Some Topics in Relativistic Heavy Ion Collision

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Theoretical overview

Towards understanding the quark gluon plasma

Some form of a quark-gluon plasma is produced in ultra-relativistic nucleus-nucleus collisions

Large energy density achieved $e_{\tau_0} >> e_c$

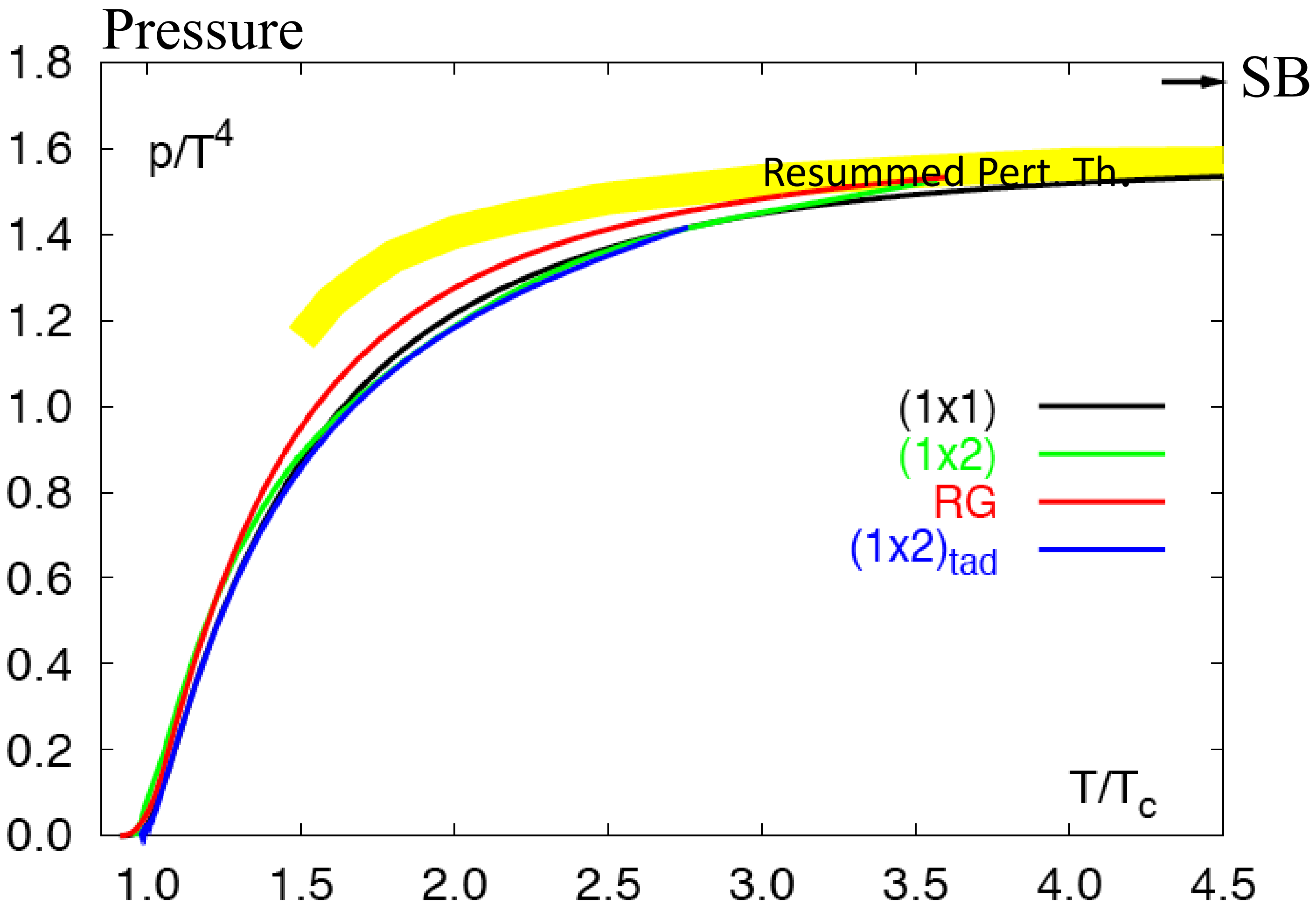
Collective behaviour observed v_2 / ε

Jet energy loss in matter $\hat{q}$

Hints of gluon saturation Q_s

→ « Perfect liquid », « sQGP »

Thermodynamical functions go to SB limit as T becomes large



(SU(3) lattice gauge calculation from Karsch et al, hep-lat/0106019)

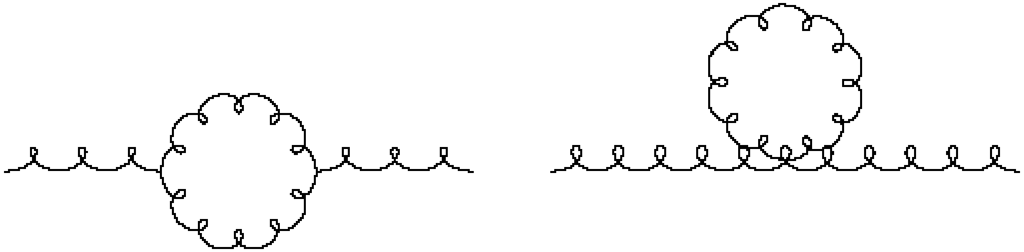
Resummed perturbation

$$D(k) \sim \frac{i}{k_0 - \Sigma(k)} \sim \frac{i}{k_0 - \omega(k) + i\gamma(k)},$$

