Thermal phenomenological AdS/QCD

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(hep-ph/0609008)

1. Thermal AdS/QCD
2. In-medium parameters
3. Chiral phase transition
4. Coupling constants

AdS/QCD (T=0)

Ingredients:

Relevant operators for chiral dynamics:		5D Theory:	$SU(3)_L \times SU(3)_R$
$\bar{q}_R q_L$	$\longrightarrow$	scalar Φ	$(\bar{3},3)$
$\bar{q}_L \gamma^\mu q_L$	$\longrightarrow$	vector L_M	(8,0)
$\bar{q}_R \gamma^\mu q_R$	$\longrightarrow$	vector R_M	(0,8)

$\hookrightarrow$ 5D gauging of $SU(3)_L \times SU(3)_R$

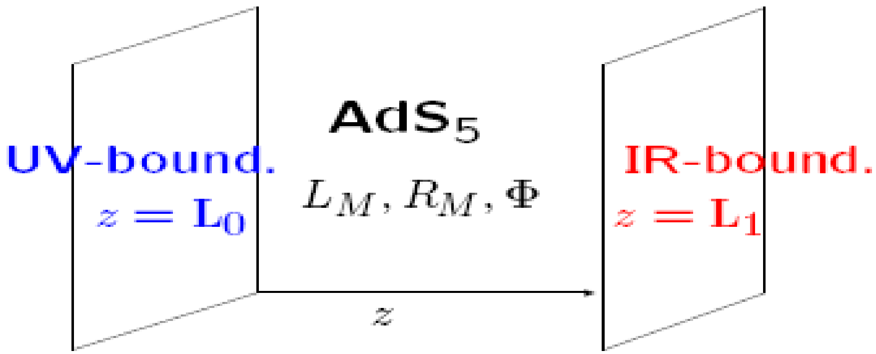
Chiral breaking: $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$

Quark masses M_q, $\langle \bar{q}_R q_L \rangle$ $\Big\}$ $\longrightarrow$ $\langle \Phi \rangle \propto \mathbb{1}$ Higgsing!

Dim[$\bar{q}_R q_L$]=3 $\longrightarrow$ $M_\Phi^2 = -3$

Leandro Da Rold

A QCD-like holographic model



AdS geommetry $\longrightarrow$ conformal 4D theory in UV
boundary at $z = L1$ $\longrightarrow$ mass gap

$$\mathcal{L}_5 = \frac{M_5}{2} \text{Tr}\left[-L_{MN}L^{MN} - R_{MN}R^{MN} + |D_M\Phi|^2 + 3|\Phi|^2\right]$$

Chiral breaking: $\langle\Phi\rangle = M_q z + \xi \frac{z^3}{L_1^3}$ $\quad M_q, \xi \neq 0$ force by b.c.

PARAMETERS:

I) $1/M_5$ Expansion parameter $\Rightarrow$ $1/N_c$
II) $1/L_1$ Mass gap $\Rightarrow$ Λ_{QCD} or M_ρ
III) M_q $\Rightarrow$ Quark masses
IV) ξ $\Rightarrow$ Condensate $\langle\bar{q}q\rangle$

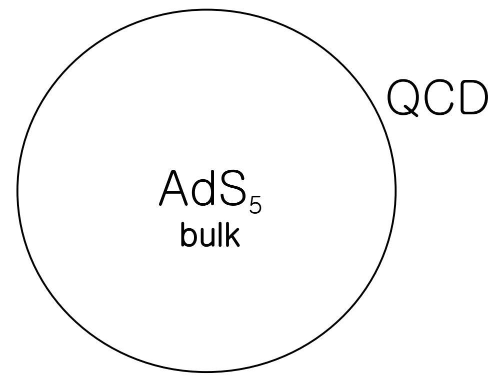
$\searrow$ The only extra parameter!

Schematic view of the holographic QCD

zero Temperature:

J. M. Maldacena (1997)

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu),$$

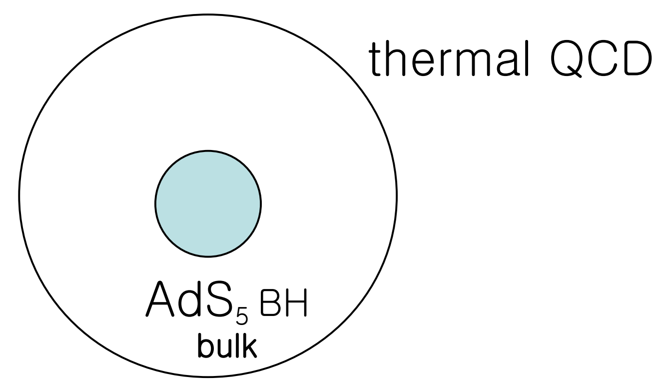


finite Temperature:

