Effective field theory approach for Fluctuations near Fermi Surface

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Problems of ChPT at finite ρ

- ChPT is an expansion scheme wrt Q/Λ_χ
  - Q: typical momentum scale, m_π, k_F
  - when k_F is large, no small scale expansion parameter
  - too large medium-modifications even at $\rho' \rho_0$ (c_1, c_2, c_3, ⋯, are unnaturally large)

$$f_\pi^t = f_\pi \,(1-0.23\ \rho/\rho_0),\ f_\pi^s = f_\pi \,(1 - 1.26\ \rho/\rho_0),$$

which means, at $\rho=\rho_0$,

$$D_F^{-1}(\omega, \mathbf{q}) = (\omega^2 - \mathbf{q}^2 - m_\pi^2)$$

$$!\ (0.77\ \omega^2 + 0.26\ \mathbf{q}^2 - m_{\pi*}^2)$$

Strategy for a new finite-density EFT

- Let's build an EFT on the Fermi surface
  - integrate out $|\mathbf{p} - k_F|$ À Λ, Λ ¿ k_F
  - chiral symmetry is still (spontaneously) broken
  - dof.s: pions and nucleons
  - LECs: ρ-dependent
  - assumption: Q ¿ k_F (okay), k_F ¿ Λ_χ (to be improved)
  - matching to χPT at some matching-density, $\rho_M < \rho_0$
- cf) D.-K. Hong's High Density ET
  - chiral symmetry is unbroken
  - d.o.f.s: (massless) quarks and gluons.
  - assumption: Q ¿ k_F, g ¿ 1.
  - matching to pQCD at some high density, ρ_M À ρ_0

Formulation

- A free Lagranian with chemical potential:

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - m_N + \mu\gamma_0\right)\psi$$

- i ∂_i' p_F, remove large momentum by defining ψ_1 as

$$\psi(x) = \sum_{\vec{v}_F} e^{i\vec{p}_F\cdot\vec{x}}\,\psi_1(x;\vec{v}_F) \qquad \vec{v}_F \equiv \frac{1}{\mu}\vec{p}_F$$

- i r= p – p_F » Q, even when p_F is large.

$$\mathcal{L} = \sum_{\vec{v}_F}\bar{\psi}_1\left(i\partial_\mu\gamma^\mu - \vec{p}_F\cdot\vec{\gamma} - m_N + \mu\gamma_0\right)\psi_1$$

- remove off-diagonal components by introducing ψ_2 as

$$\psi_1(x;\vec{v}_F) = U_F(\vec{v}_F)\,\psi_2(x;\vec{v}_F)\ , \tag{2.6}$$

where

$$U_F(\vec{v}_F) \equiv \cos\theta - \frac{\vec{v}_F\cdot\vec{\gamma}}{\bar{v}_F}\sin\theta = \cos\theta - \frac{\vec{p}_F\cdot\vec{\gamma}}{p_F}\sin\theta\ , \tag{2.7}$$

with $\bar{v}_F = |\vec{v}_F|$ and θ satisfying

$$\cos\theta = \sqrt{\frac{\mu+m_N}{2\mu}}\ , \quad \sin\theta = \sqrt{\frac{\mu-m_N}{2\mu}}\ . \tag{2.8}$$

Lagrangian (2.5) is rewritten with $\psi_2(x;\vec{v}_F)$ in Eq. (2.6) as

$$\mathcal{L} = \sum_{\vec{v}_F}\bar{\psi}_2\left[i\partial_0\gamma^0 + i\partial_j v_F^j + i\partial_j\left(\gamma_\perp^j + \frac{m_N}{\mu}\gamma_{||}^j\right) + \mu\gamma_0 - \mu\right]\psi_2\ , \tag{2.9}$$

$$\gamma_{||}^j \equiv \frac{1}{\bar{v}_F^2}v_F^j(\vec{v}_F\cdot\vec{\gamma})\ ,$$

$$\gamma_\perp^j \equiv \gamma^j - \frac{1}{\bar{v}_F^2}v_F^j(\vec{v}_F\cdot\vec{\gamma})$$

