Spontaneous isospin violation due to kaon condensate and generalized Goldstone theorem

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(hep-ph/0511024) ,

Y. Kim, " Goldstone bosons in a kaon-condensed matter," work in progress

Contents

- $\langle \bar{q}q \rangle$ in dense matter
- Kaon condensate vs. quark condensate in a chiral quark model
- Ferromagnet & antiferromagnet
- GBs in a kaon condensed phase
- Summary & more to go

Quark condensate in free space

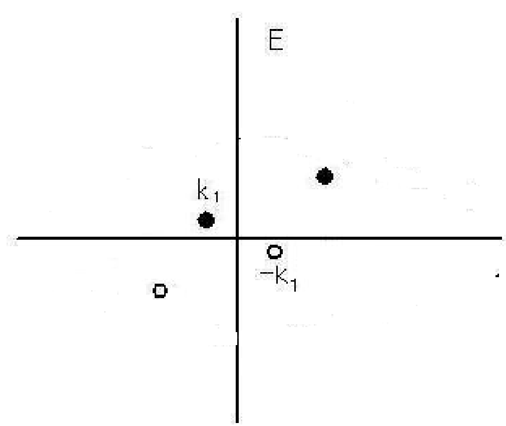
- A BCS-like trial ground state

$$|\Omega\rangle \approx \prod [1 + f(p)\,\epsilon\, b^{\dagger}(\vec{p})\, d^{\dagger}(-\vec{p})]|0\rangle$$

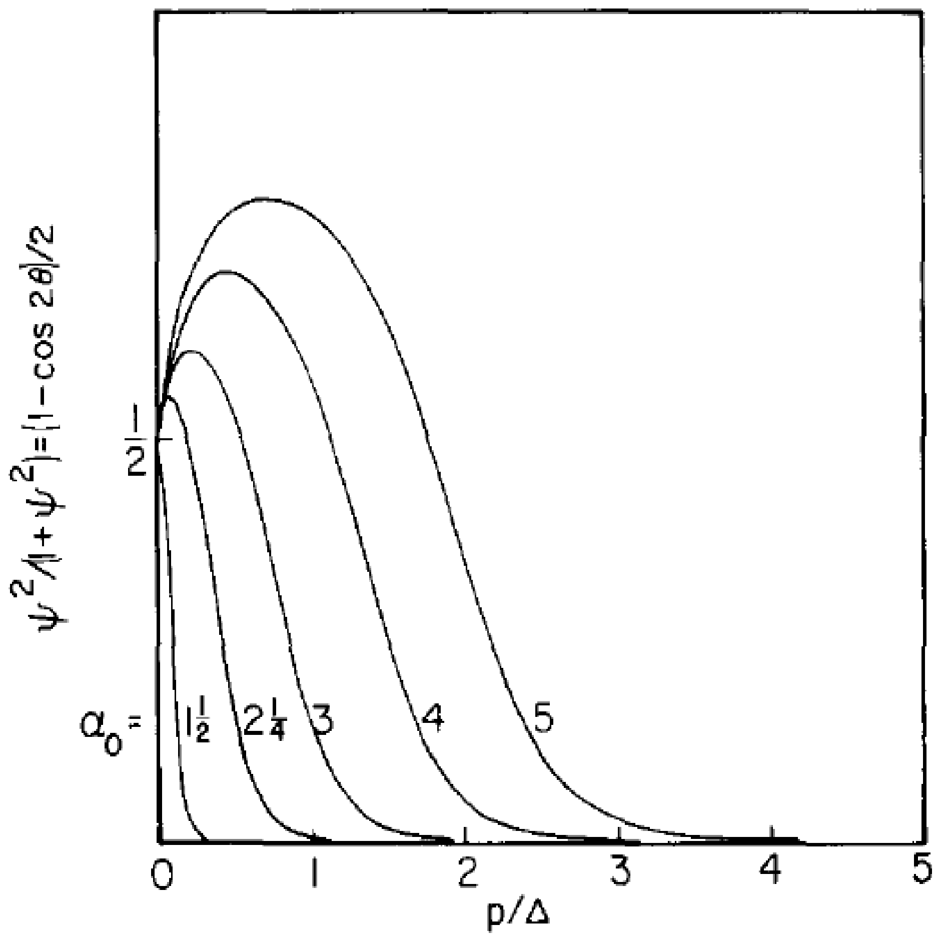
[Finger & Mandula, NPB 199, 168 (1982)]

