Some Topics on Chiral Transition and Color Superconductivity

Teiji Kunihiro (YITP)

HIM
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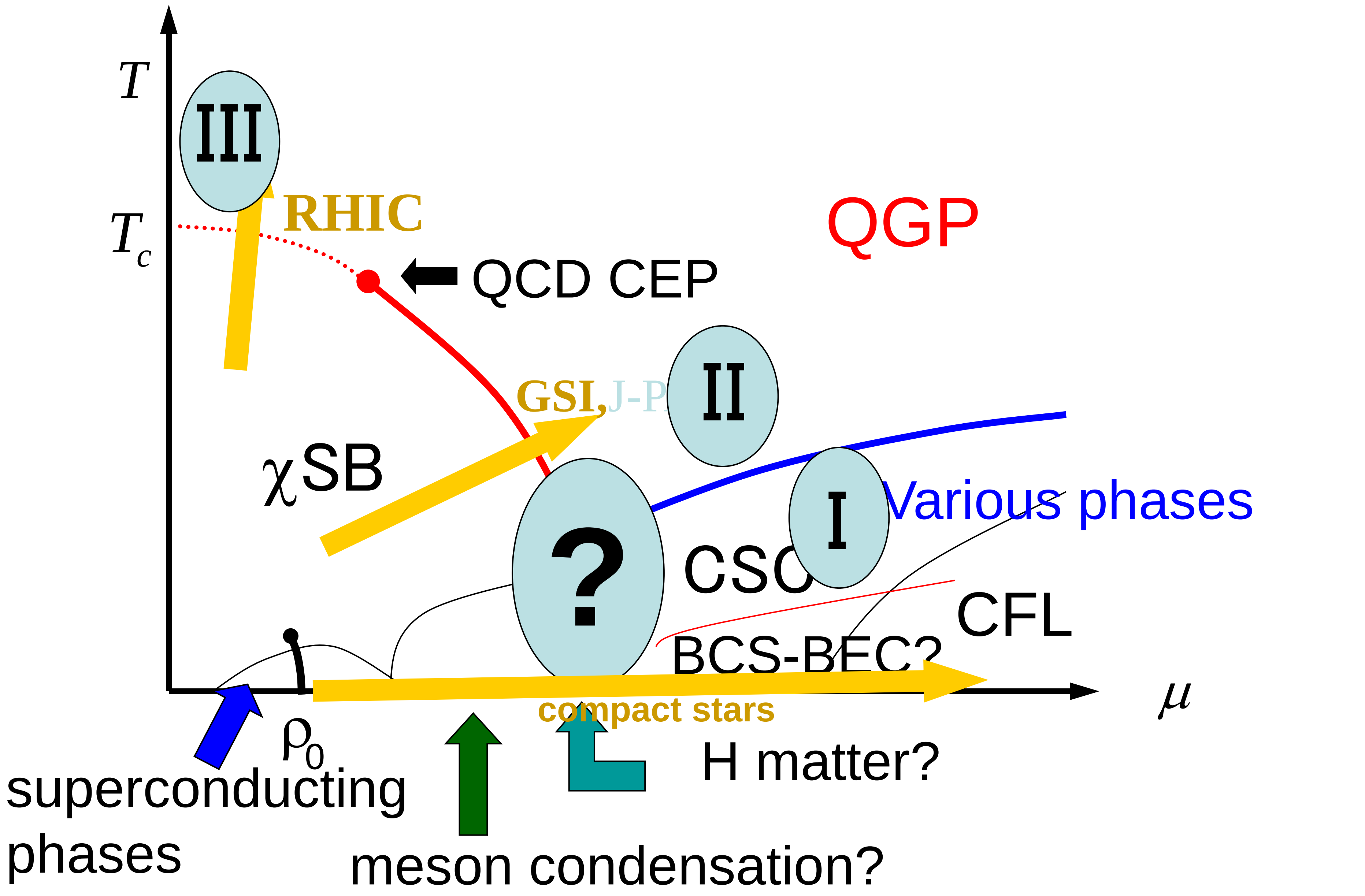
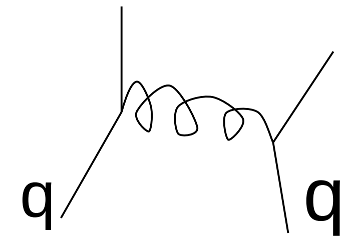
QCD phase diagram



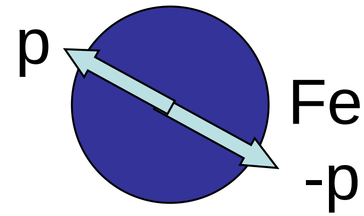
Color Superconductivity; diquark condensation

- Dense Quark Matter:
  - quark (fermion) system
  - with attractive channel in one-gluon exchange interaction.



$[3]_c \times [3]_c = [\bar{3}]_c + [6]_c$

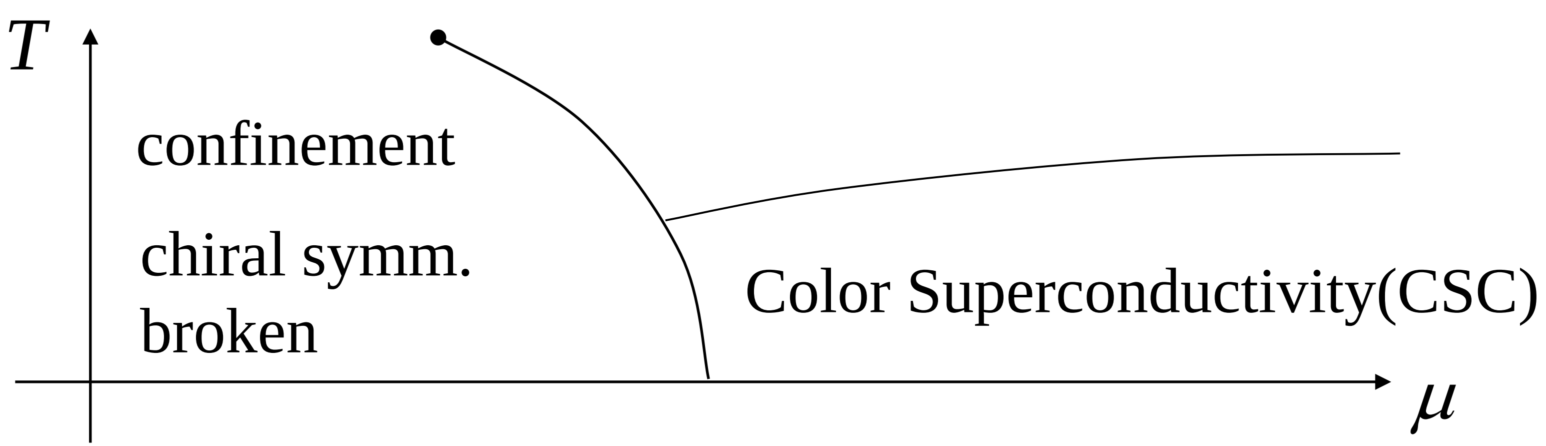
Attractive!



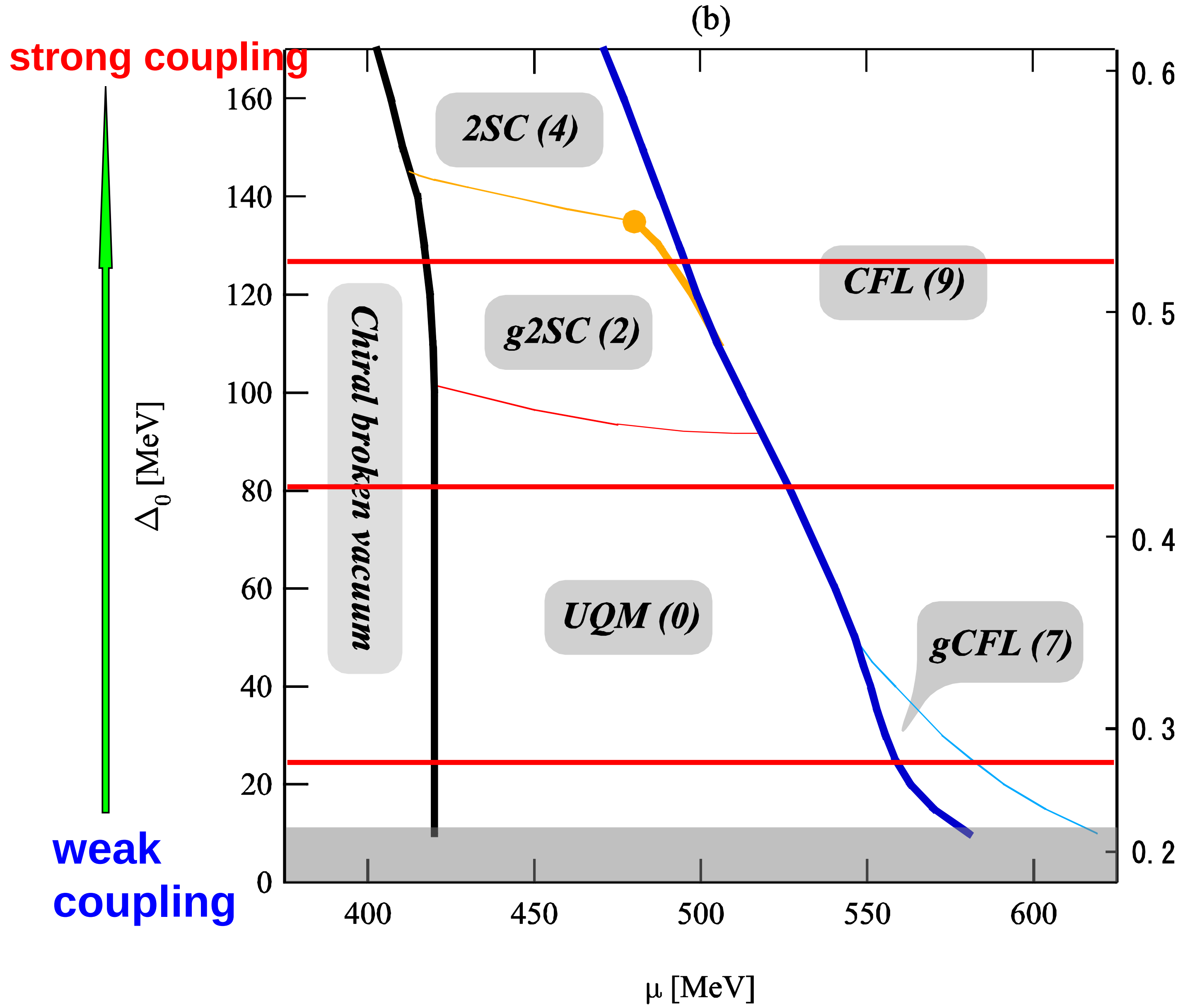
⇨ Cooper instability at sufficiently low T

⇨ $SU(3)_c$ gauge symmetry is broken!

- Δ~100MeV at moderate density μ_q~ 400MeV



(Dis)Appearance of CFL and Gapless phases in Charge- and Color-neutral System at T=0



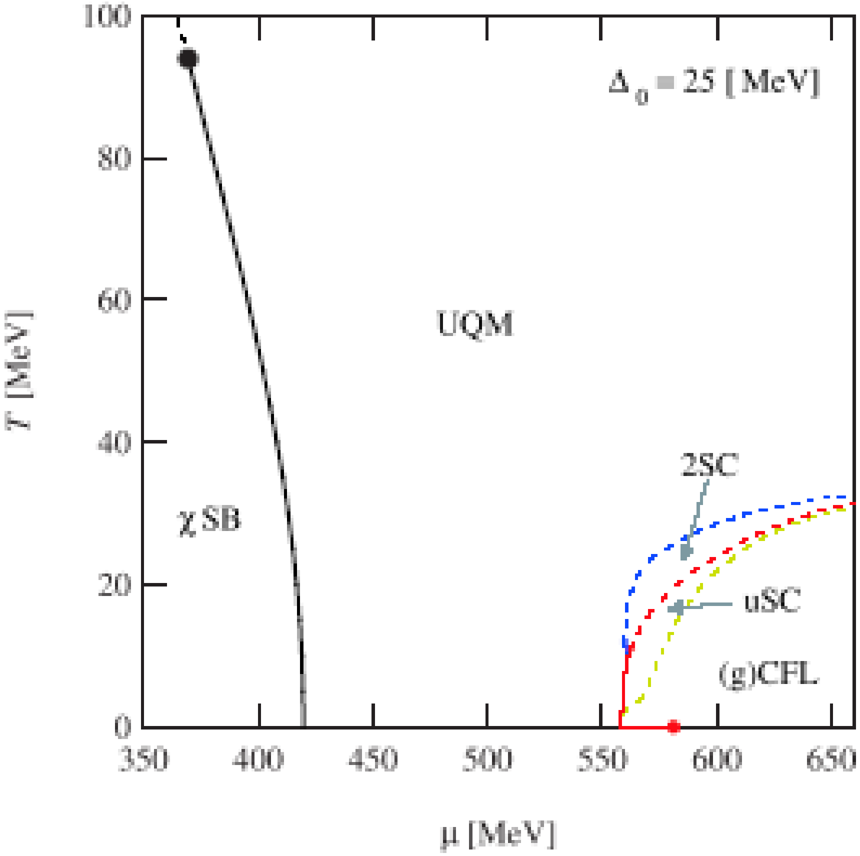
Abuki, Kitazawa and T.K.; P.L.B615,102('05)

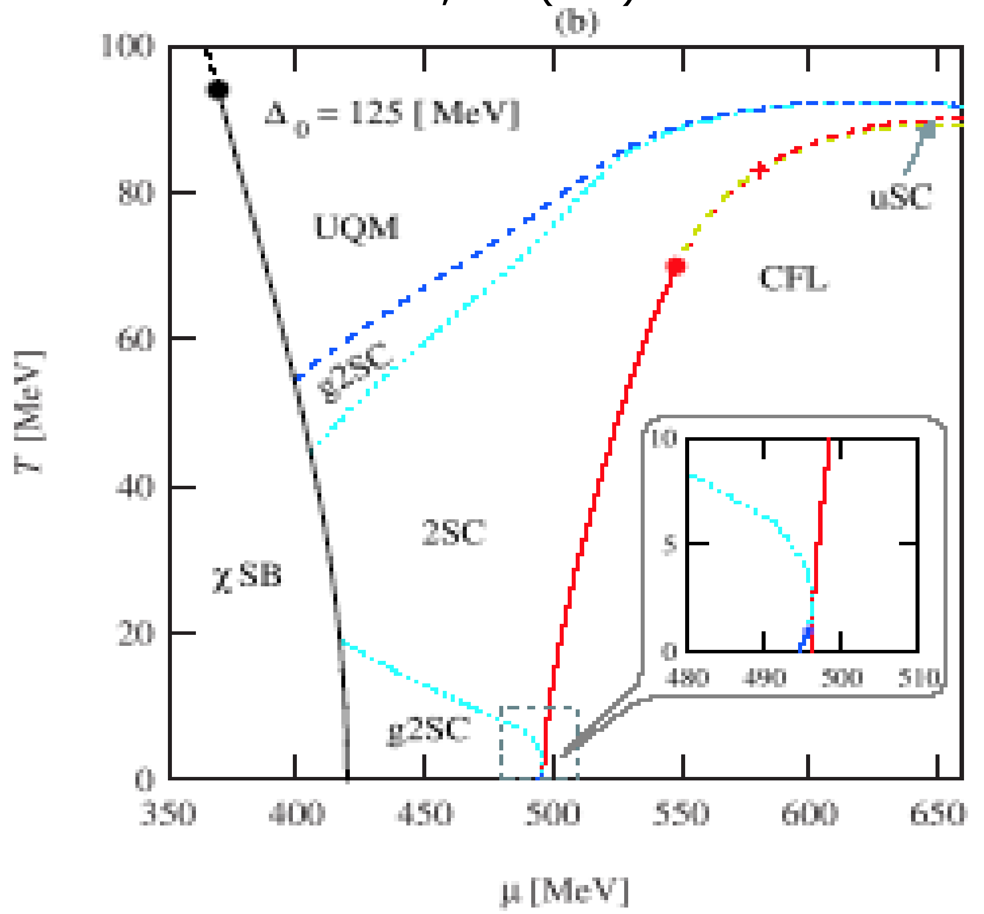
g : gapless

Various CSC phases in $T-\mu$ plane

H .Abuki and T.K. hep-ph/0509172:

Abuki, Kitazawa and T.K.; P.L.B615,102('05)



