Current Status of Numerical Approaches

Cosmological Heavy Ion Collisions:

Colliding Neutron Stars and Black Holes

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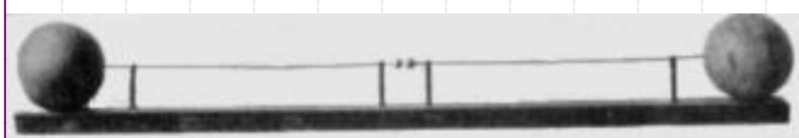
Introduction

EM wave

- Theory: Maxwell (1873)
- Acceleration of electric charge
- Detection : H. Hertz (1888)



Grav. wave

- Theory : Einstein (1916)
- Acceleration of matter (transverse & spin 2)
- Evidence : Taylor & Hulse ('79)
- Detection : K. Thorne(?)

LIGO(?)

?

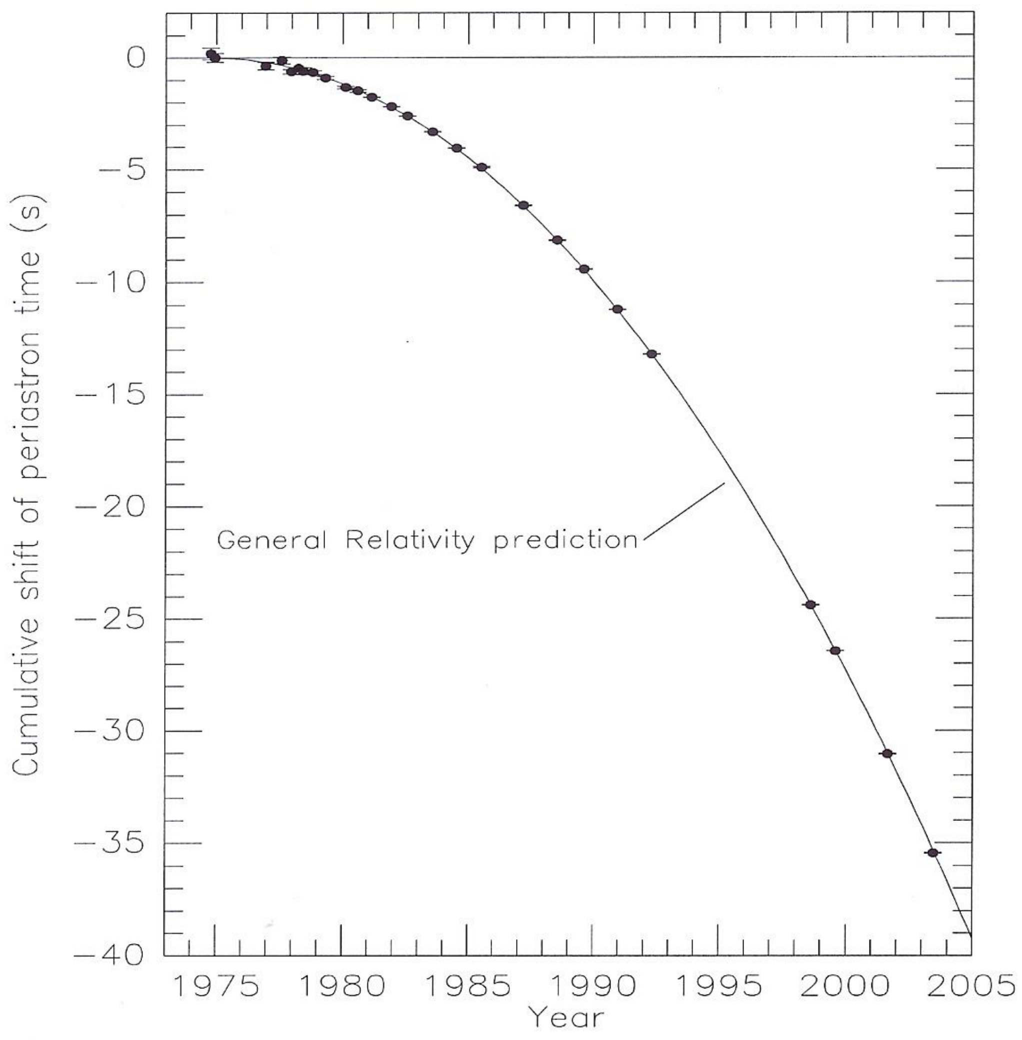
➢ Gravitation Wave from Binary Neutron Star

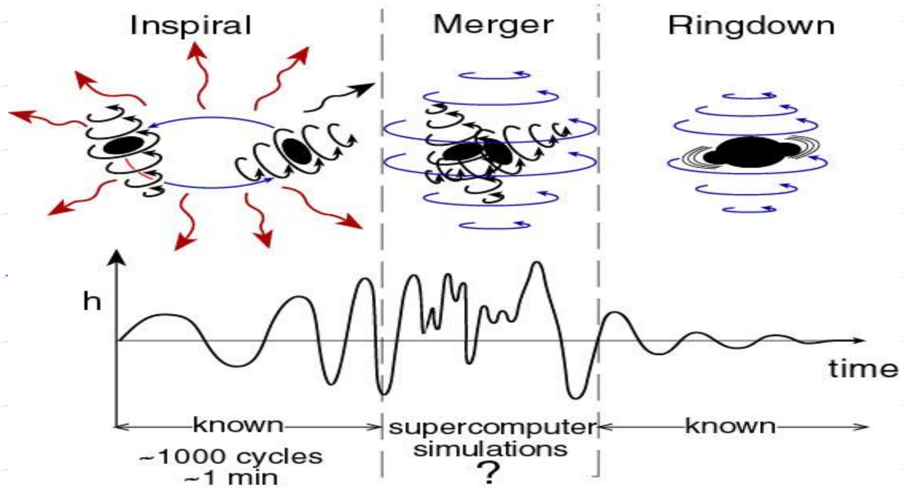
B1913+16
Hulse & Taylor (1975)

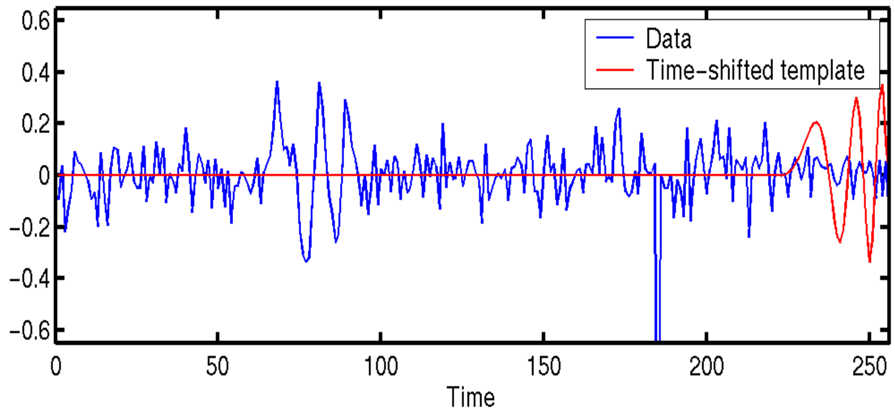
Effect of Gravitational Wave Radiation

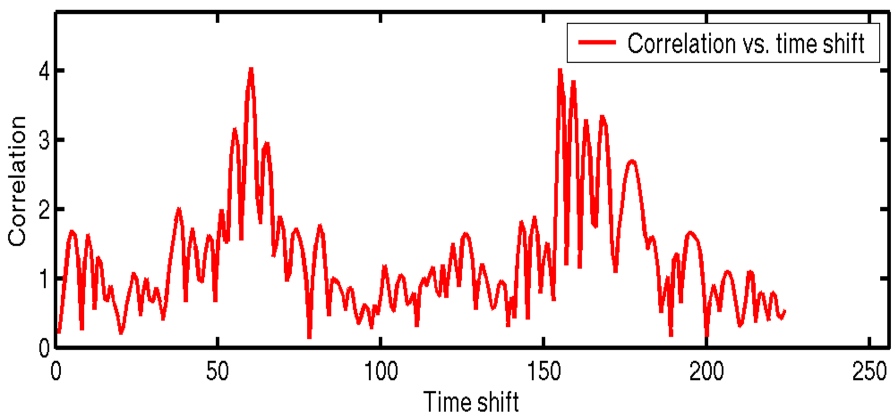
1993 Nobel Prize
Hulse & Taylor

LIGO was based on one DNS until 2002







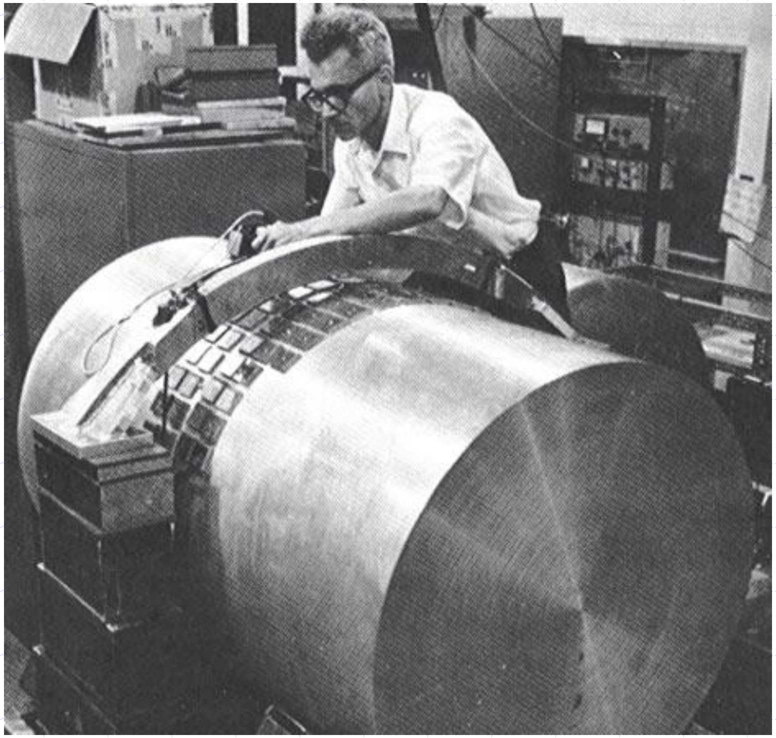


- GW Sources
  - BH-BH, NS-NS mergers
  - Cosmological perturbations
  - Supernovae

- Grav. wave pattern:

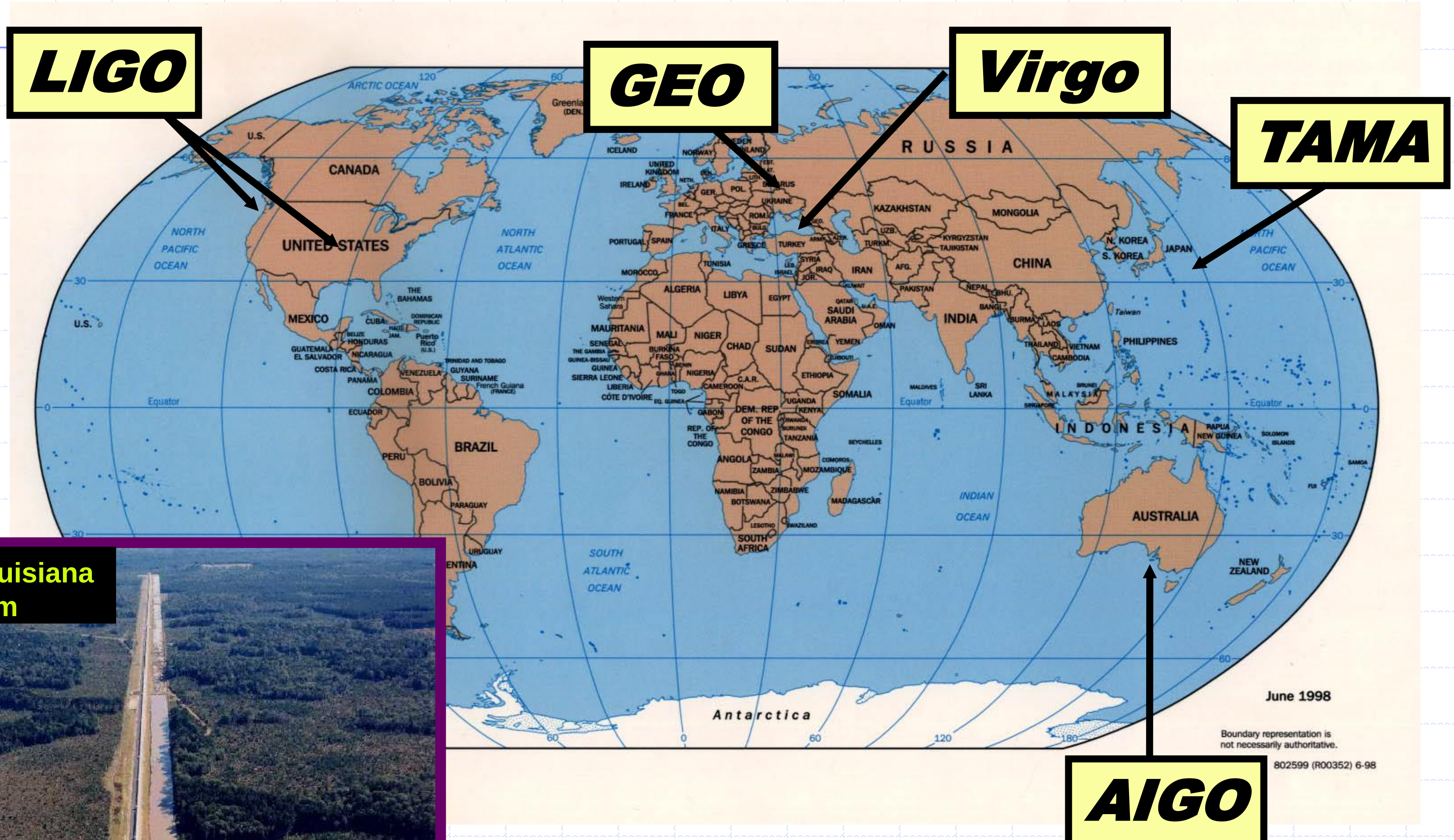
  "Urgent Demand For GW Detection !!"

NR and Gravitational Wave Detection



Joseph Weber (1960)

Network of Interferometers



NS-NS, NS-BH, BH-BH Binaries as sources for LIGO

(*Laser Interferomer Gravitational Wave Observatory*)

Observations

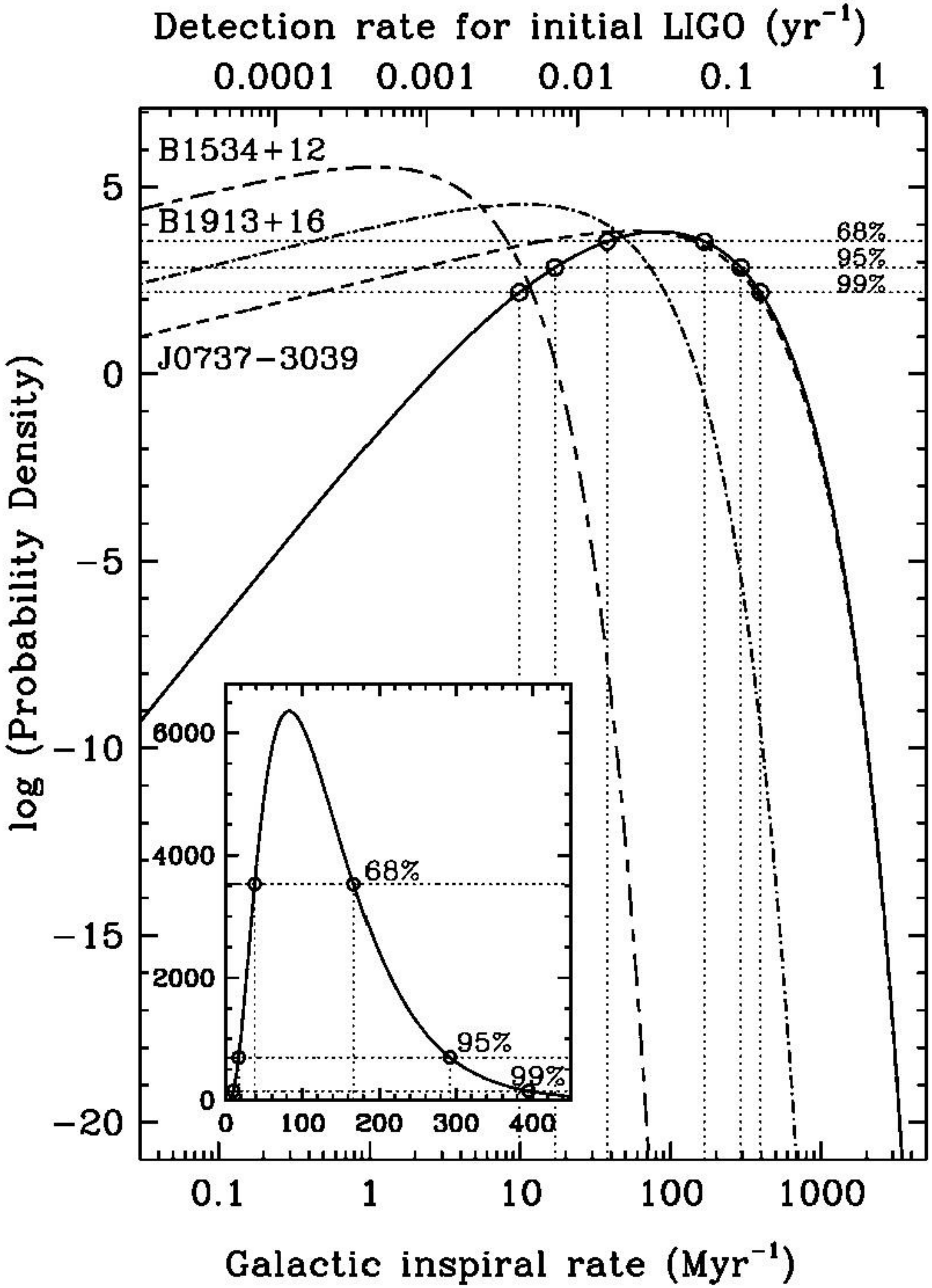
➢NS (radio pulsar) which coalesce within Hubble time

PSR	P (ms)	P_b (hr)	e	Total Mass $M_\odot$	τ_c (Myr)	τ_{GW} (Myr)	
J0737−3039A	22.70	2.45	0.088	2.58	210	87	(2003)
J0737−3039B	2773	2.45	0.088	2.58	50	87	(2004)
B1534+12	37.90	10.10	0.274	2.75	248	2690	(1990)
J1756−2251	28.46	7.67	0.181	2.57	444	1690	(2004)
B1913+16	59.03	7.75	0.617	2.83	108	310	(1975)
B2127+11C	30.53	8.04	0.681	2.71	969	220	(1990)
J1141−6545†	393.90	4.74	0.172	2.30	1.4	590	(2000)

Not important

Globular Cluster : no binary evolution

White Dwarf companion



- Due to J0737-3039 LIGO detection rate was increased by 8 !

- weak radio signal: 1/6 of B1913+16
- short coalesce time: 1/2 of B1913+16

- Initial LIGO 0.035 event/year
- Advanced LIGO 187 event/year

Kalogera et al. (2004)

Neutron Star - Neutron Star Binaries

1518+49	$1.56^{+0.13}_{-0.44}$	1518+49 companion	$1.05^{+0.45}_{-0.11}$
1534+12	$1.3332^{+0.0010}_{-0.0010}$	1534+12 companion	$1.3452^{+0.0010}_{-0.0010}$
1913+16	$1.4408^{+0.0003}_{-0.0003}$	1913+16 companion	$1.3873^{+0.0003}_{-0.0003}$
2127+11C	$1.349^{+0.040}_{-0.040}$	2127+11C companion	$1.363^{+0.040}_{-0.040}$
J0737−3039A	$1.337^{+0.005}_{-0.005}$	J0737−3039B	$1.250^{+0.005}_{-0.005}$
J1756−2251	$1.40^{+0.02}_{-0.03}$	J1756−2251 companion	$1.18^{+0.03}_{-0.02}$

- All masses are < 1.5 $M_\odot$
- 1534, 2127: masses are within 1%
- J0737, J1756: ΔM = 0.1 - 0.2 $M_\odot$

Predicted LIGO Detection Rates (yr^{-1}).

