holographic QCD

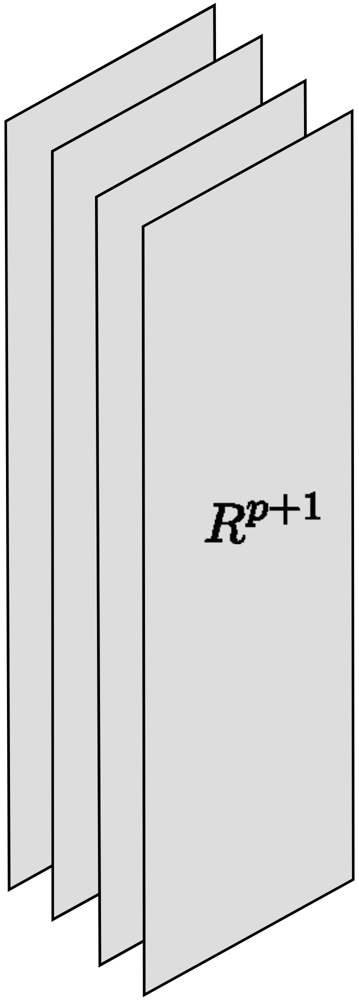
PILJIN YI
HIM, Sept. 2007

content

- D-Branes and a non-AdS/non-CFT
- Holographic QCD is a theory of the large n Master Field
- Sakai-Sugimoto & Mesons
- Baryons
- Effective Field Theory of Baryons
- Vector Dominance

General D-Brane Theory

Dp's



p+1 dim Maximally SUSY U(n) Yang-Mills

+ Chern-Simons couplings to RR-fields

$$\mu_{\rm p}\int dx^{p+1}\frac{1}{4e^{\Phi}}\sqrt{-h}\,{\rm tr}\,|2\pi\alpha' F|^2$$

$$+\quad \mu_{\rm p}\int\sum_{k=0}^{p} C_{k+1}\wedge {\rm tr}\, e^{2\pi\alpha' F}$$