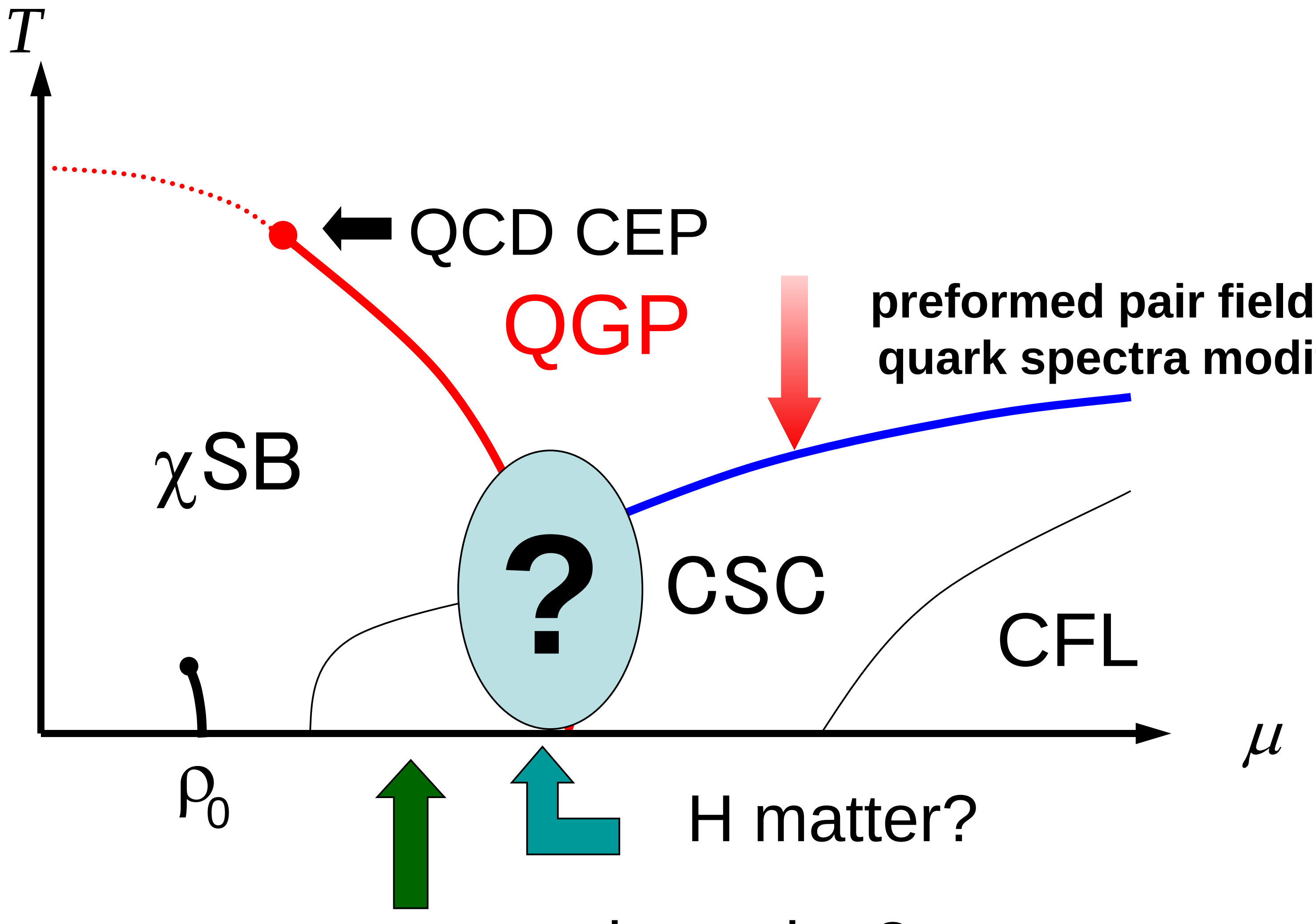


The phase in the highest temperature is 2SC or g2SC.

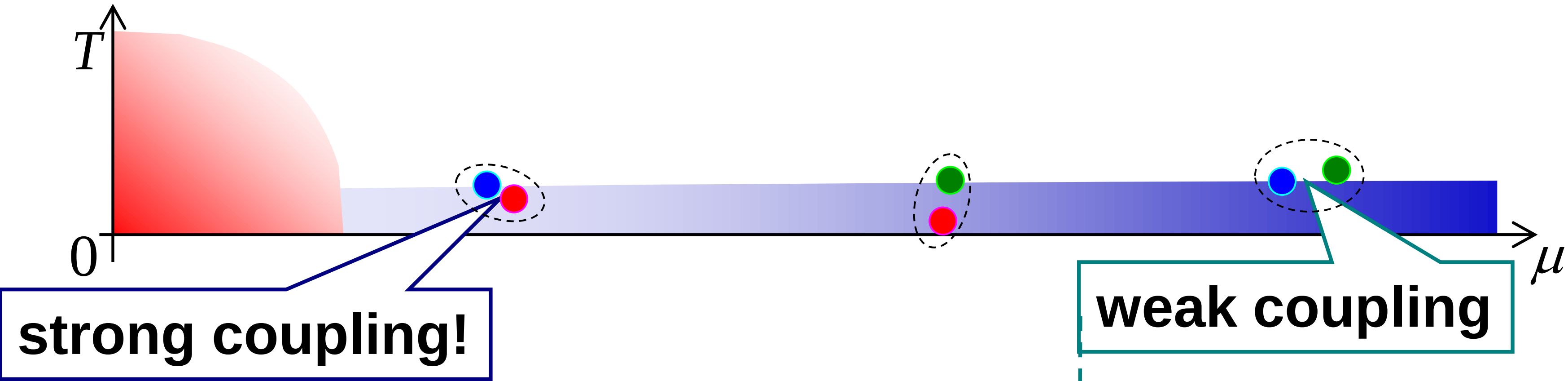
2. Precursory Phenomena of Color Superconductivity in Heated Quark Matter

Ref. M. Kitazawa, T. Koide, T. K. and Y. Nemoto
Phys. Rev. D70, 956003(2004);
Prog. Theor. Phys. 114, 205(2005),
M. Kitazawa, T.K. and Y. Nemoto,
hep-ph/0505070 , Phys. Lett.B, in press;
hep-ph/0501167

QCD phase diagram



The nature of diquark pairs in various coupling



$\Delta \sim$ **50-100MeV**

$\Delta / E_F \sim$ **0.1-0.3**

in electric SC

$\Delta / E_F \sim 0.0001$

Mean field approx. works well.

Very Short coherence length ξ.

There exist large fluctuations of pair field even at T>0.

- Large pair fluctuations can
  - invalidate MFA.
  - cause precursory phenomena of CSC.

relevant to newly born neutron stars

or intermediate states in heavy-ion collisions (GSI, J-Parc)

Collective Modes in CSC

Response Function of Pair Field

Linear Response

- external field: $H_{ex} = \int d\mathbf{x} \left(\Delta_{ex}^{\dagger} \bar{\psi}^{C} i\gamma_5 \tau_2 \lambda_2 \psi + \text{h.c.} \right)$

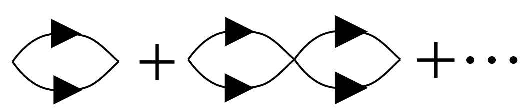
  ⇒ expectation value of induced pair field:

$$\left\langle \bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x) \right\rangle_{ex} = i \int_{t_0}^{t} ds \left\langle \left[H_{ex}(s), O(\mathbf{x}, t) \right] \right\rangle$$

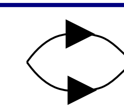
$$\begin{cases} \Delta_{ind}(x) = -2G_C \left\langle \bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x) \right\rangle_{ex} = \int dt' \int d\mathbf{x} D^{R}(x, x') \Delta_{ex}(x') \\ D^{R}(\mathbf{x}, t) = -2G_C \left\langle \left[\bar{\psi}(x) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(x), \bar{\psi}(0) i\gamma_5 \tau_2 \lambda_2 \psi^{C}(0) \right] \right\rangle \theta(t) \end{cases}$$

Retarded Green function

- Fourier transformation with Matsubara formalism ⇒ $\Delta^{\dagger}(\mathbf{k}, \omega_n)_{\text{ind}} = \mathcal{D}(\mathbf{k}, \omega_n) \Delta^{\dagger}(\mathbf{k}, \omega_n)_{\text{ext}}$

- RPA approx.: $\mathcal{D}(\mathbf{k}, \omega_n) =$ 

$$= -\frac{G_C Q(\mathbf{k}, \omega_n)}{1 + G_C Q(\mathbf{k}, \omega_n)}$$

with $Q(\mathbf{k}, \omega_n) =$ 

After analytic continuation to real time,

$$D^R(\mathbf{k},\omega) = -G_cQ(\mathbf{k},\omega)/(1+G_cQ(\mathbf{k},\omega)),$$
$$\equiv -G_cQ(\mathbf{k},\omega)\cdot\Xi(\mathbf{k},\omega)$$
$$\Xi^{-1}(\mathbf{k},\omega) \equiv 1+G_cQ(\mathbf{k},\omega).$$

The spectral function;

$$\rho(\mathbf{k},\omega) = -\frac{1}{\pi}\mathrm{Im}D^R(\mathbf{k},\omega)$$

An important obeservation: at $T=T_c$;

$$\Xi^{-1}(\mathbf{k}=\mathbf{0},\omega=0)=0$$

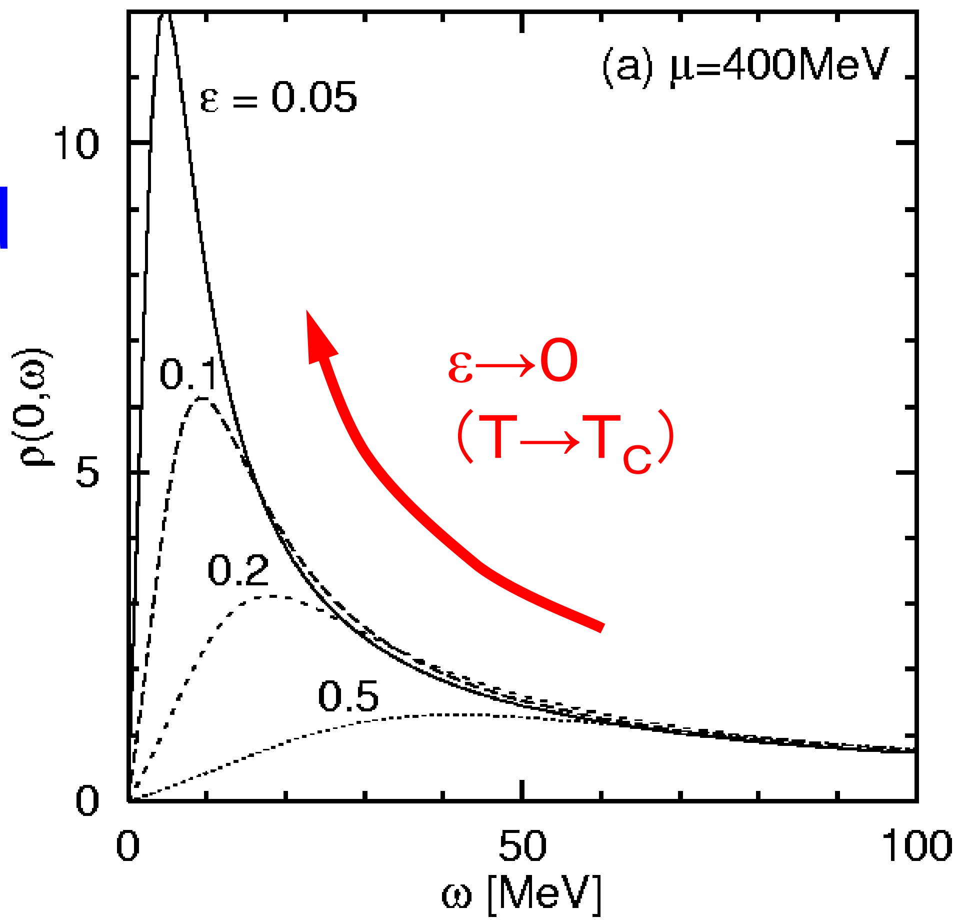
Equivalent with the gap equation (Thouless criterion)

Precursory Mode in CSC

(Kitazawa, Koide, Nemoto and T.K., PRD 65, 091504(2002))

$$\rho(\mathbf{k}=\mathbf{0},\omega)$$

Spectral function of the pair field at T>0



at k=0

$$\varepsilon = \frac{T - T_C}{T_C}$$

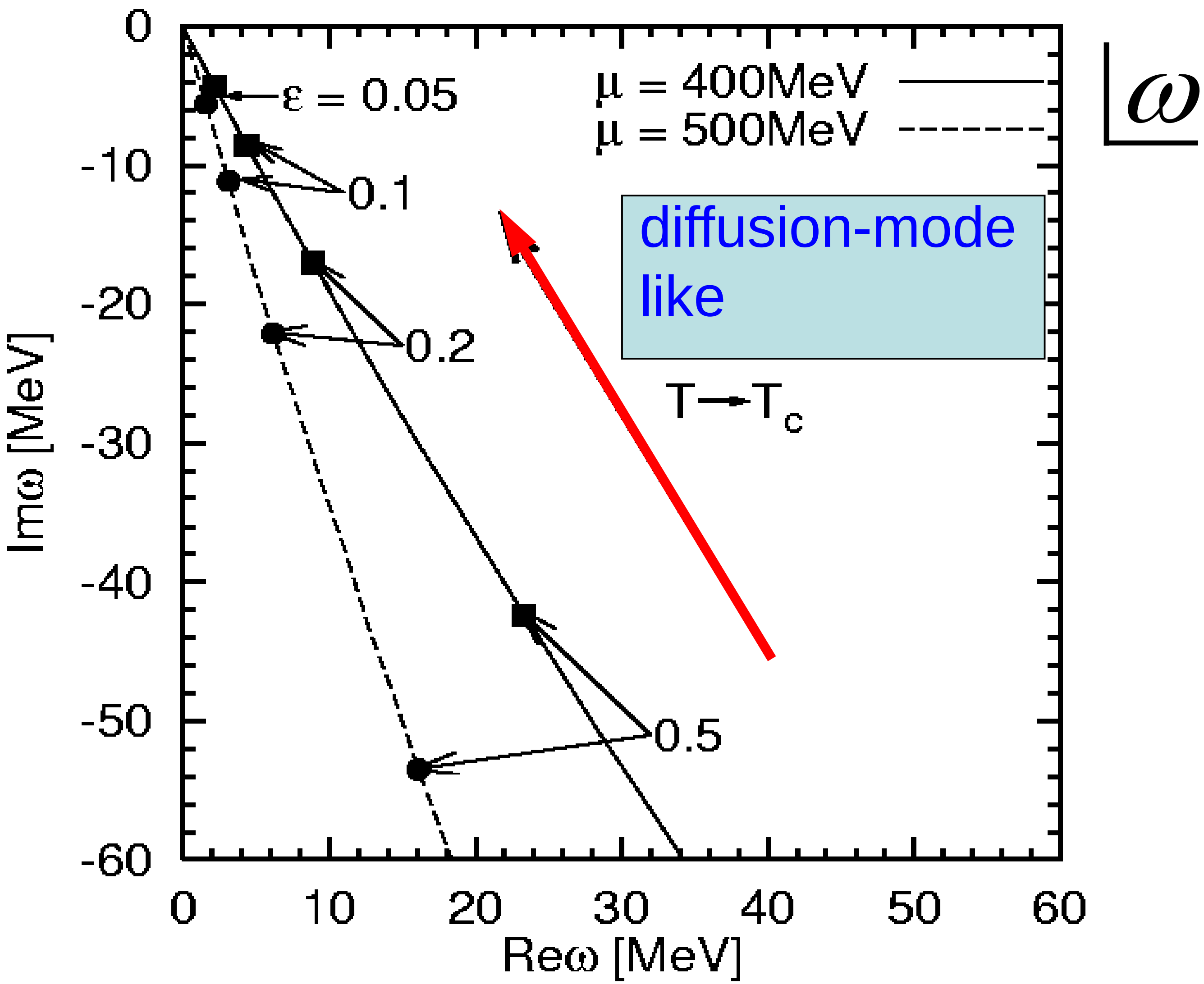
- As T is lowered toward T_C,

  The peak of ρ becomes sharp. (Soft mode) ⇨ Pole behavior

- The peak survives up to ε □ 0.2 ⇔ electric SC : ε □ 0.005

The pair fluctuation as the soft mode;

--- movement of the pole of the precursory mode---

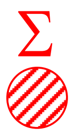


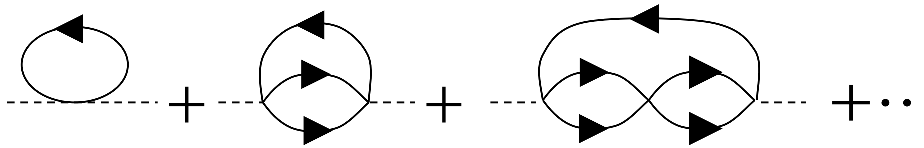
How does the soft mode affect the quark spectra?