Binary Type	Initial LIGO	Advanced LIGO	Chirp Masses ($M_\odot$)
NS-NS†	0.0348	187	1.0 - 1.3
BH-NS††	0.696	3740	1.3 - 2.7
BH-BH**	0.58	2450	~ 6
Total	1.31	6377	

$$R_{\text{eff}} = R_0 \left(\frac{M_{\text{chirp}}}{M_\odot} \right)^{5/6}, \qquad M_{\text{chirp}} = \mu^{3/5} M_{\text{tot}}^{2/5}$$

➢R_0=17 Mpc (initial LIGO), 280 Mpc (advanced LIGO)

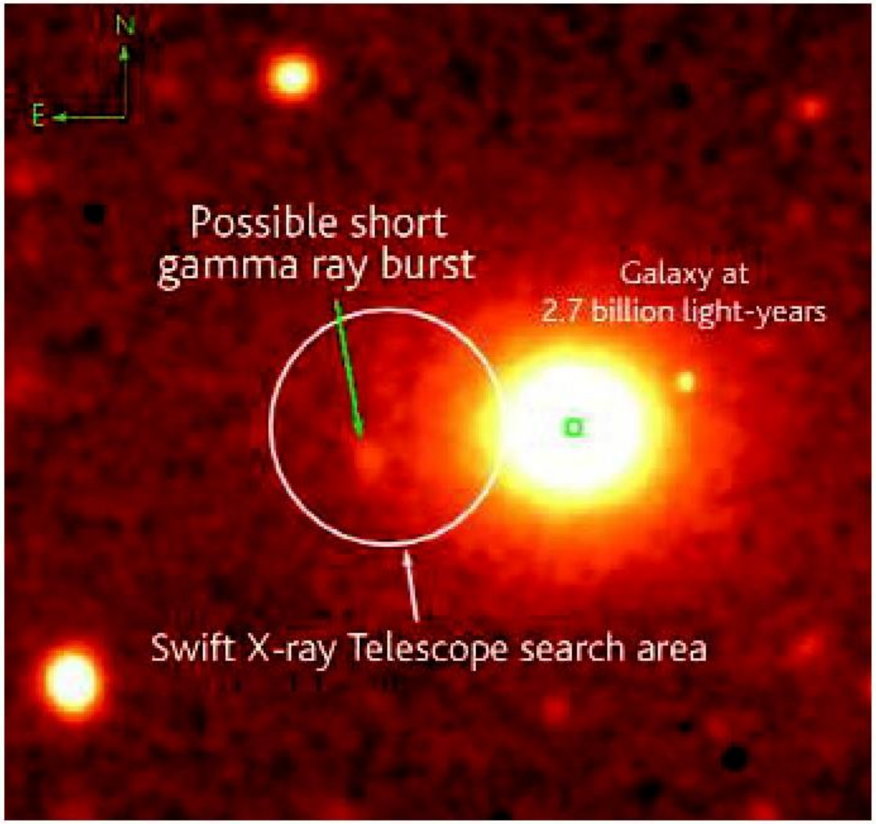
GAMMA RAY ASTRONOMY

Signs Point to Neutron-Star Crash

Astronomers think they have witnessed their first colossal crash of two neutron stars, an event that has tantalized theorists for decades.

Shortly after midnight EDT on 9 May, a NASA satellite detected a sharp flare of energy, apparently from the fringes of a distant galaxy. The news from Swift, launched in November 2004, was quickly disseminated to ground-based astronomers, triggering hours of intense research. As *Science* went to press, exhausted observers verified that their early observations look a lot like a neutron-star merger. "Prudence would say that we need a strong confirmation, but we're very excited by it," says astronomer Joshua Bloom of the University of California, Berkeley.

Colliding neutron stars would help explain a puzzling variety of the titanic explosions called gamma ray bursts (GRBs). Astronomers are



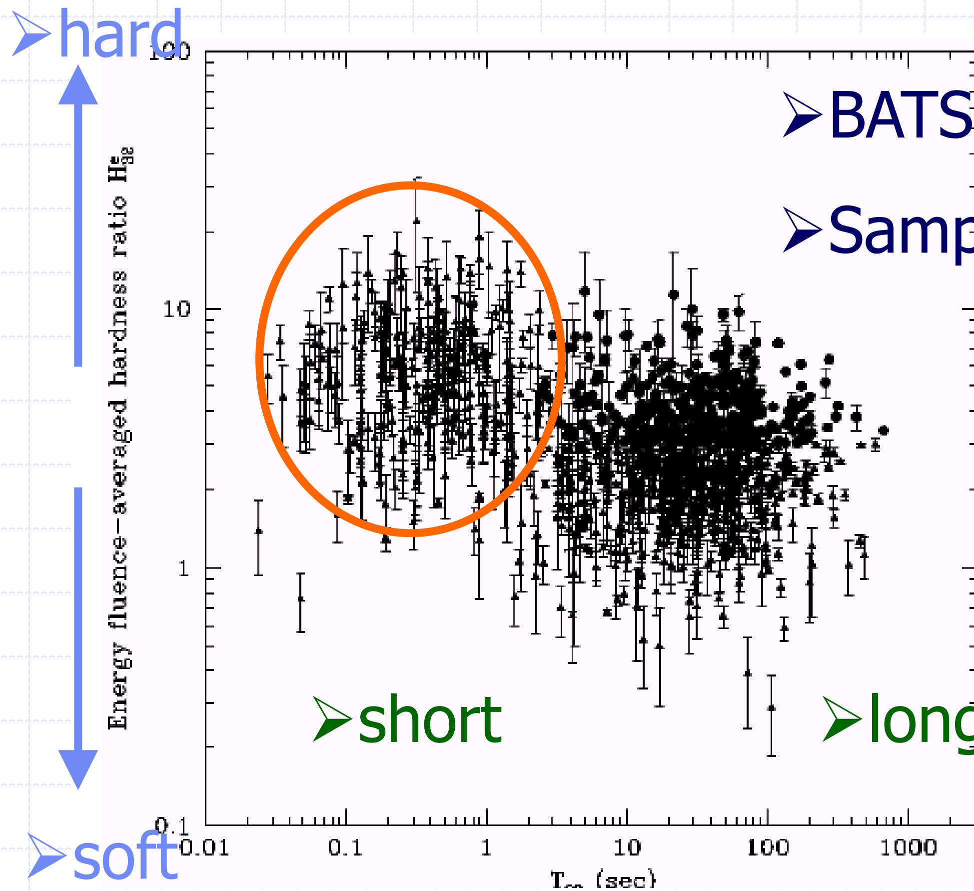
Neutron-star cataclysm? A faint patch of light (green arrow) may mark the spot where two neutron stars collided.

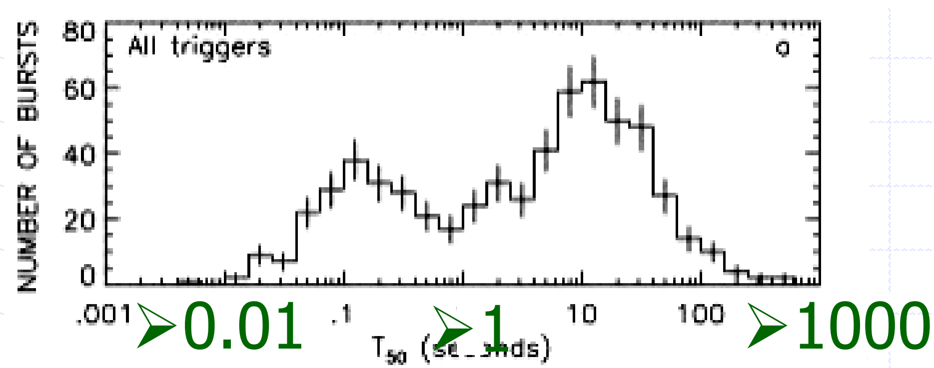
Science 308 (2005) 939

Short-hard GRBs

- No optical counterpart (?)

- Origin
  - Neutron star merger?
  - Magnetar flare?
  - Supernova?





Introduction

- NS-NS binaries : several
- NS-BH binaries : some clues
- BH-BH binaries : expected in globular clusters where old-dead stars (NS, BH) are populated.

Wanted

- How to distinguish sources from GW observations?
- What is the GW pattern ?

Why numerical approach ?

- Perturbative analytic method:
  good during the early stages of a merger & later stages of ringdown.
- Numerical solution is essential:
  during last several orbits, plunge, early stages of ring down.
- Problem:
  no code to simulate a nonaxisymmetric collisions through coalescence & ringdown

A bit of history : Simulations

- Head-on collision of two equal mass black holes: early 70's
  DeWitt ('76), Cadez ('71), Smarr ('75,'76,'77,'79), Eppley ('71), Anninos et al. ('95)
- 3-dim grazing collision of two BHs (2001)
  Alcubierre, Benger, Brugmann, Lanfermann, Nerger, Seidel, and Takahashi
- Single relativistic star (2002)
  Font, Gooddale, Iyer, Miller, Rezzolla, Seidel et al.
- Binary black hole coalescences (2005)
  Pretorius (0507), Campanelli (0511), Baker, Centrella, Dae-Il Choi, Koppitz, van Meter (0511), Diener et al. (0512)

Current Status of Numerical Approaches

2005

F. Pretorius, PRL 95, 121101 (2005)

2006

Baker, Centrella, D.-I. Choi, Koppits, van Meter, PRL 96, 111102 (2006)

2006

Campanelli, Lousto, Marronetti, Zlochower, PRL 96, 111101 (2006)

Evolution of Binary Black-Hole Spacetimes

F. Pretorius, PRL 95, 121101 (2005)

Best (?) work until 2005

Traditional treatment

Initial data by punctures (Brill-Lindquist conformal factor ψ_{BL})

- Brandt & Brugmann, PRL 78, 3606 (1997)

metric $g^{ph}_{ab} = \psi^4 g_{ab}$, $K^{ph}_{ab} = \psi^{-2} K_{ab}$ curvature

$$\psi = \frac{1}{\alpha} + u\,, \qquad \frac{1}{\alpha} = \sum_{i=1}^{N} \frac{m_{(i)}}{2|\vec{r} - \vec{r}_{(i)}|}$$

Evolved numerically

- Traditionally " $\psi_{BL} = 1/\alpha$ " is factored out & handled analytically
- Puncture remain fixed on the grid (during evolution)

- Problem in traditional treatment

1. As the distance between BHs shrinks, certain component of the metric must approach zero
   => causing other quantity diverge
   => kill the run before common horizon forms
2. Corotating coordinate frame causes superluminal coordinate speed at large distances

this paper: generalized harmonic coordinates

- Capable of evolving binary systems for enough time to extract information about the orbit, merger, and gravitational waves

- e.g., evolution of binary:
  2 equal mass, nonspinning BHs,
  ➔ single plunge orbit, merger, ringdown
  ➔ Kerr BH (a=0.7)
  ➔ 5% of initial rest mass radiated as GW

Features of generalized harmonic coordinates (I)

1. discretized scheme → minimize constraint eq (Evolved quantities → covariant metric elements, harmonic source, matter functions)
2. Compactified coordinate → boundaries at spatial infinity (correct boundary)
3. Adaptive mesh refinement → relevant length scale
4. Dynamical excision → track the motion of BH through grid (using harmonic gauge)
5. Addition of numerical dissipation → control high-frequency instabilities

Features of generalized harmonic coordinates (II)

1. Time slicing → slows down the collapse of the lapse
2. Addition of "constraint-damping" → very important effect on how long a simulation with black holes can run with reasonable accuracy.