$$\omega(k) \sim \mathrm{Re}\,\Sigma(k_0 = \omega) \qquad = O(gT)$$

$$\gamma \sim \mathrm{Im}\,\Sigma(k_0 = \omega) \qquad = O(gT^2)$$

Guage dependent

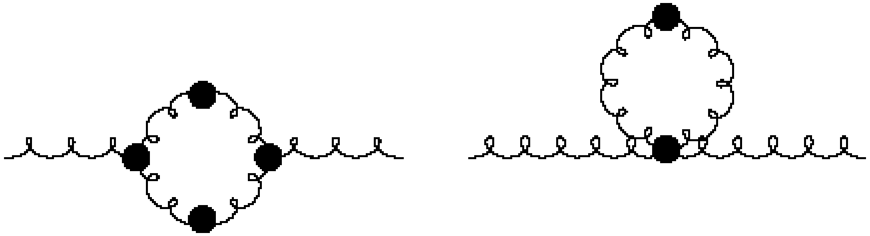


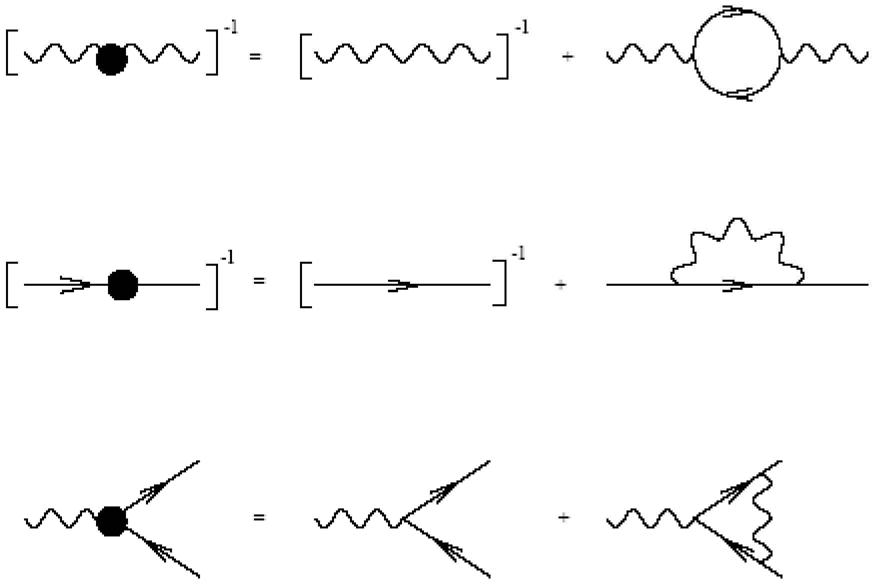
Internal loop momenta "p"