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Models

$$S_{\mathrm{hQCD-I}} = \int d^4x dz \sqrt{g} \mathcal{L}_5 \,,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[-\frac{1}{g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) - |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right],$$

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

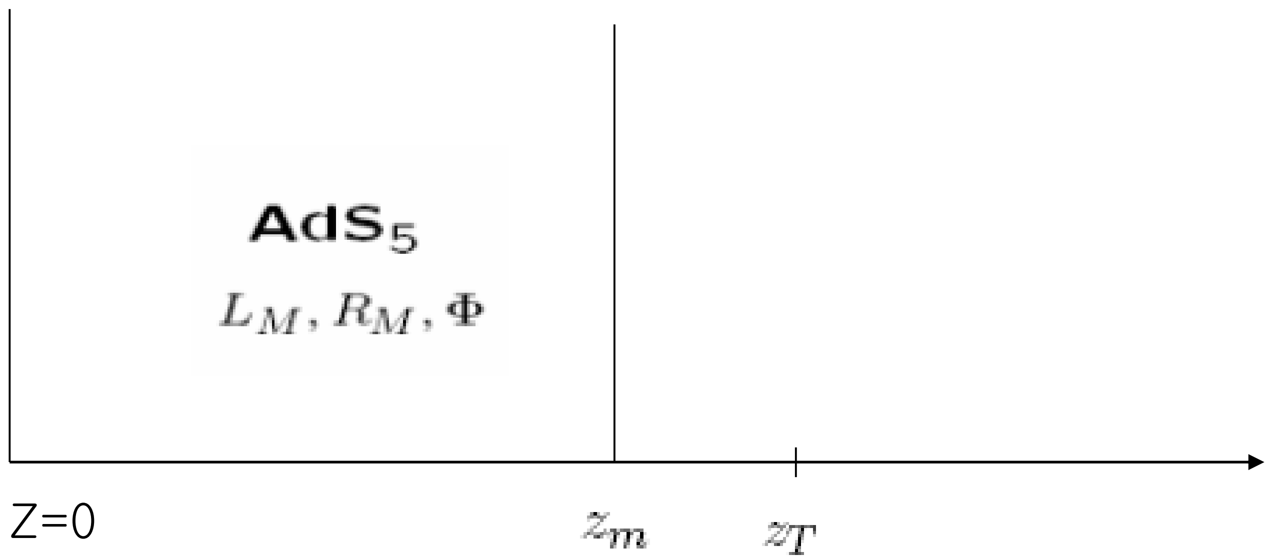
L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

$$S_{\mathrm{hQCD-II}} = \int d^4x dz \mathrm{e}^{-\Phi} \mathcal{L}_5 \,, \quad \Phi = cz^2.$$

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys.Rev.D74:015005,2006

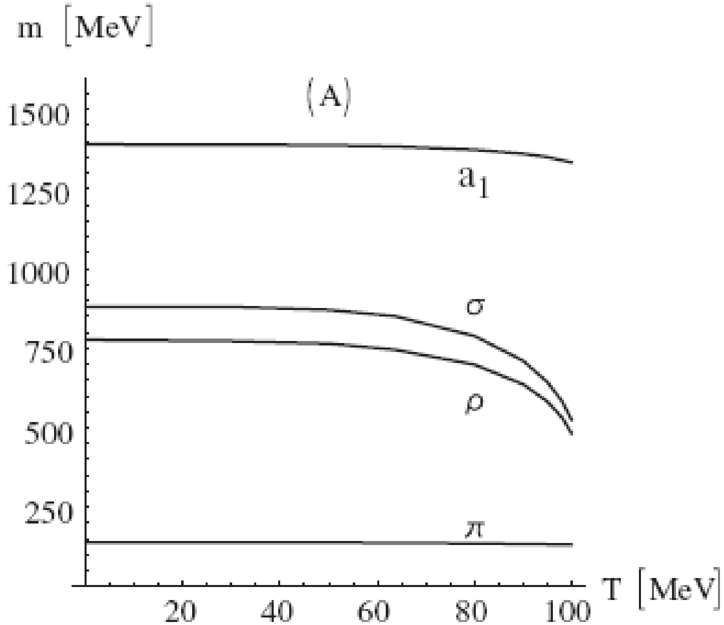
$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2 \right), \quad f^2(z) = 1 - (\frac{z}{z_T})^4,$$

Hard wall model at finite temperature



K. Ghoroku and M. Yahiro, Phys. Rev. **D73**, 125010 (2006)

Model-I (hard wall)



$$T_c = 1/(\pi z_m)$$

~102 MeV

$$\left[\frac{m^2}{f^4} + \partial_z^2 - \frac{4-3f^2}{zf^2}\partial_z\right]V_i = 0.$$