Discretize Einstein field equation

$$g^{\delta\gamma} g_{\alpha\beta,\gamma\delta} + g^{\gamma\delta}_{,\beta} g_{\alpha\delta,\gamma} + g^{\gamma\delta}_{,\alpha} g_{\beta\delta,\gamma} + 2H_{(\alpha,\beta)} - 2H_{\delta}\Gamma^{\delta}_{\alpha\beta} + 2\Gamma^{\gamma}_{\delta\beta}\Gamma^{\delta}_{\gamma\alpha}$$
$$= -8\pi(2T_{\alpha\beta} - g_{\alpha\beta}T) - \kappa(n_{\alpha}C_{\beta} + n_{\beta}C_{\alpha} - g_{\alpha\beta}n^{\gamma}C_{\gamma})$$

H: source function (gauge freedom)

T: matter stress tensor

N: unit hypersurface normal vector

Γ: Christoffel symbols

- Constraint

$$C_\mu \equiv H_\mu - g_{\mu\nu} \Box x^\nu .$$

- Evolution of source function

$$\Box H_t = -\xi_1 \frac{\alpha - 1}{\alpha^\eta} + \xi_2 H_{t,\nu} n^\nu , \qquad H_i = 0,$$

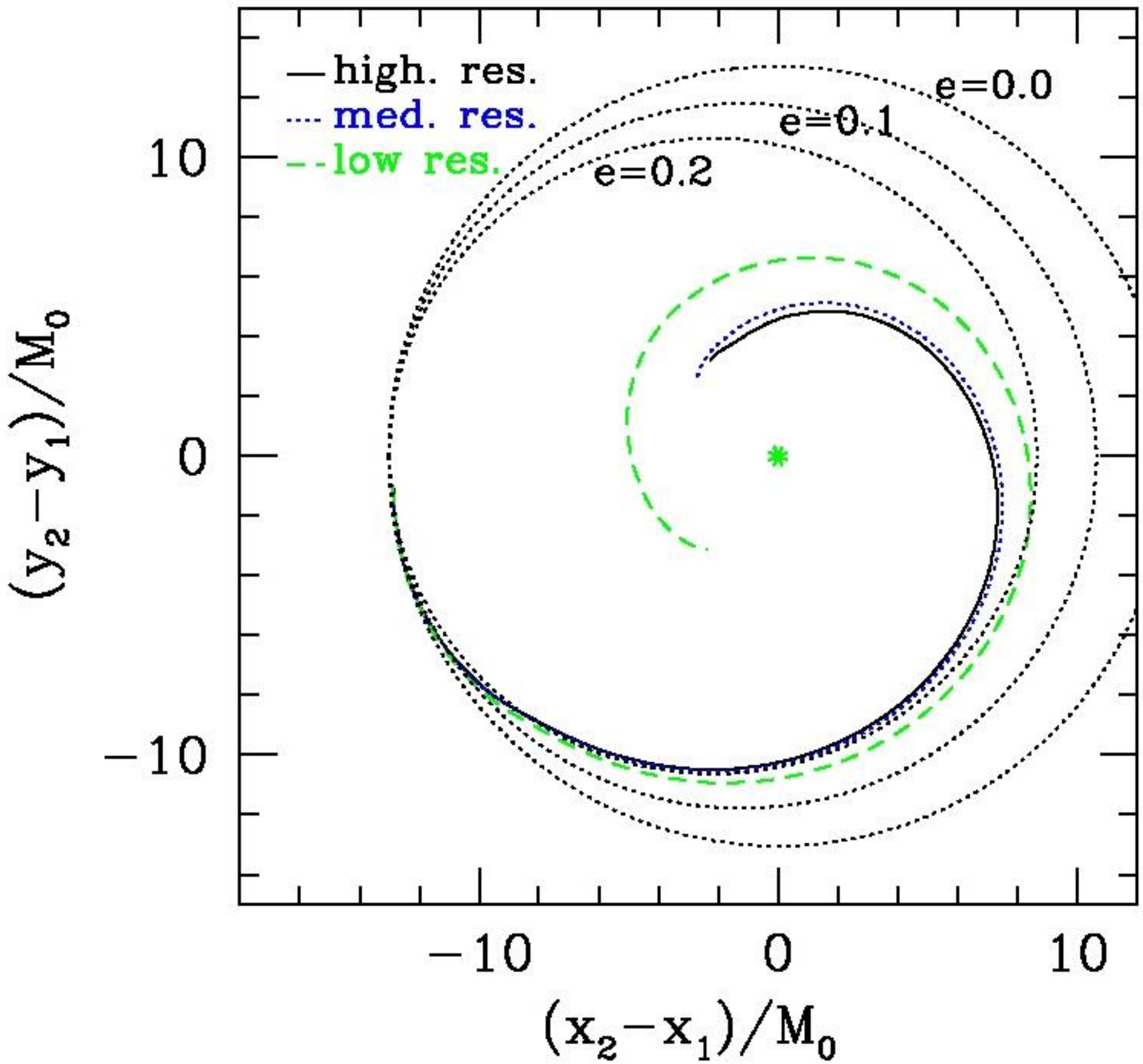
Initial conditions

- Initial data: scalar field gravitational collapse (at t=0: two Lorentz boosted scalar field profiles)
- initial spatial metric & its first time derivative: conformally flat
- Maximal condition
- Harmonic condition: $H=0$

Three different grid hierarchies: for efficiency

- Low resolution: 32^3 with up to 7 additional levels of 2:1 refinement
- Medium resolution: one additional refinement during the inspiral and early phases of the merger
- High resolution: upto 10 levels of refinement during the inspiral and early ringdown phase.

Initial parameter are chosen such that the BH would merge within roughly one orbit.



e: eccentricity

		Low Res.	Med. Res.	High Res.
	ADM Mass	$2.36M_0$	$2.39M_0$	$2.39M_0$
Initial	BH masses	$0.97M_0$	$0.99M_0$	M_0
	Orbital eccentricity	0–0.2	0–0.2	0–0.2
	Proper separation	$16.5M_0$	$16.6M_0$	$16.6M_0$
	Angular velocity $\times M_0$	0.023	0.023	0.023
Final	BH mass	$1.77M_0$	$1.85M_0$	$1.90M_0$
	BH spin parameter	0.74	0.74	0.74

ADM Mass: Arnowitt-Deser-Misner mass (total energy of binary system)

- 15% of total scalar field energy leaves the orbit in light crossing time of the orbit.

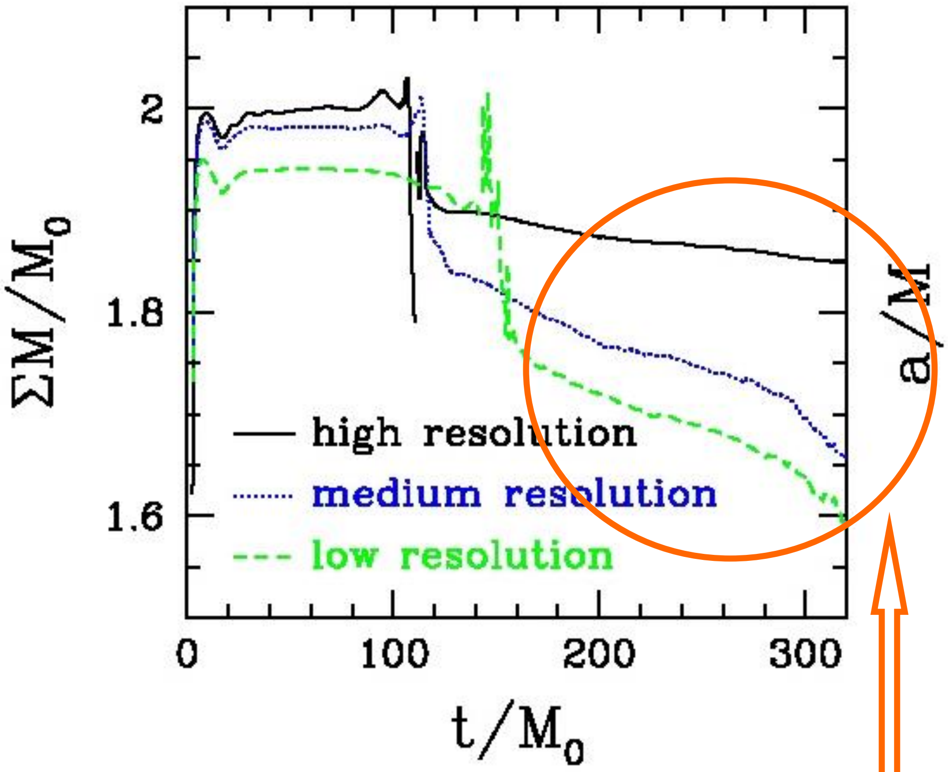
BH Mass : Smarr formula (A: horizon area)

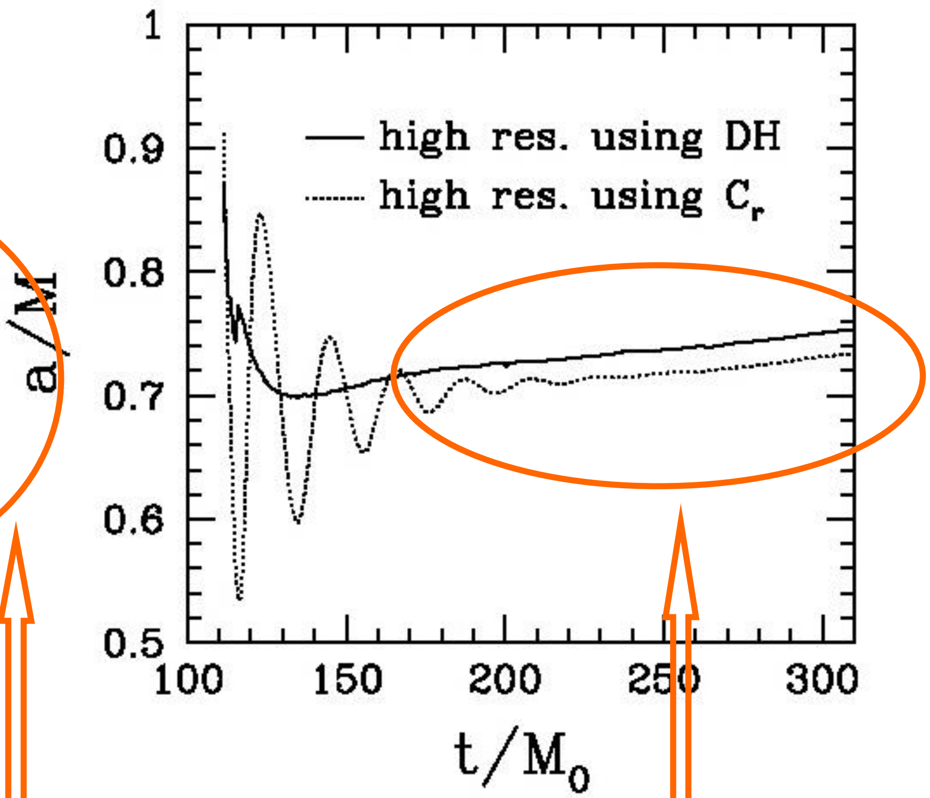
$$M = \sqrt{M_{ir}^2 + J^2/(4M_{ir}^2)}, \qquad M_{ir} \equiv \sqrt{A/16\pi}.$$