If the internal loop p is

1) Hard (T) : normal perturbation

2) Soft (gT): resummed propagator and vertex

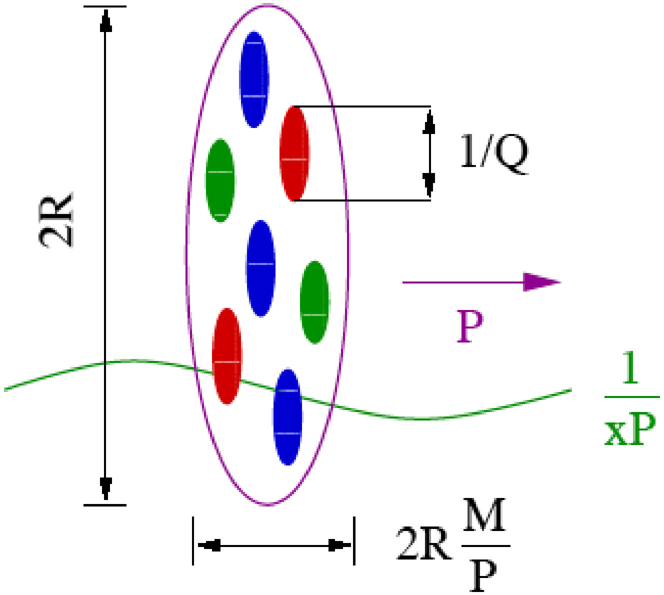


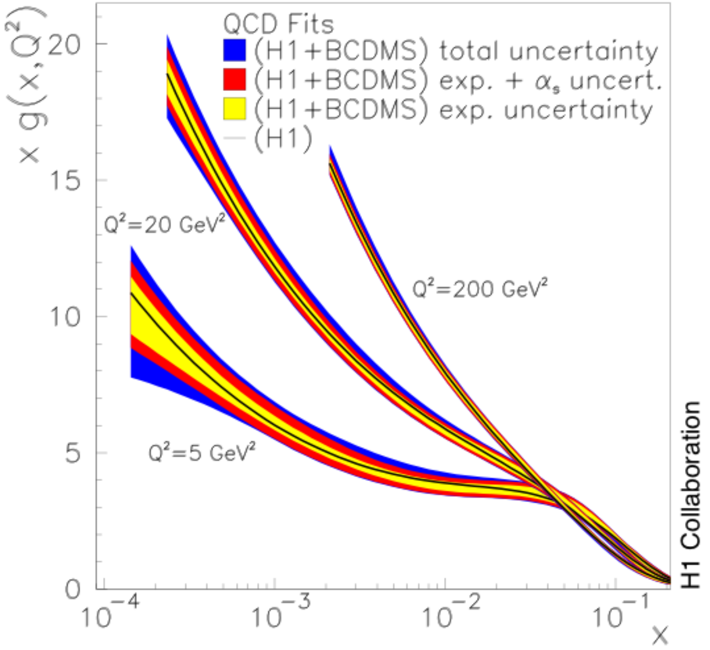


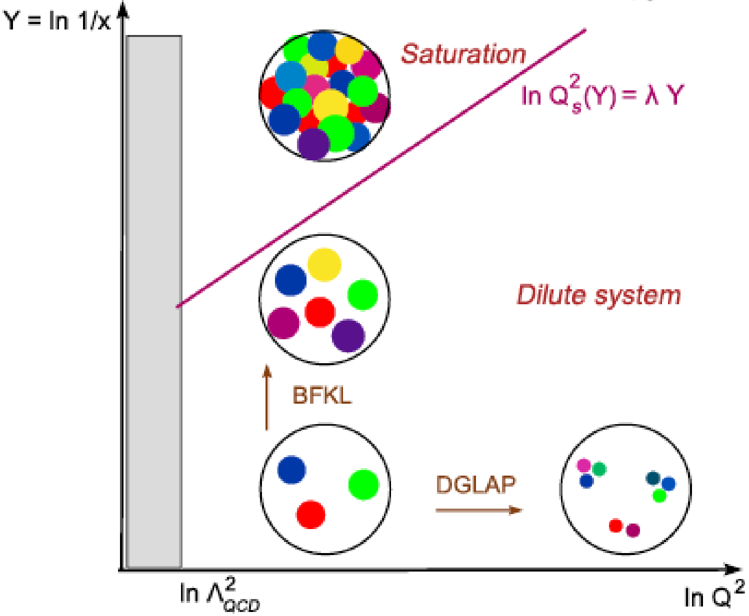
The infrared behavior strongly depends on the external momenta “k”

$$D(k) = \ldots \int d^4 q \, \frac{1}{(q+k)^2 - m_g^2} \, \frac{1}{q^2 - m_g^2}$$

Saturation


2R
1/Q
P
$\frac{1}{xP}$
$2R\frac{M}{P}$


QCD Fits
(H1+BCDMS) total uncertainty
(H1+BCDMS) exp. + α_s uncert.
(H1+BCDMS) exp. uncertainty
(H1)
$x\,g(x,Q^2)$
20
15
10
5
0
Q^2=20 GeV^2
Q^2=200 GeV^2
Q^2=5 GeV^2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x
H1 Collaboration

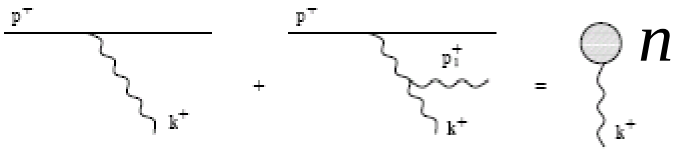

Y = ln 1/x
Saturation
$\ln Q_s^2(Y) = \lambda Y$
Dilute system
BFKL
DGLAP
$\ln \Lambda^2_{QCD}$
$\ln Q^2$

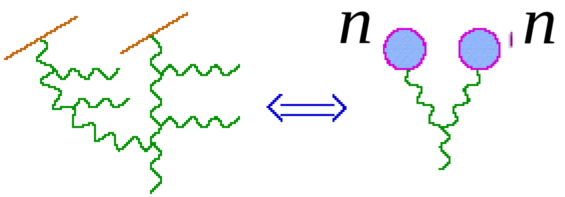
Saturation formula

The growth of the gluon density is governed by non linear evolution equations and eventually « saturates »

$$\frac{dn}{dY} \approx \alpha_s n - \alpha_s^2 n^2$$

$$\frac{dn}{dY} = 0 \Rightarrow n \approx \frac{1}{\alpha_s}$$





Whether a parton is in the saturated regime or not depends on its (transverse) momentum

$k_T \leq Q_s$ (saturated regime)

$k_T \geq Q_s$ (dilute regime)

Effective number of gluons= xG (1/Q)^2

$$Q_s^2 \approx \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2}$$

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda}$$

$$Q_0^2 \propto A^{1/3}$$

For large nucleus G=O(A)

R= O(A^1/3)

Day 1 @ LHC: event multiplicity at y=0

- generic trends in $dN^{ch}/d\eta$
  - extended longitudinal scaling
  - self-similar trapezoidal shape

$$\Rightarrow \; dN^{ch}/d\eta\big|_{\eta=0} \propto \ln\sqrt{s_{NN}}$$

- Saturation models predict

Armesto, Salgado, Wiedemann, PRL94 (2005) 022002

$$\frac{1}{N_{\rm part}}\frac{dN^{AA}}{d\eta}\bigg|_{\eta\sim 0} = N_0\sqrt{s}^{\lambda} N_{\rm part}^{\frac{1-\delta}{3\delta}}$$