$$\alpha = \frac{g^2}{4\pi} > \frac{9}{8} \rightarrow E_{<\bar{q}q>} < 0$$

->true vacuum contains the condensates



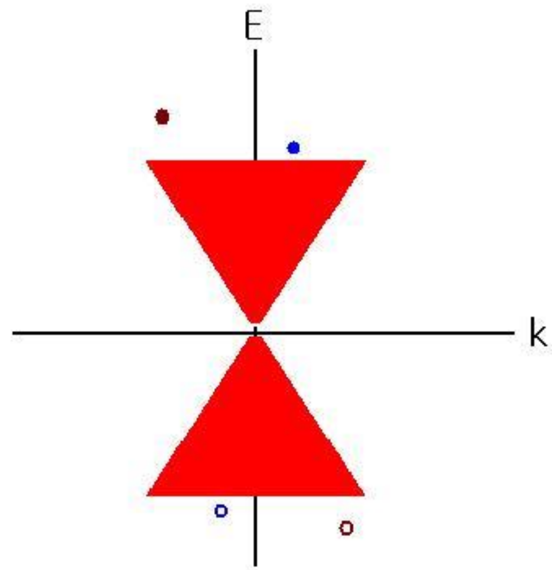
[Finger & Mandula, NPB 199, 168 (1982)]



Graph of the condensate function $\Psi^2(p)/(1+\Psi^2(p)) = \frac{1}{2}(1-\cos 2\theta(p)) = \sin^2\theta(p)$ for values of $\alpha_0 = 1.5,\ 2.25,\ 3.0,\ 4.0,\ 5.0$.

$$\alpha(\boldsymbol{p}-\boldsymbol{q}) = \frac{\alpha_0 \Delta^2}{\Delta^2 + (\boldsymbol{p}-\boldsymbol{q})^2}$$

Quark condensate in matter



$$< \bar{q}q > \approx - \int_0^\infty dp \frac{m}{E_p} [1 - \theta(k_F - p)]; \quad \bar{q}q = (\bar{u}u + \bar{d}d)/2$$

(NJL model)

Quark-antiquark condensate at low density

$$\langle \bar{q}_f q_f \rangle = \frac{\mathrm{Tr}\,\bar{q}_f q_f\, e^{-\beta(H-\mu N)}}{\mathrm{Tr}\, e^{-\beta(H-\mu N)}}$$

$$H_{QCD} = \bar{H}_{QCD} + \int dx\, m(\bar{u}(x)u(x) + \bar{d}(x)d(x))$$

$$e^{-\beta\Omega} = \mathrm{Tr}\, e^{-\beta(H_{QCD}-\mu N)}, \quad \frac{\Omega}{V} = \epsilon_0 + \epsilon$$

$$\langle \bar{u}u + \bar{d}d \rangle = \frac{\partial}{\partial m}(\epsilon_0 + \epsilon) \quad \to \quad \langle \bar{q}q \rangle_{\rho_B} = \langle \bar{q}q \rangle_{vac} + \frac{1}{2}\frac{\partial \epsilon}{\partial m}$$

$$\text{GMOR:} \quad 2m\langle \bar{q}q \rangle_{vac} = -m_\pi^2 f_\pi^2$$

$$\frac{\langle \bar{q}q \rangle_{\rho_B}}{\langle \bar{q}q \rangle_{vac}} \simeq 1 - \frac{\sigma_N}{m_\pi^2 f_\pi^2}\rho_B$$

