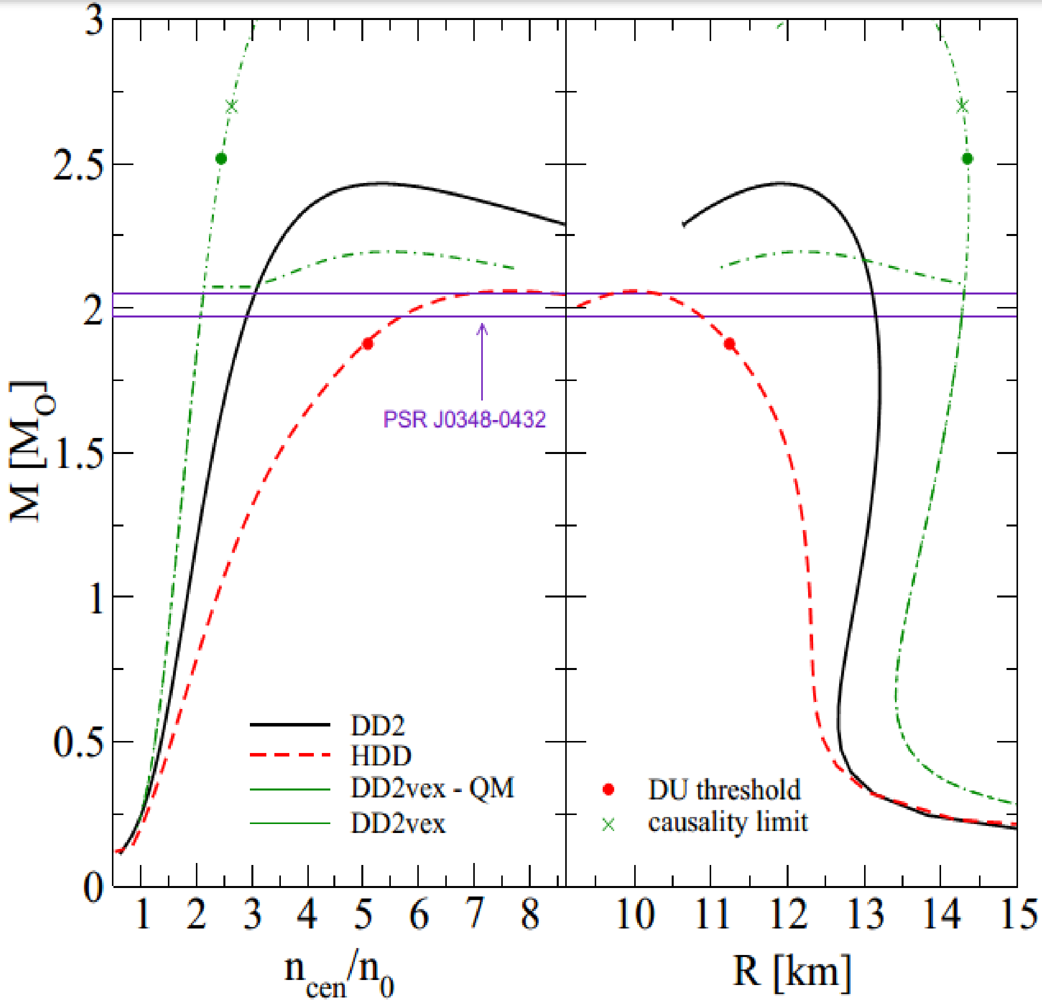
[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

EoS vs. Mass Radious of NS

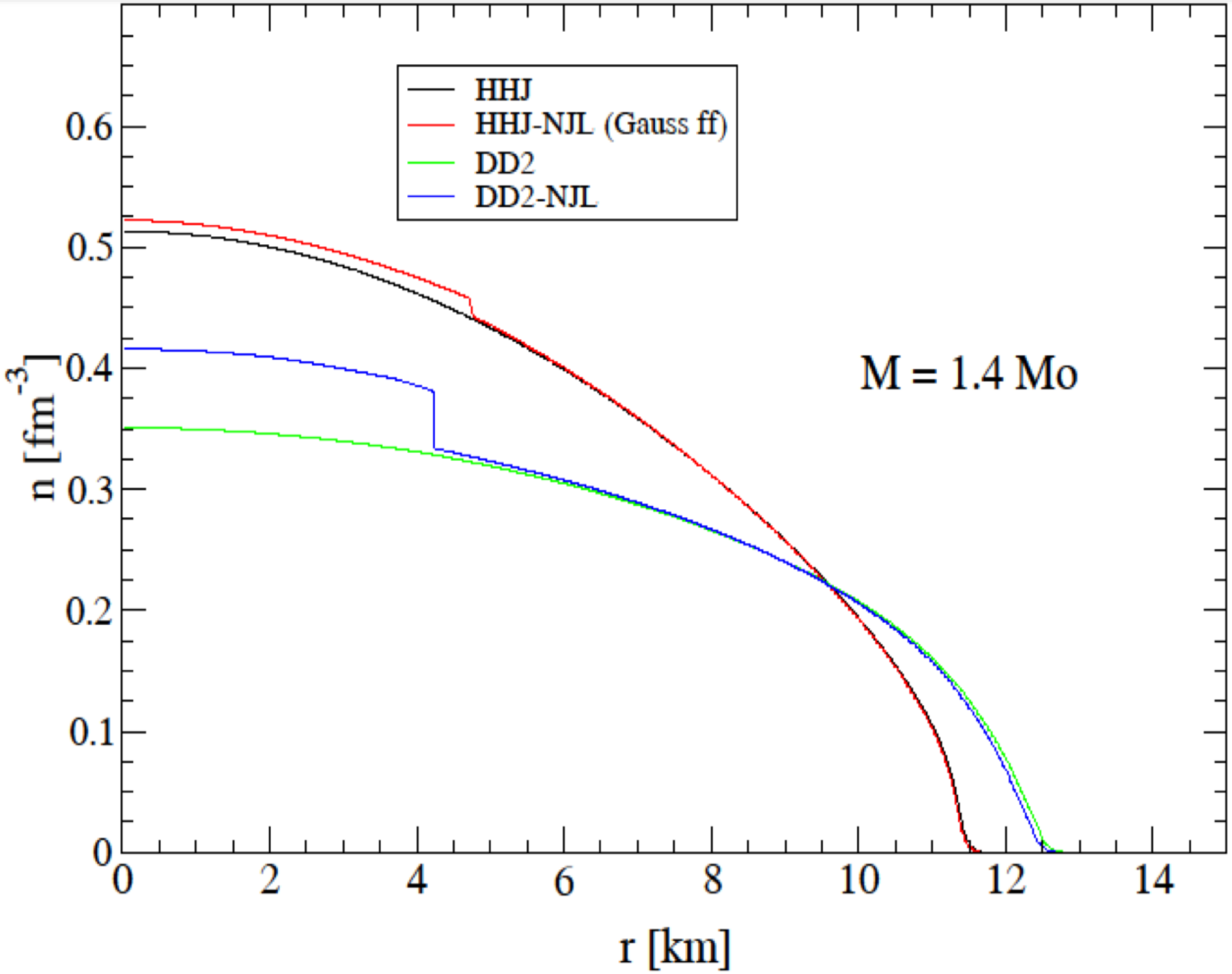


Lattimer,
Annu. Rev. Nucl. Part. Sci. 62,
485 (2012)
arXiv: 1305.3510

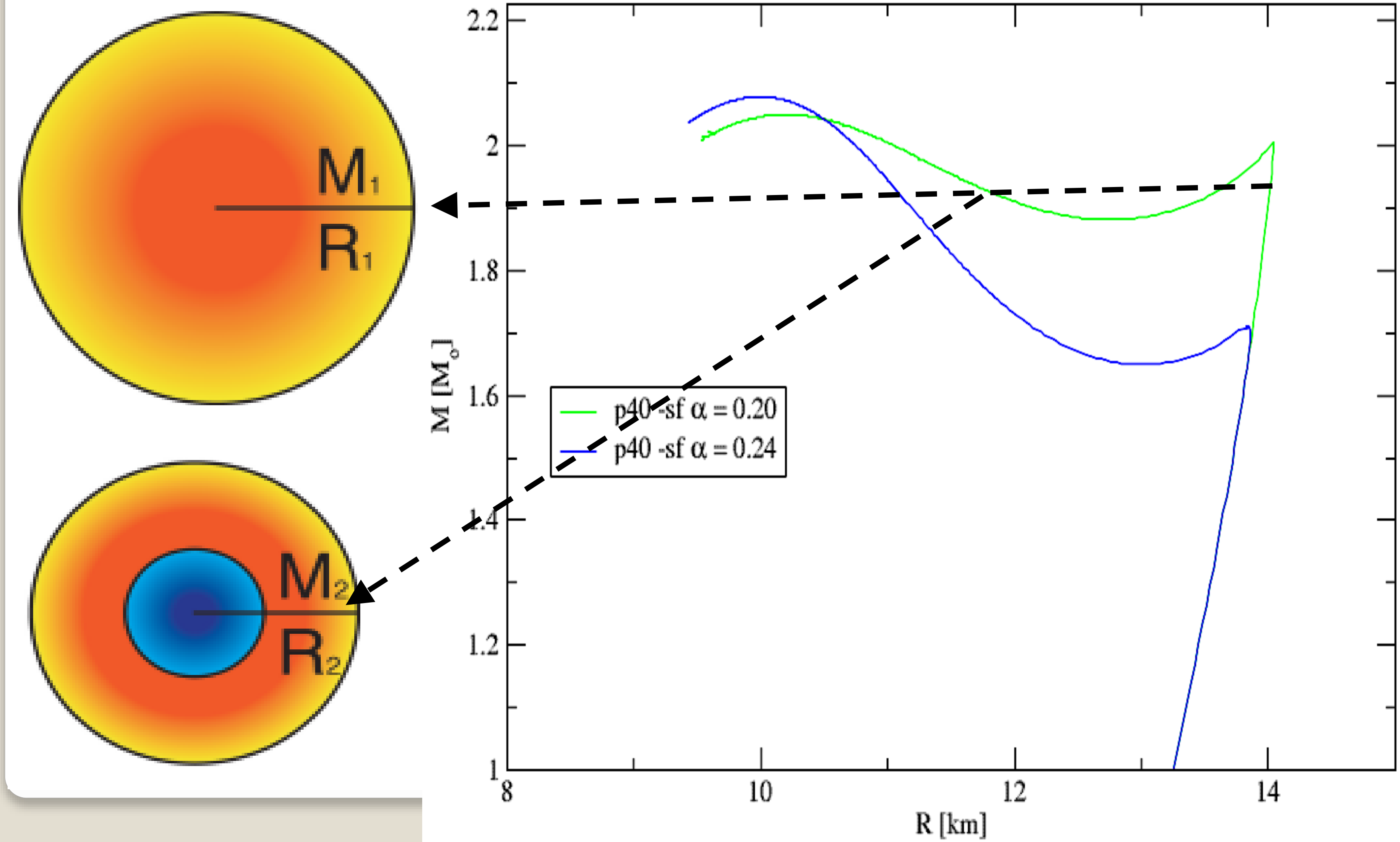
Stability of stars HDD, DD2 & DDvex-NJL EoS models



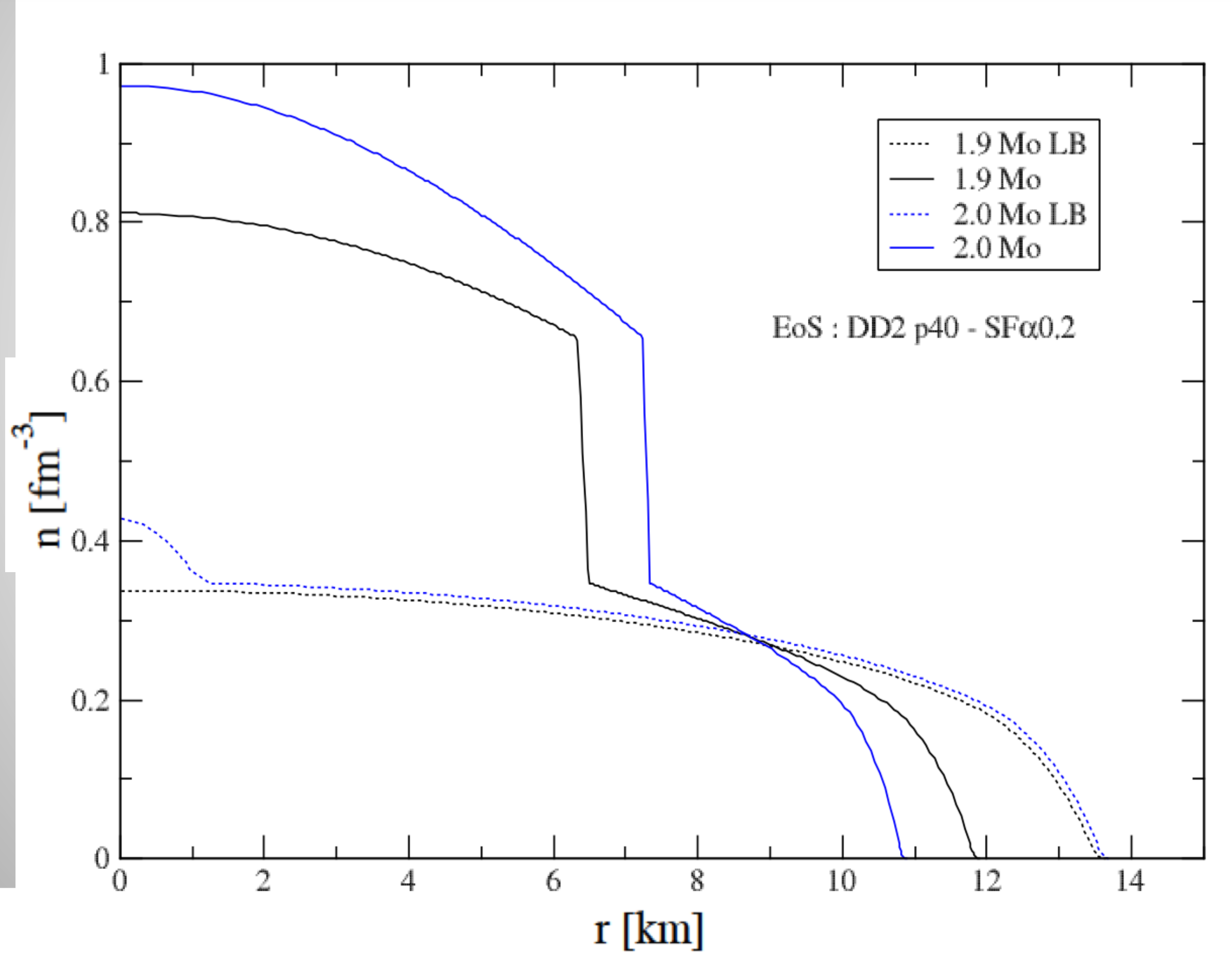
Different Configurations with the same NS mass



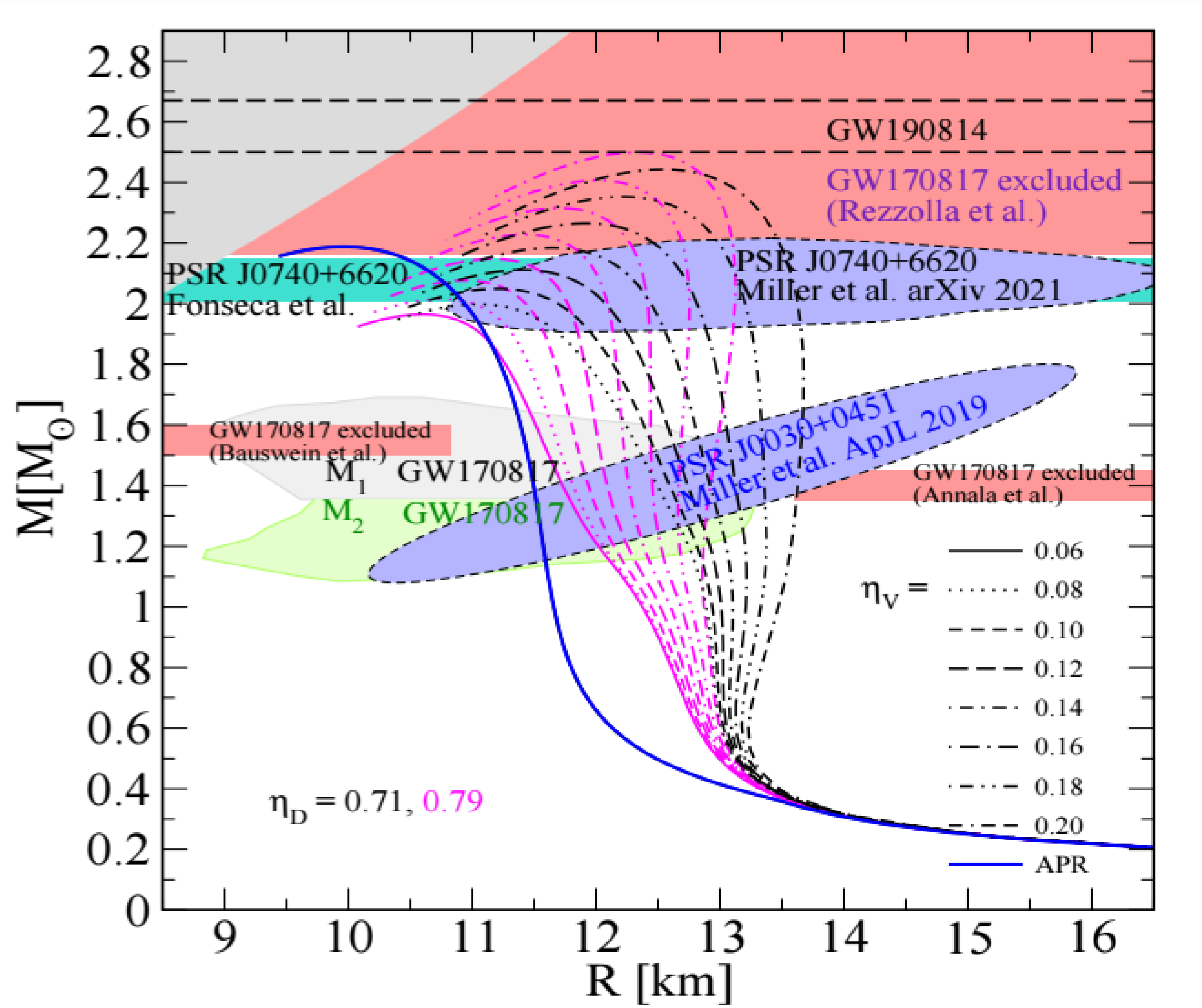
High Mass Twin CS



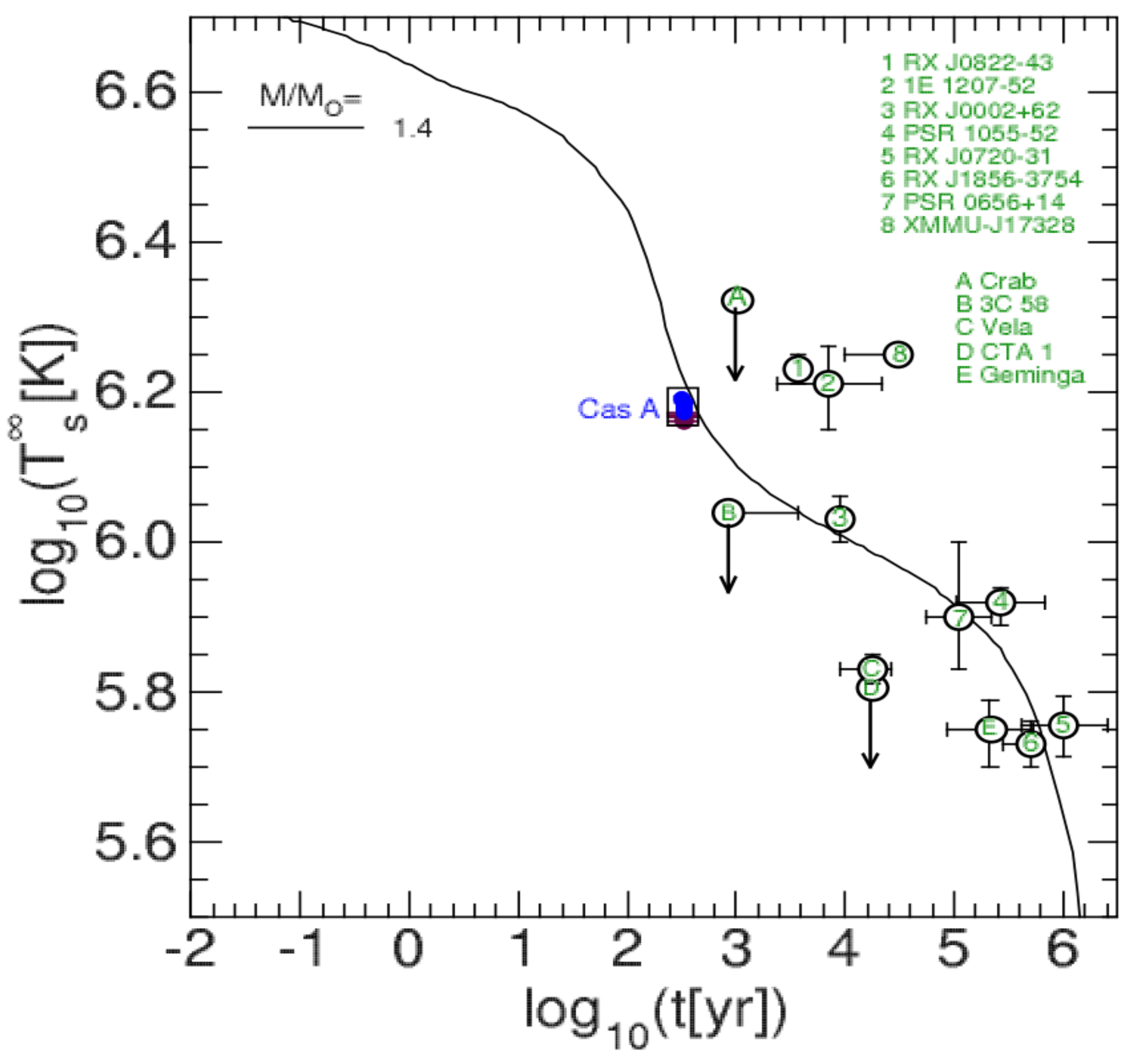
Different Configurations with the same NS mass



Modern MR Data and Models

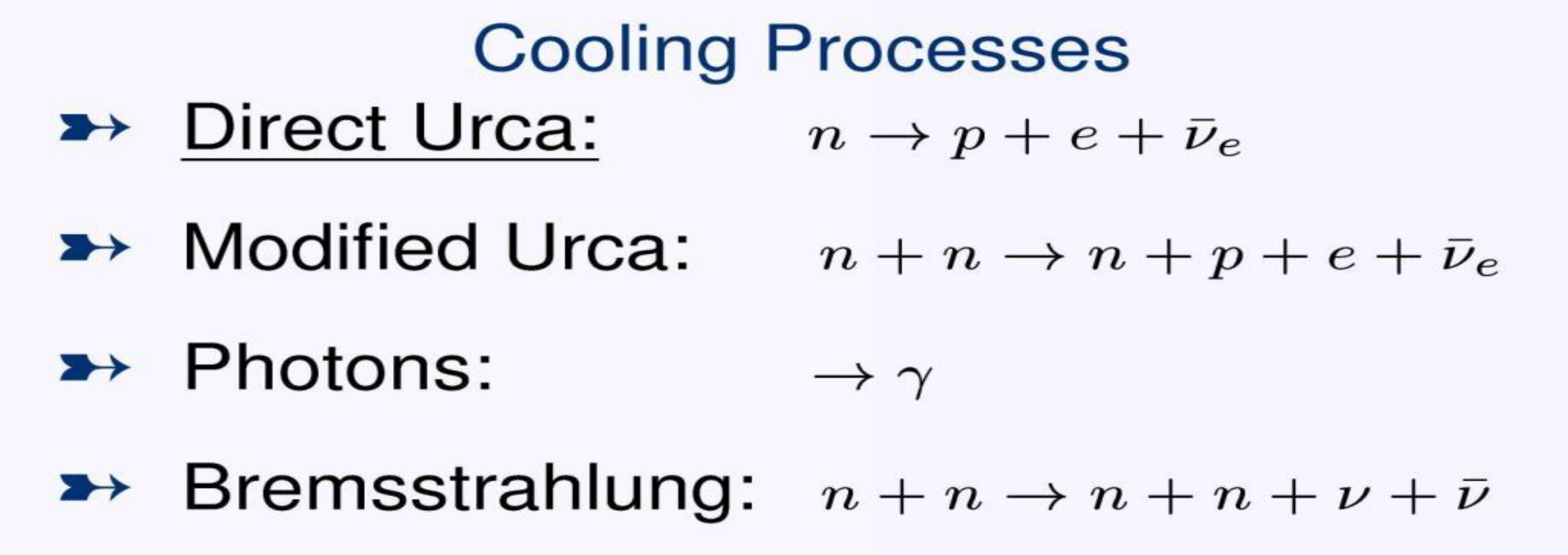


Surface Temperature & Age Data



Cooling Mechanism

$$\frac{dU}{dt} = \sum_i C_i \frac{dT}{dt} = -\varepsilon_\gamma - \sum_j \varepsilon_\nu^j$$



Cooling Processes

- **Direct Urca:** $n \rightarrow p + e + \bar{\nu}_e$
- **Modified Urca:** $n + n \rightarrow n + p + e + \bar{\nu}_e$
- **Photons:** $\rightarrow \gamma$
- **Bremsstrahlung:** $n + n \rightarrow n + n + \nu + \bar{\nu}$

Cooling Evolution

The energy flux per unit time l(r) through a spherical slice at distance r from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial (T e^{\Phi})}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(l e^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(T e^{\Phi}))$$

$$\frac{\partial}{\partial N_B}(T e^{\Phi}) = -\frac{1}{k} \frac{l e^{\Phi}}{16\pi^2 r^4 n}$$

where n = n(r) is the baryon number density, NB = NB(r) is the total baryon number in the sphere with radius r

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n (1 - \frac{2M}{r})^{-1/2}$$

F.Weber: Pulsars as Astro. Labs … (1999);

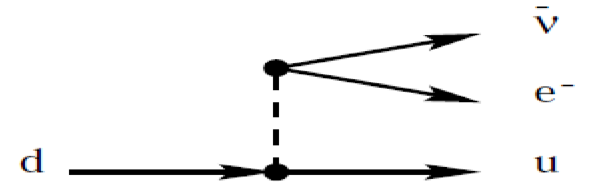
D. Blaschke Grigorian, Voskresensky, A& A 368 (2001)561.