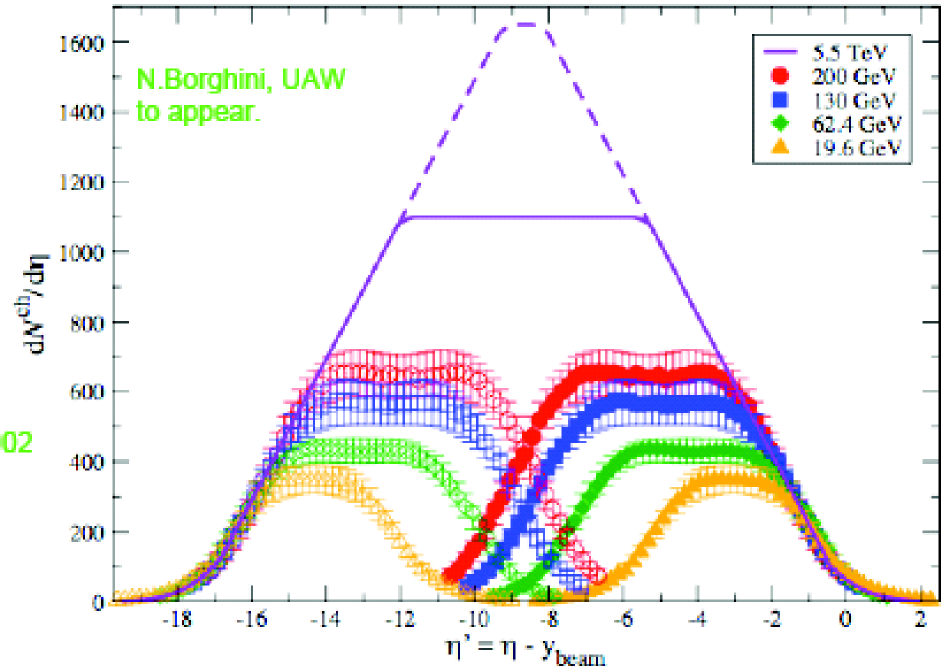
$$\Rightarrow \; dN^{ch}_{LHC}/d\eta\big|_{\eta=0} \approx 1650$$

or Kharzeev, Levin, Nardi, NPA747 (2005) 609.

$$\Rightarrow \; dN^{ch}_{LHC}/d\eta\big|_{\eta=0} \approx 1800-2100$$

Both consistent with main trends at RHIC, but …

PHOBOS, PRC74 (2006) 021901; W. Busza .



N.Borghini, UAW to appear.

Extrapolations to LHC deviate from so-far generic trends in data

→ Impact for understanding the dynamical origin of soft physics at RHIC and LHC.

Early stage of nucleus-nucleus collisions

Conventional picture (mean field): Qs sets the scale at Which partons are set free, as well as their typical transverse momentum

$$\boxed{k_T \approx Q_s} \qquad \boxed{\tau \approx Q_s^{-1}}$$

Role of fluctuations of Qs ?

Anisotropy of initial momentum distributions may lead to plasma instabilities. Role in thermalization ?

J. Kapusta
Theoretical overview

Strongly Interacting Low Viscosity Matter Created in Heavy Ion Collisions

Big Theoretical Motivation!

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics

Kovtun, Son, Starinets PRL 94, 111601 (2005)

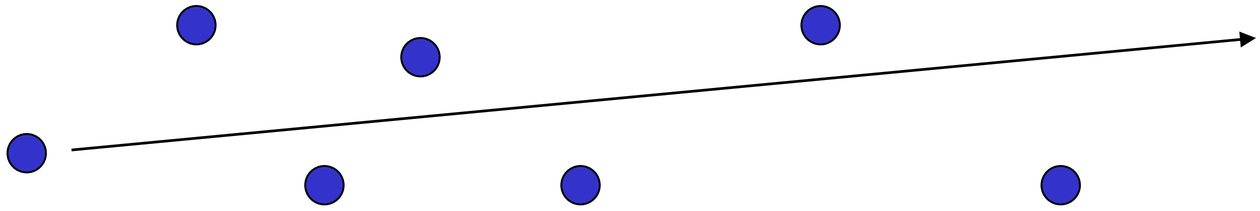
Using the Kubo formula $\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \left\langle \left[T^{ij}_{\text{traceless}}(x), T^{ij}_{\text{traceless}}(0) \right] \right\rangle$

the low energy absorption cross section for gravitons on black holes, and the black hole entropy formula they found that $\eta / s = 1/4\pi$ and conjectured that this is a universal lower bound.

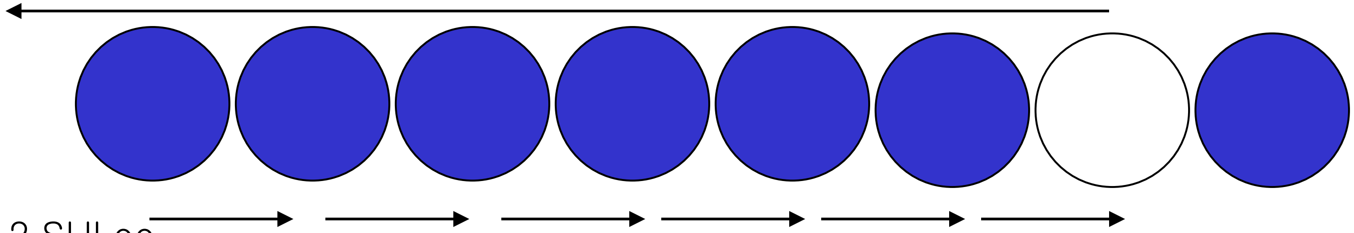
Atomic and Molecular Systems

In classical transport theory $\frac{\eta}{s} \sim T l_{\text{free}} \bar{v}$ and $l_{\text{free}} \sim \frac{1}{n\sigma}$

