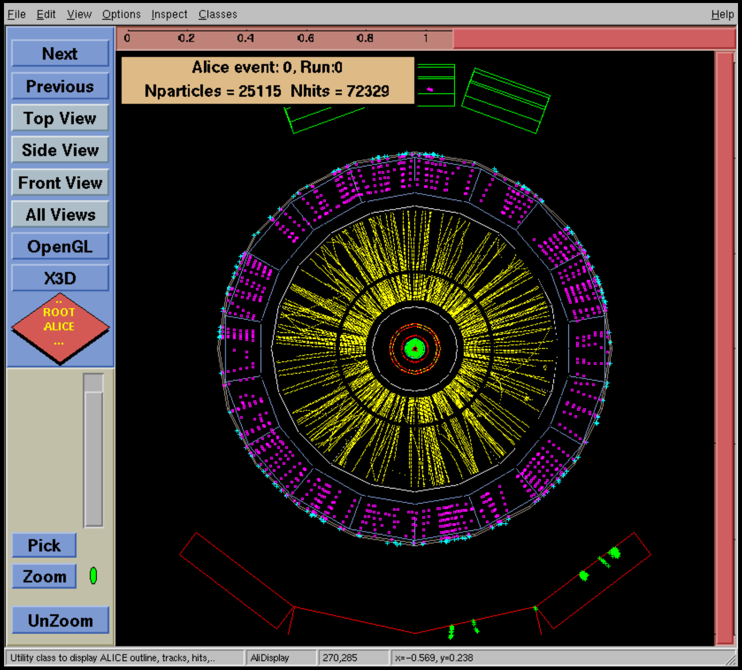
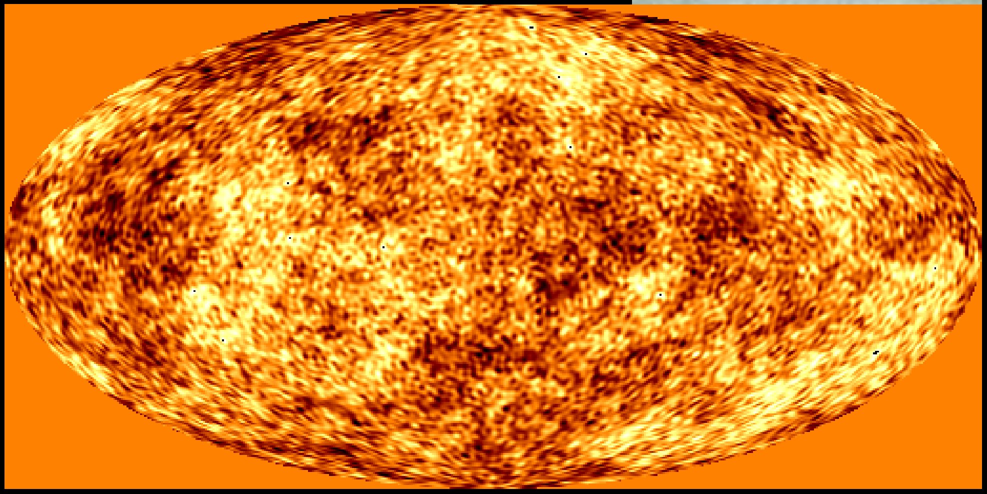
Strong Correlations in Hot/Dense QCD

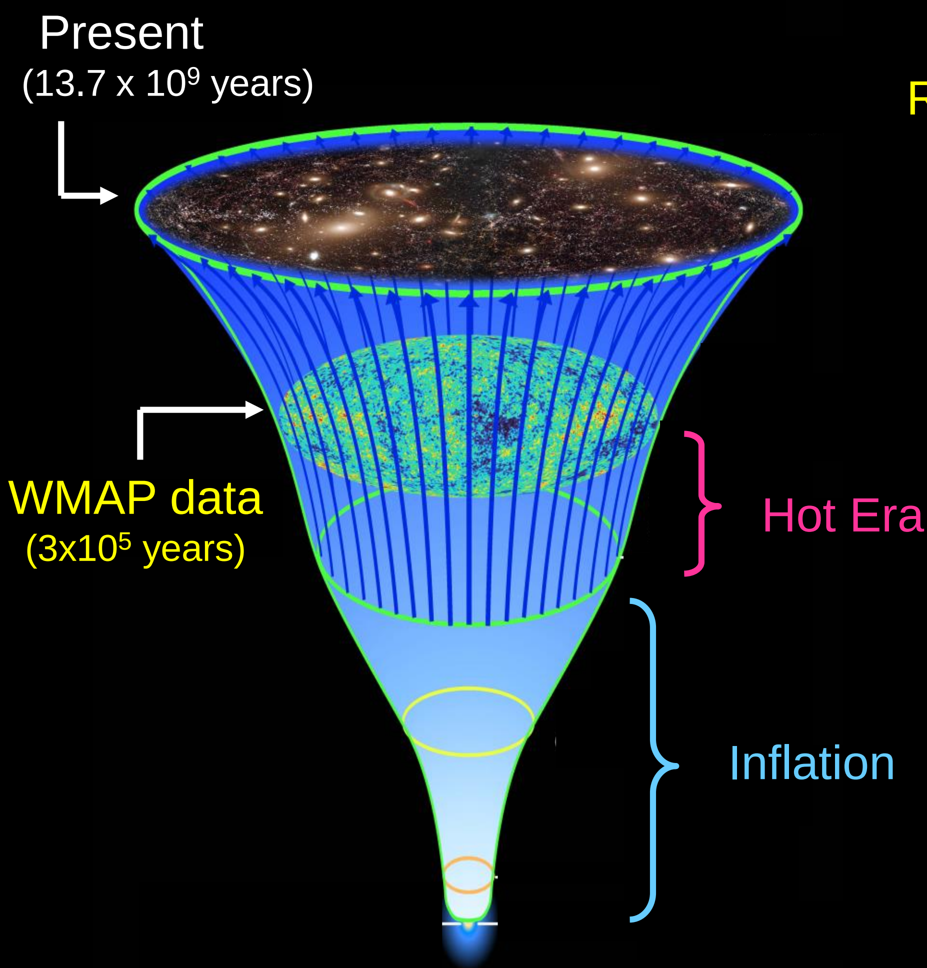
T. Hatsuda
(Univ. Tokyo)

Future (2007-):
LHC (CERN) & Planck (ESA)

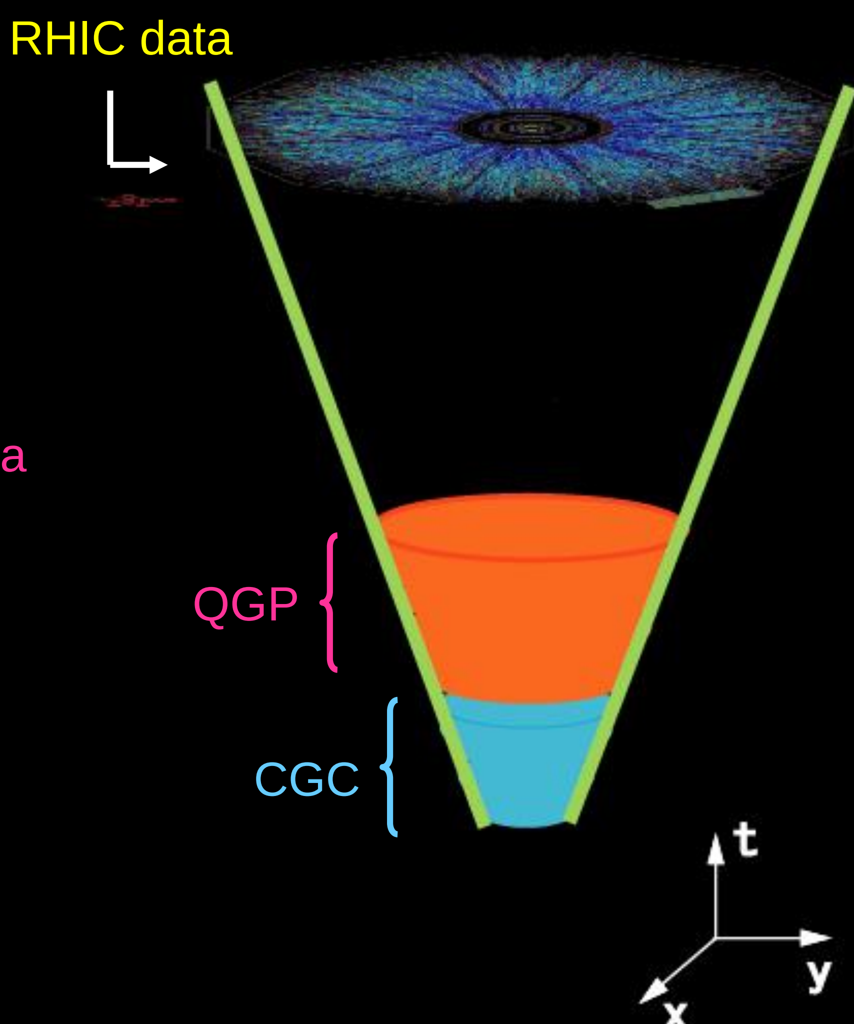




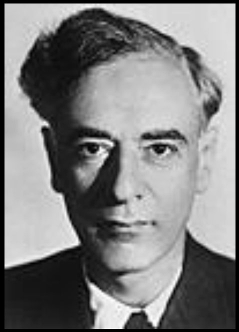
Big Bang



Little Bang

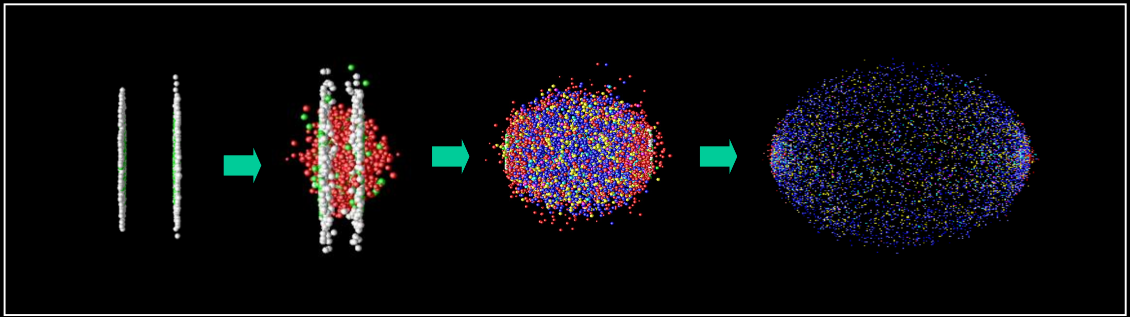


Landau's view on High-Energy Hadron Collisions (1953-1955)



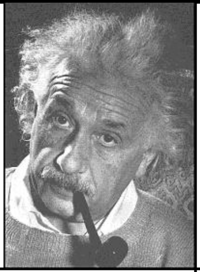
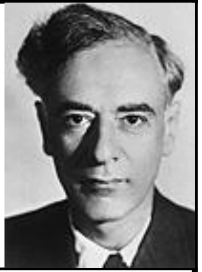
Belensky, S.Z. and Landau, L.D., Ups.Fiz.Nauk. 56 (1955) 309, reprinted in *Collected Papers of Landau, L.D.* ed. ter Haar, D.T., (Gordon & Breach, New York,1965) 665.

(i) *When two nucleons collide, a compound system is formed, and energy is released in a small volume V subject to a Lorentz contraction in the transverse direction†. At the instant of collision, a large number of "particles" are formed; the "mean free path" in the resulting system is small compared with its dimensions, and statistical equilibrium is set up.*



(ii) *The second stage of the collision consists in the expansion of the system. Here the hydrodynamic approach must be used, and the expansion may be regarded as the motion of an ideal fluid (zero viscosity and zero thermal conductivity). During the process of expansion the "mean free path" remains small in comparison with the dimensions of the system, and this justifies the use of the hydrodynamics. Since the velocities in the system are comparable with that of light, we must use not ordinary but relativistic hydrodynamics. Particles are formed and absorbed in the system throughout the first and second stages of the collision. The high density of energy in the system is of importance here. In this case, the number of particles is not an integral of the system, on account of the strong interaction between the individual particles.*

(iii) *As the system expands, the interaction becomes weaker and the mean free path becomes longer. The number of particles appears as a physical characteristic when the interaction is sufficiently weak. When the mean free path becomes comparable with the linear dimensions of the system, the latter breaks up into individual particles. This may be called the "break-up" stage. It occurs with a temperature of the system of the order* $T \approx \mu c^2$, *where* μ *is the mass of the pion. (All temperatures are in energy units.)*

	Big Bang	Mini Bang
Initial state	Inflation ? $(10^{-35}$ sec)	Color glass ? $(10^{-1}$ fm)
Thermalization	Inflaton decay ?	decoherence of CGC?
Expansion	$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu}$ 	$\partial_\mu T^{\mu\nu} = 0$ $\partial_\mu J^{\mu} = 0$ 
Freezeout	(T = 1.95 K neutrino) T = 2.73 K photon	T_{chem} = 170 MeV T_{therm} = 120 MeV
Observables	CMB & anisotropy (CvB, CGB & anisotropy)	Collective flow & anisotropy Jets, leptons, photons
Parameters to be determined	**8~10** cosmological parameters ・Initial density fluctuation ・Cosmological const. Λ etc	QGP parameters ・Initial energy density ・Equation of state etc
Evolution Code	CMBFAST	3D-hydro. code

Contents

[1] QCD Phase Structure

-- similarity to High T_c superconductivity
– various critical points

[2] Dynamics of QCD Phase Transition

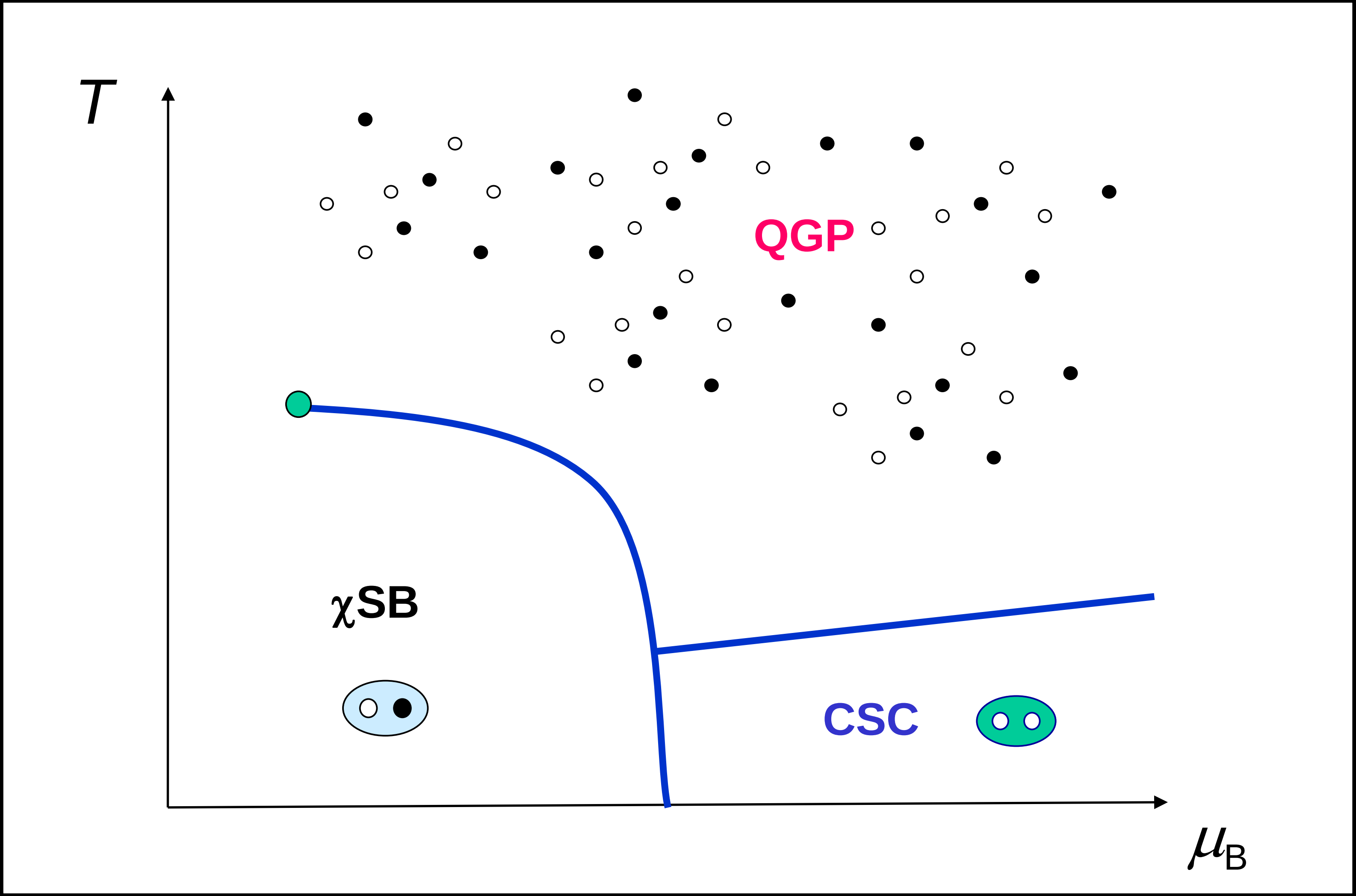
-- climbing the Hagedorn slope
-- lattice thermodynamics

[3] Strongly Correlated QCD Plasma ?