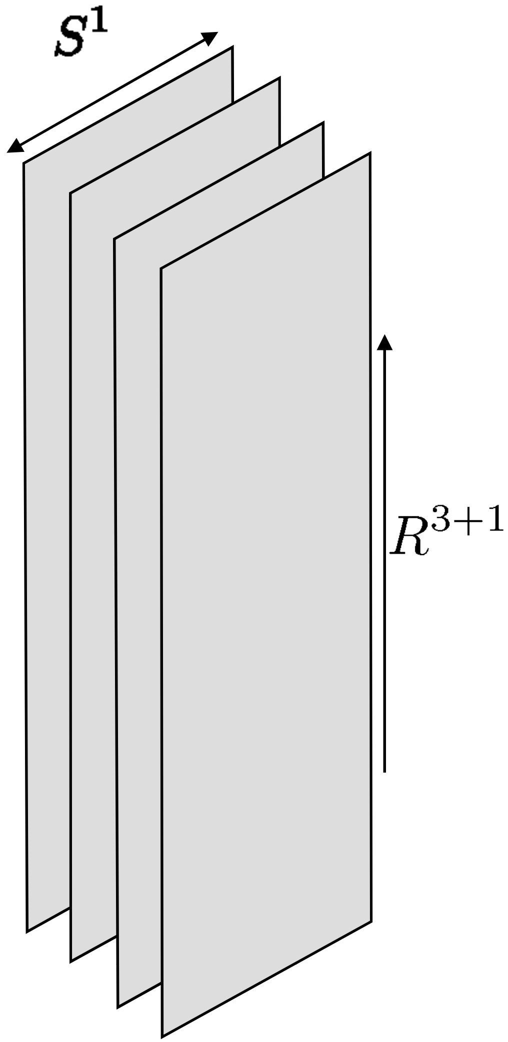
$+$ susy completion

$+$ higher dimensional corrections

$$\mu_{\rm p} = \frac{2\pi}{(4\pi^2\alpha')^{(p+1)/2}}$$

pure D=4 SU(n) Yang-Mills theory

n D4's



4+1 dim Maximally SUSY
U(n) Yang-Mills theory on $S^1 \times R^{3+1}$

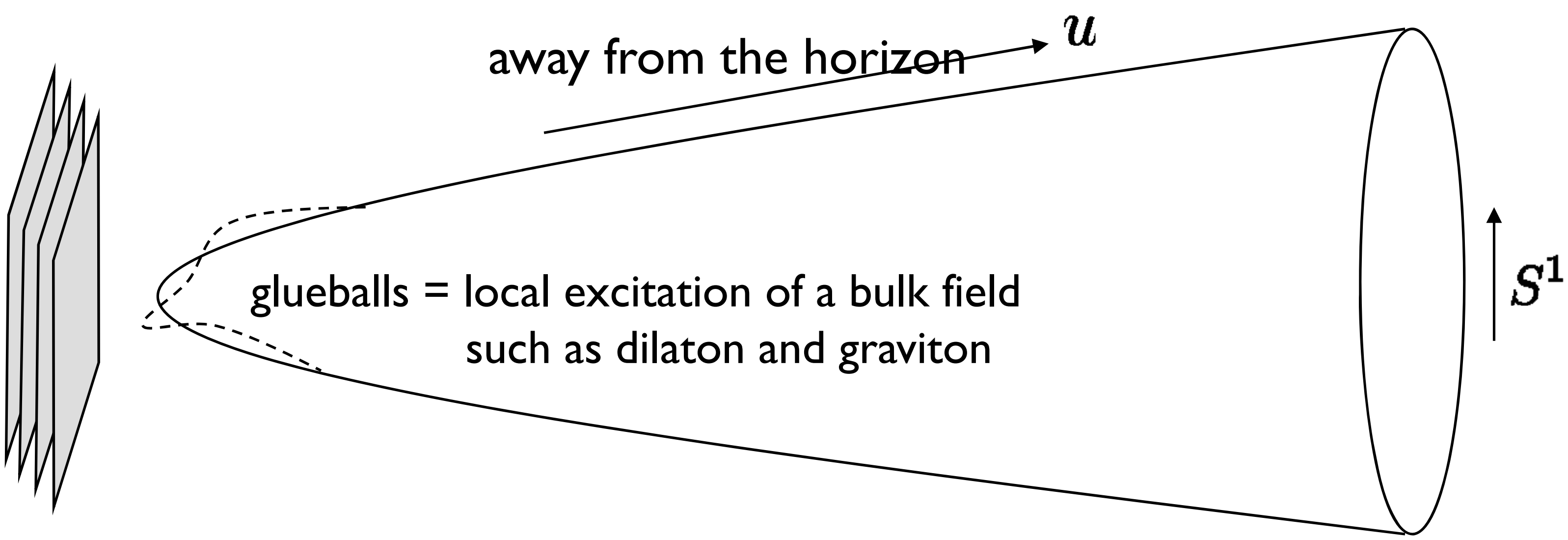
anti-periodic BC along S^1
on fermions breaks SUSY and produces

3+1-D NonSusy SU(n) Pure YM
+ towers of heavy adjoint particles.

E.Witten 1998

pure SU(n) Yang-Mills theory

$n \gg 1, \ g_{YM}^2 n \gg 1$



$$\times R^{3+1} \times S^4$$

nonAdS/nonCFT for pure SU(n) Yang-Mills theory

$n \gg 1, \;\; g_{YM}^2 n \gg 1$

$$G_{9-1} = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{3-1} + f(u) d\theta^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

