Parton Production in nn and AA collision

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Sept. 3, 2005 APCTP

0. Introduction

- QM05:
  - We have a parton system which is not just a bunch of partons but has collective effects: flow, jet suppression and very dense.
  - Which means that the classical parton cascade may have only very limited application! -> quantum Boltzmann or PC+Hydro

  - Before we move on, we need to understand very carefully the partonic system: 'How many parton', 'How are they evolving', 'How are they breaking up', 'How are the particles formed'

1. Particle production

- Depends on the colliding particles:
  - ep
  - nn
  - AA

- Depends on the available energy:
  - KeV
  - MeV
  - GeV
  - TeV

CROSS SECTION:

1) Physical meaning:

a. $$d\sigma = D(\theta,\phi)d\Omega \rightarrow D(\theta,\phi) = \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta}\left(\frac{db}{d\theta}\right) \right|$$

b. $$dN = L d\sigma \rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{L}\frac{dN}{d\Omega}$$

2) Fermi Golden Rule:

$$1 + 2 \rightarrow 3 + 4 + \ldots + n$$

$$d\sigma = |M|^2 \frac{\hbar^2 S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2}} \left[\left(\frac{c d^3 p_3}{(2\pi)^2 2E_3}\right)\right]\left[\left(\frac{c d^3 p_4}{(2\pi)^2 2E_4}\right)\right] \ldots \left[\left(\frac{c d^3 p_n}{(2\pi)^2 2En_n}\right)\right]$$

$$(2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - \ldots - p_n)$$

3. ep Collisions

Depending on energy:

- **Movie at KeV:**
- **Movie at MeV:**
- **Movie at GeV:**
- **Movie at TeV:**

- ep at KeV: see only electric field: Rutherford scattering, Mott formula

$$\frac{d\sigma}{d\Omega}\Big)_{mott} = \left(\frac{\alpha\hbar}{2\vec{p}^{\,2}\sin^2(\theta/2)} \right)^2 \left[(mc)^2 + \vec{p}^{\,2}\cos^2\frac{\theta}{2} \right]$$

$$\frac{d\sigma}{d\Omega}\Big)_{Rutherford} = \left(\frac{e^2}{2mv^2\sin^2(\theta/2)} \right)^2$$

- ep at MeV: Rosenbluth formula(Elastic scattering)

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar E'}{8\pi McE} \right)^2 \left\langle |M|^2 \right\rangle$$

$$\left\langle |M|^2 \right\rangle = \frac{g_e{}^4}{q^4} L_{electron}{}^{\mu\nu} K_{\mu\nu,proton}$$

$$L_{electron}^{\mu\nu} = 2\left\{ p_1{}^{\mu} p_3{}^{\nu} + p_3{}^{\mu} p_1{}^{\nu} + g^{\mu\nu}\left[(mc)^2 + (p_1 \cdot p_3) \right] \right\}$$

$$K_{proton}^{\mu\nu} = -K_1 g^{\mu\nu} + \frac{K_2}{(Mc)^2} p_2^{\mu} p_2^{\nu} + \frac{K_3}{(Mc)^2} q_2^{\mu} q_2^{\nu} + \frac{K_5}{(Mc)^2} (p_2^{\mu} q^{\nu} + p_2^{\nu} q^{\mu})$$

Thus,

$$\frac{d\sigma}{d\Omega})_{Rosenbluth} = \left(\frac{\alpha\hbar}{24ME\sin^2(\theta/2)} \right)^2 \frac{E'}{E}\left[2K_1\sin^2(\theta/2) + K_2\cos^2(\theta/2) \right]$$

- ep at GeV: Inelastic collisions

$$\frac{d\sigma}{dE'd\Omega} = \left(\frac{\alpha\hbar}{cq^2}\right)^2 \frac{E'}{E}\left\langle |M|^2 \right\rangle$$

$$\left\langle |M|^2 \right\rangle = \frac{g_e{}^4}{q^4} L_{electron}{}^{\mu\nu} W_{\mu\nu,proton}$$

$$W_{proton}^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{(Mc)^2} p_2^\mu p_2^\nu + \frac{W_4}{(Mc)^2} q_2^\mu q_2^\nu + \frac{W_5}{(Mc)^2}(p_2^\mu q^\nu + p_2^\nu q^\mu)$$

$$\frac{d\sigma}{dE'd\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 \left[2W_1(q^2,x)\sin^2(\theta/2) + W_2(q^2,x)\cos^2(\theta/2)\right]$$

$$where...x = -\frac{q^2}{2q\cdot p}$$

<Bjorken function>

$$MW_1(q^2, x) \to F_1(x),$$

$$\frac{-q^2}{2Mc^2x} W_2(q^2, x) \to F_2(x)$$

<Callan & Gross relation>

$$2xF_1(x) = F_2(x)$$

- ep at TeV:

  Hard particles: More than 80%, comming from elastic collision between e-partons.