K. Ghoroku and M. Yahiro, Phys. Rev. **D73**, 125010 (2006)

Pion decay constant in AdS/QCD at zero temperature :

$$F_\pi^2 = -\frac{1}{g_5^2}\,\frac{1}{z}\partial_z A(0,z)\Big|_{z=\epsilon}$$

* Boundary (QCD) side :

$$\Pi_A(k^2) = k^2 \sum_n \frac{F_{A_n}^2}{k^2 + M_{A_n}^2} + F_\pi^2 \ ,$$

* Bulk (AdS) side :

$$\Pi_A(k^2) = -\frac{1}{g_5^2}\,\frac{1}{z}\partial_z A(k^2,z)\Big|_{z=\epsilon}$$

Pion decay constant in AdS/QCD at finite temperature :

$$(F_\pi^{t,s})^2 = \frac{1}{g_5^2}\frac{\partial_z A_{0,i\perp}^{(0)}}{z}, \;\; z = z_0$$

K. Ghoroku and M. Yahiro, Phys. Rev. **D73**, 125010 (2006)

$$[\frac{m_a^2}{f^4} + \partial_z^2 - \frac{4-3f^2}{zf^2}\partial_z - g_5^2\frac{v^2}{z^2f^2}]A_{i\perp} = 0,$$

$$[\partial_z^2 - \frac{1}{z}\partial_z - g_5^2\frac{v^2}{z^2f^2}]A_{0\perp} = 0,$$

$$A_{i,0}|_{z=0} = 1 \text{ and } \partial_z A_{i,0}|_{z=z_m} = \epsilon,$$

$$(F_\pi^t)^2 = \frac{1}{g_5^2}\frac{\epsilon}{z_m},$$

$$(F_\pi^s)^2 = \frac{1}{g_5^2}\frac{\epsilon}{z_m}(1-(\frac{z_m}{z_T})^4)$$

The pion velocity

$$v_\pi^2 \equiv \frac{F_\pi^s(T)}{F_\pi^t(T)} = \sqrt{1-(\frac{T}{T_c})^4}$$

K. Ghoroku and M. Yahiro, Phys. Rev. **D73**, 125010 (2006)

0, SS, 2002: pion only

1 (0.83-0.99), HKRS,2004:pion+rho-meson,

0.6 : RHIC

Chiral/deconfinement transition in thermal AdS/QCD

* Chiral symmetry restoration (CSR) through the v.e.v. of the scalar field:

Chiral breaking: $\langle \Phi \rangle = M_q z + \xi \frac{z^3}{L_1^3}$

$$\left[\partial_z^2 - \frac{4-f^2}{z f^2}\partial_z + \frac{3}{z^2 f^2}\right] v(z) = 0\,,$$

$$v(z) = z\left(M_q\ {}_2F_1\left(\frac{1}{4},\frac{1}{4},\frac{1}{2},\frac{z^4}{z_T^4}\right) + \Sigma_q z^2\ {}_2F_1\left(\frac{3}{4},\frac{3}{4},\frac{3}{2},\frac{z^4}{z_T^4}\right)\right).$$

* CSR in the hard-wall mode: $M_q = 0, \qquad \Sigma_q = 0.$

Quark number SUS as a chiral symmetry order parameter

$$r = \rho_B^{CS}/\rho_B^{CB}$$

In an ideal QGP

$$\rho_B^{CS} = \frac{4}{\pi^2}\mu T^2$$

In the hadron gas phase

$$\rho_B^{CB} = \frac{1}{2\pi^{3/2}}(2mT)^{3/2}\frac{\mu}{T}e^{-\beta m} \; .$$

For example, if we take $T \sim 150$ MeV, and $m \sim 1$ GeV, then $r \sim 10^2$.

$$\rho_B = \mu \left(\frac{\partial}{\partial \mu}\rho_B \right) \Bigg|_{\mu=0} \equiv \mu \kappa_B$$

L. McLerran, Phys. Rev. **D36**, 3291 (1987).

1. $$\chi_{\mathrm{q}}^{(1)}(T,\mu)=\left(\frac{\partial}{\partial\mu_{\mathrm{u}}}+\frac{\partial}{\partial\mu_{\mathrm{d}}}\right)(\rho_{\mathrm{u}}+\rho_{\mathrm{d}})=\frac{\langle\langle \hat{N}_{\mathrm{q}}^{2}\rangle\rangle}{VT},$$

$$\hat{N}_{\mathrm{q}}\equiv\hat{N}_{\mathrm{u}}+\hat{N}_{\mathrm{d}}=\int \mathrm{d}\boldsymbol{x}\, j_0(t,x) \qquad j_\mu(t,\boldsymbol{x})=\bar{\psi}(t,\boldsymbol{x})\gamma_\mu\psi(t,\boldsymbol{x})$$

$$\rho_i=\mathrm{Tr}\,\hat{N}_i\exp\left[-\beta\left(\hat{H}-\sum_{i=\mathrm{u,d}}\mu_i\hat{N}_i\right)\right]\Big/V\equiv\frac{\langle\langle \hat{N}_i\rangle\rangle}{V}$$

