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Hadronic multiplicity at RHIC and LHC

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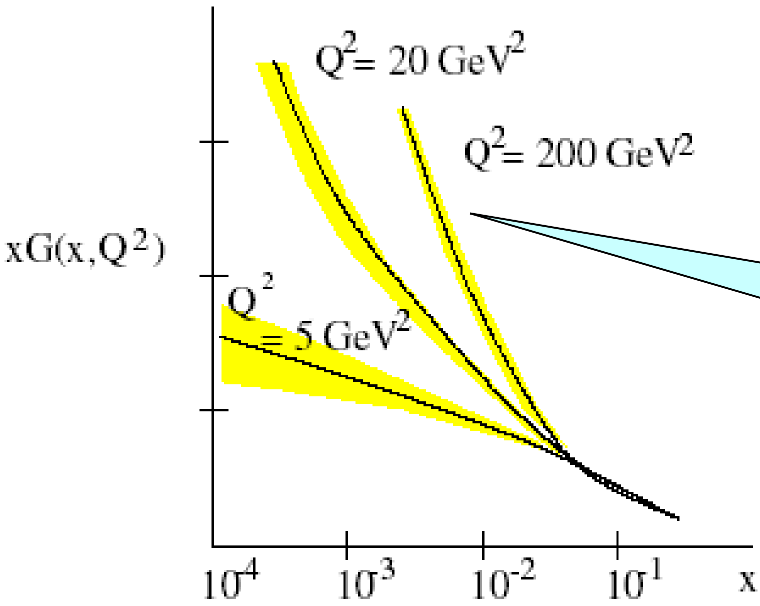
Outline

Introduction on the **Color Glass Condensate** and results on hadron multiplicity for:

- Au-Au and d-Au collisions at RHIC, $\sqrt{s_{NN}}=20\div200$ GeV
- Pb-Pb and p-Pb collisions at LHC, $\sqrt{s_{NN}}= 5500$ GeV
  - total multiplicity
  - centrality dependence
  - rapidity dependence

Hadron scattering at high energy



$$x = \frac{E_{constituent}}{E_{hadron}}$$

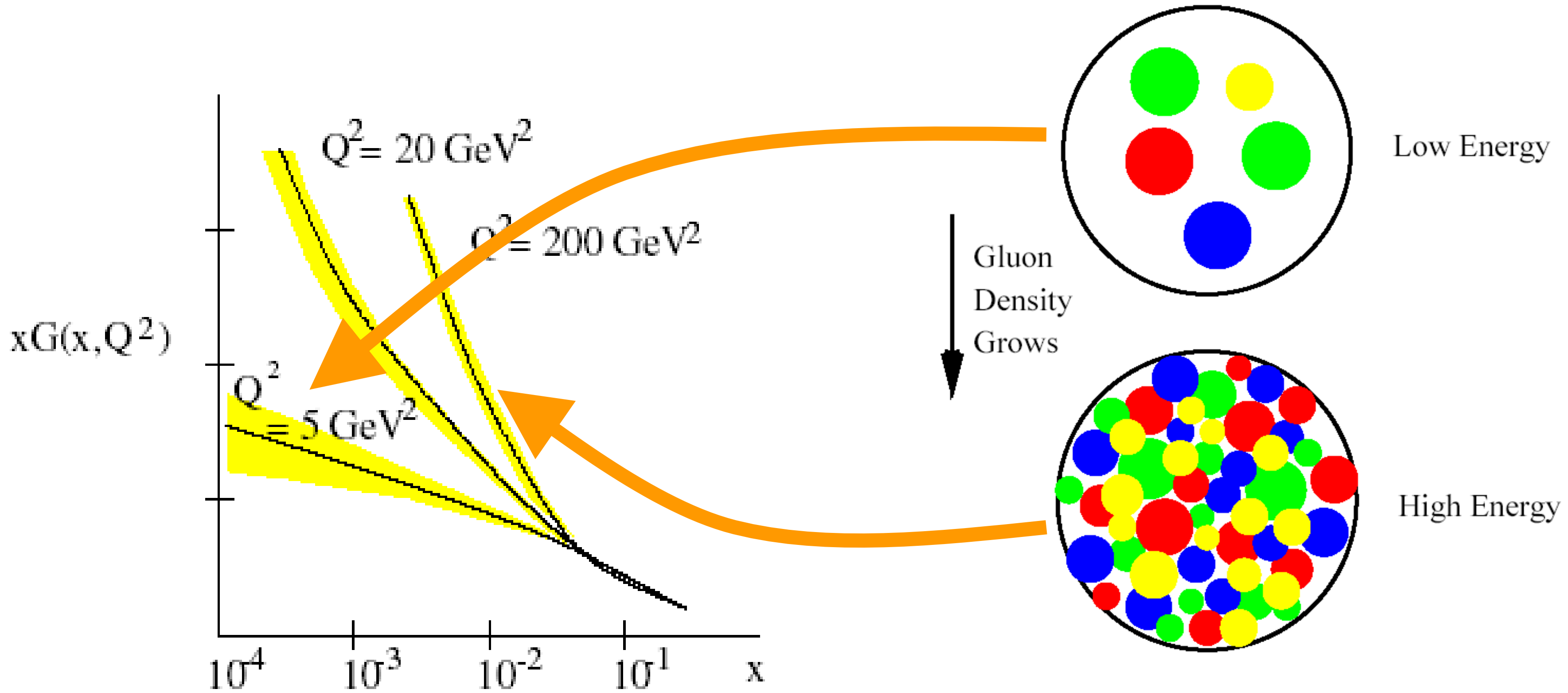
Small-x problem

New regime of QCD: α_s is small but perturbative theory is not valid, due to strong non-linear effects

A new phenomenon is expected in these conditions:

parton saturation

Gluon density in hadrons



McLerran, hep-ph/0311028

Color Glass Condensate

Classical effective theory originally proposed by McLerran and Venugopalan to describe the gluon distribution in large nuclei.

The valence quarks of the hadrons (*fast partons*) are treated as a source for a classical color field representing the small-x (*slow*) gluons. The classical approximation is appropriate since the slow gluons have large occupation numbers.

The theory implies a non-linear renormalization group equation [JIMWLK]

Color Glass Condensate

- **Color** : gluons are colored
- **Glass** : the gluons at small x are emitted from other partons at larger x. In the infinite momentum frame the fast partons are Lorentz dilated, therefore the low x gluons evolve very slowly compared to their natural time scale.
- **Condensate** : balance between gluon emission and gluon recombination : $\rho \sim \alpha_s \rho^2$, or $\rho \sim 1/\alpha_s$, (Bose condensate)

Color Glass Condensate

- Universal form of matter controlling the high energy limit of all strong interactions
- First principle description of
  - high energy cross-sections
  - parton distribution functions at small x
  - initial conditions for heavy ion collisions
  - **distribution of produced particles**

Mathematical formulation of the CGC

$$\mathcal{Z} = \int [dA][d\rho]exp\left(iS[A,\rho] - F[\rho]\right)$$

Effective theory defined below some cutoff X_0 : gluon field in the presence of an external source ρ.

The source arises from quarks and gluons with $x \geq X_0$

The weight function $F[\rho]$ satisfies renormalization group equations (theory independent of X_0).

The equation for F (JIMWLK) reduces to BFKL and DGLAP evolution equations.

Bibliography on CGC

MV Model

- McLerran, Venugopalan, Phys.Rev. D 49 (1994) 2233, 3352; D50 (1994) 2225
- A.H. Mueller, hep-ph/9911289

JIMWLK Equation

- Jalilian-Marian, Kovner, McLerran, Weigert, Phys. Rev. D 55 (1997) 5414;
- Jalilian-Marian, Kovner, Leonidov, Weigert, Nucl. Phys. B 504 (1997) 415; Phys. Rev. D 59 (1999) 014014

Saturation scale

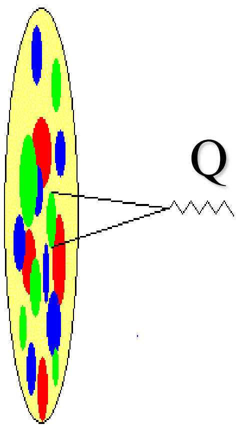
There is a critical momentum scale Q_s which separates the two regimes : ***Saturation scale***

- For $p_T < Q_s$ the gluon density is very high, they can not interact independently, their number saturates
- For $p_T > Q_s$ the gluon density is smaller than the critical one, perturbative region

The CGC approach is justified in the limit $Q_s \gg \Lambda_{QCD}$:

- ok at LHC
- ~ ok at RHIC

Saturation scale



- Boosted nucleus interacting with an external probe
- Transverse area of a parton ~ 1/Q
- Cross section parton-probe : $\sigma \sim \alpha_s/Q^2$
- Partons start to overlap when $S_A \sim N_A \sigma$
- The parton density saturates

- Saturation scale : $Q_s^2 \sim \alpha_s(Q_s^2) N_A/\pi R_A^2 \sim A^{1/3}$
- At saturation N_{parton} is proportional to $1/\alpha_s$
- Q_s^2 is proportional to the density of participating nucleons; larger for heavy nuclei.