-- plasma viscosity
-- heavy flavor as a plasma probe

[4] Summary

QCD Phase Structure



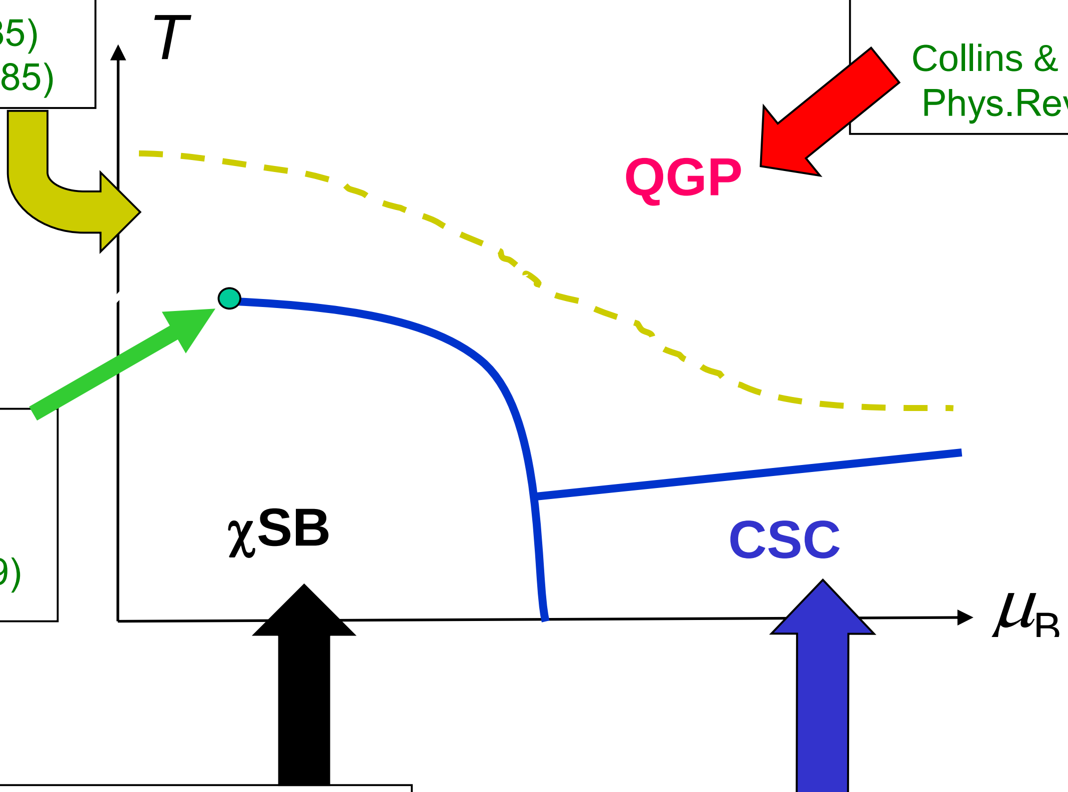
Origin of various phases

strong residual int.
→ **pre-formed pairs**

Hatsuda & Kunihiro,
Phys. Rev. Lett. 55 ('85)
DeTar, Phys.Rev. D32 ('85)

Asymptotic freedom
+ Debye screening
→ **deconfinement**

Collins & Perry,
Phys.Rev.Lett. 34 ('75)



Critical point

Asakawa & Yazaki,
Nucl. Phys. A504 ('89)

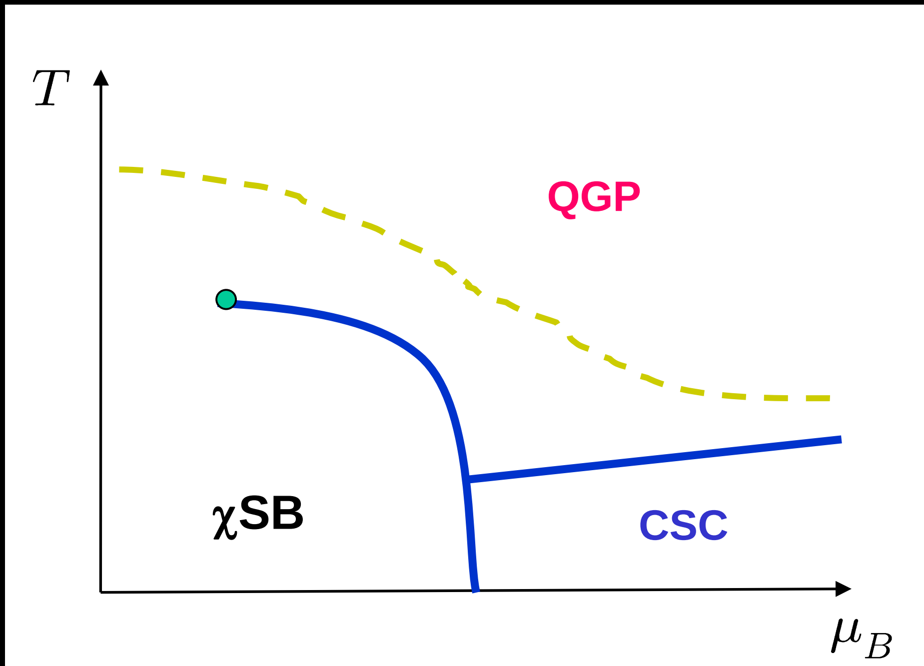
quark-anti-quark pairing
→ **Chiral instability**

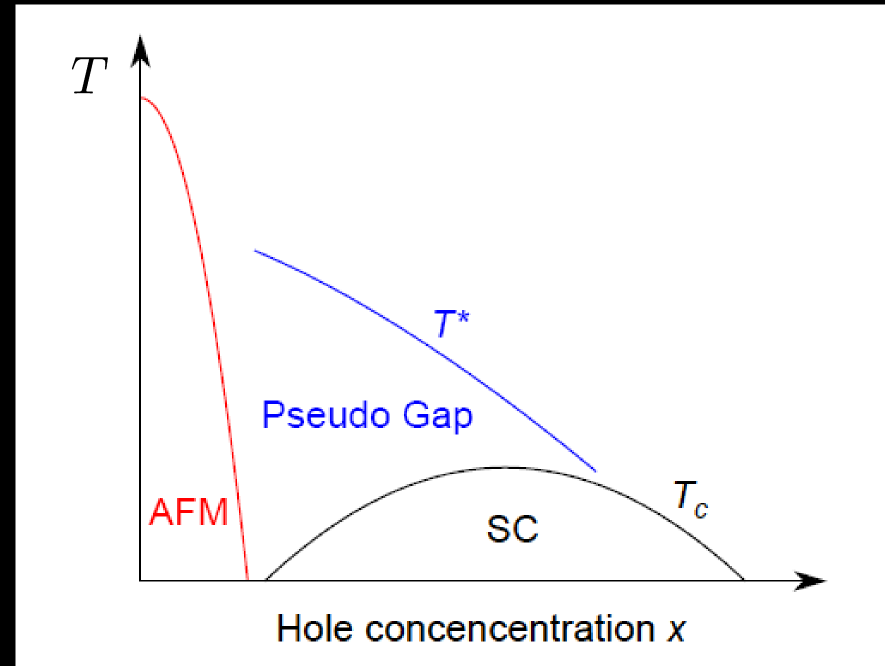
Nambu & Jona-Lasinio,
Phys.Rev. 122 ('61)

quark-quark pairing
→ **Cooper instability**

Bailin & Love,
Phys.Rep.107 ('84)

Similarity with high T_c superconductivity



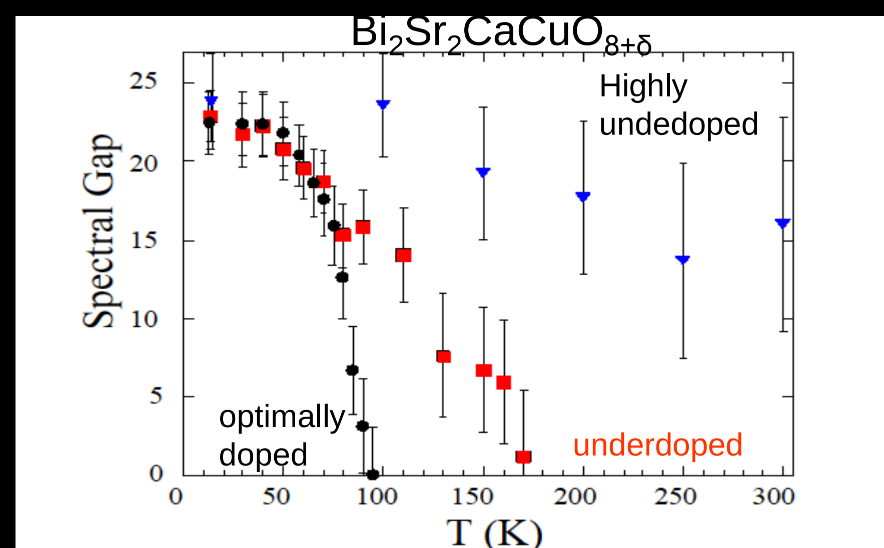


1. Competition of different orders
2. Strong coupling

→ pre-formed pairs in $T_c < T < T^*$?

decoherence at T_c

pair breaking at T*



HTS – BEC/BCS – QCD connection ?

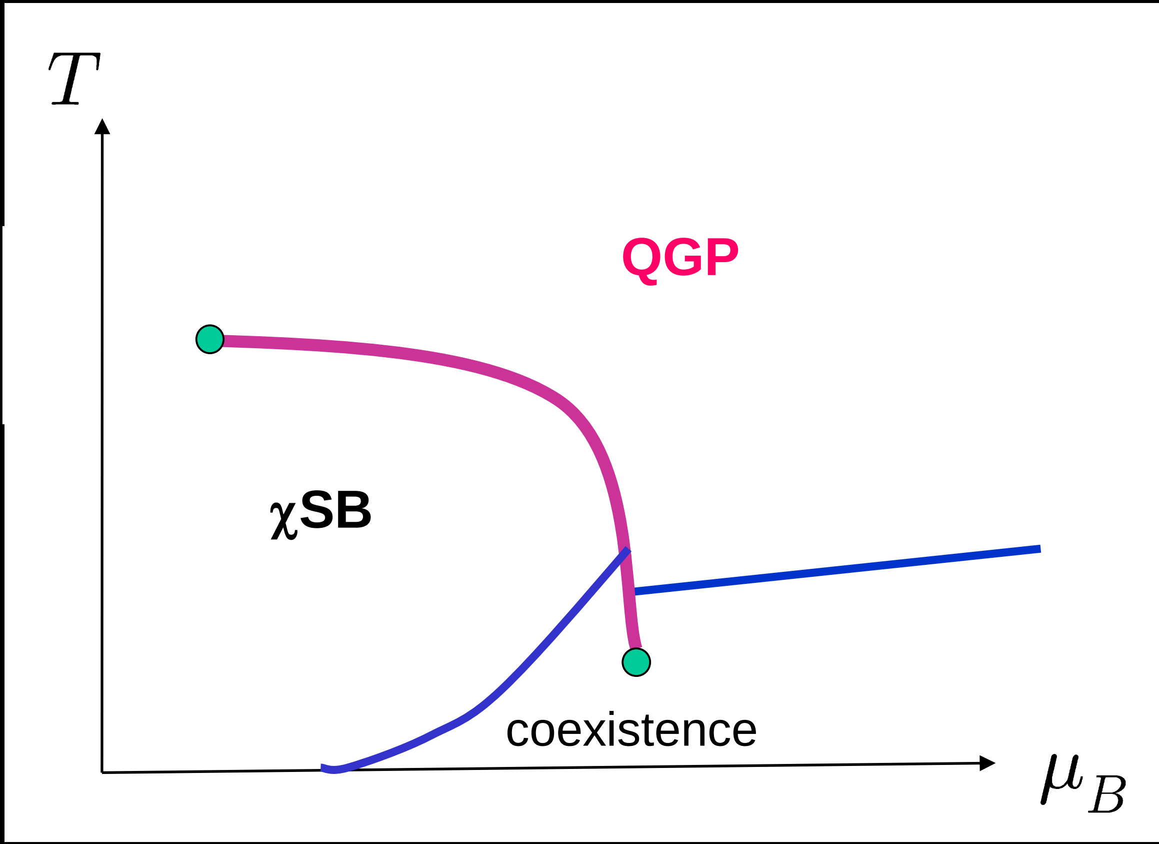
- Babaev, PRD ('00)
- Abuki, Itakura & Hatsuda, PRD ('02)
- Kitazawa, Koide, Kunihiro & Nemoto, PRD ('02)
- Chen, Stajic, Tan & Levin, Phys. Rep. ('05)

New critical point induced by axial anomaly ?

General Ginzburg-Landau analysis based on QCD symmetry:

$$SU(3)_L \times SU(3)_R \times U(1)_B \times \cancel{U(1)_A} \times SU(3)_C$$

$$\longrightarrow \quad \Omega_{\chi d} = \gamma_1 \, \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger] + \cdots$$



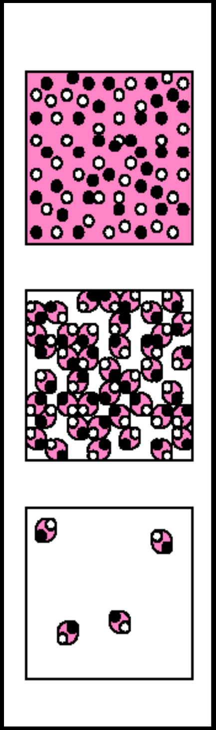
New critical point

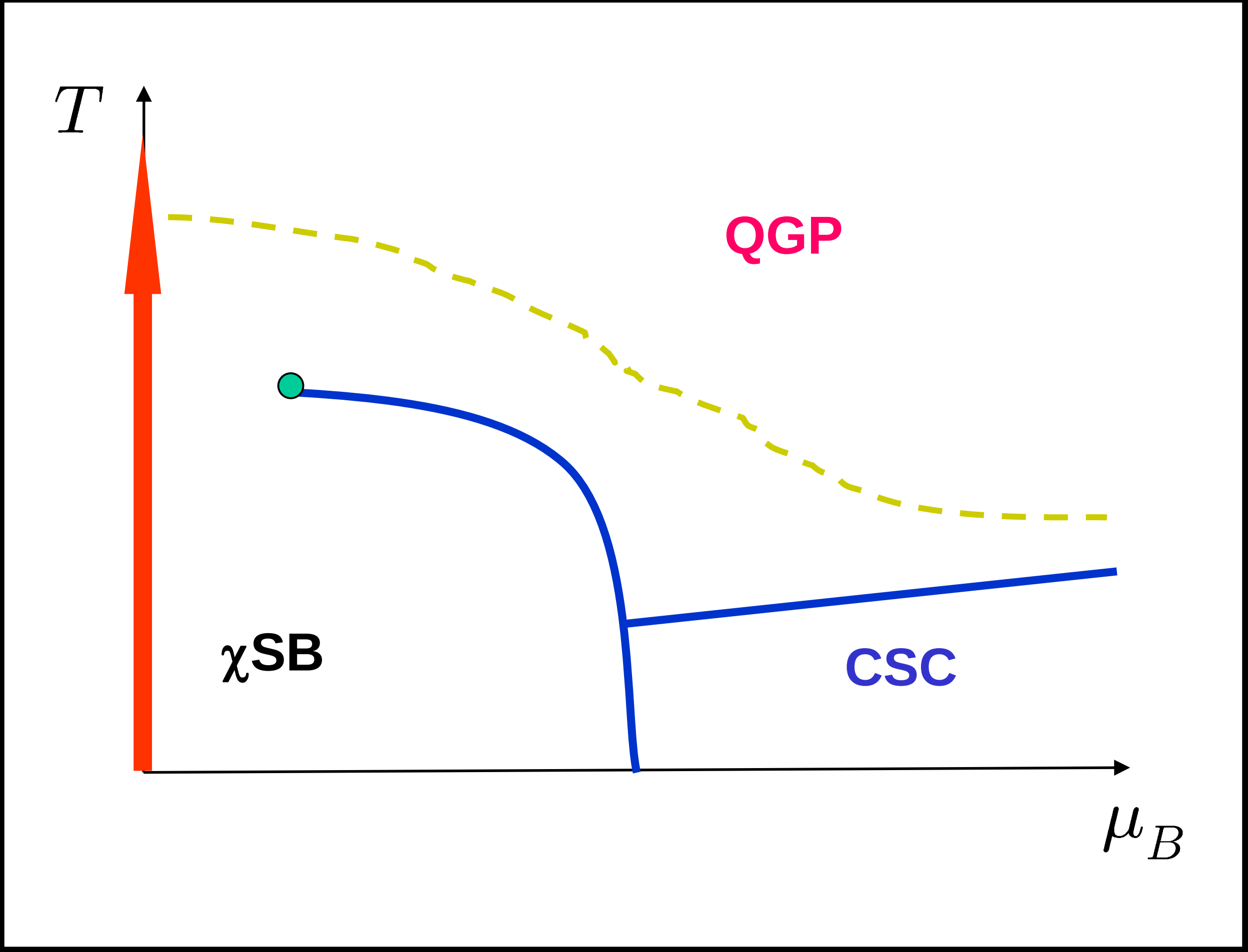
Yamamoto, Tachibana, Baym & Hatsuda, PRL 97 ('06)

Hadron-quark continuity

Schafer & Wilczek, PRL 82 ('99)

Dynamics of QCD Phase Transition





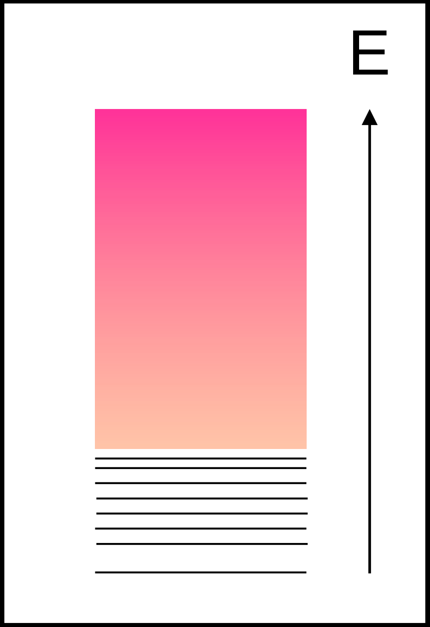
Climbing up the Hagedorn slope
-- micro-canonical view of hot QCD --

Ejiri & Hatsuda, hep-lat/0509119

* QCD in a finite box : *V*

$$H_{QCD}\Psi_n = E_n\Psi_n$$

$$\sigma(E,V) \equiv \sum_n \delta(E-E_n) \geq 0$$



* Z(T,V) = Laplace transform of the level density σ(E,V)

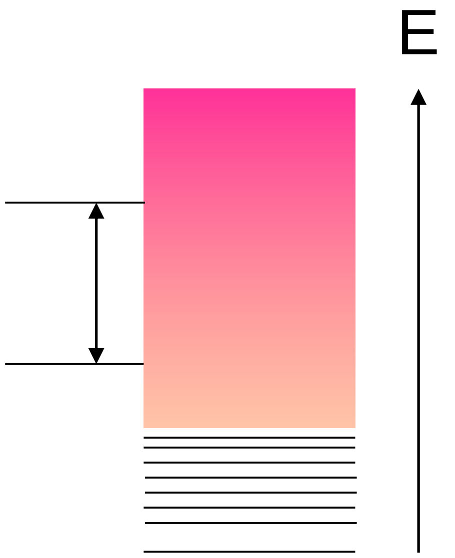
$$\begin{aligned} Z(T,V) &= \int_0^\infty dE\ \sigma(E,V)\ e^{-E/T} \\ &= \mathrm{Tr}[e^{-H_{QCD}/T}] \end{aligned}$$