2. $$\chi_{\mathrm{q}}^{(\tau)}(T,\mu)=\left(\frac{\partial}{\partial\mu_{\mathrm{u}}}-\frac{\partial}{\partial\mu_{\mathrm{d}}}\right)(\rho_{\mathrm{u}}-\rho_{\mathrm{d}})=\frac{\langle\langle \hat{I}_z^{2}\rangle\rangle}{VT}$$

$$\hat{I}_z\equiv\hat{N}_{\mathrm{u}}-\hat{N}_{\mathrm{d}}=\int \mathrm{d}\boldsymbol{x}\, j_{z0}(t,\boldsymbol{x}) \qquad j_{i\mu}(t,\boldsymbol{x})=:\bar{\psi}(t,\boldsymbol{x})\tau_i\gamma_\mu\psi(t,\boldsymbol{x})$$

$\chi_q^{(1)}=\chi_q^{(\tau)}$ is a good approximation, since the flavor mixing between u and d quarks in the vector channel is almost zero, $<\bar{u}\gamma_0 u\bar{d}\gamma_0 d>\approx 0$.

VSUS near T_c

$$\chi_V(T_c) = aT_c^2$$

pQCD, NJL : a=1.3

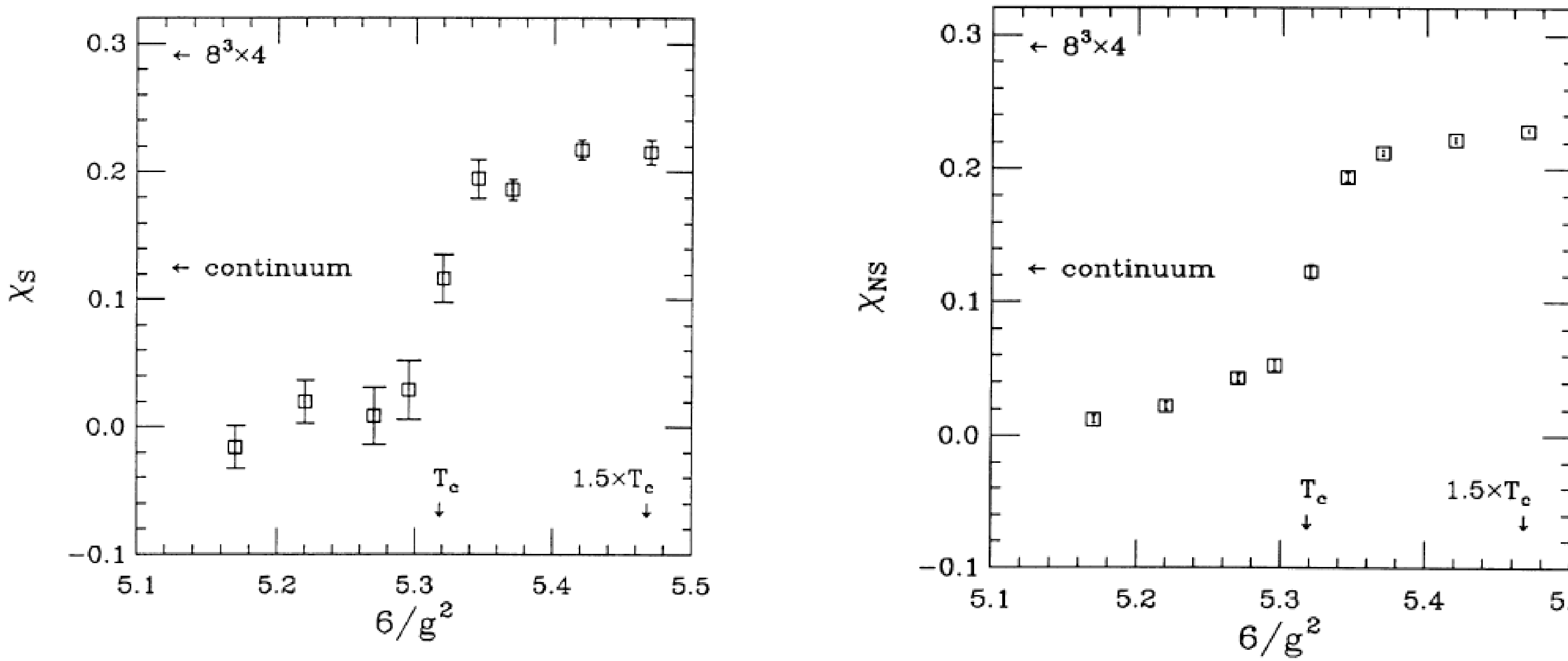
P. Chakraborty, M. G. Mustafa and M. H. Thoma, Eur. Phys. J. C **23**, 591 (2002)