Saturation scale

$$Q_s^2 = \frac{8\pi^2 N_c}{N_c^2 - 1}\ \alpha_s(Q_s^2)\ xG(x, Q_s^2)\ \frac{n_{part}(b)}{2}$$

[A.H. Mueller Nucl. Phys. B558 (1999) 285]

- Q_s^2 depends on the impact parameter and on the nuclear atomic number through $n_{part}(b)$
- Self-consistent solution:

$Q_S^2 = 2$ GeV^2 $xG(x, Q_S^2) = 2$ $x = 2Q_S/\sqrt{s}$

$\alpha_s = 0.6$ $b = 0$ $\sqrt{s} = 130$ GeV $|\eta| < 1$

Parton production

We assume that the number of produced particles is :

$$\left.\frac{d^2N}{d^2b\,d\eta}\right|_{|\eta|<1} = c\,\frac{N_c^2-1}{4\pi^2 N_c}\,\frac{1}{\alpha_s}\,Q_s^2$$

centrality dependence !

or

$$\left.\frac{dN}{d\eta}\right|_{|\eta|<1} = c\,N_{part}\,xG(x,Q_s^2)$$

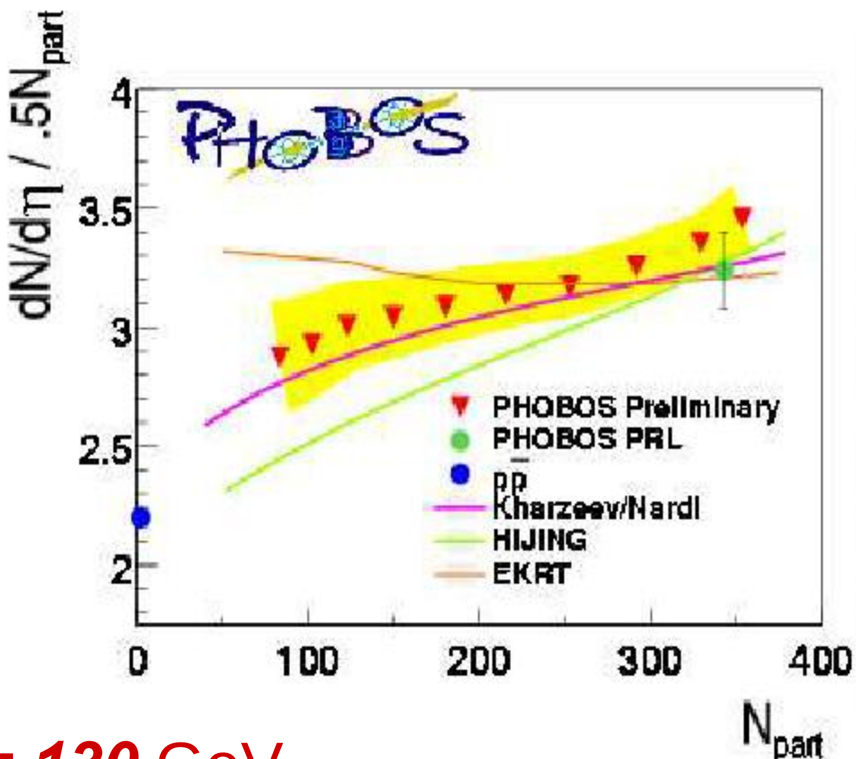
c is the "parton liberation coefficient";

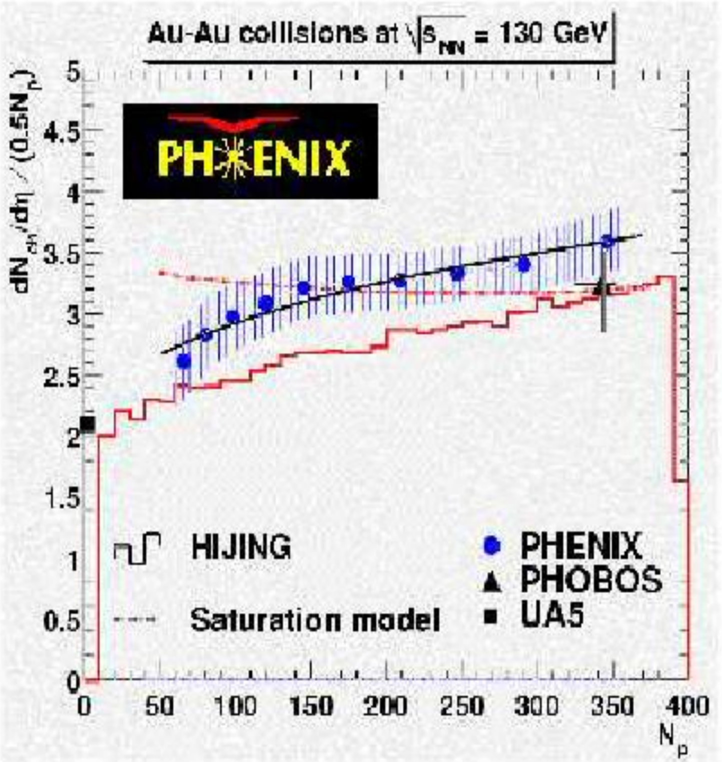
$xG(x, Q_s^2) \sim 1/\alpha_s(Q_s^2) \sim \ln(Q_s^2/\Lambda_{QCD}^2)$.

The multiplicative constant is fitted to data (PHOBOS,130 GeV, charged multiplicity, Au-Au 6% central): $c = 1.23 \pm 0.20$