Neutrino emissivities in quark matter:

- Quark direct Urca (QDU) the most efficient processes

$d \rightarrow u + e + \bar{\nu}$ and $u + e \rightarrow d + \nu$

$$\epsilon_\nu^{\mathrm{QDU}} \simeq 9.4 \times 10^{26} \alpha_s u Y_e^{1/3} \zeta_{\mathrm{QDU}}\, T_9^6 \ \mathrm{erg\ cm^{-3}\ s^{-1}},$$

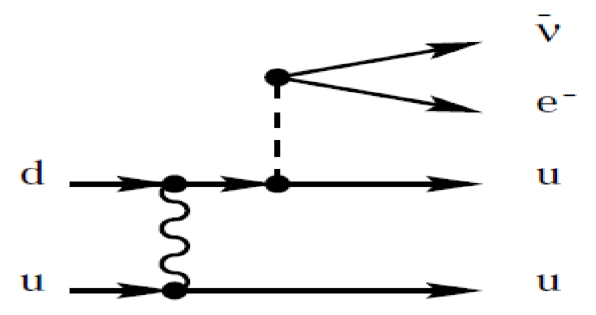


Compression n/no ≃ 2 , strong coupling αs ≈ 1

- Quark Modified Urca (QMU) and Quark Bremsstrahlung

$d + q \rightarrow u + q + e + \bar{\nu}$ and $q_1 + q_2 \rightarrow q_1 + q_2 + \nu + \bar{\nu}$

$$\epsilon_\nu^{\mathrm{QMU}} \sim \epsilon_\nu^{\mathrm{QB}} \simeq 9.0 \times 10^{19} \zeta_{\mathrm{QMU}}\, T_9^8 \ \mathrm{erg\ cm^{-3}\ s^{-1}}.$$



- Suppression due to the pairing

QDU : $\zeta_{\mathrm{QDU}} \sim \exp(-\Delta_q/T)$

QMU and **QB** : $\zeta_{\mathrm{QMU}} \sim \exp(-2\Delta_q/T)$ for $T < T_{\mathrm{crit},q} \simeq 0.57\,\Delta_q$

- Enhanced cooling due to the pairing

- $e + e \rightarrow e + e + \nu + \bar{\nu}$ (becomes important for $\Delta_q/T >> 1$)

$$\epsilon_\nu^{ee} = 2.8 \times 10^{12}\, Y_e^{1/3} u^{1/3} T_9^8 \ \mathrm{erg\ cm^{-3}\ s^{-1}},$$

Quark PBF

Neutrino emissivities in hadronic matter:

- Direct Urca (DU) the most efficient processes

$$\begin{aligned} \epsilon_{DU} &= M_{DU} \quad *(m_p^*)(m_n^*) * \Gamma^2_{wN} * (n_e)^{1/3}(T_9)^6 * R_D; \\ M_{DU} = & \quad 4 \times 10^{27} \quad erg/s/cm^3 \end{aligned}$$

- Modified Urca (MU) and Bremsstrahlung

$$\epsilon_{MUp} \quad = \quad F_M * M_p * (m_p)^3 (m_n^*)(T_9)^8 (n_e)^{1/3} * R_{MUp}(v_n, v_p);$$

$$\epsilon_{nnBS} \quad = \quad P_{nnBS} * R^{nn}_{BS}(v_n) * \Gamma^2_w \Gamma^4_s (n_b)^{4/3} (T_9)^8 (m_n^*)^4/(\omega)^3;$$

- Suppression due to the pairing

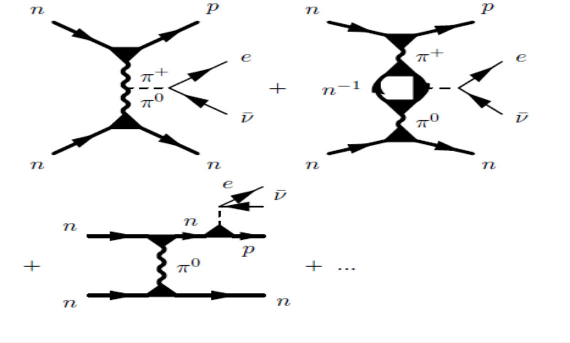
$$v_N \quad = \Delta_N(T)/T \quad = \sqrt{1-\tau_N}\left(1.456 - \frac{0.157}{\sqrt{\tau_N}} + \frac{1.766}{\tau_N}\right)$$

- Enhanced cooling due to the pairing

$$\begin{aligned} \epsilon_\nu^{\mathrm{NPBF}} &= 6.6 \times 10^{28} (m_n^*/m_n)(\Delta_n(T)/\mathrm{MeV})^7 \ u^{1/3} \\ &\times \xi \ I(\Delta_n(T)/T) \ \mathrm{erg\ cm^{-3} s^{-1}}, \\ \epsilon_\nu^{\mathrm{PPBF}} &= 0.8 \times 10^{28} (m_p^*/m_p)(\Delta_p(T)/\mathrm{MeV})^7 \ u^{2/3} \\ &\times \ I(\Delta_p(T)/T) \ \mathrm{erg\ cm^{-3} s^{-1}}, \end{aligned}$$

Medium Effects In Cooling Of Neutron Stars

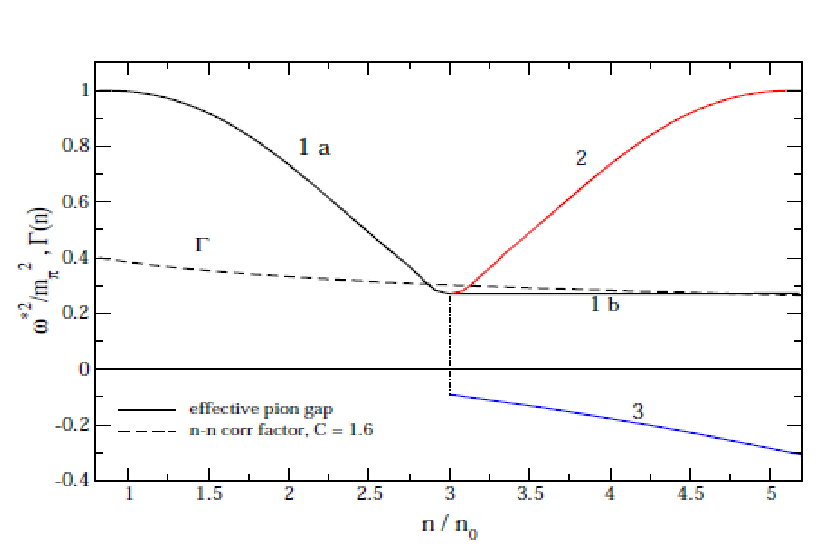
- Based on Fermi liquid theory (Landau (1956), Migdal (1967), Migdal et al. (1990))
- MMU – insted of MU



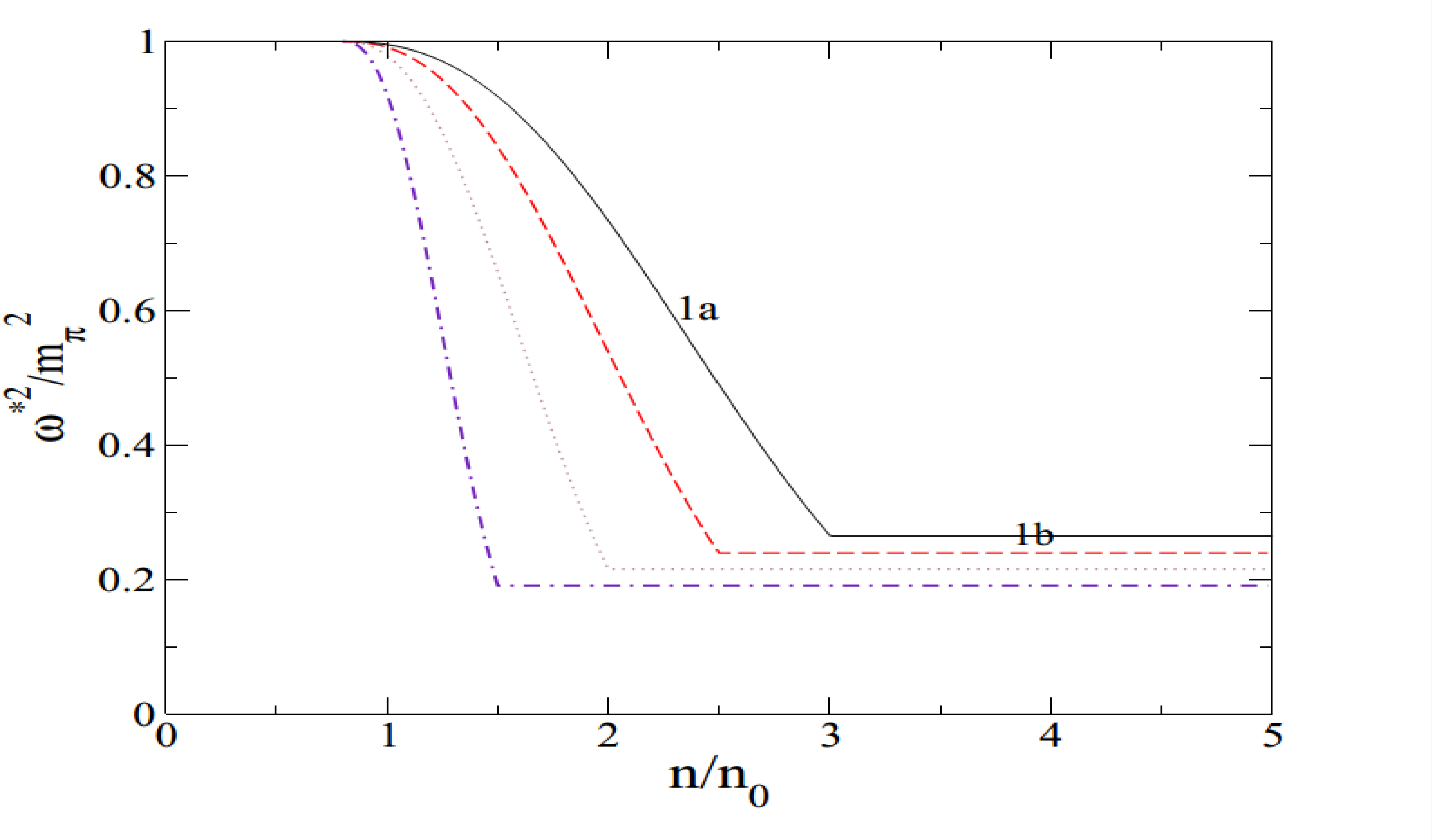
$$\frac{\varepsilon_\nu[\mathrm{MMU}]}{\varepsilon_\nu[\mathrm{MU}]} \sim 10^3\ (n/n_0)^{10/3}\,\frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8},$$

- Main regulator in Minimal Cooling

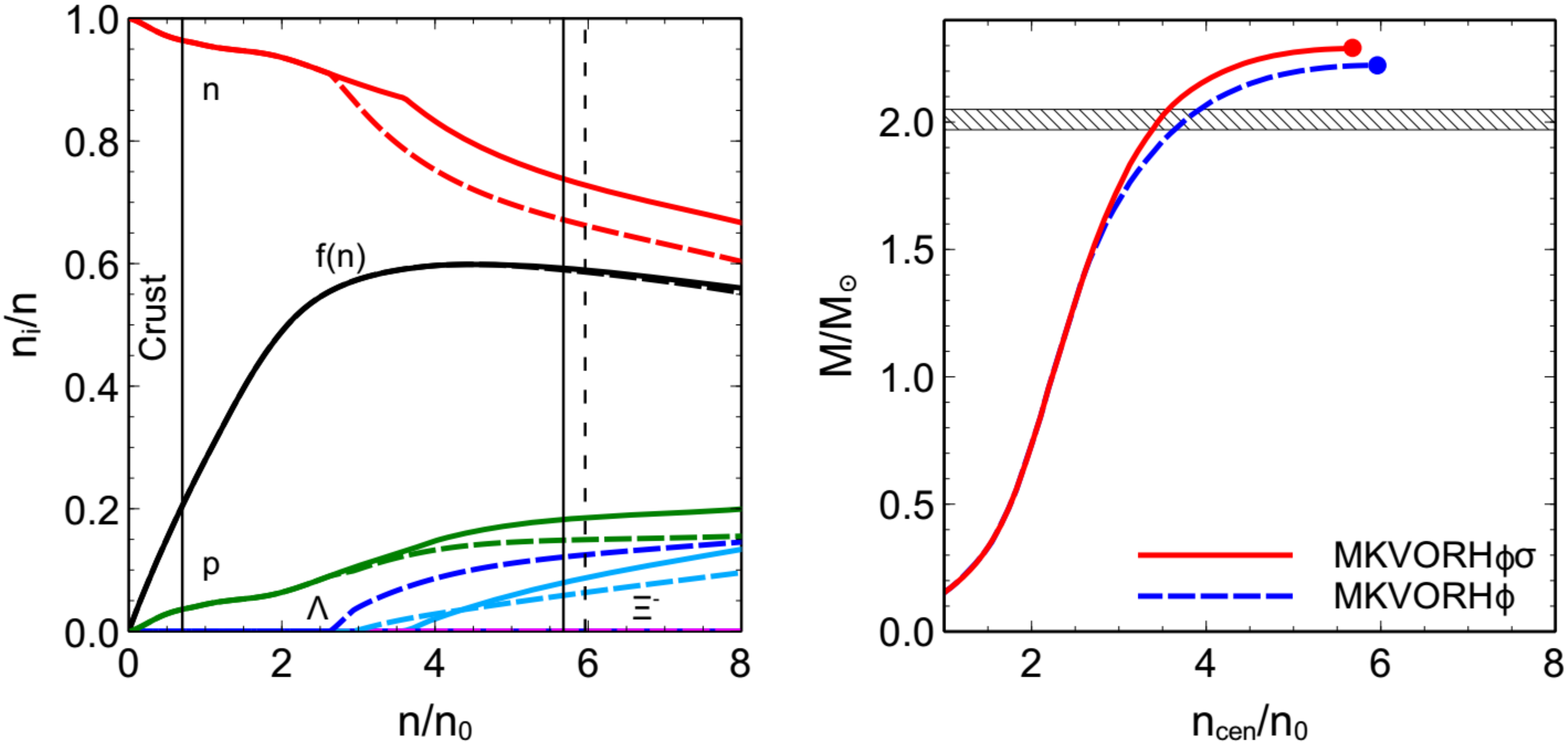
$$\varepsilon_\nu[\mathrm{MpPBF}] \sim 10^{29}\,\frac{m_N^*}{m_N}\left[\frac{p_{Fp}}{p_{Fn}(n_0)}\right]\left[\frac{\Delta_{pp}}{\mathrm{MeV}}\right]^7 \times\left[\frac{T}{\Delta_{pp}}\right]^{1/2}\xi_{pp}^2\,\frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{sec}},\quad T<T_{cp}.$$



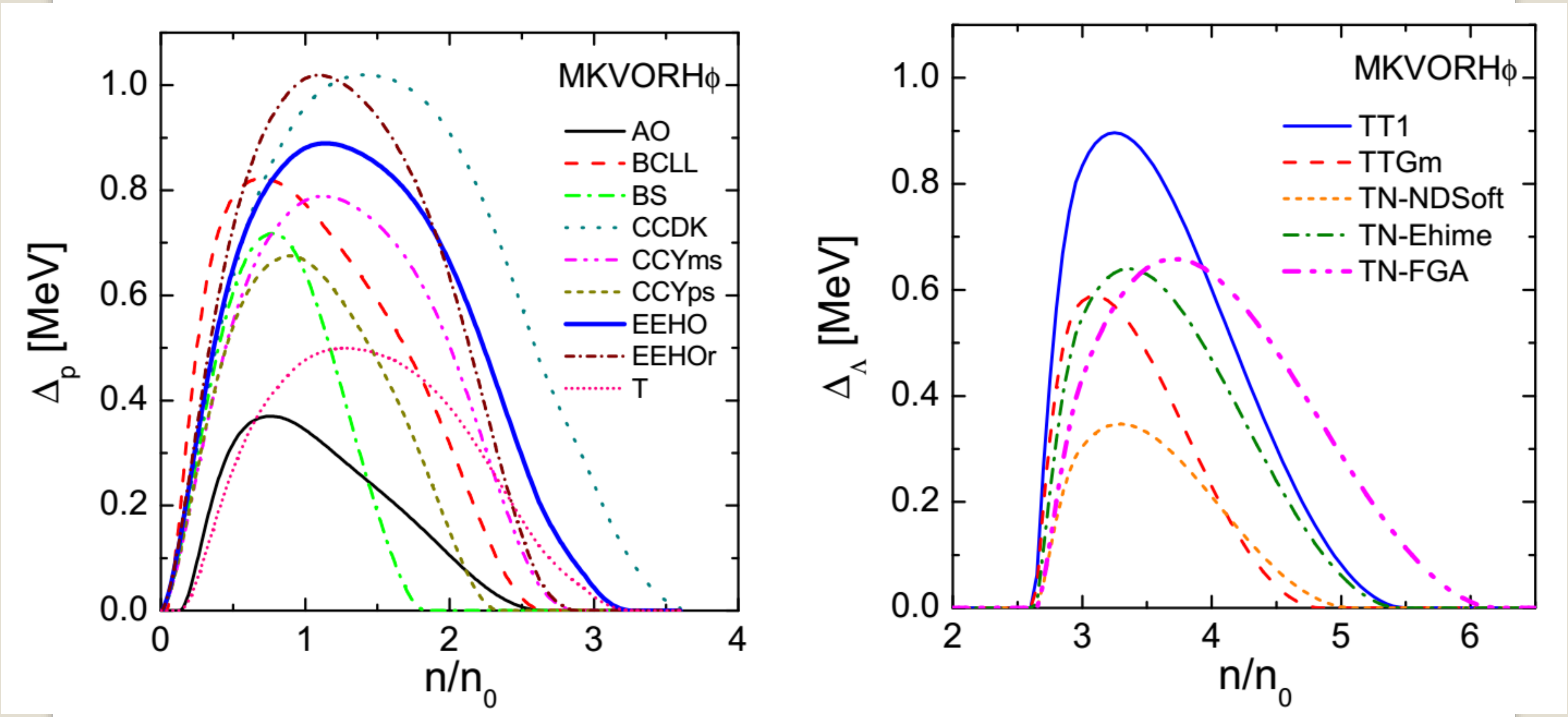
Medium Effects In Cooling Of Neutron Stars



MKVOR – EoS model



MKVORHp – Gap models



Crust Model

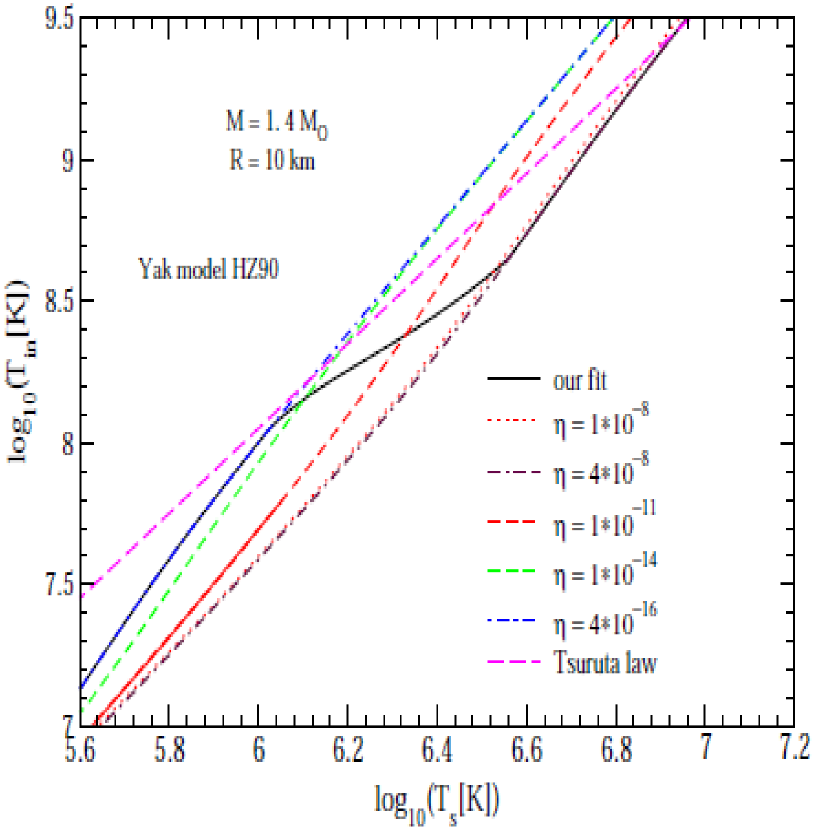
Time dependence of the light element contents in the crust

$$\Delta M_{\mathrm{L}}(t) = e^{-t/\tau}\,\Delta M_{\mathrm{L}}(0)$$

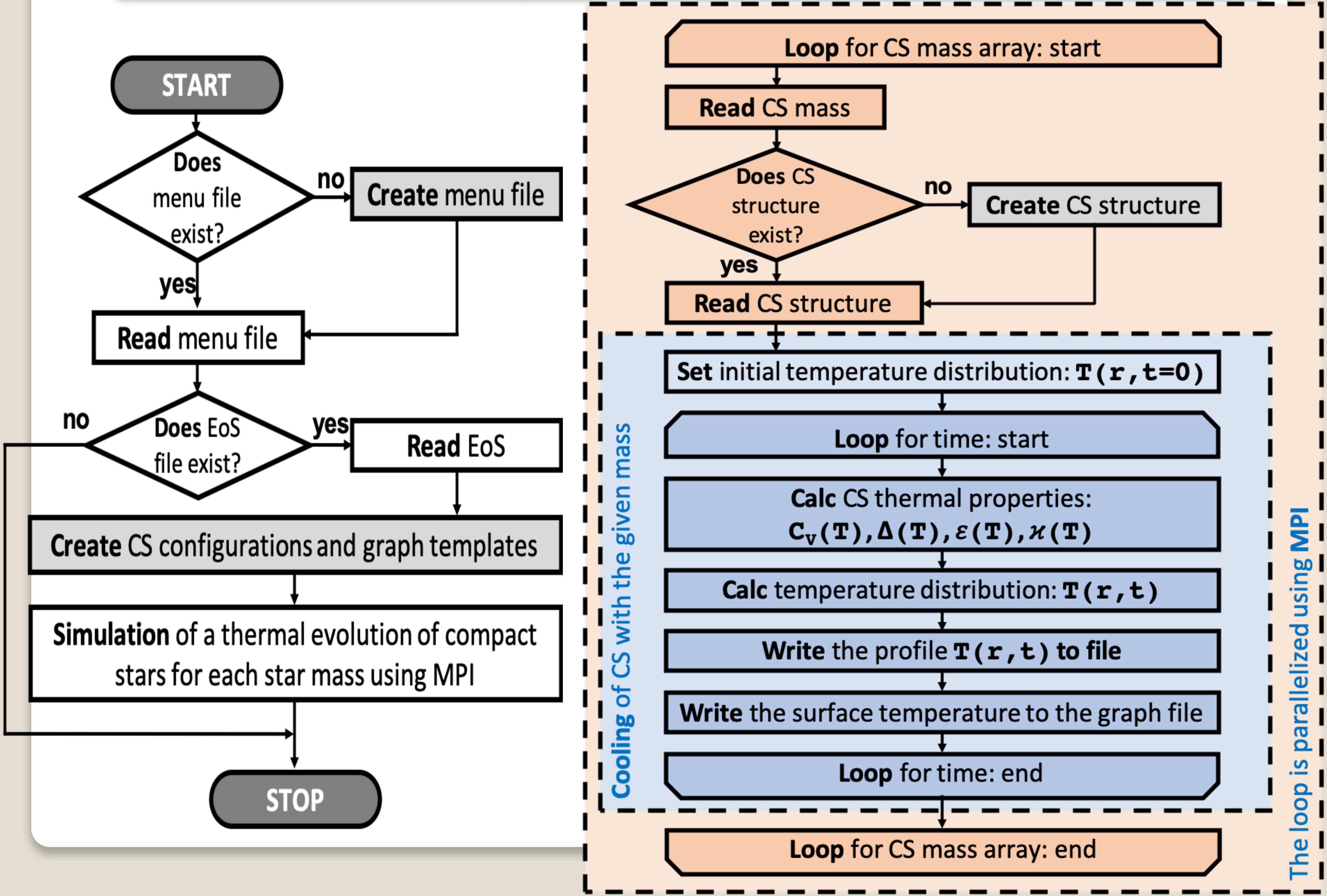
Blaschke, Grigorian, Voskresensky, A& A 368 (2001)561.