T. Kunihiro, Phys. Lett. B **271**, 395 (1991).

HLS/VM :
a=0.7, pion + rho−meson
a=1.3, pion + rho−meson + cQuark

M. Harada, Y. Kim, M. Rho and C. Sasaki, Nucl. Phys. **A730**, 379 (2004)

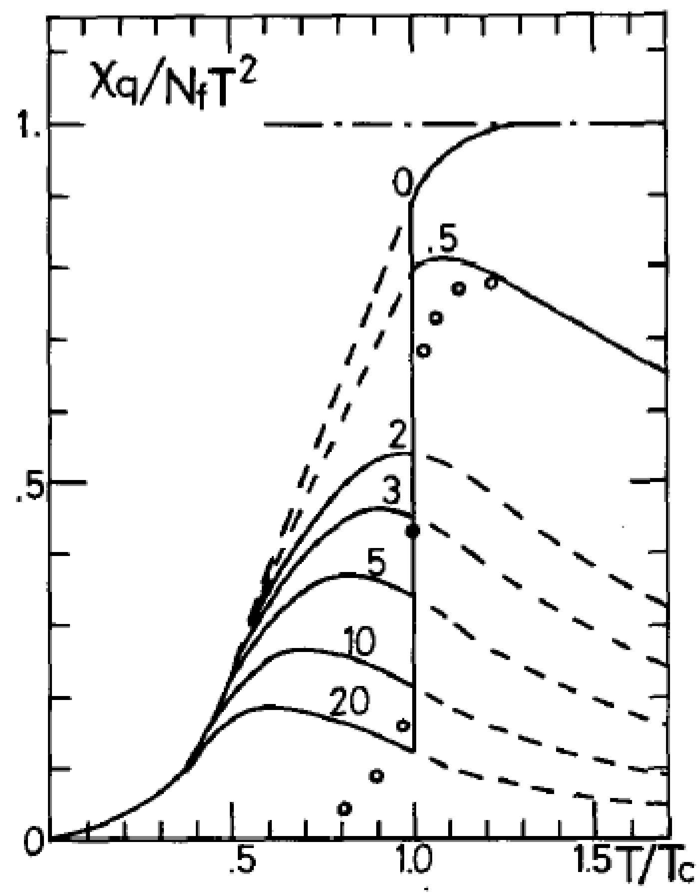
Full/Quenched Lattice QCD



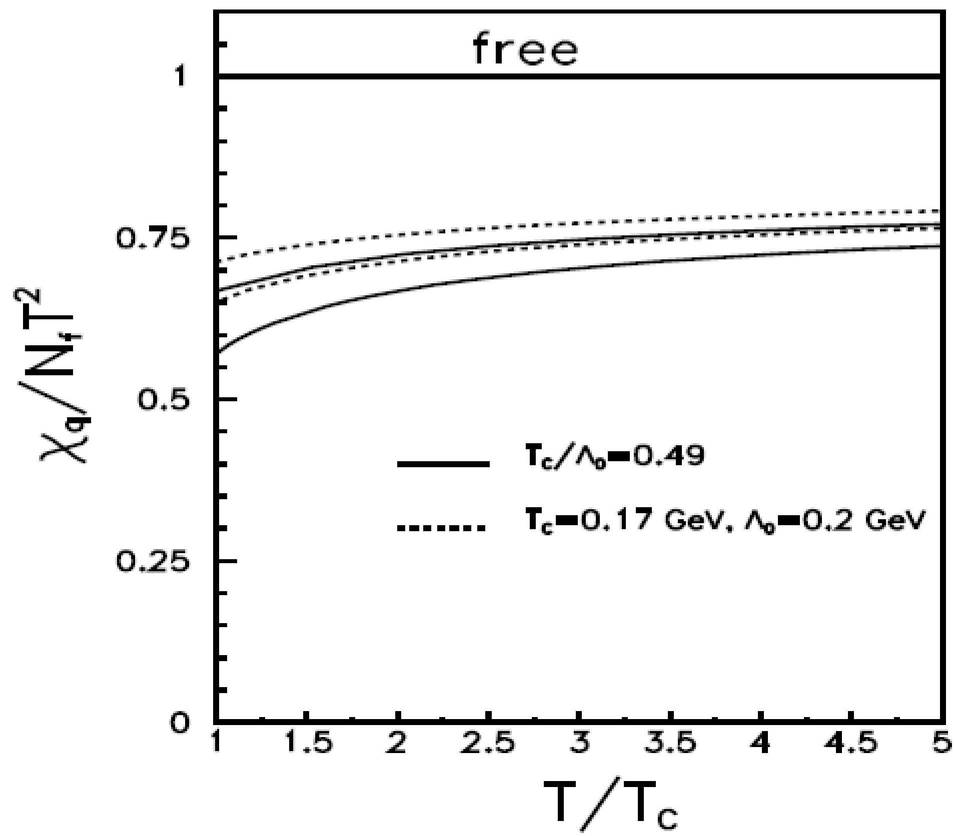
S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken and R. L. Sugar, Phys. Rev. Lett. **59** (1987) 2247.