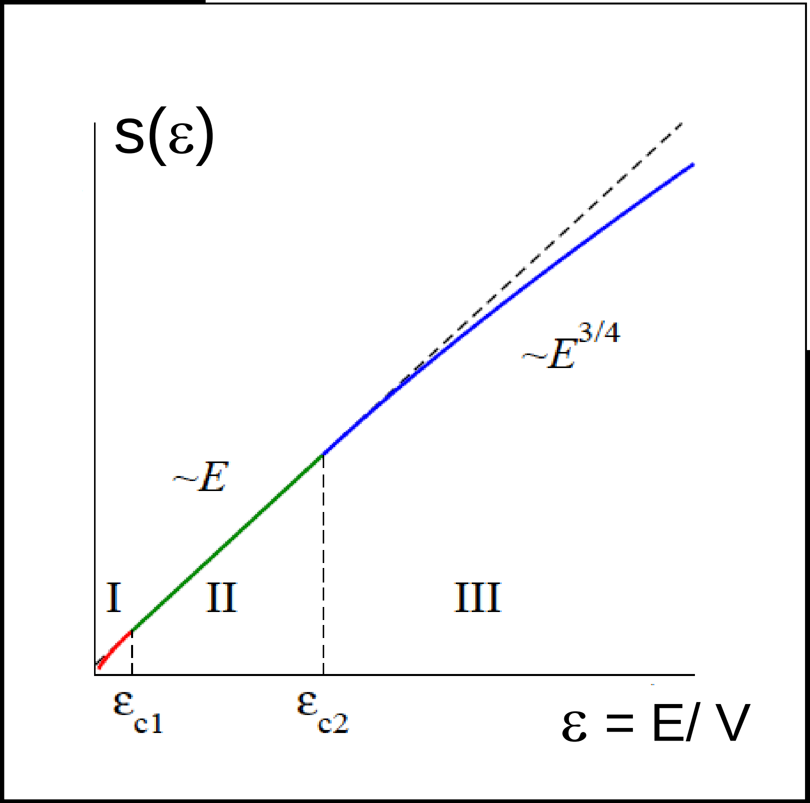
Information of the Phase Transition is encoded in the QCD leverl density $\sigma(E,V)$

$$s(\varepsilon) \propto \ln \sigma(E,V) \sim \varepsilon^{3/4} \qquad \text{III}$$

$$\sim \varepsilon/T_0 \qquad \text{II}$$

$$\sim \varepsilon^{3/4} \qquad \text{I}$$



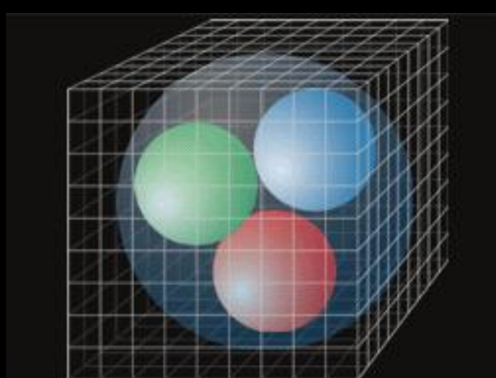


Hagedorn (1965)
Ejiri & Hatsuda, hep-lat/0509119

QCD thermodynamics

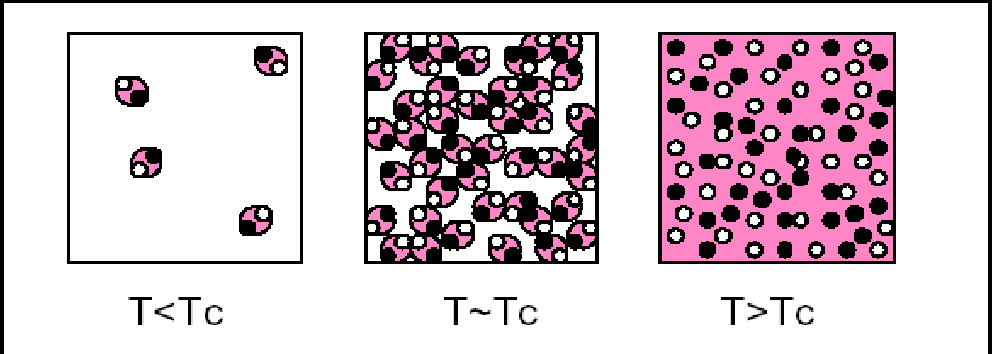
$$Z(T,V) = \mathrm{Tr}\left[e^{-H_{\mathrm{QCD}}/T}\right]$$
$$= \int [dU]\, e^{-[S_g(U)+\bar{S}_q(U)]}$$

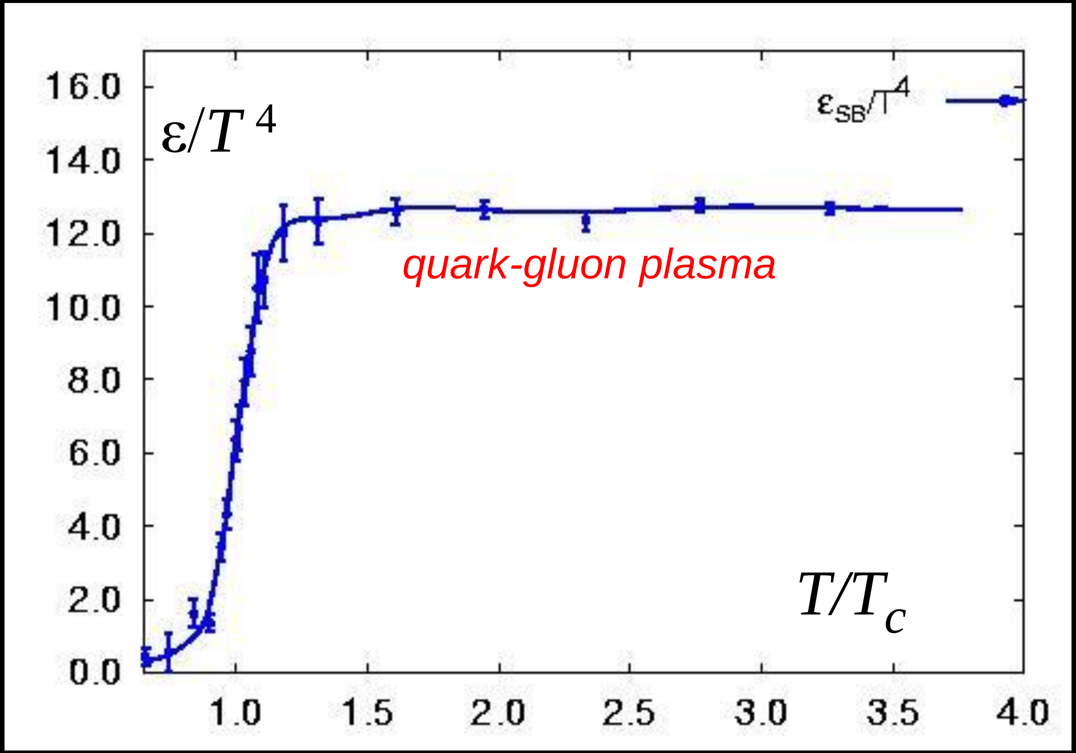
→ *Monte Carlo integration*



IBM BlueGene at KEK
(March, 2006-) 57.3 Tflops

QCD phase transition on the lattice





Critical temperature

T_c = 192(7)(4) MeV

Cheng et al., hep-lat/0608013

Critical energy density

ε_c : 2–3 GeV/fm^3

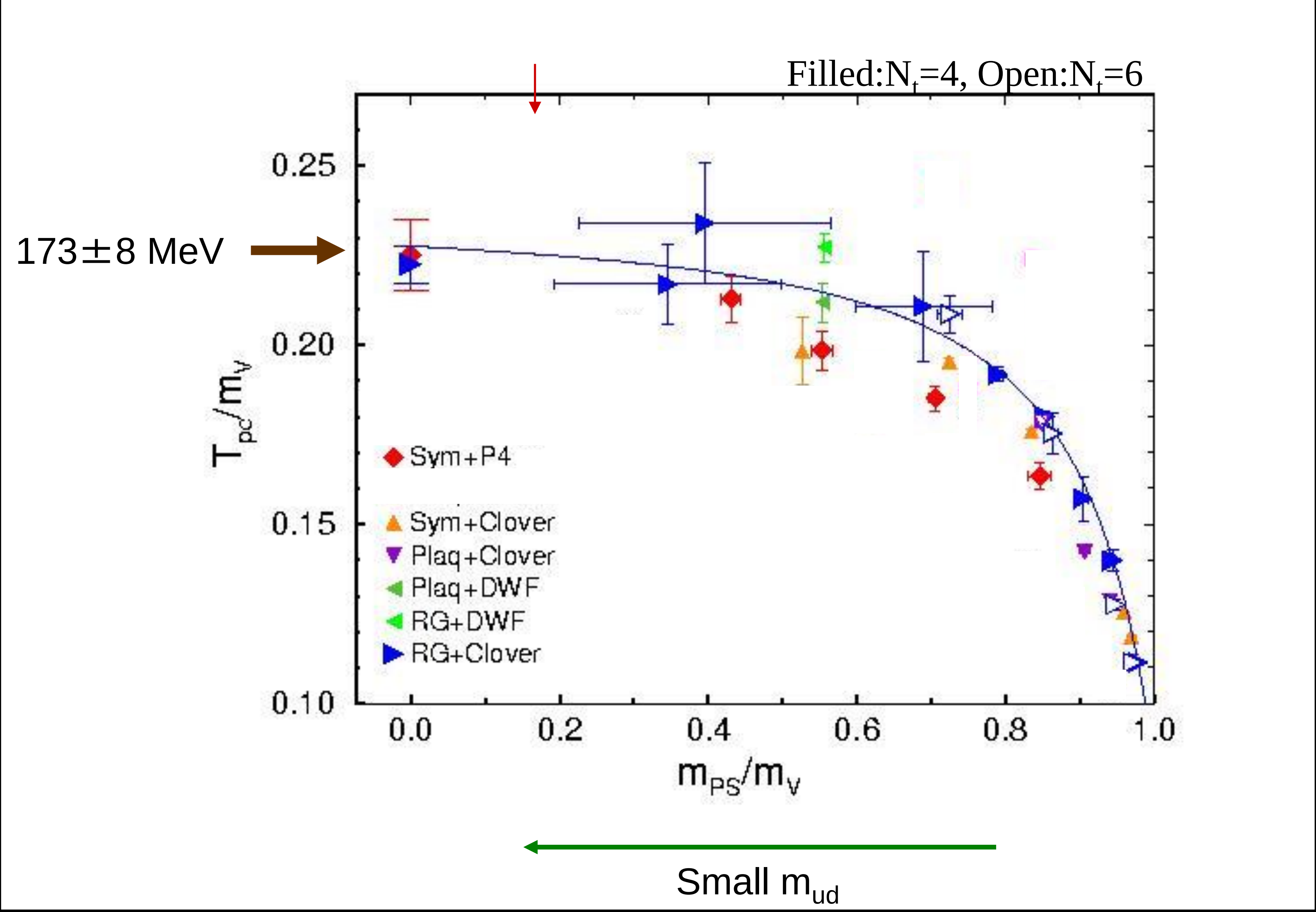
$\sim$ (10–20) ε_{nm}

Orders

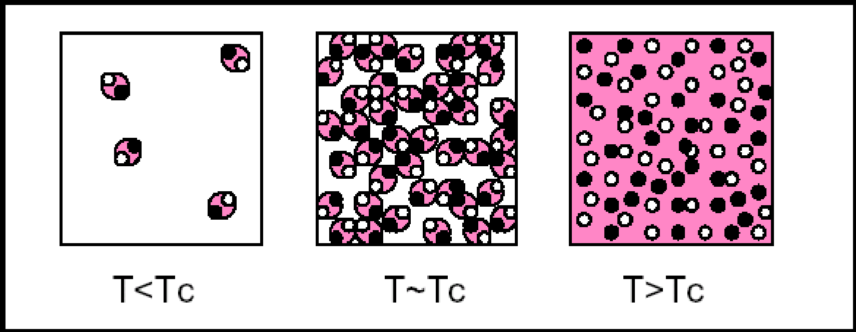
crossover	(real world)
2nd order	(u,d)
1st order	(u,d,s)

T_c in 2-favor lattice QCD

Ejiri ('04)

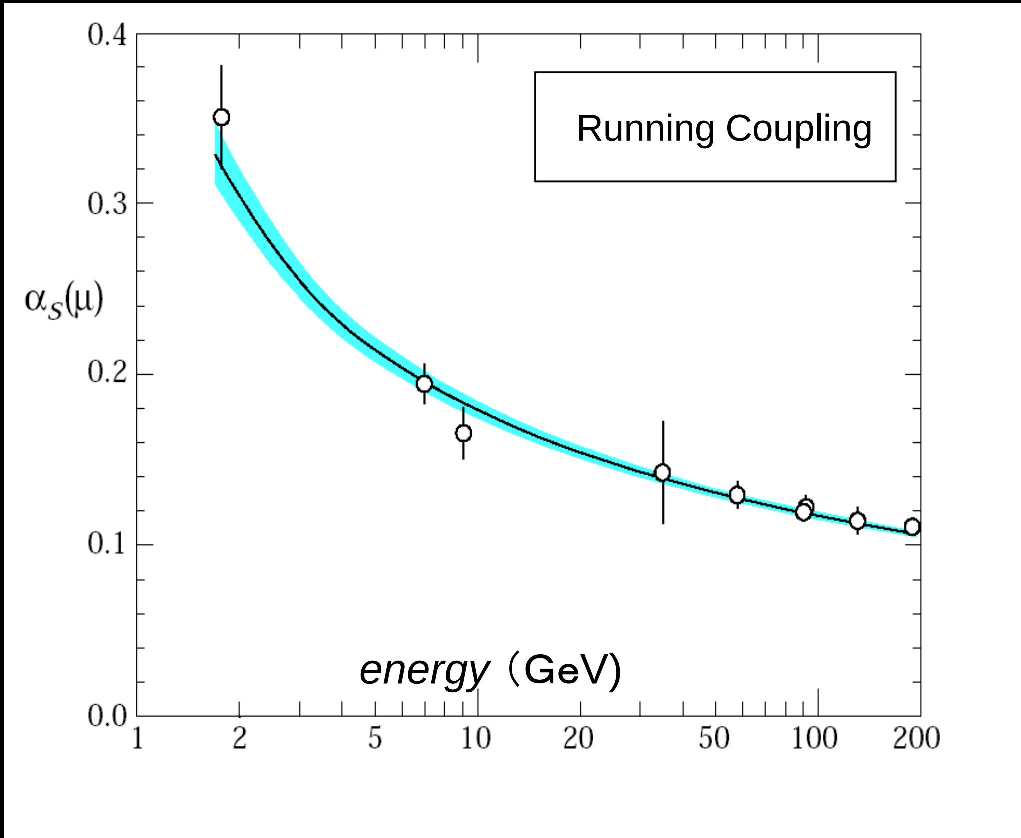


Strongly Correlated QGP?



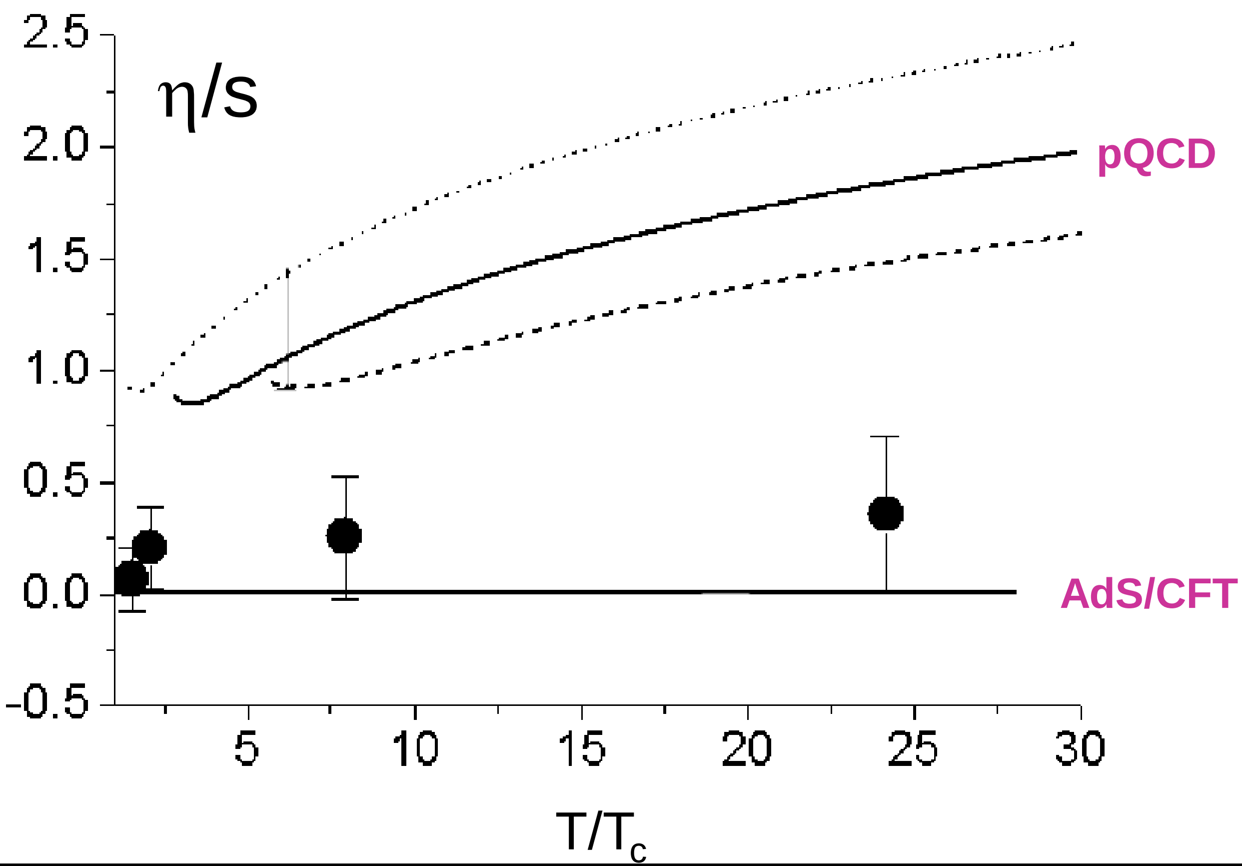
kinetic energy at $T = 2T_c$
~ 1 GeV

→ $\alpha_s \sim O(1)$



$$\frac{P}{P_{SB}} = 1 - 2.76\left(\frac{\alpha_s}{\pi}\right) + 17.8\left(\frac{\alpha_s}{\pi}\right)^{3/2} + \left(81.2 + 15.9\ln\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha_s}{\pi}\right)^2 - 327\left(\frac{\alpha_s}{\pi}\right)^{5/2} + \cdots$$