- Decompose ψ_2 into two parts, $\psi_2 = \psi_+ + \psi_-$

$$\psi_\pm(x;\vec{v}_F) \equiv P_\pm\,\psi_2(x;\vec{v}_F)\ , \qquad P_\pm \equiv \frac{1 \pm \gamma_0}{2}$$

$$\mathcal{L} = \sum_{\vec{v}_F}\Big[\bar{\psi}_+\,V_F^\mu\,i\partial_\mu\,\psi_+ + \bar{\psi}_-\left(-\tilde{V}_F^\mu\,i\partial_\mu - 2\mu\right)\psi_- + \bar{\psi}_-\left(\gamma_\perp^j + \frac{m_N}{\mu}\gamma_{||}^j\right)i\partial_j\psi_+ + \bar{\psi}_+\left(\gamma_\perp^j + \frac{m_N}{\mu}\gamma_{||}^j\right)i\partial_j\psi_-\Big]\ ,$$

where

$$V_F^\mu = (1,\vec{v}_F)\ , \qquad \tilde{V}_F^\mu = (1,-\vec{v}_F)\ .$$

- Integrate out "anti-particle" part (ψ₋) :

$$\mathcal{L} = \sum_{\vec{v}_F} \left[\bar{\psi}_+ V_F^\mu \, i\partial_\mu \, \psi_+ - \bar{\psi}_+ \frac{(\tilde{\gamma}^j \partial_j)^2}{2\mu} \sum_{n=0}^{\infty} \left(-\frac{i\tilde{V}_F^\mu \partial_\mu}{2\mu} \right)^n \psi_+ \right];$$

with $\tilde{\gamma} = \gamma_\perp + \frac{m_N}{\mu}\gamma_{||}$

- leading order propagator :

$$i\,S_F(p) = \frac{1}{-V_F \cdot p - i\epsilon p_0} P_+$$

Power counting

- Very similar to the χPT, but

nucleonic loop integrals ~ $4\pi^2 k_F^2 Q^2$ (in χPT, loops ~ $\pi^2 Q^4$)

$$\int d^4 l = \sum_{\vec{v}_F} \int d\vec{l}_\perp^2 \int dl_{||} dl_0 \sim 4\pi p_F^2 \int dl_{||} dl_0$$

$$M \sim \left(\frac{Q}{\Lambda_\chi}\right)^\nu \left(\frac{2p_F}{\Lambda_\chi}\right)^{2L_N} ,$$

$$\nu = 2 - \left(\frac{E_N}{2} + E_E\right) + 2(L - L_N) + \sum_i \nu_i ,$$

$$\nu_i \equiv d_i + \frac{n_i}{2} + e_i - 2 .$$

Building blocks

- ξ=exp(i π/F_π^t), $\pi = \sum_{a=1}^3 \pi^a T^a$

  ξ ! $\xi' = h\, \xi\, g_R^y = g_L\, \xi\, h^y$, $h = h(\pi, g_L, g_R)$
- covariant derivatives of pion

$$\alpha_A^\mu \equiv \frac{1}{2i}\left(\mathcal{D}^\mu\xi\cdot\xi^\dagger - \mathcal{D}^\mu\xi^\dagger\cdot\xi\right)$$

$$\alpha_V^\mu \equiv \frac{1}{2i}\left(\mathcal{D}^\mu\xi\cdot\xi^\dagger + \mathcal{D}^\mu\xi^\dagger\cdot\xi\right)$$

$$\alpha_A^\mu \to h(\pi, g_\mathrm{R}, g_\mathrm{L})\, \alpha_A^\mu\, h^\dagger(\pi, g_\mathrm{R}, g_\mathrm{L})\ ,$$

$$\alpha_V^\mu \to h(\pi, g_\mathrm{R}, g_\mathrm{L})\, \alpha_V^\mu\, h^\dagger(\pi, g_\mathrm{R}, g_\mathrm{L}) - \frac{1}{i} h(\pi, g_\mathrm{R}, g_\mathrm{L})\, \partial^\mu\, h^\dagger(\pi, g_\mathrm{R}, g_\mathrm{L})$$

- Ψ ! $h(\pi, g_L, g_R)\, \Psi$
- covariant derivatives of nucleon

$$D_\mu \Psi = (\partial_\mu - i\alpha_{V\mu})\, \Psi\ ,$$

Leading order Lagrangian

- pion sector:

$$\mathcal{L}_{A0} = \left[\left(F_\pi^t\right)^2 u_\mu u_\nu + \left(F_\pi^t F_\pi^s\right)(g_{\mu\nu} - u_\mu u_\nu) \right] \mathrm{tr}\,[\alpha_A^\mu \alpha_A^\nu] \;, \quad u_\mu = (1, \vec{0})$$