Page,Lattimer,Prakash & Steiner, Astrophys.J. 155,623 (2004)

Yakovlev, Levenfish, Potekhin, Gnedin & Chabrier , Astron. Astrophys , 417, 169 (2004)



Program Algorithm



START

Does menu file exist?

no

Create menu file

yes

Read menu file

Does EoS file exist?

no

yes

Read EoS

Create CS configurations and graph templates

Simulation of a thermal evolution of compact stars for each star mass using MPI

STOP

Loop for CS mass array: start

Read CS mass

Does CS structure exist?

no

Create CS structure

yes

Read CS structure

Set initial temperature distribution: $T(r,t=0)$

Loop for time: start

Calc CS thermal properties: $C_v(T), \Delta(T), \varepsilon(T), \varkappa(T)$

Calc temperature distribution: $T(r,t)$

Write the profile $T(r,t)$ **to file**

Write the surface temperature to the graph file

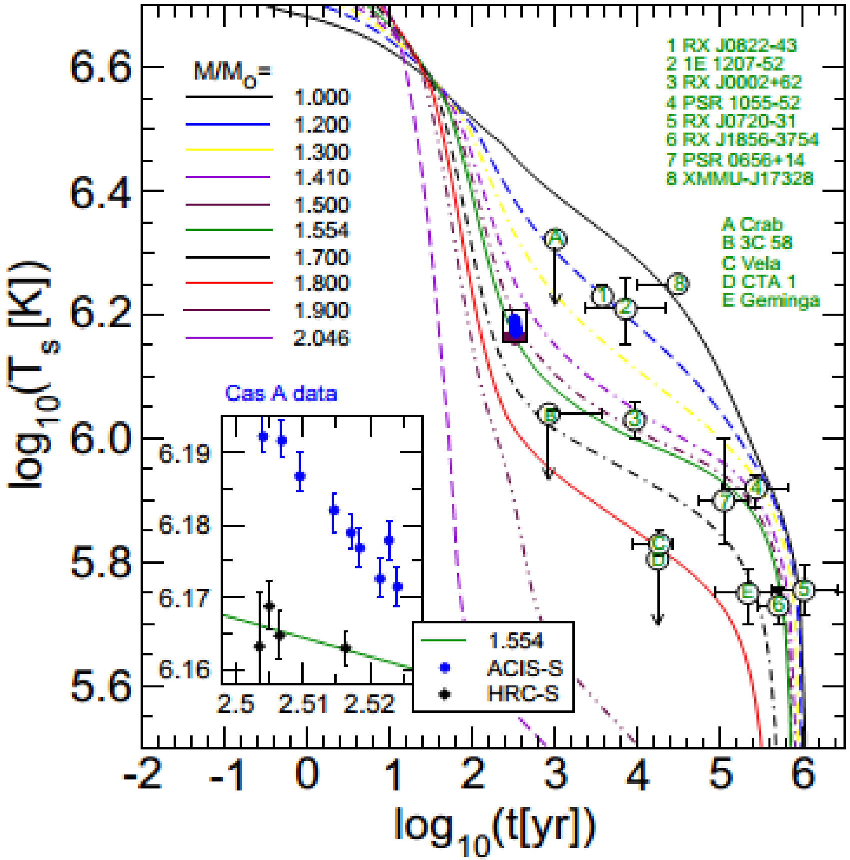
Loop for time: end

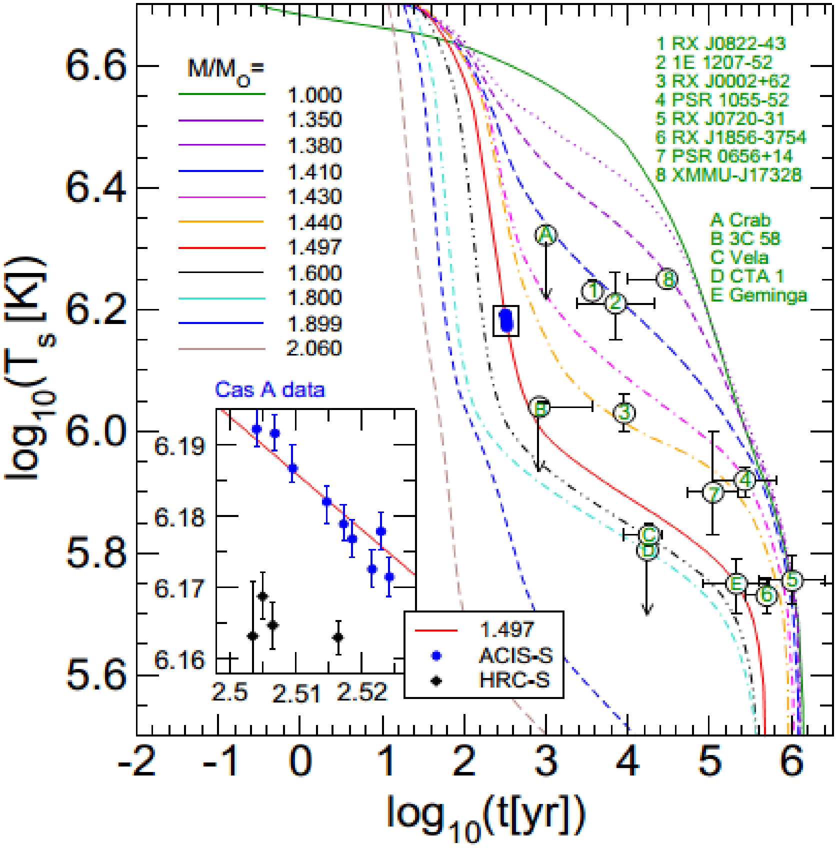
Loop for CS mass array: end

Cooling of CS with the given mass

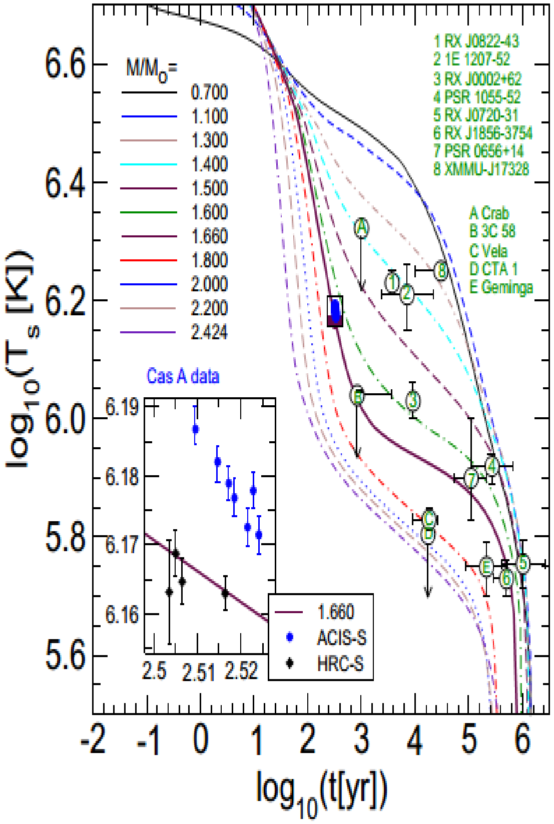
The loop is parallelized using **MPI**

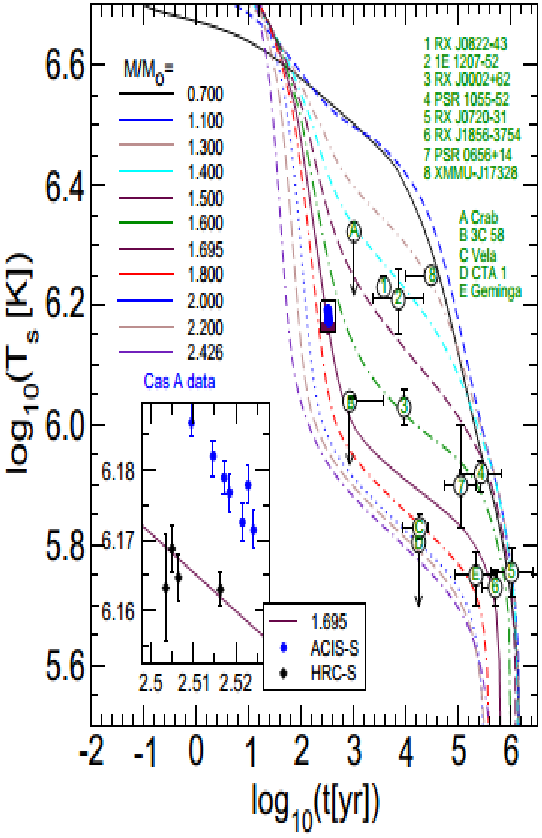
HDD – AV18 , Yak.
ME nc = 3 n0

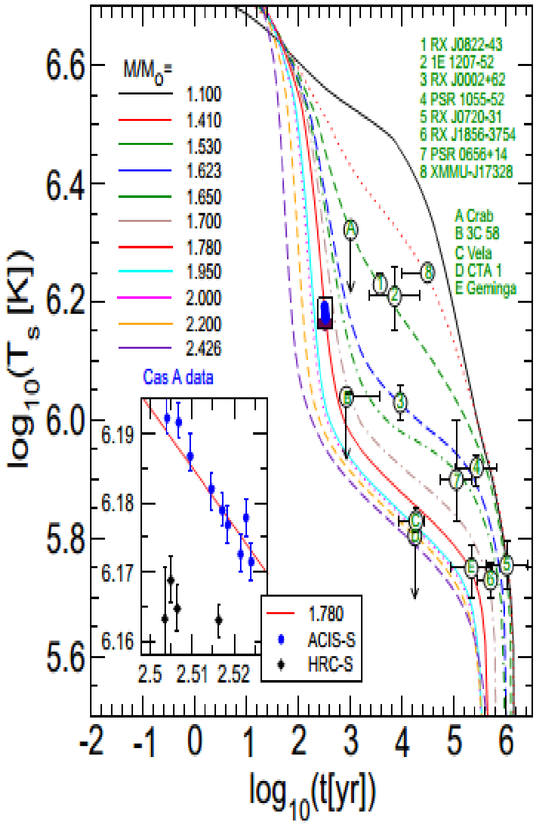




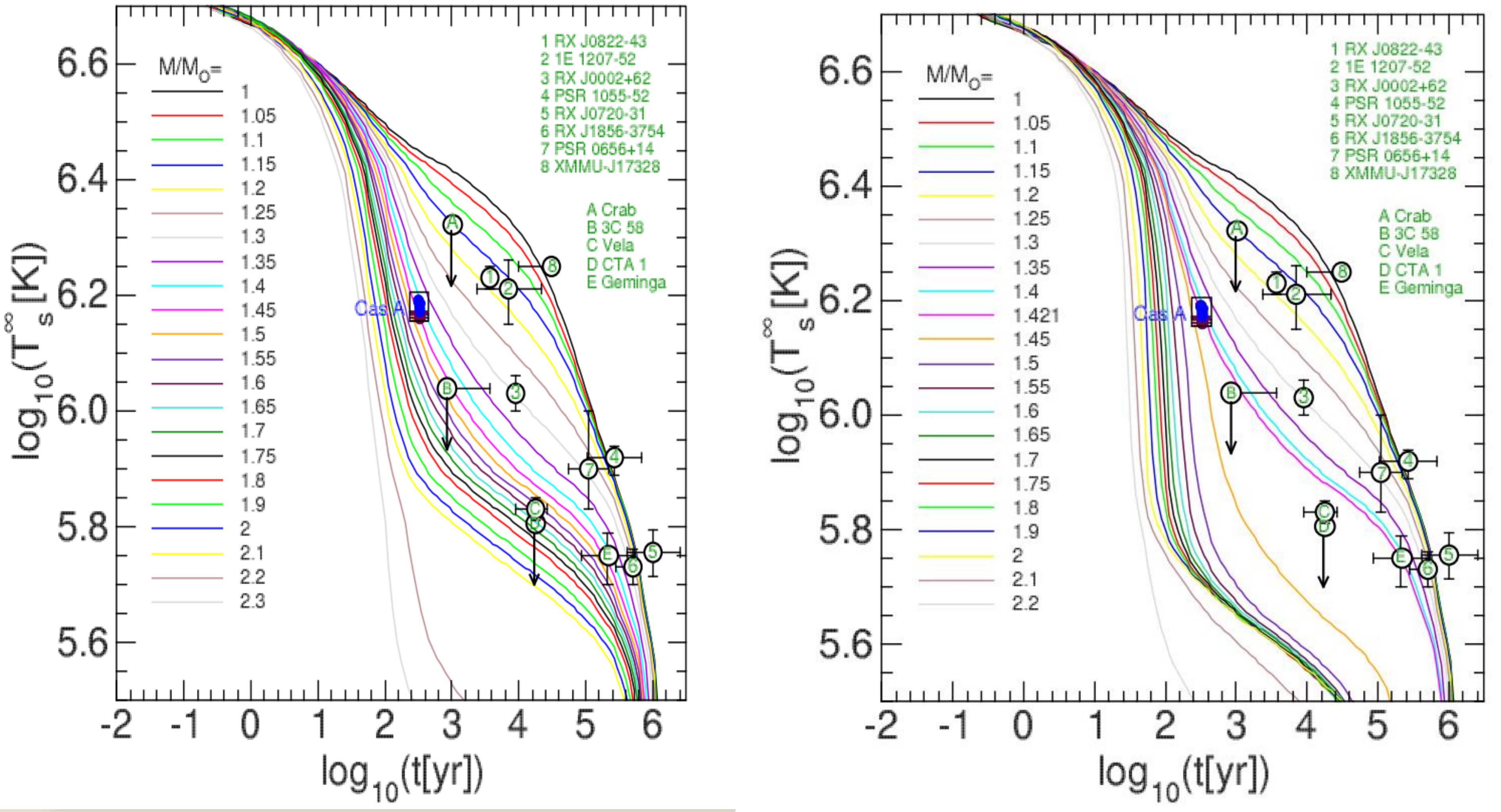
DD2 – EEHOr
ME-nc=1.5,2.0,2.5n0



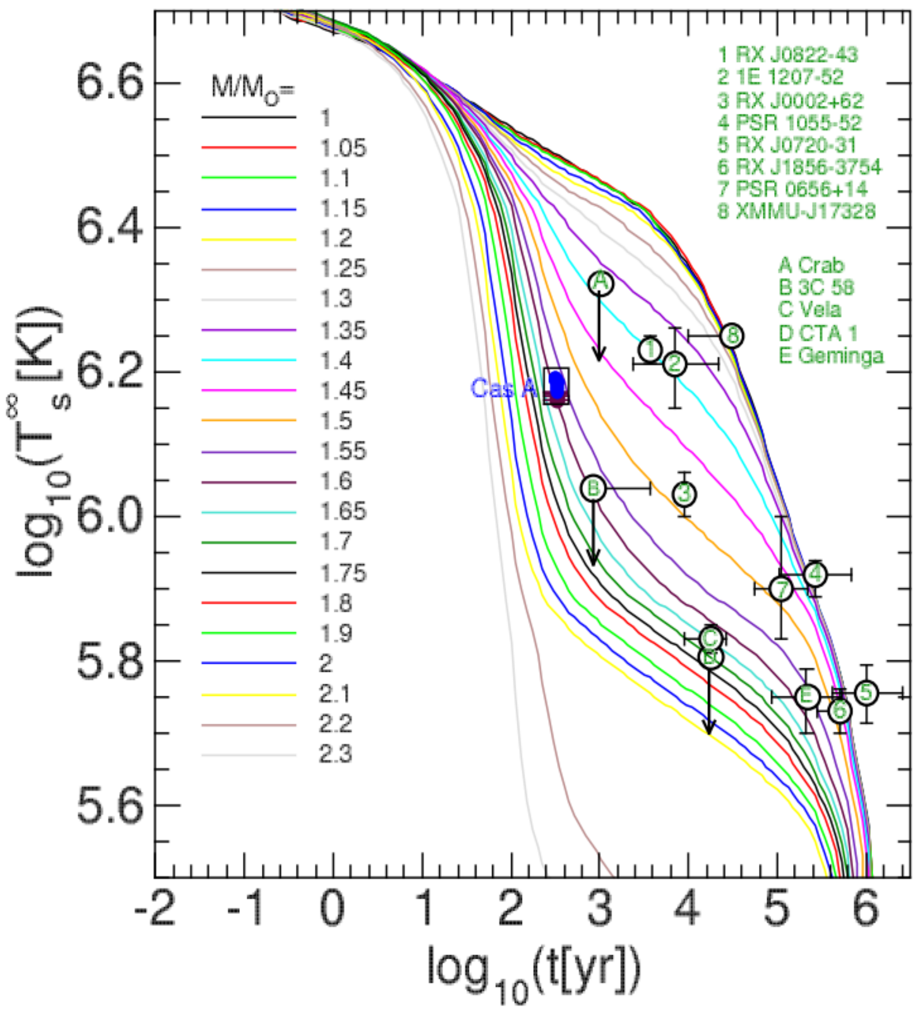


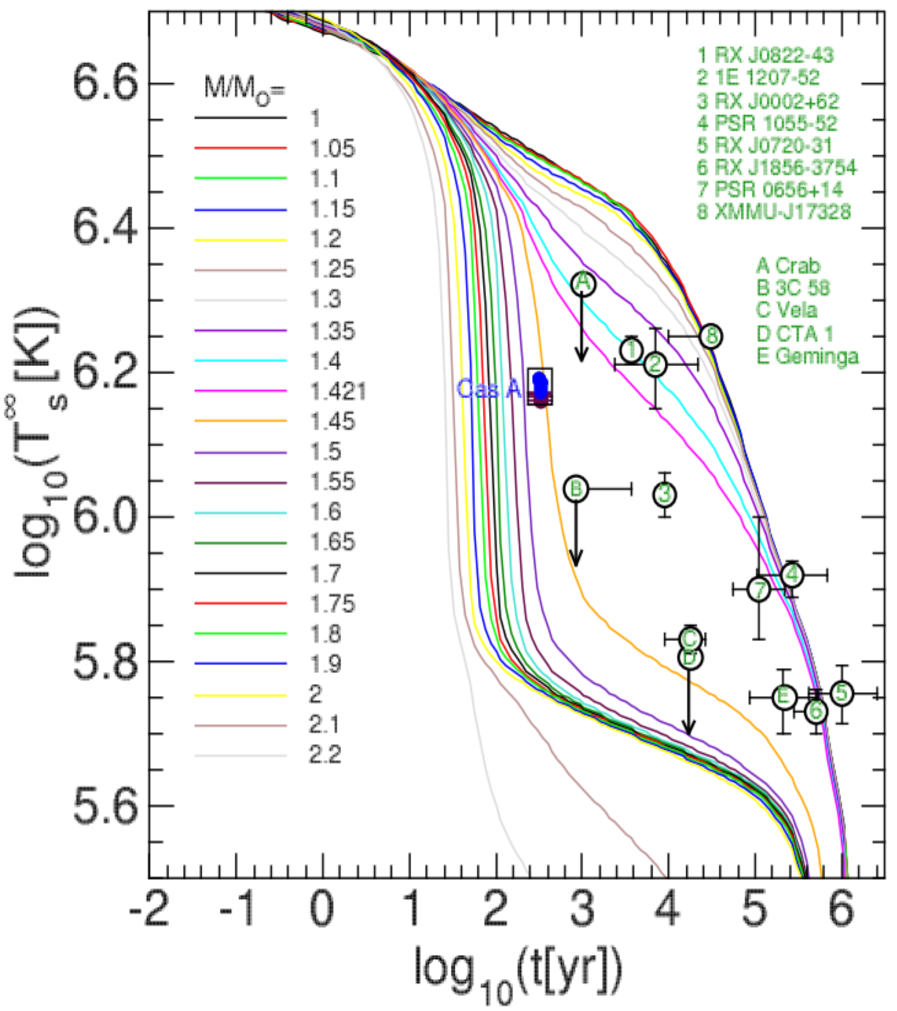


MKVOR – BCLL, TN-FGA ME-nc=3.0n0

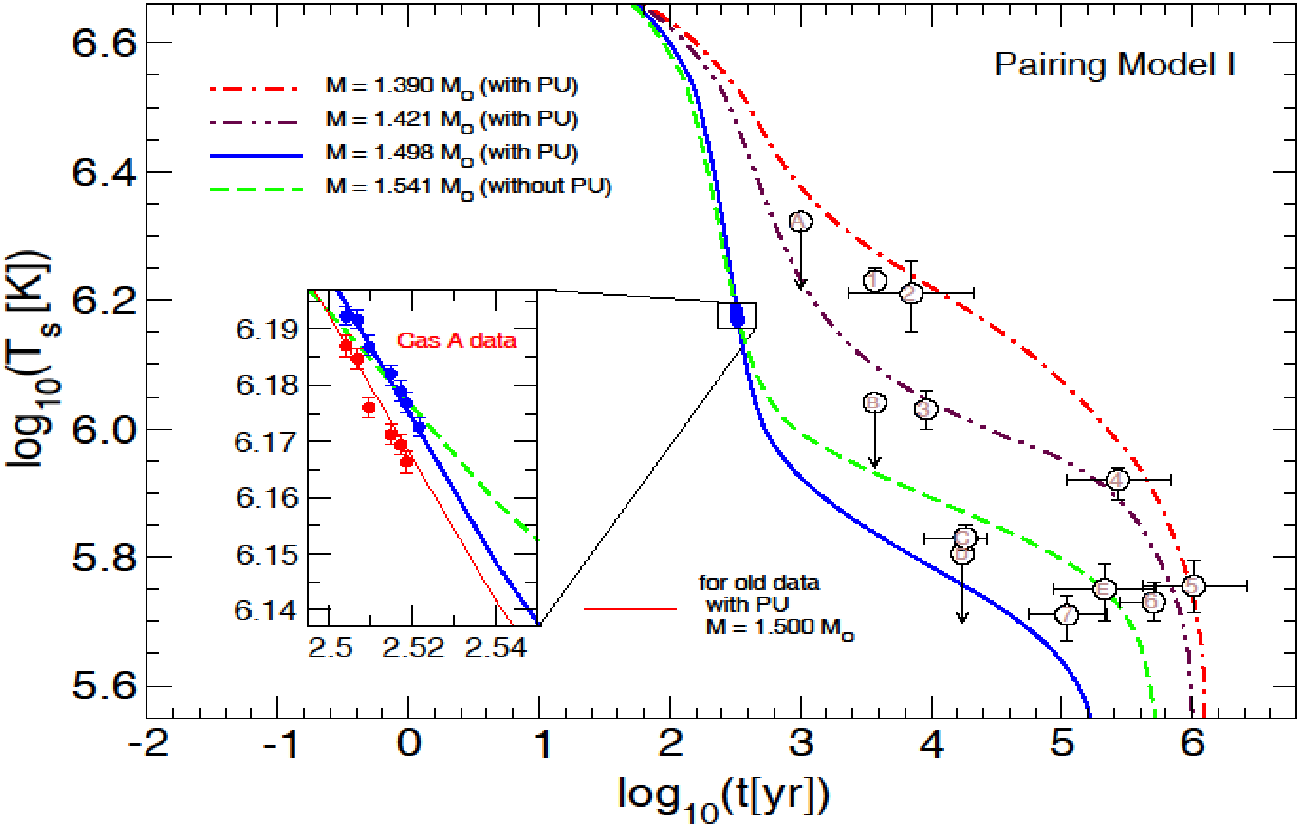


MKVOR Hyp – EEHOr, TN-FGA
ME-nc=3.0n0



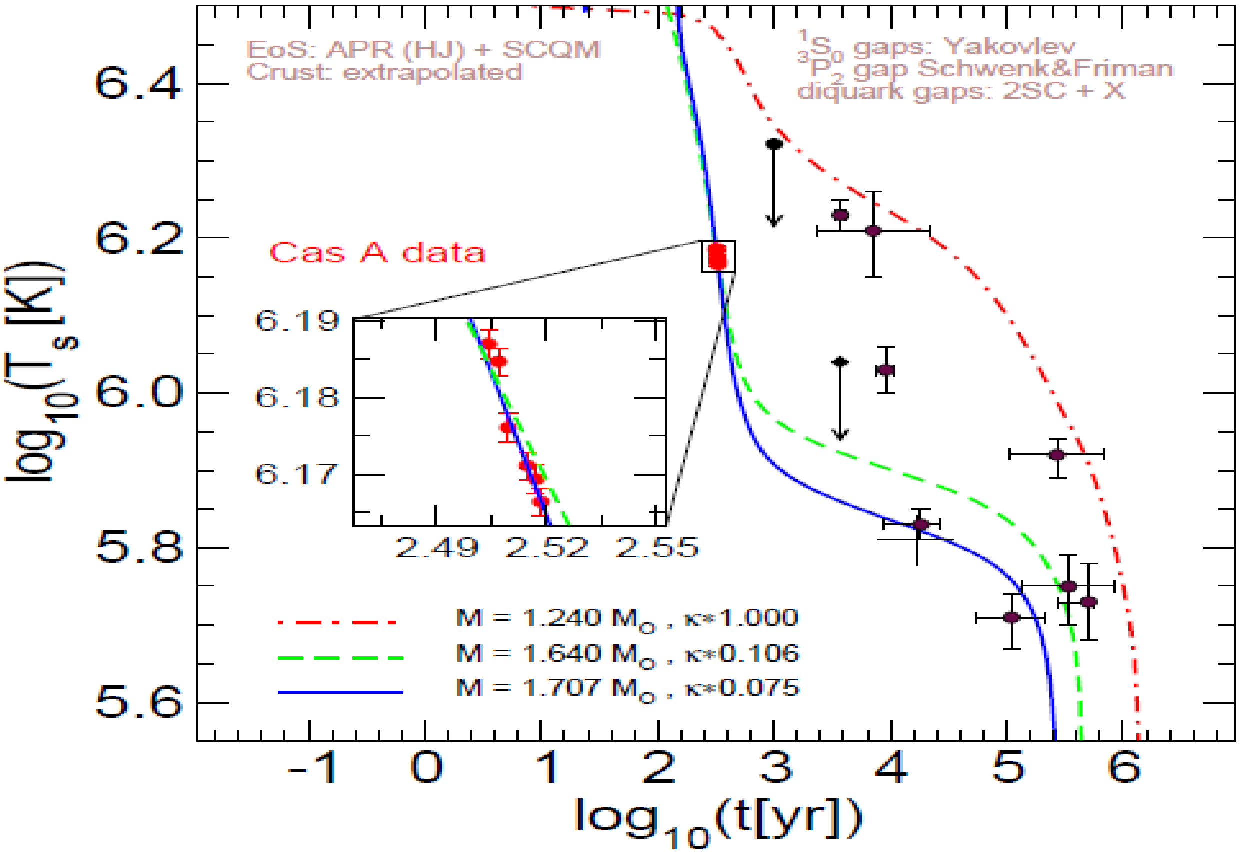