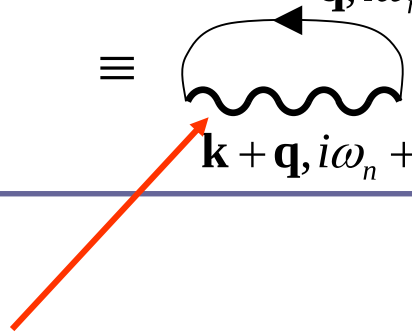
---- formation of pseudogap ----

T-matrix Approximation for Quark Propagator

$$G(\mathbf{k},\omega_n)=\frac{1}{G^0(\mathbf{k},i\omega_n)-\Sigma(\mathbf{k},i\omega_n)}\qquad G^0(\mathbf{k},i\omega_n)=\left[(i\omega_n+\mu)\gamma^0-\mathbf{k}\cdot\vec{\gamma}\right]^{-1}=$$

$\Sigma(\mathbf{k},\omega_n)=$ 

$=$ 

$\equiv$ 

$$=T\sum_m\int\frac{d^3q}{(2\pi)^3}\Xi(\mathbf{k}+\mathbf{q},\omega_n+\omega_m)G^0(\mathbf{q},\omega_m)$$

Soft mode

Density of States $N(\omega)$:

$$N=\int d^3x\left\langle\bar{\psi}\gamma^0\psi\right\rangle$$

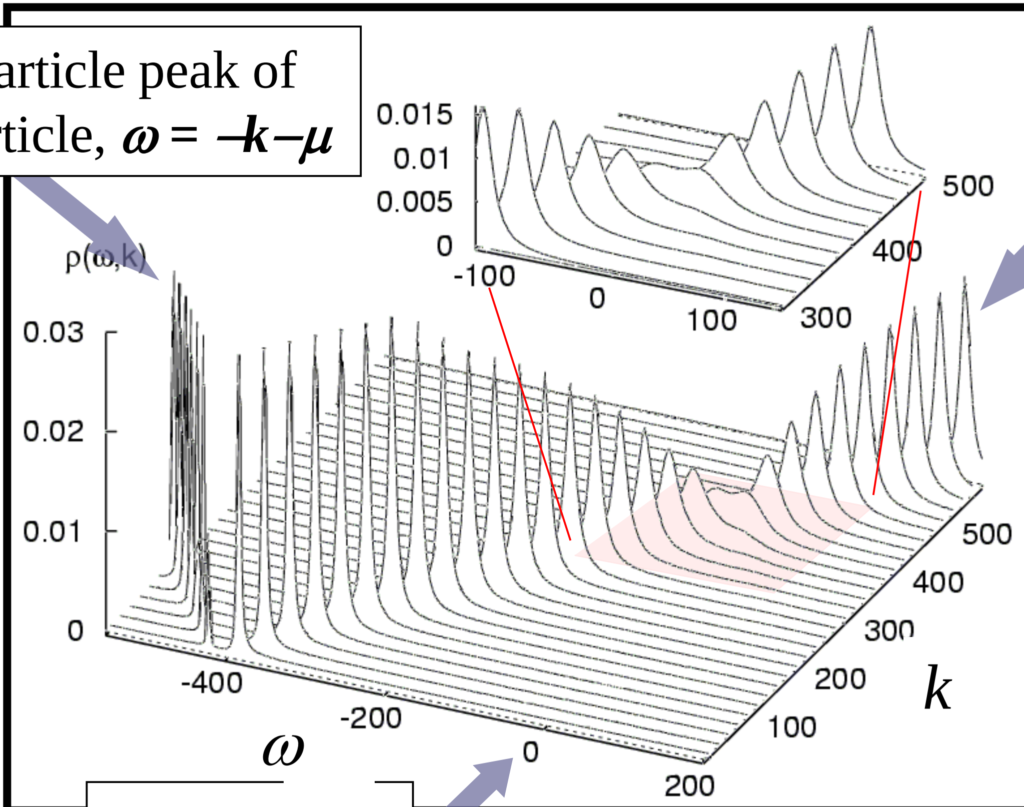
$$N(\omega)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\rho^0(\mathbf{k},\omega)\Leftarrow\rho^0(\mathbf{k},\omega)=\frac{1}{4}\mathrm{Tr}\left[\gamma^0\,\mathrm{Im}\,G^R(\mathbf{k},\omega)\right]$$

1-Particle Spectral Function

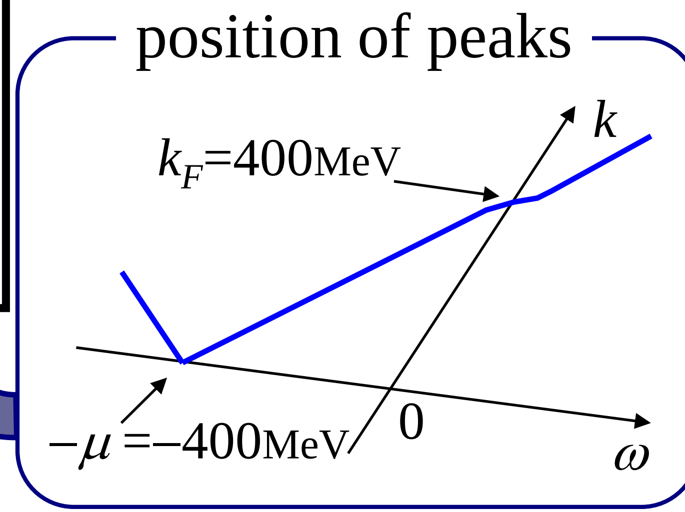
μ= 400 MeV
ε=0.01

quasi-particle peak of anti-particle, $\omega = -k-\mu$

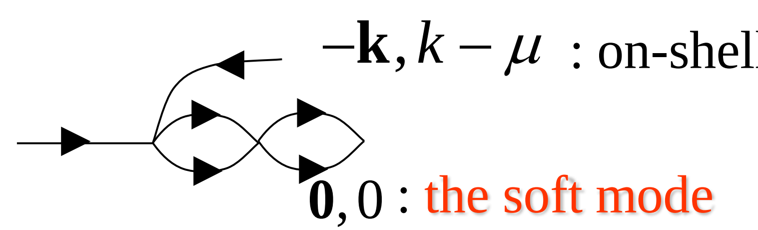
quasi-particle peak, $\omega = \omega\_(k) \sim k-\mu$



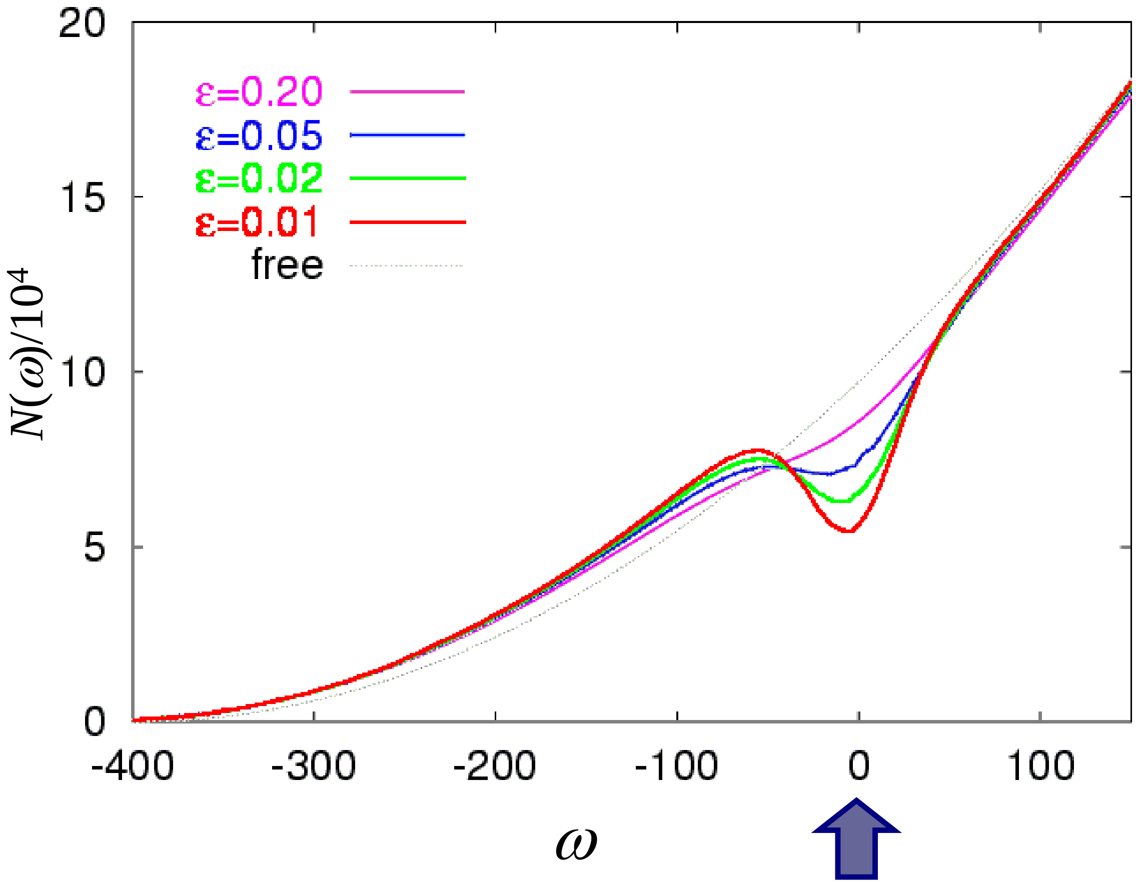
Fermi energy



- quasi-particle peaks at $\omega = \omega\_(k) \sim k-\mu$ and $\omega = -k-\mu$.
- Quasi-particle peak has a depression around the Fermi energy due to **resonant scattering**.



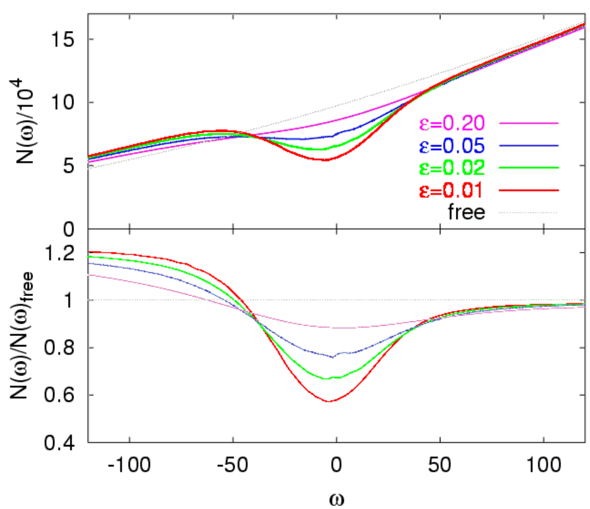
Density of states of quarks in heated quark matter



Fermi energy

μ = 400 MeV

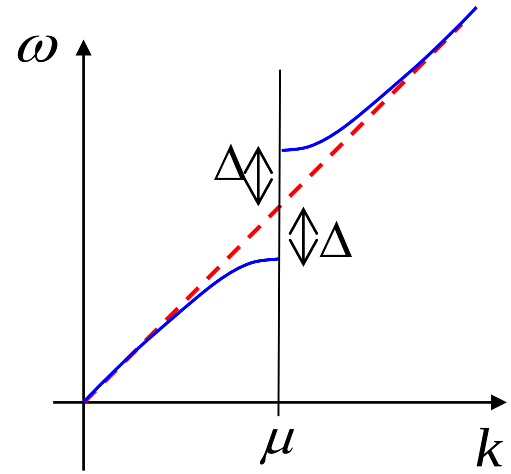
- **Pseudogap structure appears in $N(\omega)$.**
- The pseudogap survives up to $\varepsilon \sim 0.05$ (5% above T_C).



Density of States in Superconductor

- Quasi-particle energy:

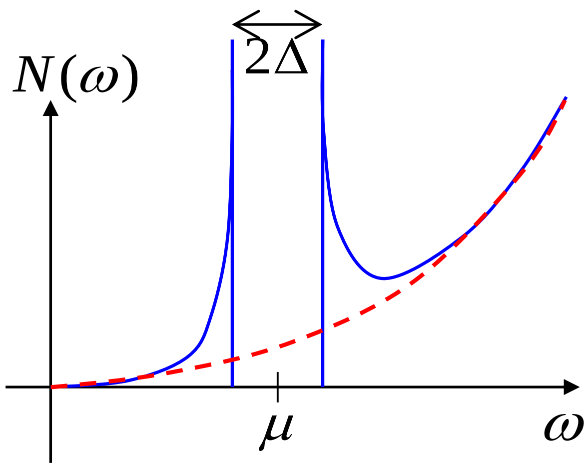
$$\omega = \mathrm{sgn}(k-\mu)\sqrt{(k-\mu)^2+\Delta^2}$$



- Density of States:

$$N(\omega) \propto k^2 \frac{dk}{d\omega}$$

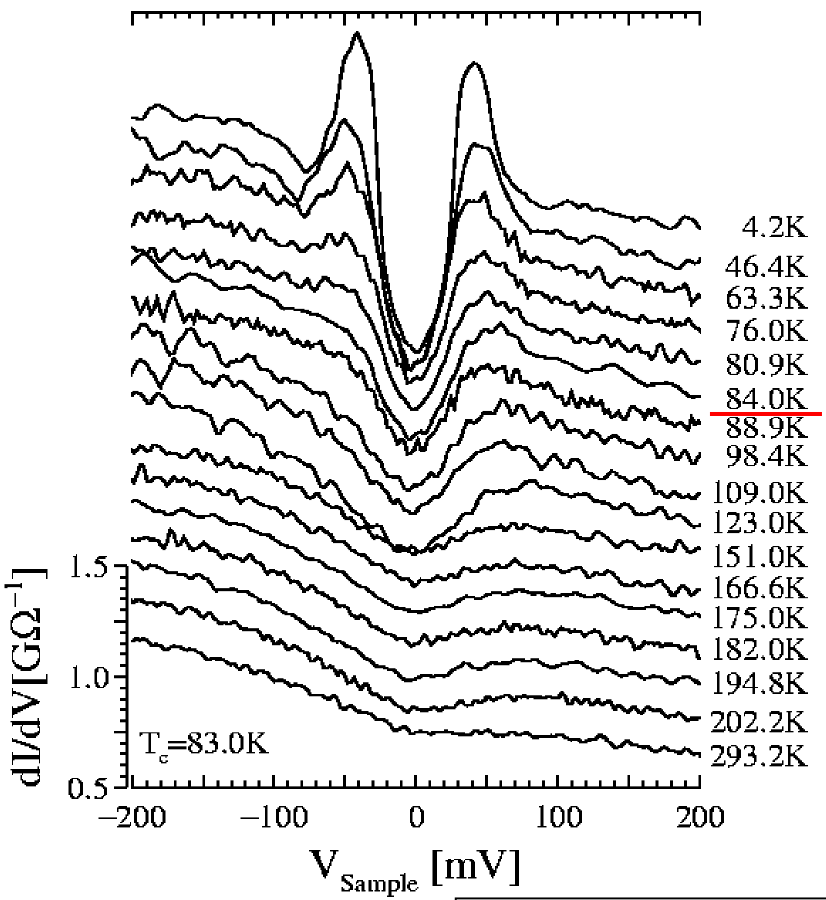
$$\frac{d\varepsilon}{dk} = \frac{k-\mu}{\sqrt{(k-\mu)^2+\Delta^2}}$$



⇨ The gap on the Fermi surface becomes smaller as T is increased, and it closes at T_c.

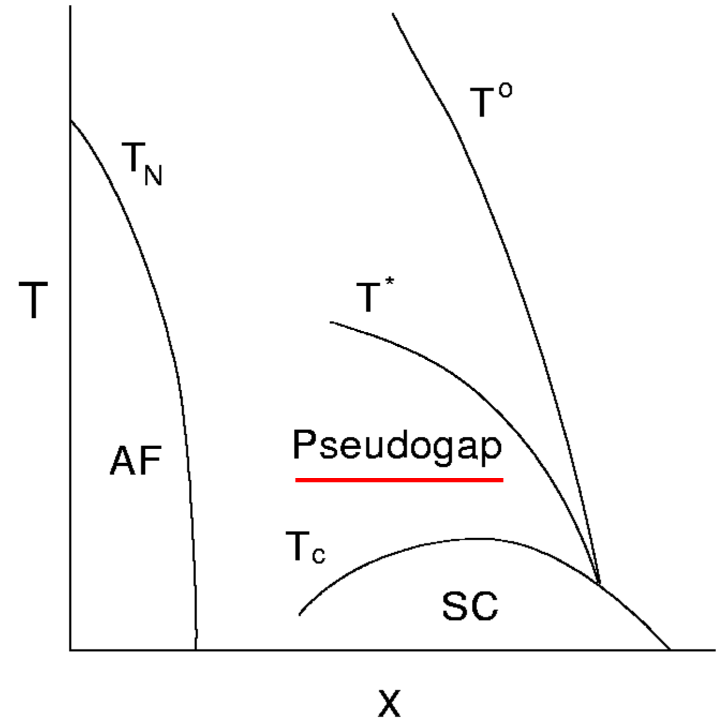
Pseudogap

:Anomalous depression of the density of state near the Fermi surface in the normal phase.



Renner et al.(‘96)

Conceptual phase diagram of HTSC cuprates



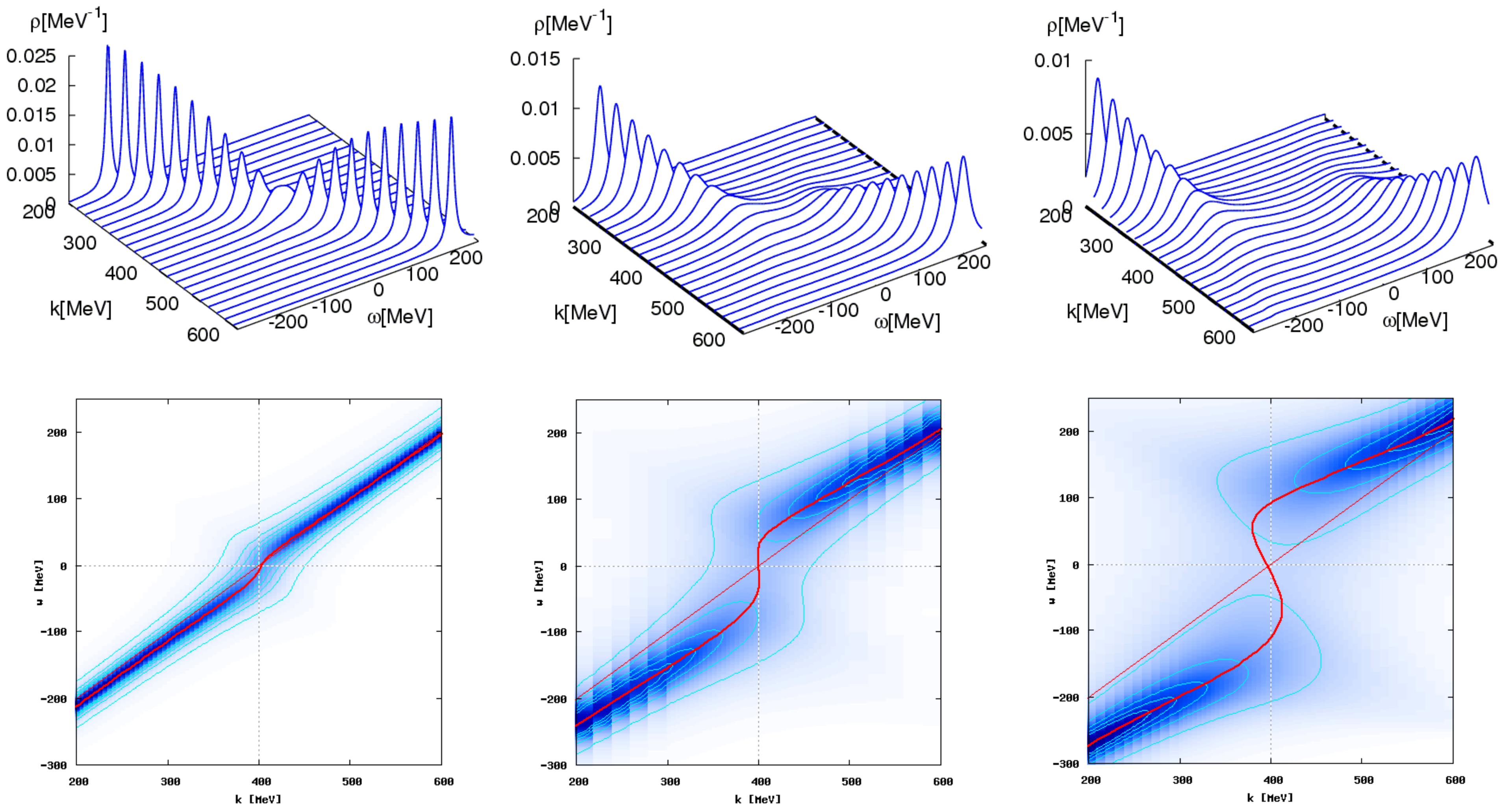
The origin of the pseudogap in HTSC is **still controversial**.