Plasma viscosity on the lattice



viscous fluid

R << 1

perfect fluid

R >> 1

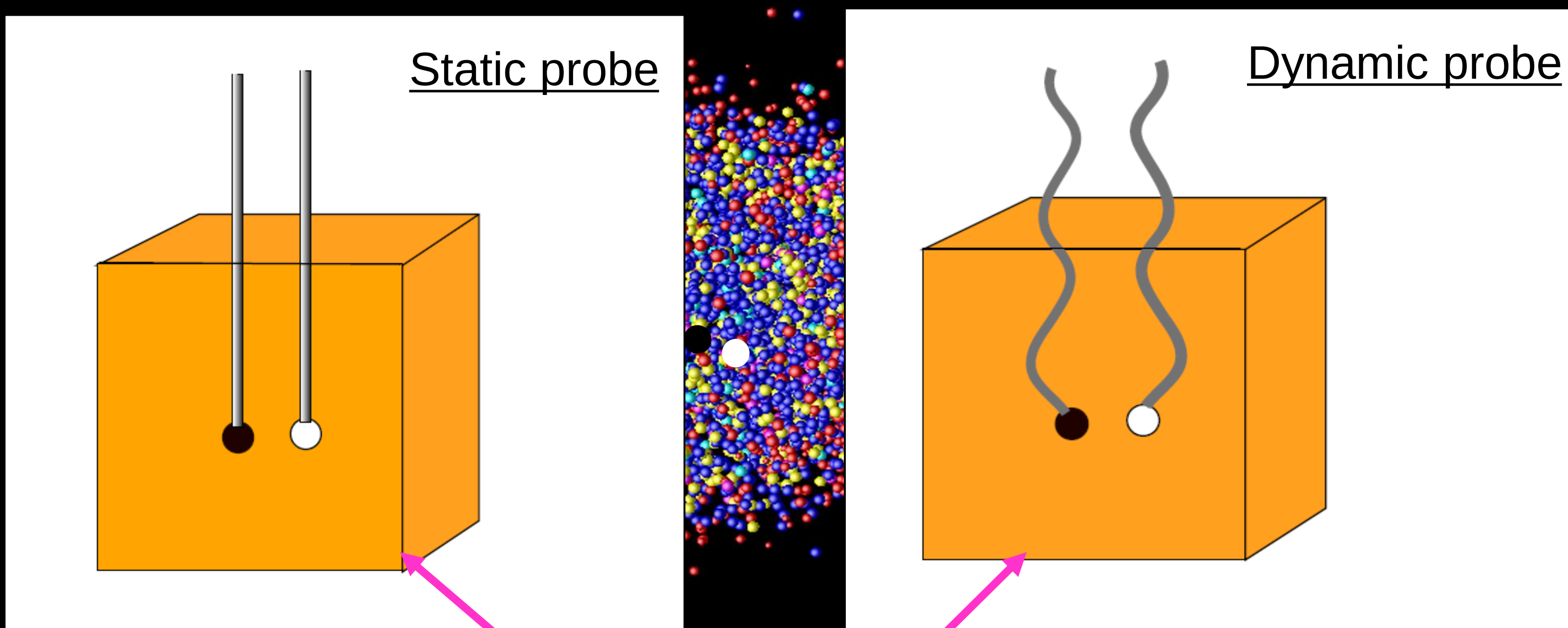
"Reynolds number"

$$R = \left(\frac{\frac{4}{3}\eta + \zeta}{s} \cdot \frac{1}{T\tau} \right)^{-1}$$

24^3x8 (quenched lattice QCD)

Nakamura & Sakai, hep-lat/0510100

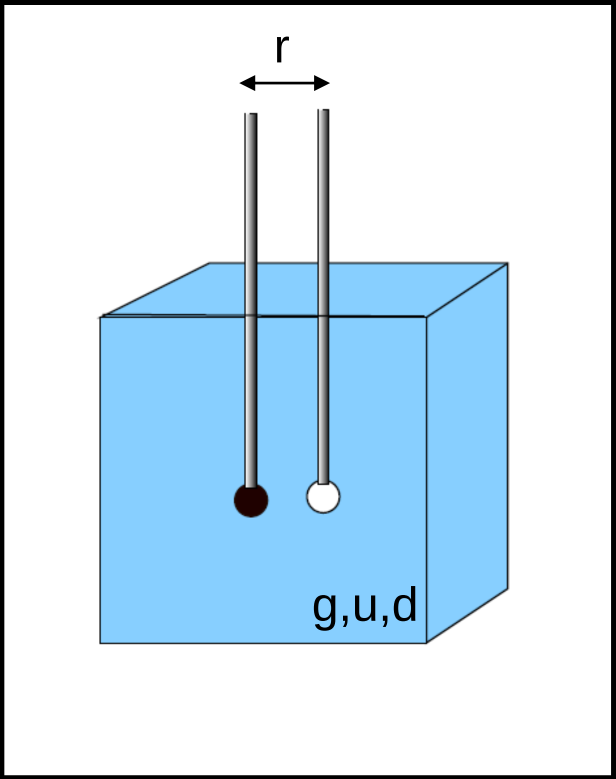
Heavy flavor as a plasma probe

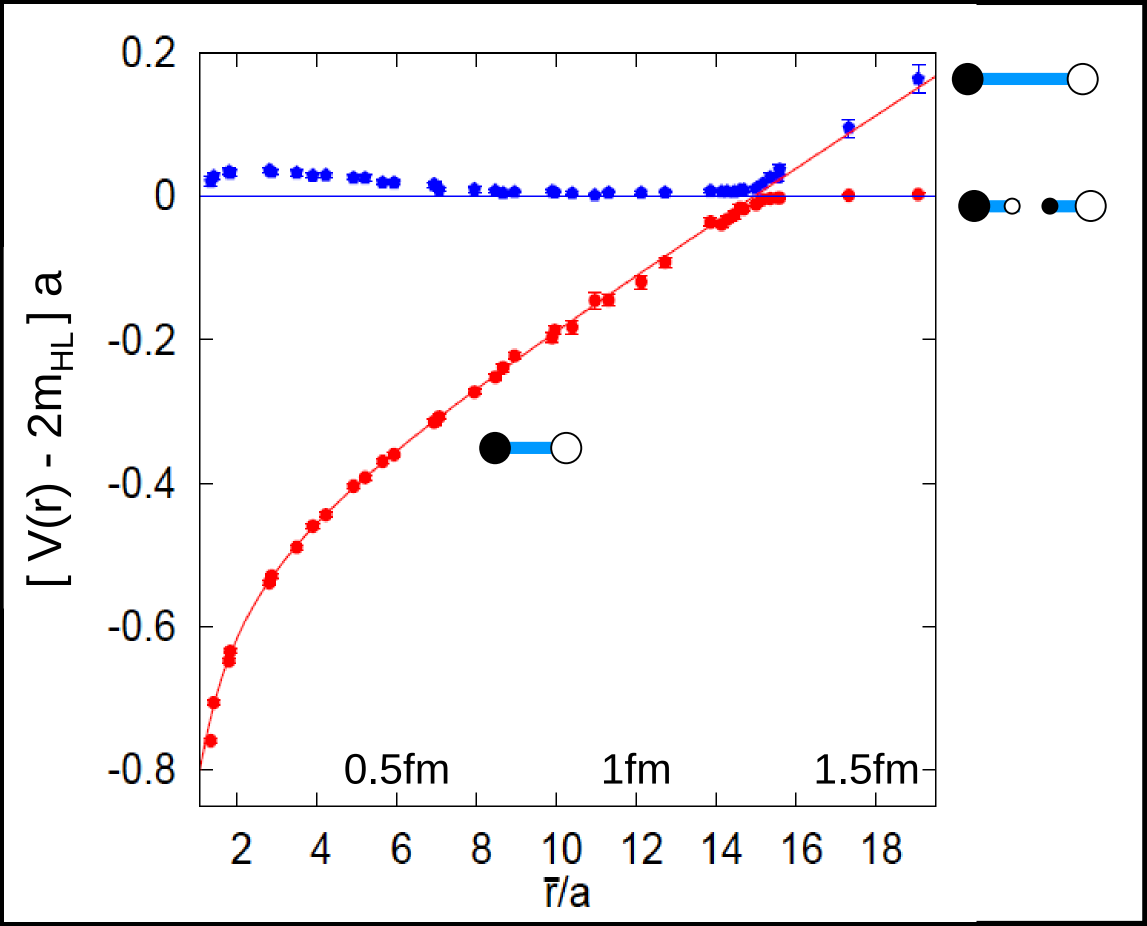


Matsui & Satz, PLB178 ('86)

Gluon matter (quenched QCD)
Quark-gluon matter (full QCD)

Static Probe at T=0 : heavy-quark potential (full QCD)

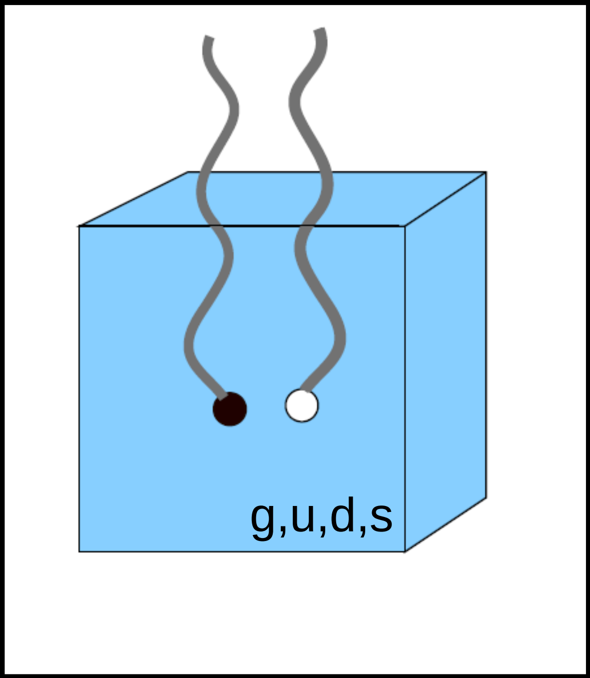


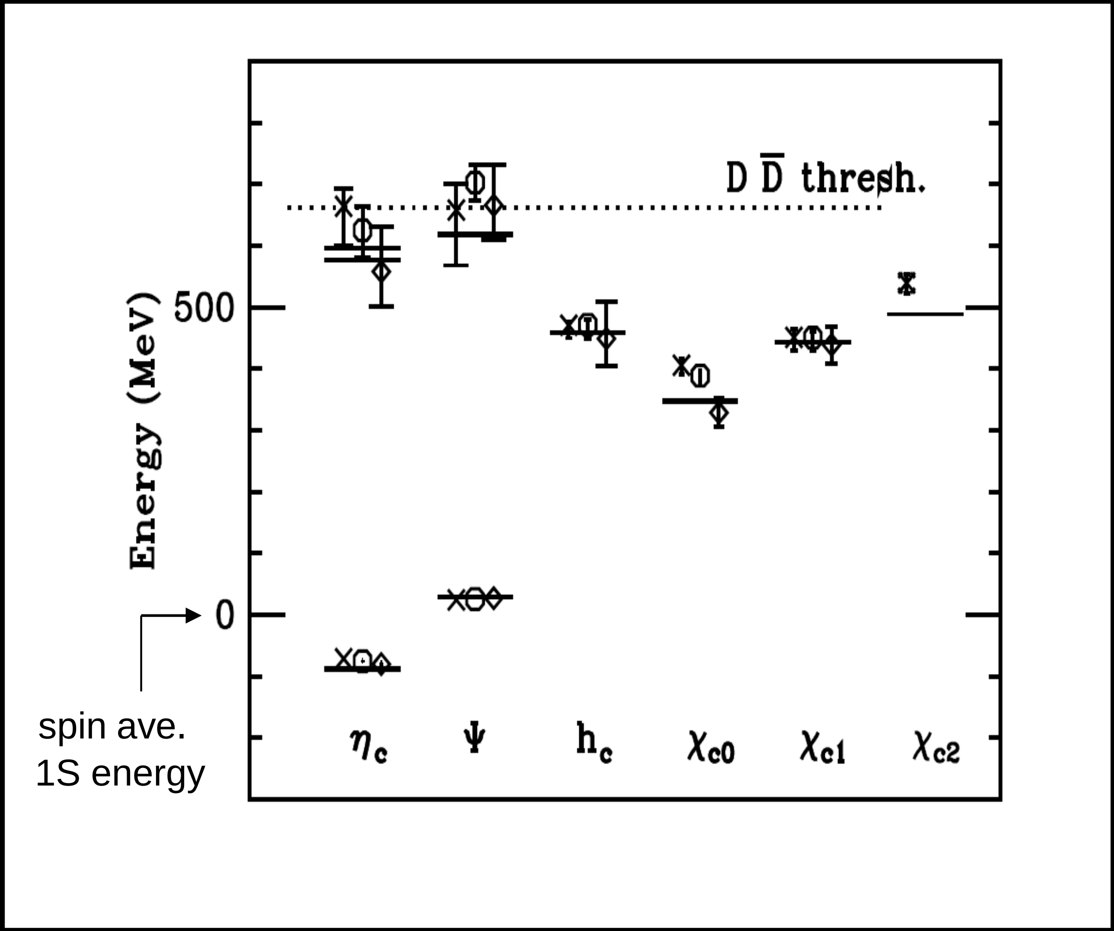


N_f= 2, Wilson sea-quarks, 24^3x40
a= 0.083 fm, L= 2 fm, m_p/m_r= 0.704

SESAM Coll., Phys.Rev.D71 (2005) 114513

Dynamic Probe at T=0 : charmonia spectra (full QCD)



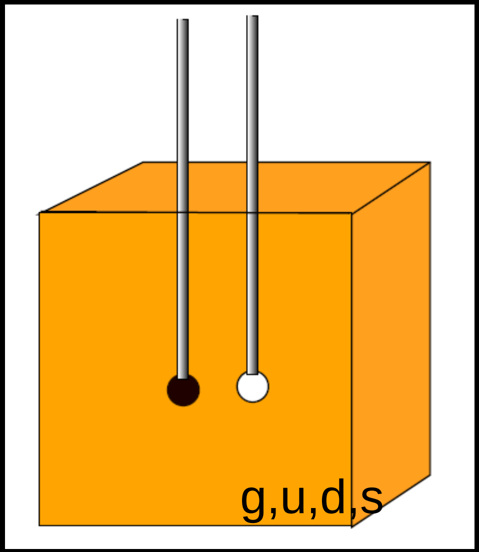


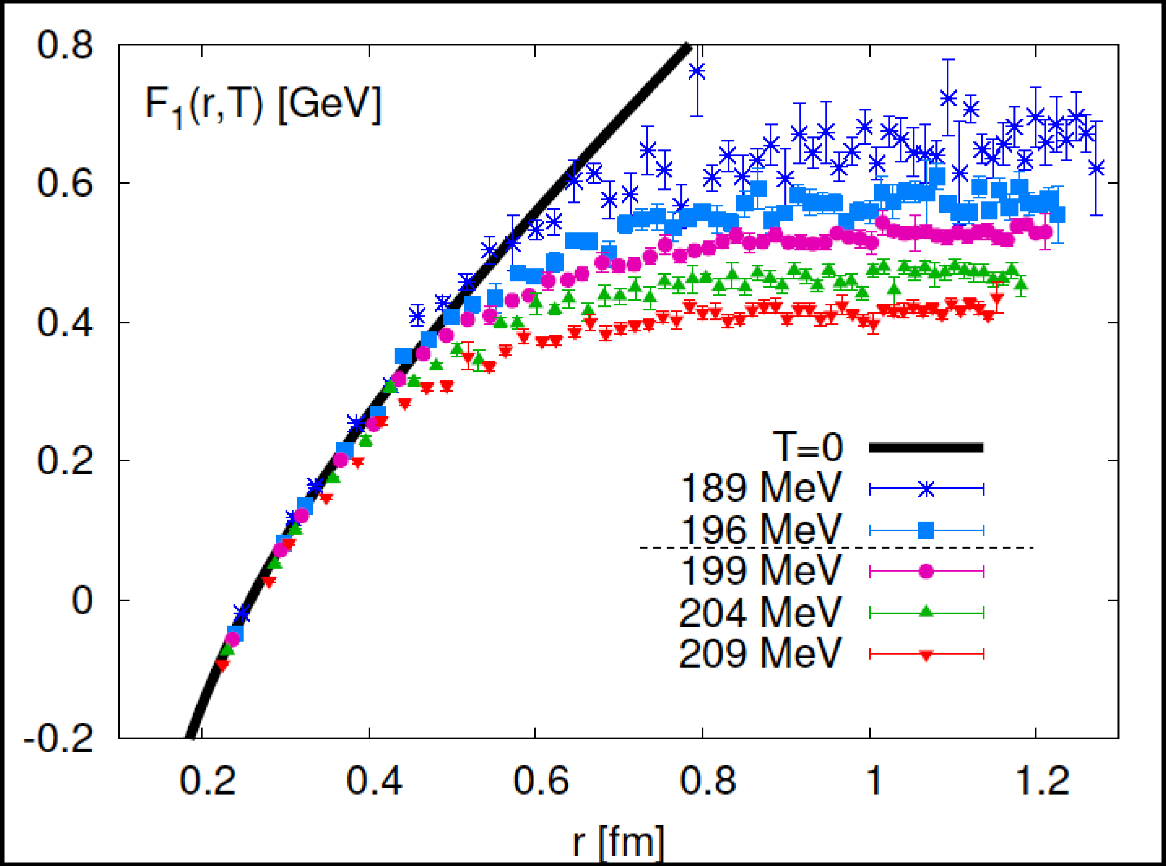
Note:
connection between spectroscopy and V(r) through $1/m_c$ expansion:
Eichten-Feinberg ('79)
Brown-Weisberger('79)

N_f= 2+1, staggered sea-quarks, 16^3x48, 20^3x64, 28^3x96
a = 0.18, 0.12, 0.086 fm, L= 2.8, 2.4, 2.4 fm

MILC Coll., PoS (LAT2005) 203 [hep-lat/0510072]

Free energy of quark + anti-quark (full QCD)