First comparison to data

$dN/d\eta$ vs Centrality at $\eta=0$

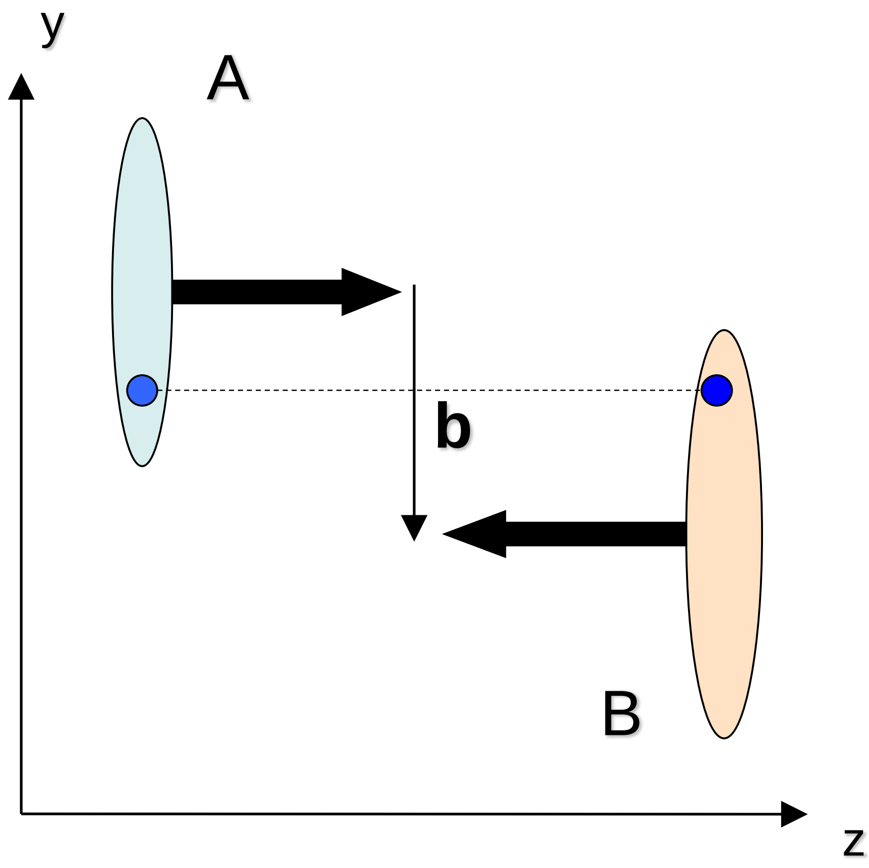




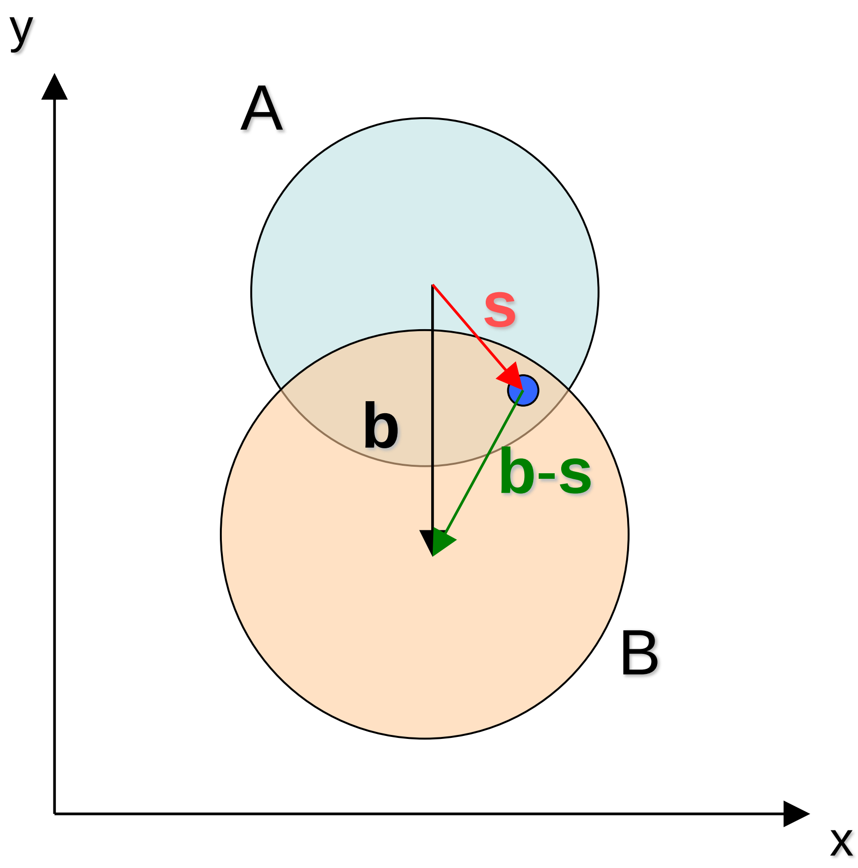
$\sqrt{s}$ ***= 130*** GeV

Number of participants: variables

Side view



Front view



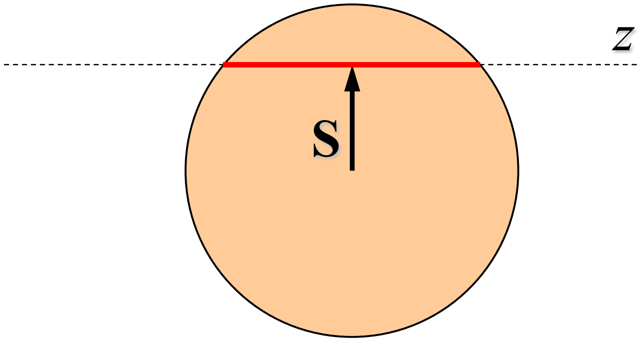
Number of participants: definitions

- Nuclear profile function (cilindrical coordinates)

- Thickness function :

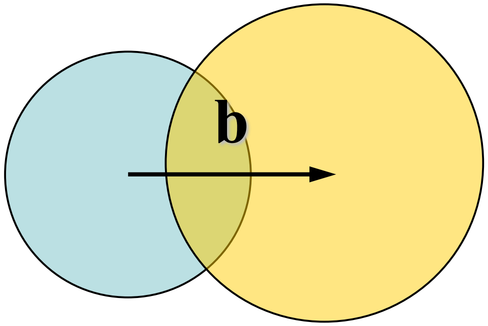
$$T_A(\mathbf{s}) = \int_{-\infty}^{+\infty} dz \; \rho_A(\mathbf{s},z)$$

norm. : $\int d\mathbf{s}\, T_A(\mathbf{s}) = 1$



- Overlap function :

$$T_{AB}(\mathbf{b}) = \int d\mathbf{s} \; T_A(\mathbf{s}) T_B(\mathbf{b}-\mathbf{s})$$



Number of participants: calculation

• Eikonal approximation: interacting nucleons do not deviate from original trajectory (early stages of A-B collision)

• Pointlike nucleons

•The number of participants (wounded nucleons) is:

$$N^{AB}_{part}(\mathbf{b}) = N^{A}_{part}(\mathbf{b}) + N^{B}_{part}(\mathbf{b}) =$$
$$= A\int ds^2 T_A(\mathbf{s})\left\{1-\left[1-\sigma_N T_B(\mathbf{b}-\mathbf{s})\right]^B\right\}+$$
$$+B\int ds^2 T_B(\mathbf{b}-\mathbf{s})\left\{1-\left[1-\sigma_N T_A(\mathbf{s})\right]^A\right\}$$

A nucleon of B at b-s

Probability of no interaction

At least one interaction

•The density is:

$$n^{AB}_{part}(\mathbf{b},\mathbf{s}) = AT_A(\mathbf{s})\left\{1-\left[1-\sigma_N T_B(\mathbf{b}-\mathbf{s})\right]^B\right\}+BT_B(\mathbf{b}-\mathbf{s})\left\{1-\left[1-\sigma_N T_A(\mathbf{s})\right]^A\right\}$$

Energy dependence

We assume the same energy dependence used to describe HERA data;

at y=0: $$Q_s^2(x) = Q_{s0}^2 \left(\frac{x}{x_0}\right)^{-\lambda} = Q_{s0}^2 \left(\frac{\sqrt{s}}{\sqrt{s_0}}\right)^{\frac{\lambda}{1+\lambda/2}}$$

with $\lambda = 0.288$ (HERA)

The same energy dependence was obtained in Nucl.Phys.B 648 (2003) 293; 640 (2002) 331; with $\lambda \sim 0.30$ [Triantafyllopoulos , Mueller]