Kerr parameter (ratio of the polar to equatorial proper radius of the horizon)

$$a \approx \sqrt{1 - (2.55C_r - 1.55)^2}.$$

- Two methods to determine "a" ➔ agree in average *"Dynamical horizon framework" (rotation axis is orthogonal to BH) & using "Cr"*



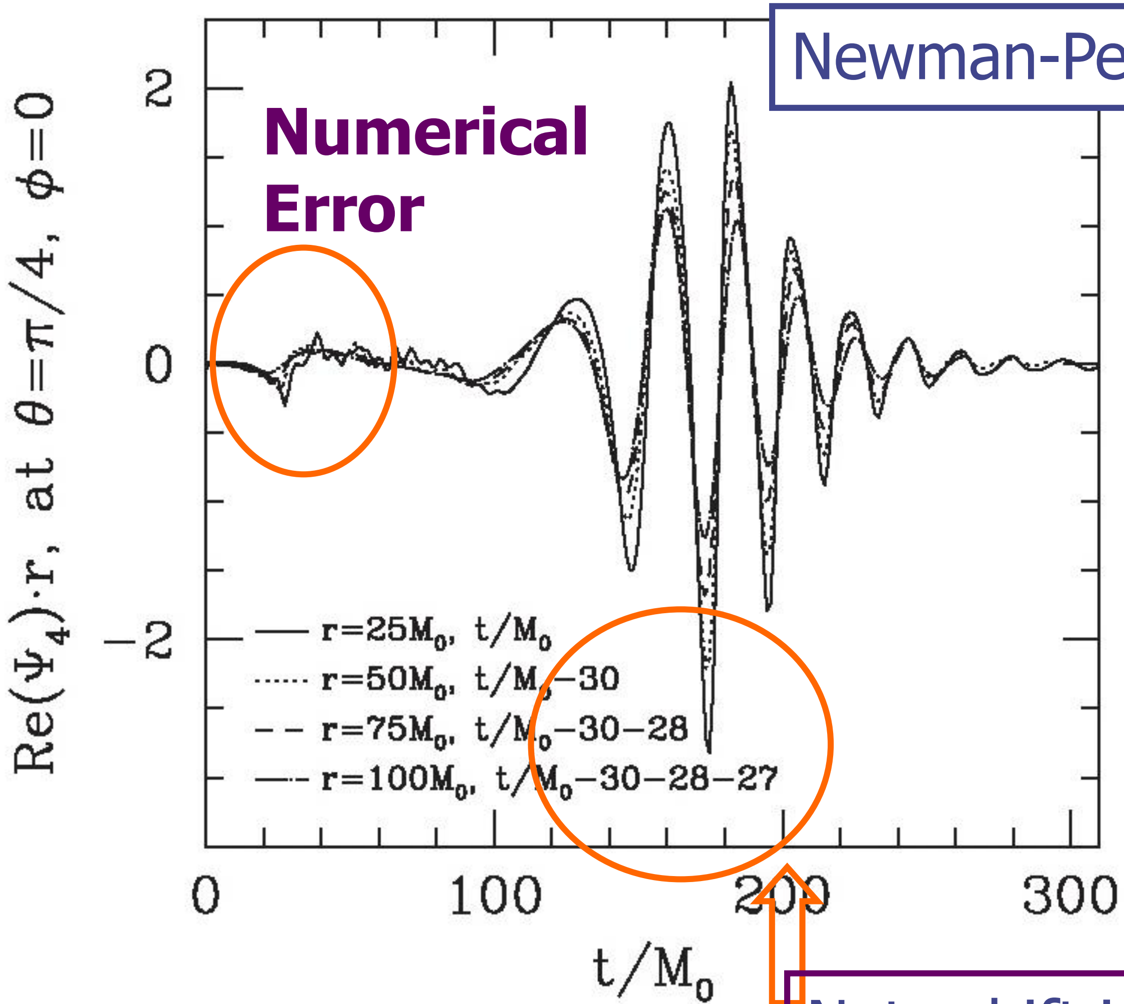


Numerical errors

2005

➢Emitted gravitational wave (medium resolution)

Newman-Penrose scalar



25 M_0=light travel time

Note shift in time !
(not light travel time)

Total energy emitted

$$\frac{dE}{dt} = \frac{R^2}{4\pi} \int p d\Omega, \qquad p = \int_0^t \Psi_4 dt \int_0^t \bar{\Psi}_4 dt,$$

- Numerical error in Ψ will inflate !
- To reduce error: filter high spherical harmonics (> l=6)

Radiated GW energy

- From summation from Ψ :
  4.7% (r=25), 3.2% (r=50),
  2.7% (r=75), 2.3% (r=100)
- From final & initial horizon mass difference
  ➔ 5% (high resolution), 11% (low resolution)

works to be done (as of 2005)

- Inprovement of the accuracy
  (in particupar the gravitational wave)
- explore large classes of initial conditions
  (separation, initial mass, initial BH spin, …)
- Extract more geometric informations about the nature of the merger event from the simulation.

New Achievement in 2006

2006

Baker, Centrella, D.-I. Choi, Koppits, van Meter, PRL 96, 111102 (2006)

Campanelli, Lousto, Marronetti, Zlochower, PRL 96, 111101 (2006)

2006

New features in these two works

- Moving black holes through grid without excision

2006

Work by D.-I. Choi group

- Evolution by HAHNDOL code

Gravitational-Wave Extraction from an Inspiraling Configuration of Merging Black Holes

Baker, Centrella, Choi, Koppitz, van Meter

PRL 96, 111102 (2006)

Initial data by punctures (Brill-Lindquist conformal factor ψ_{BL})

- Brandt & Brugmann, PRL 78, 3606 (1997)

$$g_{ab}^{ph} = \psi^4 g_{ab}, \qquad K_{ab}^{ph} = \psi^{-2} K_{ab}$$

$$\psi = \frac{1}{\alpha} + u, \qquad \frac{1}{\alpha} = \sum_{i=1}^{N} \frac{m_{(i)}}{2|\vec{r} - \vec{r}_{(i)}|}$$

Evolved numerically

- Traditionally " $\psi_{BL} = 1/\alpha$ " is factored out & handled analytically
- Puncture remain fixed on the grid (during evolution)

- Problem in traditional treatment

1. As the distance between BHs shrinks, certain component of the metric must approach zero
=> causing other quantity diverge
=> kill the run before common horizon forms

2. Corotating coordinate frame causes superluminal coordinate speed at large distances

- New approach

Evolve full conformal factor ψ

- Initial setup: centers of BHs are not at the grid points
- Initially, effectively regularize the puncture singularity by taking numerical derivatives of conformal factor
- During evolution: BHs remain in the z=0 plane
- grids points in cell-centered implementation.

HANDOL code : cell-centered implementation

- Innermost refinement region is a cube stretching from -2M to 2M in all 3-direction
- Punctures are placed in the z=0 plane
- Impose equatorial symmetry
- Resolution M/16, M/24, M/32
- Outer boundary 128M
- 4th-order finite differentiating
- Highest resolution: 40 hours on 256 processors of SGI Altix 3000 machine

Free evolution of punctures

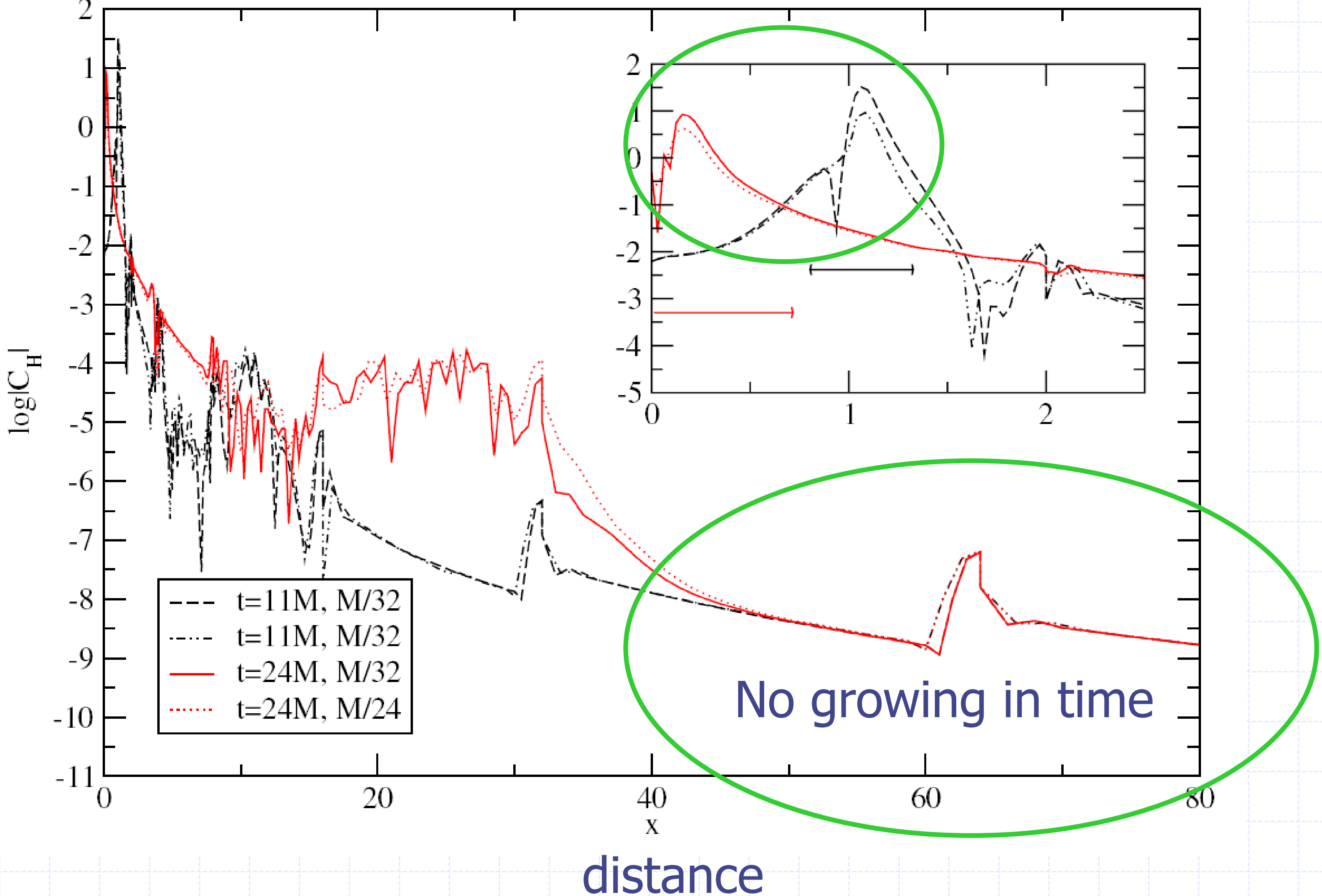
- Possible by Gamma-freezing shift vector which drives coordinates towards quiescence as the merged remnant BHs also becomes physically quiescent.

