Quantum Opacity and Refractivity in HBT Puzzle

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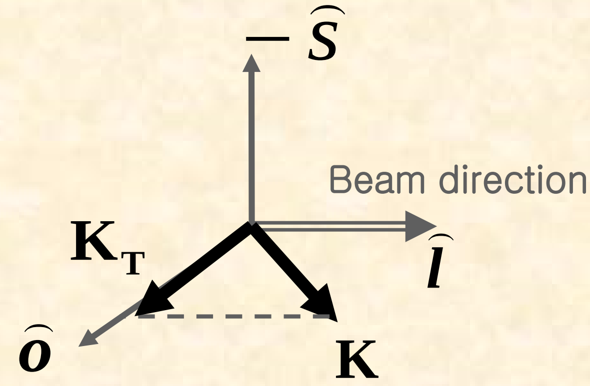
Correlations in Phase-Space

Space–time structure of fireball can be studied by HBT interferometry

Correlation function

$$C(\mathbf{p}_1,\mathbf{p}_2) = N\frac{P_2(\mathbf{p}_1,\mathbf{p}_2)}{P_2(\mathbf{p}_1)P_1(\mathbf{p}_2)}$$



is typically parametrized as

$$C(\mathbf{p}_1,\mathbf{p}_2) \approx 1 + \lambda \exp[-R_o^2 q_o^2 - R_s^2 q_s^2 - R_l^2 q_l^2]$$

$$\mathbf{K} = (\mathbf{p}_1 + \mathbf{p}_2)/2 \qquad \mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$$

Correlations in Phase-Space

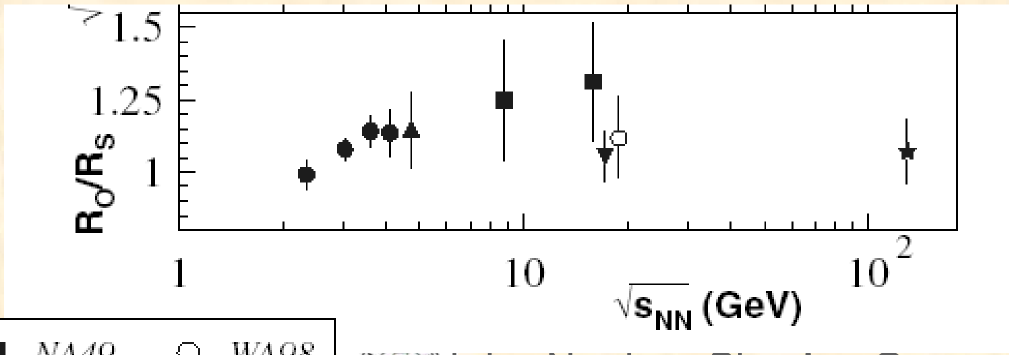
Hydrodynamic calculation predicts $R_o / R_s \approx 1.5 \sim 10$

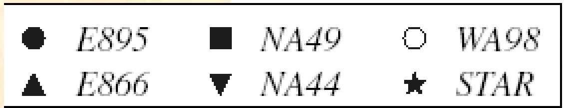
(D. H. Rischke and M. Gyulassy, Nucl. Phys. A608, 479 (1996);
P. F. Kolb and U. Heinz, ***Quark Gluon Plasma 3***, World Scientific, Singapore, 2004)

But, experimental results are $R_o / R_s \approx 1 \sim 1.2$

(STAR Collaboration, C. Adler et al., Phys. Rev. Lett. 87, 082301 (2001);
PHENIX Collaboration, K. Adcox et al., Phys. Rev. Lett. 88, 192302 (2002);
PHENIX Collaboration, A. Enokizono, Nucl. Phys. A715, 595 (2003))

⟹ HBT Puzzle





Correlations in Phase-Space

Experimental Data(Au+Au@200 GeV)
shows **Dense Medium**

⟶ **OPACITY** and **Refractive Effects**

Our Purpose : Quantum mechanical treatment of
Opacity & Refractive effects
which reproduces $R_o,\ R_s,\ R_l$

Correlations in Phase-Space

Theoretically, the observables are expressed by

$$S_0(x,K)=\int \frac{d^4y}{2(2\pi)^3}\exp(-iK\cdot y) < J^*(x+y/2)J(x-y/2) >$$

$$C_0(\mathbf{p}_1,\mathbf{p}_2)=C_0(\mathbf{q},\mathbf{K})=1+\frac{|\int d^4x\, S_0(x,K)e^{-iq\cdot x}|^2}{\int d^4x\, S_0(x,p_1)\int d^4x\, S_0(x,p_2)}$$

Subscript 0 means **no final state interaction** (FSI).