Cas A as an Hadronic Star

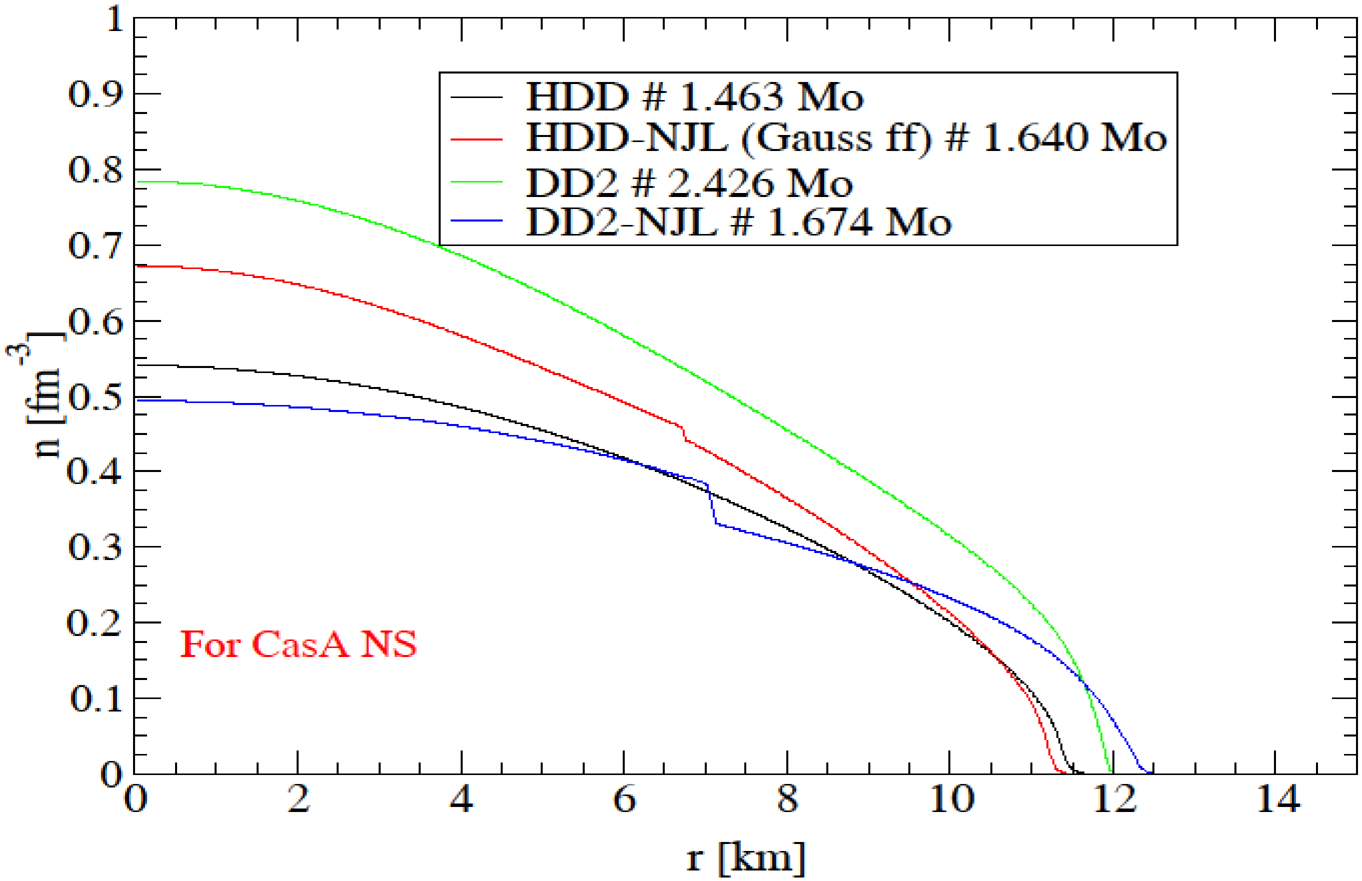


Cas A As An Hybrid Star

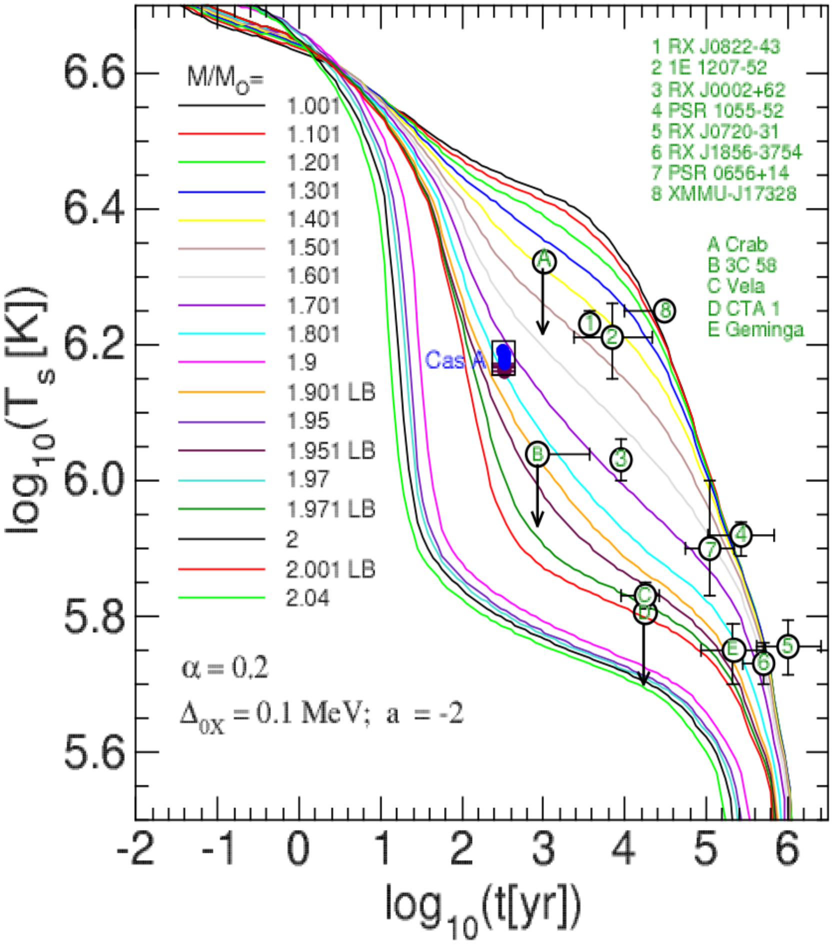
H. Grigorian, D. Blaschke, D.N. Voskresensky, Phys. Rev. C 71, 045801 (2005)

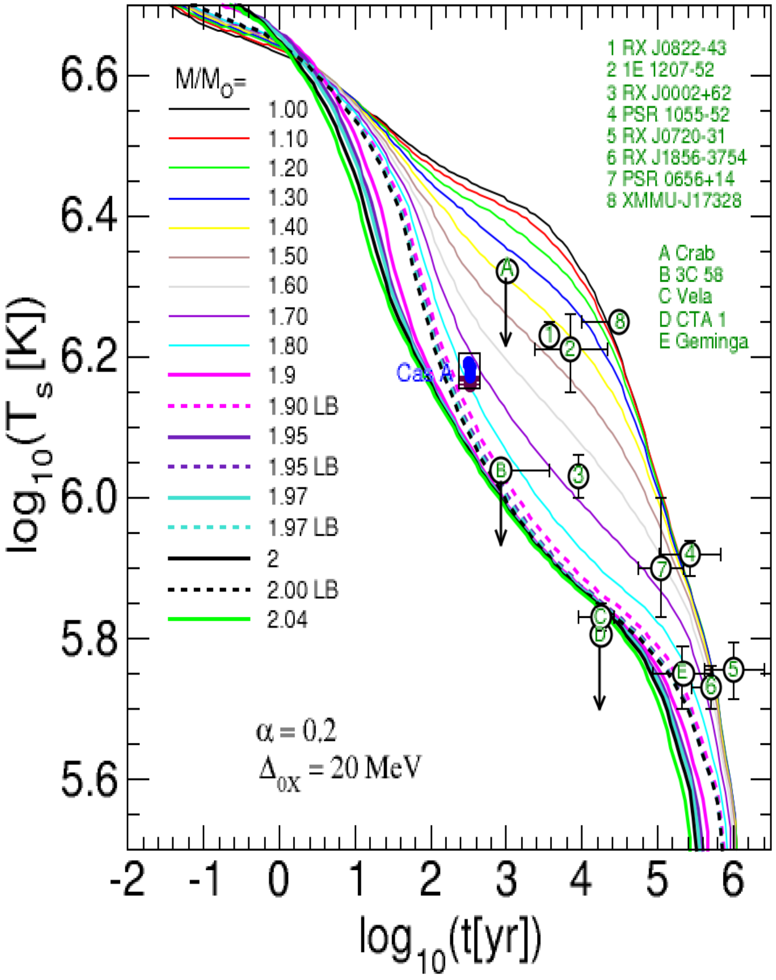


Possible internal structure of CasA

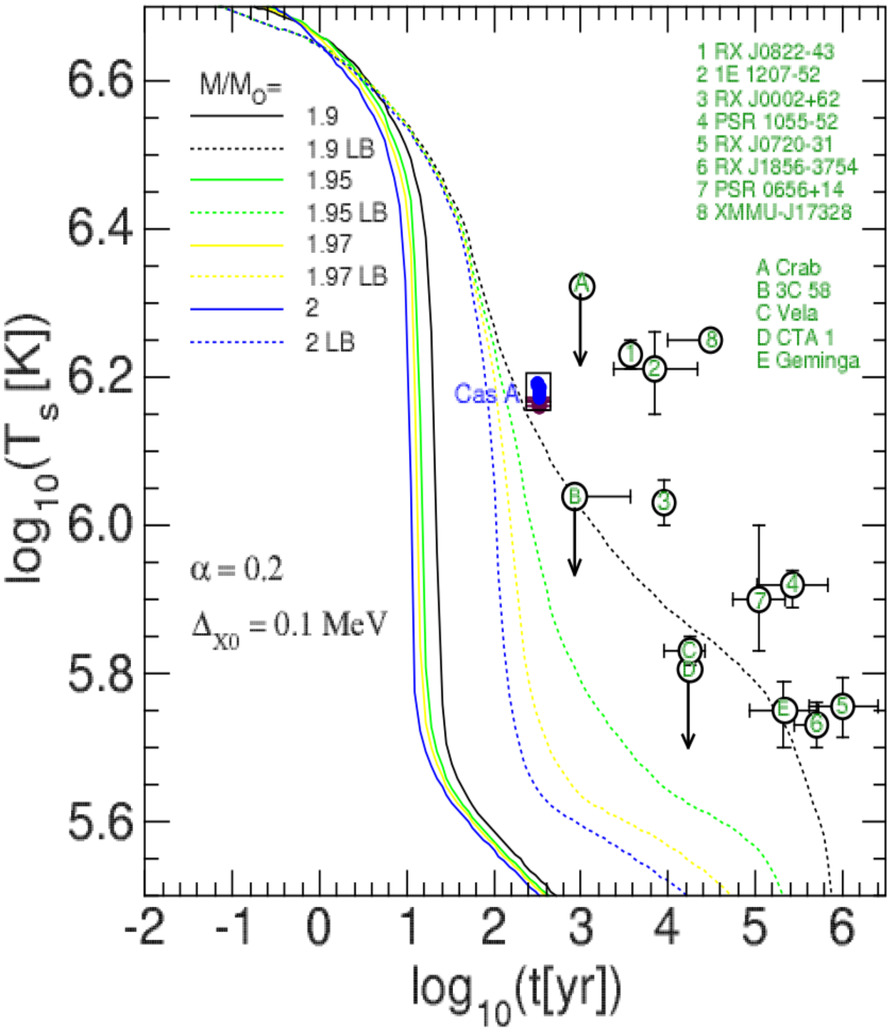


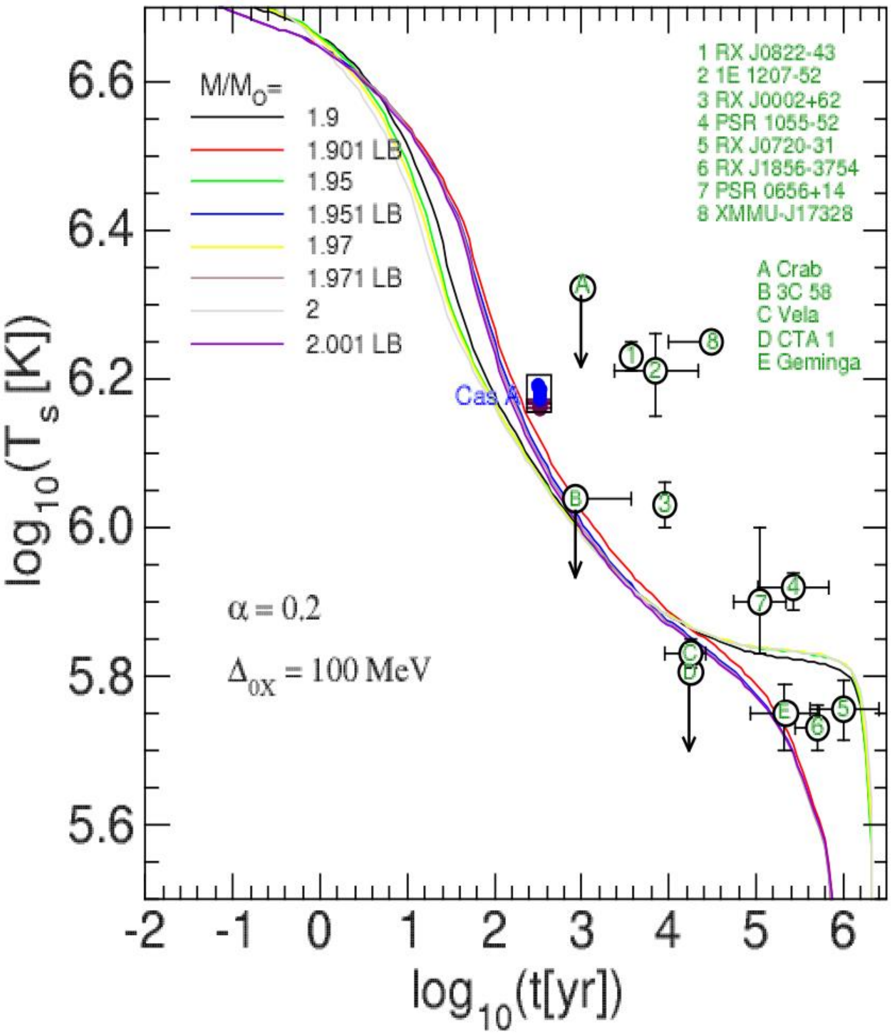
Cooling of Twin CS

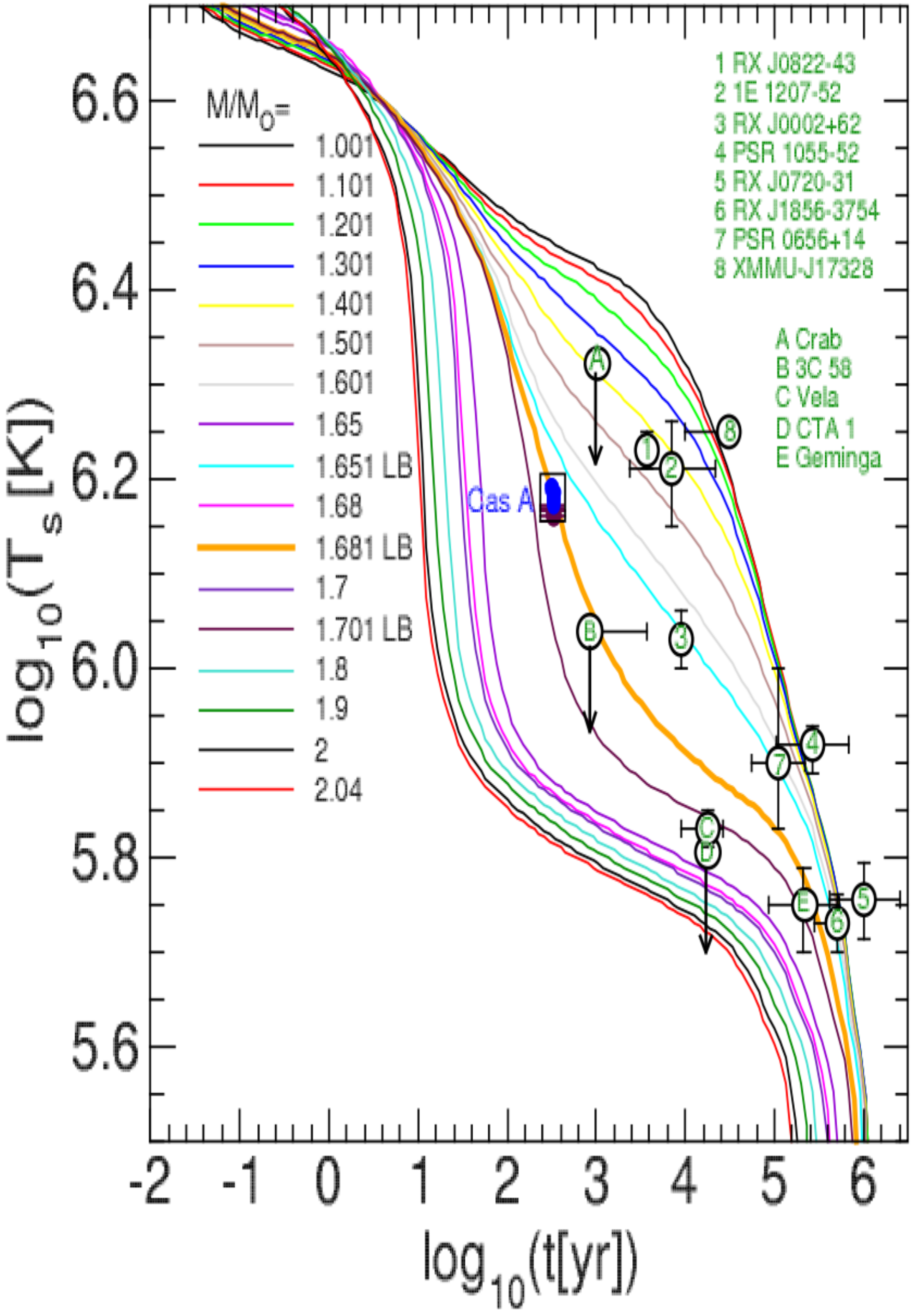




Highmass Twins: QM SC Effect






log10(Ts [K])
log10(t[yr])
6.6
6.4
6.2
6.0
5.8
5.6
-2
-1
0
1
2
3
4
5
6
M/MO=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
1 RX J0822-43
2 1E 1207-52
3 RX J0002+62
4 PSR 1055-52
5 RX J0720-31
6 RX J1856-3754
7 PSR 0656+14
8 XMMU-J17328
A Crab
B 3C 58
C Vela
D CTA 1
E Geminga
Cas A
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A
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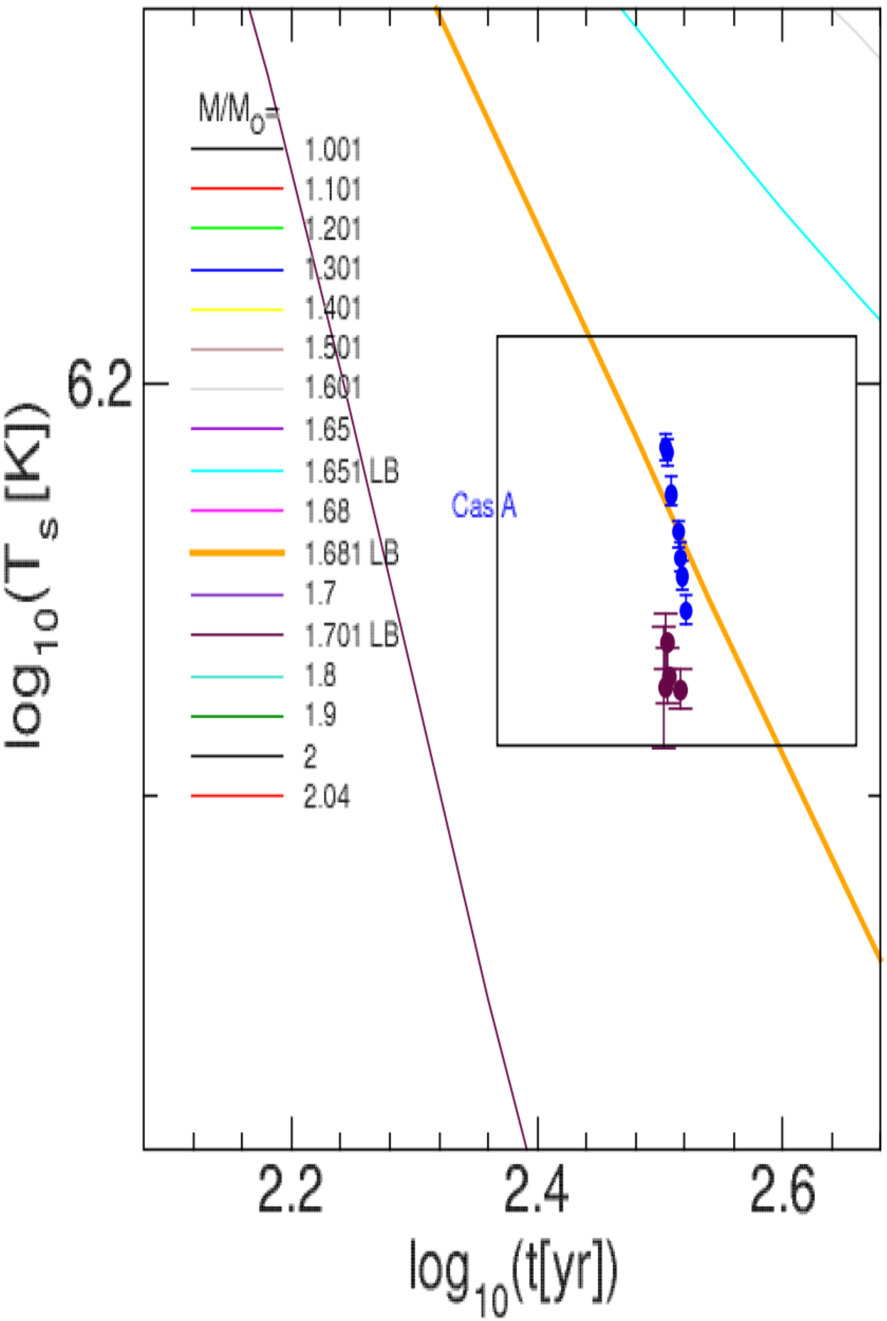
E


log10(Ts [K])
log10(t[yr])
6.2
2.2
2.4
2.6
M/MO=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
Cas A

Cooling of Neutron Stars admixed with Light Dark Matter

$$\frac{e^{-\lambda-2\Phi}}{4\pi r^2}\frac{\partial}{\partial r}\left(e^{2\Phi}L\right) = -Q + Q_h - \frac{c_V}{e^{\Phi}}\frac{\partial T}{\partial t},$$

$$\frac{L}{4\pi\kappa r^2} = e^{-\lambda-\Phi}\frac{\partial}{\partial r}\left(Te^{\Phi}\right)$$

$$N_\chi(t) \simeq N_{\chi,0} + \frac{dN_\chi}{dt}(t-t_0),$$

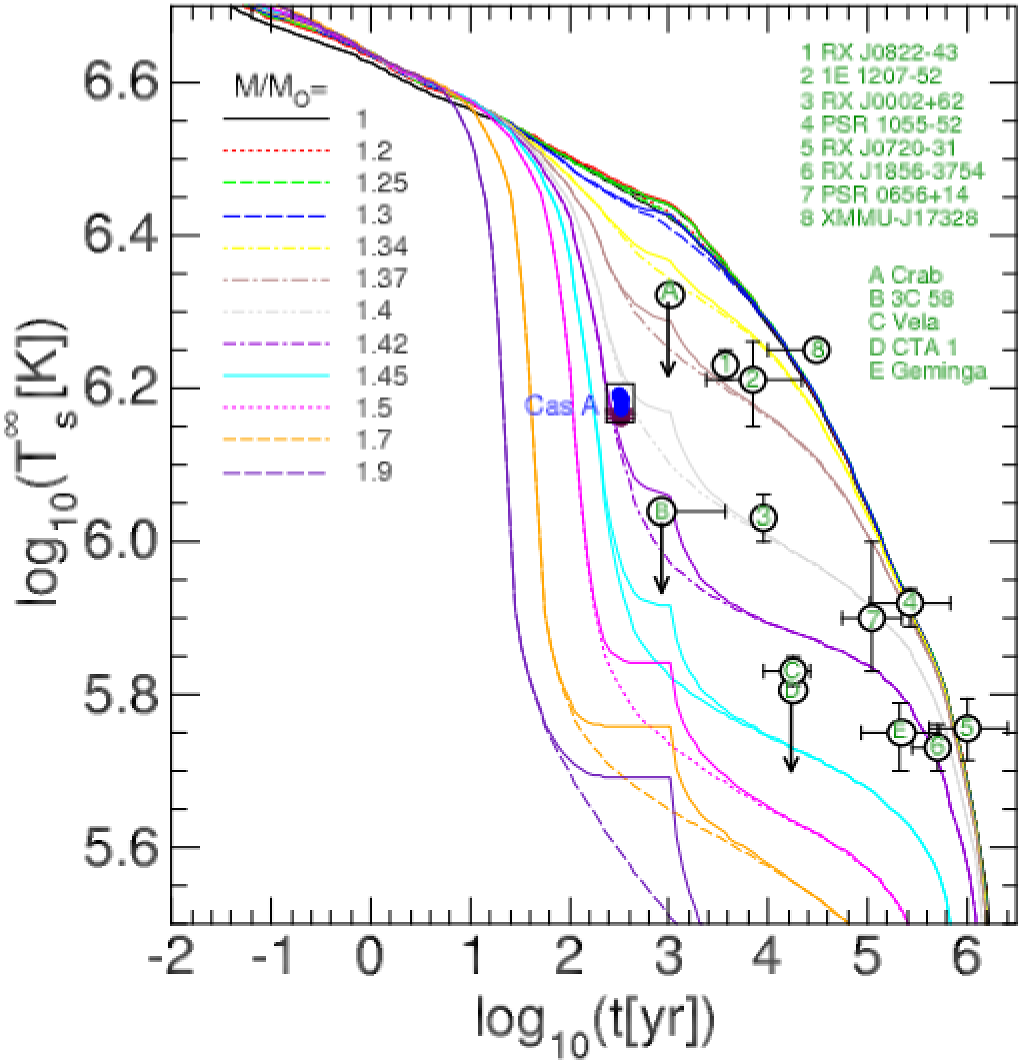
$$\frac{dN_\chi}{dt} = C_\chi - C_a N_\chi^2.$$