- nucleon's kinetic term

$$\mathcal{L}_{\mathrm{kin}} = \sum_{\vec{v}_F} \bar{\Psi}\, V_F^\mu\, i D_\mu \Psi$$

- πNN term

$$\mathcal{L}_A = \sum_{\vec{v}_F} \left[\kappa_{A0} \bar{\Psi} (\vec{v}_F \cdot \vec{\gamma}) \gamma_5 \alpha_A^0 \Psi + i \kappa_{A||} \bar{\Psi} \gamma_{||}^i \gamma_5 \alpha_{Ai} \Psi + \kappa_{A\perp} \bar{\Psi} \gamma_\perp^i \gamma_5 \alpha_{Ai} \Psi \right]$$

$$g_A \bar{\psi} \gamma^\mu \gamma_5 \alpha_{A\mu} \psi$$

$$\kappa_{A0} = \kappa_{A||} = g_A \;, \quad \kappa_{A\perp} = \frac{m_N}{\mu} g_A$$

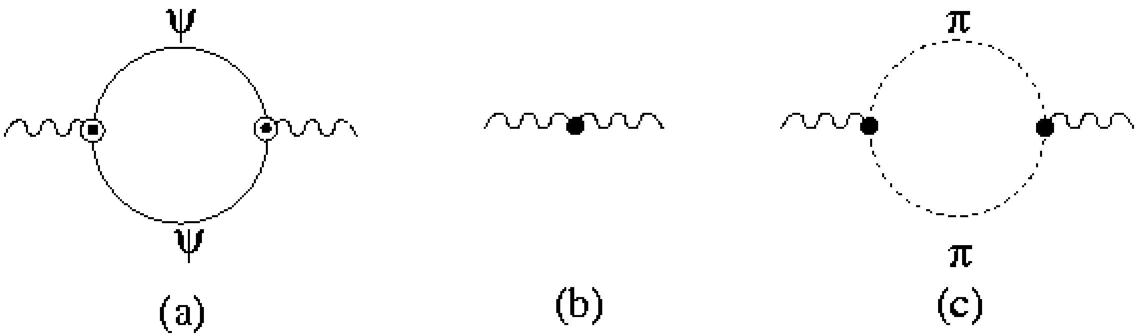
cf) axial-charge term has been promoted to LO from NLO

Leading order Lagrangian

- 4-nucleon contact terms :

$$\mathcal{L}_{4F} = \frac{F_S}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\Psi\right)^2 + \frac{F_A}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\gamma_\mu\gamma_5\Psi\right)^2$$
$$+ \frac{F_T}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}V_F^\mu\gamma_\mu\gamma_5\Psi\right)^2 + \frac{G_S}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\vec{\tau}\Psi\right)^2$$
$$+ \frac{G_A}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}\gamma_\mu\gamma_5\vec{\tau}\Psi\right)^2 + \frac{G_T}{(F_\pi^t)^2}\left(\sum_{\vec{v}_F}\bar{\Psi}V_F^\mu\gamma_\mu\gamma_5\vec{\tau}\Psi\right)^2$$

VV correlator



- (a): ν=0, L_N=1, (b,c): ν=2, L_N=0 .

$$\delta_{ab}\Pi_{\mathcal{V}}^{(1)\mu\nu}(p_0,\vec{p}) = -\mathrm{tr}[T_aT_b]\sum_{\vec{v}_F} 2V_F^\mu V_F^\nu \int \frac{d^4l}{i(2\pi)^4}\frac{1}{[-V_F\cdot(l-\eta_1 p)-i\epsilon(l_0-\eta_1 p_0)]}$$
$$\times\frac{1}{[-V_F\cdot(l+\eta_2 p)-i\epsilon(l_0+\eta_2 p_0)]}\,, \qquad (5.3)$$

$$\Pi_V^{(1)\mu\nu}(p_0,\vec{p}) \equiv u^\mu u^\nu \Pi_V^t(p_0,\vec{p}) + (g^{\mu\nu} - u^\mu u^\nu)\Pi_V^s(p_0,\vec{p}) + P_L^{\mu\nu}\Pi_V^L(p_0,\vec{p}) + P_T^{\mu\nu}\Pi_V^T(p_0,\vec{p}) \ ,$$