Diquark Coupling Dependence

μ= 400 MeV
ε=0.01

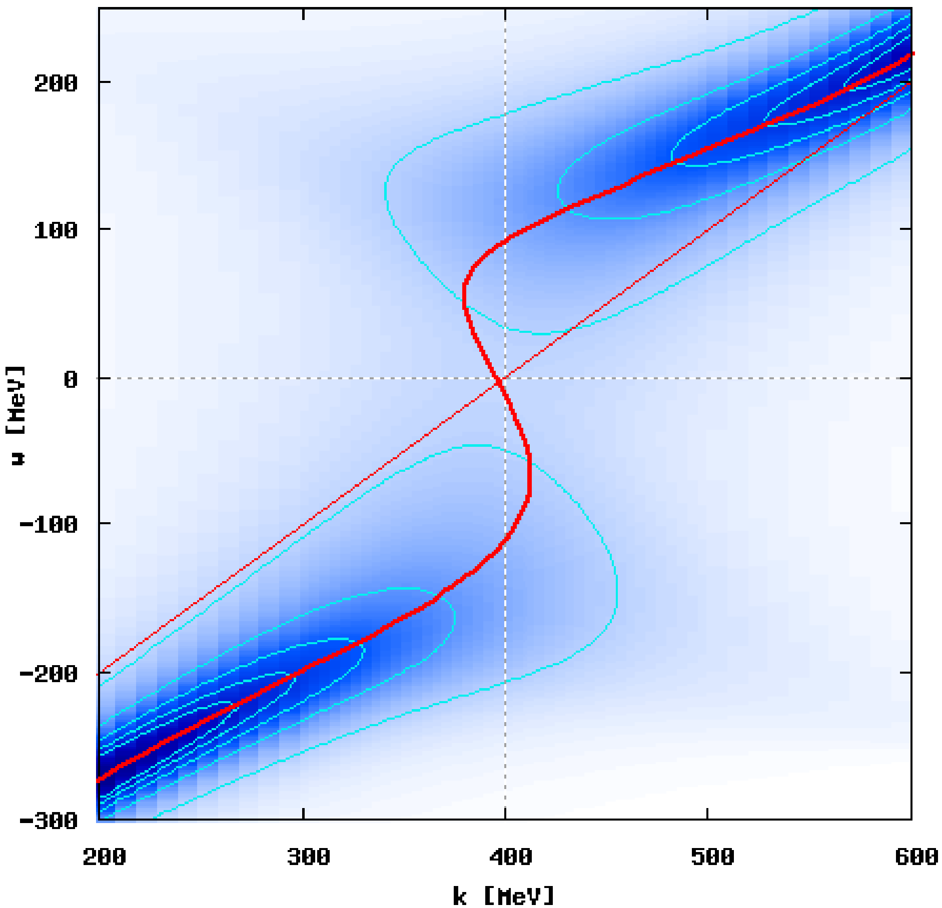
stronger diquark coupling G_C

G_C $\times 1.3$ $\times 1.5$

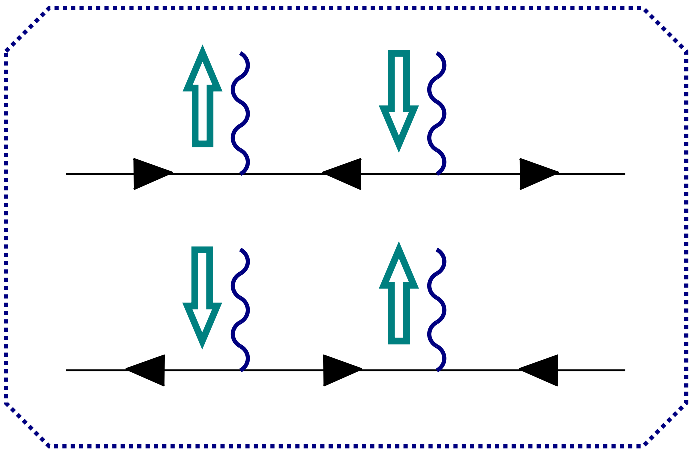


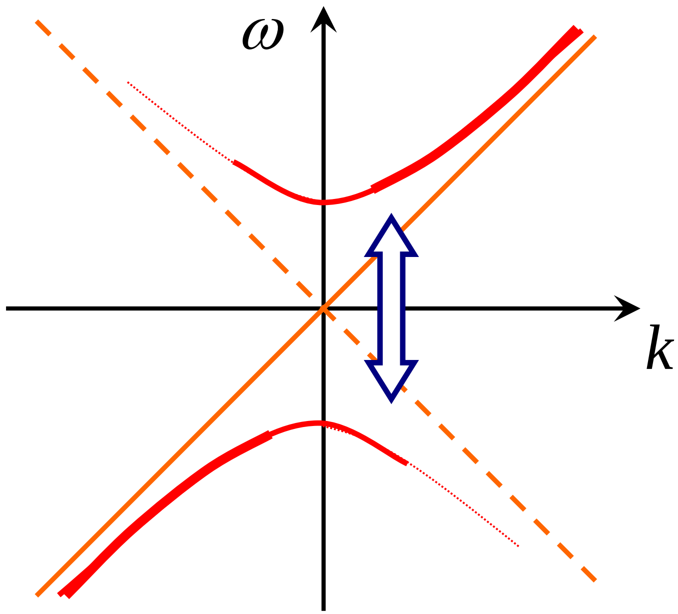
Resonance Scattering of Quarks

G_C=4.67GeV^{-2}



Mixing between quarks and holes





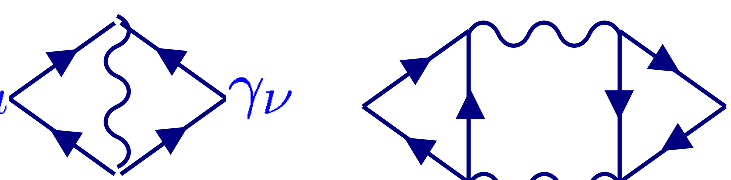
(M.Kitazawa, Y. Nemoto, T.K.
hep-ph/0505070; Phys. Lett.B , in press)

Summary of this section

- There may exist a wide T region where the precursory soft mode of CSC has a large strength.
- The soft mode induces the pseudogap, Typical Non-Fermi liquid behavior ⟸ resonant scattering

Future problems:

effects of the soft mode on . **H-I coll. & proto neutron stars**

⇒ eg.1) γ_μ  γ_ν ⇒ anomalous lepton pair production

collective mode:  (ω,k)

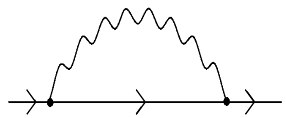
eg.2) C_V ; cooling of newly born stars

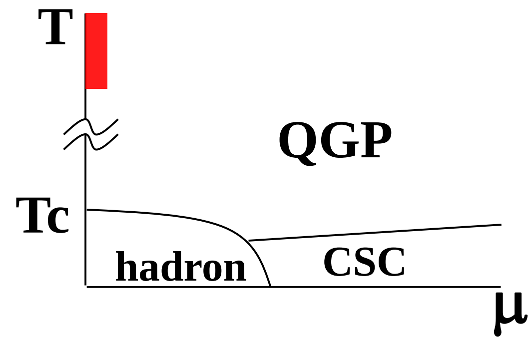
(M.Kitazawa and T.K in progress.)

3. Precursory Hadronic Mode and Single Quark Spectrum above Chiral Phase Transition

Quarks at very high T (T>>>Tc)