The DM capture rate can be approximated by

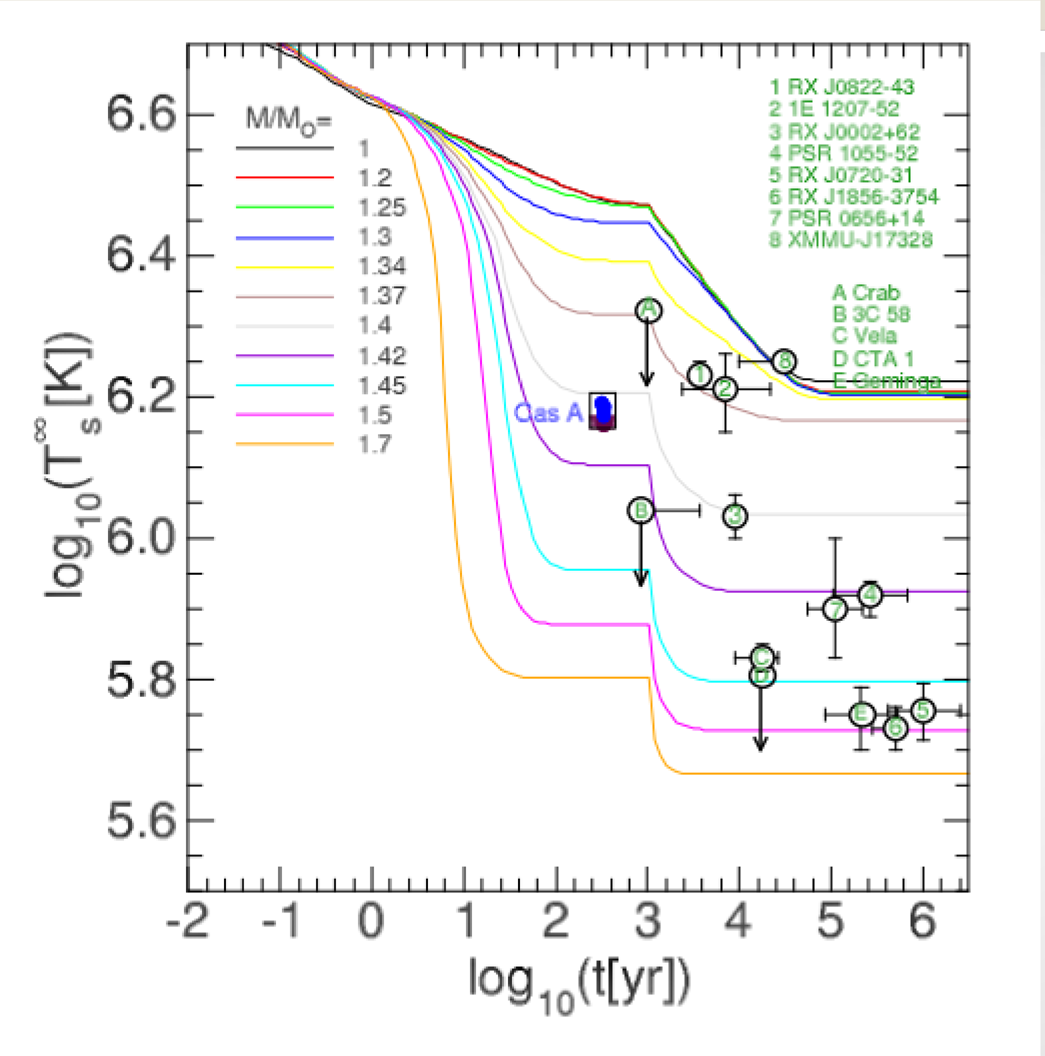
$$C_\chi \simeq 5.6\times10^{26}\left(\frac{M}{1.5M_\odot}\right)\left(\frac{R}{14\ \mathrm{km}}\right)\left(\frac{0.1\mathrm{GeV}}{m_\chi}\right)\left(\frac{\rho_\chi}{0.4\frac{\mathrm{GeV}}{\mathrm{cm}^3}}\right)\mathrm{s}^{-1}$$

the thermally-averaged self-annihilation rate $\langle\sigma v\rangle \sim$ $10^{-26}\,\mathrm{cm}^3\mathrm{s}^{-1}$,

$$C_a \simeq 2\times10^{-42}\left(\frac{0.1\ \mathrm{GeV}}{m_\chi}\frac{2\rho_0}{\rho_N}\frac{T}{0.5\,\mathrm{MeV}}\right)^{-3/2}\mathrm{s}^{-1}.$$



NSs with masses $M \in [1, 1.9]M_\odot$ with the effect of self-annihilating LDM ($m_\chi = 0.1$ GeV) originating a plateau or without LDM (continuous decline). Existing series of cooling



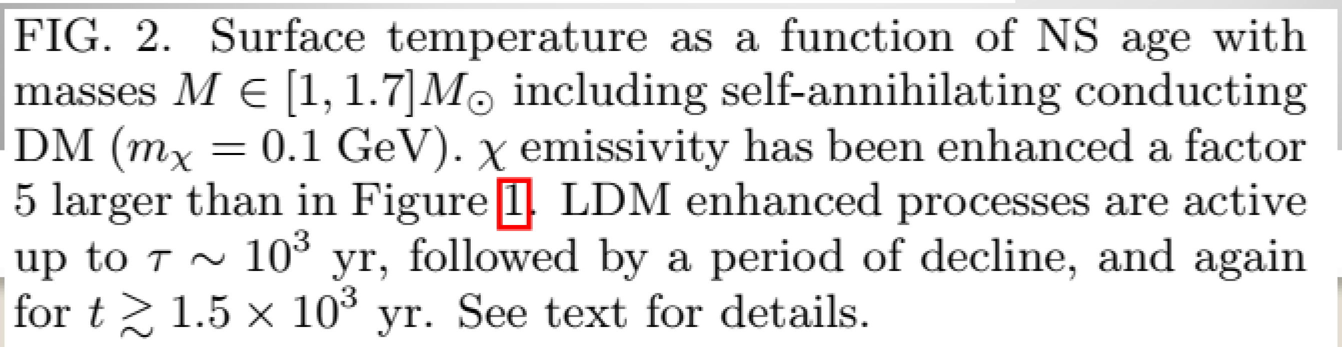
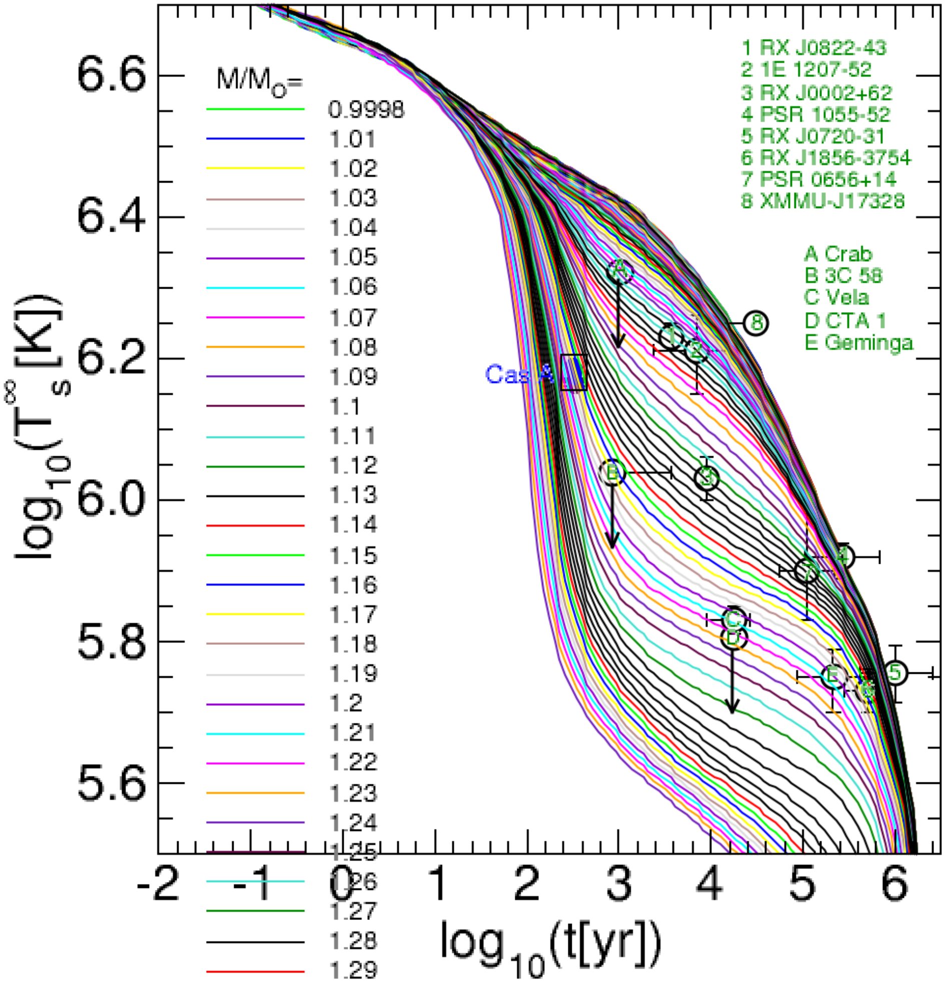


FIG. 2. Surface temperature as a function of NS age with masses $M \in [1, 1.7]M_\odot$ including self-annihilating conducting DM ($m_\chi = 0.1$ GeV). χ emissivity has been enhanced a factor 5 larger than in Figure 1. LDM enhanced processes are active up to $\tau \sim 10^3$ yr, followed by a period of decline, and again for $t \gtrsim 1.5 \times 10^3$ yr. See text for details.

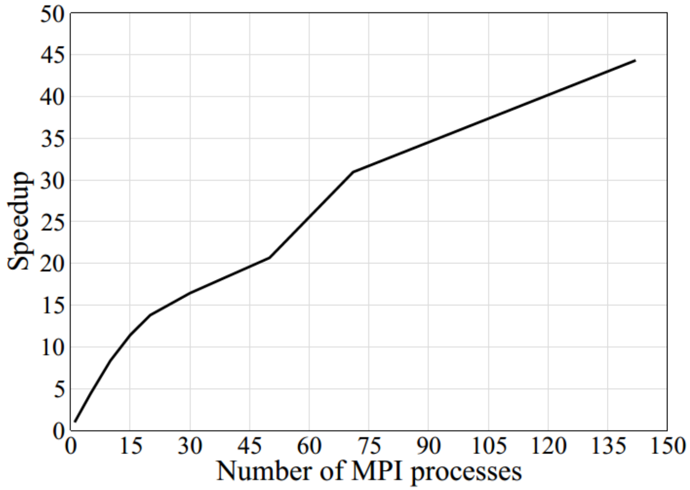
M. Ángeles Pérez-García, H. Grigorian, C. Albertus, D. Barba, J. Silk

Physics Letters B 827(2022)136937

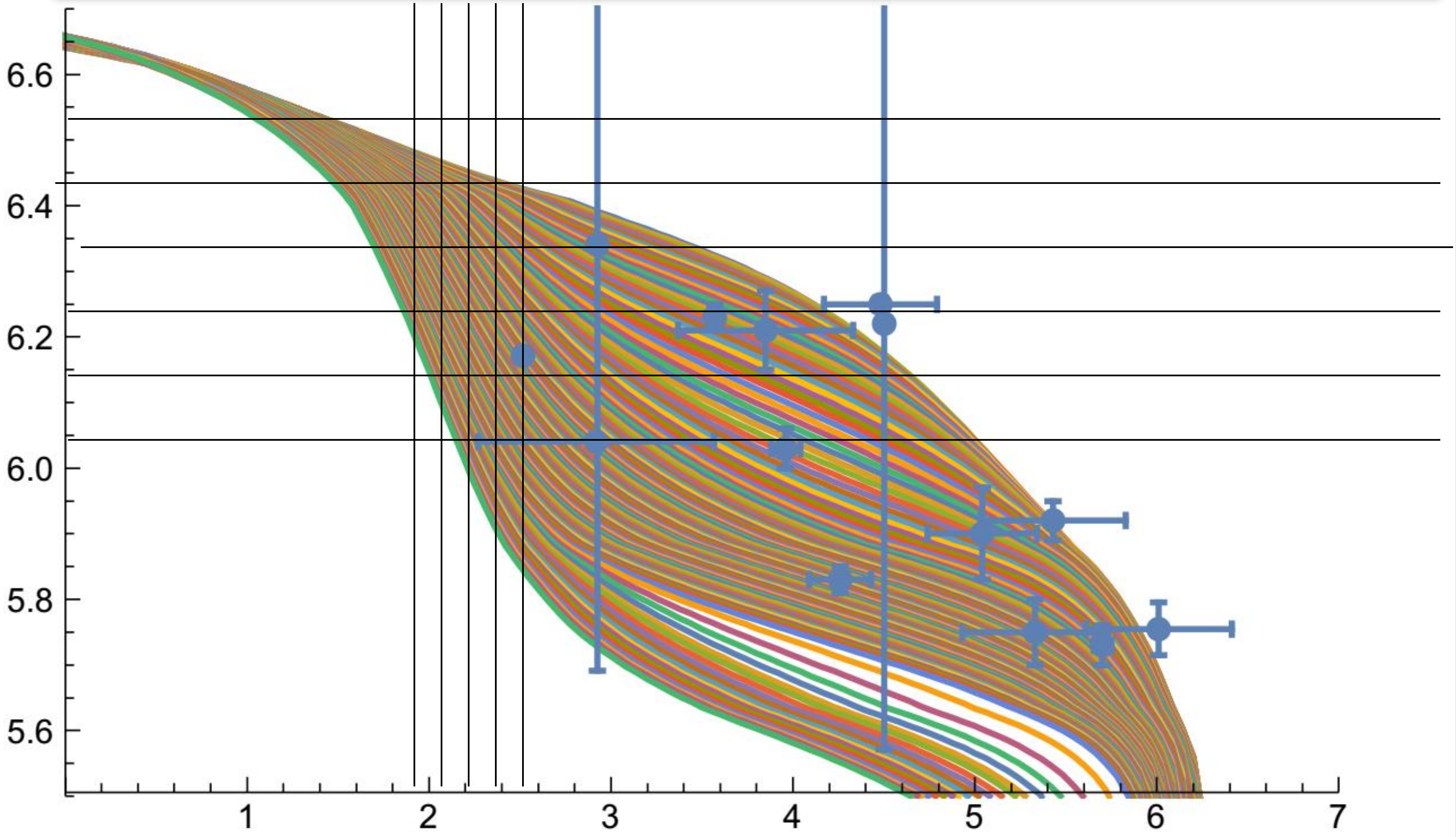
Results produced with use of MPI Technology



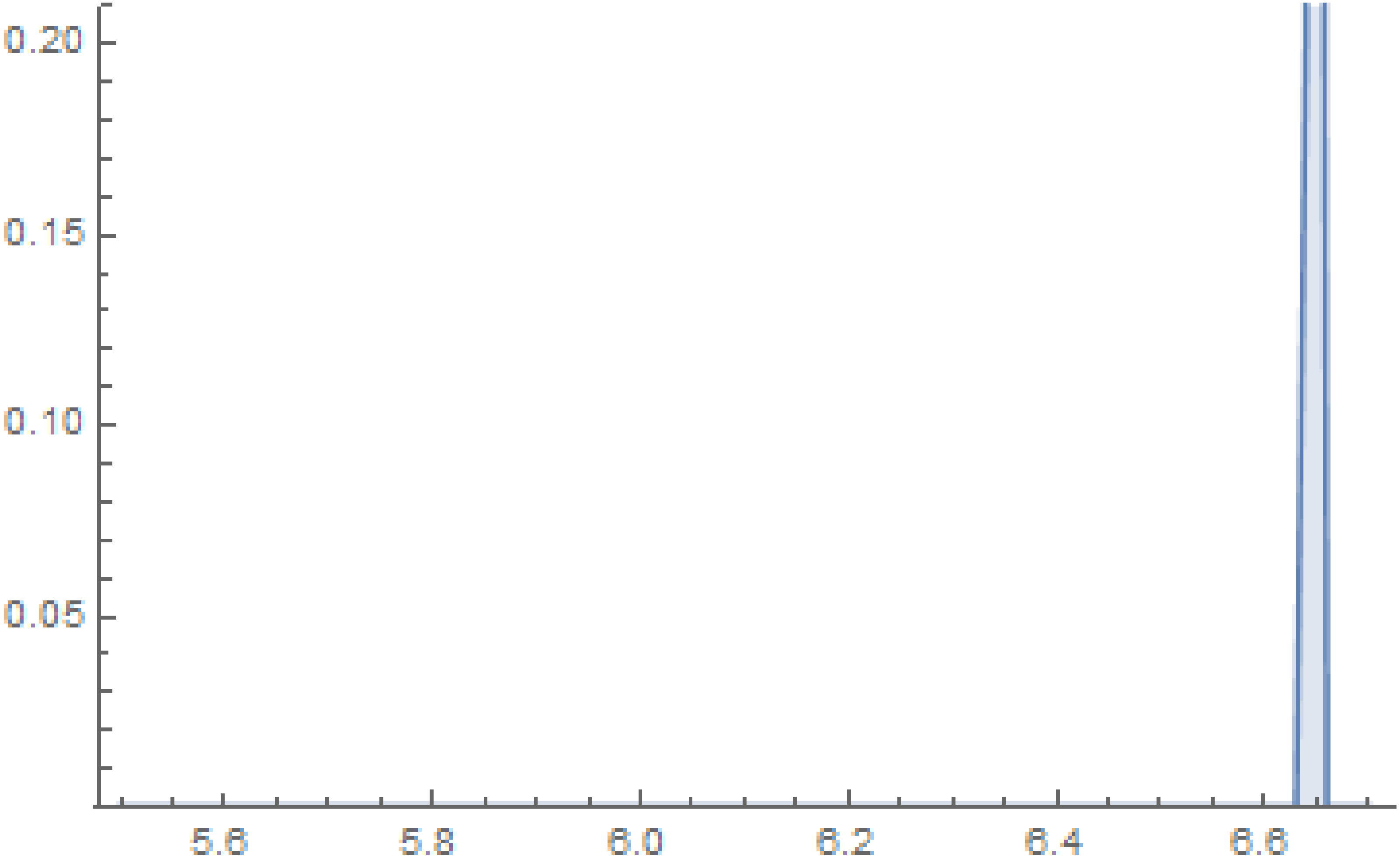
142 configurations has been calculated in 0m49s on the 142 processes.
On 1 process it takes 36m14s
– acc is ~ 44 times



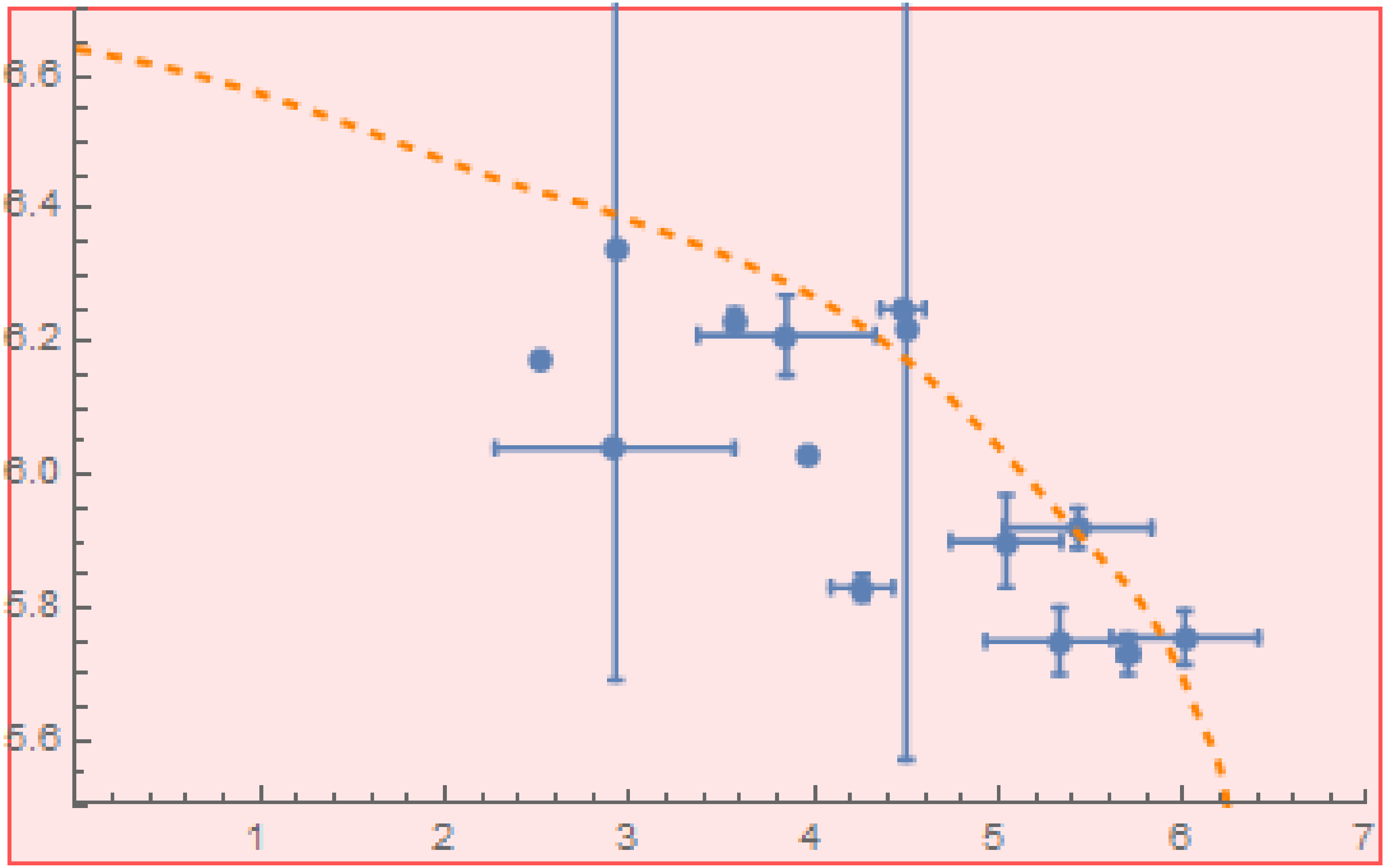
Distribution of Evolution tracks via Temperature at given Time



Distribution of Evolution tracks via Temperature at given Time

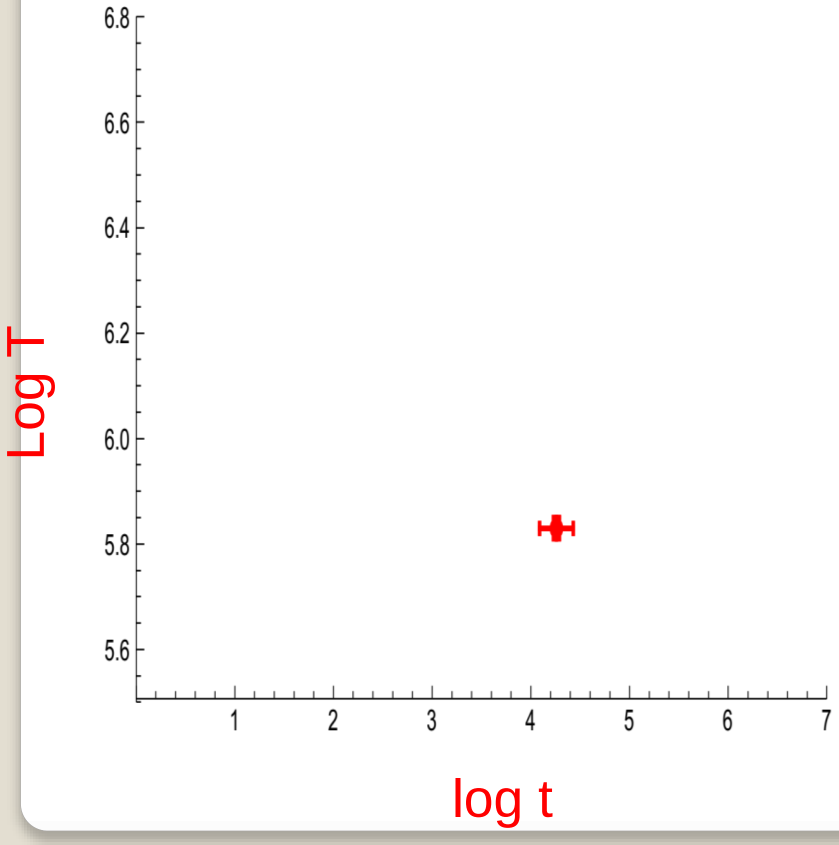


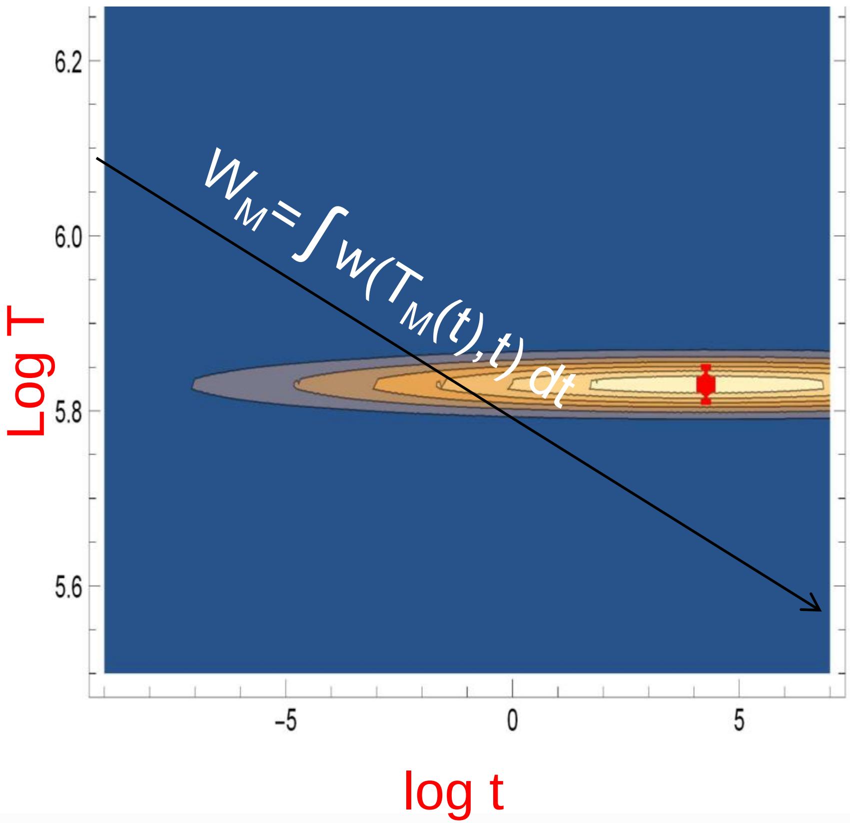
Evolution tracks for different NS Masses