P. Chakraborty, M. G. Mustafa and M. H. Thoma, Eur. Phys. J. C **23**, 591 (2002)

pQCD



NJL model



T. Kunihiro, Phys. Lett. B **271**, 395 (1991).

Vector susceptibility in AdS/QCD models

$$\chi_V(T) = 2N_f \lim_{\bar{p}\to 0}\lim_{p_0\to 0}\left[G_V^{00}(p_0,\vec{p};T)\right] \ ,$$
$$\chi_A(T) = 2N_f \lim_{\bar{p}\to 0}\lim_{p_0\to 0}\left[G_A^{00}(p_0,\vec{p};T)\right] \ ,$$

$$G_A^{\mu\nu}(p_0 = i\omega_n, \vec{p}; T)\delta_{ab} = \int_0^{1/T} d\tau \int d^3\vec{x} e^{-i(\vec{p}\cdot\vec{x}+\omega_n\tau)} \left\langle J_{5a}^{\mu}(\tau,\vec{x}) J_{5b}^{\nu}(0,\vec{0})\right\rangle_\beta \ ,$$
$$G_V^{\mu\nu}(p_0 = i\omega_n, \vec{p}; T)\delta_{ab} = \int_0^{1/T} d\tau \int d^3\vec{x} e^{-i(\vec{p}\cdot\vec{x}+\omega_n\tau)} \left\langle J_{a}^{\mu}(\tau,\vec{x}) J_{b}^{\nu}(0,\vec{0})\right\rangle_\beta \ ,$$

VSUS in the hard wall model

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + \frac{\vec{q}^2}{f^2(z)}\right] V_0(z, \vec{q}) = 0\,, \qquad V_0 = a_1 + a_2 z^2$$

BC at IR: $\partial_z V_0|_{z=z_m} = \epsilon.$

The limit $\epsilon \to 0$ will be taken after calculation. Then the vector susceptibility is given by

$$\begin{aligned} \chi_V(T) &= -2N_f \left(\frac{1}{g_5^2}\frac{\partial_z V_0}{z}\right)_{z=z_0} \\ &= -2N_f \frac{1}{g_5^2}\frac{\epsilon}{z_m}\,, \end{aligned}$$

VSUS in the hard wall model

BC at IR: $V_0(z_m) = h$

$$\begin{aligned} \chi_V(T) &= 2N_f \frac{1}{g_5^2} \frac{2(1-h)}{z_m^2} \\ &= 2N_f \frac{2\pi^2}{g_5^2} (1-h) T_c^2 \, . \end{aligned}$$

VSUS in the soft wall model

$$\partial_z \left(\frac{1}{z} \mathrm{e}^{-cz^2} \partial_z V_0\right) = 0\,,$$

We impose the following boundary conditions:

$$V_0(0) = 1, \;\; V_0(z_T) = 0.$$

$$\begin{aligned}\chi_V(T) &= 2N_f \frac{2}{g_5^2} \frac{1}{\mathrm{e}^{cz_T^2} - 1} c, \\ &= 2N_f \frac{2}{g_5^2} \frac{1}{\mathrm{e}^{T_c^2/T^2} - 1} \pi^2 T_c^2\end{aligned}$$