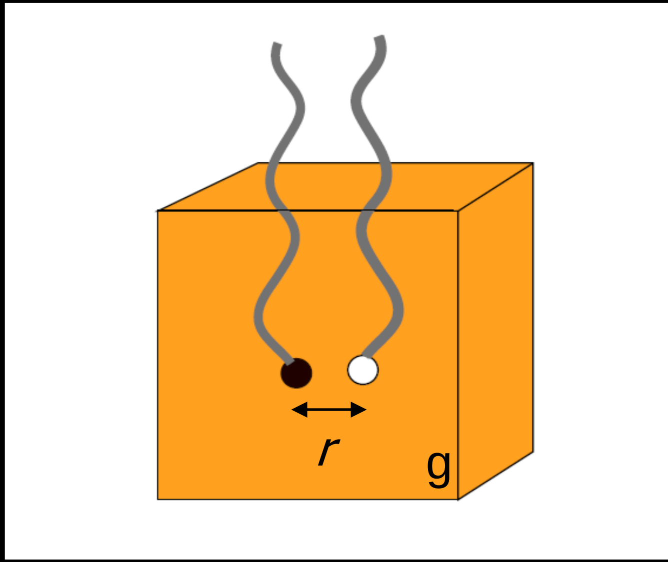


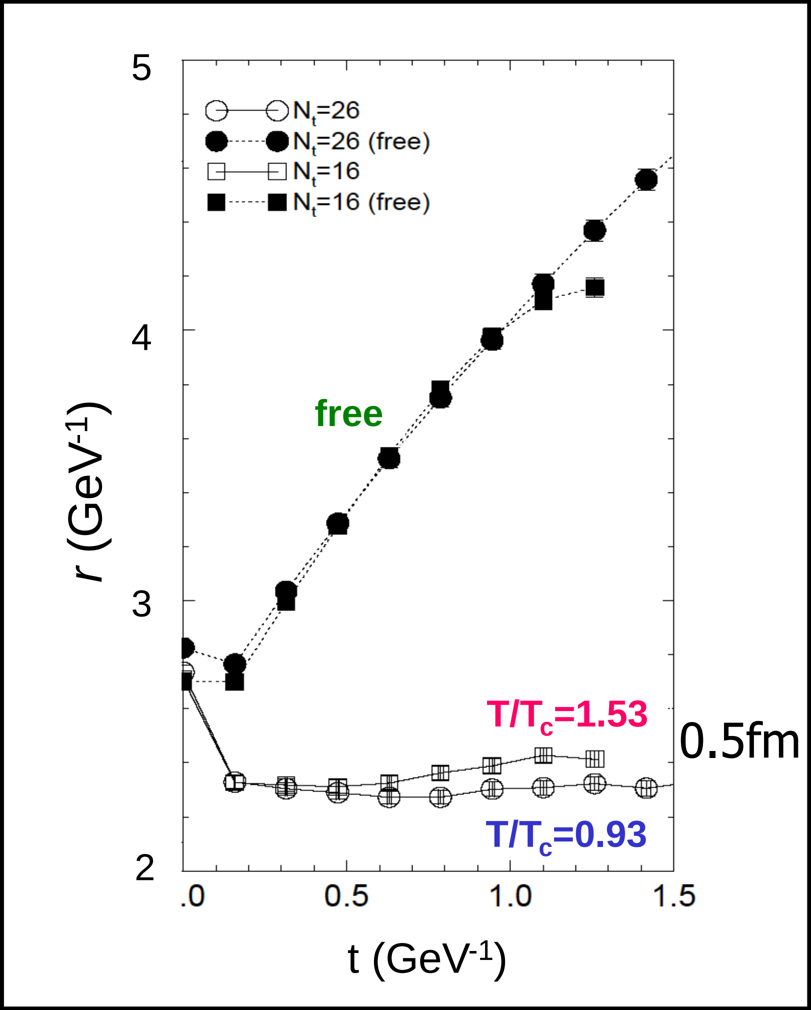


16^3x6, p4Fat3 action, m_{ud}/m_s=0.1, physical m_s

K. Petrov and RBC-Bielefeld Coll.

Charmonium "wave function" (quenched QCD)

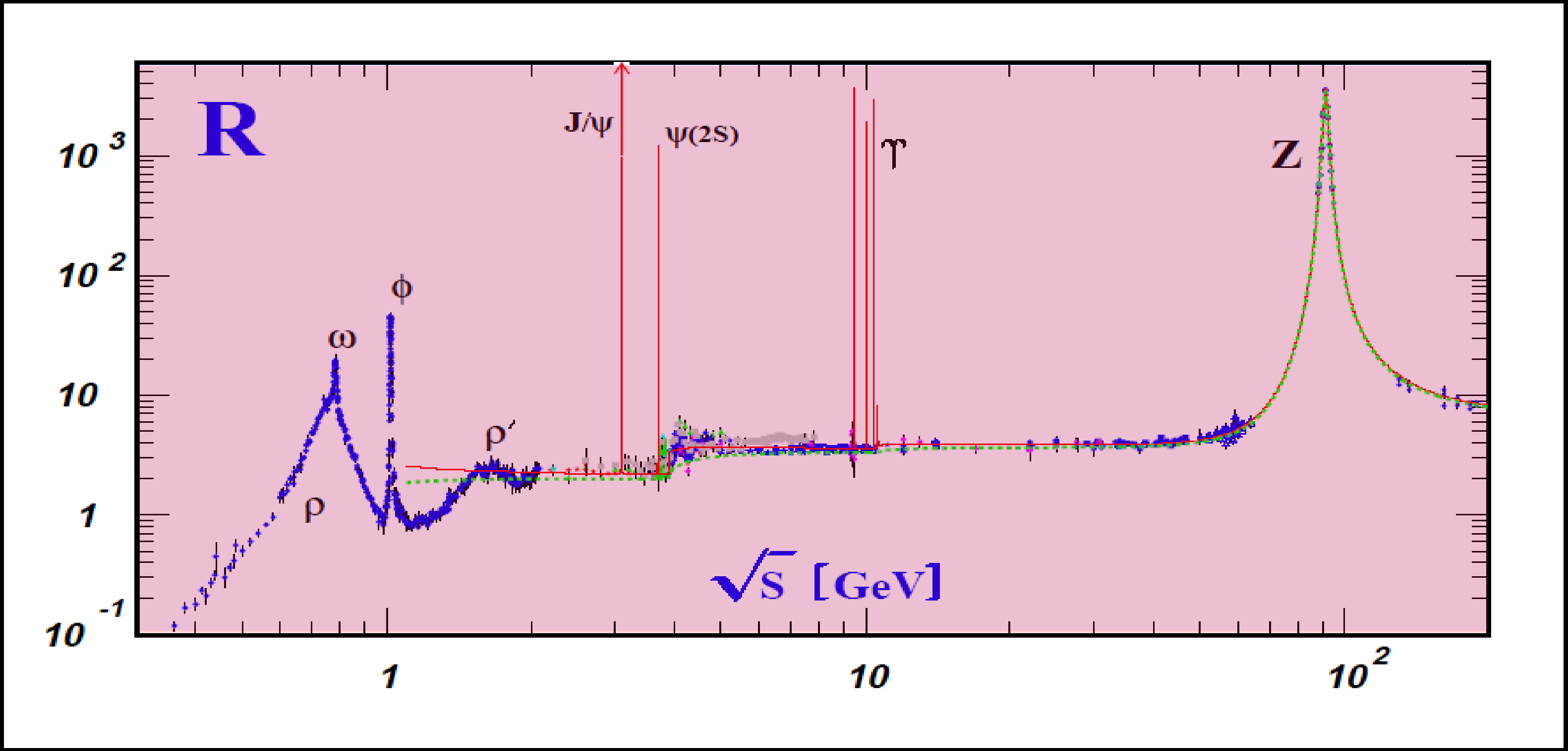




QCD-TARO Coll., Phys. Rev. D63 ('01)

QCD spectral Function

PDG('06)



$$D(\tau,\vec{p}) = \int \left\langle J(\tau,\vec{x}) J^{+}(0,0) \right\rangle e^{i\vec{p}\vec{x}} d^3x$$

$$= \int K(\tau,\omega)\, A(\omega,\vec{p})\, d\omega$$

MEM (Maximum Entropy Method): D → A

MEM (Maximum Entropy Method)

$$D(\tau, \vec{p}) = \int \left\langle J(\tau, \vec{x}) J^{+}(0,0) \right\rangle e^{i\vec{p}\vec{x}} d^3x$$

$$= \int K(\tau, \omega)\, A(\omega, \vec{p})\, d\omega$$

Lattice data

“Laplace” kernel

$$K(\tau, \omega) = e^{-\omega\tau} / (1 \mp e^{-\omega/T})$$

Spectral function

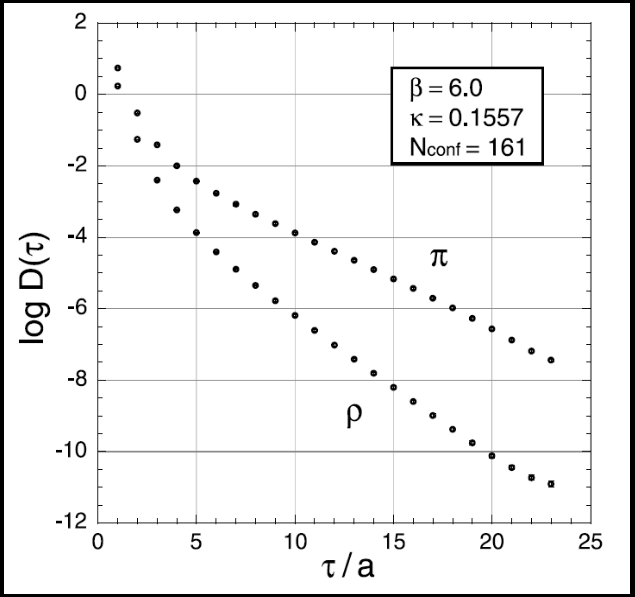
All the information of hadronic correlations

1. No parameterization necessary for A
2. Unique solution for A
3. Error estimate for A possible

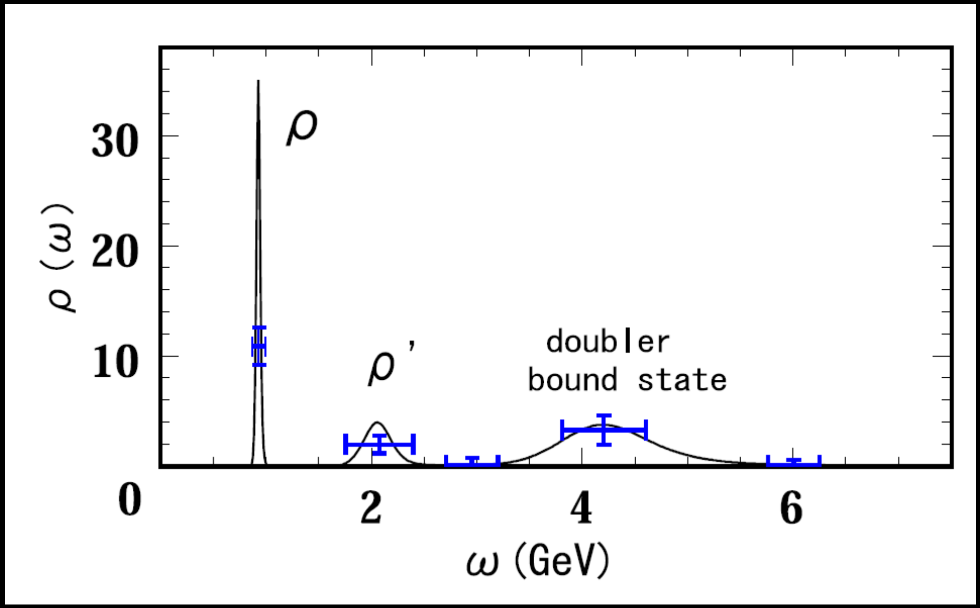
review:

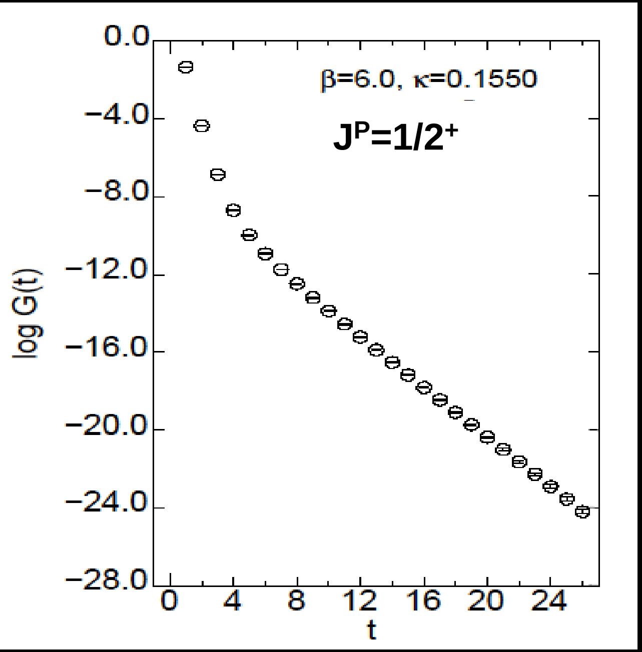
Asakawa, Nakahara & Hatsuda, hep-lat/0011040

Applications of MEM at T=0

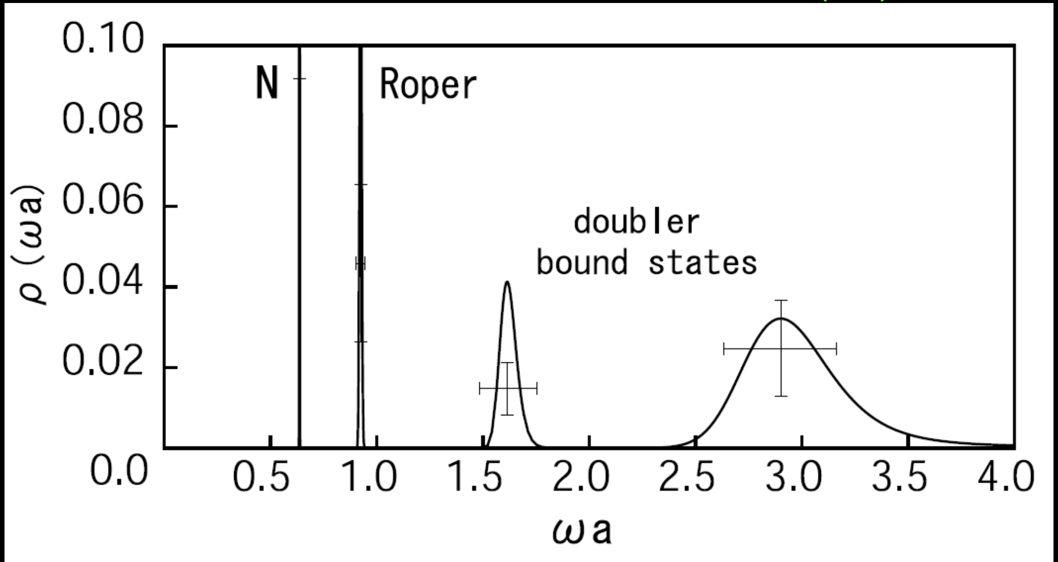


Asakawa, Nakahara & Hatsuda, PRD ('99)

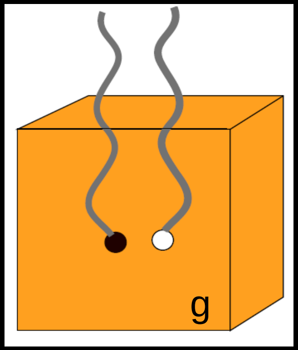


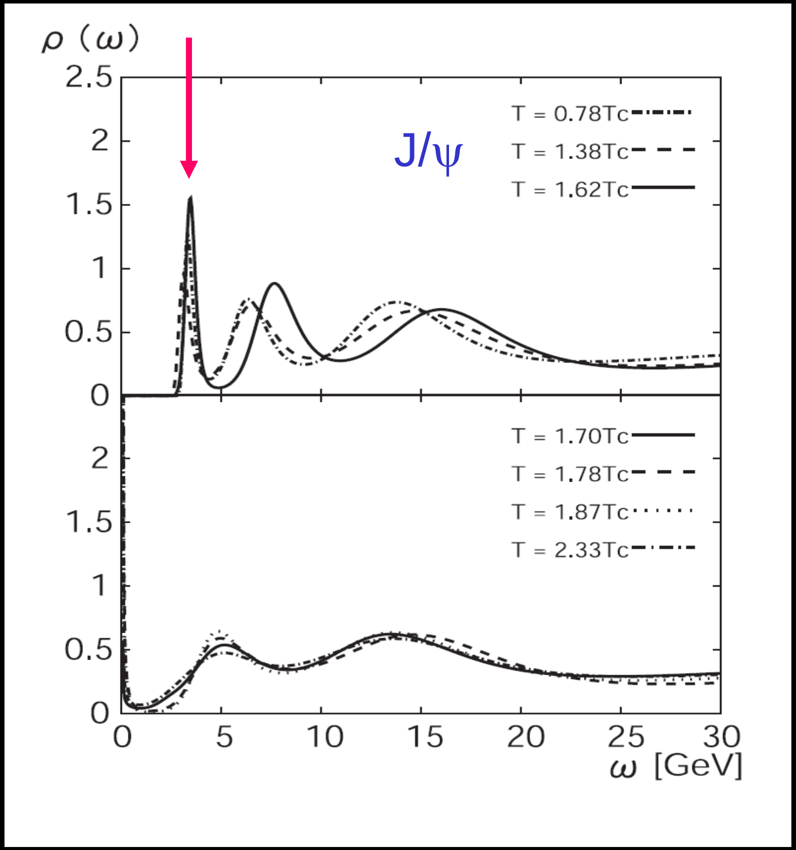


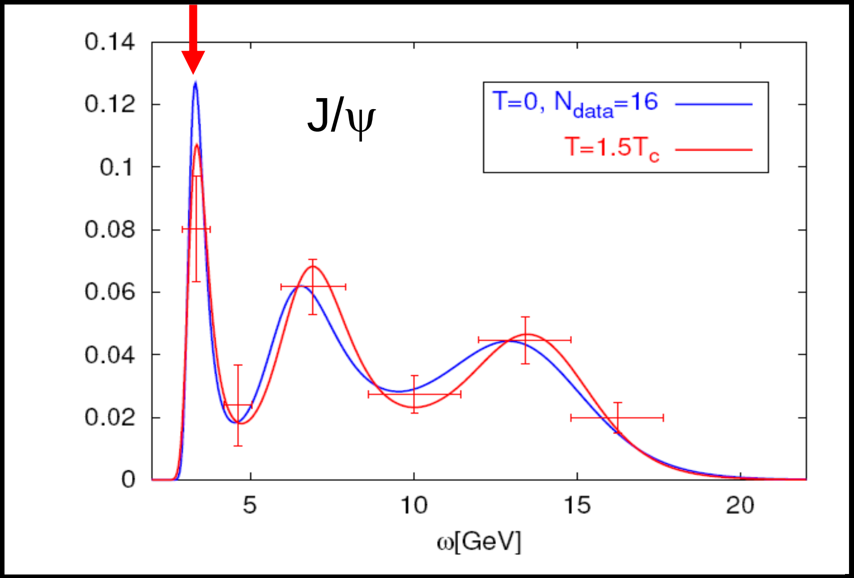
Sasaki, Sasaki & Hatsuda, PLB ('06)



Charmoniums at finiteT (quenched QCD)