Energy dependence/ HERA

HERA data exhibit scaling when plotted as a function of the variable

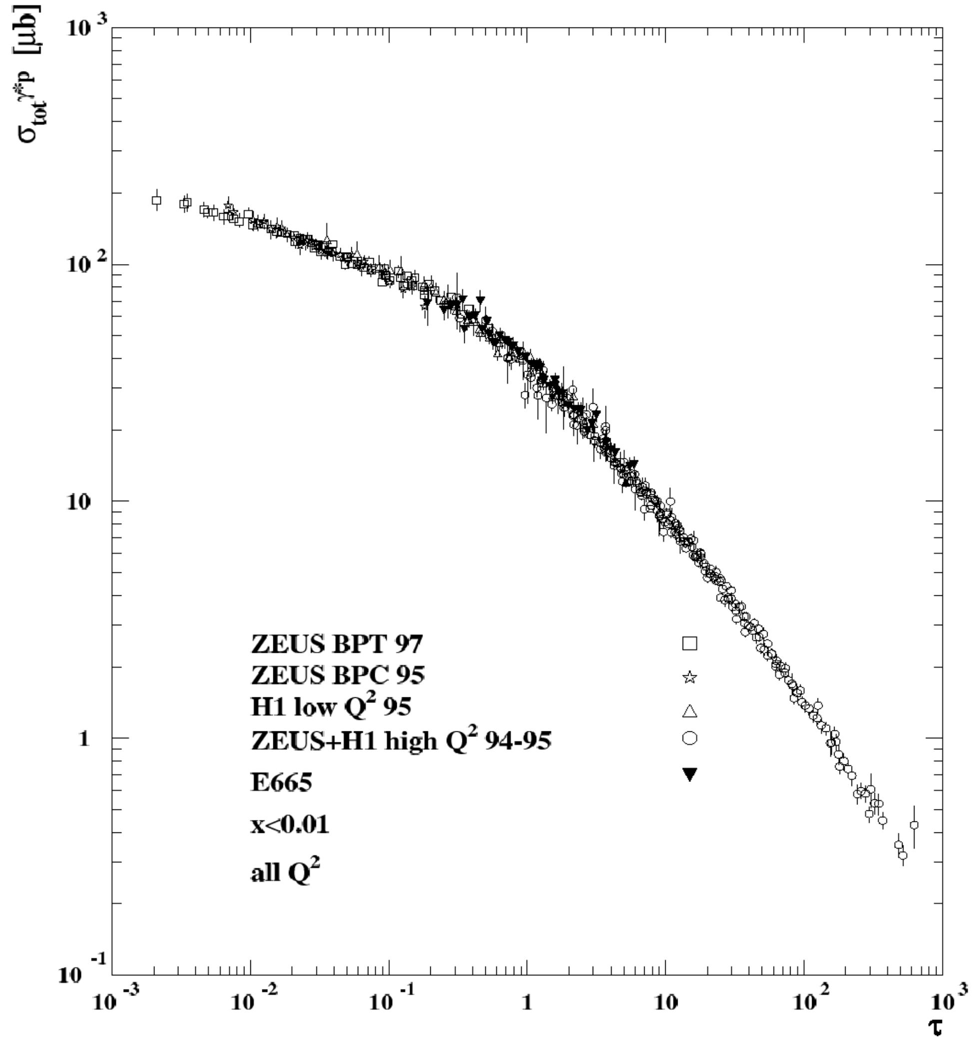
$$\tau = Q^2/Q_s^2$$

where

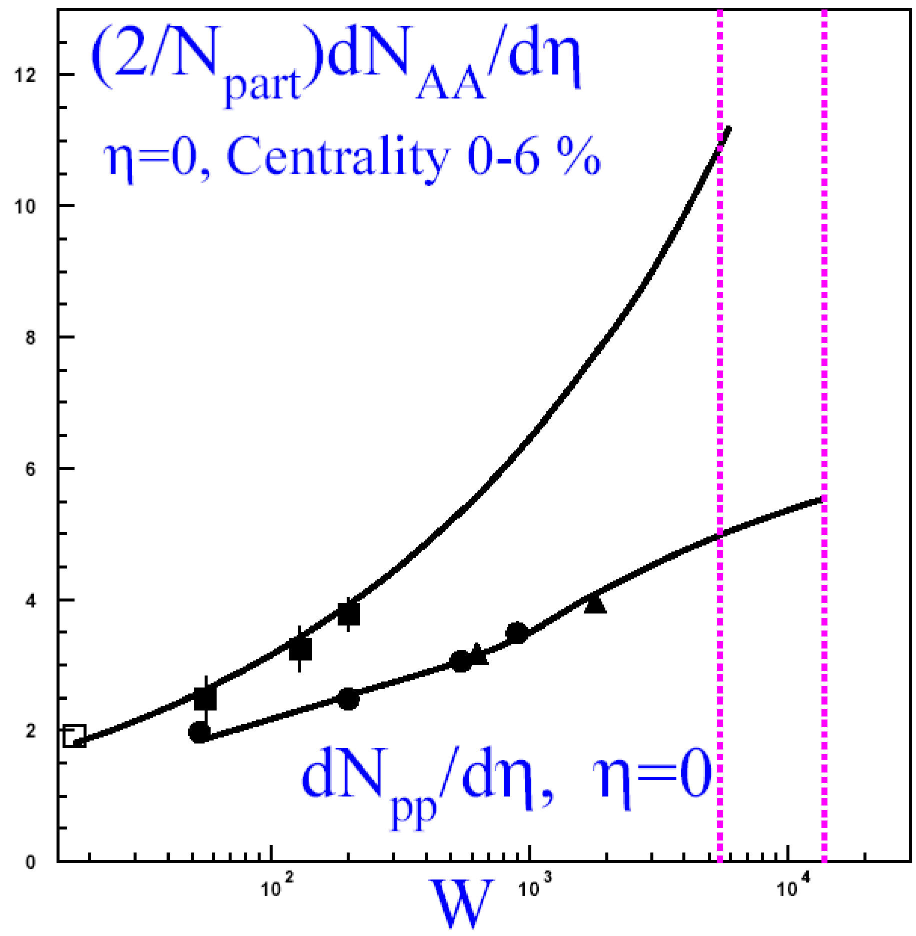
$$Q_s^2 = Q_{s0}^2 (x_0/x)^{\lambda}$$

and $\lambda \sim 0.288$

[Golec-Biernat, Wuesthoff, Phys. Rev. D59 (1999) 014017; 60 (1999) 114023]

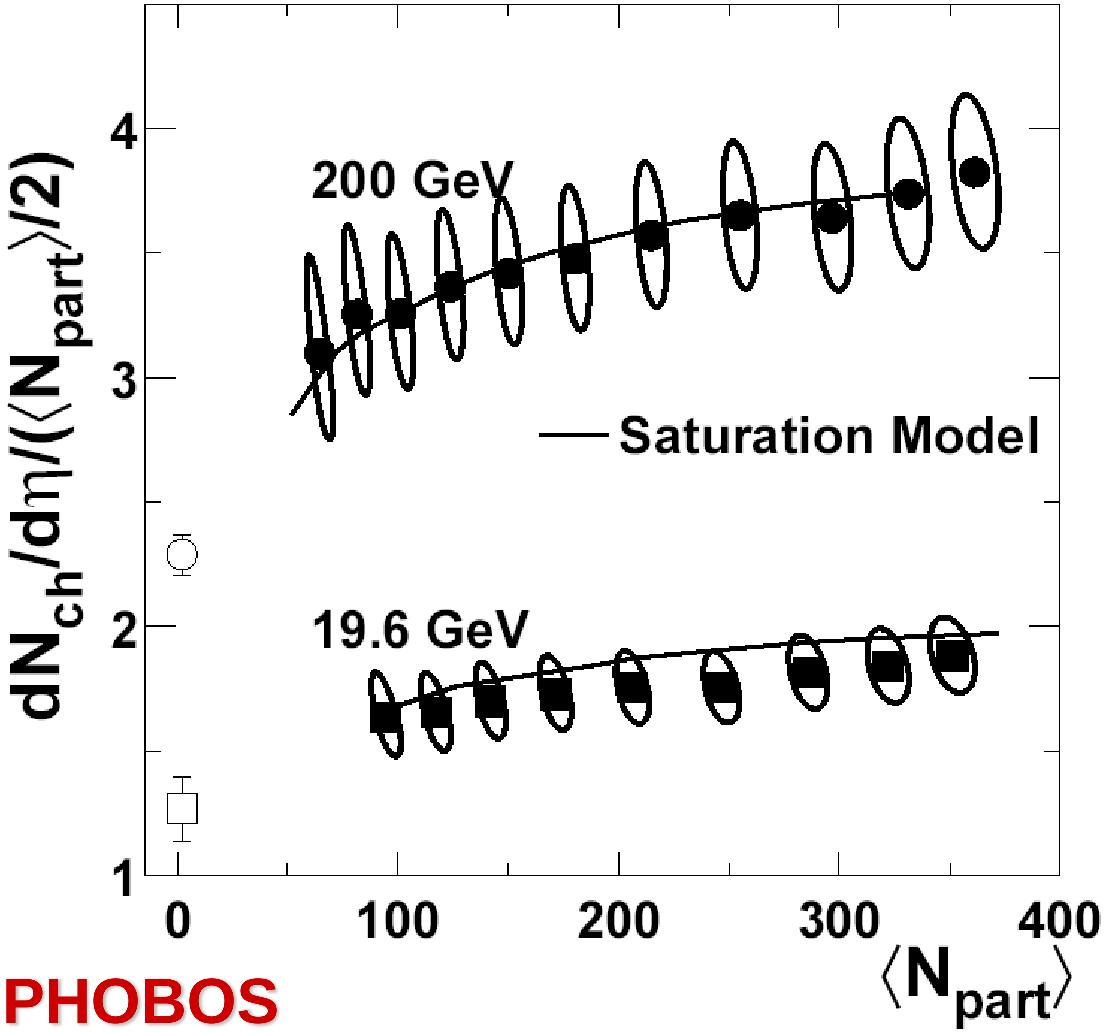


Energy dependence : pp and AA

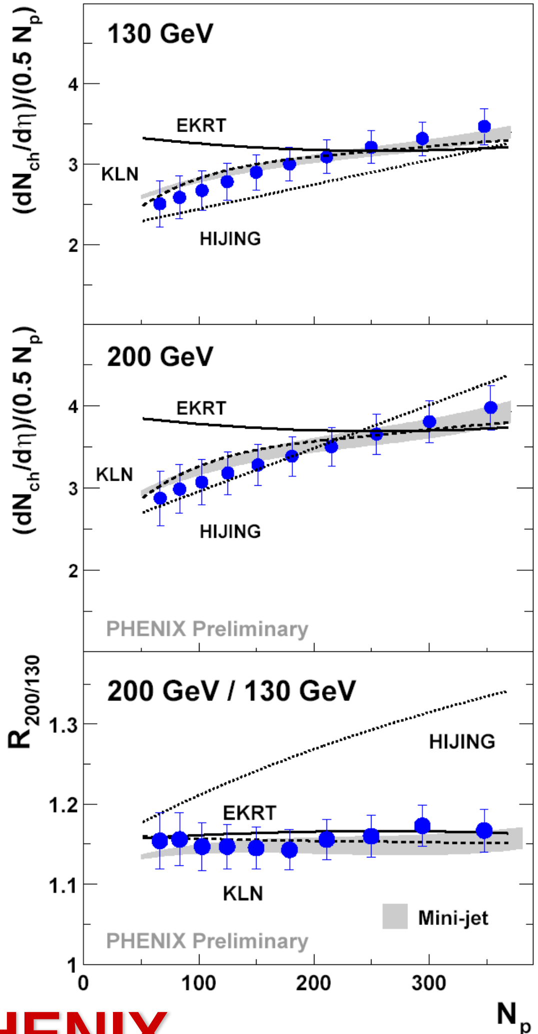


D. Kharzeev, E. Levin, M.N.
hep-ph / 0408050

Energy and centrality dependence / RHIC



PHOBOS



PHENIX

Rapidity dependence

Formula for the inclusive production:

$$E\frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2-1}\,\frac{1}{p_t^2}\;\times\int^{p_t} dk_t^2\,\alpha_s\;\varphi_{A_1}(x_1,k_t^2)\;\varphi_{A_2}(x_2,(\mathbf{p}-\mathbf{k})_t^2)$$