  Soft particles: relatively small number.

  -> the parton distribution of nucleon is very important.

  At RHIC energy, about half of them are hard particles and the other half soft particles.

4. nn, AA scattering

- pp scattering at MeV:
  - Mostly elastic collision
- pp scattering at GeV:
  - Inelastic collision with lots of soft particles
- pp scattering at TeV:
  - Elastic collisions among partons with minijets

Models

- Schwinger mechanism:
- Inside-Outside cascade:

- Classical String Model:
- Dual Parton Model:

- Parton-Parton elastic scattering (factorization):
- CGC shattering(Yang-Mills equation):

Toy models

- Schwinger mechanism:

  -two infinite plates with charges(condensers):

  -constant electric field between them

  -vacuum persistance probability: P_V

  -particle production=1-P_V

Assume the constant color electric field(flux) between q and q_bar.

If field energy > threshold energy, (qq_bar) pair produced.->

Production rate: quark field equation in the potential -> Klein-Gordon equation -> Scheodinger equation to identify the effective potential -> sea quark tunnels the potential to leave antiquark -> WKB approximation gives the tunneling rate.

- String model(Lund model):
  - Meson: q-q_bar connected with string in yo-yo state.
  - Suppose a string connected to quark and antiquark separating each other

    - See Figure
    - Quark-antiquark born at V_i
    - V_i = (x,t)-> (t+x,t-x) = $(\Gamma, y), \Gamma = \sqrt{\kappa\tau}, y = rapidity$
    - Verticies are on the proper time: At high energy, the produced particles are independent of rapidity
    - The probability element: for example, 2 vertices production
      - V_0: $dP_0 = \rho(\Gamma_0, y_0) d\Gamma_0 dy_0$
      - V_1: $dP_1 = f_{01}(z_+, M_{01}^2 g(M_{01}^2) dz_+ dM_{01}^2$

      Where

$$z_+ = (p_{0+} - p_{1+}) / p_{0+}, \qquad M_{01}^2 = (p_{0+} - p_{1+})(p_{1-} - p_{0-})$$

figure1

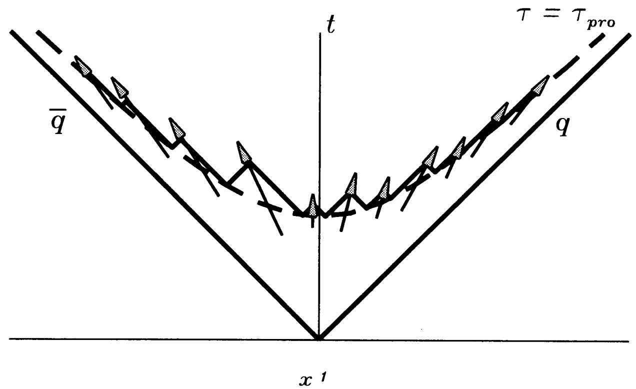


Fig. 6.4 A schematic picture of particle production in the *inside − outside cascade* picture.

- The solution:

$$g(M_{01}^2) = \sum_{\lambda} c_{\lambda} \delta(M_{01}^2 - m_{\lambda}^2)$$

$$f(z_+, m^2) = N \frac{(1 - z_+)^a}{z_+} e^{-bm^2/z_+}$$

$$\rho(\Gamma) = C\Gamma^a e^{-b\Gamma}$$

- FRITOF etc

- Dual Resonance Model:
  - Consider H-H collision at high energy. The exchange of a particle of mass M, spin J: $A \propto s^J$
  - Sum of J: Veneziano scattering amplitude