$$\partial_t \beta^i = \frac{3}{4} \alpha B^i$$

new

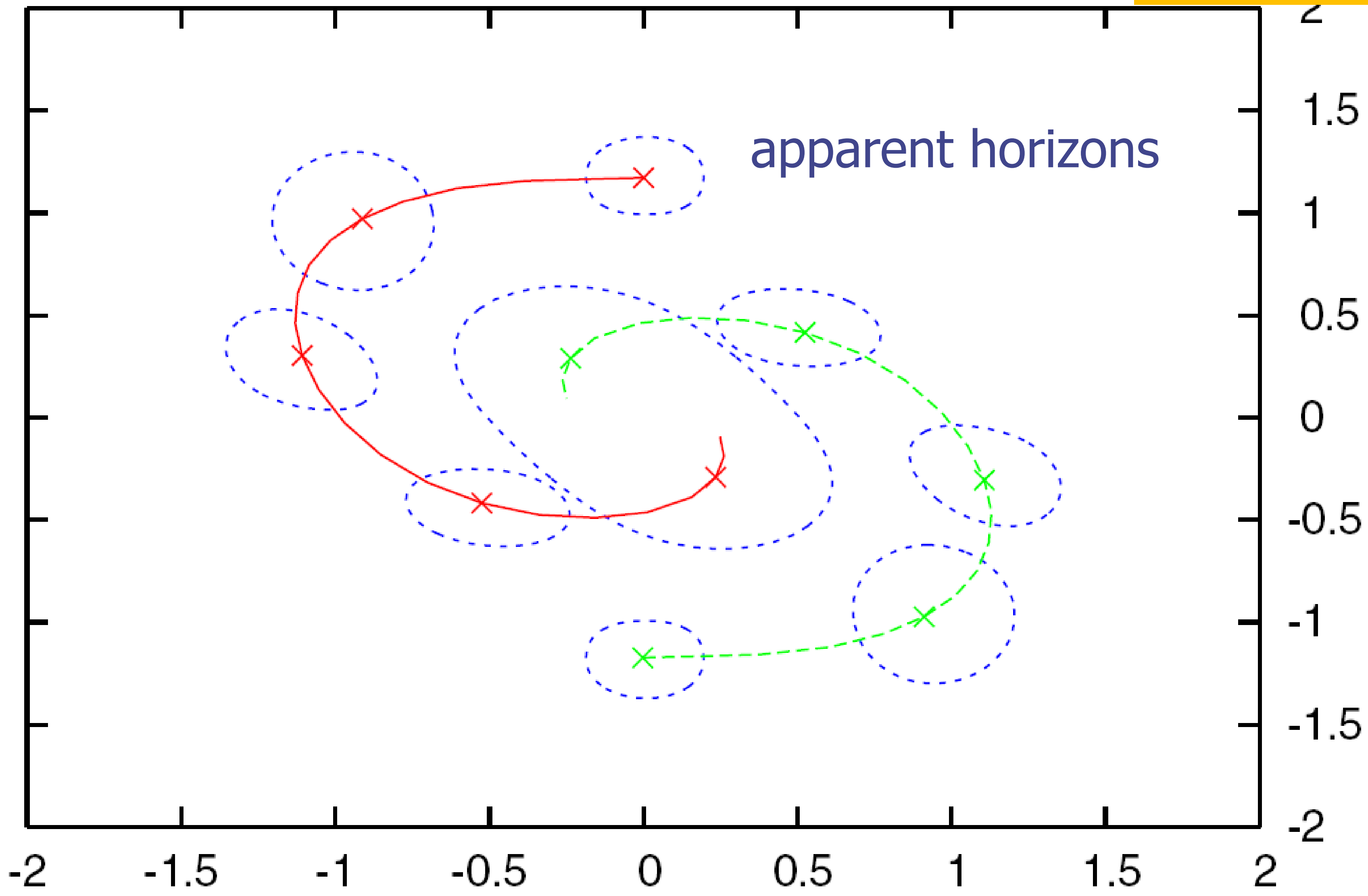
$$\partial_t B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$$

Eliminate zero-speed mode
(to destroy "puncture memory" effect)



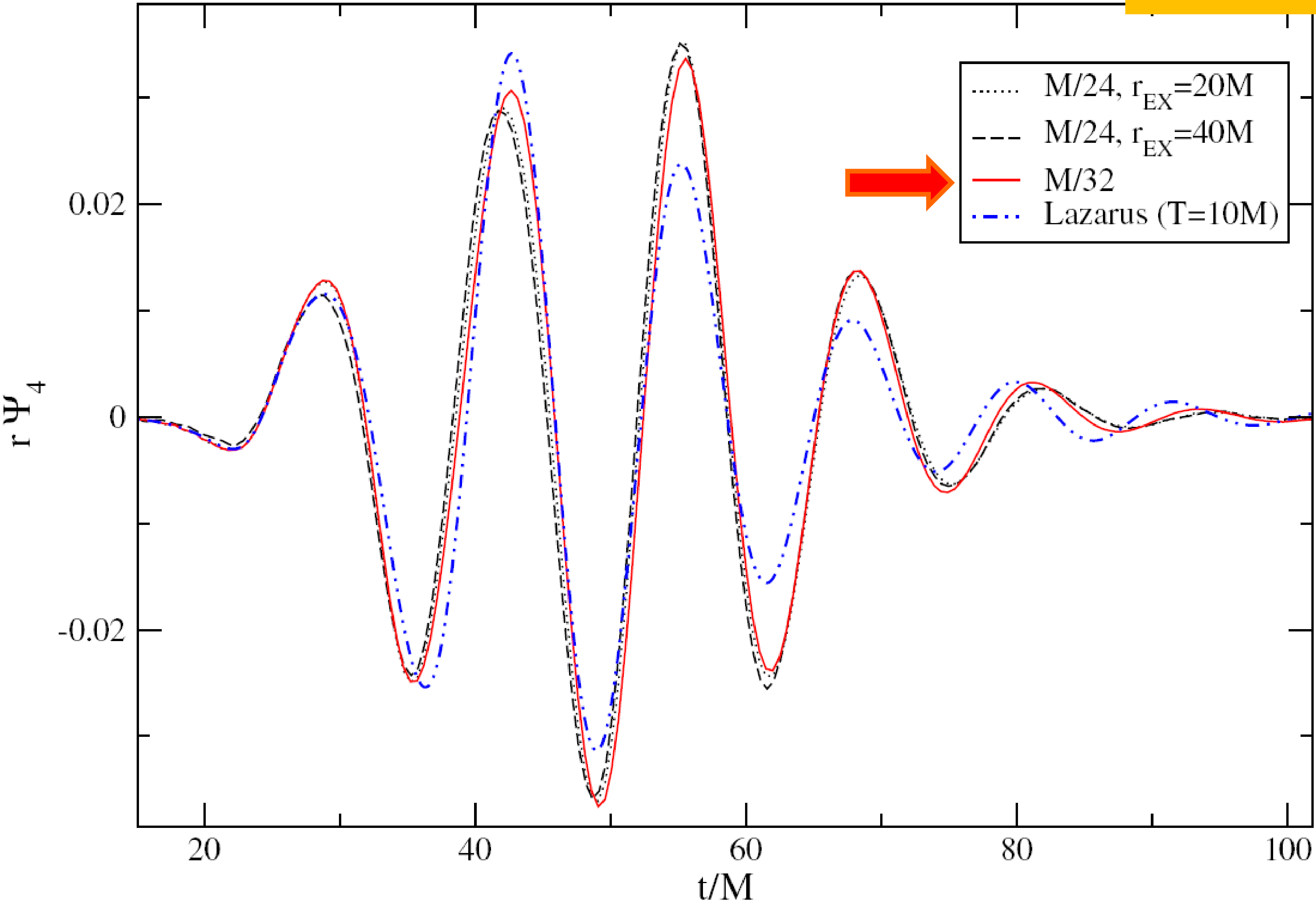
- Hamiltonial constraint error at two times

2006



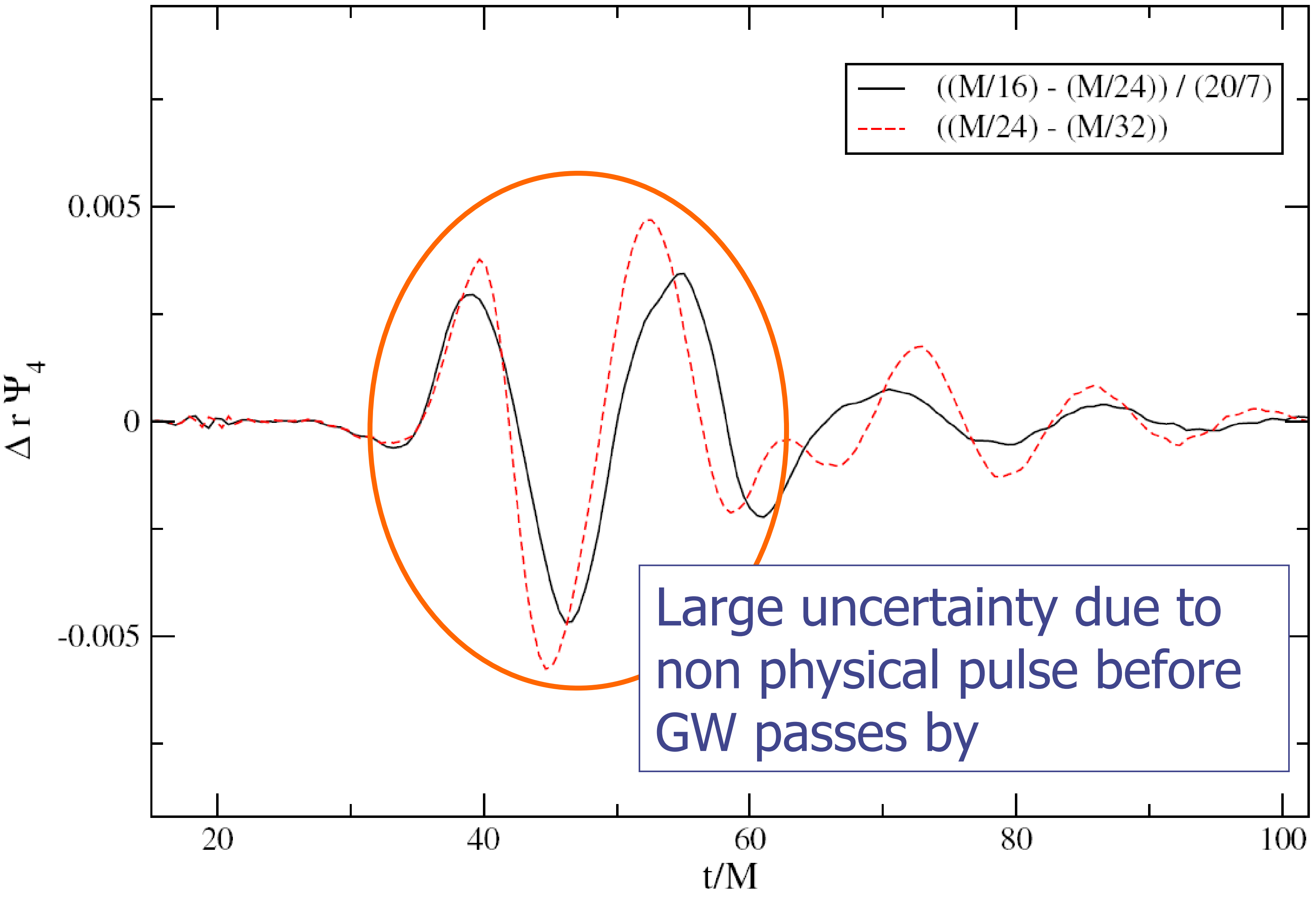
Position of apparent horizons at t=0,5,10,15,20M