[Gribov, Levin, Ryskin, Phys. Rep.100 (1983),1]

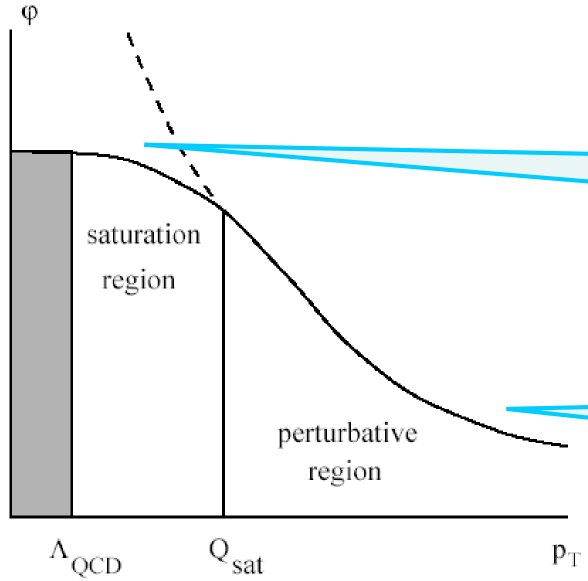
Multiplicity distribution: $$\frac{dN}{dy} = \frac{1}{S}\int d^2p_t E\frac{d\sigma}{d^3p}$$

S is the inelastic cross section for min.bias mult. (or a fraction corresponding to a specific centrality cut)

φ_A is the unintegrated gluon distribution function: $$xG(x,Q^2) = \int^{Q^2} dk_t^2\,\varphi(x,k_t)$$

Simple form of φ_A

$$\varphi_A(x,k_t^2) = \begin{cases} \kappa'\kappa \dfrac{S_A}{\alpha_s}(1-x)^4 & \text{for} \quad k_t < Q_s(x) \\ \kappa \dfrac{\alpha_s}{\pi} \dfrac{1}{k_t^2}(1-x)^4 & \text{for} \quad k_t > Q_s(x) \end{cases}$$



Saturation region: S_A/α_s

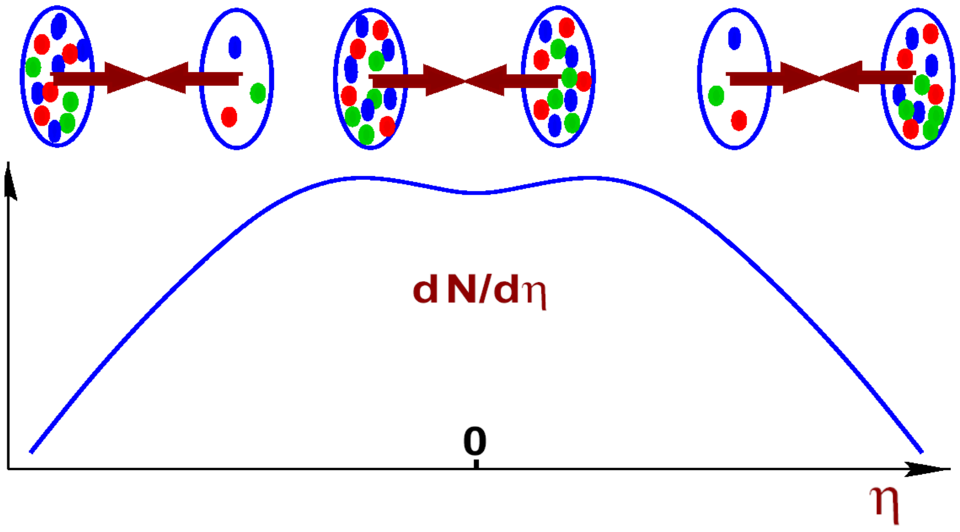
Perturbative region: α_s/p_T^2

Rapidity dependence in nuclear collisions

$x_{1,2}$ =longit. fraction of momentum carried by parton of $A_{1,2}$

At a given y there are, in general, two saturation scales:

$$Q_s^2(x_{1,2}) = Q_{s0}^2 \left(\frac{x_{1,2}}{x_0}\right)^{-\lambda} = Q_{s0}^2 \left(\frac{\sqrt{s}}{\sqrt{s_0}}\right)^{\frac{\lambda}{1+\lambda/2}} \exp\left\{\mp \frac{\lambda\, y}{1+\lambda/2}\right\}$$



$$x_1 = \frac{2Q}{\sqrt{s}}\,\mathrm{e}^{-y}$$

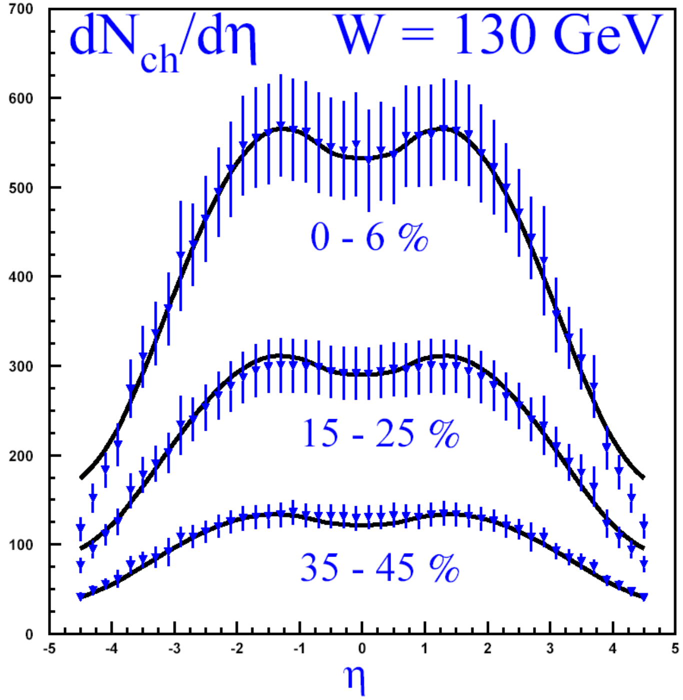
$$x_2 = \frac{2Q}{\sqrt{s}}\,\mathrm{e}^{y}$$