- 1-loop (g<<1) + HTL approx. $(p, \omega, m_q << T)$

$$\Sigma(\omega, p) =$$ 



thermal masses $m_f^2 = \frac{1}{6} g^2 T^2$

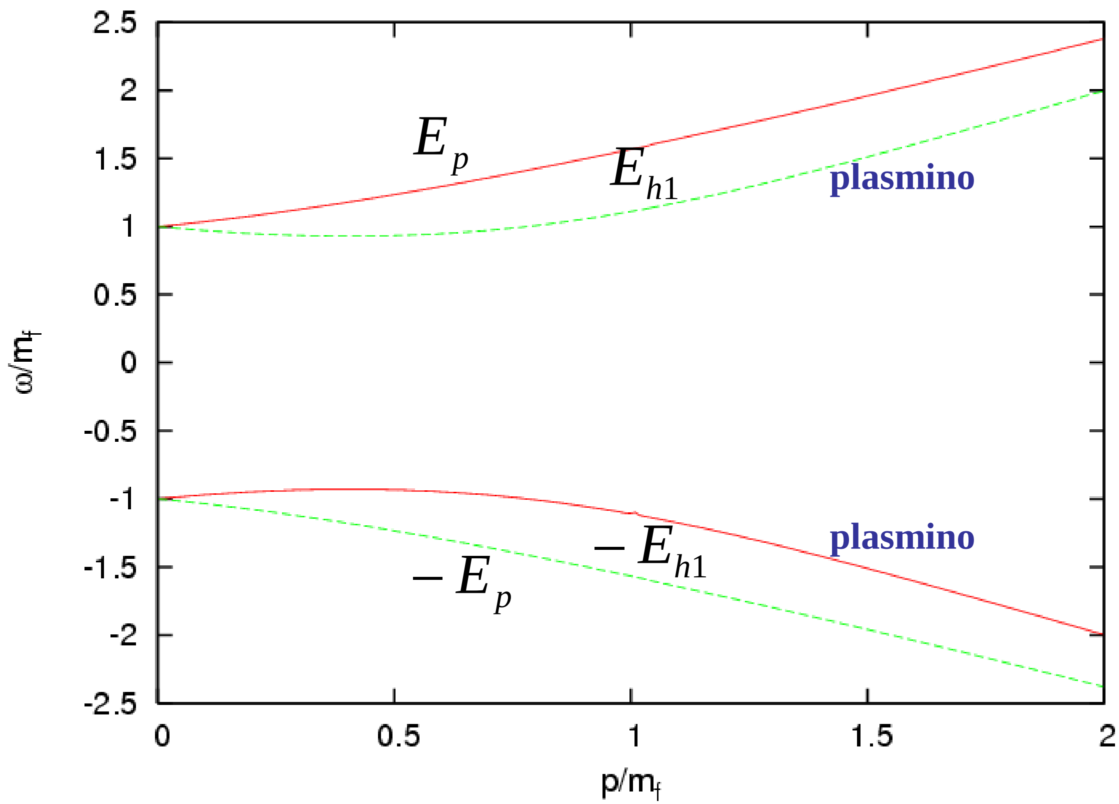
dispersion relations

$$\mathrm{Re}[D_+(\omega, p)] = 0$$

$$\omega = E_p, -E_{h1}, -E_{h2}$$

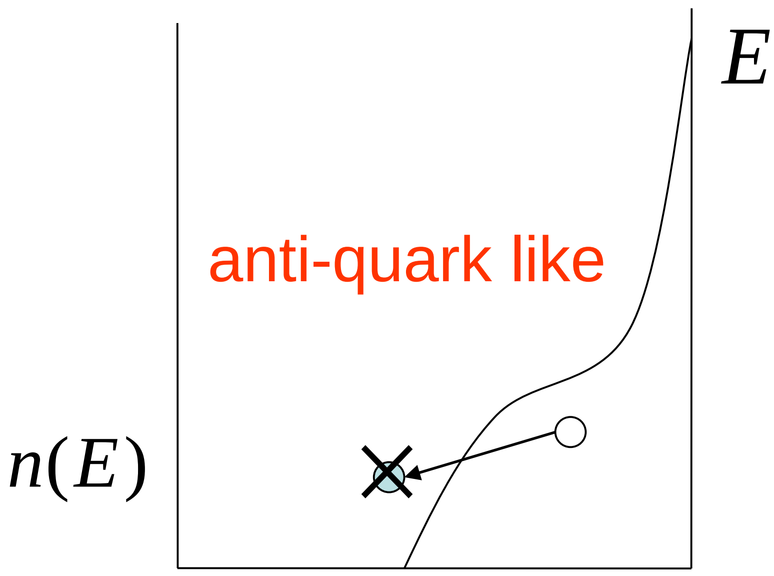
$$\mathrm{Re}[D_-(\omega, p)] = 0$$

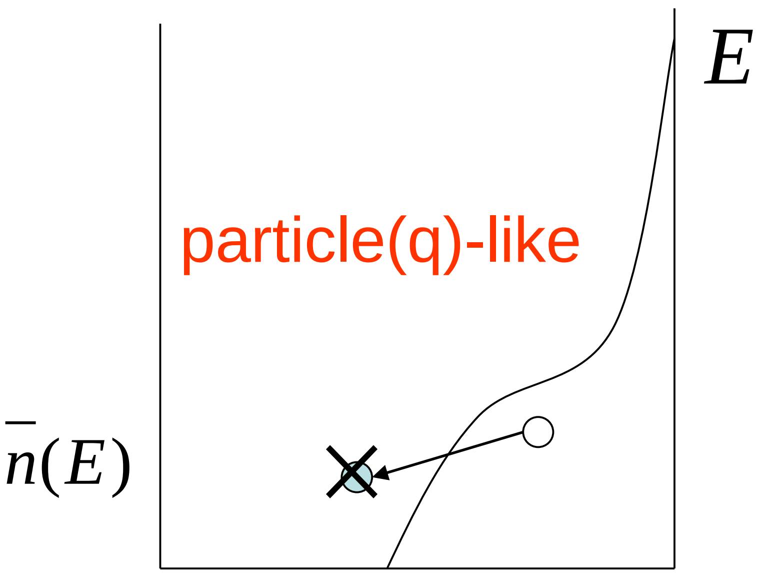
$$\omega = -E_p, E_{h1}, E_{h2}$$



quark distribution

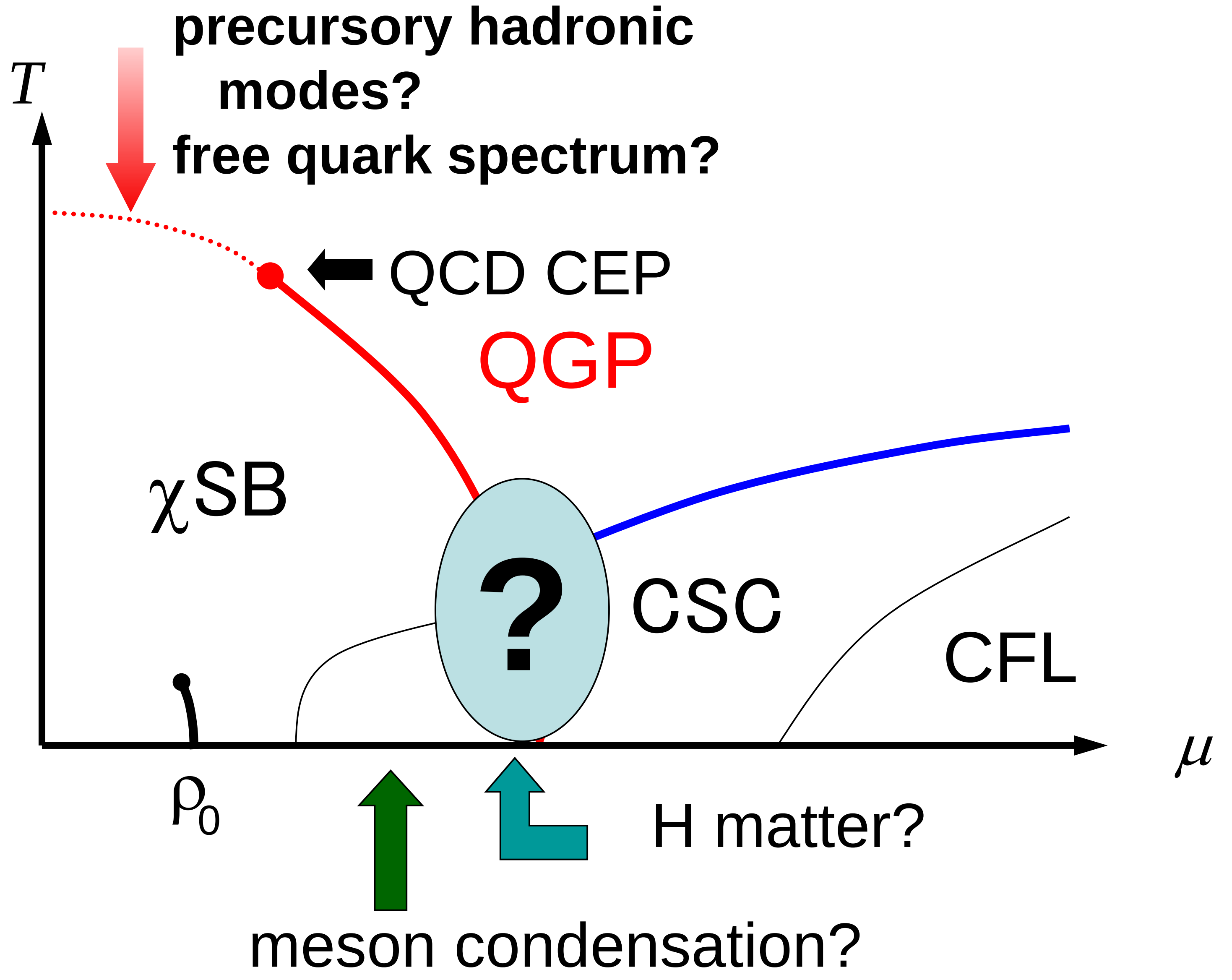
anti-q distribution





Plasmino excitation

QCD phase diagram and quasi-particles



Interest in the particle picture in QGP

RHIC experiments

robust collective flow

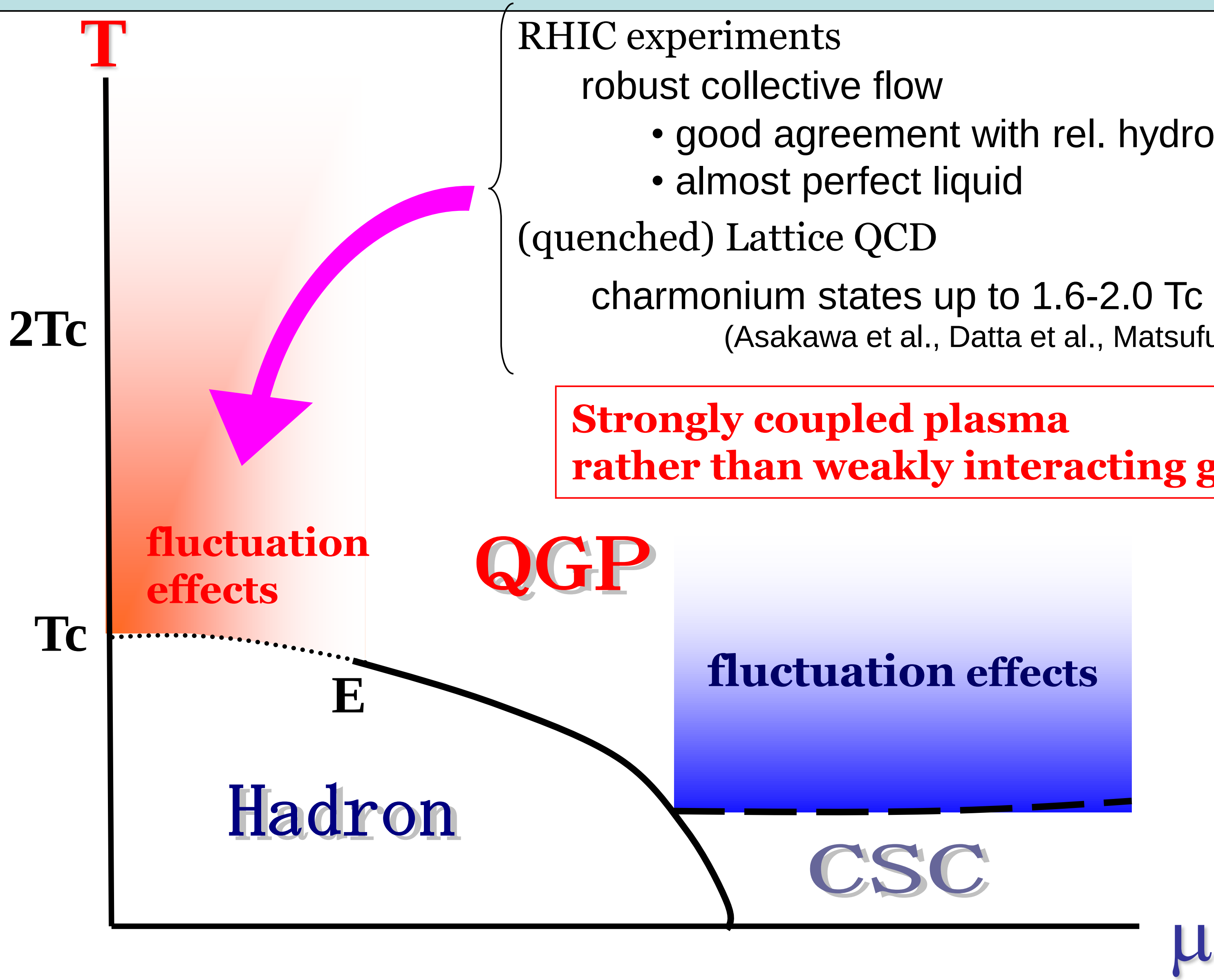
- good agreement with rel. hydro models
- almost perfect liquid

(quenched) Lattice QCD

charmonium states up to 1.6-2.0 Tc

(Asakawa et al., Datta et al., Matsufuru et al. 2004)

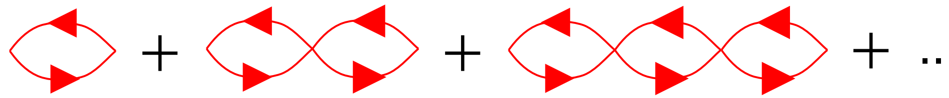
Strongly coupled plasma rather than weakly interacting gas



The spectral function of the degenerate hadronic ``para-pion” and the ``para-sigma” at T>Tc for the chiral transition: Tc=164 MeV

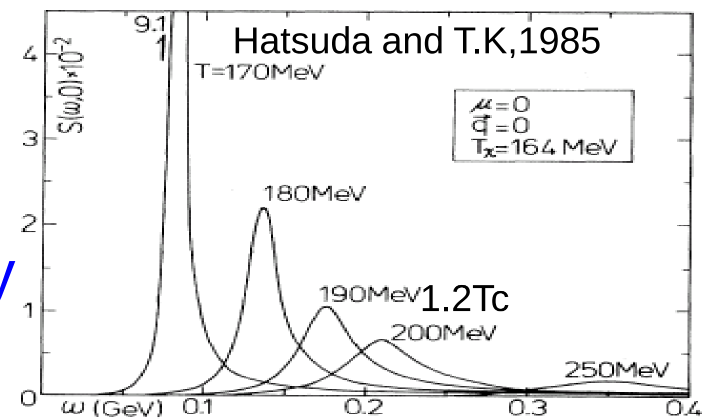
T. Hatsuda and T.K. (1985)

- response function in RPA

$D(\mathbf{k},\omega) =$ 

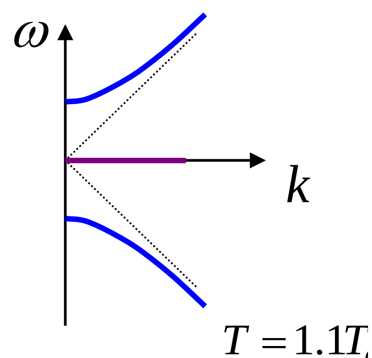
- spectral function

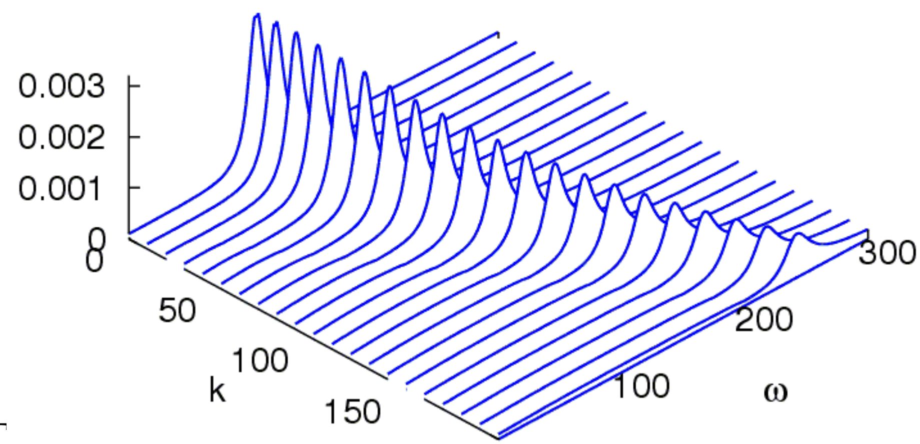
$$A(\mathbf{k}\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,D(\mathbf{k}\omega)$$



$T \to T_c$, they become elementary modes with small width!

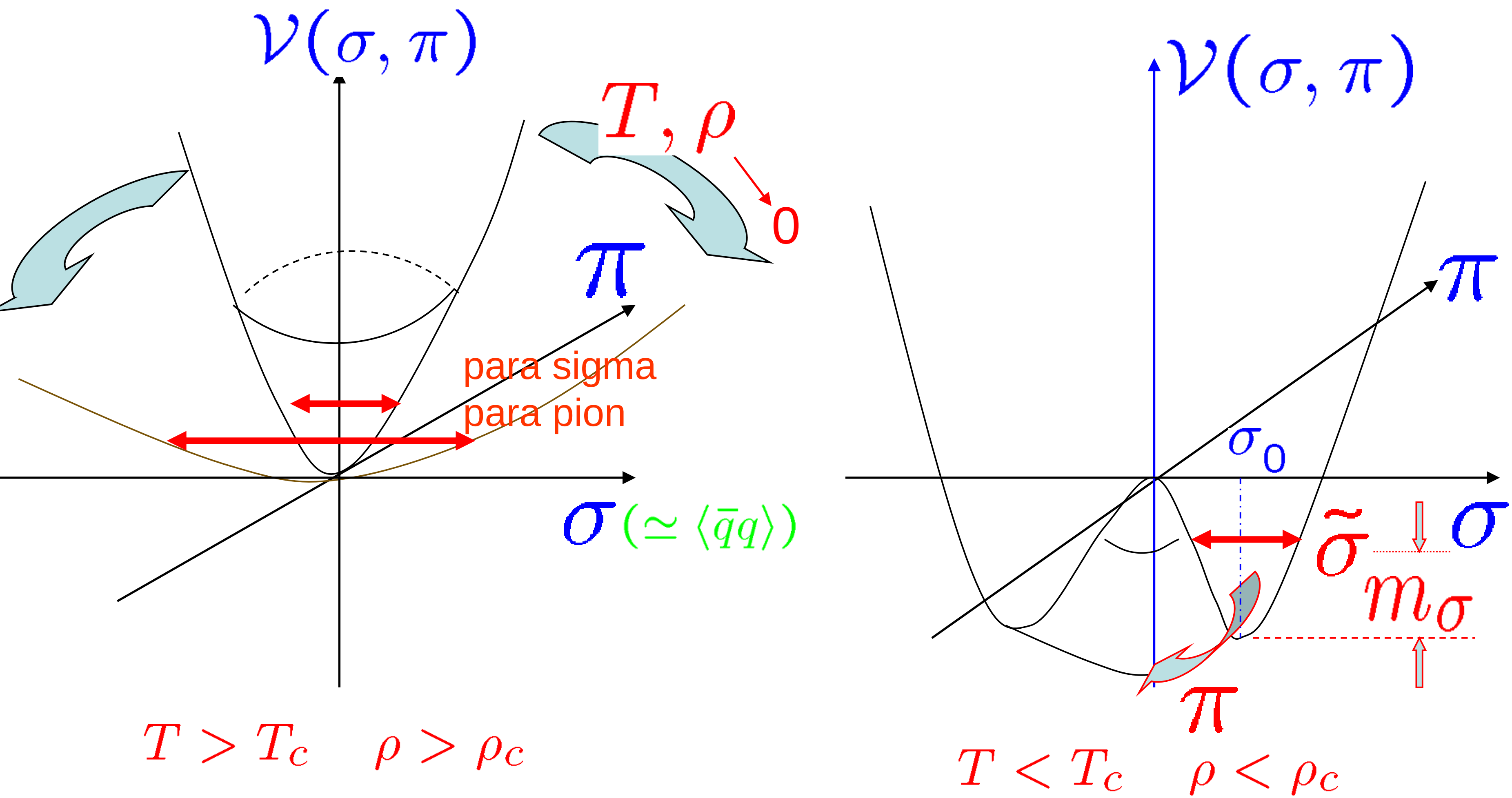
sharp peak in time-like region





M.Kitazawa, Y.Nemoto and T.K. (05)

Chiral Transition and the collective modes



$$\sigma = \sigma_0 + \tilde{\sigma}$$

c.f. Higgs particle in WSH model

ϕ ; Higgs field ⇒ $\phi = \langle \phi \rangle + \tilde{\phi}$

Higgs particle

How does the soft mode affect a single quark spectrum near Tc?

Y. Nemoto, M. Kitazawa ,T. K.

hep-ph/0510167

Model

- **low-energy effective theory of QCD**
  4-Fermi type interaction (Nambu-Jona-Lasinio with 2-flavor)

$$L = \bar{q} i\gamma \cdot q + G_S[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

τ : SU(2) Pauli matrices

$G_S = 5.5 \cdot 10^{-6}\,\mathrm{GeV}^{-2}, \Lambda = 631\,\mathrm{MeV}$

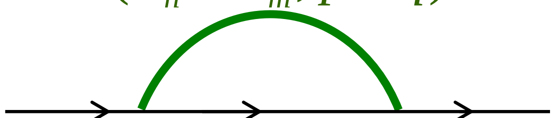
$m_u = m_d = 0$ chiral limit   finite m_u, m_d : future work

- **Chiral phase transition takes place at Tc=193.5 MeV(2nd order).**

- **Self-energy of a quark (above Tc)**

$$\Sigma(\omega_n, \vec{p}) = \mathrm{T}\sum_m \int \frac{d^3q}{(2\pi)^3} D(\omega_n - \omega_m, \vec{p} - \vec{q}) G_0(\omega_m, \vec{q})$$

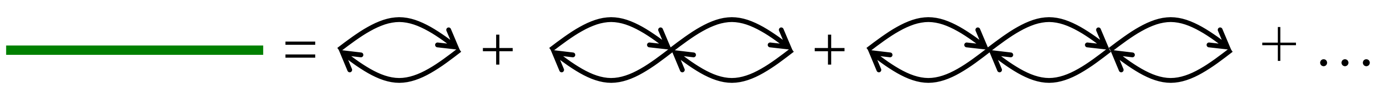
$D(\omega_n - \omega_m, \vec{p} - \vec{q})$



$G_0(\omega_m, \vec{q})$

T->Tc, may be well described with a Yukawa coupling.