2006

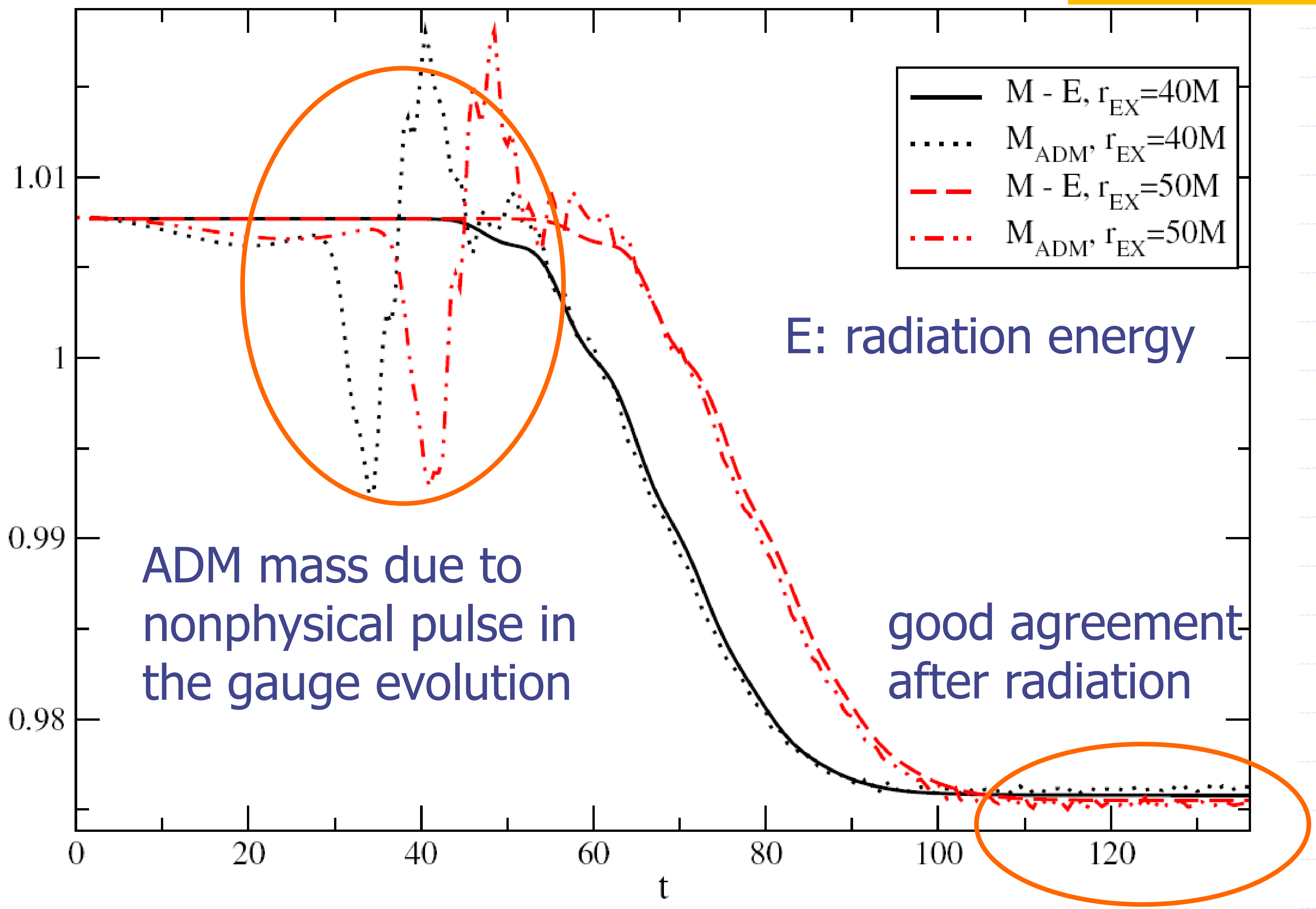


For M/32 resolution, no difference at r=20, 40M !

2006



convergence

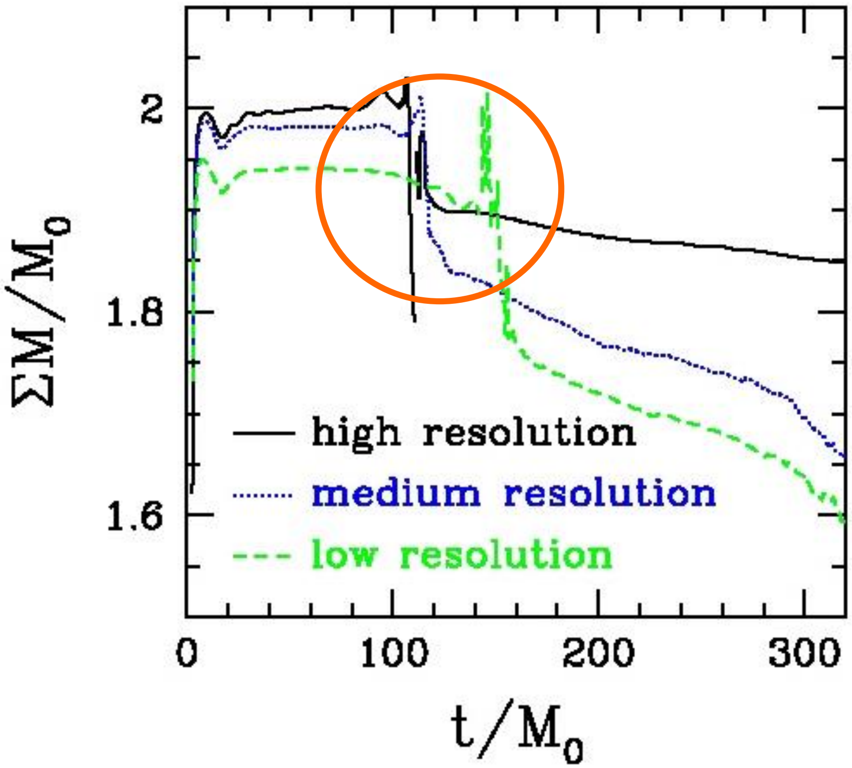

M - E, r_{EX}=40M
M_{ADM}, r_{EX}=40M
M - E, r_{EX}=50M
M_{ADM}, r_{EX}=50M
1.01
1
0.99
0.98
0
20
40
60
80
100
120
t
E: radiation energy
ADM mass due to nonphysical pulse in the gauge evolution
good agreement after radiation

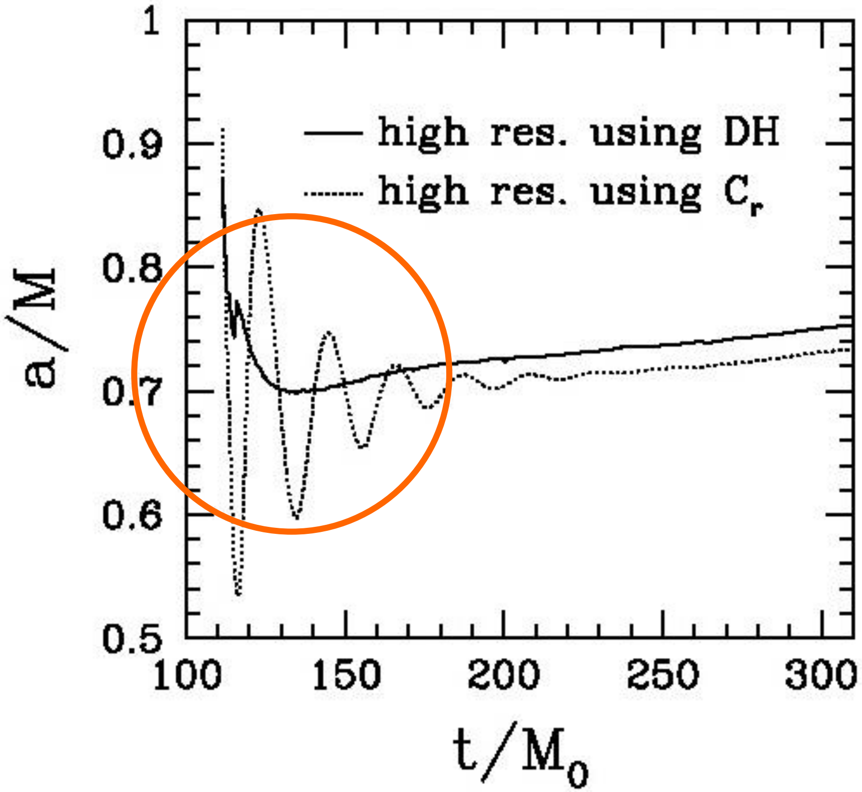
Mass-Energy

2006

For comparison

2005 Pretorius's results





C.-H. Choi's code (2006) gives better results (later t ?)