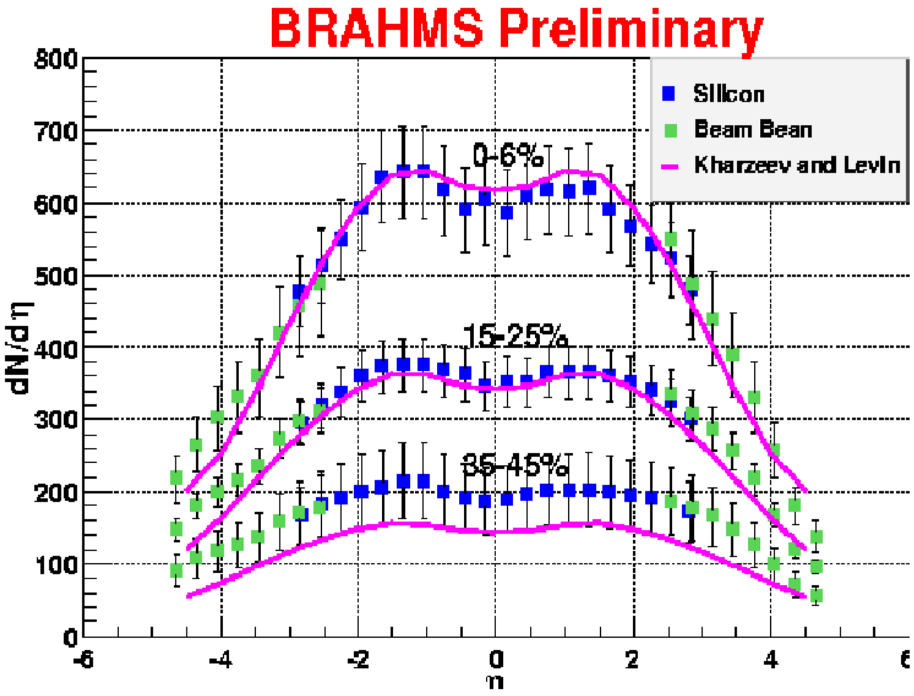
Results : rapidity dependence

Au-Au Collisions at RHIC

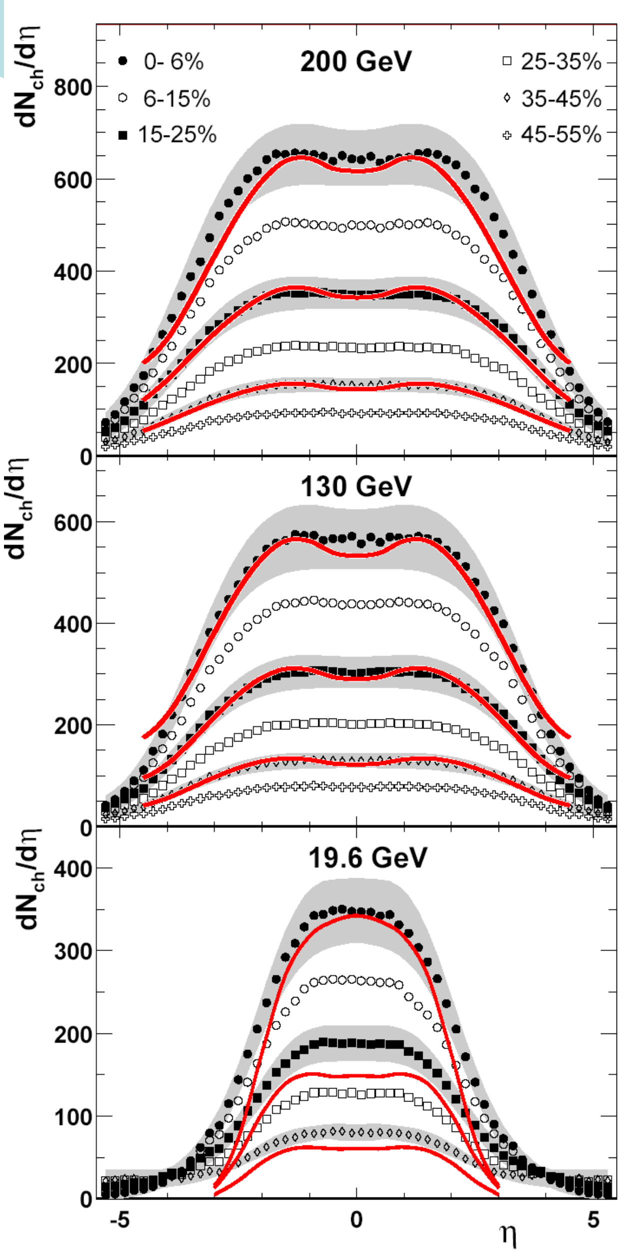
PHOBOS

W=200 GeV



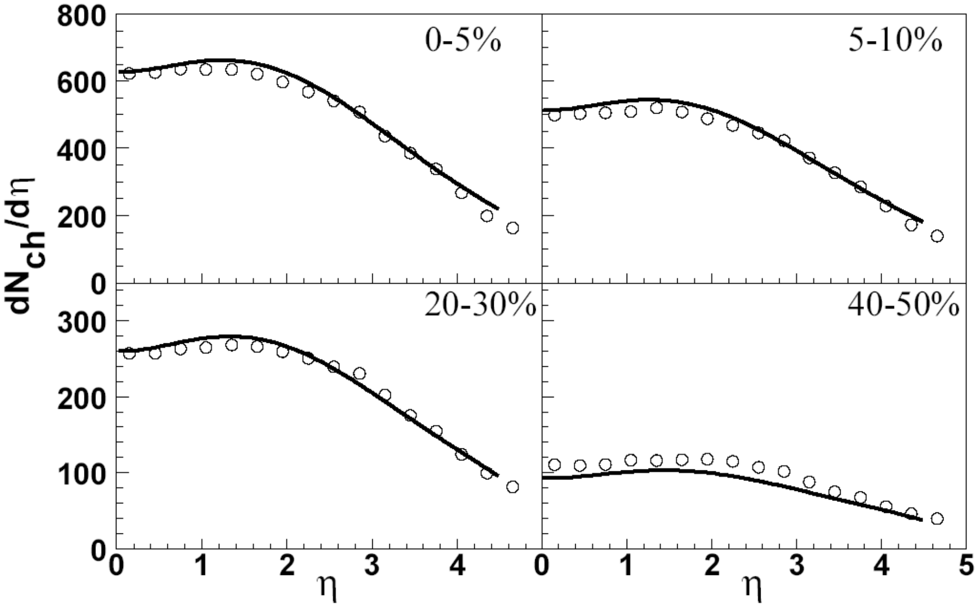


Au-Au Collisions at RHIC



PHOBOS, Phys. Rev. Lett. 91 (2003) 052303

BRAHMS (200 GeV)
Phys.Rev. Lett. 88 (2002)202301



Parton saturation at lower energies?

Is the CGC theory applicable at SPS energy ?

Condition of validity : $Q_s^2 \gg \Lambda_{QCD}^2$.

At (normal) RHIC energies: $Q_s^2 \sim 1 \div 2\ GeV^2$

Central Pb-Pb at $\sqrt{s_{NN}}=17$ GeV : $Q_s^2 \sim 1.2\ GeV^2$, comparable to peripheral (b~9 fm) Au-Au at $\sqrt{s_{NN}}=130$ GeV.

RHIC: run at $\sqrt{s_{NN}} \sim 20$ GeV, comparable to SPS energy:

test of saturation at low energy.

Test of CGC :

d-Au collisions

Deuteron wave function

$$\psi_{J_z}(\mathbf{r}) = \frac{u(r)}{r}\Phi_{1J_z0}(\Omega) + \frac{w(r)}{r}\Phi_{1J_z2}(\Omega)$$

where [Huelthen, Sugawara, "Handbuck der Physik", vol.39 (1957)]:

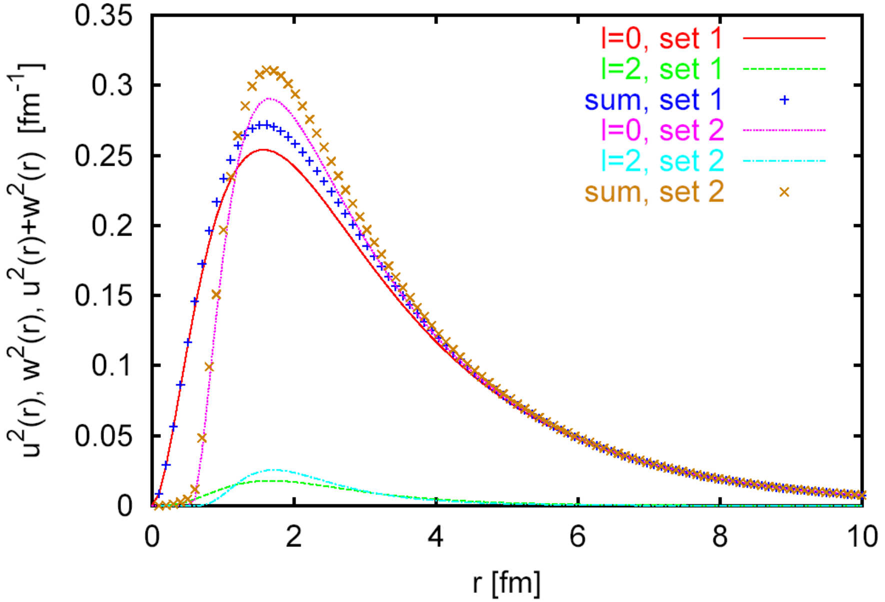
$$u(r) = N\sqrt{1-\epsilon^2}[1-\mathrm{e}^{-\beta(\alpha r - x_c)}]\,\mathrm{e}^{-\alpha r}\,\theta(\alpha r - x_c);$$

$$w(r) = N\epsilon[1-\mathrm{e}^{-\gamma(\alpha r - x_c)}]^2\mathrm{e}^{-\alpha r}\left[1 + \frac{3(1-\mathrm{e}^{-\gamma\alpha r})}{\alpha r} + \frac{3(1-\mathrm{e}^{-\gamma\alpha r})^2}{(\alpha r)^2}\right]\theta(\alpha r - x_c)$$

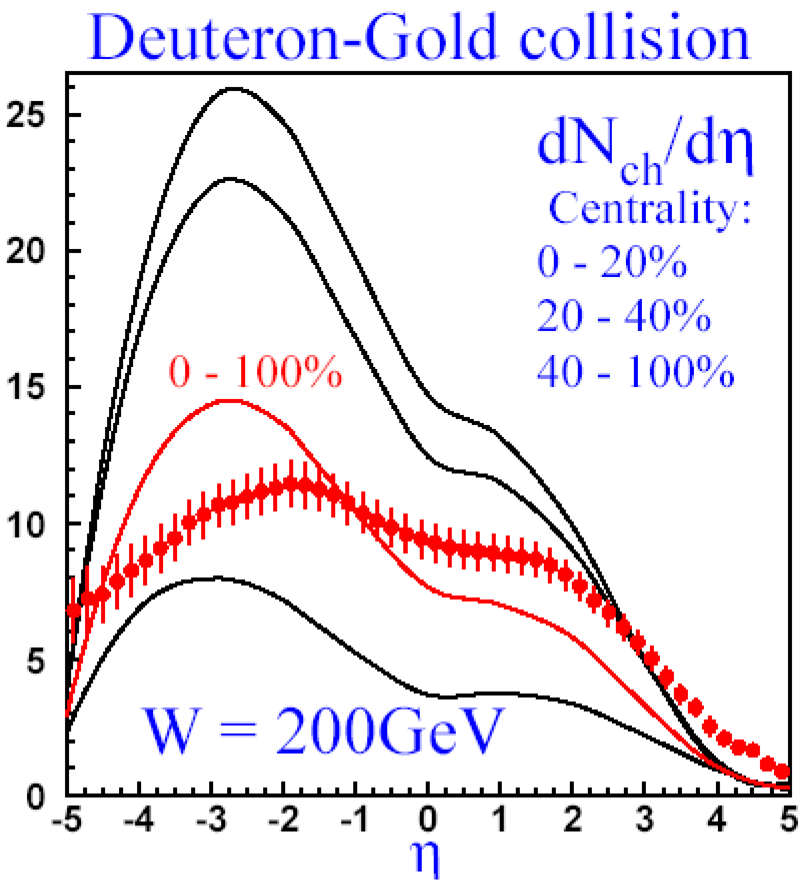
$$N^2 \equiv \frac{2\alpha}{1-\alpha\rho},$$

α is derived from the experimental binding energy:

$$\alpha = \sqrt{\frac{ME_D}{\hbar^2}} = [4.316\ \mathrm{fm}]^{-1}.$$



Predictions for d-Au



Predictions in disagreement with PHOBOS data !!!