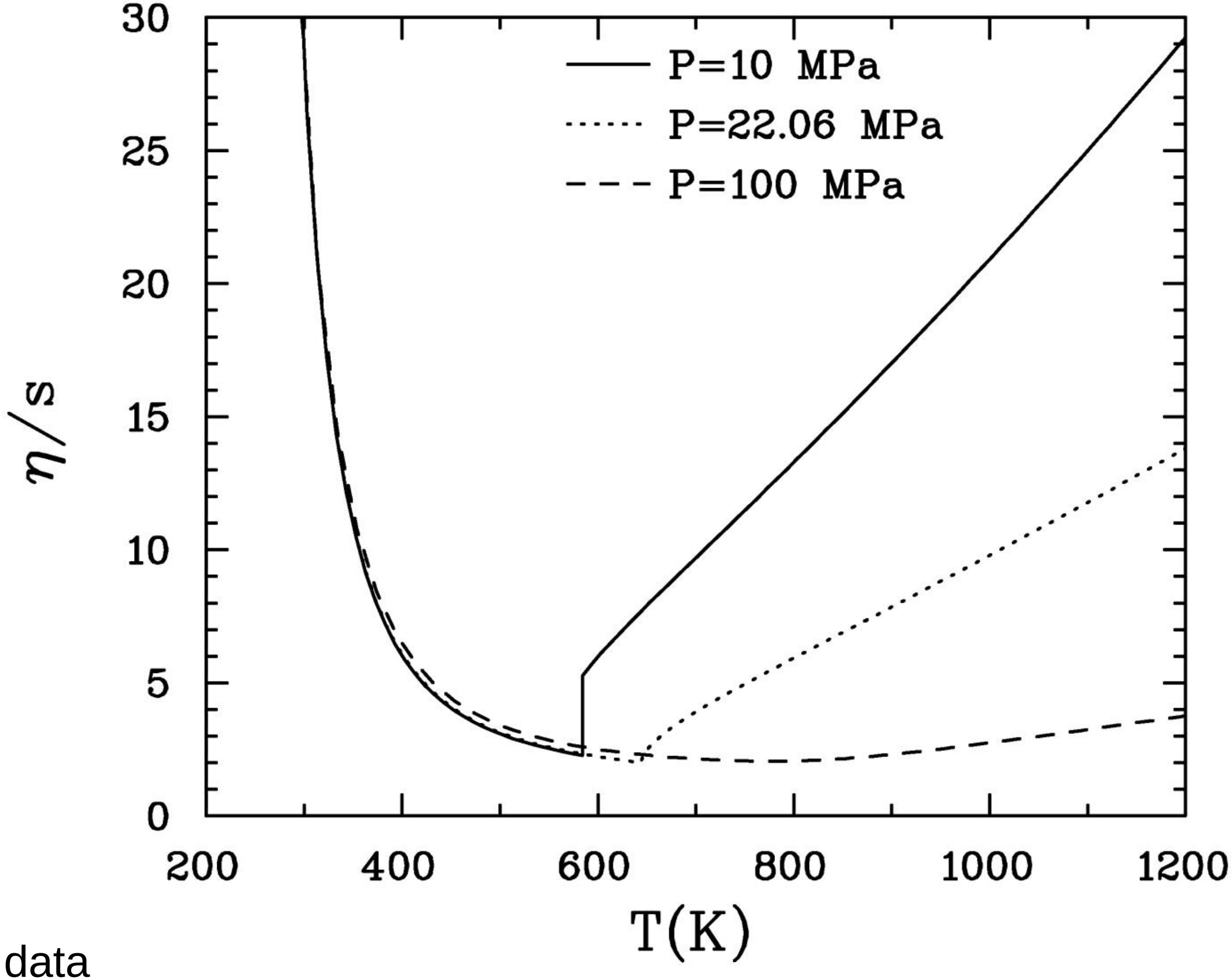
so that as the density and/or cross section is reduced (dilute gas limit) the ratio gets larger.



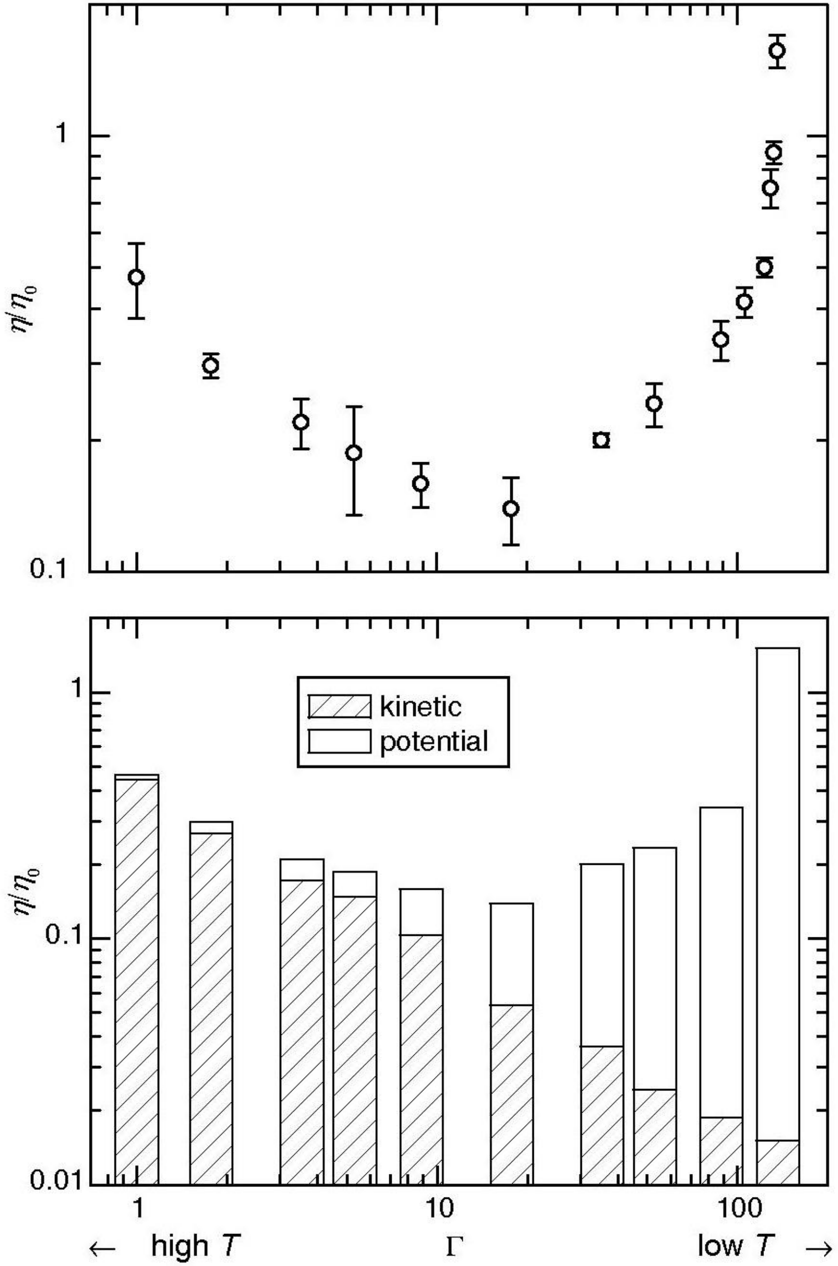
In a liquid the particles are strongly correlated. Momentum transport can be thought of as being carried by voids instead of by particles (Enskog) and the ratio gets larger.