What if

Kaon condensate (<K⁻>)?

$$< K^{-} > \to < \bar{u} u >, \; < \bar{d} d >$$

As a first attempt,

- Only effect of kaon condensate on quark condensate, not the other way around
- CQM could be relevant near chiral phase transition
  [Riska&Brown, NPA679, 577 (2001),
  Brown &Rho, Phys. Rept.'96;'02;'04]
- Symmetric nuclear matter $(K_F^u = K_F^d, K_F^s = 0)$
- No antikaon condensate

$$3.1 \le \rho_c^{\langle K^0 \rangle} \le 11.1, \quad 2.2 \le \rho_c^{\langle K^- \rangle} \le 4.2 \quad (\times \rho_0)$$

[S. Pal, et al, NPA 674, 553 (2000)]

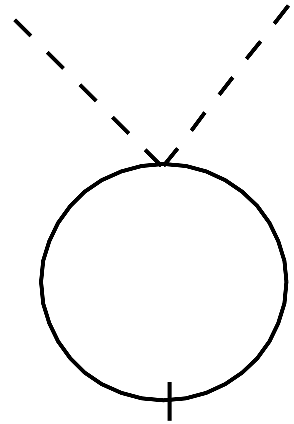
<K⁻> in the CQM

- ¤ QCD < E_{CQM} < ¤ ÂSB [Manohar & Georgi ’84]

$$\mathcal{L}_0 = \bar\psi(i\not{D} + \not{V})\psi + g_A\bar\psi\not{A}\gamma_5\psi - M_0\bar\psi\psi + \frac{1}{4}f_\pi^2\mathrm{Tr}(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma) - \frac{1}{2}\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) + \ldots$$

L M = i ½Å(»yM »y + »M »)Ã

$$\mathcal{L}_K = \frac{i}{4f_\pi^2}[\bar u(K^+\not\partial K^- - (\not\partial K^+)K^-)u + \bar s(K^-\not\partial K^+ - (\not\partial K^-)K^+)s] + \frac{1}{2f_\pi^2}(m_u + m_s)[\bar u K^+K^-u + \bar s K^-K^+s]$$



$c = 1: m^*_{K^-} \simeq 410\text{MeV}, \quad c = 2: m^*_{K^-} \simeq 330\text{MeV}$
$c = 3: m^*_{K^-} \simeq 260\text{MeV}, \quad c = 4: m^*_{K^-} \simeq 193\text{MeV}:$

Note that $200 \text{ MeV} < \mu_e < 300 \text{ MeV}$ in the density range $\rho_B = (2-4)\rho_0$.

$$\rho_c^K = (3-4)\rho_0$$

$$\rho_c^K \sim 3\rho_0$$

C.-H. Lee, Phys. Rept.275, 255 (1996)