Problems and solutions

- Present approximation not accurate for deuteron
  - **we use Monte Carlo results for N_{part}**
- proton saturation momentum more uncertain
  - **we use the same Q_{sat} as in the Golec-Biernat, Wuesthoff model**

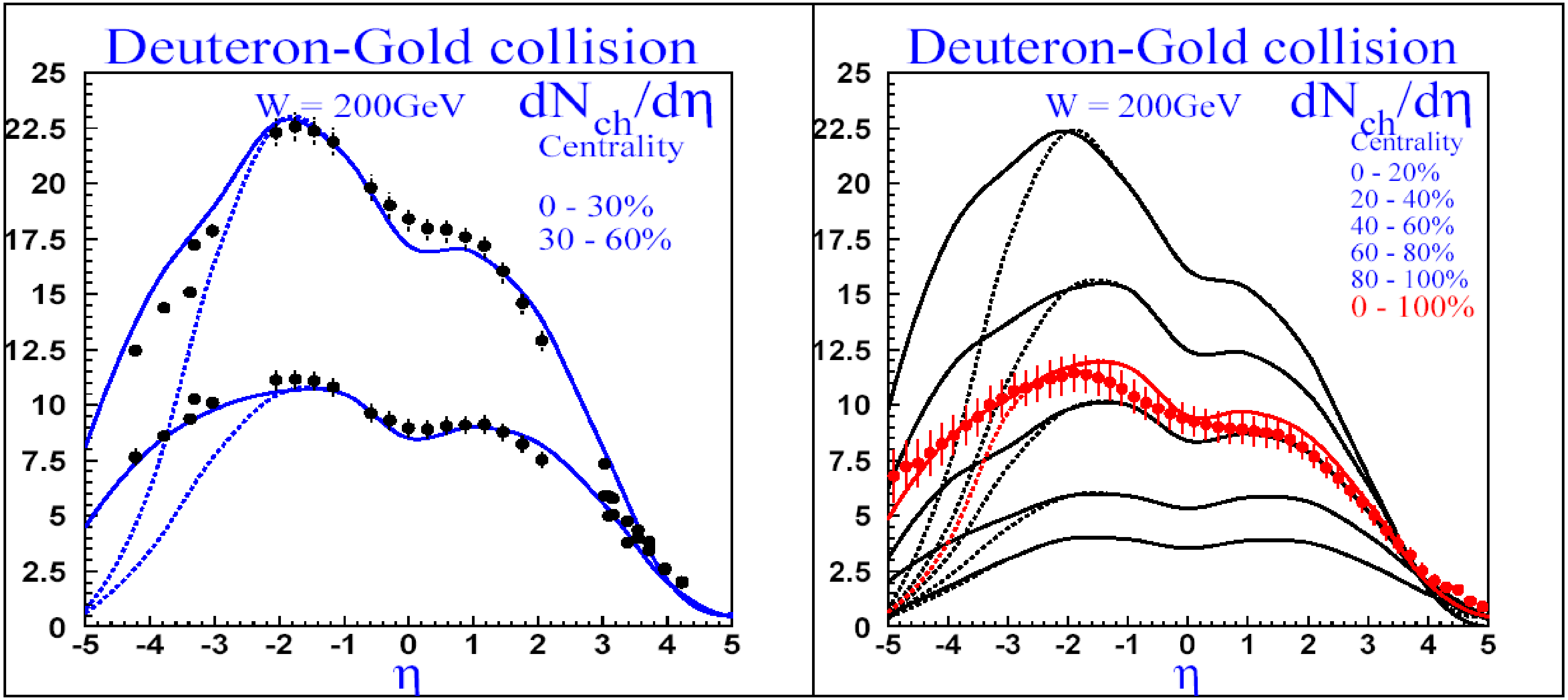
[dashed line, next plot]

- CGC not valid in the Au fragmentation region
  - **we assume $dN/d\eta = N_{part}^{Au}\, dN_{pp}/d\eta$ in the Au fragmentation region** **[solid line, next plot]**

After the corrections…

BRAHMS, nucl-ex/0401025

PHOBOS, nucl-ex/0311009



Predictions for LHC

Our main uncertainty : the energy dependence of the saturation scale.

- Fixed α_s :

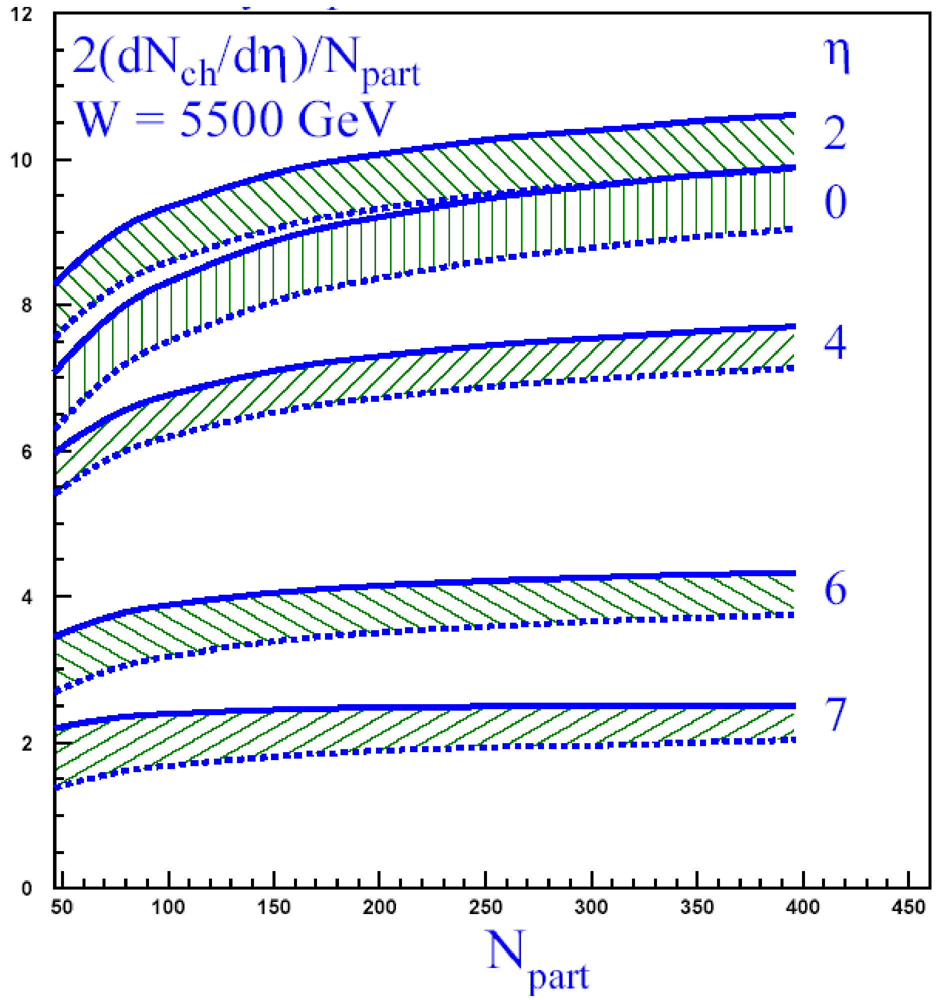
$$Q_s^2(x) = Q_{s0}^2 \left(\frac{x}{x_0}\right)^{-\lambda} = Q_{s0}^2 \left(\frac{\sqrt{s}}{\sqrt{s_0}}\right)^{\frac{\lambda}{1+\lambda/2}}$$