Final State Interaction

FSI replaces $\boldsymbol{J(x) \rightarrow J(x)\ \Psi_{\mathrm{P}}^{(-)}*(x)\ e^{-ip\cdot x}}$

$\boldsymbol{\Psi_{\mathrm{P}}^{(-)}*}$: full scattering outgoing wave function

$$C(\mathrm{p}_1,\mathrm{p}_2) = C(\mathrm{q},\mathrm{K}) = 1 + \frac{|\int d^4x\, S(x,K)|^2}{\int d^4x\, S(x,p_1) \int d^4x\, S(x,p_2)}$$

$$\boldsymbol{S(x,K) = \int d^4K'\, S_0(x,K') \int \frac{d^4y}{(2\pi)^4} \exp(iK'\cdot y)\, \Psi_{\mathrm{p}_1}^{(-)}(x+y/2)\, \Psi_{\mathrm{p}_2}^{(-)}*(x-y/2)}$$

Includes **two 4-dimensional integration**

Wigner Emission Function

Using the hydrodynamic source parameterization

(B. Tomasik and U. W. Heinz, Eur. Phys. J. C4, 327 (1998);
U. A. Wiedermann and U. W. Heinz, Phys. Rep. 319, 145 (1999))

With boost-invariant longitudinal dynamics

$$S_0(x,K) = Z_0(\tau,\eta) B_\eta(\mathbf{b},\mathbf{K_T}) / (2\pi)^3$$

$$Z_0(\tau,\eta) \equiv \frac{\cosh(\eta - Y)}{\sqrt{2\pi(\Delta\tau)^2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2(\Delta\tau)^2} - \frac{\eta^2}{2(\Delta\eta)^2} \right)$$

$$B_\eta(\mathbf{b},\mathbf{K_T}) \equiv M_T \left\{ \exp\left(\frac{K\cdot u - \mu_\pi}{T} \right) - 1 \right\}^{-1} \rho(b)$$

$$M_T^2 = K_T^2 + m_\pi^2,$$

μ_π : chemical potential of π

$$\tau = \sqrt{t^2 - z^2} \quad \eta = \frac{1}{2} \ln \frac{t+z}{t-z} \quad \mathbf{b} \equiv (x_1, x_2)$$

$$\phi = \theta(\mathbf{K_T}, \mathbf{b})$$

$$K \cdot u = M_T \cosh\eta \cosh\eta_t(b) - K_T \sinh\eta_t(b) \cos\phi$$

Wigner Emission Function

$$\rho(b) = \left(\exp\left[(b - R_{WS})/a_{WS}\right] + 1\right)^{-2}$$

: cylindrically symmetric source density

$$\eta_t(b) = \eta_f \frac{b}{R_{WS}}$$

: transverse flow rapidity

$$Y = \frac{1}{2}\ln\frac{E_K + K_L}{E_K - K_L} = 0$$

Since K_L=0 for midrapidity data

Parameters : R_{WS}, a_{WS}, t_0 , Dt , Dh , m_p , h_f , T

Full Scattering Wavefunction

Assumption : Matter is **cylindrically symmetric** with a long axis in a central collision region

$$\Psi^{(-)}_{\mathbf{P}_{1,2}}(x) = e^{-i\omega_p x_0}\, e^{\mp i q_l z/2}\, \psi^{(-)}_{\mathbf{P}_{1,2}}(\mathbf{x}_\perp = \mathbf{b})$$

$$\mathbf{P}_{1,2} = \mathbf{K} \pm \mathbf{q}_T / 2 \pm \mathbf{q}_l / 2$$

Reduced 2-dimensional Klein-Gordon Eq.

$$(-\nabla_\perp^2 + U(b))\psi^{(-)}_{\mathbf{p}} {}^*(\mathbf{b}) = p^2 \psi^{(-)}_{\mathbf{p}} {}^*(\mathbf{b})$$

Full Scattering Wavefunction

Optical Potential $U_p(b) = -(w_0 + w_2 p^2)\rho(b)$

At p=0, no opacity ⟶ w_0:real

Parameters : w_0, w_2

Using partial wave expansion,

$$\psi_{\mathbf{p}}^{(-)}{}^*(\mathbf{b}) = f_0(p,b) + 2\sum_{m=1,\infty} f_m(p,b)(-i)^m \cos m\phi$$

we can solve K-G Eq. exactly.