H_2O



NIST data



2D Yukawa Systems in the Liquid State

Minimum located at

$$\Gamma = \frac{Q^2}{aT} = \text{Coulomb coupling parameter} \approx 17$$

$$a^2 = \frac{1}{\pi n} = \text{Wigner-Seitz radius}$$

Applications to dusty-plasmas and many other 2D condensed matter systems.

Liu & Goree

Relativistic Dissipative Fluid Dynamics

$$\boxed{\begin{aligned} &T^{\mu\nu} = -Pg^{\mu\nu} + wu^{\mu}u^{\nu} + \Delta T^{\mu\nu} \\ &J_B^{\mu} = n_B u^{\mu} + \Delta J_B^{\mu} \end{aligned}}$$

In the Landau-Lifshitz approach u is the velocity of energy transport.

$$\boxed{\Delta T^{\mu\nu} = \eta\left(\Delta^{\mu}u^{\nu} + \Delta^{\nu}u^{\mu}\right) + \left(\tfrac{2}{3}\eta - \zeta\right)H^{\mu\nu}\partial_{\rho}u^{\rho}}$$

$$H^{\mu\nu} \equiv u^{\mu}u^{\nu} - g^{\mu\nu}, \quad \Delta_{\mu} \equiv \partial_{\mu} - u_{\mu}u^{\beta}\partial_{\beta}, \quad Q_{\alpha} \equiv \partial_{\alpha}T - Tu^{\rho}\partial_{\rho}u_{\alpha}$$

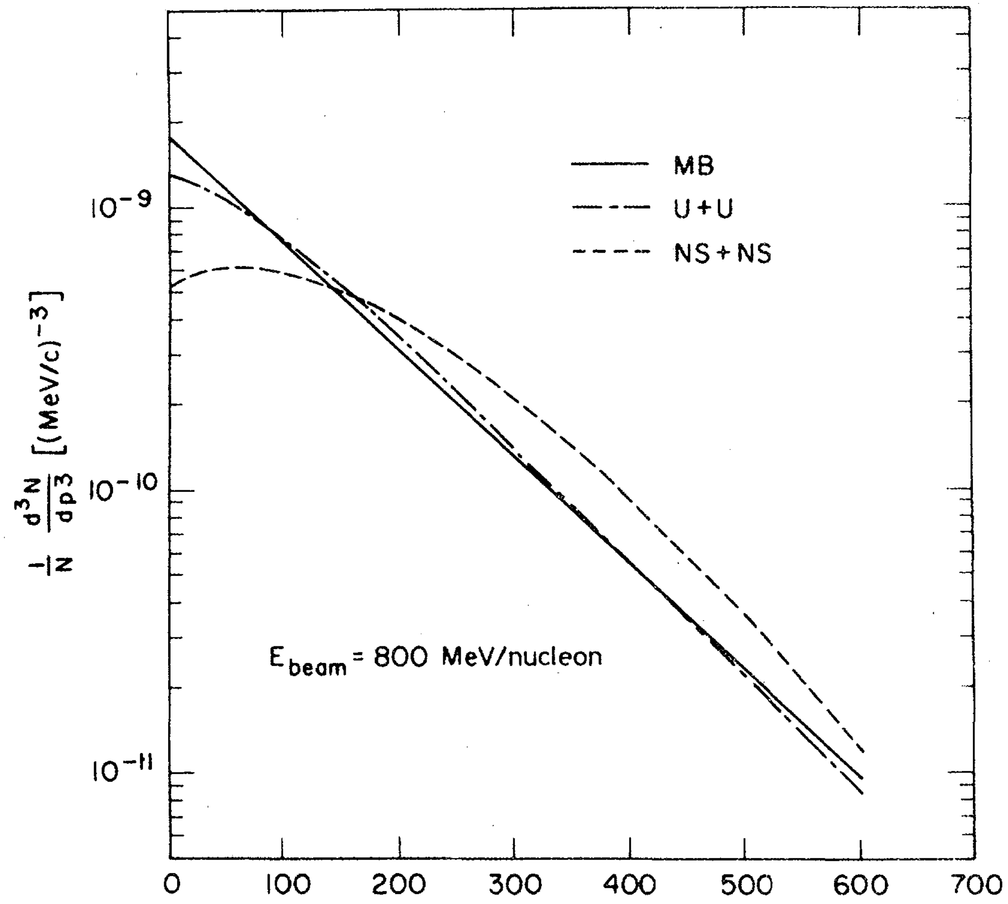
$$\Delta J_B^{\mu} = \chi\left(\frac{n_B T}{w}\right)^2 \Delta^{\mu}\left(\frac{\mu_B}{T}\right), \quad s^{\mu} = su^{\mu} - \frac{\mu_B}{T}\Delta J_B^{\mu}$$