$$P_L^{\mu\nu} = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) - P_T^{\mu\nu} \ ,$$

$$P_T^{\mu\nu} = g_i^\mu \left(\delta_{ij} - \frac{\vec{p}_i \vec{p}_j}{\vec{p}^2}\right) g_j^\nu \ ,$$

$$\Pi_V^t(p_0,\bar{p}) = 0 \ ,$$

$$\Pi_V^s(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{6\pi^2} \ ,$$

$$\Pi_V^L(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{2}\frac{p^2}{\vec{p}^2} X(\bar{v}_F \bar{p}/p_0) \ ,$$

$$\Pi_V^T(p_0,\bar{p}) = -\frac{p_F^2 \bar{v}_F}{4\pi^2} + \frac{p_F^2}{4\bar{v}_F}\frac{p_0^2 - \bar{v}_F^2 \vec{p}^2}{\vec{p}^2} X(\bar{v}_F \bar{p}/p_0) \ ,$$

$$X(r) \equiv \frac{1}{\pi^2}\left[-1 + \frac{1}{2r}\ln\left|\frac{1+r}{1-r}\right| + i\pi\theta(r-1)\right]$$

$$V(r) = q_1 q_2 \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p}\cdot\vec{r}}}{\vec{p}^2 + \Pi_V^{00}(p_0 \to 0, \vec{p}^2)} \, ,$$

$$m_D^2 = e^2 \frac{\mu p_F}{\pi^2}$$

$$\mathcal{L}_{CT} = \frac{p_F^2}{4\pi^2 \bar{v}_F} \int \frac{d\Omega_{\vec{v}_F}}{4\pi} \left(\delta_{ij} - \frac{v_F^i v_F^j}{\bar{v_F}^2} \right) \mathrm{tr}\; [\alpha_V^\mu \alpha_V^\mu]$$

$$= \frac{p_F^2}{6\pi^2 \bar{v}_F} (g_{\mu\nu} - u_\mu u_\nu) \mathrm{tr}\, [\alpha_V^\mu \alpha_V^\nu] \ .$$

$$\Pi_V^{(1)\mu\nu}(p_0,\vec{p}) = -\sum_{\vec{v}_F} V_F^\mu V_F^\nu \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \frac{1}{2\pi\bar{v}_F} \frac{\vec{v}_F\cdot\vec{p}}{V_F\cdot p + i\epsilon p_0}$$

$$\sum_{\vec{v}_F} \int \frac{d^2\vec{l}_\perp}{(2\pi)^2} \Rightarrow \frac{p_F^2}{\pi} \int \frac{d\Omega_{\vec{v}_F}}{4\pi}$$

$$\Pi_V^{(1)\mu\nu}(p_0,\vec{p}) = -V_F^\mu V_F^\nu \frac{p_F^2}{2\pi^2\bar{v}_F} \int \frac{d\Omega_{\vec{v}_F}}{4\pi} \frac{\vec{v}_F\cdot\vec{p}}{V_F\cdot p + i\epsilon p_0}$$

AA correlator

$$[f_\pi^t]^2 = (F_\pi^t)^2 + \frac{p_F}{2\bar{v}_F}\left(\kappa_{A0}^2 \bar{v}_F^2 - \kappa_{A0}\kappa_{A||}\right) \mathrm{Re}[X(\bar{v}_F/V_\pi)] \ ,$$

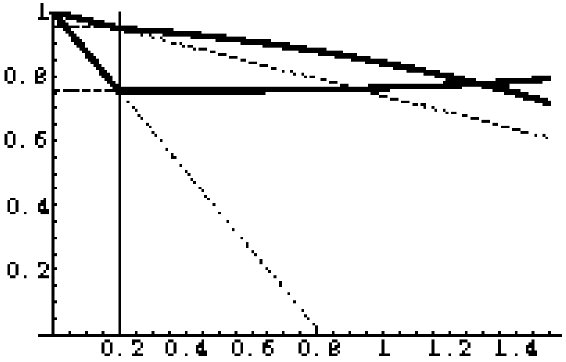
$$f_\pi^t f_\pi^s = F_\pi^t F_\pi^s + \frac{p_F^2}{6\pi^2 \bar{v}_F}\left(\kappa_{A||}^2 - \kappa_{A\perp}^2\right) + \frac{p_F^2}{2\bar{v}_F}\left\{V_\pi^2\left(\kappa_{A0}\kappa_{A||} - \frac{\kappa_{A||}^2 - \kappa_{A\perp}^2}{\bar{v}_F^2}\right) - \kappa_{A\perp}^2\right\} \mathrm{Re}[X(\bar{v}_F/V_\pi)]$$