Full Scattering Wavefunction

In Impulse Approximation, central optical potential

$$U_0 = -4\pi f \rho_0$$

f : complex forward scattering amplitude

ρ_0 : central density

For low energy **p-p** interaction

$$4\pi \,\mathrm{Im}[\,f(p)] = p\sigma$$

Using $\sigma \approx 1\,\mathrm{mb}, \quad \mathrm{p} = 1\,\mathrm{fm}^{-1}, \quad \rho_0 \approx 1.5\mathrm{fm}^{-3}$

$\mathrm{Im}[U_0] = -0.15\,\mathrm{fm}^{-2}$ ⟶ Significant opacity

Correlation Function

Now our Emission Function is

$$\boldsymbol{S}(\boldsymbol{x},\boldsymbol{K}) = \frac{1}{(2\pi)^2}\boldsymbol{Z}_0(\tau,\eta)\boldsymbol{e}^{iq^0t-iq_lz}\int \boldsymbol{d}^2\boldsymbol{b}'\widetilde{\boldsymbol{B}}_\eta(\mathbf{b},\mathbf{b}')$$

$$\times\psi_{\mathbf{p}_1}^{(-)}(\mathbf{b}+\mathbf{b}'/2)\psi_{\mathbf{p}_2}^{(-)}*(\mathbf{b}-\mathbf{b}'/2)$$

with $\widetilde{\boldsymbol{B}}_\eta(\mathbf{b},\mathbf{b}') = \int \boldsymbol{d}^2\boldsymbol{K}_T{}'\boldsymbol{B}_\eta(\mathbf{b},\mathbf{K_T}')\exp[-\boldsymbol{i}\mathbf{K_T}'\cdot\mathbf{b}']$

Large Source Approximation : $\mathbf{b}'\sim 1/T \sim 1$ fm $<< \mathbf{R}_{WS}$

$$\psi_{\mathbf{p}_1}^{(-)}\left(\mathbf{b}+\frac{\mathbf{b}'}{2}\right)\psi_{\mathbf{p}_2}^{(-)}*\left(\mathbf{b}-\frac{\mathbf{b}'}{2}\right) \approx \psi_{\mathbf{p}_1}^{(-)}(\mathbf{b})\psi_{\mathbf{p}_2}^{(-)}*(\mathbf{b})\boldsymbol{e}^{-i\mathbf{K_T}\cdot\mathbf{b}'/2}$$

Correlation Function

$$C(\mathbf{q},\mathbf{K_T}) = 1 - q_l^2 R_l^2 - q_o^2 \beta^2 \tilde{\Delta}_\tau^2 \frac{|\Phi_{12}|^2}{\Phi_{11}\Phi_{22}}$$

$$\Phi_{ij} = \sum_n \int d^2b\ f_0(\xi_n(b))\,\psi_{\mathbf{p_i}}^{(-)}(\mathbf{b})\,\psi_{\mathbf{p_j}}^{(-)*}(\mathbf{b})\,B_n(\mathbf{b},\mathbf{K_T})$$

$$B_n(\mathbf{b},\mathbf{K_T}) = \exp\left(\frac{\mu_\pi + K_T \sinh\,\eta_t(b)\cos\phi}{T_n}\right) M_T\,\rho(b)$$

$$R_l^2 = \frac{(3\Delta\tau^2 + \tau_0^2)F_1(K)}{F_0(K)}$$

$$\tilde{\Delta}_\tau^2 = (3\Delta\tau^2 + \tau_0^2)\frac{F_3(K)}{F_0(K)} - \left|\frac{(\Delta\tau^2 + \tau_0^2)F_2(K)}{\tau_0 F_0(K)}\right|^2$$

Correlation Function

$$F_m(K) = \sum_n \int d^2b\, B_n(\mathbf{b}, \mathbf{K_T})\; f_m(\xi_n(b)) \left|\psi^{(-)}_{\mathbf{K_T}}(\mathbf{b})\right|^2$$

$$\xi_n(b) = M_T \cosh\eta_t(b)/T_n + 1/\Delta\eta^2$$

$$f_0(\xi) \equiv 2K_1(\xi)$$

$$f_1(\xi) \equiv 2K_0(\xi)/\xi + 4K_1(\xi)/\xi^2$$

$$f_2(\xi) \equiv K_0(\xi) + K_2(\xi)$$

$$f_3(\xi) \equiv 2\left[K_1(\xi) + K_2(\xi)/\xi\right]$$

Here, $K_i(\xi)$ Are Modified Bessel function .