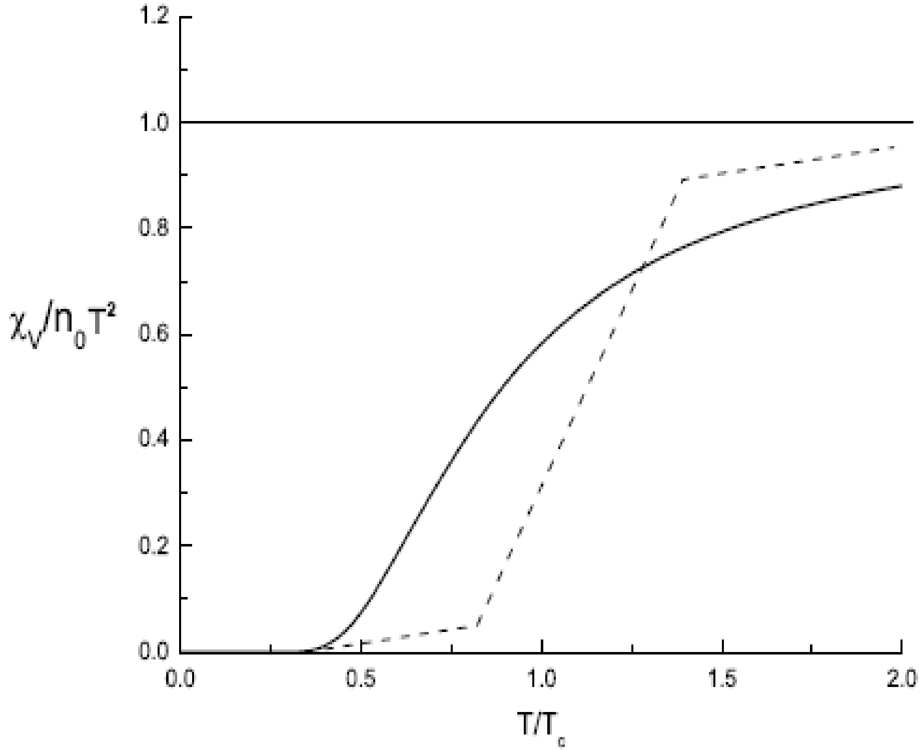


Figure 1: Solid line is for χ_V in Model-II, where $n_0 \equiv 4N_f\pi^2/g_5^2$. Note that $n_0 = N_f$. At high temperature χ_v/N_fT^2 is saturated to the ideal-gas value, the horizontal solid line. Dashed line is for schematic behavior of χ_V near T_c taken from lattice calculations