$$A(s,t) = g^2 \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

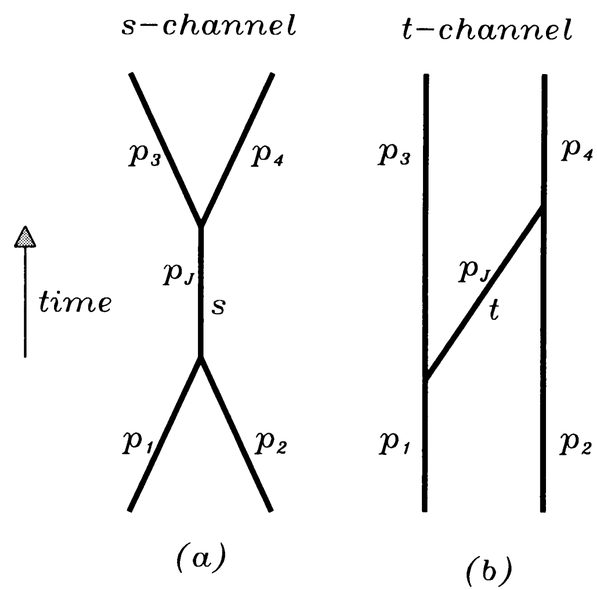
$$where...\alpha(x) = \alpha_0 + \alpha' x.$$

  - For fixed t, expand in s. Each term corresponds to a t-channel Feynman diagram with exchange of a particle:

$$J = \alpha_0 + \alpha' M_J^2$$

    - Regge trajectory -> Reggeon
  - Same for s-channel:

- figure2



Symmetrize over possible channels:

$$A_4 = A(s,t) + A(t,u) + A(u,s)$$
$$\approx -g^2 \pi e^{-\alpha(t)} \frac{(1+e^{-i\pi\alpha(t)})}{\Gamma(1+\alpha(t))} \frac{(\alpha' s)^{\alpha(t)}}{\sin(\pi\alpha(t))}$$

- Basic properties of HH collisions at high energy:
  - Inelastic processes with no quantum number flow=dominant
  - Forward scattering amplitude ~ purely imaginary
  - Total cross section ~ constant.

- Out of Regge trajectory, $\alpha_0 = 1$ satisfies these.
  - => Pomeranchuk trajectory: Pomeron

Elastic scattering between partons:

- Simple and Clear!
- Inclusive gluon production:

$$\frac{d\sigma}{dyd^2p_t} = \frac{16N_c}{N_c^2-1}\frac{1}{p_t^2}\int \alpha_s(\Omega^2)F_1(x_1,k_1^2)F_2(x_2,k_2^2)\delta(\vec{k_1}+\vec{k_2}-\vec{p_t})d^2k_1d^2k_2$$

where

$$x_{1/2} = \frac{p_t}{\sqrt{s}}\exp(\pm y)$$

$$\Omega^2 = \max(k_1^2, k_2^2, p_t^2)$$

Pros: Pretty good with minijets

Cons: no soft particles

- Minijets:

$$\frac{dN^{jet}}{dydp_t} = KT(b)\int dy_2 \frac{2\pi p_t}{\hat{s}} \sum_{ijkl} x_1 f_{i/A}(x_1, p_t^2) x_2 f_{j/B}(x_2, p_t^2) \sigma_{ij\to kl}(\hat{s}, \hat{t}, u)$$

- Unintegrated gluon distribution:
  - BFKL: solution of BFKL equation

$$f(x,k_t^2)=\frac{C}{x^{\lambda}}\left(\frac{k_t^2}{q_0^2}\right)^{1/2}\frac{\phi_0}{\sqrt{2\pi\lambda^{''}\ln(1/x)}}\exp\left[-\frac{\ln^2(k_t^2/\overline{q}^2)}{2r\lambda^{''}\ln(1/x)}\right],$$

$$where$$

$$\lambda=4\overline{\alpha}_s\ln 2,\lambda^{''}=28\overline{\alpha}_s\varsigma(3),\overline{\alpha}_s=3\alpha_s/\pi,\varsigma(3)=1.202,$$

$$C\overline{\phi}_0=1.19,r=0.15$$

  - Kimber-Martin-Ryskin: DGLAP evolution

$$F(x,k^2)=\frac{\partial}{\partial Q^2}\left[xg(x,Q^2)\right]_{Q^2=k^2},$$

$$f_a(x,k^2,\mu^2)=T_a(k^2,\mu^2)\frac{\alpha_s(k^2)}{2\rho}\sum_{a'}\int_x^{1-\delta}P_{aa'}(z)\frac{x}{z}a'(\frac{x}{z},k^2)dz$$