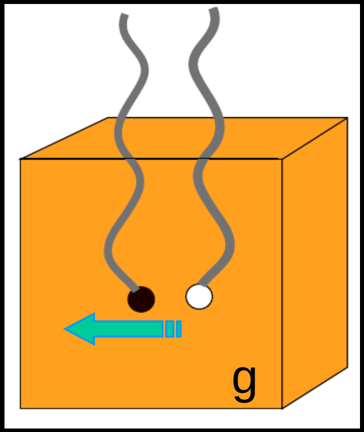


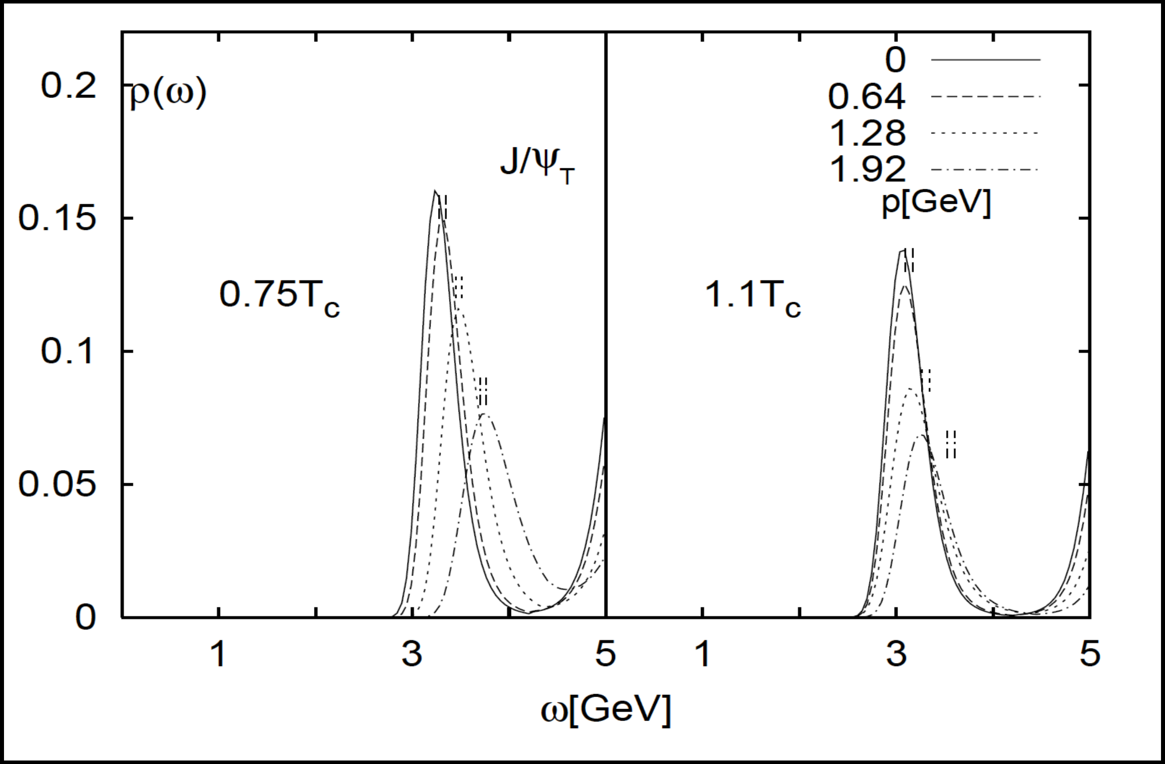


anisotropic lattice, 32^3 x (96-32)
ξ=4.0, a_t=0.01 fm, (L_s=1.25fm)
Asakawa and Hatsuda, PRL ('04)

anisotropic lattice, 24^3 x (160-34)
ξ=4.0, a_t=0.056 fm, (L_s=1.34 fm)
Jakovac, Petreczky, Petrov & Velytsky
Hep-lat/0603005

Moving J/Ψ at finite T (quenched QCD)

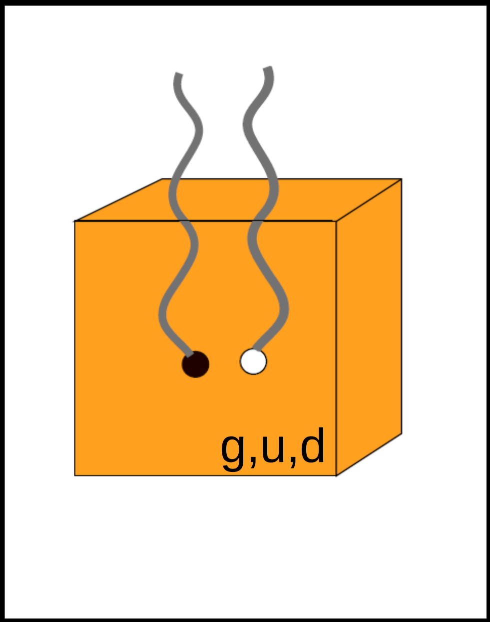


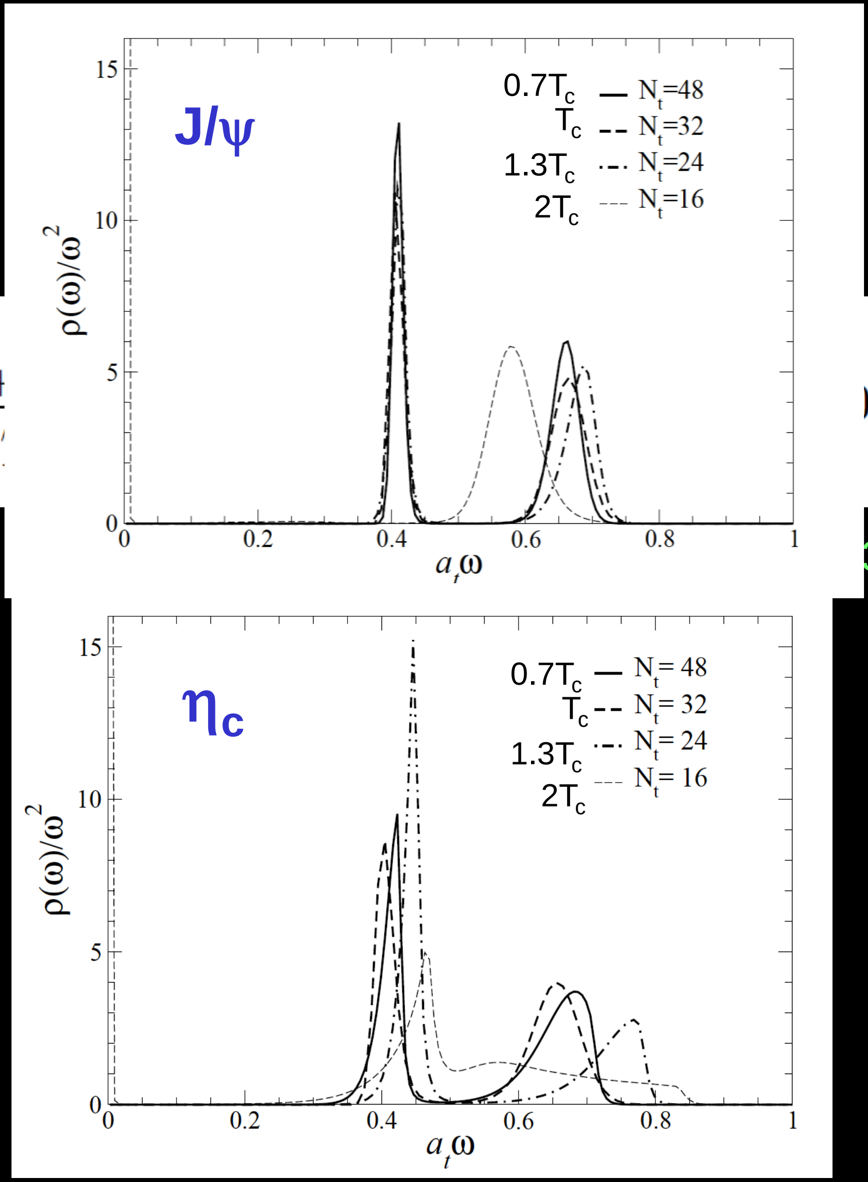


quenched

Datta, Karsch, Wissel, Petreczky & Wetzorke, [hep-lat/0409147]

Charmoniums at finiteT (full QCD)

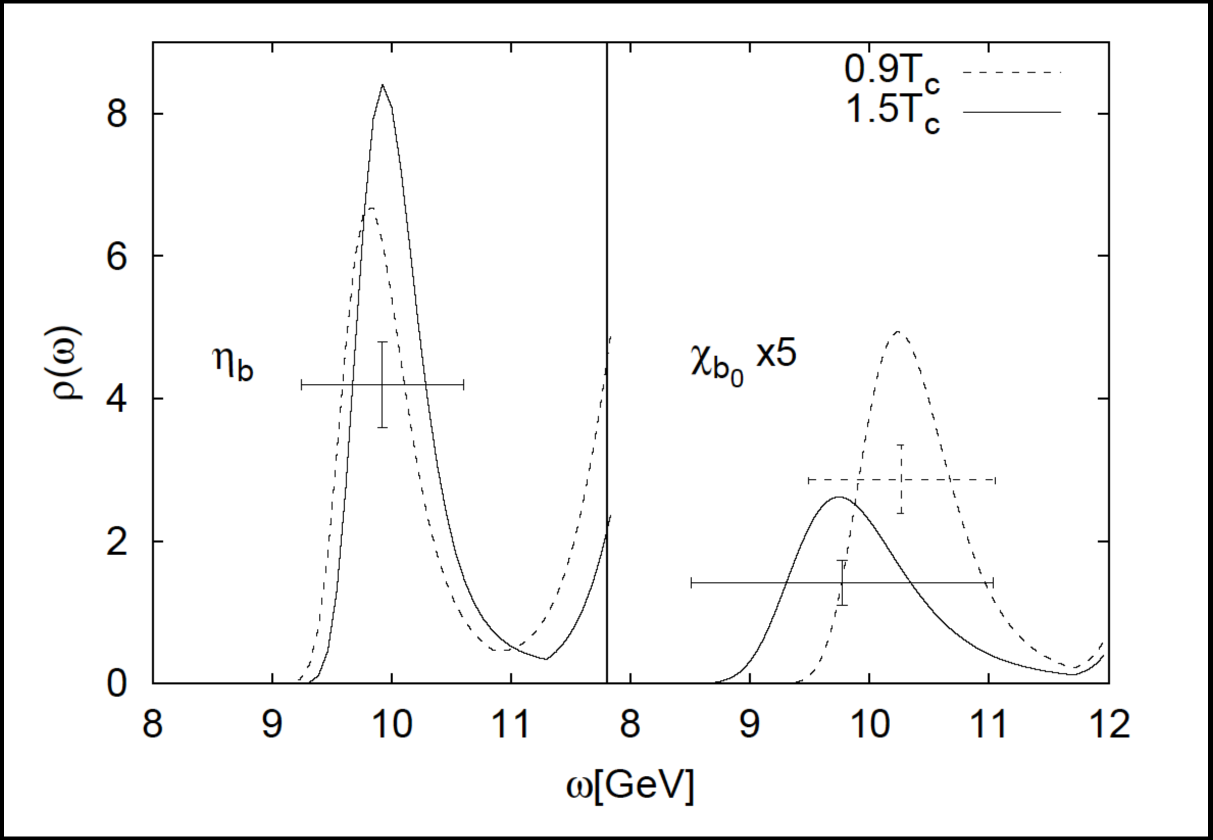




N_f=2, anisotropic lattice, 8^3 x (48,32,24,16)
ξ=6.0, a_s=0.2 fm, a_t=0.033 fm, (L_s=1.6 fm)
m_π/m_ρ=0.55

Aarts, et.al., [hep-lat/0511028]

Bottomoniums at finiteT (quenched QCD)

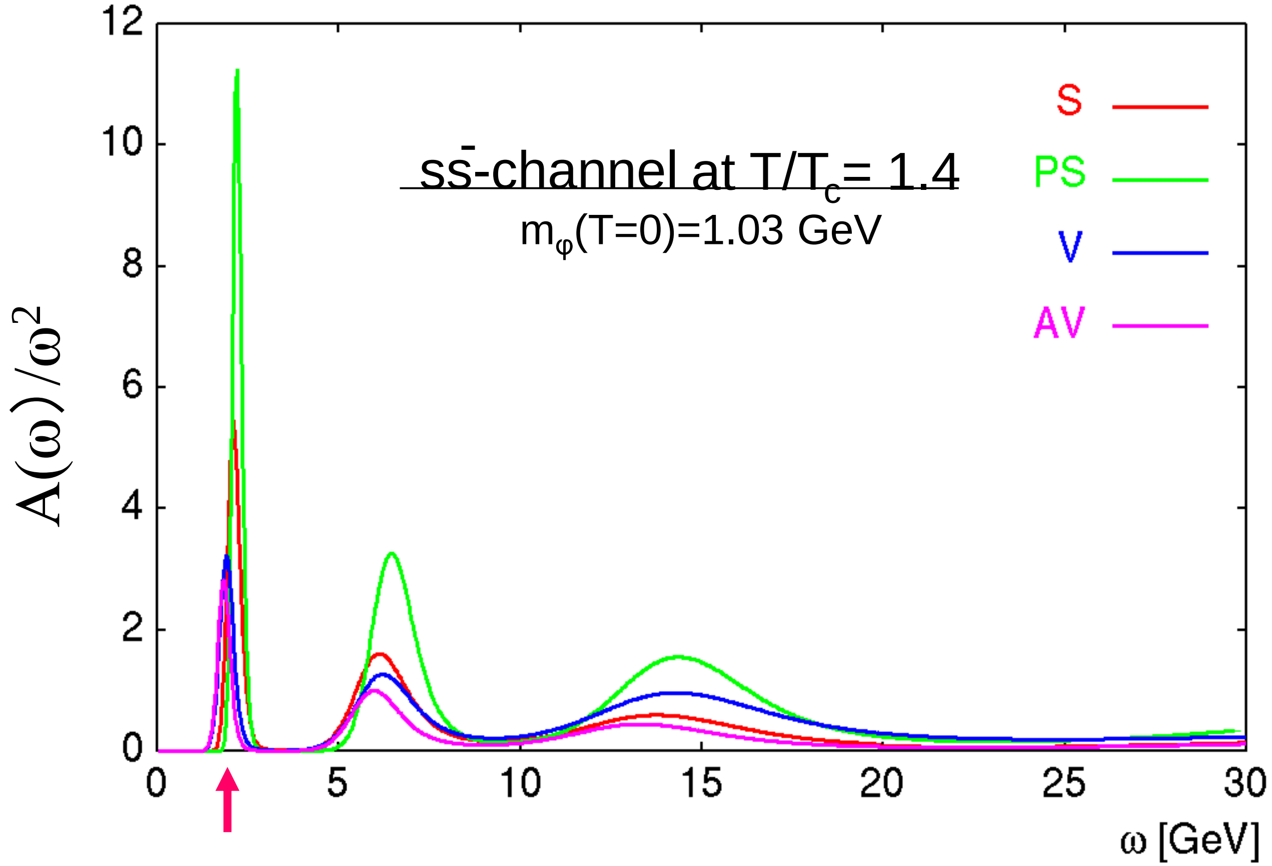


quenched, a = 0.02 fm

Datta, Jakovac, Karsch & Petreczky, [hep-lat/0603002]

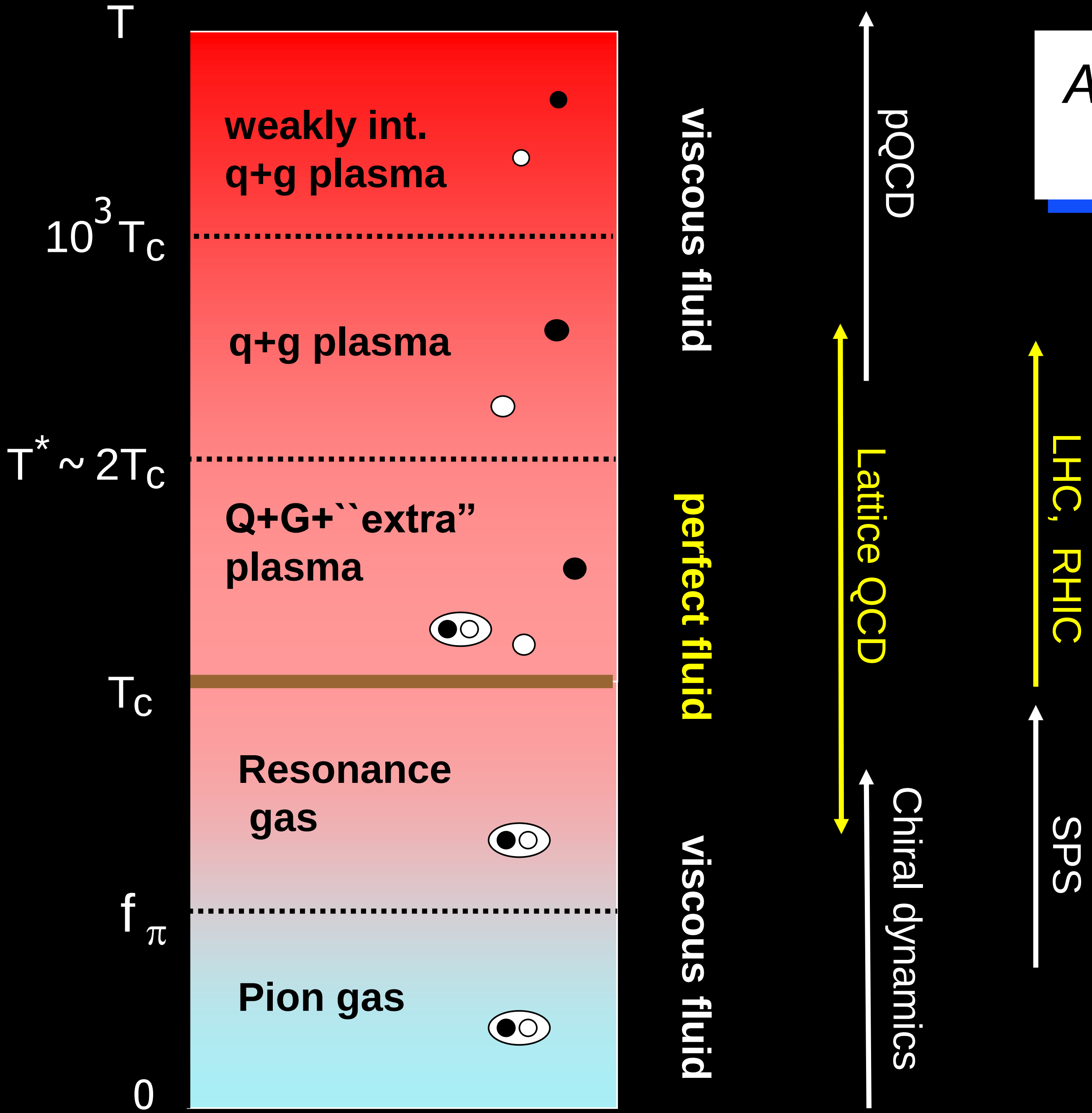
Light mesons at finiteT (quenched QCD)

$m_{ud} << m_s \sim T_c << m_c < m_b$



Asakawa, Nakahara & Hatsuda, [hep-lat/0208059]

A new “paradigm” of hot QCD



Summary

1. Hot QCD is strongly interacting at $T_c < T < T^*$?

Just like high T_c superconductor

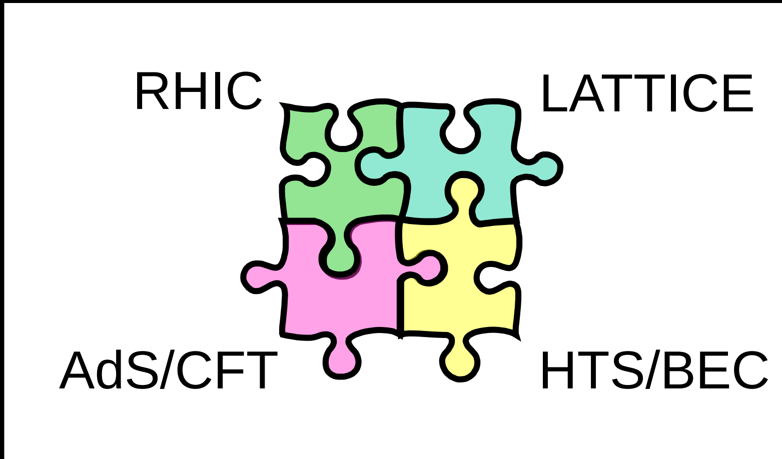
BEC regime of systems of atomic fermions