- Matching to ChPT 1-loop results:

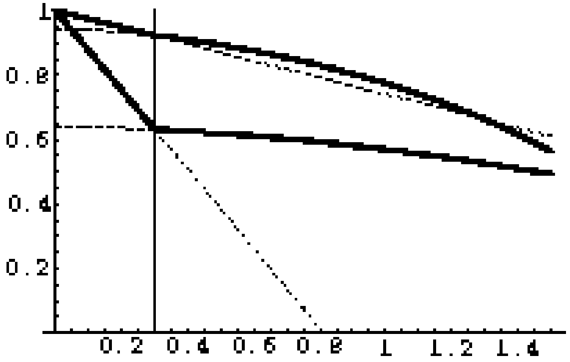
$$\kappa_{A0} = \kappa_{A||} = g_A \ , \quad \kappa_{A\perp} = \frac{m_N}{\mu} g_A$$

$$(F_\pi^t, F_\pi^s) = \begin{cases} (88.3\,\mathrm{MeV},\ 69.6\,\mathrm{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.2\,, \\ (87.1\,\mathrm{MeV},\ 59.1\,\mathrm{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.3\,, \\ (87.3\,\mathrm{MeV},\ 50.6\,\mathrm{MeV}) & \text{for} \quad \rho_M/\rho_0 = 0.4\,. \end{cases}$$

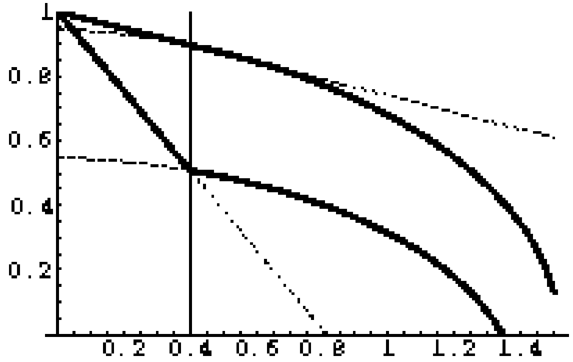
Results (f_π^t/f_π & f_π^s/f_π)



$\rho_M = 0.2\ \rho_0$

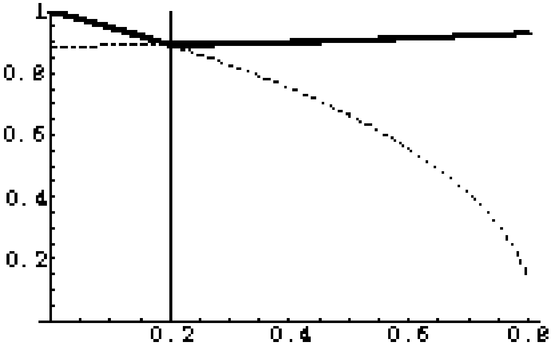


$\rho_M = 0.3\ \rho_0$

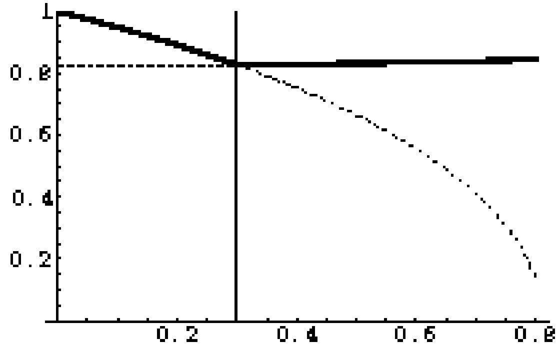


$\rho_M = 0.4\ \rho_0$

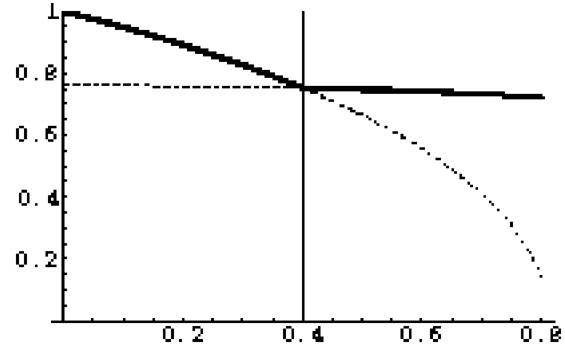
Results (v_π)