$$\boxed{\partial_{\mu}s^{\mu} = \frac{\eta}{2T}\left(\partial_i u^j + \partial_j u^i - \tfrac{2}{3}\delta^{ij}\partial_k u^k\right)^2 + \frac{\zeta}{T}\left(\partial_k u^k\right)^2 + \frac{\chi}{T^2}\left(\partial_k T + T\dot{u}_k\right)^2}$$

Viscosity smoothes out gradients in temperature, velocity, pressure, etc.

Viscous Heating of Expanding Fireballs
JK, PRC 24, 2545 (1981)

$$\eta = \frac{2}{3\sigma}\sqrt{\frac{mT}{\pi}} \qquad \zeta = 0 \qquad \chi = \frac{1}{\sigma}\sqrt{\frac{T}{\pi m}}$$



The nucleon momentum distribution for a Maxwell-Boltzmann (straight line), for a viscous uranium plus uranium expansion, and for a pure hydrodynamical expansion represented by neutron star collisions.

U. A. Wiedemann

Physics opportunities at the LHC

Parton energy loss depends on parton identity

- Vacuum and medium radiation is suppressed due to **quark mass**

Dokshitzer, Kharzeev, PLB 519 (2001) 199

$$\frac{1}{k_T^2} \Rightarrow \frac{k_T^2}{\left(k_T^2 + \frac{m^2}{E^2}\omega^2\right)^2}$$