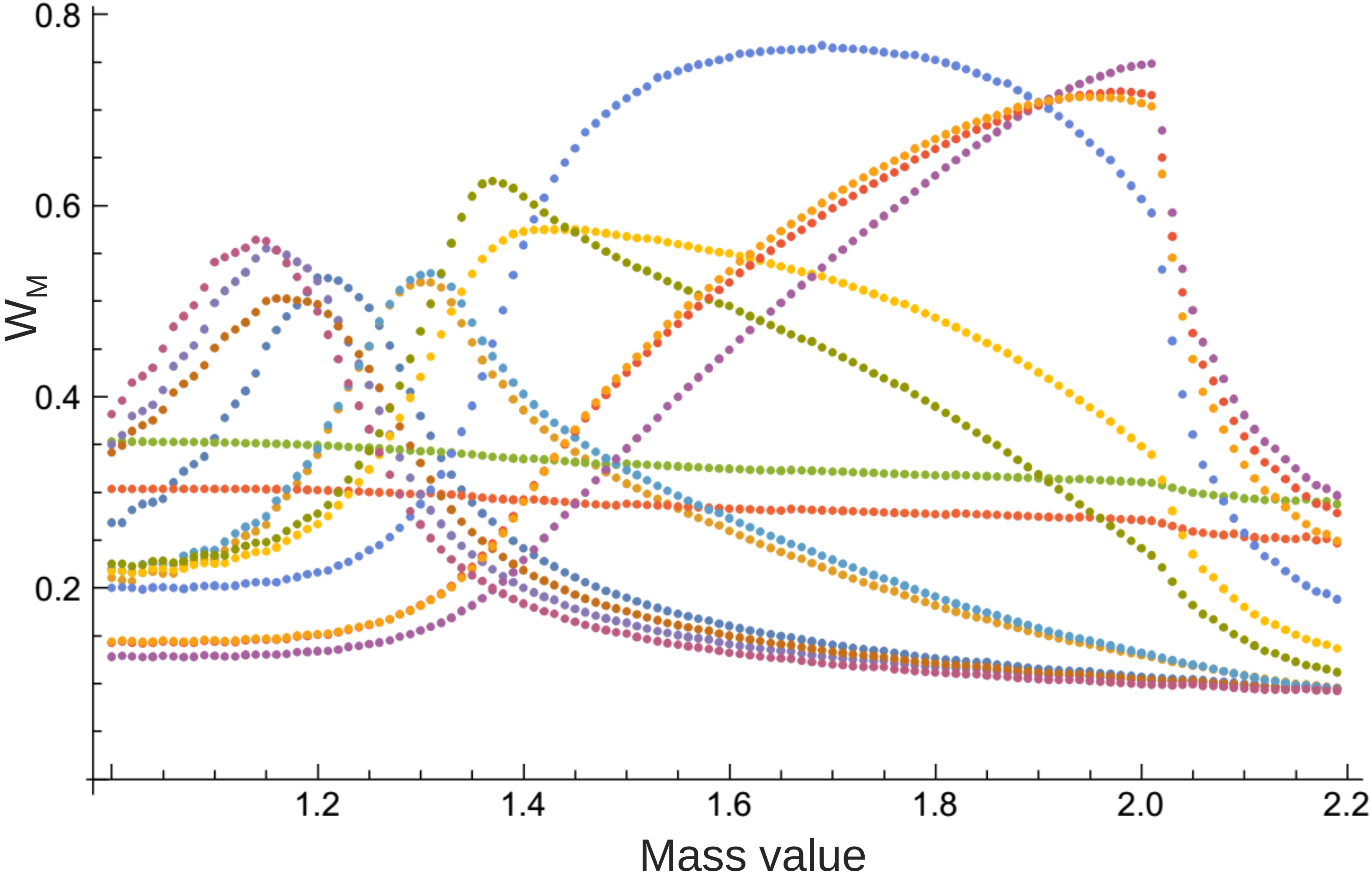
Weighting of Data point on the Temperature - Age Diagram

$$w(T,t) = \mathrm{Exp}\left\{(\log T - \log T_D)^2/\sigma_T + (\log t - \log t_D)^2/\sigma_t\right\}$$

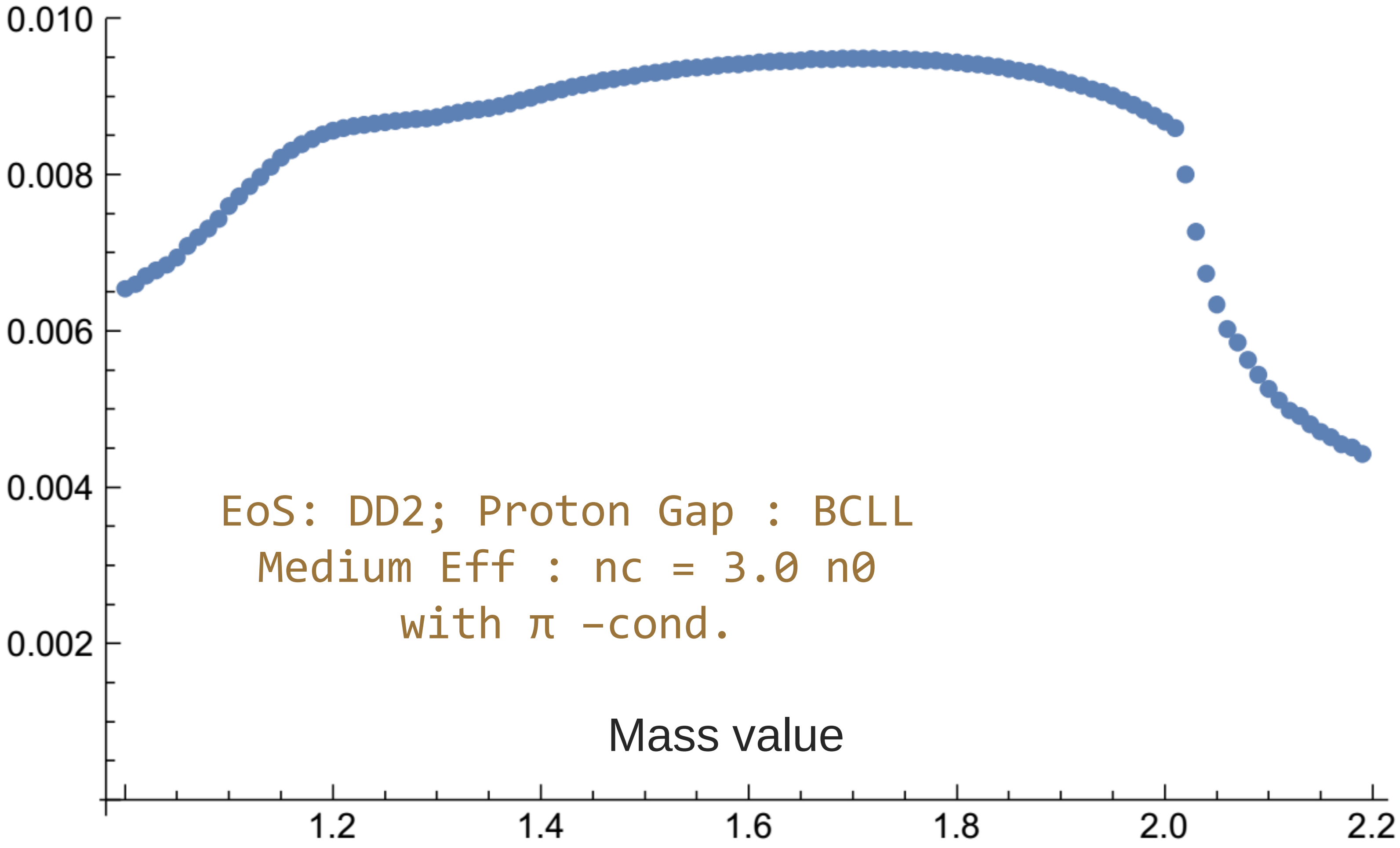




Expected Mass value for the Data points on the T - t Diagram

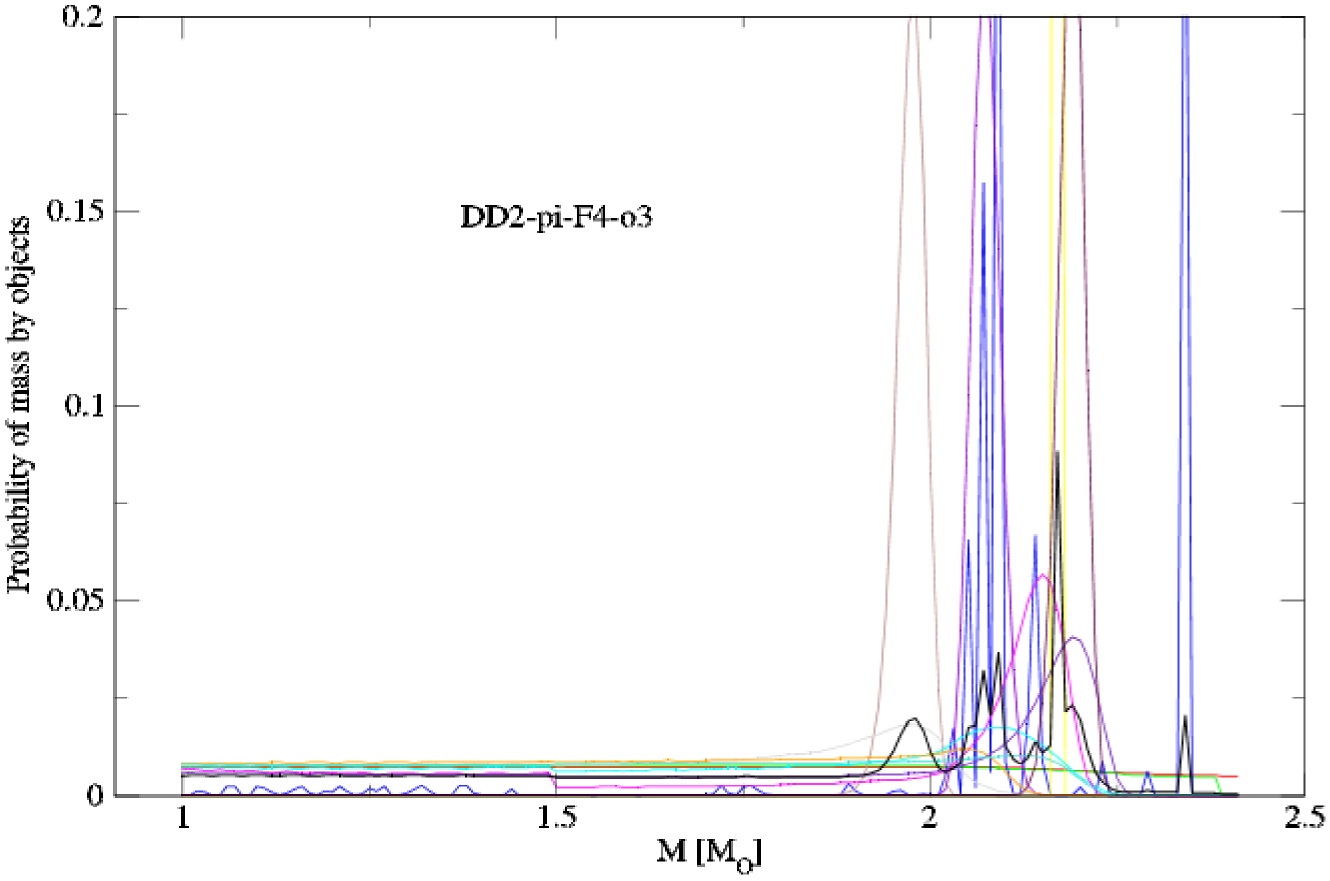


Expected Mass value for the Data points on the T - t Diagram

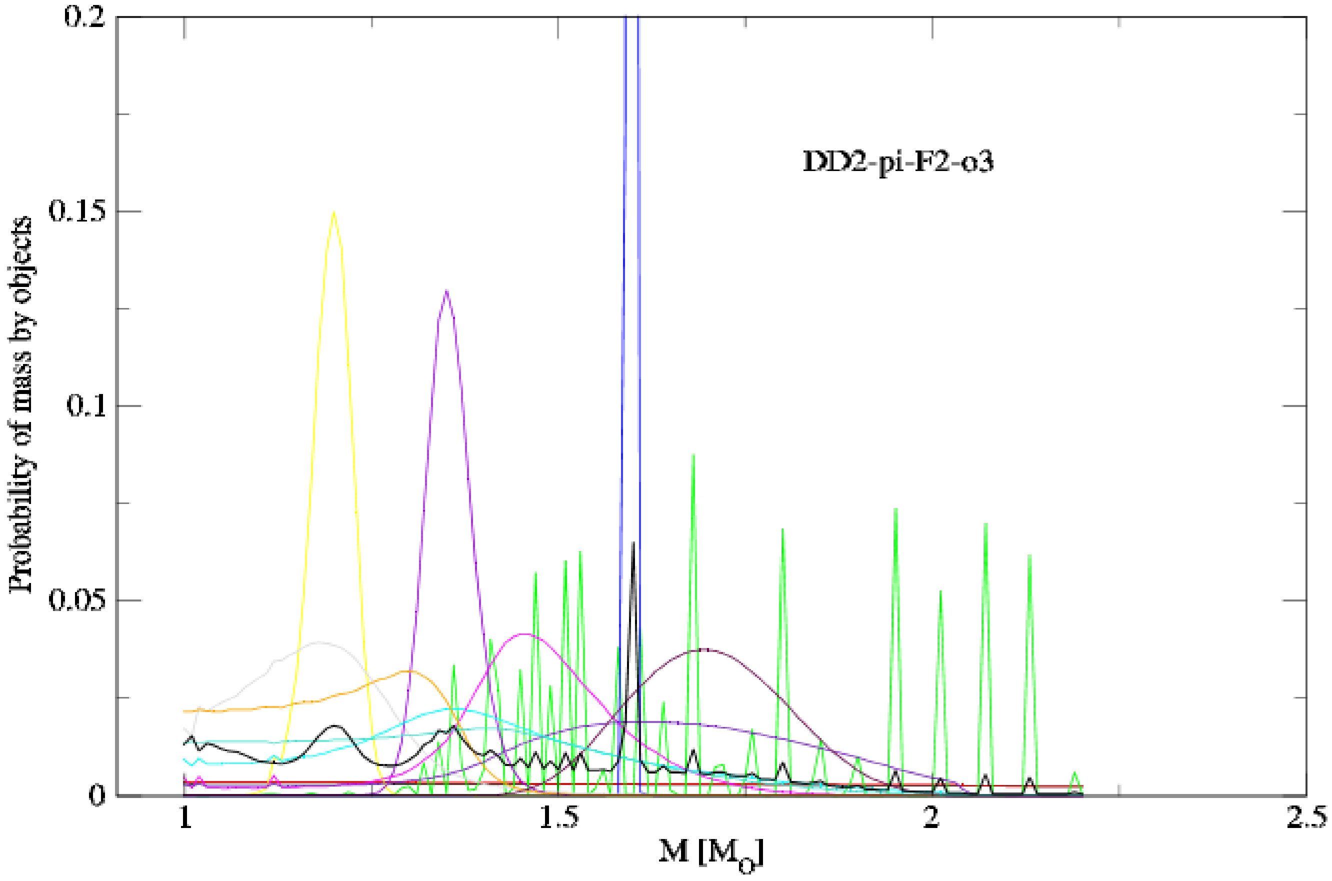


EoS: DD2; Proton Gap : BCLL
Medium Eff : nc = 3.0 n0
with π -cond.

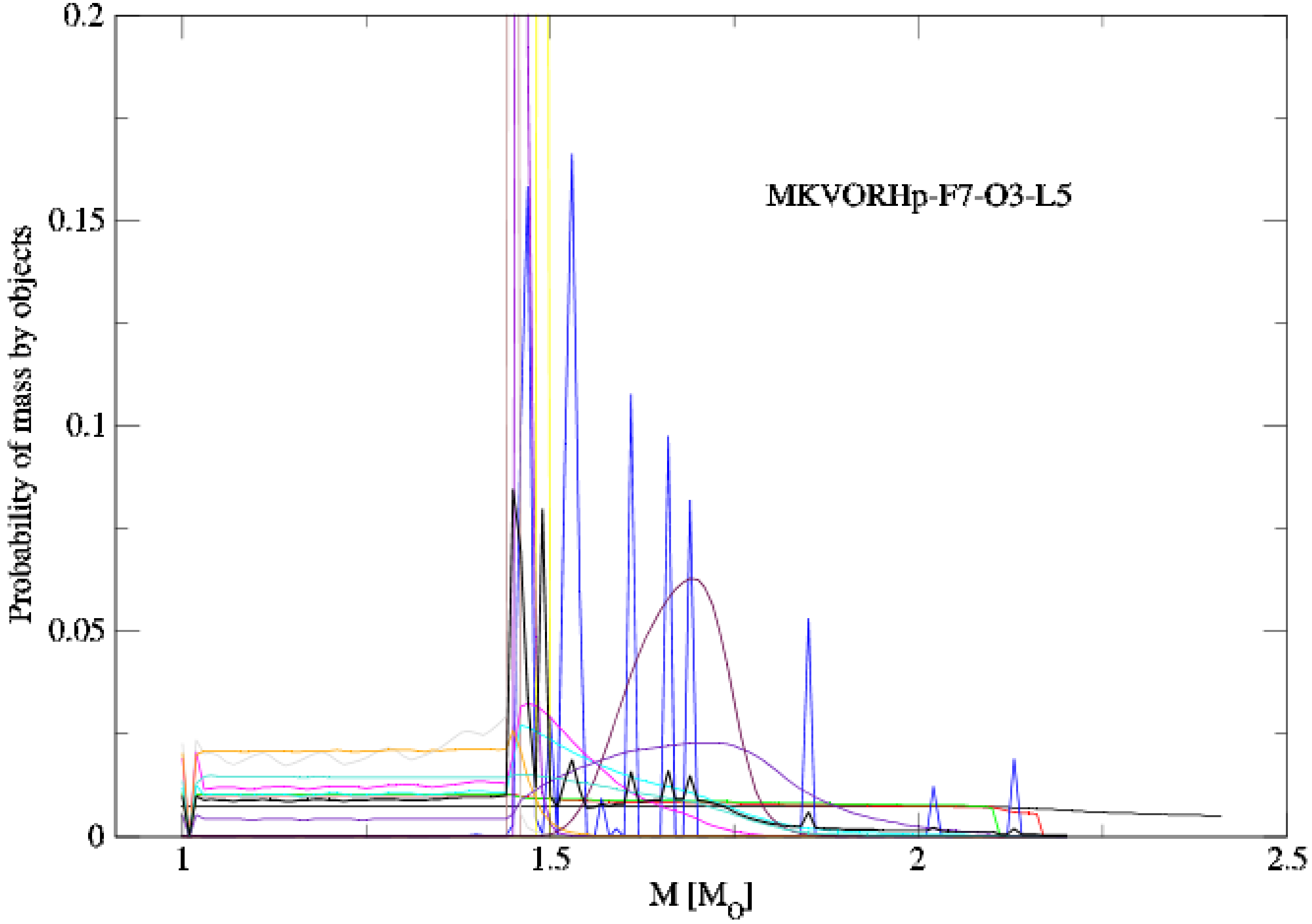
Expected Mass value for the Data points on the T - t Diagram



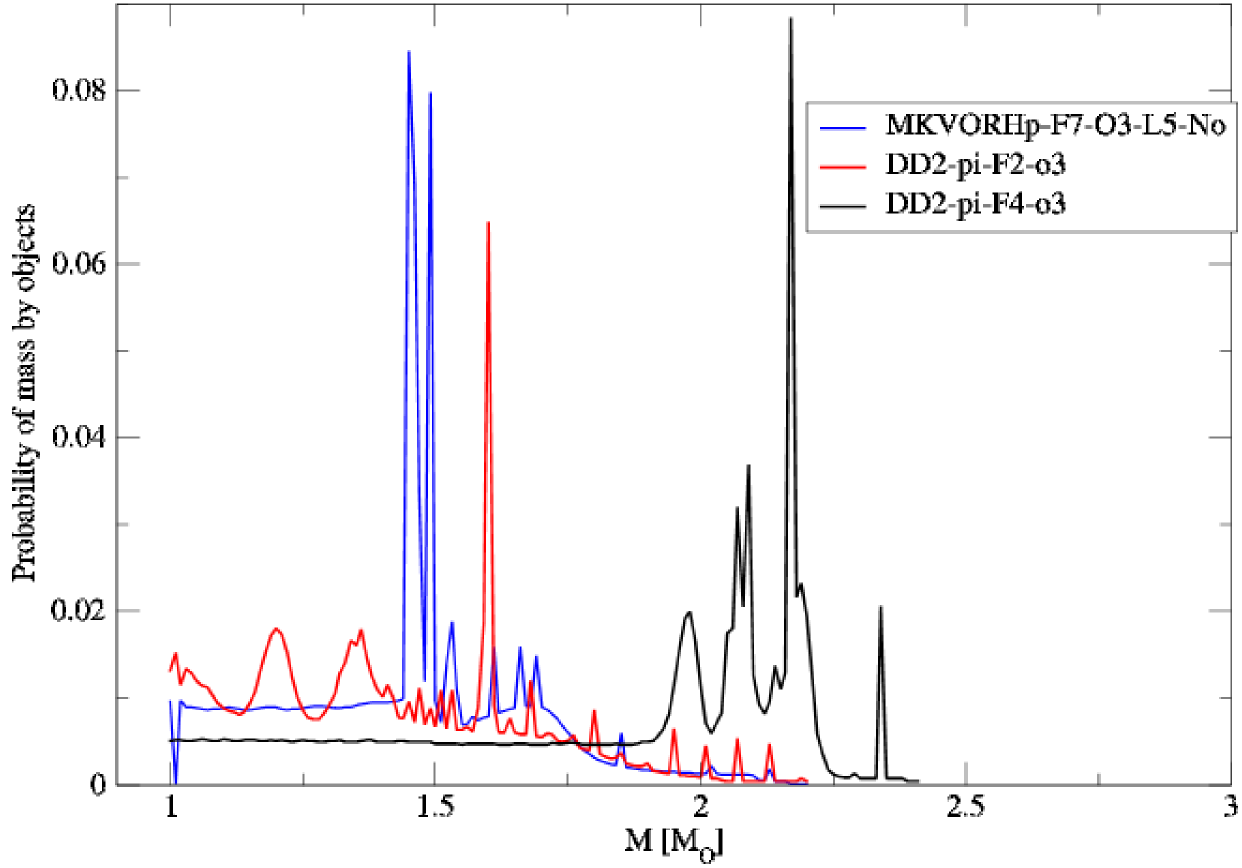
Expected Mass value for the Data points on the T - t Diagram



Expected Mass value for the Data points on the T - t Diagram



Expected Mass value for the Data points on the T - t Diagram

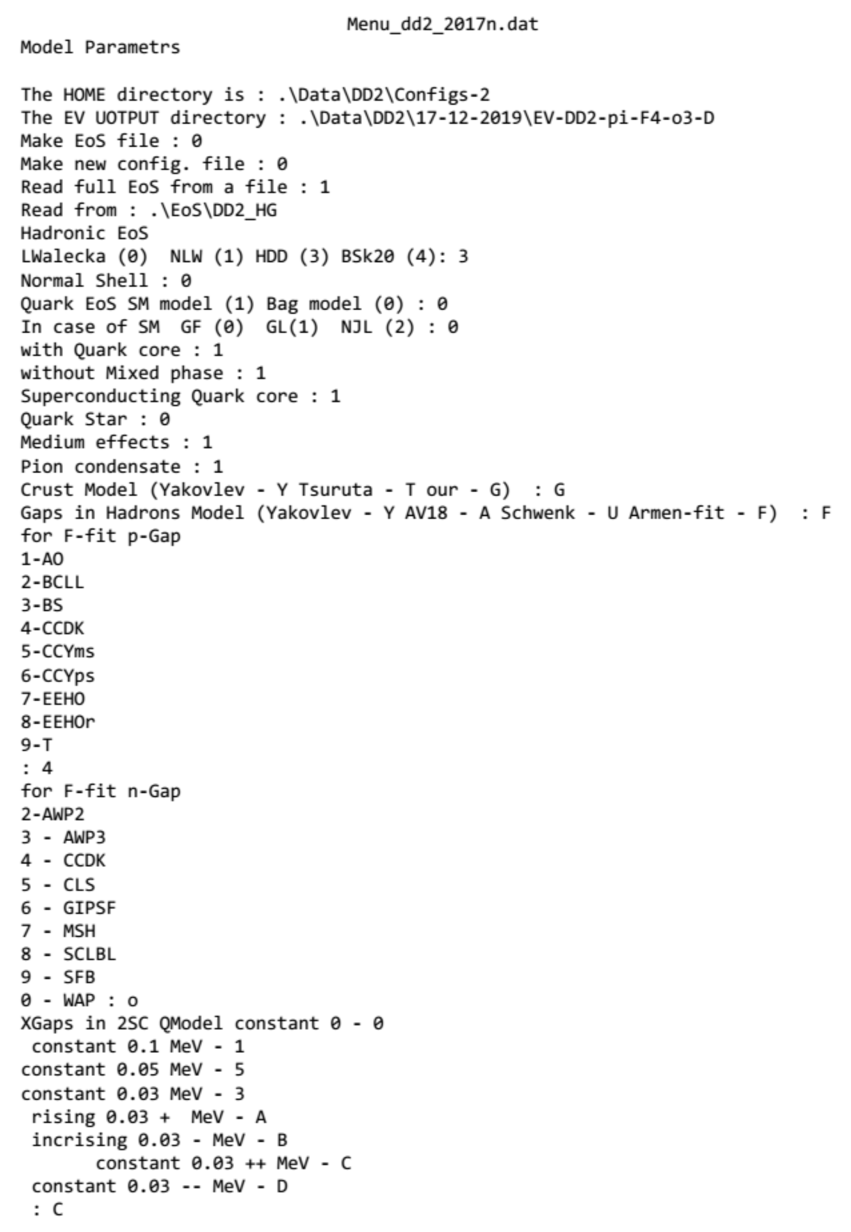


Conclusions

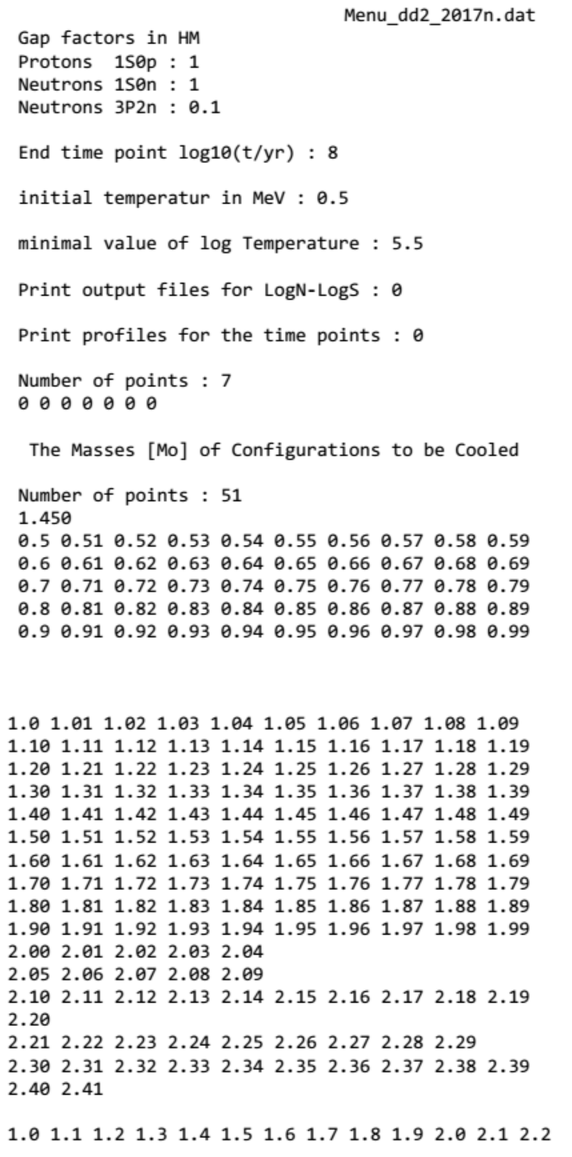
- All known cooling data including the Cas A rapid cooling consistently described by the “nuclear medium cooling” scenario
- Influence of stiffness on EoS and cooling can be balanced by the choice of corresponding gap model.
- Parallelization allowed to make the calculations for statistical analyses of models in reasonable time,
- it allows to estimate the masses of observed objects.
- The cases of existence of Hyperons and/or Quarks or Dark-Matter in high-mass stars could be discussed for extraction of stars masses.