- Running α_s :

$$Q_s^2(W) = \Lambda_{QCD}^2 \exp\left(\sqrt{2\,\delta\,\ln(W/W_0) + \ln^2(Q_s^2(W_0)/\Lambda_{QCD}^2)}\right)$$

we give results for both cases…

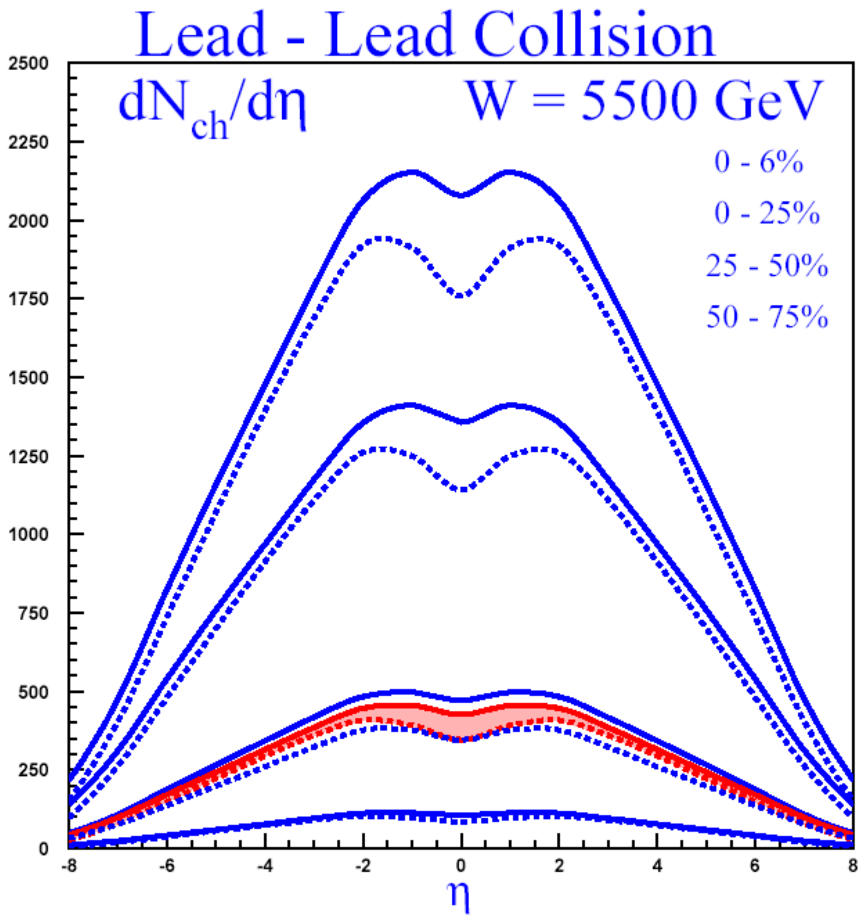
Centrality dependence / LHC

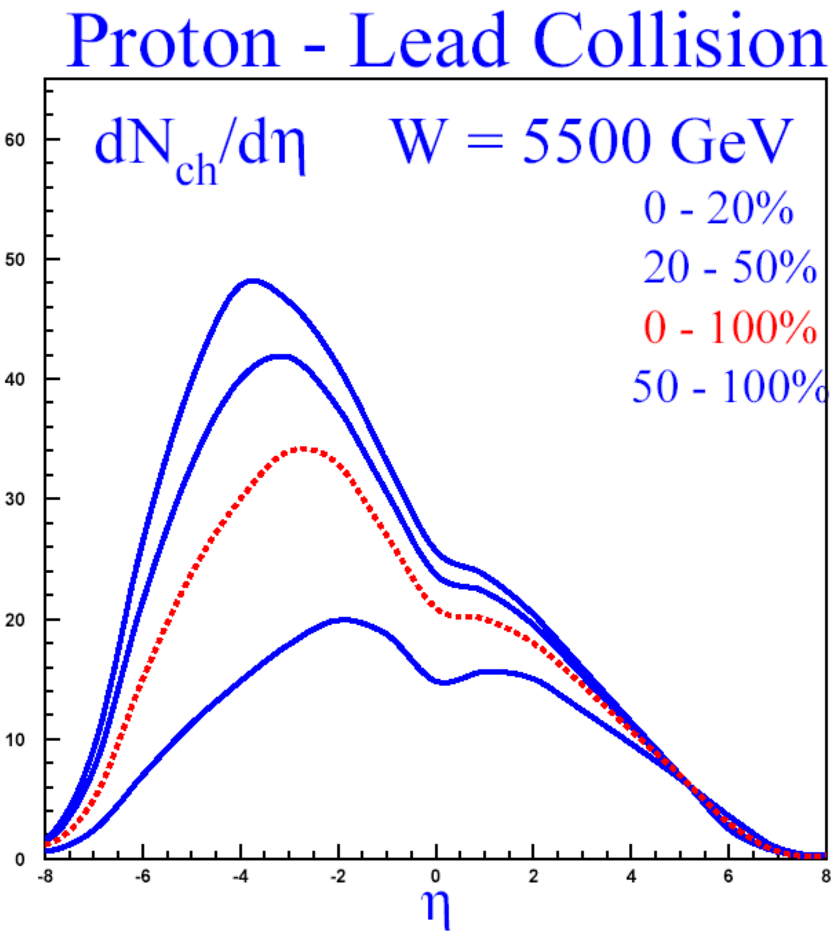


Solid lines : constant α_s
dashed lines : running α_s

Pb-Pb collisions at LHC

Pseudo-rapidity dependence

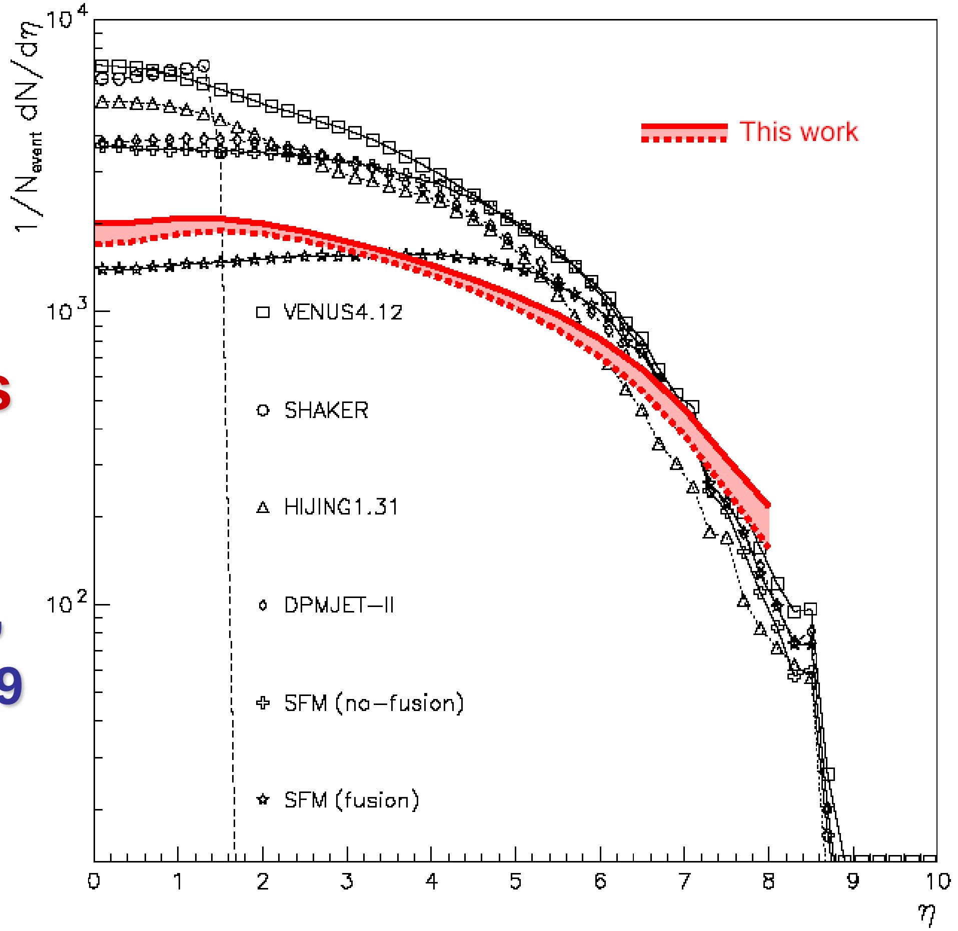




Other models…

Central Pb-Pb collisions at LHC energy

from: N. Armesto, C.Pajares, Int.J.Mod.Phys. A15(2000)2019



Conclusions

- The parton saturation model gives a reasonnable description of hadron multiplicity at RHIC for high energies (130, 200 GeV), centrality and rapidity dependence
- Lower energy collisions and different interacting systems (d-Au) useful to define its limits of applicability
- LHC will provide the best opportunity to study CGC

Thank you !

Bibliography

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D.Kharzeev, E.Levin, M.N.	Color Glass Condensate at the LHC: hadron multiplicity in pp,pA and AA coll.	hep-ph/0408050