Conclusion

	$M/16$	$M/24$	$M/32$	Lazarus	AEI
E/M	0.0516	0.0342	0.0330	0.025	0.030
J/M^2	0.208	0.140	0.138	0.10	0.17

Radiated energy (E) & angular momentum (J) by gravitional wave

- 3% of initial mass-energy is in gravitational wave
- Good energy conservation during the evolution
- Future work: adaptive mesh refinement implementation

Accurate Evolutions of Orbiting Black-Hole Binaries without Excision

Campanelli, Lousto, Marronetti, Zlochower

PRL96, 111101 (2006)

2006

Gauge condition

$$\partial_0 \alpha = -2\alpha K,$$

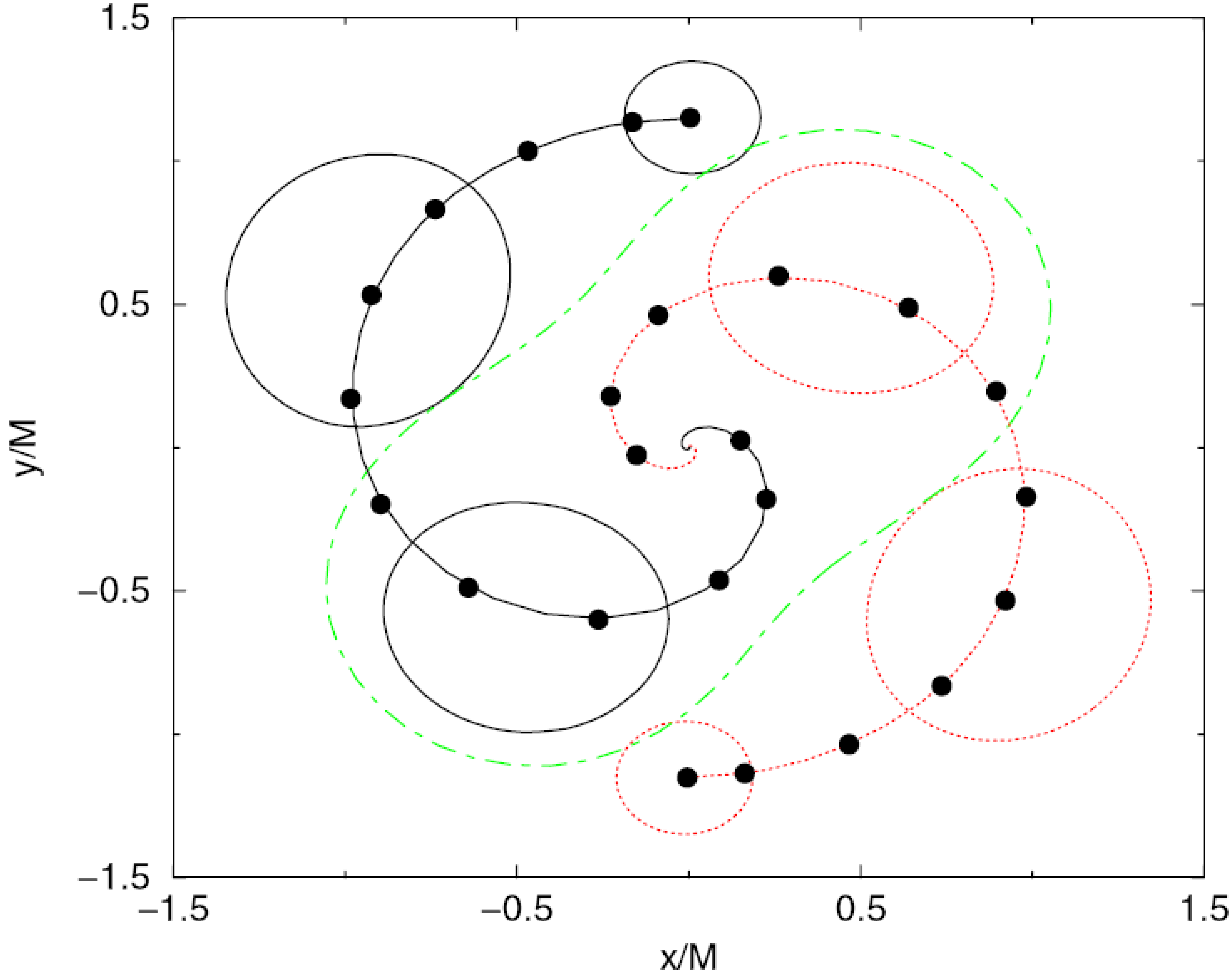
$$\partial_t \beta^a = B^a, \qquad \partial_t B^a = 3/4\partial_t \tilde{\Gamma}^a - \eta B^a.$$

2006

Cf) Dr. Choi's group

$$\partial_t \beta^i = \frac{3}{4}\alpha B^i$$

$$\partial_t B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$$

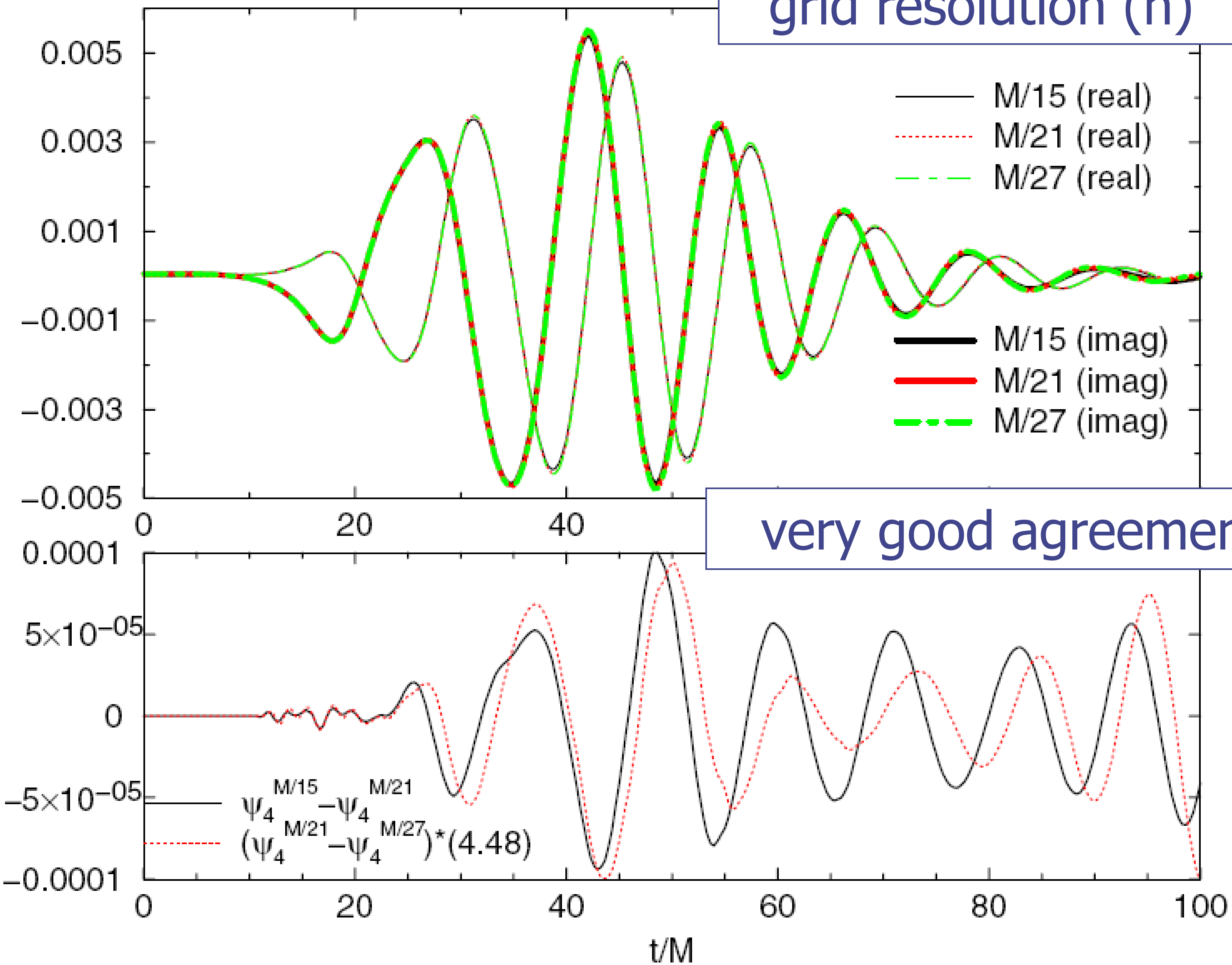


trajectories from t=0 to 18.8 M (in 2.5 M step)

2006

grid resolution (h)

very good agreement



Weyl scalar $(\ell = 2,\, m = 2)$ mode of ψ_4 at $r = 15M$

TABLE I. Results of the evolution.

Method	$E_{\rm rad}/M$	$J_{\rm rad}/M^2$	$t_{\rm merger}/M$	$a/M_{\mathcal{H}}$
This Letter	2.8 ± 0.2	$15 \pm 1\%$	$T_{\rm CAH} \approx 18.8$	0.677 ± 0.006
Lazarus[a]	2.5 ± 0.2	$13 \pm 2\%$	$T_{\rm Tran} \approx 10$	0.70 ± 0.02

$$M_{\mathcal{H}} = (M_{\rm irr}/\tilde{a})\sqrt{2(1-\sqrt{1-\tilde{a}^2})}$$

Horizon mass reduction is in excellent agreement with the calculated radiated energy !

Computing time

Largest run (h=M/27)

- 2882 X 576 grid points (64 GB)
- 2 weeks on 16 nodes (dual 3.2 GHz Xeon processors)

Dr. Choi's group

2006

- Highest resolution: 40 hours on 256 processors of SGI Altix 3000 machine
- Twice computing time per processor

Future plan

- Larger initial separations (several orbits before merging)
- Thin-sandwidth & post-Newtonian initial data set
- Unequal-mass black-holes & their gravitational kick
- Highly spinning black-holes

Prospects

- Two recent works without excision give more stable (reliable) results !!
- Future possibilities in numerical relativity !
- Colliding Neutron Stars:
  - Equation of States
  - QGP formation in the process of collision (?)

Physics of Heavy Ion Collisions

NR Group vs HIM (since 2004)

- ✓ 2004: Gravitational Wave Working Group
- ✓ 2005.3.16: Kick-off meeting
- ✓ 2005.6.28: Korean Numerical Relativity Group
- ✓ Monthly mini-workshops and Schools
- ✓ KISTI (Super Computing Center)
- ✓ APCTP Topical Research Program

- ✓ 2004.12 first HIM meeting
- ✓ Bimonthly meeting
- ✓ 1st ATHIC (2006.06)
- ✓ 2nd ATHIC (2008.10)
- ✓ 2005-2007: APCTP Topical Research Program