HBT Radii

Then our transverse radii can be calculated by

$$R_{o,s}^2(K_T) = \frac{2 - C(\Delta q_{o,s}, K_T)}{\Delta q_{o,s}^2}$$

with $\Delta q_{o,s} \approx K_T / 40$

Fitting Parameters

$$\mathbf{STAR@}\sqrt{s_{NN}} = 200\,\mathbf{GeV\ Au + Au}$$

T(MeV)	h_f	R_{WS}(fm)	a_{WS}(fm)	Dt(fm/c)
173.2 $\pm$1.6	1.314 $\pm$0.025	11.782 $\pm$0.056	0.725 $\pm$0.015	2.852 $\pm$0.067
w_0(fm^{-2})	w_2	t_0(fm/c)	Dh	m_p(MeV)
0.137 $\pm$0.046	$0.582 + i0.121$ $\pm 0.014 \pm i0.002$	8.23 $\pm$0.10	1.063 $\pm$0.032	123.2 $\pm$1.1

$c^2/N \sim 7.8$

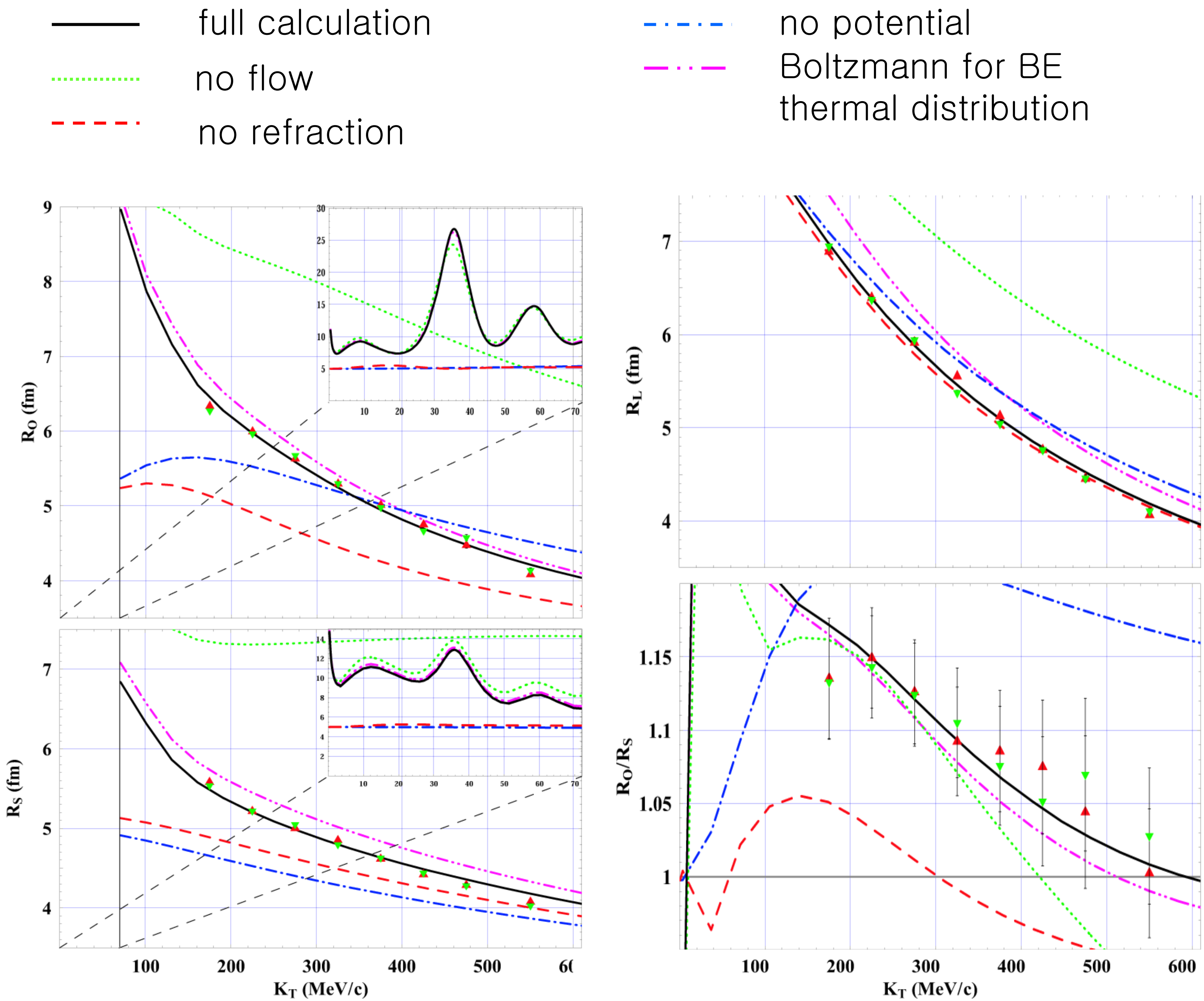
Fitting Parameters Check

- Temperature *T* (173 MeV) ~ *Tc* (160 MeV)
- h_f=1.31 ⟶ maximum flow velocity ~ 0.85c
- Source Size R_{WS} (11.7 fm) ~ R_{Au} (7.3 fm)+4.4 fm
- Expansion time t_0 (8.2 fm/c)

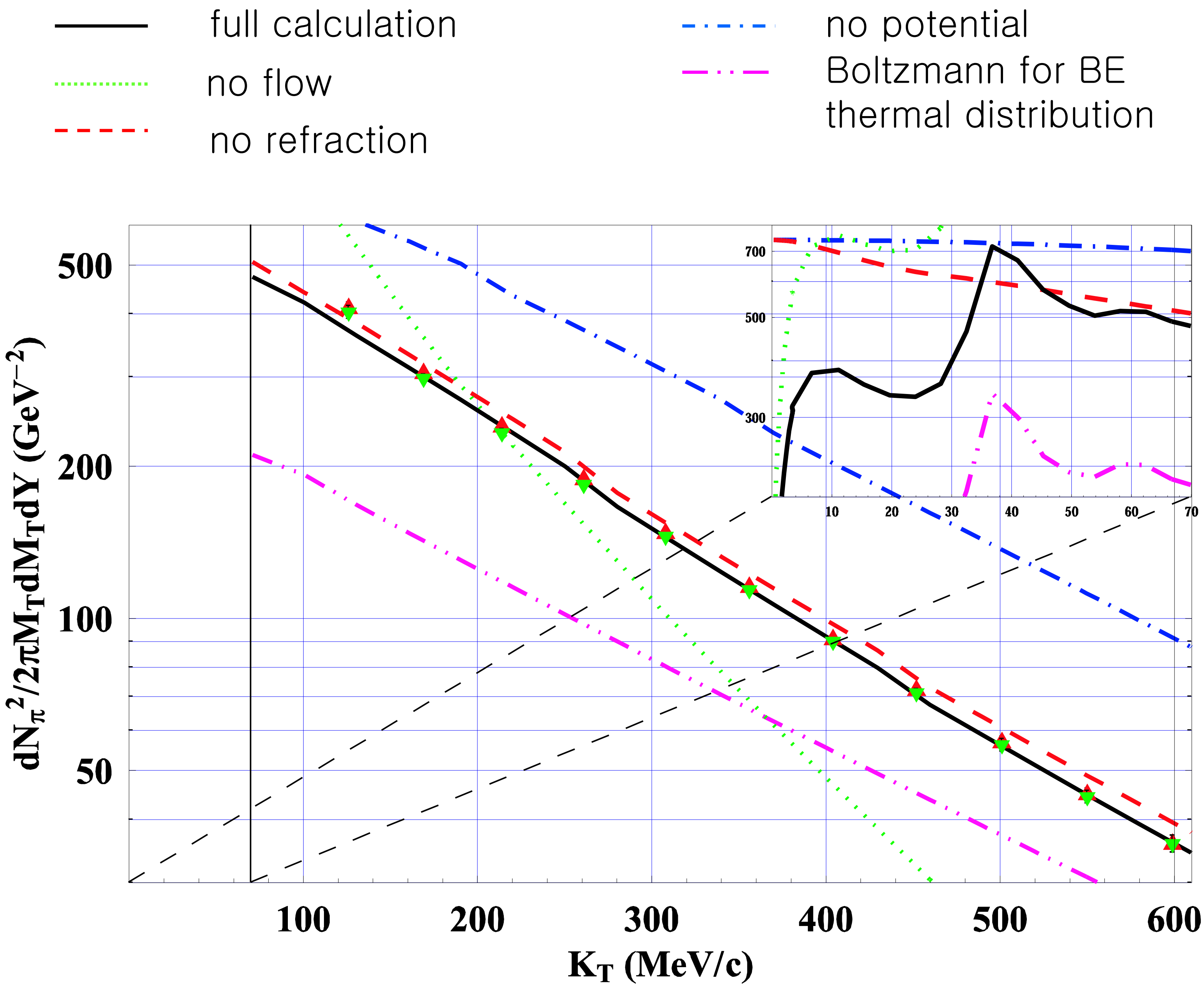
  average expansion velocity ~ 0.5c
- Emission duration **Dt**(2.9 fm/c) << t_0(8.2 fm/c)
- Longitudinal length **Dh(1.06)**