ρ_M=0.2 ρ_0



ρ_M=0.3 ρ_0

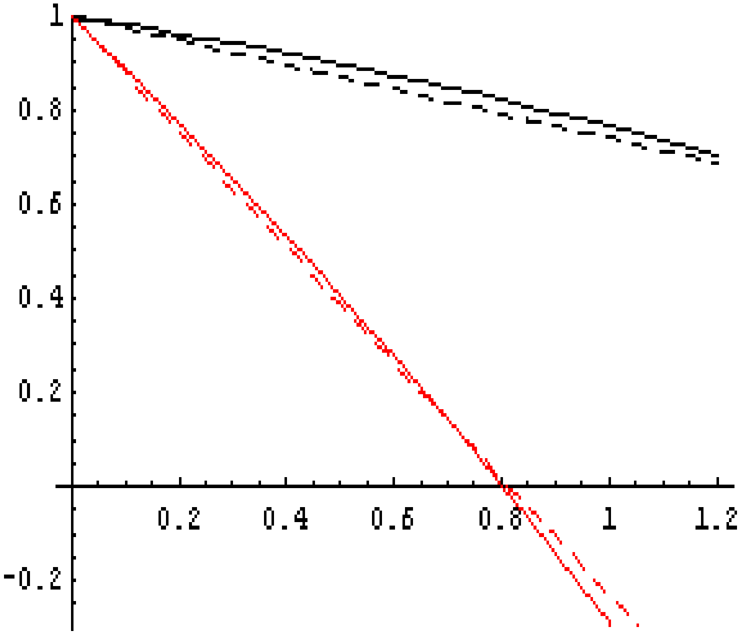


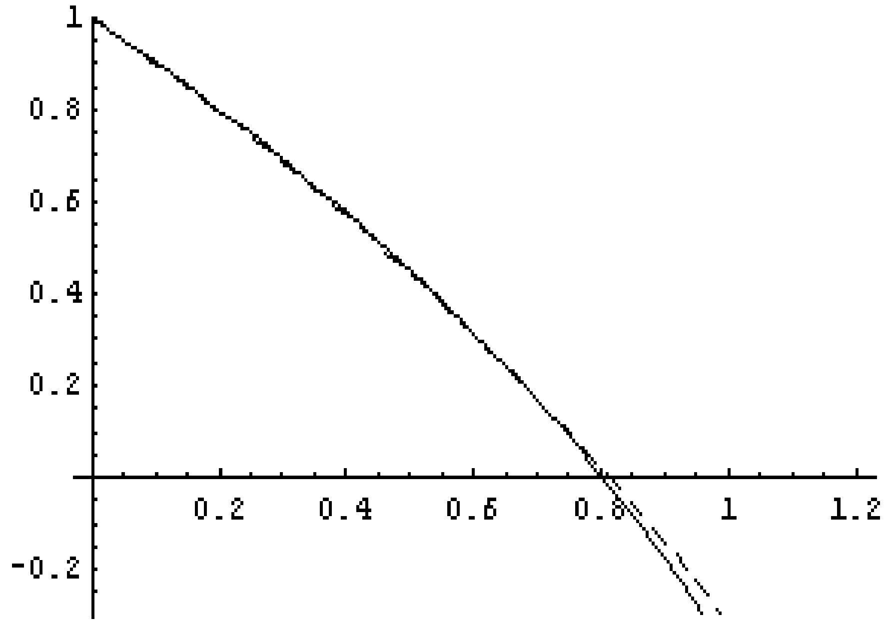
ρ_M=0.4 ρ_0

Discussions/Problems/Future works

- Resummation scheme for all orders in (p_F/Λ_χ) is needed.
- Requires input from ChPT, progresses in ChPT at finite-density are needed.
- How to treat density-dependences of LECs ?
- Search for applications.

Results ($f_\pi^{t,s}/f_\pi$ (Δ_0=10 MeV), $v_\pi^2=f_\pi^s/f_\pi^t$)





dotted: NLO (1-loop)

sold : NNLO (2-loop)

unfortunately, 2-loop contributions are neglegible ... ...