- Kharzeev-Levin: based on CGC

$$F(x,k^2)=\begin{cases} f_0 \text{..........} if \,..k^2 > Q_s^2 \\ f_0 \dfrac{Q_s^2}{k^2} \text{...} if \,..k^2 > Q_s^2 \end{cases}$$

where

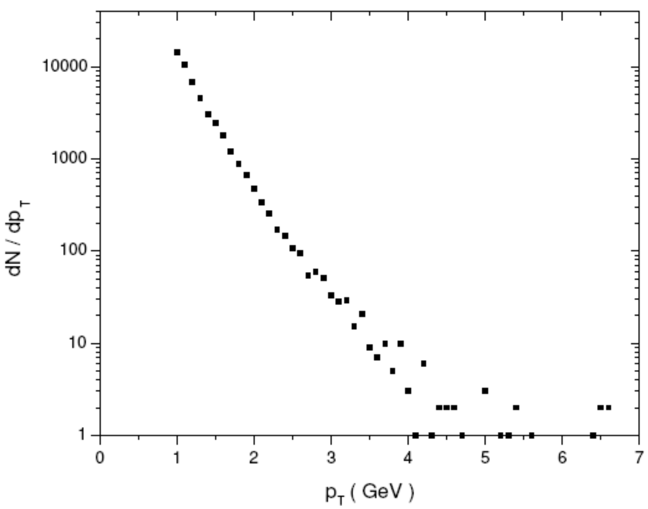
$$Q_s^2(x)=1(GeV)^2\left(\frac{x_0}{x}\right)^2,\ f_0=170mb$$

- GRV98: we have the code!

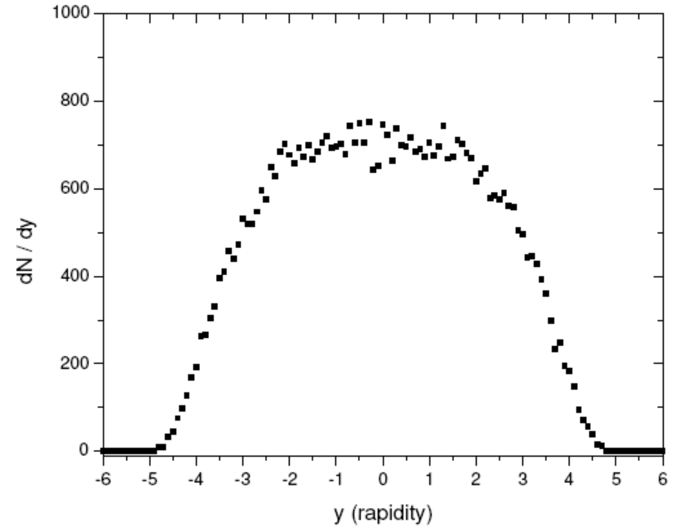
– CGC distribution: Triantafyllopoulos

$$\varphi(k_t, y) = \begin{cases} \dfrac{1}{\alpha_s} \ln \dfrac{Q_s^2}{k_t^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots if\ .k_t << Q_s \\ \dfrac{1}{\alpha_s} \left(\dfrac{Q_s^2}{k_t^2} \right) \left(\ln \dfrac{k_t^2}{Q_s^2} + \Delta \right) \ldots\ldots if\ .k_t \geq Q \\ \dfrac{Q_s^2}{k_t^2} I_0 \left(\sqrt{4 \bar{\alpha}_s y \ln \dfrac{k_t^2}{Q_s^2}} \right) \ldots\ldots k_t >> Q_s \end{cases}$$

- GRV98 & EKS98+minijets:



P_T distribution



rapidity distribution

- DGLAP evolution:(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{\partial a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \sum_{b=q,g} \int_x^1 dz P_{ab}(z,\alpha_s(\mu^2)) b(\frac{x}{z},\mu^2),$$

$where$

$P_{ab} : b \rightarrow ac, branching..\ker nal,$

$a \,\&\, b : parton..distrstribution$

- Modified DGLAP:

$$\frac{\partial a(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} \sum_{b=q,g} [\int_x^1 dz P_{ab}(z,\alpha_s(\mu^2)) b(\frac{x}{z},\mu^2) - a(x,\mu^2)\int_0^1 d\varsigma \varsigma P_{ba}(\varsigma)]$$

- *BFKL equation:(Balitsky-Fadin-Kuraev-Lipatov)*

$$\frac{\partial f_g(x,k_t^2)}{\partial \ln(1/x)} = \frac{C_A\alpha_s}{\pi} k_t^2 \int_0^\infty \frac{dk_t^2}{k_t^2}\left[\frac{f_g(x,k_t^{'2}) - f_g(x,k_t^2)}{\left|k_t^{'2} - k_t^2\right|} + \frac{f_g(x,k_t^2)}{(4k_t^{'4} + k_t^4)^{1/2}}\right]$$

Where

$$a(x,\mu^2) = \int_0^{\mu^2} \frac{dk_t^2}{k_t^2} f_a(x,k_t^2)$$

Yang-Mills Equation:

- Yang-Mills equation of motion:
  - Krasnitz, Nara and Venugopalan; Lappi

$$\left[D_\mu, F^{\mu\nu} \right]^a = J^{\nu,a}$$

where

$$J^\nu = \rho_1 \delta(x^-) \delta^{\nu +} + \rho_2 \delta(x^+) \delta^{\nu -}$$

- Solve the equation and identify the number density:

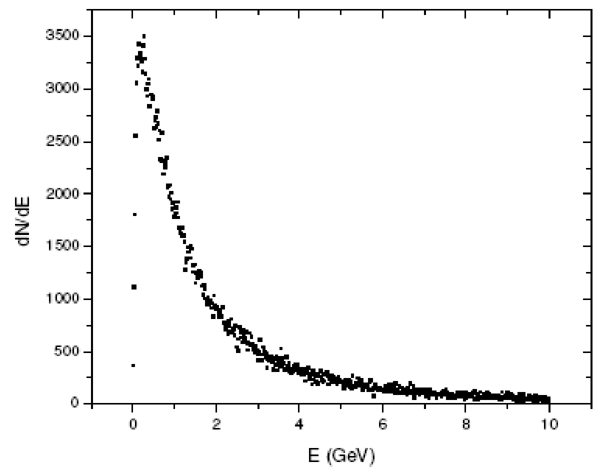
$$f^{glue} = \frac{1}{g^2} \begin{cases} a_1 \left[\exp(\sqrt{k_t^2 + m^2} / T_{eff}) - 1 \right]^{-1} \text{..}(k_t / \Lambda_s > 1.5) \\ a_2 \Lambda_s^4 \log(4\pi k_t / \Lambda_s) k_t^{-4} \text{...........}(k_t / \Lambda_s > 1.5) \end{cases}$$

where

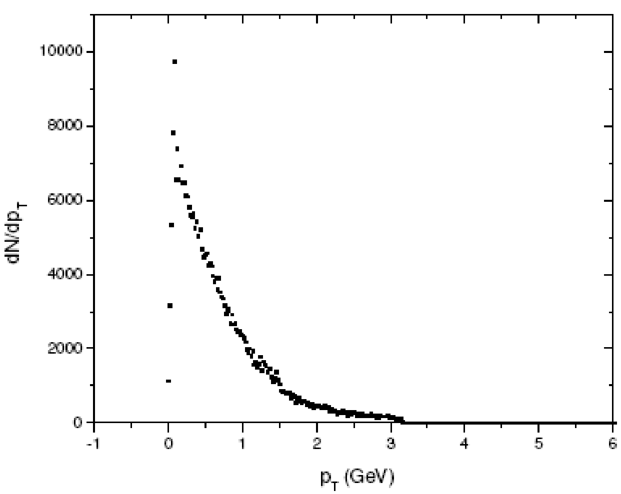
$$a_1 = 0.137, a_2 = 0.0087,$$

$$m = 0.0358 \Lambda_s, T_{eff} = 0.465 \Lambda_s$$

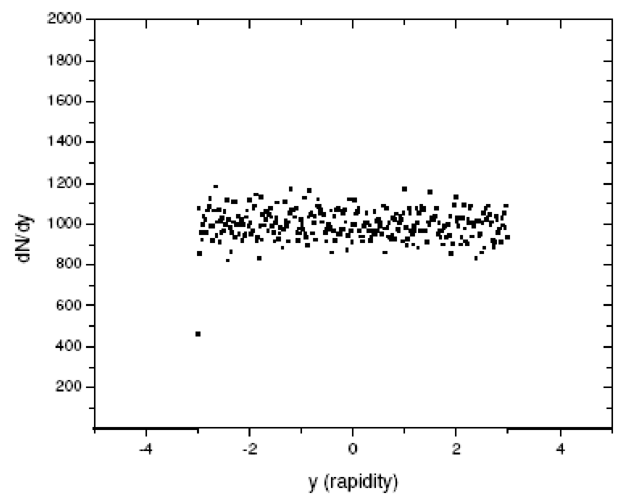
- KNV:



energy distribution



Number distribution



rapidity distribution

- Pros and cons:
  - Good: based on the first principle(QCD)
  - Bad: infinite flat plates, only gluons,
    no information on space

5. Summary

- Needs lots more work on particle production.
- Of cause the work is one of great problems to challenge.

- Need a new method to solve physics problems? I.e. we have a finite proton-proton cross section but we cannot calculate that in current formalism.