- Color charge dependence dominates

$$R_{D/h} = R_{AA}^{D} / R_{AA}^{h}$$

$$\boxed{\Delta E_{gluon} > \Delta E_{quark}}$$

- Mass dependence dominates

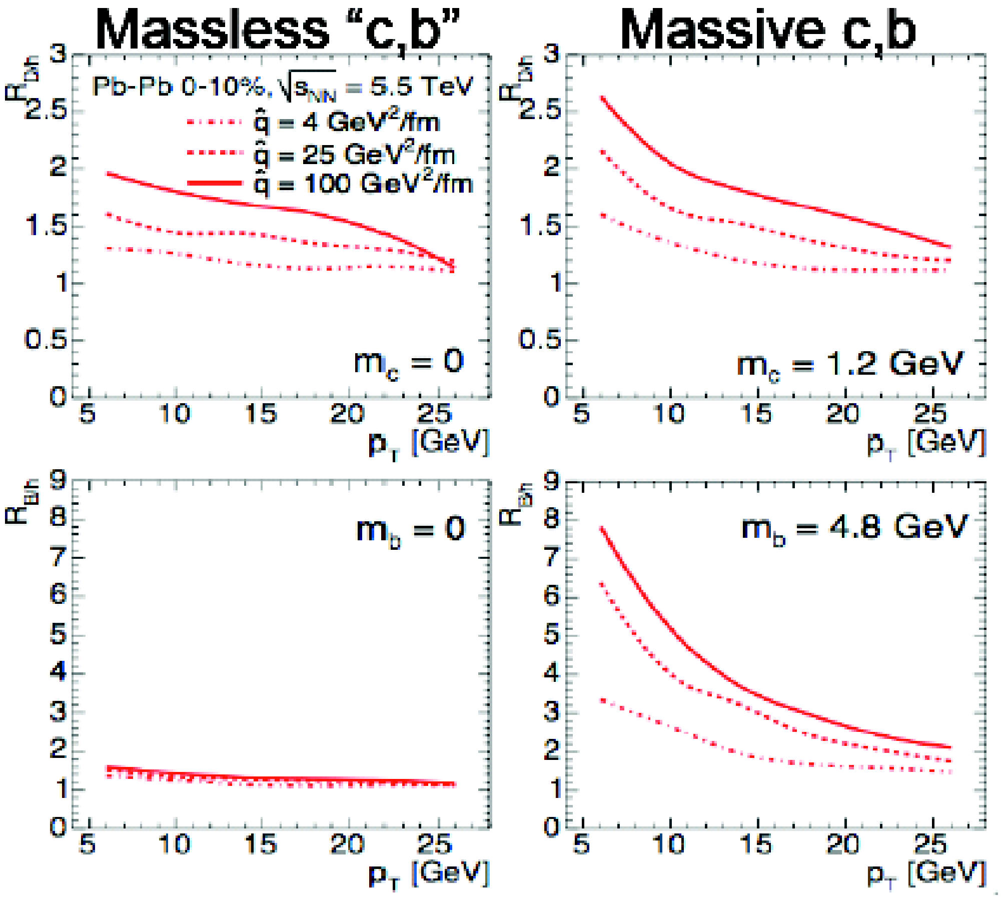
$$R_{B/h}$$

$$\boxed{\Delta E_{quark,m=0} > \Delta E_{quark,m>0}}$$

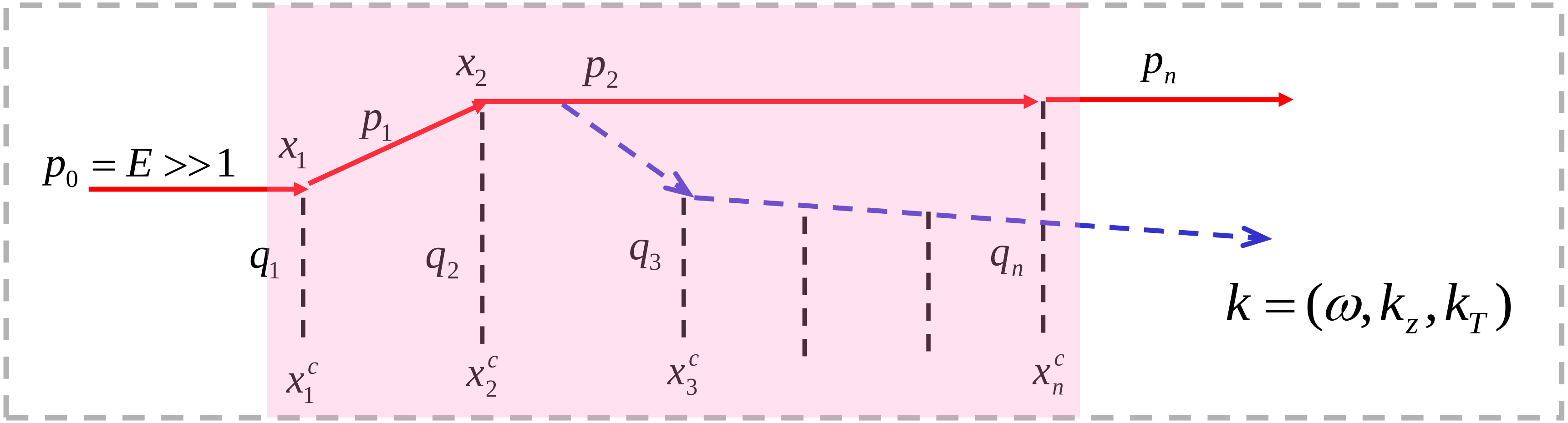
- To test this at the LHC, exploit:

light-flavored mesons	- gluon parents
D - mesons	- quark parents (m_c~0)
B - mesons	- quark parents (m_b>0)

Armesto, Dainese, Salgado, Wiedemann, PRD71:054027, 2005



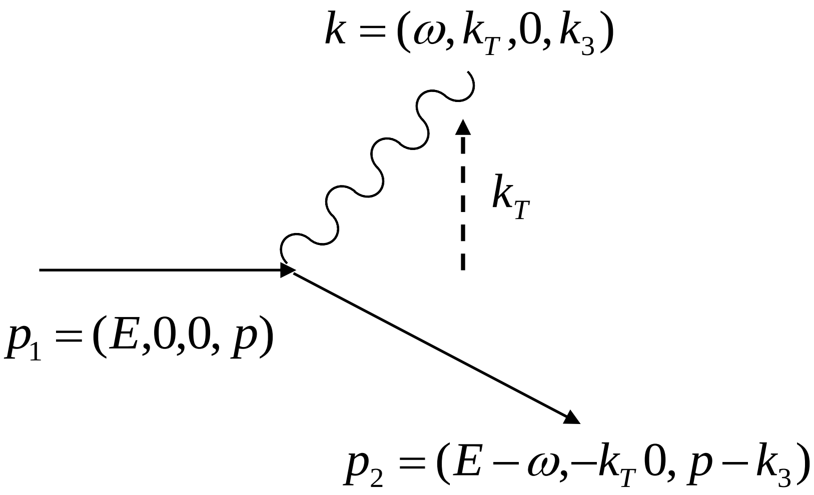
Some famous result for QCD



$$\lim_{R\to\infty} \omega \frac{dI}{d\omega} \approx \frac{2\alpha_s C_R}{\pi} \begin{cases} \sqrt{\dfrac{\omega_c}{\omega}} & \text{for} \quad \omega < \omega_c \\ \dfrac{1}{12}\left(\dfrac{\omega_c}{\omega}\right)^2 & \text{for} \quad \omega \geq \omega_c \end{cases}$$

$$\langle \Delta E \rangle_{R\to\infty} = \lim_{R\to\infty} \int_0^\infty \omega \frac{dI}{d\omega} d\omega \;\propto\; \alpha_s C_R \omega_c \;\propto\; \alpha_s C_R \hat{q} L^2$$

The probability to radiate a gluon



The probability to radiate a gluon is inversely proportional to its virtuality

$$dP \propto \frac{1}{-(p_1^2 - p_2^2)} = \frac{1}{k_T^2 + \omega^2 (\frac{m^2}{E^2})}$$

For same energy loss, minium virtuality is larger → dead cone

Some form of a quark-gluon plasma is produced in ultra-relativistic nucleus-nucleus collisions

Large energy density achieved $e_{\tau_0} >> e_c$

Collective behaviour observed v_2 / ε

Jet energy loss in matter $\hat{q}$

Hints of gluon saturation Q_s

→ « Perfect liquid », « sQGP »