2. Several critical points in (T,μ)-plane ?

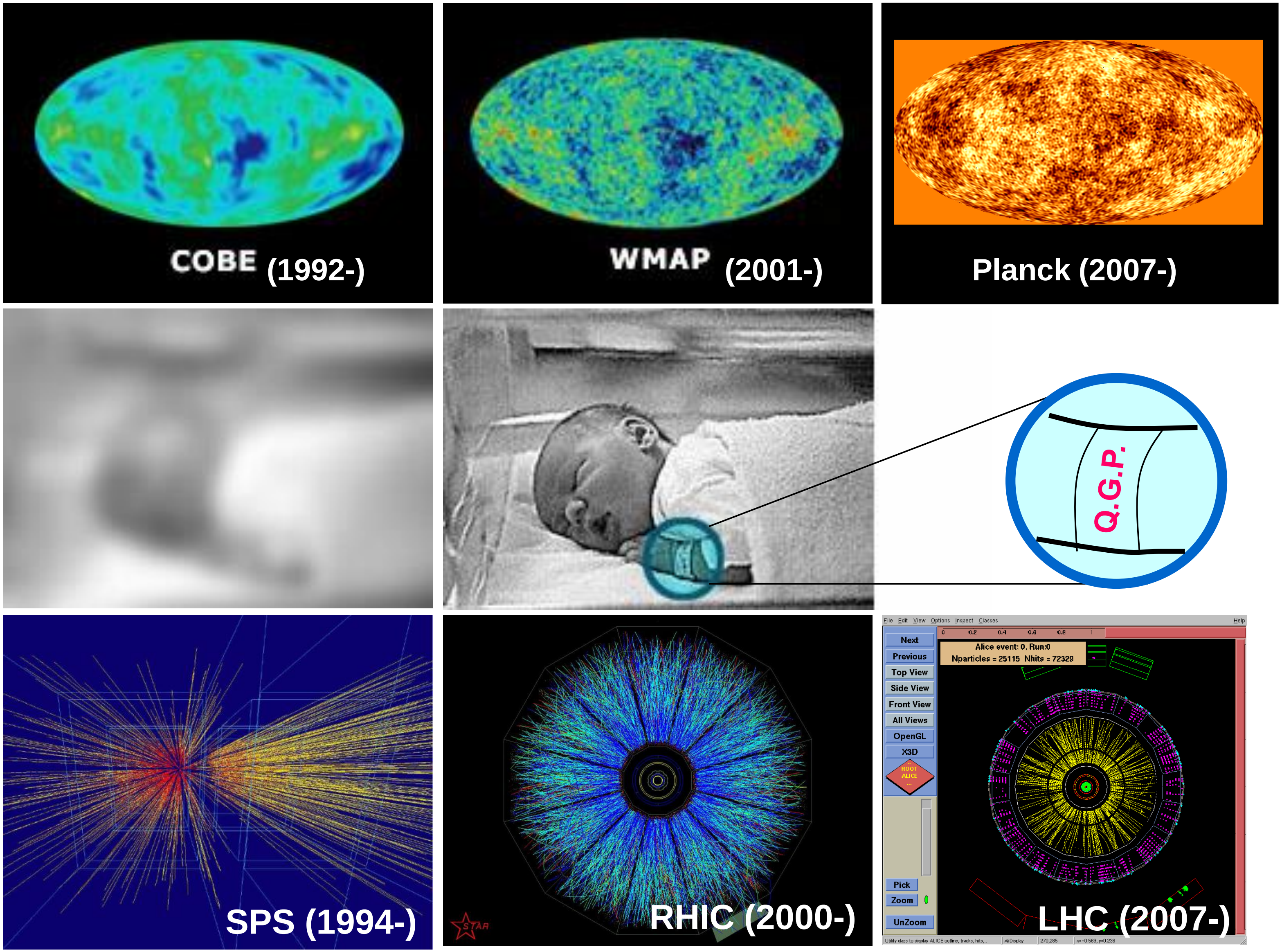
CP at high T and CP at high μ

3. Progress in spectral analysis on the lattice

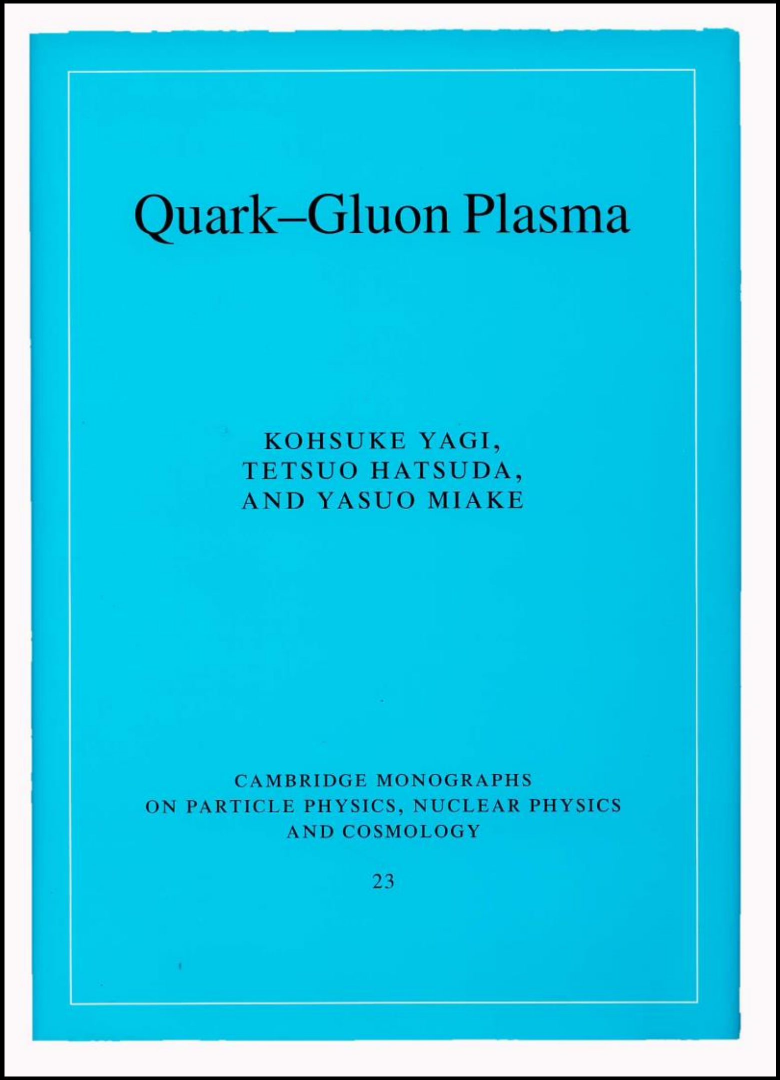
Heavy and light bound states above T_c
Small viscosity even up to 30 T_c ?
Full QCD studies are started



Big Bang



Little Bang



(Cambridge Univ. Press, 2005)

1. What is quark-gluon plasma

Part I. Basic Concept of Quark-Gluon Plasma:

2. Introduction to QCD
3. Physics of quark-hadron phase transition
4. Field theory at finite temperature
5. Lattice gauge approach to QCD phase transitions
6. Chiral phase transition
7. Hadronic states in hot environment

Part II. QGP in Astrophysics:

8. QGP in the early universe
9. Compact stars

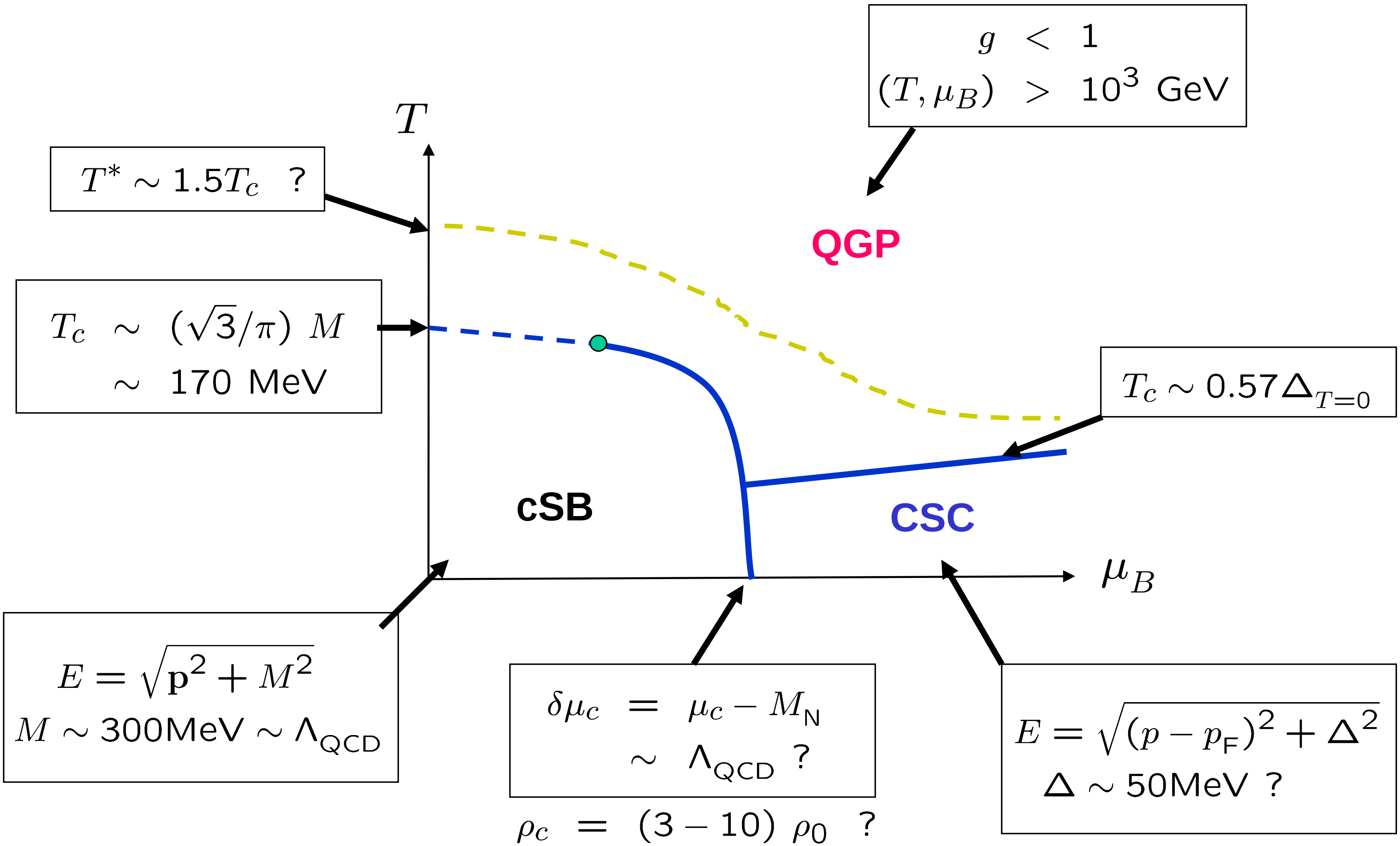
Part III. QGP in Relativistic Heavy Ion Collisions:

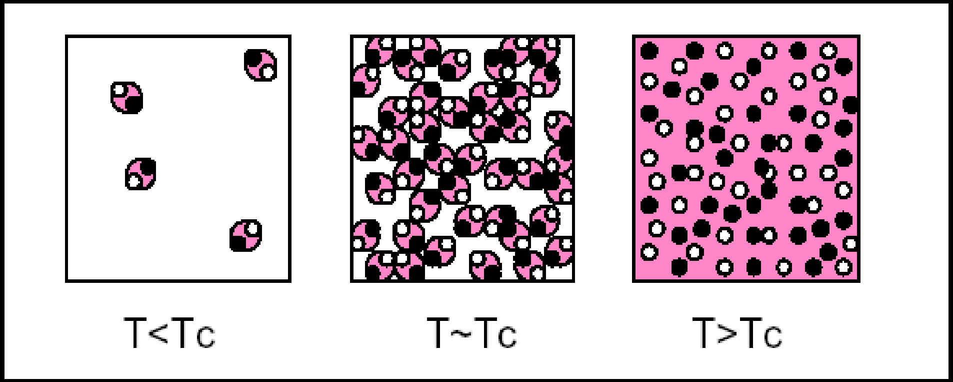
10. Introduction to relativistic heavy ion collisions
11. Relativistic hydrodynamics for heavy ion collisions
12. Transport theory for pre-equilibrium process
13. Formation and evolution of QGP
14. Fundamentals of QGP diagnostics
15. Results from CERN-SPS experiments
16. First results from BNL-RHIC
17. Detectors in relativistic heavy ion experiments

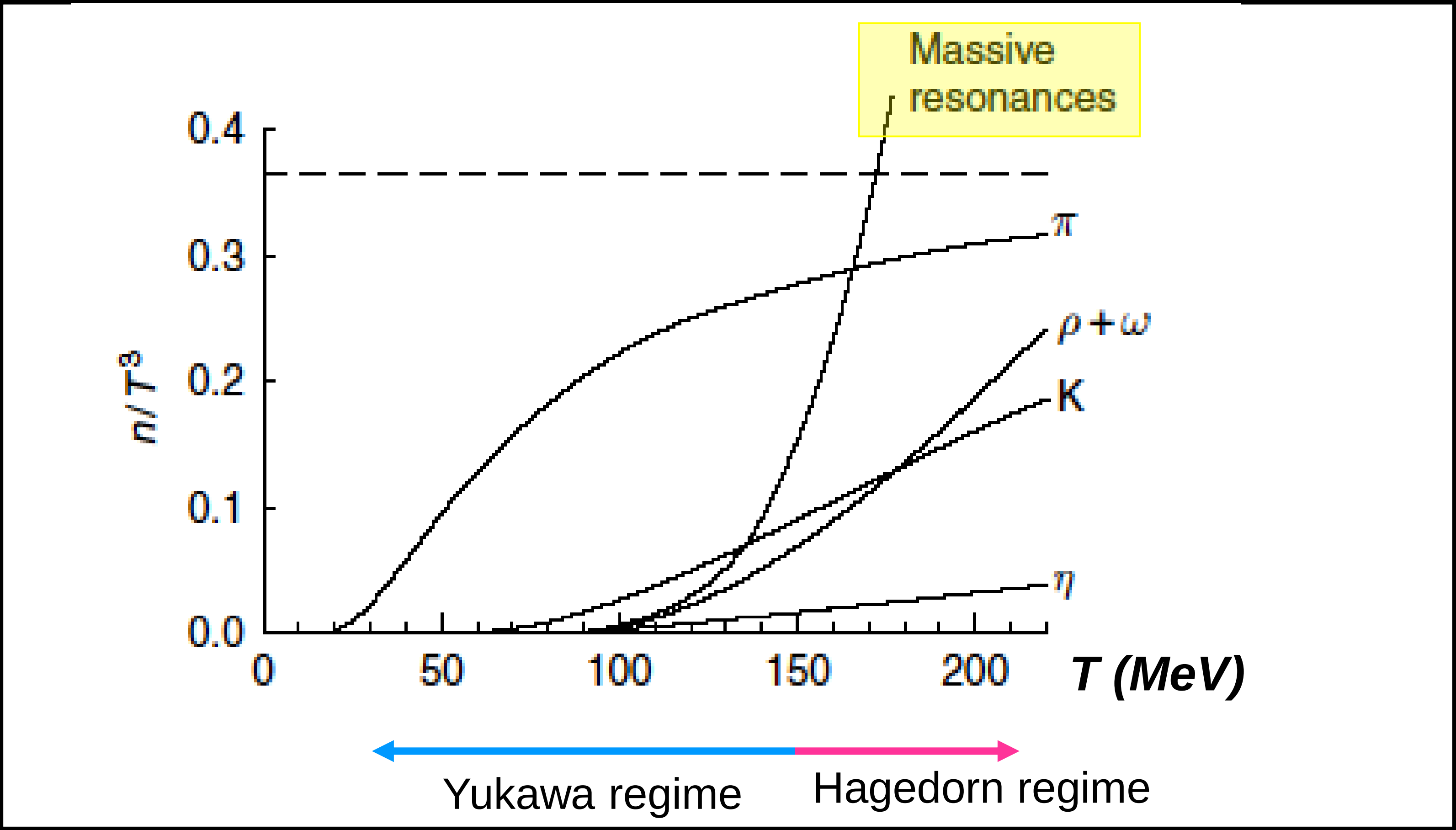
http://utkhii.px.tsukuba.ac.jp/cupbook/index.html

Back up slides

Scale of each “phase”



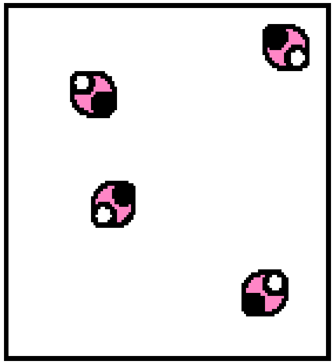

T<Tc
T~Tc
T>Tc


Massive resonances
0.4
0.3
0.2
0.1
0.0
n/T3
π
ρ+ω
K
η
0
50
100
150
200
T (MeV)
Yukawa regime
Hagedorn regime

Percolation picture

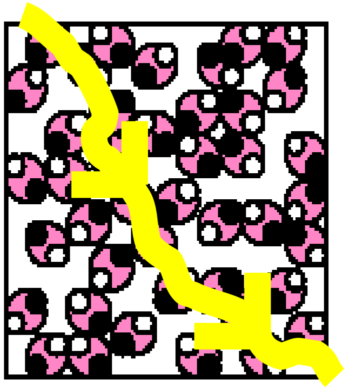
Dilute gas

$T \sim 0$



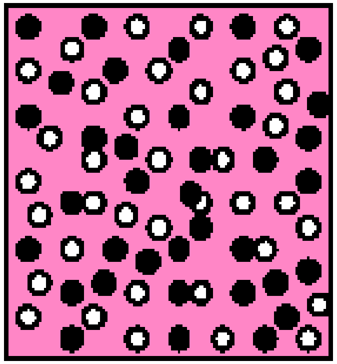
Percolation transition

$T \sim T_c$



Closely packed

$T \sim T_c^*$



Volume fraction: $\frac{N_\pi(T_c)\cdot v_\pi}{V} \simeq 0.35$ $\quad \frac{N_\pi(T_c^*)\cdot v_\pi}{V} \simeq 1$