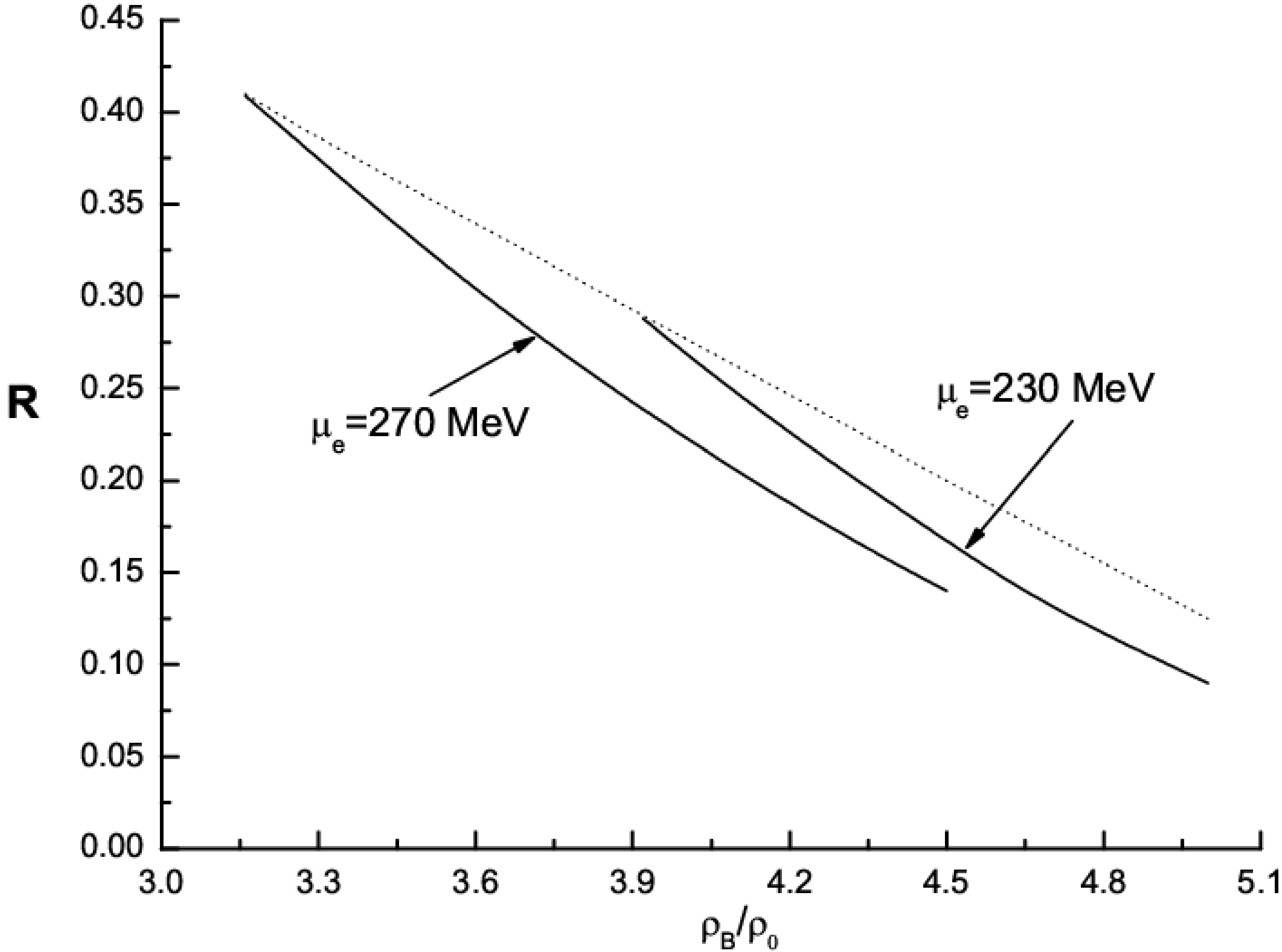
Kaon condensate and quark condensate

$$\langle \bar{q}_f q_f \rangle_{\rho_B} = \langle \bar{q}_f q_f \rangle_{vac} + \frac{1}{2}\left[\frac{\partial \Omega}{\partial m_f} + \frac{dM_f}{dm_f}\frac{\partial \Omega}{\partial M_f} + \frac{dm_\phi^2}{dm_f}\frac{\partial \Omega}{\partial m_\phi^2}\right]$$

$$\Omega = \frac{3}{4\pi^2}\sum \left[k_F^q (k_F^{q2} + M_q^2)^{3/2} - \frac{1}{2} M_q^2 k_F^q \sqrt{k_F^{q2} + M_q^2} - \frac{1}{2} M_q^4 \ln \frac{k_F^q + \sqrt{k_F^{q2} + M_q^2}}{M_q}\right]$$
$$- \frac{1}{2} f_\pi^2 \mu_e^2 \sin^2\theta + 2 m_K^2 f_\pi^2 \sin^2\frac{\theta}{2} - \mu_e \rho_u \sin^2\frac{\theta}{2} - m' \rho_s \sin^2\frac{\theta}{2}$$