$D(\omega_n, \vec{p}) =$



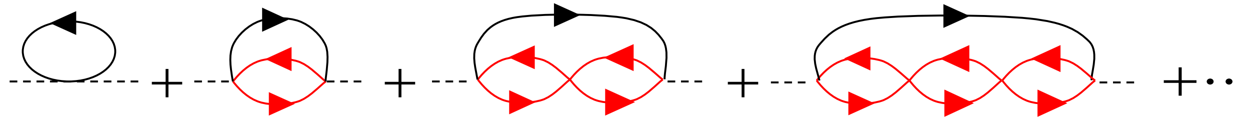
scalar and pseudoscalar parts

$$\Sigma^R(\omega, p) = \Sigma(\omega_n, p)|_{i\omega_n = \omega + i\varepsilon}$$: imaginary time → real time

Spectral Function of Quark

- **Quark self-energy**

$\Sigma(\mathbf{k}, i\omega_n) =$

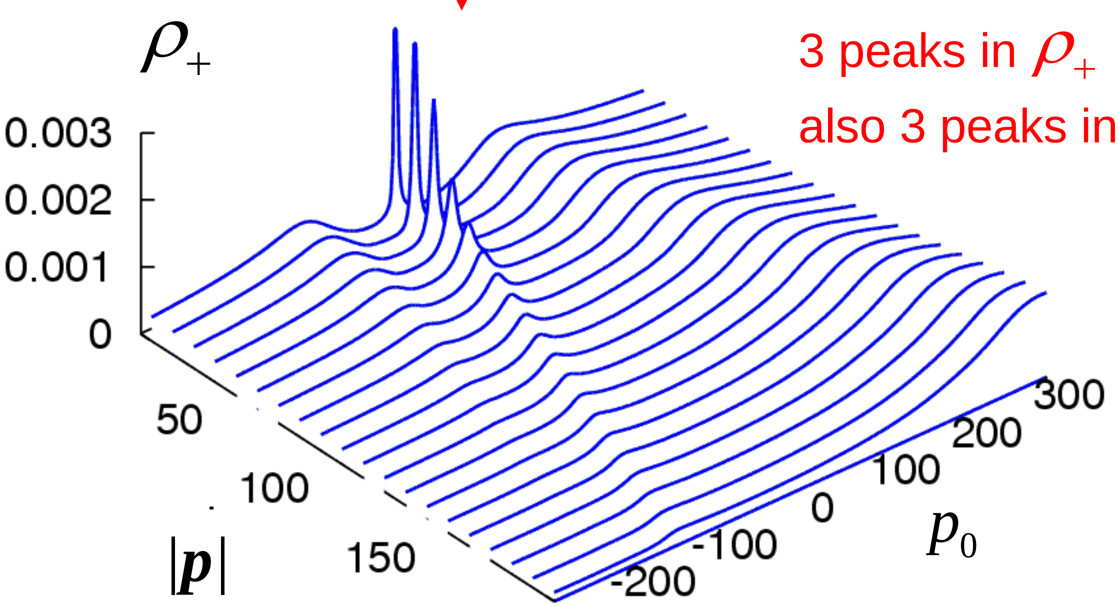


- **Spectral Function**

$$A(\mathbf{p}, p^0) = \underbrace{\rho_+(\mathbf{p}, p^0)}_{\text{quark}}\Lambda_+\gamma^0 + \underbrace{\rho_-(\mathbf{p}, p^0)}_{\text{antiquark}}\Lambda_-\gamma^0$$

$$\Lambda_\pm(p) = \frac{1}{2}(1 \pm \gamma^0 \vec{\gamma} \cdot \hat{p})$$

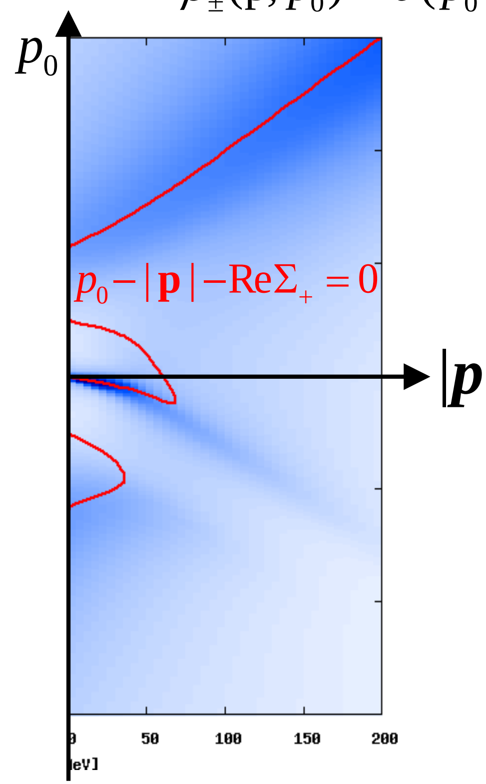
for free quarks,
$\rho_\pm(\mathrm{p}, p_0) \propto \delta(p_0 \mp |\mathrm{p}|)$



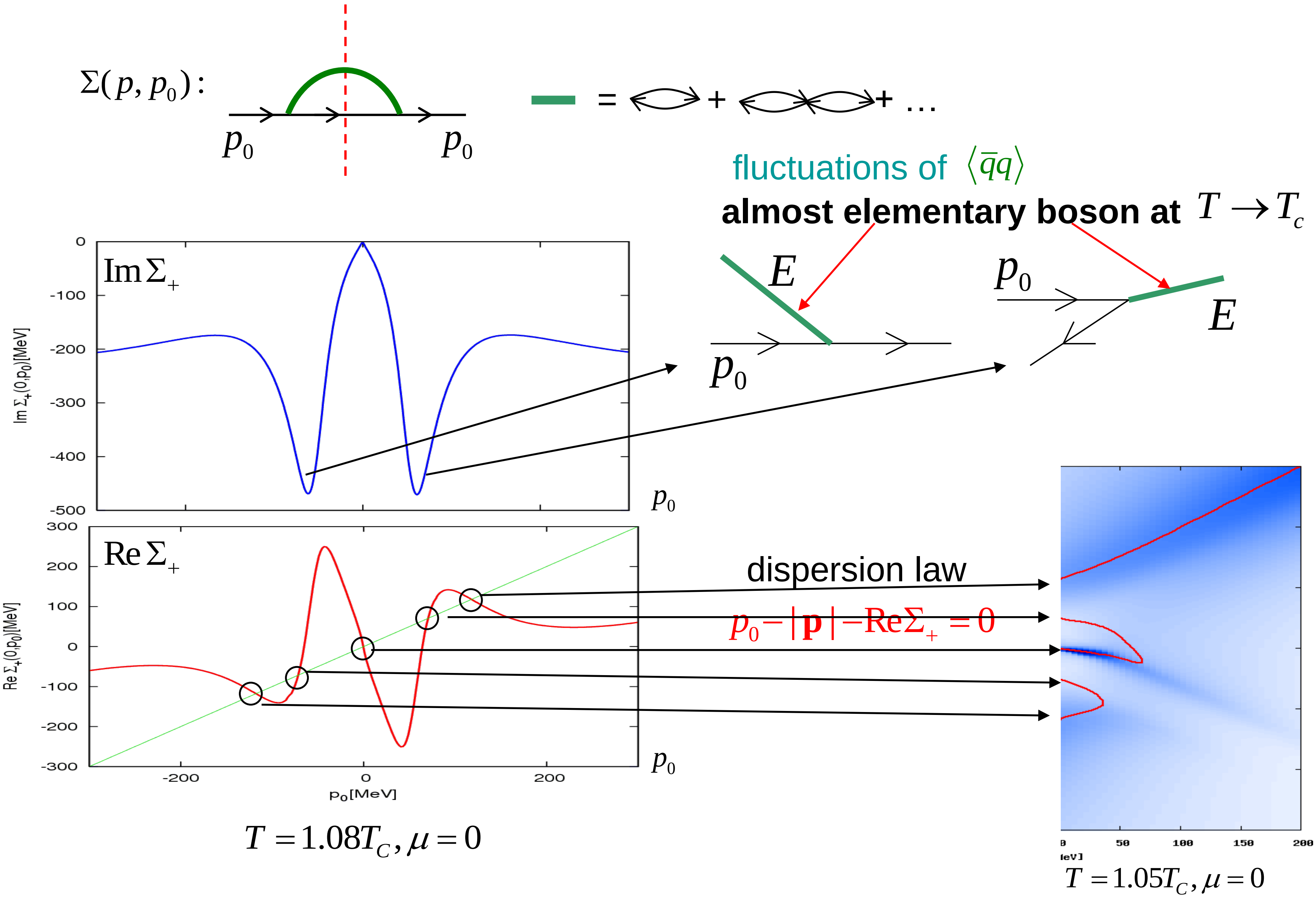
3 peaks in ρ_+

also 3 peaks in ρ_-

$T = 1.05T_C, \mu = 0$



Resonant Scatterings of Quark for CHIRAL Fluctuations

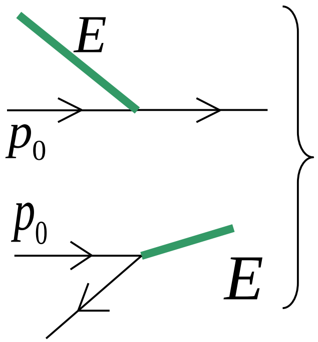


Resonant Scatterings of Quark for CHIRAL Fluctuations

“quark hole”: annihilation mode of a thermally excited quark

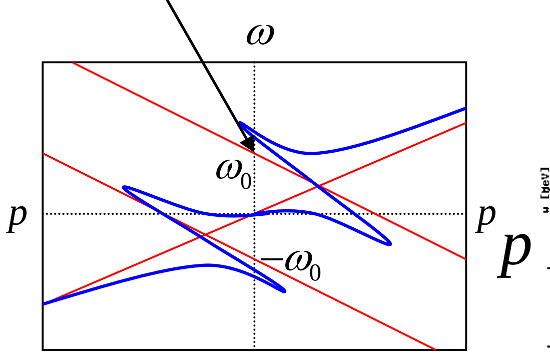
“antiquark hole”: annihilation mode of a thermally excited antiquark

(Weldon, 1989)

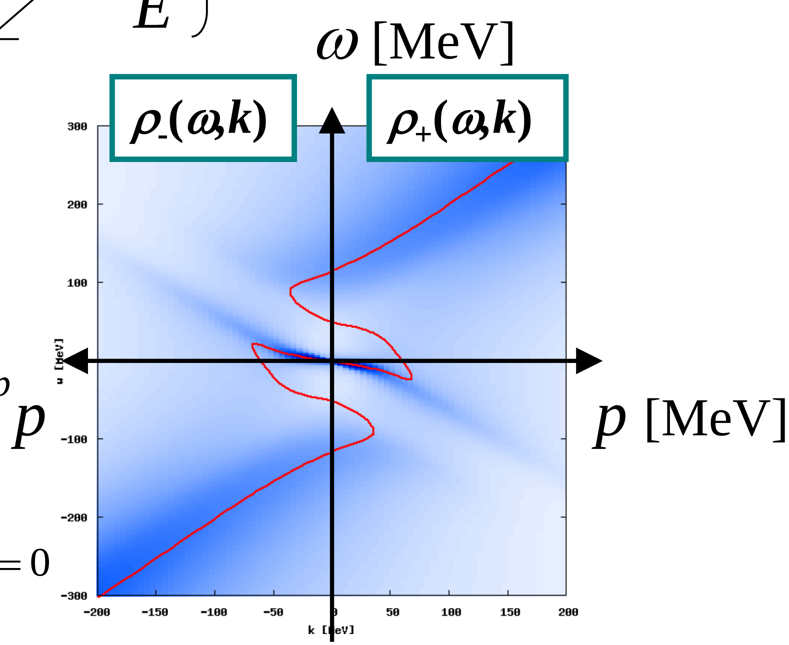


lead to quark-”antiquark hole” mixing

the ‘mass’ of the elementary modes



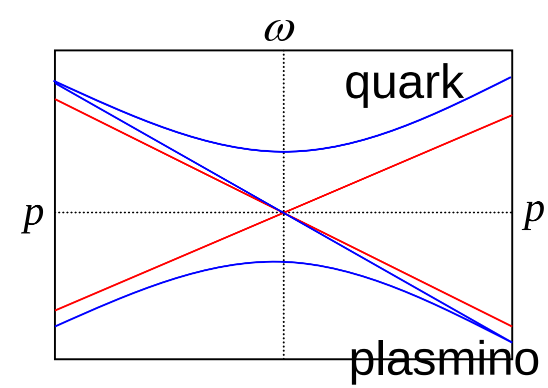
$T = 1.05T_C, \mu = 0$



cf: hot QCD

(HTL approximation)

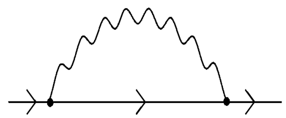
(Klimov, 1981)

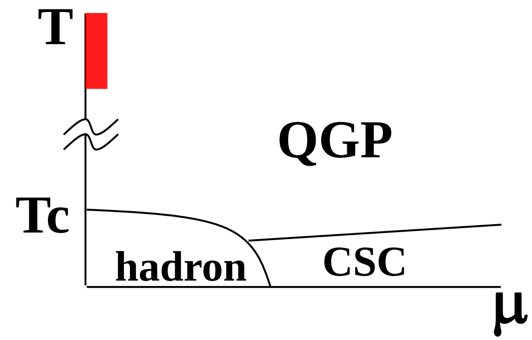


Quarks at very high T (T>>>Tc)

- 1-loop (g<<1) + HTL approx. $(p, \omega, m_q << T)$

$$\Sigma(\omega, p) =$$





thermal masses $m_f^2 = \frac{1}{6} g^2 T^2$

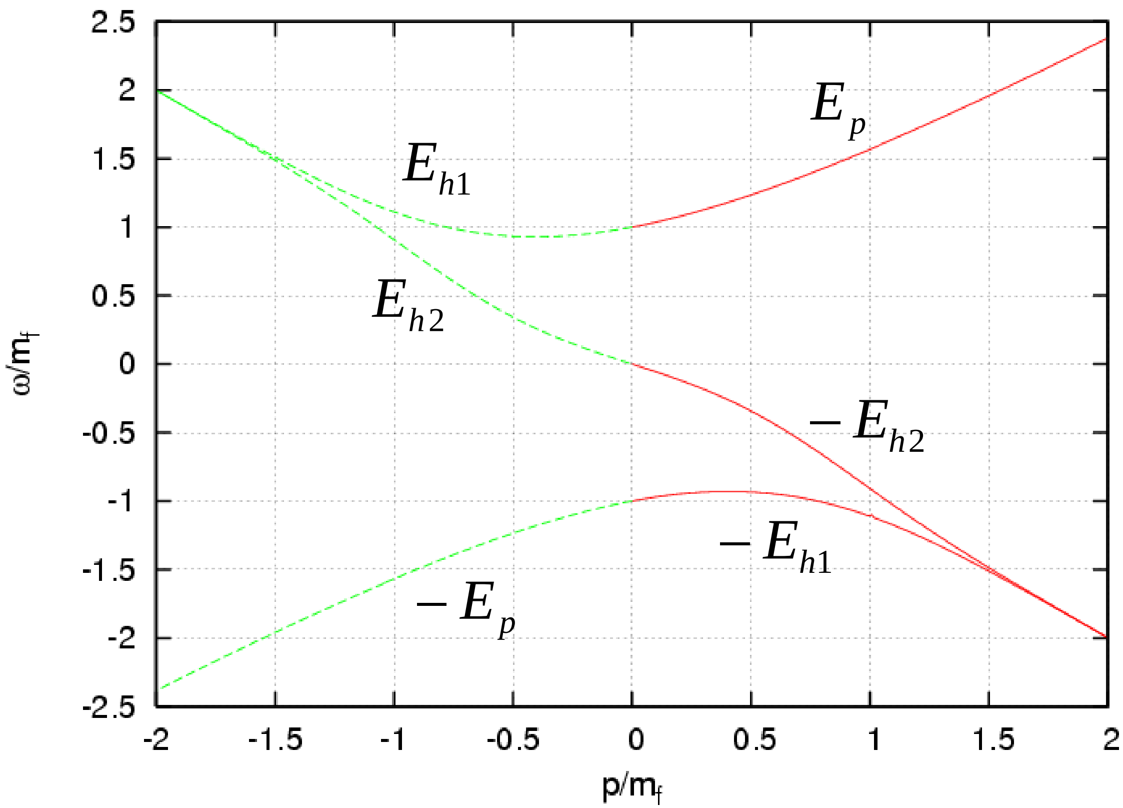
dispersion relations

$$\mathrm{Re}[D_+(\omega, p)] = 0$$

$$\omega = E_p, -E_{h1}, -E_{h2}$$

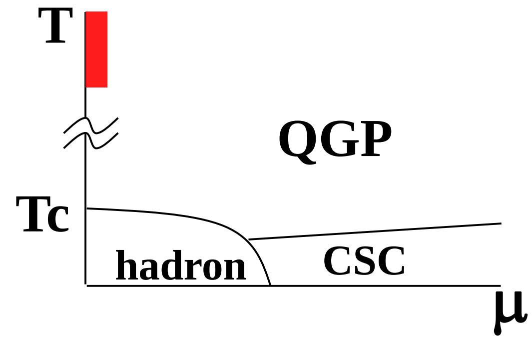
$$\mathrm{Re}[D_-(\omega, p)] = 0$$

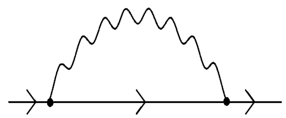
$$\omega = -E_p, E_{h1}, E_{h2}$$



Quarks at very high T (T>>>Tc)

- 1-loop (g<<1) + HTL approx. $(p, \omega, m_q << \mathrm{T})$



$$\Sigma(\omega, p) =$$ 

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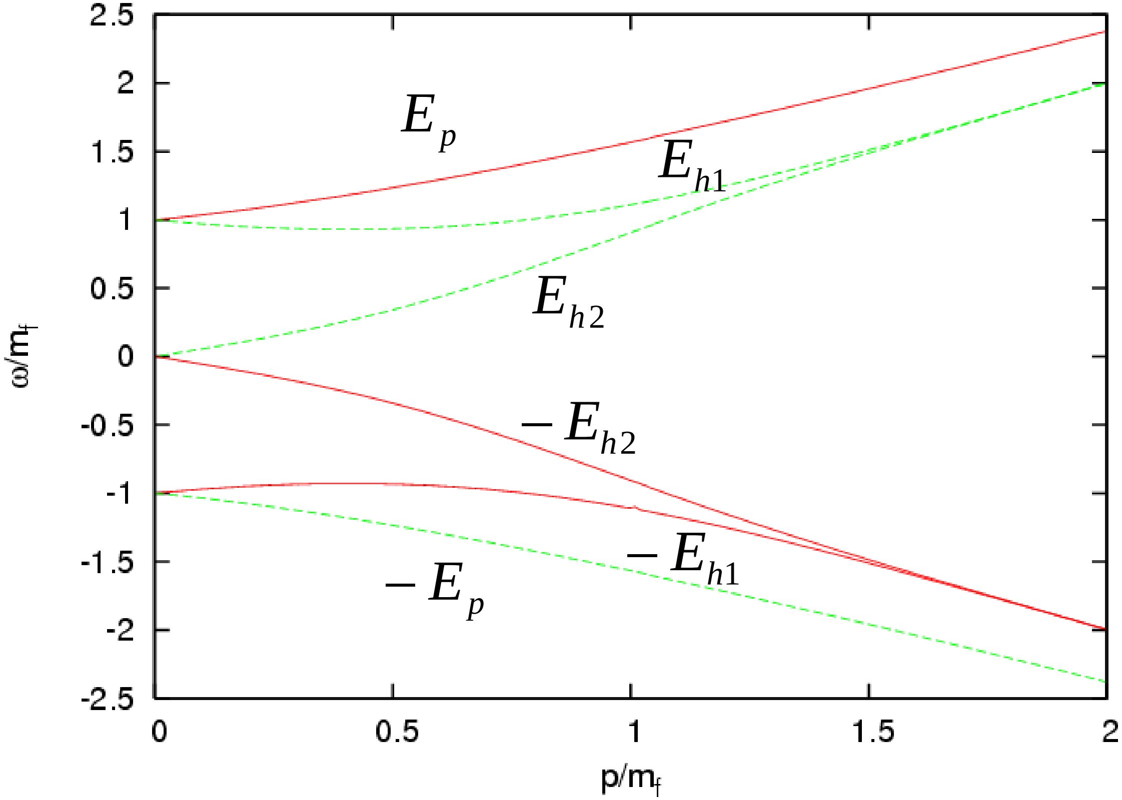
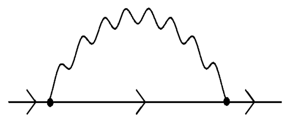
$$\omega = -E_p, E_{h1}, E_{h2}$$