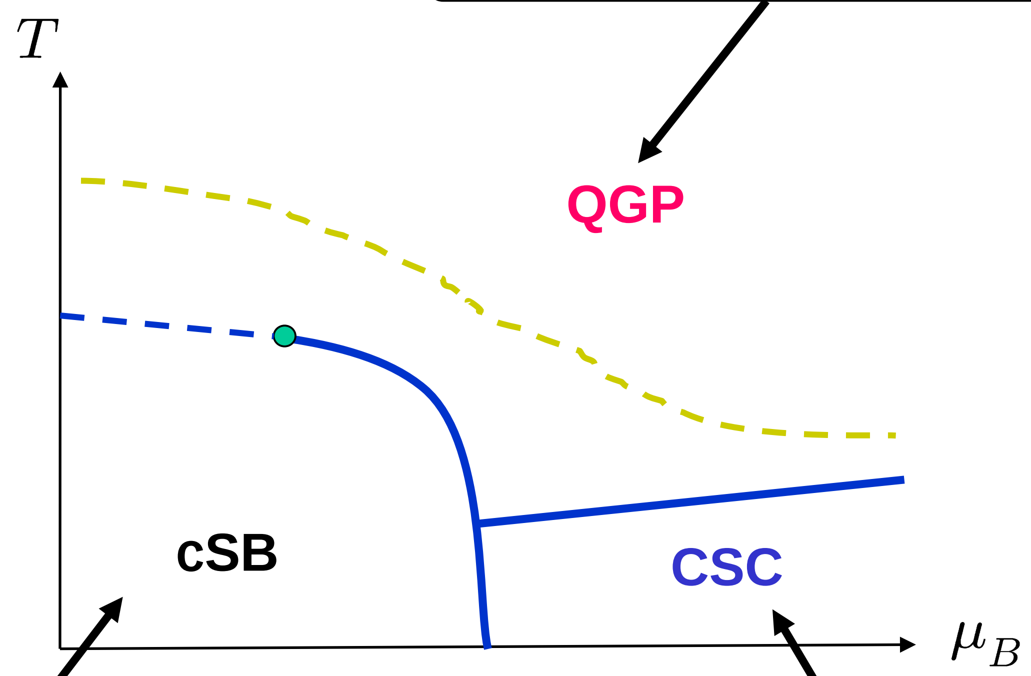
$$T_c = \left(\frac{\pi}{4\zeta(3)}\right)^{1/3} \frac{0.35^{1/3}}{R_\pi} \simeq 186 \text{ MeV}$$

$$T_c^* \simeq \left(\frac{1}{0.35}\right)^{1/3} T_c = 1.4 T_c$$

T.H., ('97)

Symmetry of each "phase" (case for small m_{ud} with $m_s=\infty$)

$$SU_C(3)\times[SU_L(2)\times SU_R(2)]\times U_B(1)$$



$$SU_C(3)\times SU_{L+R}(2)\times U_B(1)$$

$$\langle \bar{q}q \rangle \neq 0$$

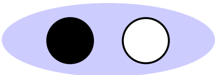
$$SU_C(2)\times[SU_L(2)\times SU_R(2)]\times \tilde{U}_{B+C}(1)$$

$$\langle qq \rangle \neq 0$$

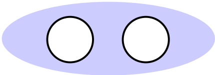
· Ginzburg-Landau Potential (3-flavor, chiral limit)

Symmetry: $$\mathrm{G} = SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A \times SU(3)_C$$

Chiral modes: $$\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}_{\mathrm{R}}{}^j q^i_{\mathrm{L}}$$



Diquark modes: $$[d_L^\dagger]_{ai} \sim \epsilon_{abc}\epsilon_{ijk} \langle (q_L)^j_b C (q_L)^k_c \rangle$$



G	$SU(3)_L$	$SU(3)_R$	B#	A#	$SU(3)_C$
Φ	**3**	**3***	0	2/3	**1**
d_L	**3**	**1**	2/3	-2/3	**3**
d_R	**1**	**3**	2/3	2/3	**3**

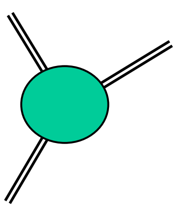
- **Ginzburg-Landau Potential (3-flavor, chiral limit)**

Yamamoto, Tachibana, Baym & Hatsuda, hep-ph/0605018

$$\Omega_\chi = \frac{a_0}{2}\mathrm{tr}\ \Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\ \Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$
$$\boxed{-\frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right)},$$

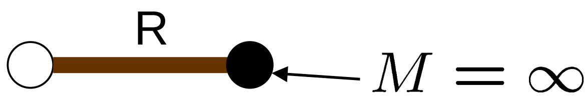
$$\Omega_d = \alpha_0\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]$$
$$+\ \beta_1\left([\mathrm{tr}(d_L d_L^\dagger)]^2 + [\mathrm{tr}(d_R d_R^\dagger)]^2\right) + \beta_2\left(\mathrm{tr}[(d_L d_L^\dagger)^2] + \mathrm{tr}[(d_R d_R^\dagger)^2]\right)$$
$$+\ \beta_3\ \mathrm{tr}[(d_R d_L^\dagger)(d_L d_R^\dagger)] + \beta_4\ \mathrm{tr}(d_L d_L^\dagger)\mathrm{tr}(d_R d_R^\dagger)$$

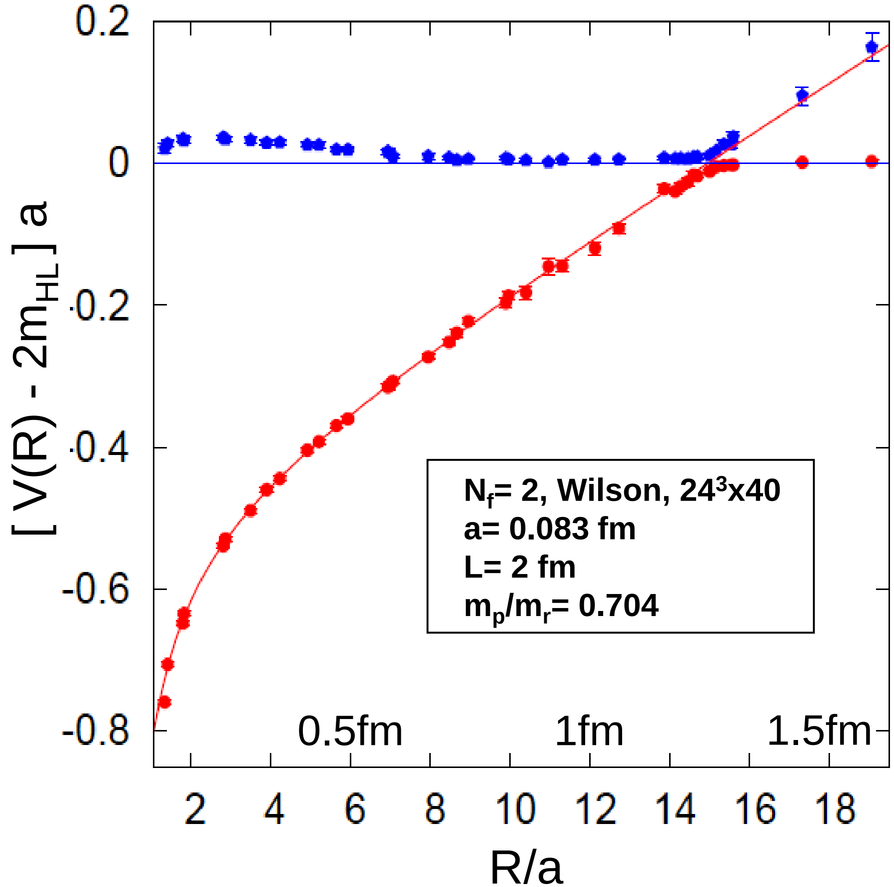
$$\Omega_{\chi d} = \boxed{\gamma_1\ \mathrm{tr}[(d_R d_L^\dagger)\Phi + (d_L d_R^\dagger)\Phi^\dagger]}$$
$$+\lambda_1\ \mathrm{tr}[(d_L d_L^\dagger)\Phi\Phi^\dagger + (d_R d_R^\dagger)\Phi^\dagger\Phi] + \lambda_2\ \mathrm{tr}[d_L d_L^\dagger + d_R d_R^\dagger]\cdot\mathrm{tr}[\Phi^\dagger\Phi]$$
$$+\lambda_3\left(\det\Phi\cdot\mathrm{tr}[(d_L d_R^\dagger)\Phi^{-1}] + h.c\right)$$

$\boxed{}$ = $U(1)_A$ breaking terms = 

Examples in full lattice QCD

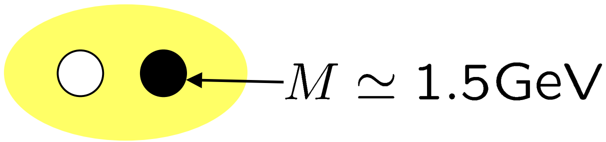
Confining string

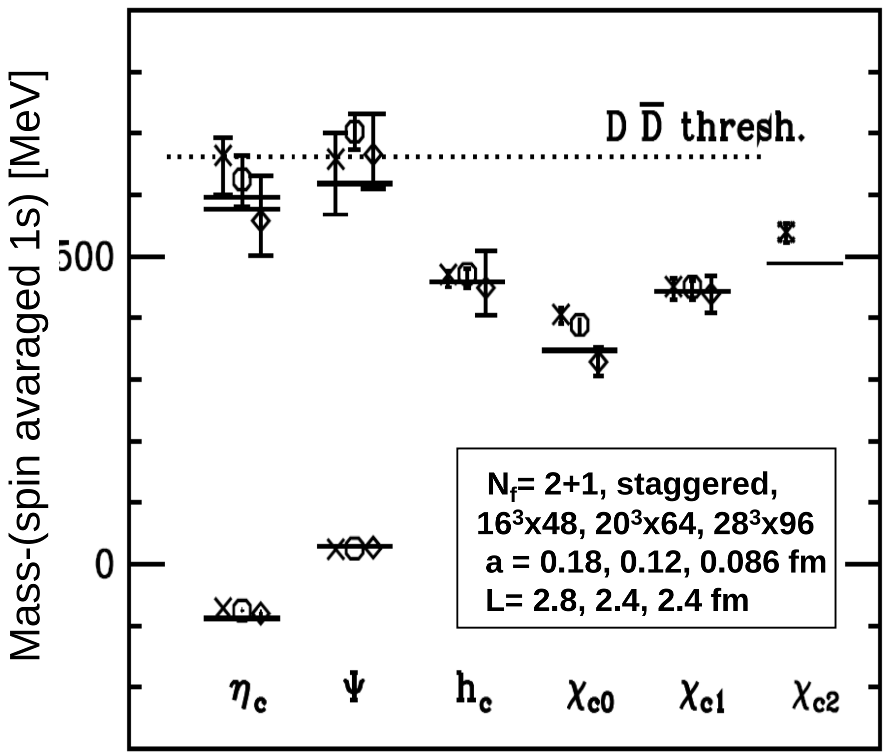




SESAM Coll., Phys.Rev.D71 (2005) 114513

Heavy bound states



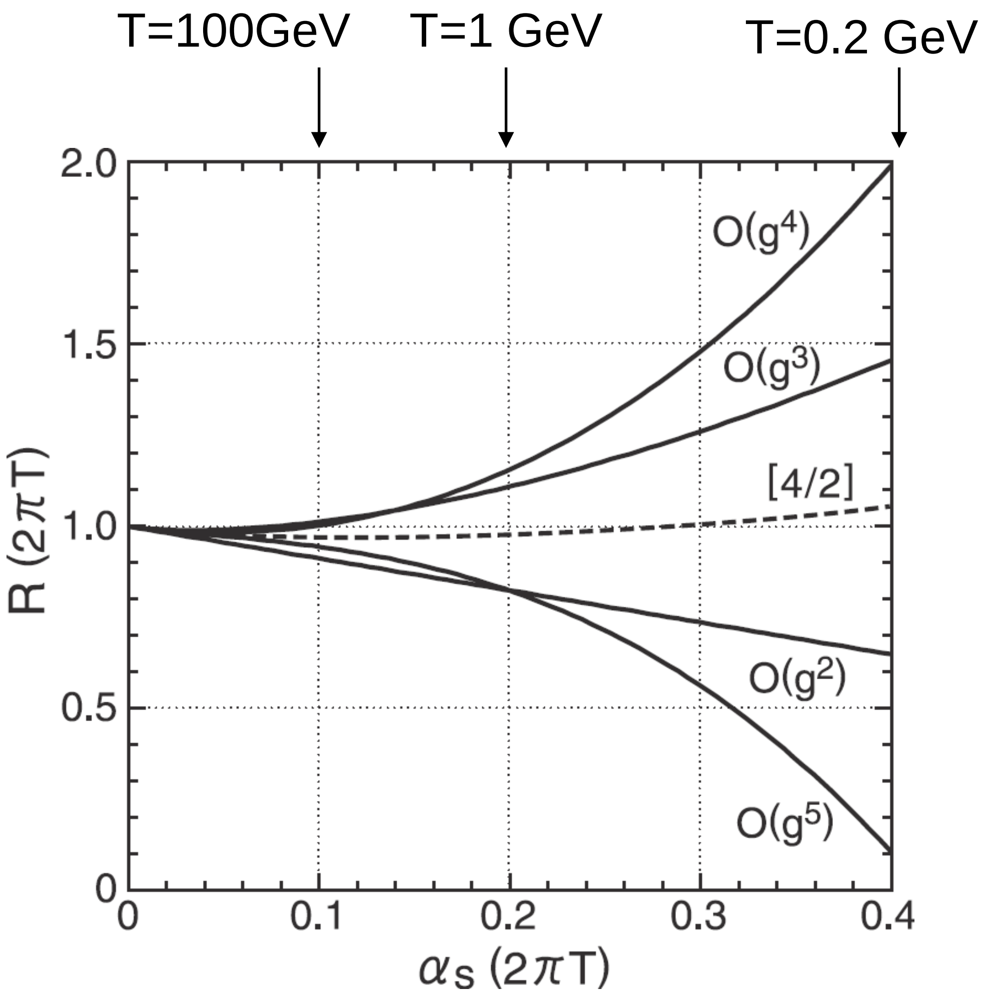


MILC Coll., hep-lat/0510072

QCD Pressure near T_c

QCD Pressure (N_f=4)

$$R = \frac{P}{P_{\rm ideal}}$$
$$= 1 - 2.76\left(\frac{\alpha_s}{\pi}\right) + 17.8\left(\frac{\alpha_s}{\pi}\right)^{3/2} + \left(81.2 + 15.9 \ln\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha_s}{\pi}\right)^2 - 327\left(\frac{\alpha_s}{\pi}\right)^{5/2} + \cdots$$



- naive perturbation: meaningful only for T>100 GeV
- resummation may improve the situation

QGP for g << 1 (T >> 100 GeV)

Relativistic plasma :

$$n \sim T^3 \;\rightarrow\; r \sim 1/T$$

$$\lambda_d \sim 1/T, \quad \lambda_D \sim 1/(gT)$$

$$\frac{1}{T} \ll \frac{1}{gT} \ll \frac{1}{g^2T}$$

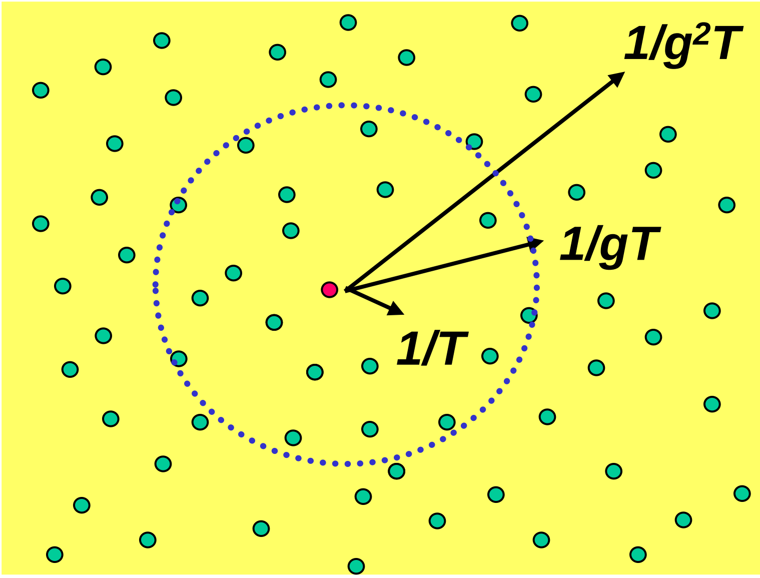
Inter-particle distance | Electric screening | Magnetic screening

Debye number :

$$N_D = \frac{4\pi}{3}\lambda_D^3 n \sim (2/g)^3$$

"Coulomb" coupling parameter :

$$\Gamma = \frac{\text{Coulomb}}{\text{Kinetic}} \sim \frac{\alpha_s T}{T} = \frac{g^2}{4\pi}$$



S. Ichimaru, Rev. Mod. Phys. 54 ('82) 1071

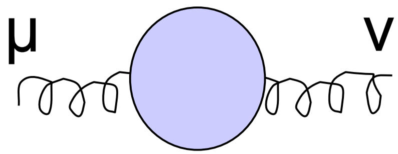
Non-Abelian magnetic problem

EOS :

$$P = \frac{8\pi^2}{45} T^4 \left[1 + \sum_{n=2}^{5} c_n g^n + C' \right]$$

A. Linde,
Phys. Lett. B96 ('80) 289

$$\langle A_\mu(\mathbf{x}) A_\nu(\mathbf{0}) \rangle \sim e^{-m|\mathbf{x}|}$$



magnetic screening :

$$\omega_{\mathrm{m}} \simeq C \, (g^2 T)$$

"Debye" screening :

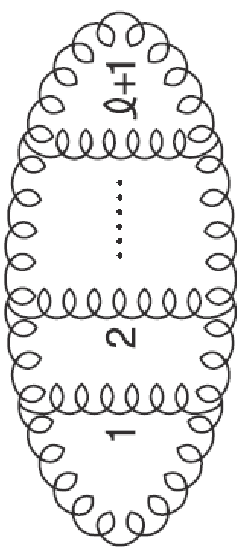
$$\omega_{\mathrm{e}} = gT \sqrt{1 + \frac{3}{2\pi} g \left(\ln \frac{2gT}{\omega_{\mathrm{m}}} - 0.5 \right)}$$

Kraemmer & Rebhan,
Rept.Prog.Phys.67 ('04)351

QCD is non-perturbative even at T = ∞

soft magnetic gluons are always non-perturbative
even if g → 0 (T → ∞)

⇒ ~~pertubation theory~~ from O(g^6)



$$P_{(2l)} \sim g^{2l} \left(T \int_m^T d^3k \right)^{l+1} \left(\frac{1}{k^2} \right)^{3l} k^{2l}$$

$$\begin{cases} l < 3: & g^{2l} T^4, \\ l = 3: & g^6 T^4 \ln(T/m), \\ l > 3: & g^6 T^4 (g^2 T/m)^{l-3} \end{cases}$$

($\omega_m \sim g^2 T$)