$$\theta = \sqrt{2} v / f_\pi, \qquad \langle K^- \rangle = v e^{-i\mu_e t}$$

$$R = \frac{<\bar{u}u>_{\rho_B}}{<\bar{u}u>_{vac}} = 1 - \frac{m_u}{m_\pi^2 f_\pi^2}\left[\frac{\partial\Omega}{\partial m_u} + \frac{\sigma_u}{m_u}\frac{\partial\Omega}{\partial M_u} + \frac{m_K^2}{m_0}\frac{\partial\Omega}{\partial m_K^2}\right]$$



0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
R
3.0
3.3
3.6
3.9
4.2
4.5
4.8
5.1
ρ_B/ρ_0
μ_e=270 MeV
μ_e=230 MeV

A partial chiral symmetry restoration in medium may be flavor dependent:

$$< \overset{1}{u}u > =< \overset{1}{d}d > < 1$$

Spontaneous isospin violation

$$M_u^{?} = M_0 + m_u \text{ ¡ } \frac{m^0}{2f_{¼}^2} v^2; \quad M_d^{?} = M_0 + m_d$$

Check-up I

- Quark condensate in HBChPT

$$\text{²}_{HB} = \frac{3}{5} E_F^{(0)} u^{\frac{5}{3}} \text{½}_0 + V(u) + u\text{½}_0 (1 \text{¡} 2x)^2 S(u)$$

$$\text{¡} \; \frac{1 \;\; 2}{2} f_{\text{¼}}^2 \sin^2 \text{µ} + 2m_K^2 f_{\text{¼}}^2 \sin^2 \frac{\text{µ}}{2} + {}^{1} u\text{½}_0 x \; \text{¡} \; {}^{1} u\text{½}_0 (1 + x) \sin^2 \frac{\text{µ}}{2}$$

$$+ (2a_1 x + 2a_2 + 4a_3) m_s u\text{½}_0 \sin^2 \frac{\text{µ}}{2} \; \text{¡} \; \frac{1 \;\; 4}{12\text{¼}^2},$$

$$\text{½}_p = x\text{½}_B, \; \text{½}_B = u\text{½}_0$$

V(u): charge-symmetric contribution of the nuclear interactions,
S(u): symmetry energy

$$S(u) = (2^{2=3} \; \text{¡} \; 1) \frac{3}{5} E_F^{(0)} (u^{2=3} \; \text{¡} \; F(u)) + S_0 F(u); \; S_0 \; \text{'} \; 30\text{Mev}; \; F(u) = u$$

$a_3 m_s$	U	R_u	R_d
−134	7.18	0.834	» 1
−134	7.68	0.822	» 1
−222	4.08	0.865	» 1
−222	4.58	0.812	» 1
−310	2.92	0.875	» 1