Thank YOU!!!!!

Model parameters – DD2

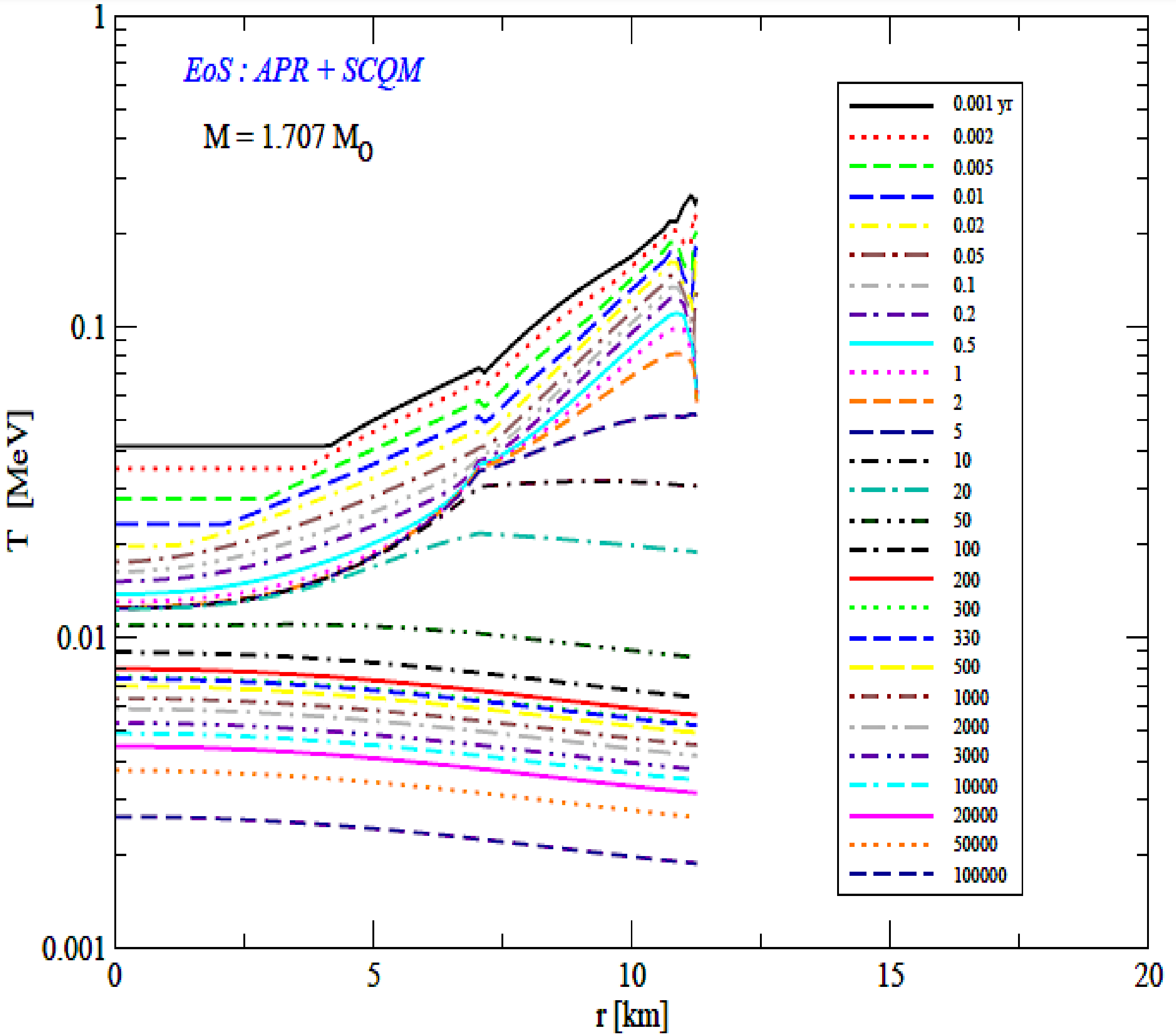


```
                              Menu_dd2_2017n.dat
Model Parametrs

The HOME directory is : .\Data\DD2\Configs-2
The EV UOTPUT directory : .\Data\DD2\17-12-2019\EV-DD2-pi-F4-o3-D
Make EoS file : 0
Make new config. file : 0
Read full EoS from a file : 1
Read from : .\EoS\DD2_HG
Hadronic EoS
LWalecka (0)  NLW (1) HDD (3) BSk20 (4): 3
Normal Shell : 0
Quark EoS SM model (1) Bag model (0) : 0
In case of SM  GF (0)  GL(1)  NJL (2) : 0
with Quark core : 1
without Mixed phase : 1
Superconducting Quark core : 1
Quark Star : 0
Medium effects : 1
Pion condensate : 1
Crust Model (Yakovlev - Y Tsuruta - T our - G)  : G
Gaps in Hadrons Model (Yakovlev - Y AV18 - A Schwenk - U Armen-fit - F)  : F
for F-fit p-Gap
1-AO
2-BCLL
3-BS
4-CCDK
5-CCYms
6-CCYps
7-EEHO
8-EEHOr
9-T
: 4
for F-fit n-Gap
2-AWP2
3 - AWP3
4 - CCDK
5 - CLS
6 - GIPSF
7 - MSH
8 - SCLBL
9 - SFB
0 - WAP : o
XGaps in 2SC QModel constant 0 - 0
 constant 0.1 MeV - 1
constant 0.05 MeV - 5
constant 0.03 MeV - 3
 rising 0.03 +  MeV - A
 incrising 0.03 - MeV - B
       constant 0.03 ++ MeV - C
 constant 0.03 -- MeV - D
 : C
```

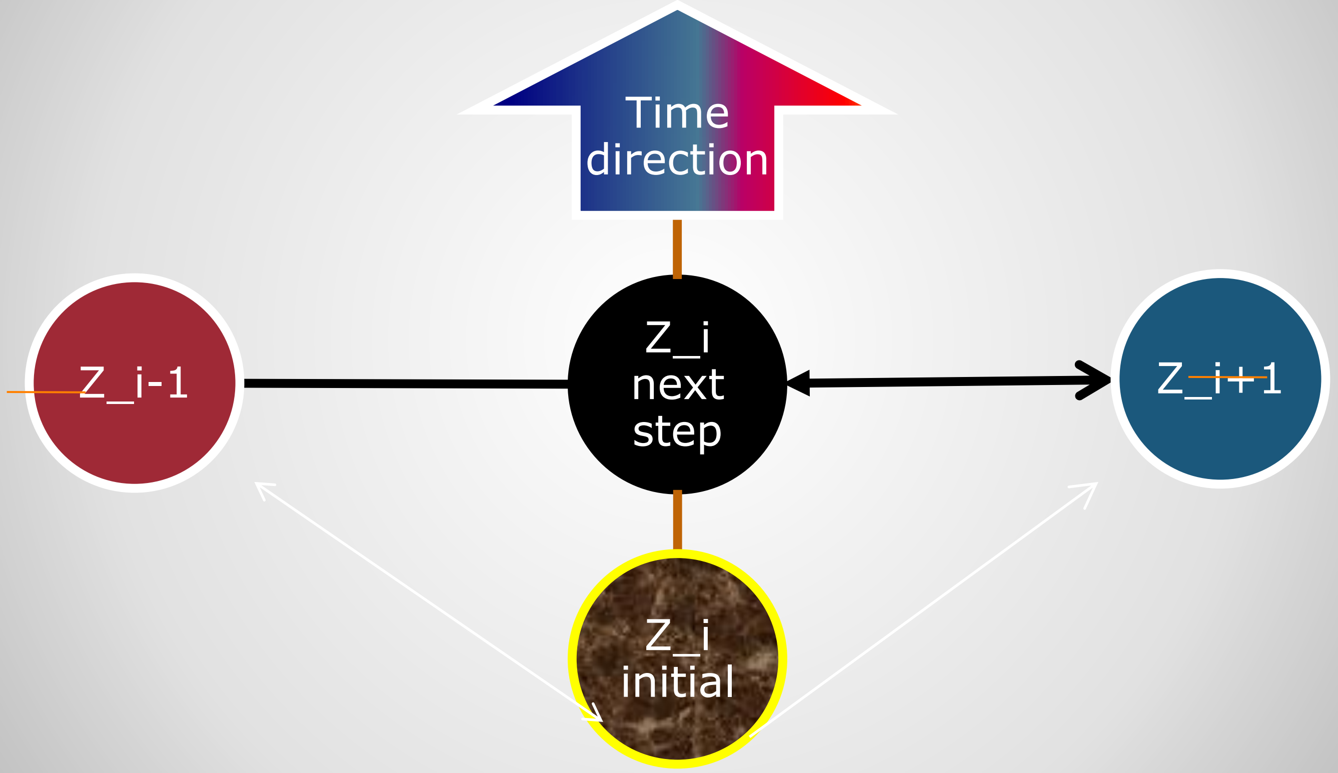


```
                              Menu_dd2_2017n.dat
Gap factors in HM
Protons  1S0p : 1
Neutrons 1S0n : 1
Neutrons 3P2n : 0.1

End time point log10(t/yr) : 8

initial temperatur in MeV : 0.5

minimal value of log Temperature : 5.5

Print output files for LogN-LogS : 0

Print profiles for the time points : 0

Number of points : 7
0 0 0 0 0 0 0

 The Masses [Mo] of Configurations to be Cooled

Number of points : 51
1.450
0.5 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69
0.7 0.71 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79
0.8 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99



1.0 1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19
1.20 1.21 1.22 1.23 1.24 1.25 1.26 1.27 1.28 1.29
1.30 1.31 1.32 1.33 1.34 1.35 1.36 1.37 1.38 1.39
1.40 1.41 1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49
1.50 1.51 1.52 1.53 1.54 1.55 1.56 1.57 1.58 1.59
1.60 1.61 1.62 1.63 1.64 1.65 1.66 1.67 1.68 1.69
1.70 1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79
1.80 1.81 1.82 1.83 1.84 1.85 1.86 1.87 1.88 1.89
1.90 1.91 1.92 1.93 1.94 1.95 1.96 1.97 1.98 1.99
2.00 2.01 2.02 2.03 2.04
2.05 2.06 2.07 2.08 2.09
2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19
2.20
2.21 2.22 2.23 2.24 2.25 2.26 2.27 2.28 2.29
2.30 2.31 2.32 2.33 2.34 2.35 2.36 2.37 2.38 2.39
2.40 2.41

1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
```

Temperature in the Hybrid Star Interior



Finite difference scheme



Time direction

Z_i-1

Z_i next step

Z_i+1

Z_i initial

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Finite difference scheme

$$\begin{pmatrix} \beta_{0,j-1} & \alpha_{0,j-1} & & & 0 \\ \gamma_{1,j-1} & * & * & & \\ & * & * & * & \\ & & * & * & \alpha_{N-1,j-1} \\ 0 & & & \gamma_{N,j-1} & \beta_{N,j-1} \end{pmatrix} \begin{pmatrix} z_{0,j} \\ z_{1,j} \\ * \\ * \\ z_{N,j} \end{pmatrix} = \begin{pmatrix} \delta_{0,j-1} \\ \delta_{1,j-1} \\ * \\ * \\ \delta_{N,j-1} \end{pmatrix}$$

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Equations for Cooling Evolution

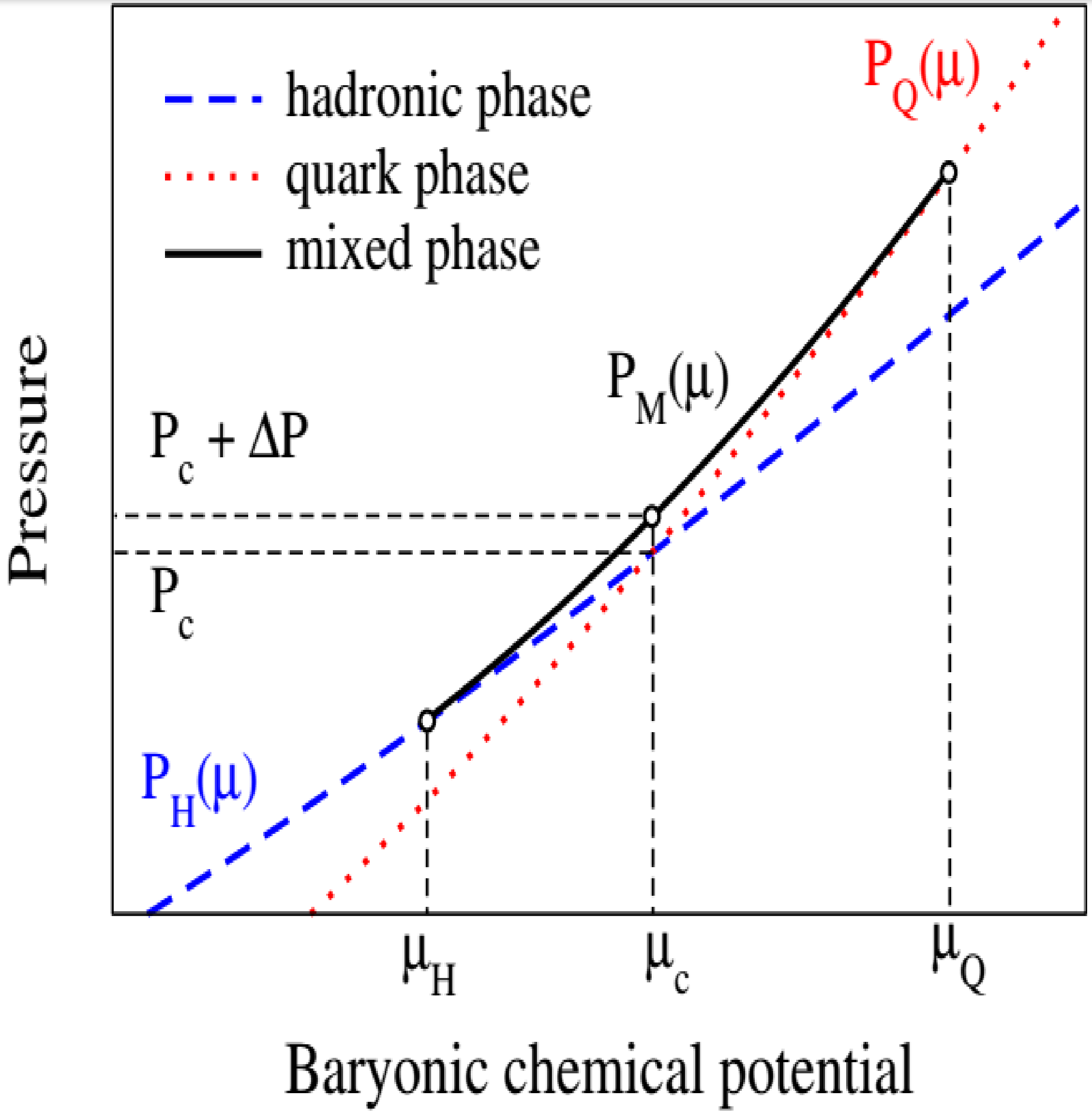
$$\begin{cases} \dfrac{\partial z(\tau,a)}{\partial \tau} = A(z,a)\dfrac{\partial L(\tau,a)}{\partial a} + B(z,a) \\ L(\tau,a) = C(z,a)\dfrac{\partial z(\tau,a)}{\partial a} \end{cases}$$

$$z(\tau,a) = \log T(\tau,a)$$

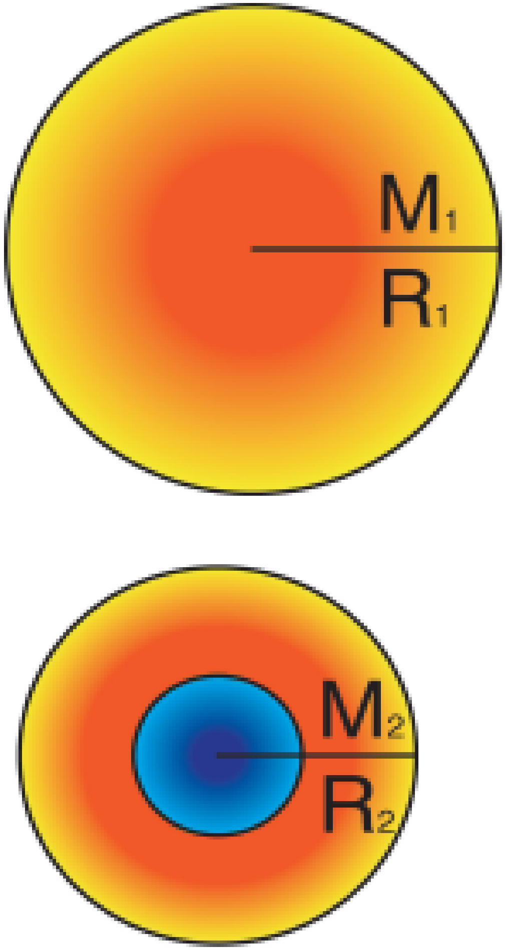
$$L_{i\pm1/2} = \pm\frac{C_i + C_{i\pm1}}{2}\,\frac{z_{i\pm1} - z_i}{\Delta a_{i-1/2(1\mp1)}}$$

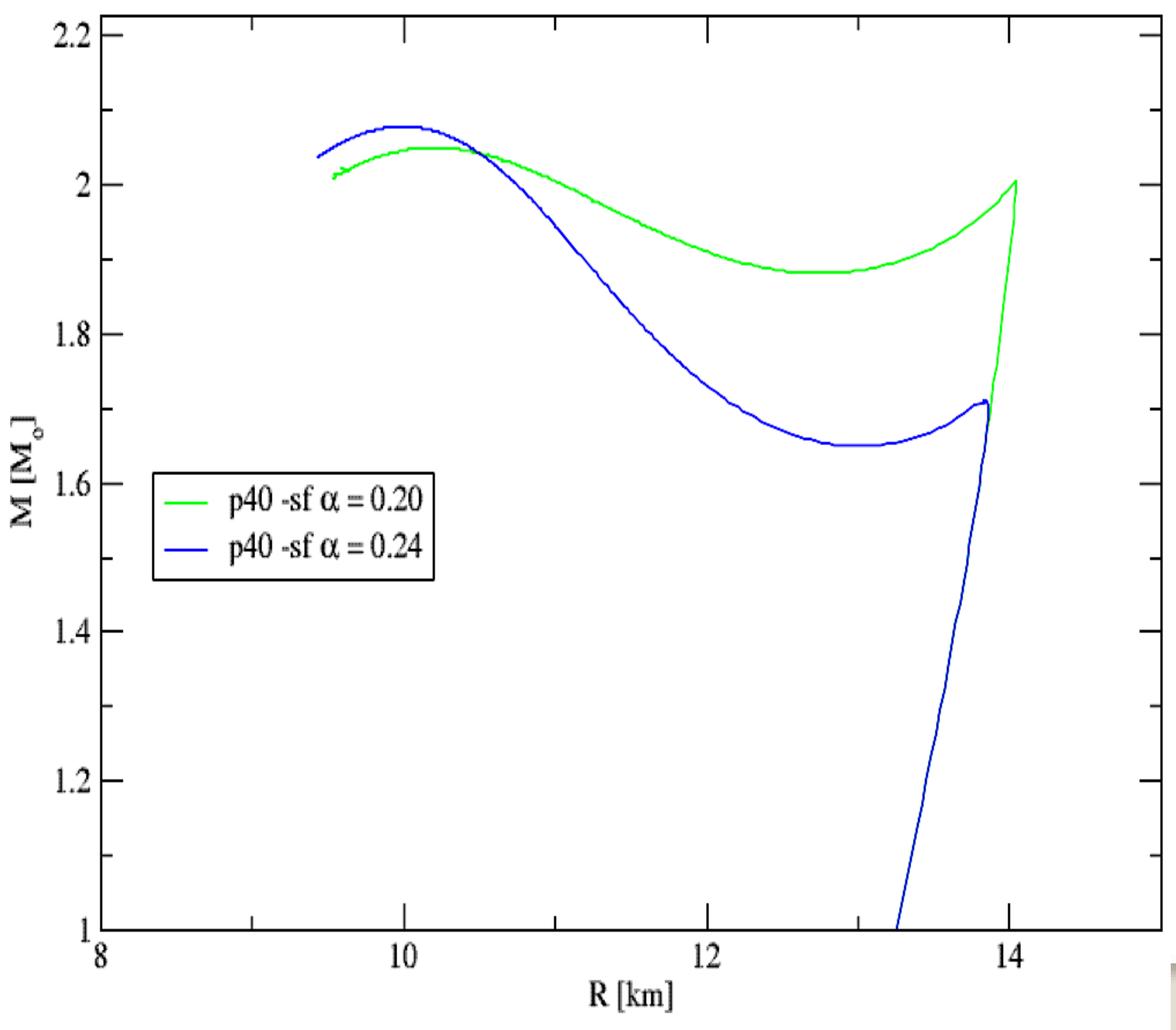
$$\frac{\partial L_i}{\partial a} = 2\,\frac{L_{i+1/2} - L_{i-1/2}}{\Delta a_i + \Delta a_{i-1}}$$