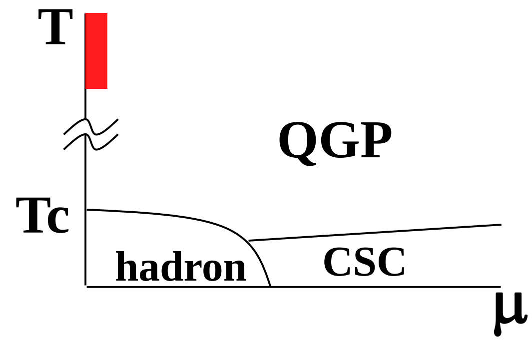


Quarks at very high T (T>>>Tc)

- 1-loop (g<<1) + HTL approx. (p, ω, m_q << T)

$$\Sigma(\omega, p) =$$





thermal masses $m_f^2 = \frac{1}{6} g^2 T^2$

dispersion relations

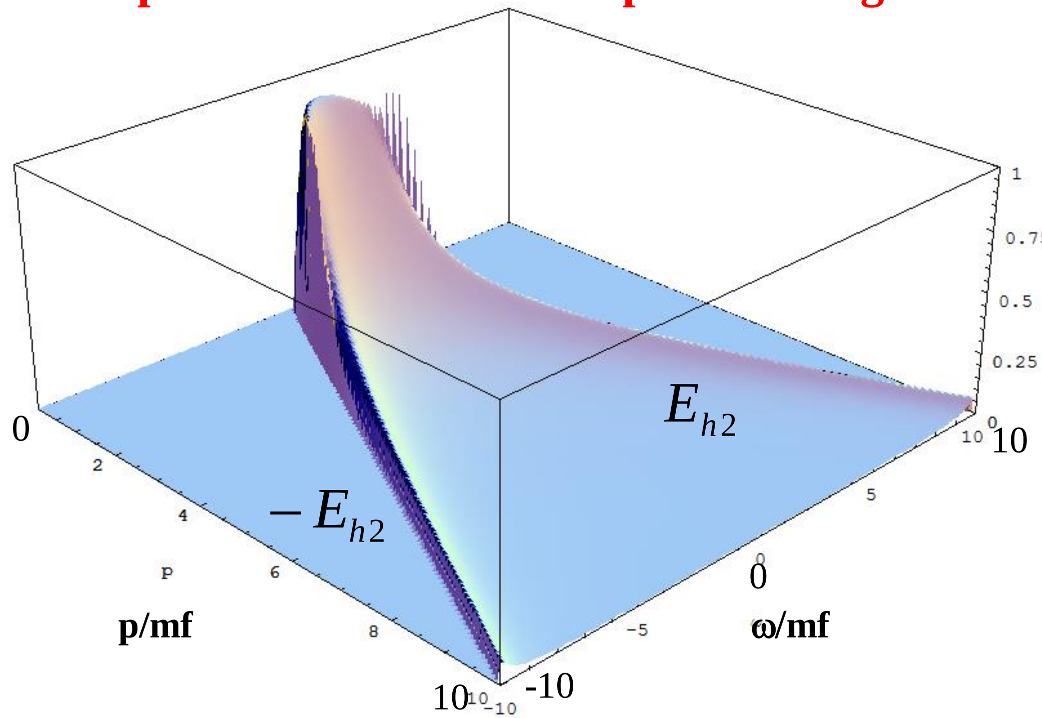
$$\mathrm{Re}[D_+(\omega, p)] = 0$$

$$\omega = E_p, -E_{h1,} - E_{h2}$$

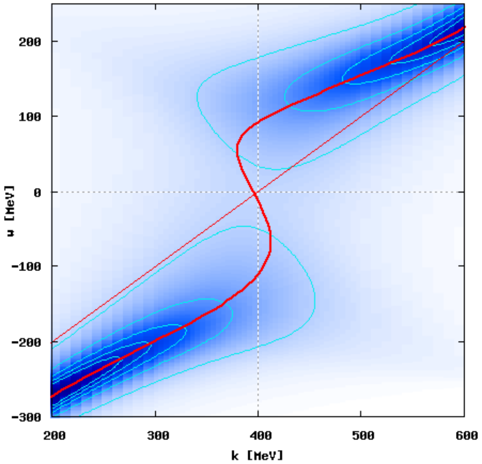
$$\mathrm{Re}[D_-(\omega, p)] = 0$$

$$\omega = -E_p, E_{h1,} E_{h2}$$

spectral function of the space-like region



Difference between CSC and CHIRAL

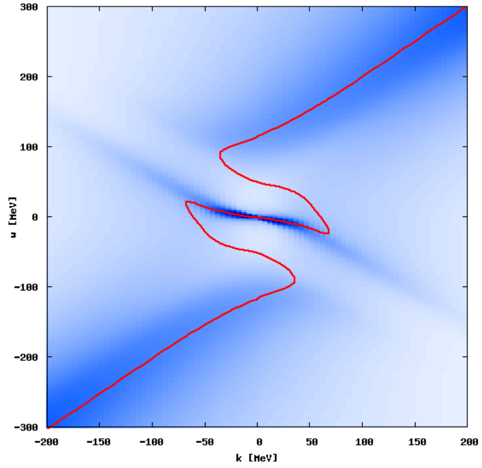


- <u>above CSC phase:</u>

  One resonant scattering

  fluctuations of the order parameter ~ diffusion-like

  $$\omega(p) \sim p^2 \quad (p \sim 0)$$



- <u>above chiral transition:</u>

  Two resonant scatterings

  fluctuations of the order parameter

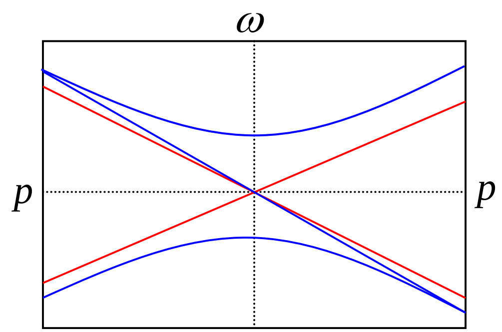
  ~ propagating-like

  $$\omega(p) \sim \pm\omega_0 \, (\neq 0) \quad (p \sim 0)$$

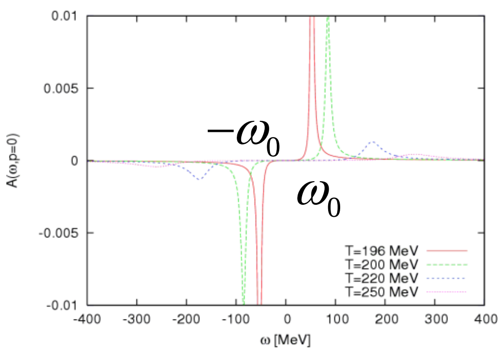
Level Repulsions

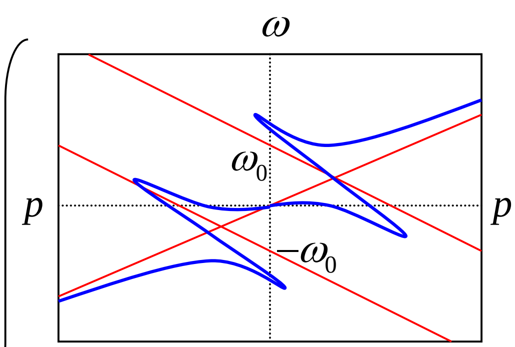
For massless gauge field,

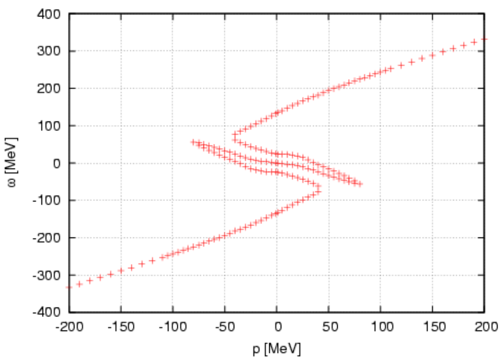
$$A(\omega, p) \sim \delta(\omega - p) - \delta(\omega + p)$$

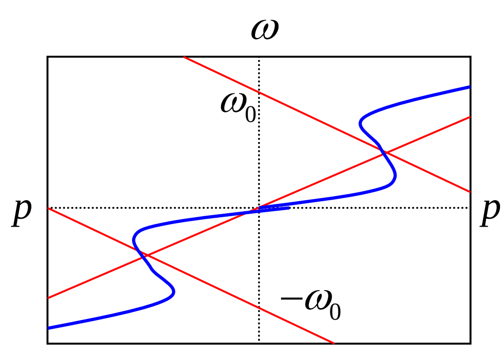


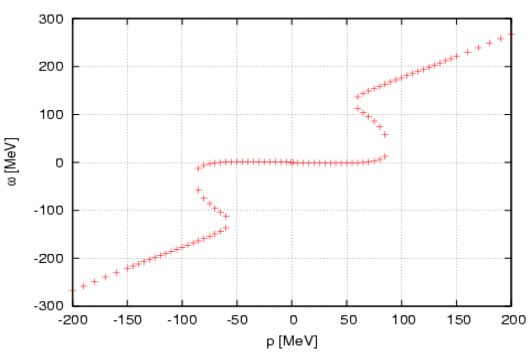
$$A(\omega, p) \sim \delta(\omega - \sqrt{p^2 + \omega_0^2}) - \delta(\omega + \sqrt{p^2 + \omega_0^2})$$



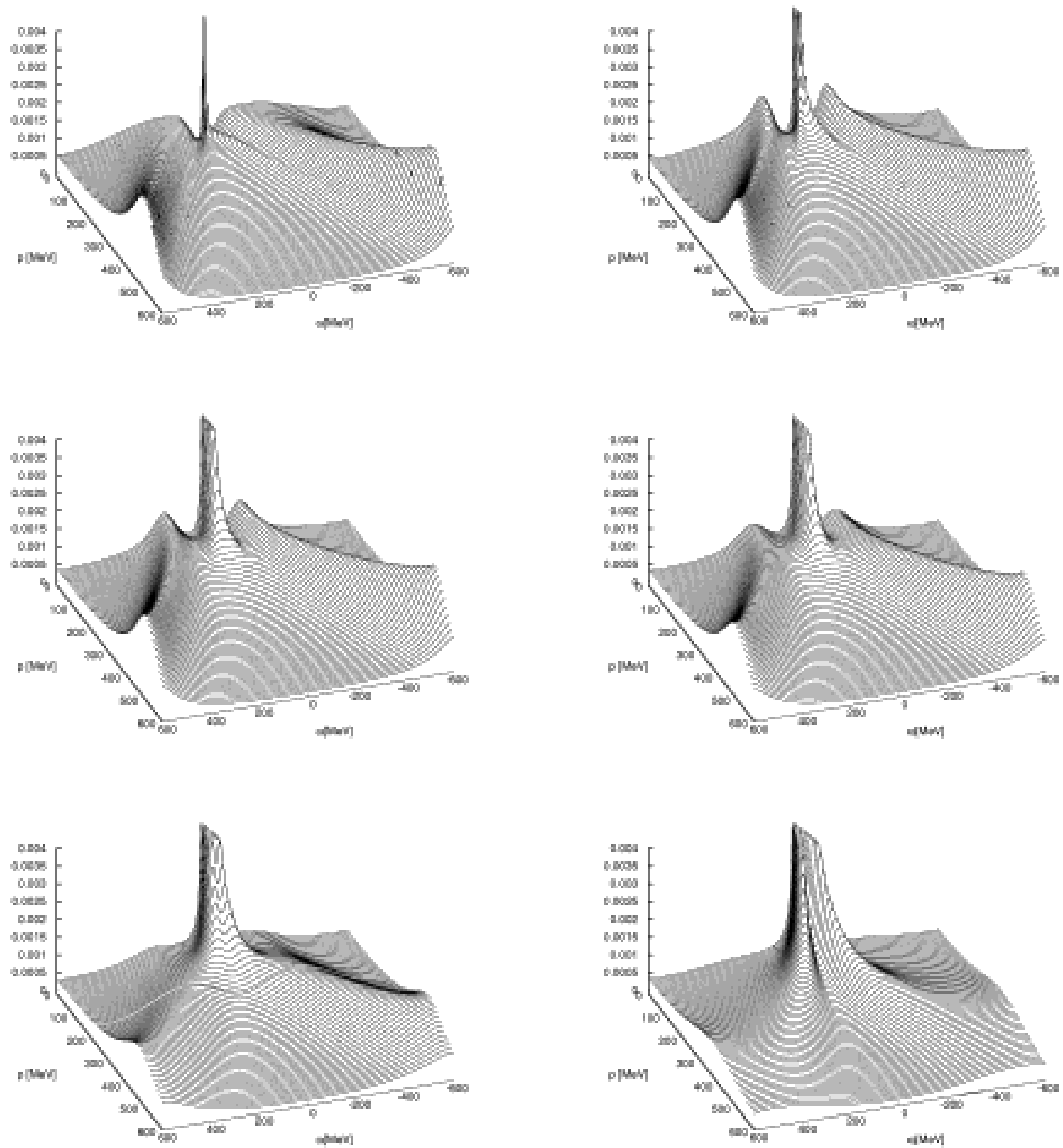




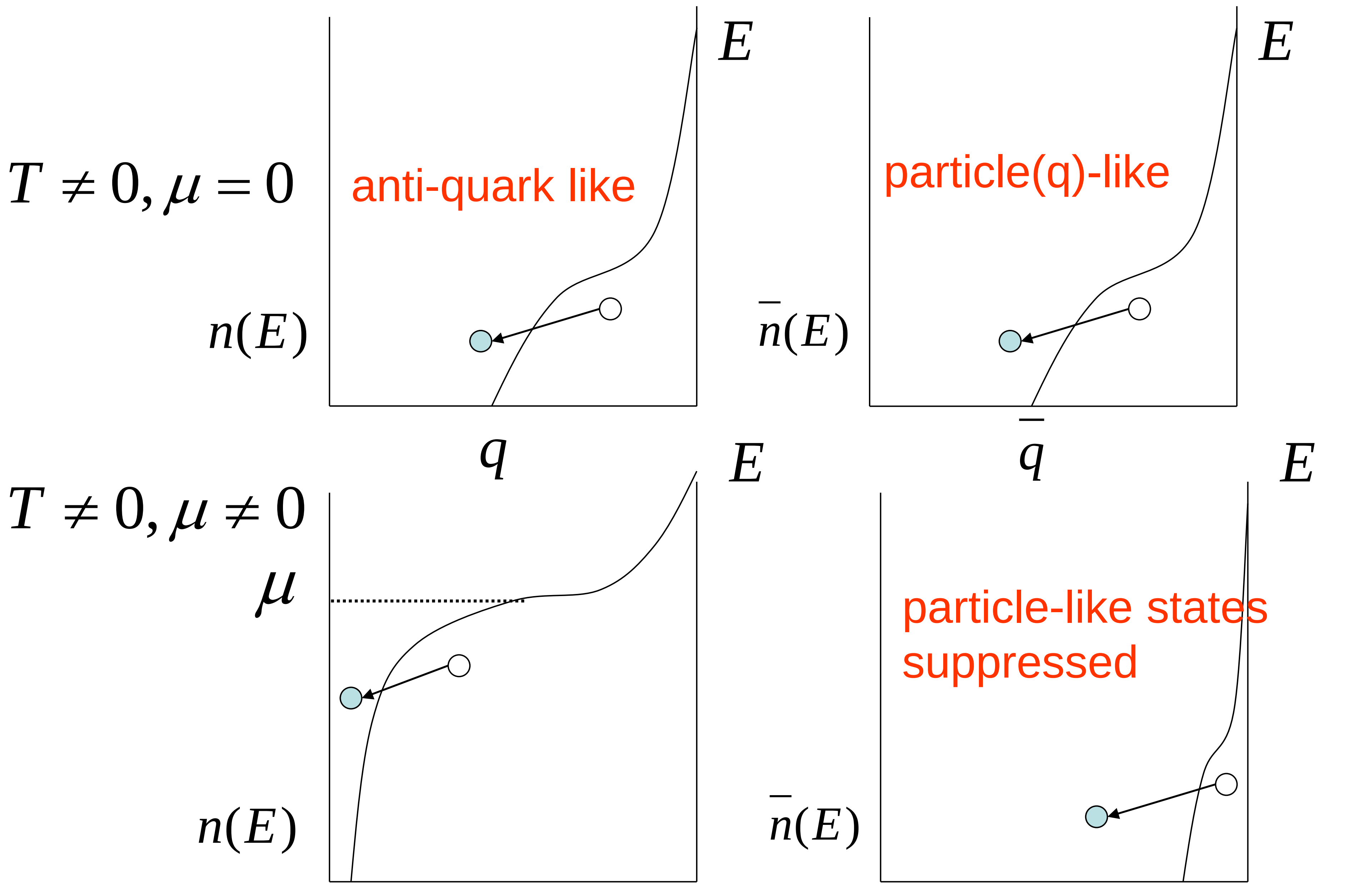




Spectral function of the quarks



Finite μ dependence; asymmetry between q and $\bar{q}$



Summary of the second part

- **We have investigated how the fluctuations of $\langle \bar{q}q \rangle$ affect the quark spectrum in symmetry-restored phase near Tc.**

- **Near (above) Tc, the quark spectrum at long-frequency and low momentum is strongly modified by the fluctuation of the chiral condensate, $\langle \bar{q}q \rangle$.**

- **The many-peak structure of the spectral function can be understood in terms of two resonant scatterings at small ω and p of a quark and an antiquark off the fluctuation mode.**

- **This feature near Tc is model-independent if the fluctuation of $\langle \bar{q}q \rangle$ is dominant over the other degrees of freedom.** ⇨ **can be reproduced by a Yukawa theory with the boson being a scalar/pseudoscalar or vector/axial vector one**

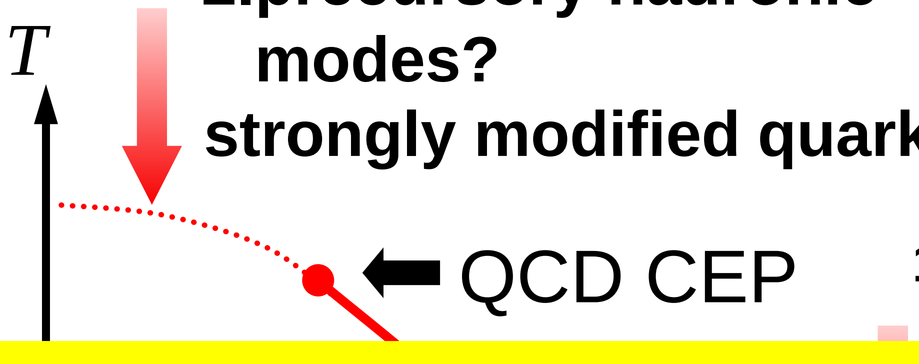
(Kitazawa, Nemoto and T.K.,in preparation)

Future

- finite quark mass effects. (2nd order → crossover)
- finite μ ⇨ coupling with density fluctuation; CEP?