Check-up II

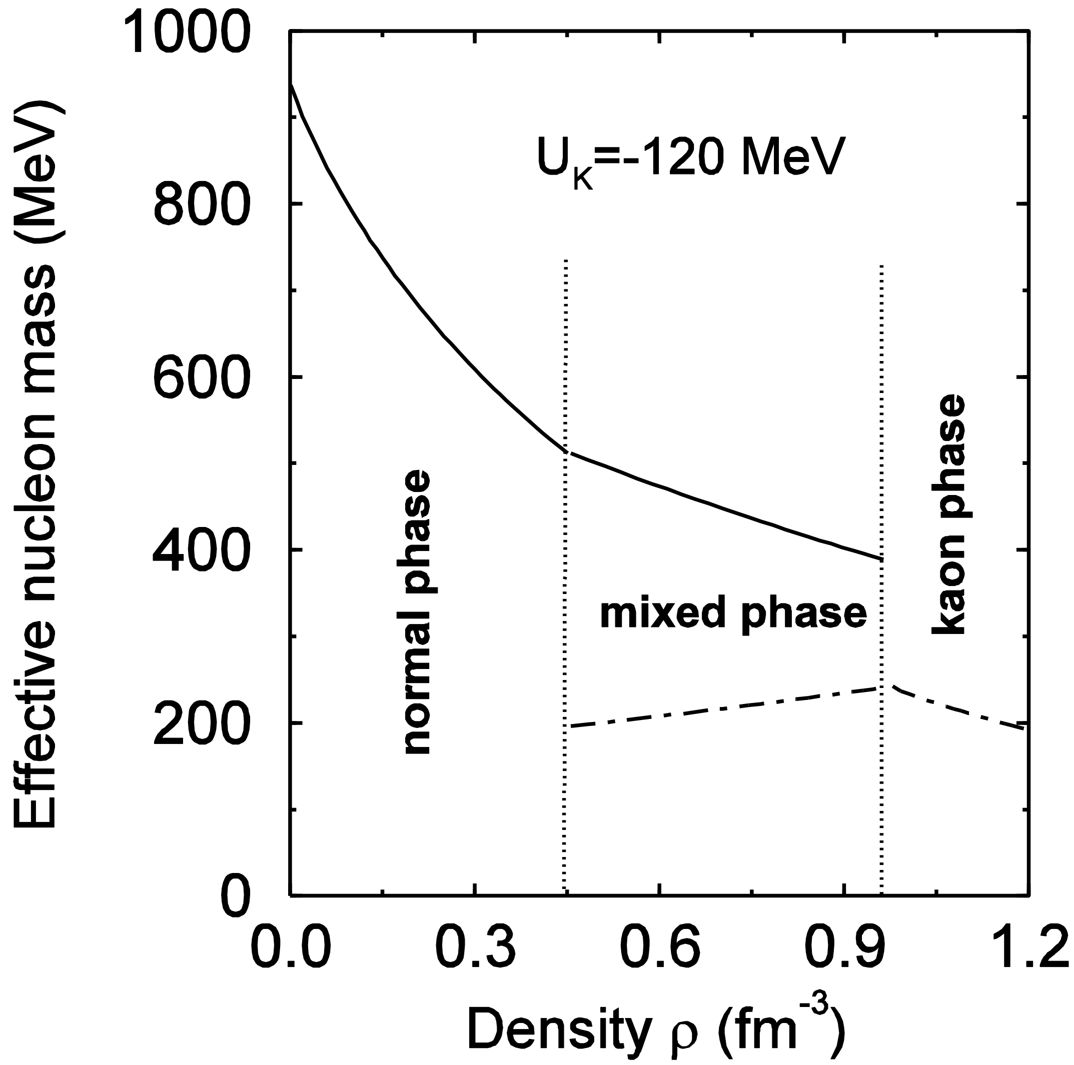
(Predictions)

- Neutron–Proton mass difference

p » uud, n » udd

The in-medium nucleon masses in a kaon condensed phase decreases with the density faster than in a normal phase

In a kaon condensed phase the in-medium proton mass will be much smaller than neutron mass



Glendenning&Schaffner-Bielich, PRL81, 4564 (1998)

Kubodera, JKPS26, S171(1993), M. Prakash, et al, Phys. Rept.280, 1 (1997)

$$¢\ M^{?}_{np}(\mu) =\ ¡\ 2a_1 m_s \sin^2 \frac{\mu}{2}\ ,\quad a_1 m_s =\ ¡\ 67\ \text{MeV}$$

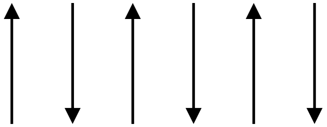
$a_3 m_s$	u	¢ $M^{?}_{np}(\mu)$
−134	4.68	4.8
−134	7.18	18.3
−222	3.58	7.6
−222	4.08	14.8
−310	2.92	13.6