Mixed Phase in Quark-Hadron Phase Transition

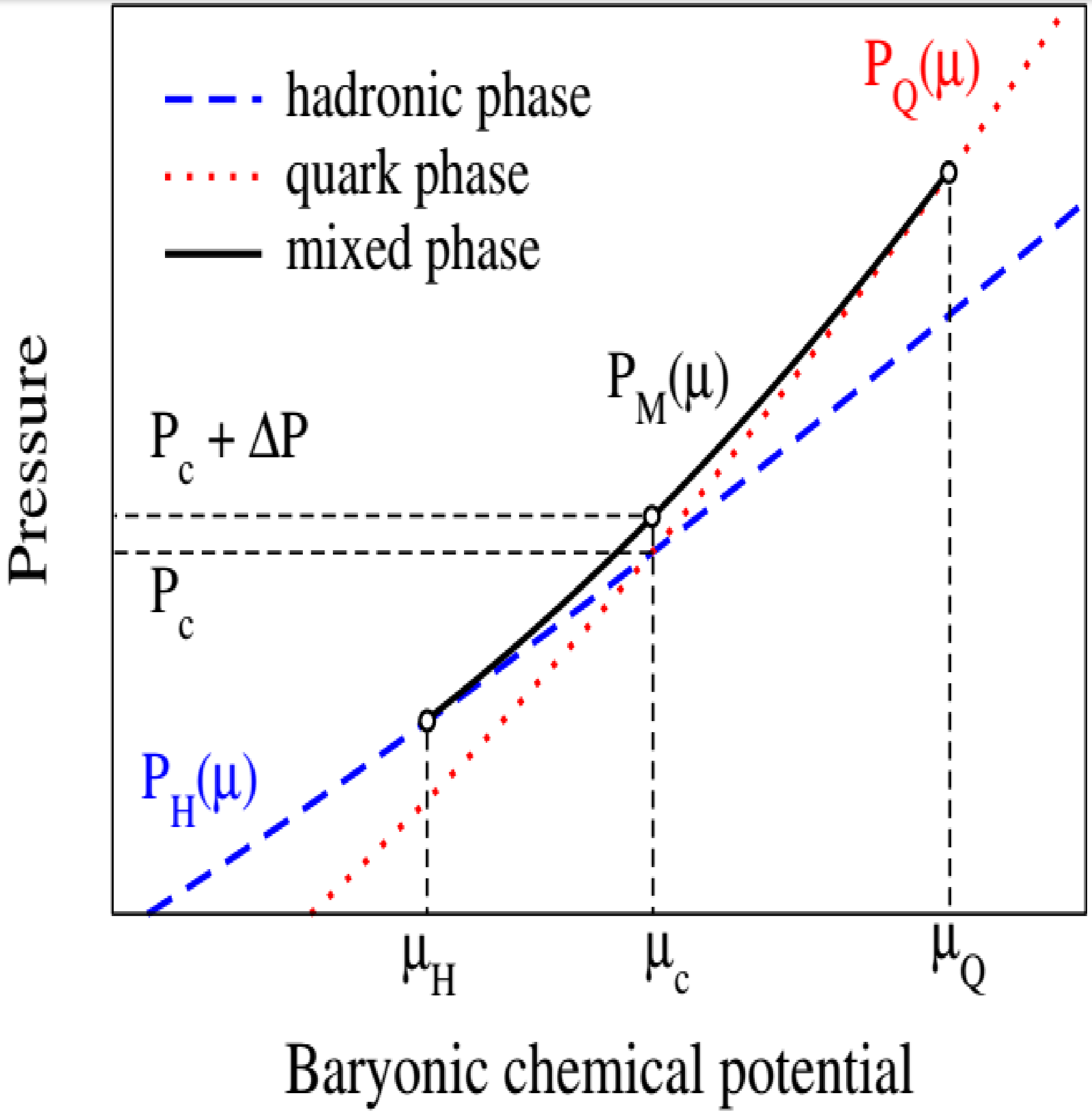


High Mass Twin CS

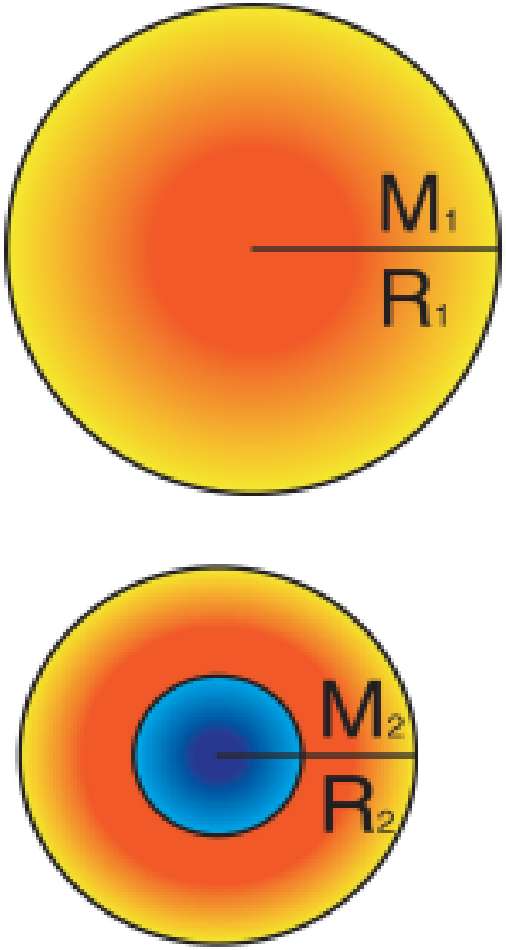


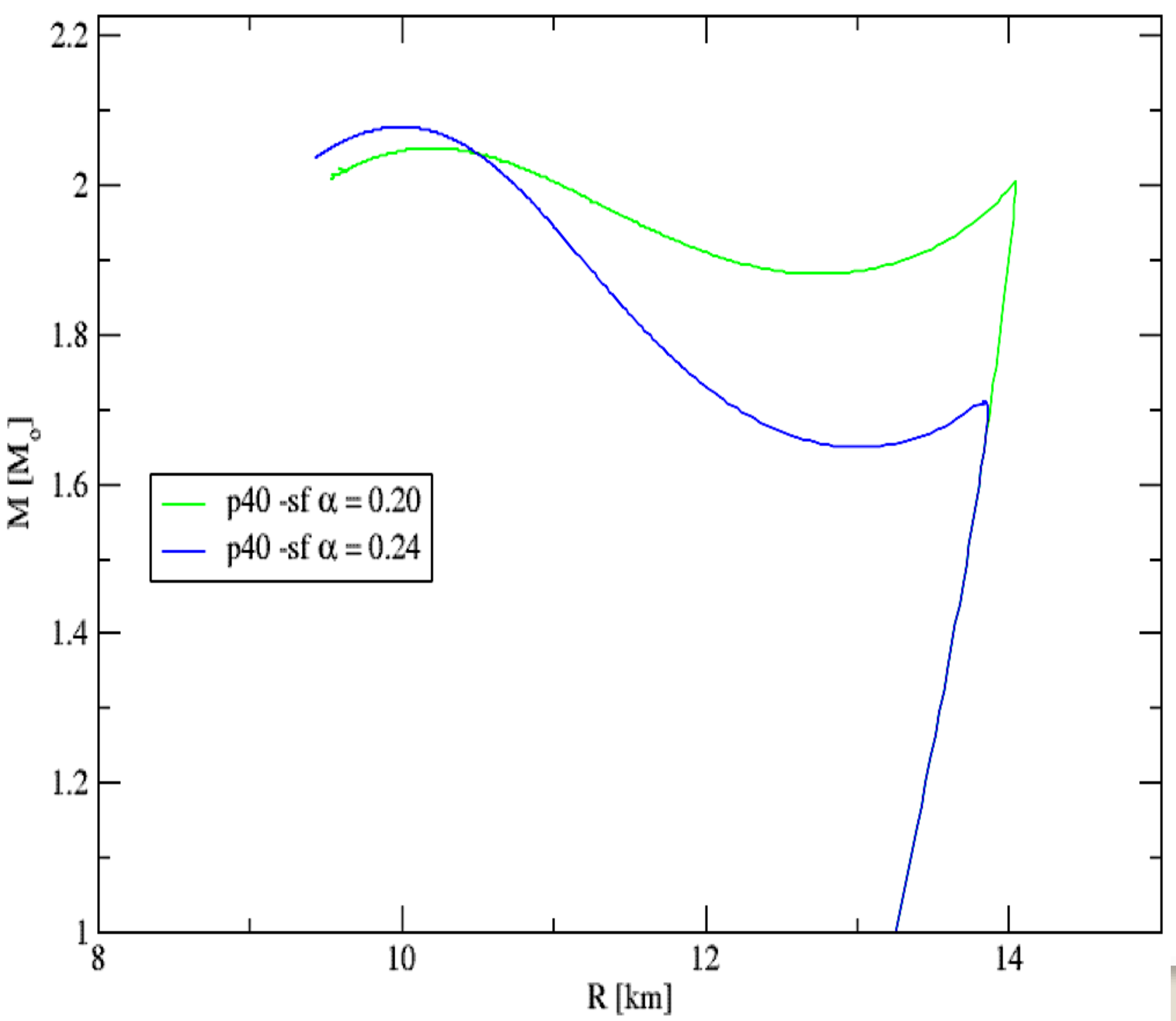


Mixed Phase in Quark-Hadron Phase Transition

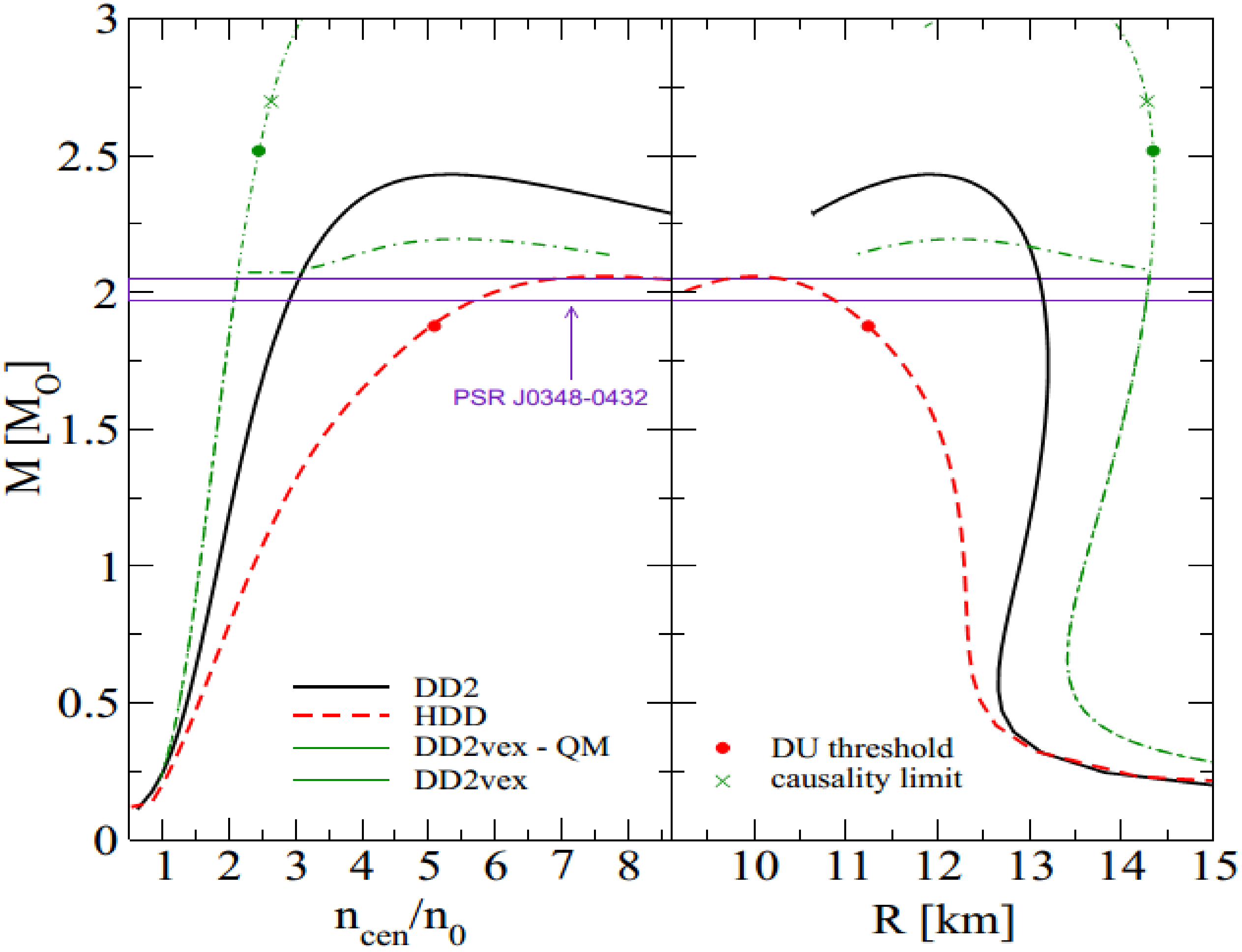


High Mass Twin CS

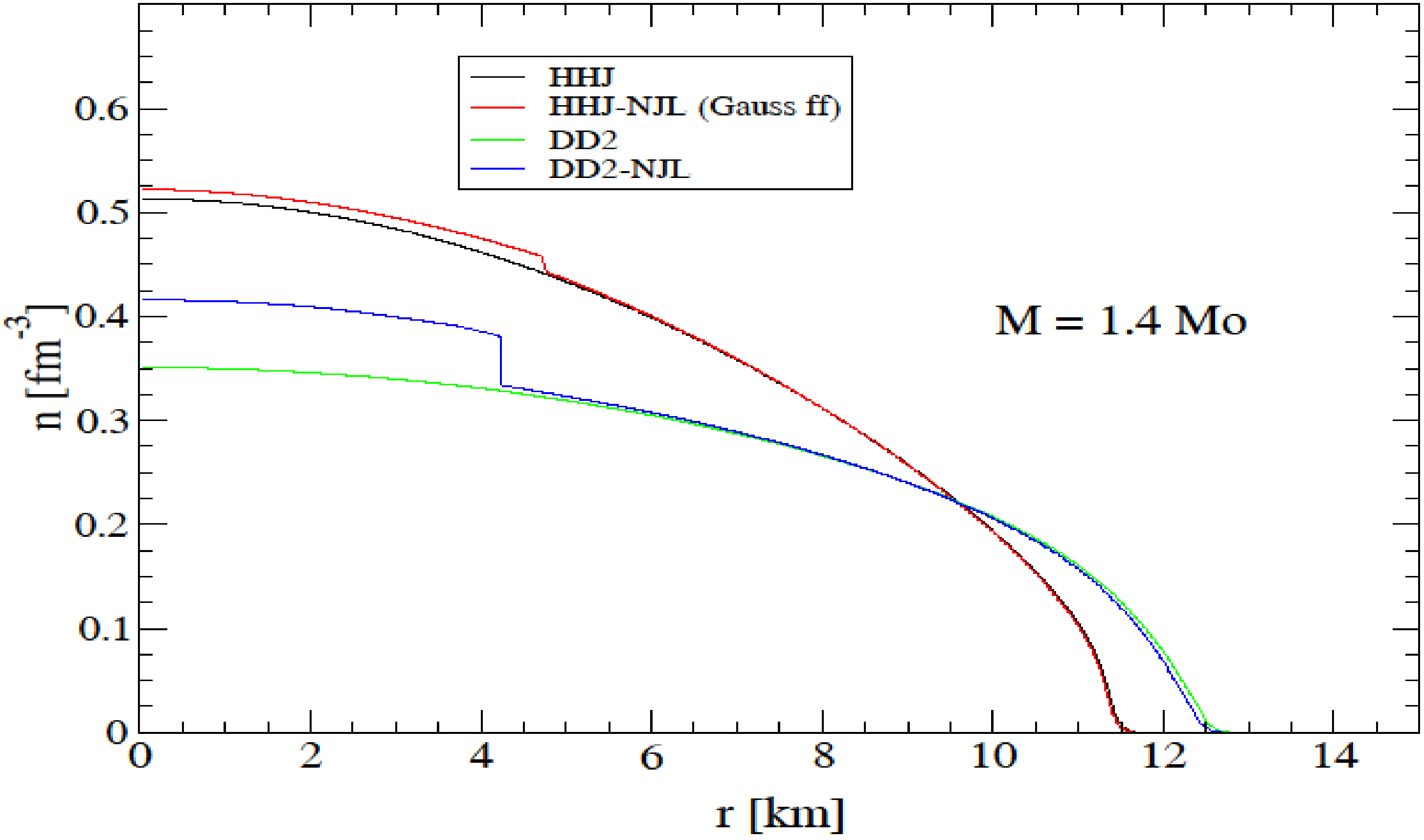




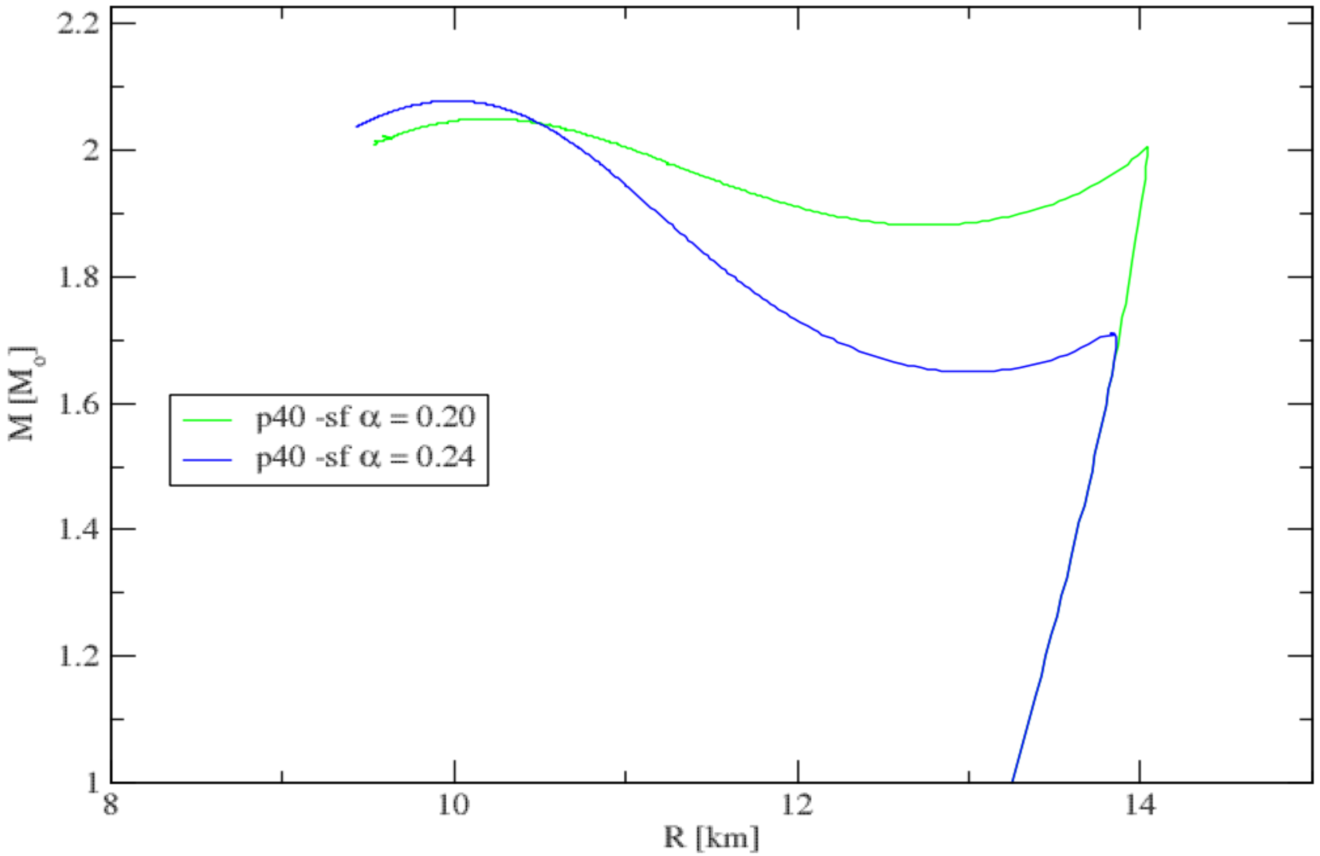
Stability of stars HDD, DD2 & DDvex-NJL EoS model



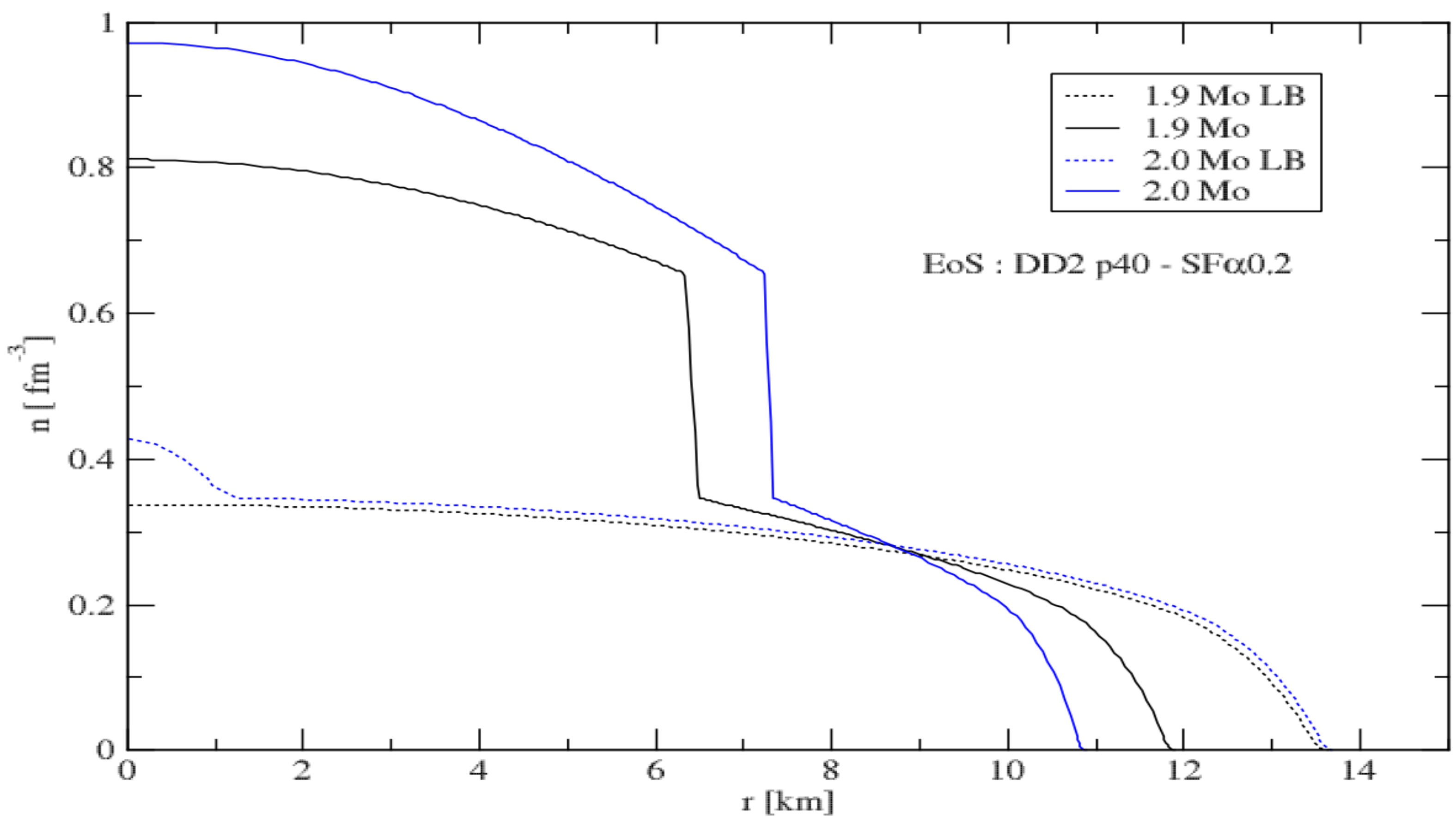
Different Configurations with the same NS mass



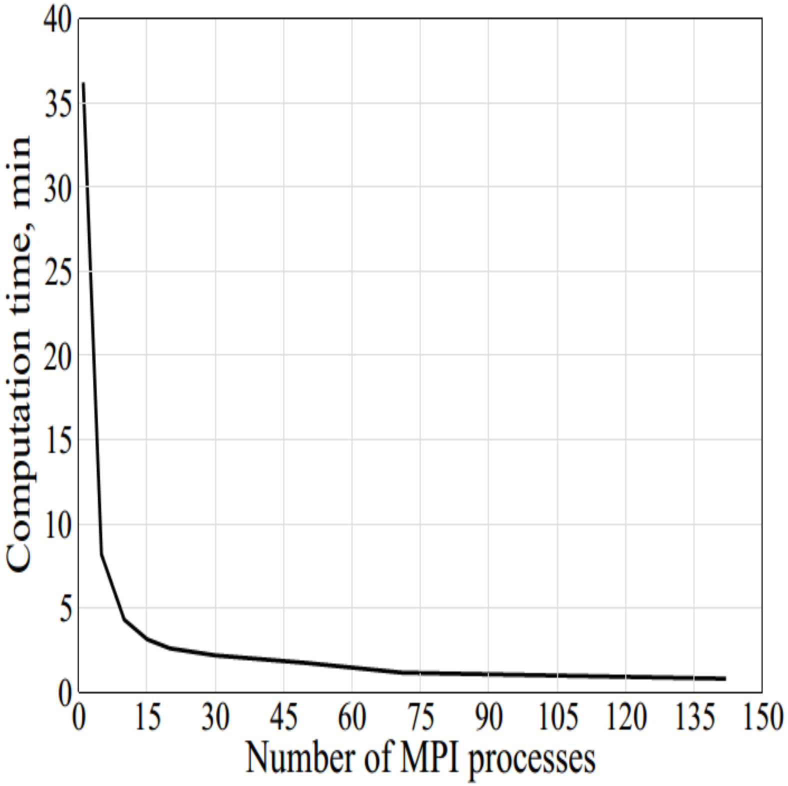
High Mass Twin CS

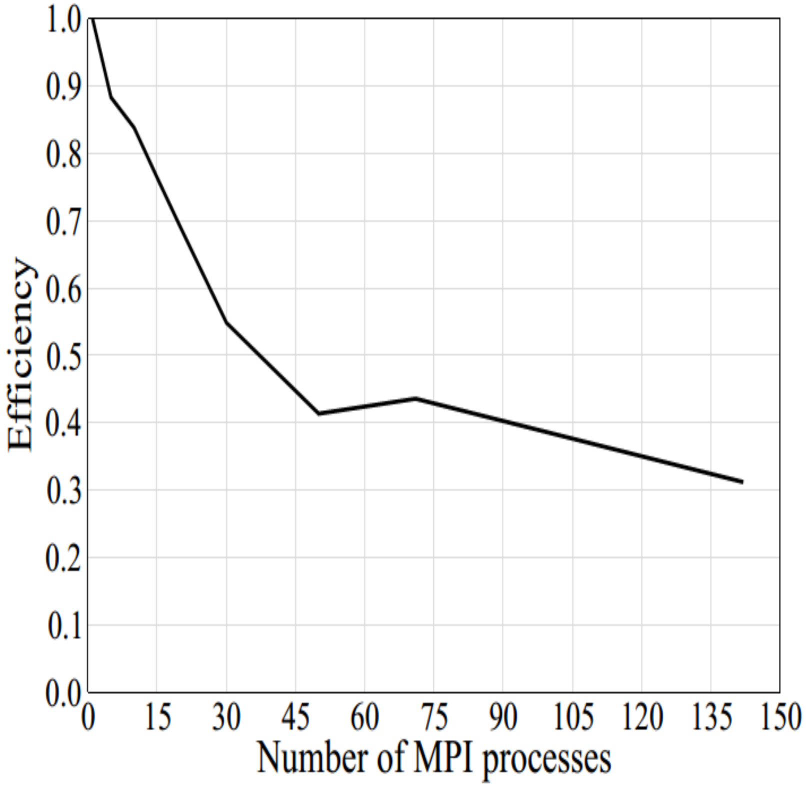


Different Configurations with the same NS mass

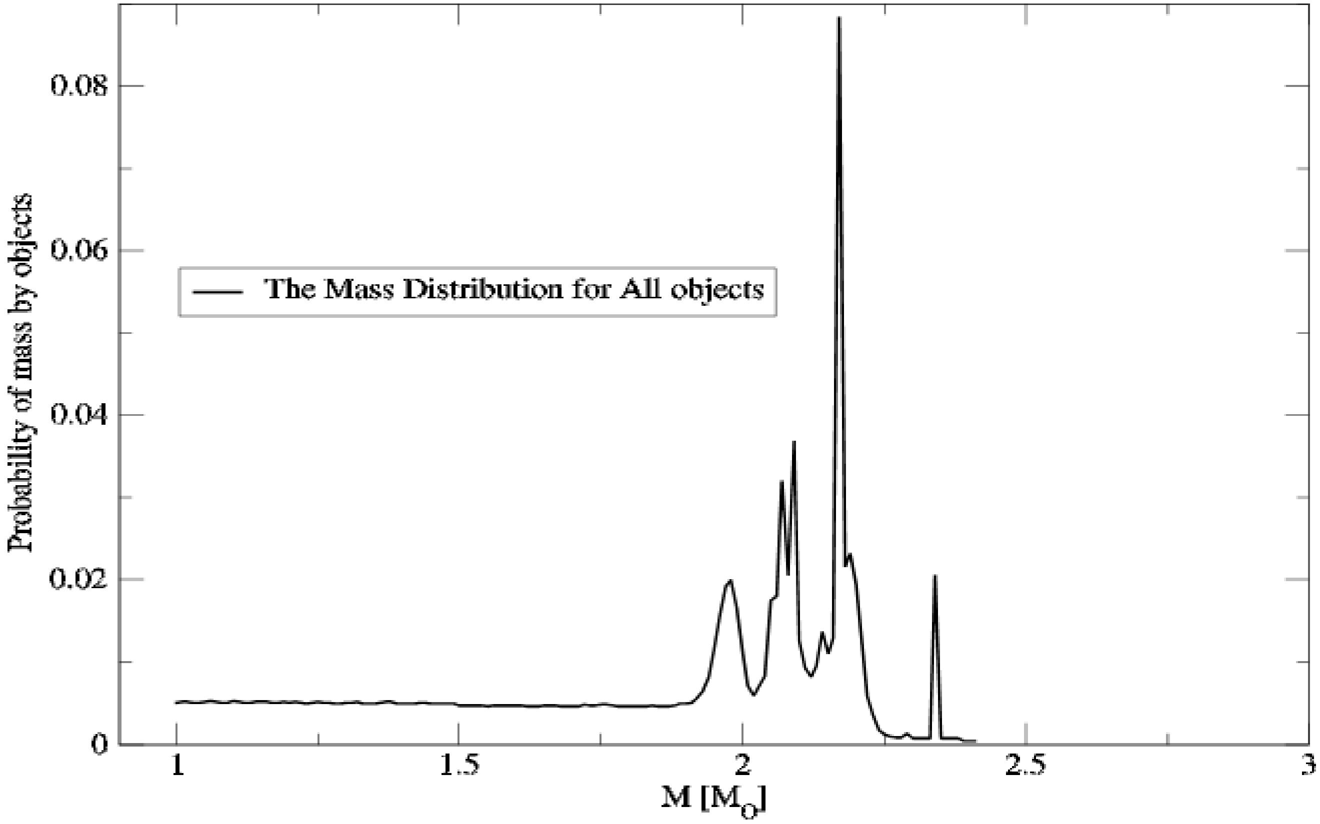


Calculation Time and efficiency





Expected Mass value for the Data points on the T - t Diagram



EoS: DD2; Proton Gap : BCLL Medium Eff : nc = 3.0 n0 with π – cond.