  system's axial length ~ $2t_0$Dh(17.5 fm)

  : large enough for long cilyndrical symmetry



full calculation
no flow
no refraction
no potential
Boltzmann for BE
thermal distribution
R_O (fm)
R_S (fm)
R_L (fm)
R_O/R_S
K_T (MeV/c)
K_T (MeV/c)



full calculation
no flow
no refraction
no potential
Boltzmann for BE
thermal distribution
500
200
100
50
$dN_\pi^2/2\pi M_T dM_T dY$ (GeV^{-2})
100
200
300
400
500
600
K_T (MeV/c)
700
500
300
10
20
30
40
50
60
70

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Quantum Opacity, the RHIC Hanbury Brown–Twiss Puzzle, and the Chiral Phase Transition

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We present a relativistic quantum-mechanical treatment of opacity and refractive effects that allows reproduction of observables measured in two-pion Hanbury Brown–Twiss (HBT) interferometry and pion spectra at RHIC. The inferred emission duration is substantial. The results are consistent with the emission of pions from a system that has a restored chiral symmetry.

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An emptier emptiness?

Frank Wilczek

Temperatures similar to those reached an instant after the Big Bang can be created in collisions of gold atoms. The resulting fireballs may allow us a glimpse of a world that is more symmetrical than our own.

The concept that what we ordinarily perceive as empty space is in fact a complicated medium is a profound and pervasive theme in modern physics. This invisible, inescapable medium alters the behaviour of the matter that we do see. Just as Earth's gravitational field allows us to select a unique direction as up, and thereby locally reduces the symmetry of the underlying equations of physics, so cosmic fields in 'empty' space lower the symmetry of these fundamental equations everywhere. Or so theory has it. For although this concept of a symmetry-breaking aether has been extremely fruitful (and has been demonstrated indirectly in many ways), the ultimate demonstration of its validity — cleaning out the medium and restoring the pristine symmetry of the equations — has never been achieved: that is, perhaps, until now.

In a new paper, Cramer *et al.*[1] claim to have found evidence that — for very brief moments, and over a very small volume — experimentalists working at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory in New York have vaporized one symmetry-breaking aether, and produced a more perfect emptiness. This pioneering attempt to decode the richly detailed (in other words, complicated and messy) data emerging from the RHIC experiments is intricate[2], and it remains to be seen whether the interpretation Cramer *et al.* propose evolves into a consensus. In any case, they've put a challenge on the agenda, and suggested some concrete ways to tackle it.

But what exactly is this underlying symmetry of nature that is broken by the aether? How is it broken, and how might it be restored? The symmetry in question is called chiral symmetry, and it involves the behaviour of quarks, the principal constituents of the protons and neutrons in atomic nuclei (among other things).

Chiral symmetry is easiest to describe if we adopt the slight idealization that the lightest quarks, the up quark (u) and down quark (d), are massless. (In reality their masses are small, on the scale of the energies in play, but not quite zero.) According to the equations of quantum chromodynamics (QCD), the theory that describes quarks and their interactions via the strong nuclear

1. Cramer, J., Miller, G., Wu, J. & Yoon, J.-H. preprint at www.arxiv.org/nucl-th/0411031 (2004).

Thank you.