Summary of the Talk

QCD phase diagram



`QGP' itself seems surprisingly rich in physics!

Condensed matter physics of strongly coupled Quark-Gluon systems will constitute a new field of fundamental physics.

Back Upps

Pairing patterns of CSC

- for $J^P=0^+$ pairing $\Delta_{ij}^{\alpha\beta} = \langle \psi_i^{\alpha} C i \gamma_5 \psi_j^{\beta} \rangle$

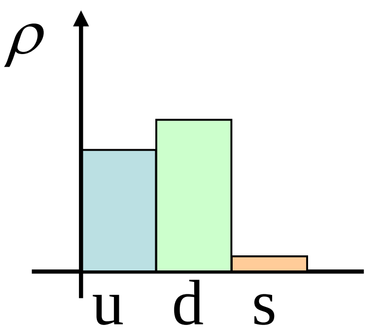
a,b : color
i,j : flavor

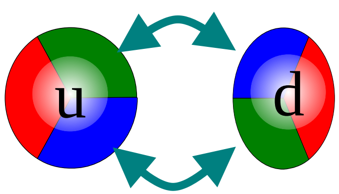
- attractive channel : color anti-symm.
  → flavor anti-symm.

→ $\Delta_{ij}^{\alpha\beta} = \varepsilon^{\alpha\beta\gamma} \varepsilon_{ijk} \mathbf{d}_k^{\gamma}$

- $\mu < M_s$

Two Flavor Superconductor (2SC)



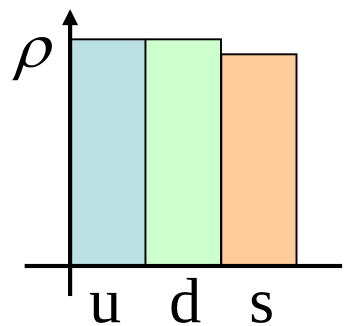


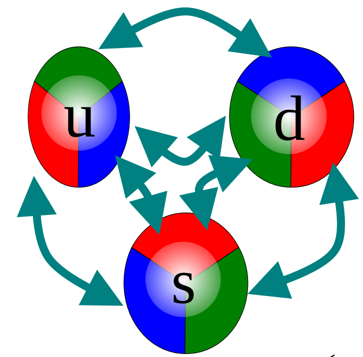
$$\mathbf{d} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Delta \end{pmatrix}$$

$$SU(3)_c \rightarrow SU(2)_c$$

- $\mu >> M_s$

Color-Flavor Locked (CFL)

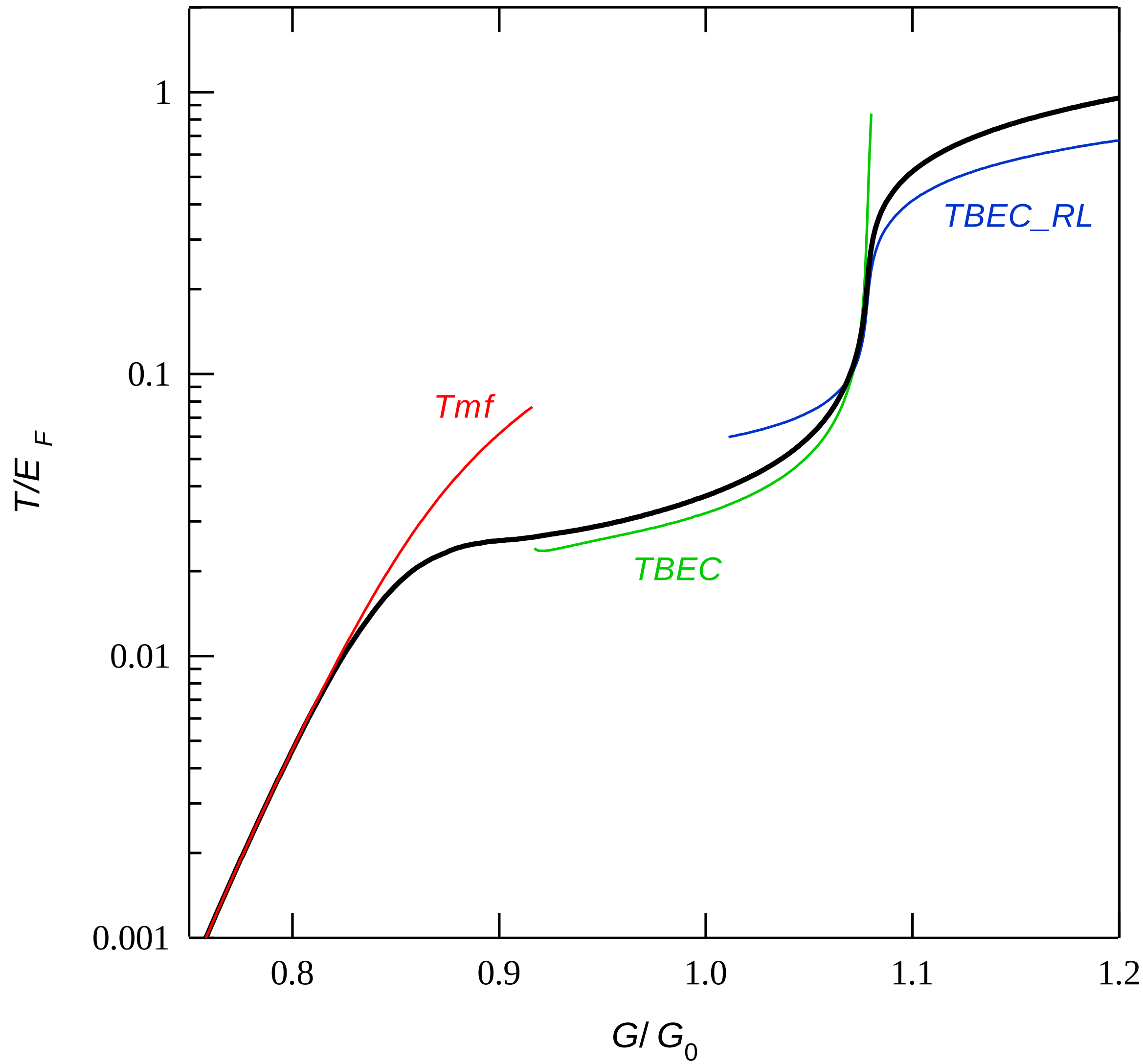




$$\mathbf{d} = \begin{pmatrix} \Delta_1 & & \\ & \Delta_2 & \\ & & \Delta_3 \end{pmatrix}$$

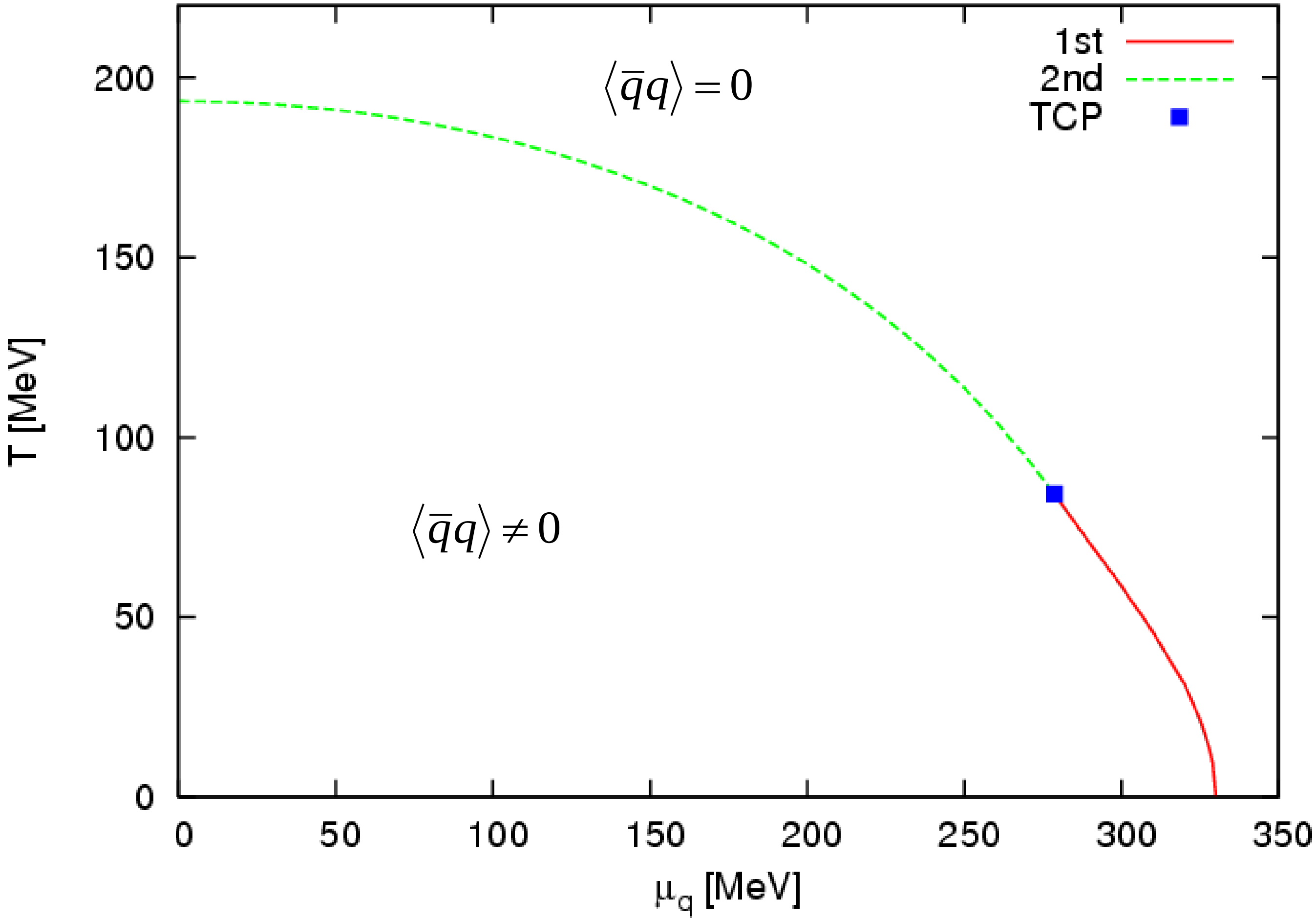
$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{c+L+R}$$

BCS-BEC transition in QM



Y.Nishida and H. Abuki, hep-ph/0504083

Calculated phase diagram



Fermions at finite T

- **free massless quark at T=0**

$$S_0(\omega, p) = \frac{1}{\not p} = \frac{1}{2}\frac{\gamma_0 - \vec{\gamma}\cdot\hat{p}}{\omega - |\vec{p}|} + \frac{1}{2}\frac{\gamma_0 + \vec{\gamma}\cdot\hat{p}}{\omega + |\vec{p}|}$$

$\omega = \pm|\vec{p}|$ quark and antiquark

- **quark at finite T (massless)**

$$S(\omega, p) = \frac{1}{A(\omega, p)\gamma_0 - C(\omega, p)\vec{\gamma}} = \frac{1}{2}\frac{\gamma_0 - \vec{\gamma}\cdot\hat{p}}{D_+(\omega, p)} + \frac{1}{2}\frac{\gamma_0 + \vec{\gamma}\cdot\hat{p}}{D_-(\omega, p)}$$

$$D_\pm = A \mp C$$

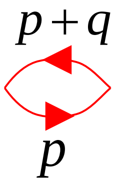
$D_+(\omega, p) = 0$ **several solutions**

$D_-(\omega, p) = 0$ **several solutions**

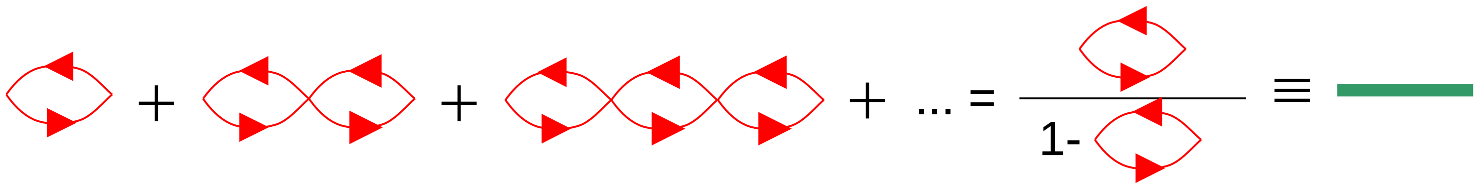
$$D_+(\omega, p) = 0 \longrightarrow \omega = E_p > 0, \omega = -E_h < 0$$

$$D_-(\omega, p) = 0 \longrightarrow \omega = E_h > 0, \omega = -E_p < 0$$

Formulation (Self-energy)

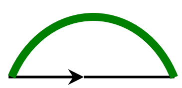


$$(\vec{p}, p_0) = \int \frac{d^3p}{(2\pi)^3} \frac{N_f N_C}{E_{q+p}E_p} \Bigg[(E_{q+p}E_p - \vec{p}\cdot(\vec{q}+\vec{p})) \left(\frac{f(E_p) - f(E_{q+p})}{p_0 + E_p - E_{q+p} + i\varepsilon} - \frac{f(E_p) - f(E_{q+p})}{p_0 - E_p + E_{q+p} + i\varepsilon} \right)$$
$$+ (E_{q+p}E_p + \vec{p}\cdot(\vec{q}+\vec{p})) \left(\frac{1 - f(E_p) - f(E_{q+p})}{p_0 - E_p - E_{q+p} + i\varepsilon} - \frac{1 - f(E_p) - f(E_{q+p})}{p_0 + E_p + E_{q+p} + i\varepsilon} \right) \Bigg]$$



$$= D(\mathbf{k}, \omega)$$

Quark self-energy:

$\Sigma(\vec{p}, p_0) =$ 

$$= -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{Im}\, D(\vec{p}-\vec{q}, q_0)}{q_0 - p_0 + |\vec{p}| + i\varepsilon} (\gamma^0 - \hat{q}\cdot\vec{\gamma}) \left[\coth\frac{q_0}{2T} + \tanh\frac{|\vec{q}|}{2T} \right]$$
$$-\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{Im}\, D(\vec{p}-\vec{q}, q_0)}{q_0 - p_0 - |\vec{p}| + i\varepsilon} (\gamma^0 + \hat{q}\cdot\vec{\gamma}) \left[\coth\frac{q_0}{2T} - \tanh\frac{|\vec{q}|}{2T} \right]$$

Formulation (Spectral Function)

Spectral function:

$$A(\vec{p}, p_0) = \rho_0(\vec{p}, p_0)\gamma^0 - \rho_V(\vec{p}, p_0)\vec{\gamma}\cdot\hat{p}$$
$$= [\underbrace{\rho_+(\vec{p}, p_0)}_{\text{quark}}\Lambda_+(p) + \underbrace{\rho_-(\vec{p}, p_0)}_{\text{antiquark}}\Lambda_-(p)]\gamma^0$$

$$\Lambda_\pm(p) = \frac{1}{2}(1 \pm \gamma^0\vec{\gamma}\cdot\hat{p})$$

$$\rho_\pm(\vec{p}, p_0) = -\frac{1}{\pi}\mathrm{Im}\frac{1}{p_0 \mp |\vec{p}| - \Sigma_\pm(\vec{p}, p_0) + i\varepsilon}$$

Spectral Contour and Dispersion Relation