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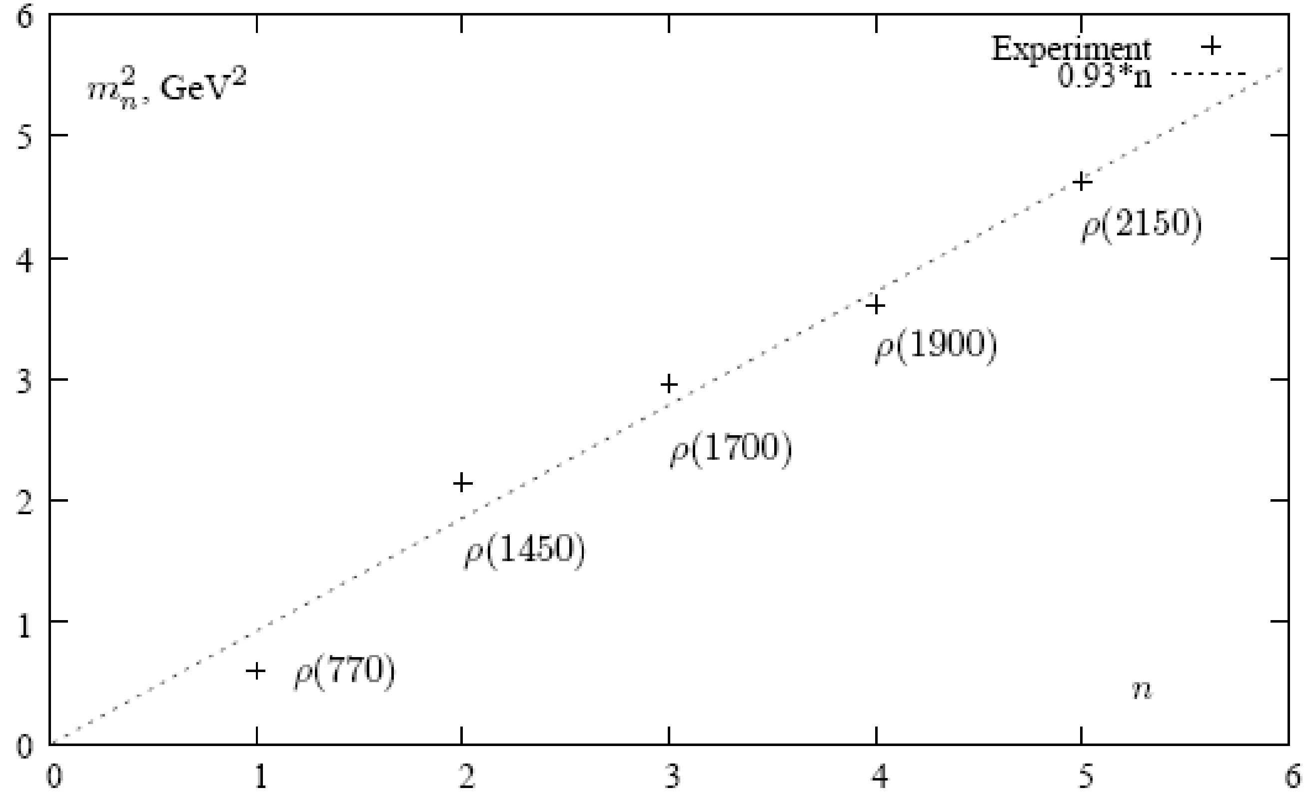
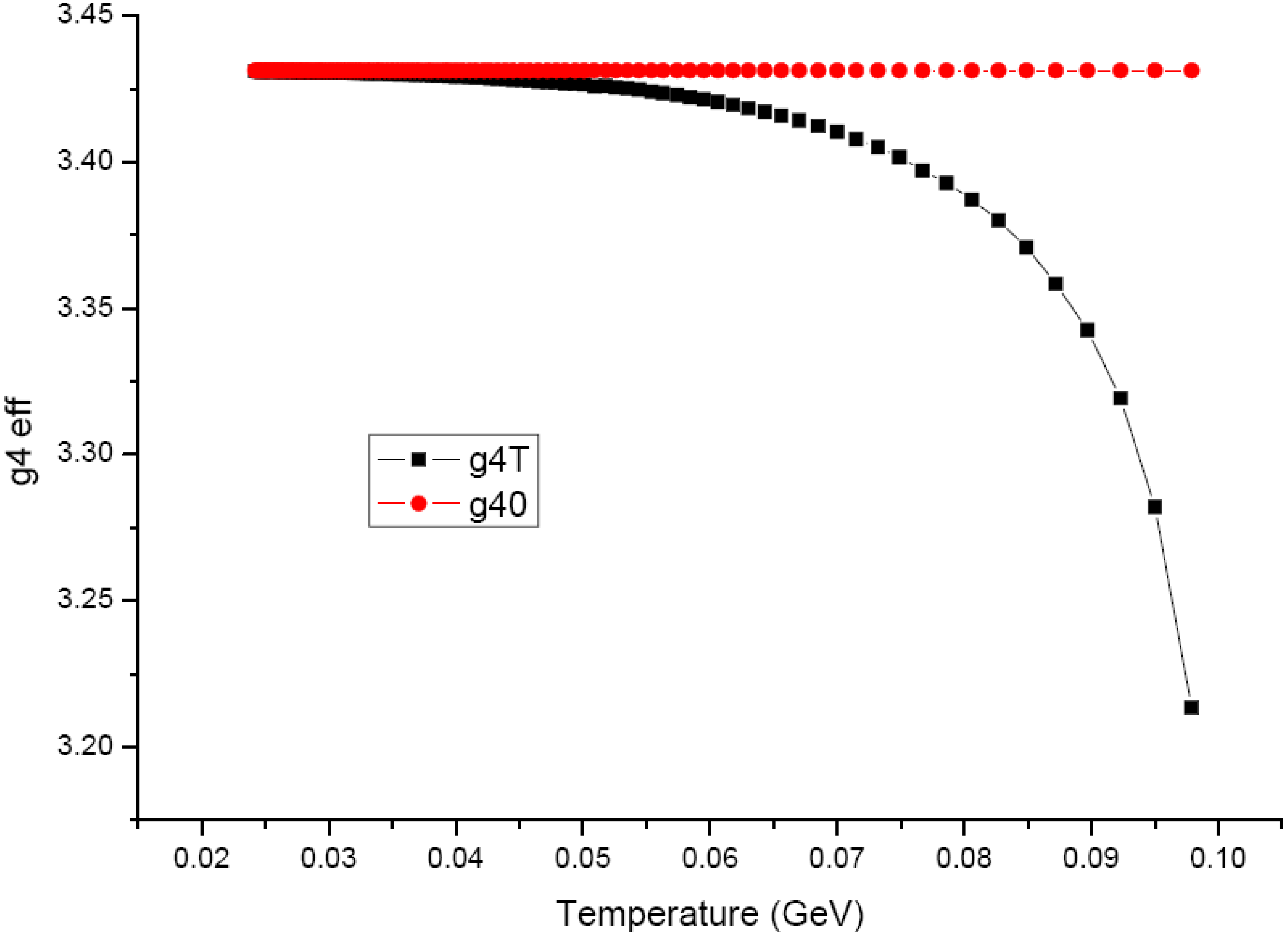
$$\chi_V(T_c) \approx 1.2 T_c^2 \qquad\qquad T_c = \sqrt{c}/\pi.$$

The value of c is determined by the slope of Regge trajectory, and we obtain the critical temperature ~195 (158) MeV.



m_n^2, GeV^2
Experiment
0.93*n
$\rho(770)$
$\rho(1450)$
$\rho(1700)$
$\rho(1900)$
$\rho(2150)$
n
0
1
2
3
4
5
6

Coupling constants



Kwang Hyun Jo, et al, work in progress

Summary

- We can calculate *nonperturbatively* the temperature dependent masses and coupling constant in AdS/QCD.
- vSUS from AdS/QCD is compatible with LQCD or EFTs of QCD or pQCD results.

*AdS/QCD predicts a reasonable value of the critical temperature.