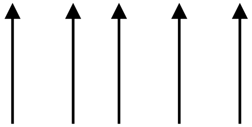
The Heisenberg model

$$H = g \sum_{\mathrm{mn}} \vec{s}_{\mathrm{m}} \cdot \vec{s}_{\mathrm{n}}$$

$$\mathrm{G} = \mathrm{O}(3) \qquad \vec{Q} = \sum_{\mathrm{n}} \vec{s}_{\mathrm{n}}$$

$g>0$: antiferromagnet **the ground state** $g<0$: ferromagnet





$$\mathrm{H} = \mathrm{O}(2)$$

two GBs (two magnons)

Antiferromagnet	Ferromagnet
two magnons	one magnon
E ~ p	E ~ p^2
$\langle \vec{Q} \rangle = 0$	$\langle \vec{Q} \rangle = V \langle \vec{M} \rangle$

The charges of the symmetry group have nonzero density in the ground state, which cannot happen in a Lorentz invariant ground state.

The generators Q_i of G are space integrals over the corresponding charge densities,

$$Q_i = \int d^3x J_i^0(x)$$

$$\mathcal{L}_{eff}^{(0,1)} = \Sigma\,(1+U^3)^{-1}(\partial_0 U^1 U^2 - \partial_0 U^2 U^1) + \Sigma f_0^i\, U^i$$

H. Leutwyler, Phys. Rev. D49, 3033 (1994)

NGs and Kaon condensate

- A toy model: SU(2)xU(1)

$$\mathcal{L} = (\partial_0 + i\mu)\Phi^\dagger(\partial_0 - i\mu)\Phi - \partial_i\Phi^\dagger\partial_i\Phi - m^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

$\longrightarrow$ $(\mu^2 - m^2)\Phi^\dagger\Phi$ causes an instability.

$$\Phi = \begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \tilde{\varphi}_1 + i\tilde{\varphi}_2 \end{pmatrix}, \quad \text{with} \quad \varphi_0^2 = \frac{\mu^2 - m^2}{2\lambda}$$

$$\omega_2 \simeq \frac{k^2}{2\mu}, \quad \text{for} \quad k \to 0,$$

$$\tilde{\omega}_2 \simeq \sqrt{\frac{\mu^2 - m^2}{3\mu^2 - m^2}}k, \quad \text{for} \quad k \to 0$$

only two massless NGs

V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. 88, 111601 (2002)

In CFL phase, $\langle \psi_L^{\alpha a} \psi_L^{\beta b} \rangle \propto \Delta \epsilon^{\alpha\beta A} \epsilon^{abA}$, $\mu = m_s^2/2p_F$

Kaon condensate

↓

SU(n−1)xU(1) −> SU(n−2)xU(1)

Only (n−1) GBs < 2n−3

T. Schäfer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. Verbaarschot, Phys. Lett. B **522**, 67 (2001)

Goldstone's theorem by Nielsen and Chadha

- The sum of twice the number of Goldstone modes with $E \sim p^2$ and the number of Goldstone modes with $E \sim p$ is equal to or greater than the number of independent broken generators.

[H. B. Nielsen & S. Chada, NPB105, 445 (1976)]

In electron-aided kaon condensate,

We have $1 \gg 1_e$,

and so we expect abnormal number of GBs.

(work in progress)

Summary & more to go

- The partial chiral symmetry restoration in dense matter could be flavor dependent.

  But, we have to let them compete.

  $< K^{i} > \$ < \bar{u}u >$

How?

–the extended chiral quark model, Andrianov & Espriu '98

◎ $\langle K^{i} \rangle$ & $\langle K^{1}_{0} \rangle$

may make it flavor-independent.

◎ Abnormal number of GBs and any physical consequence?