E.Witten 1998

R^{3+1} $\quad S^1$

occupied by n D4's

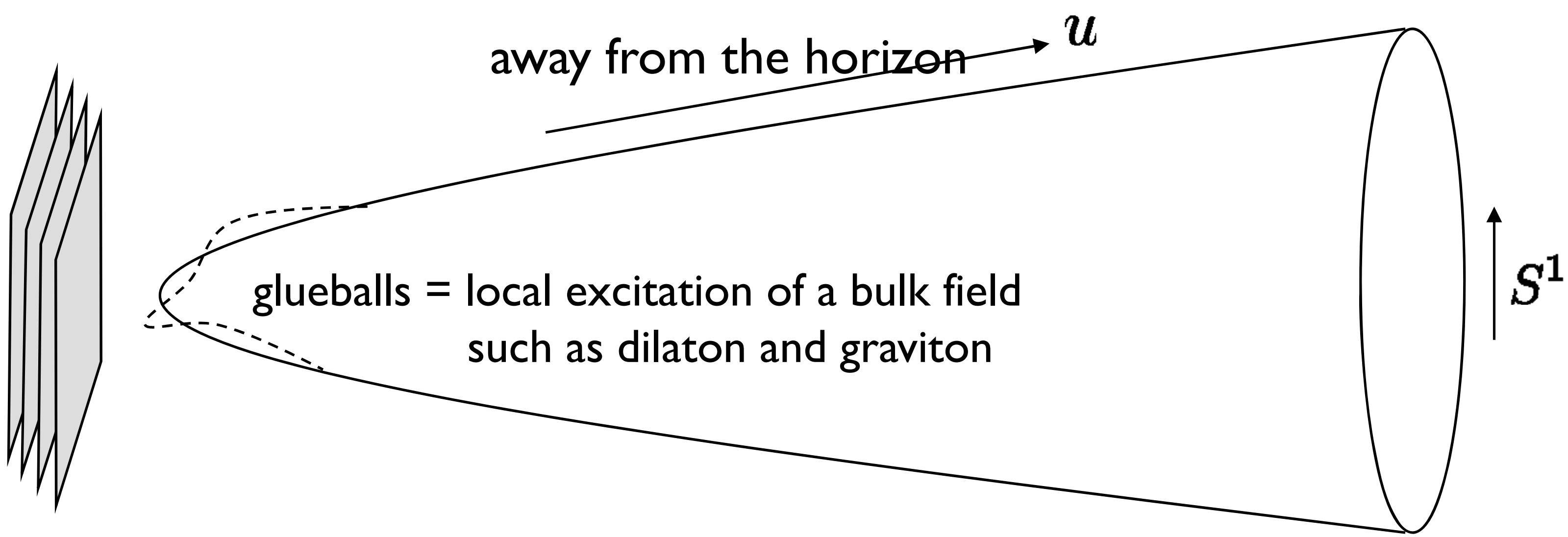
$u_0 \leq u < \infty$

$f(u) = 1 - \left(\frac{u_0}{u}\right)^3$

Gravity in the dual nonAdS geometry = A Subsector of the Dual nonCFT

nonAdS/nonCFT for pure SU(n) Yang-Mills theory

$n \gg 1, \ g_{YM}^2 n \gg 1$



$$\times R^{3+1} \times S^4$$

nonAdS/nonCFT for pure SU(n) Yang-Mills theory

$n \gg 1, \; g_{YM}^2 n \gg 1$

$$G_{9-1} = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{3-1} + f(u) d\theta^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

R^{3+1} $\quad$ S^1

$$u_0 \leq u < \infty$$

$$f(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

Gravity in the dual nonAdS geometry = A Subsector of the dual nonCFT

states with spin no larger than 2,
including some low-lying glueballs

10% match with lattice results on glueball mass ratios had been reported.

C. Csaki, H. Ooguri, Y. Oz, J. Terning 1998

Holographic QCD is a theory of the large n Master Field

?

Holographic QCD is a theory of the large n Master Field

Toy model: hermitian matrix model

$$Z = \int \prod_i dM_i e^{-S(M_i)}$$

M_i n x n hermitian matrices

$$S(M_i) = S(U^\dagger M_i U)$$

any large n theory of matrices
with U(n) (gauge) symmetry

Fact:

$$O_a = \frac{1}{n}\mathrm{tr}\prod_{a_i} M_{a_i} \longrightarrow \langle O_1 O_2\rangle = \langle O_1\rangle\langle O_2\rangle + O(\frac{1}{n^2})$$

In the large n limit, the invariant operators behaves as classically,
i.e. , certain classical configurations dominate the evaluation of
the path integral for these operators.

Holographic QCD is a theory of the large n Master Field

$$\langle \prod_a O_a \rangle = \frac{1}{Z} \int \prod_i dM_i e^{-S(M_i)} \prod_a O_a \sim \prod_a O_a^{classical} + \cdots$$

Holographic QCD is a theory of the large n Master Field

$$\langle \prod_a O_a \rangle = \frac{1}{Z} \int \prod_i dM_i e^{-S(M_i)} \prod_a O_a \sim \prod_a O_a^{classical} + \cdots$$

However, the situation is too complicated to expect the classical configuration be expressed in terms of the original matrix variables. In other words, while classical O's exist, classical M's may not exist.

Holographic QCD is a theory of the large n Master Field

$$\langle \prod_a O_a \rangle = \frac{1}{Z} \int \prod_i dM_i e^{-S(M_i)} \prod_a O_a \sim \prod_a O_a^{classical} + \cdots$$

However, the situation is too complicated to expect the classical configuration be expressed in terms of the original matrix variables. In other words, while classical O's exist, classical M's may not exist.

Note that above would be true only for (gauge-)invariant object, whereas M's are not gauge invariant by themselves. If we insert M's in place of O's, the above approximation breaks down.

Holographic QCD is a theory of the large n Master Field

$$\langle \prod_a O_a \rangle = \frac{1}{Z} \int \prod_i dM_i e^{-S(M_i)} \prod_a O_a \sim \prod_a O_a^{classical} + \cdots$$

However, the situation is too complicated to expect the classical configuration be expressed in terms of the original matrix variables. In other words, while classical O's exist, classical M's may not exist.

Note that above would be true only for (gauge-)invariant object, whereas M's are not gauge invariant by themselves. If we insert M's in place of O's, the above approximation breaks down.

Instead, we may anticipate a new classical field for (some of) the gauge-invariant operators. The collection of such classical fields are known in large n matrix models as Master Field.

Holographic QCD is a theory of the large n Master Field

$$\langle\prod_a O_a\rangle = \frac{1}{Z}\int\prod_i dM_i e^{-S(M_i)}\prod_a O_a \sim \prod_a O_a^{classical} + \cdots$$

However, the situation is too complicated to expect the classical configuration be expressed in terms of the original matrix variables. In other words, while classical O's exist, classical M's may not exist.

Note that above would be true only for (gauge-)invariant object, whereas M's are not gauge invariant by themselves. If we insert M's in place of O's, the above approximation breaks down.

Instead, we may anticipate a new classical field for (some of) the gauge-invariant operators. The collection of such classical fields are known in large n matrix models as Master Field.

The IIB gravity theory on AdS is a theory of these Master Fields for the large n maximally SUSY YM in the conventional AdS/CFT duality, which was conjectured from the study of the D3-brane theory.

Holographic QCD is a theory of the large n Master Field

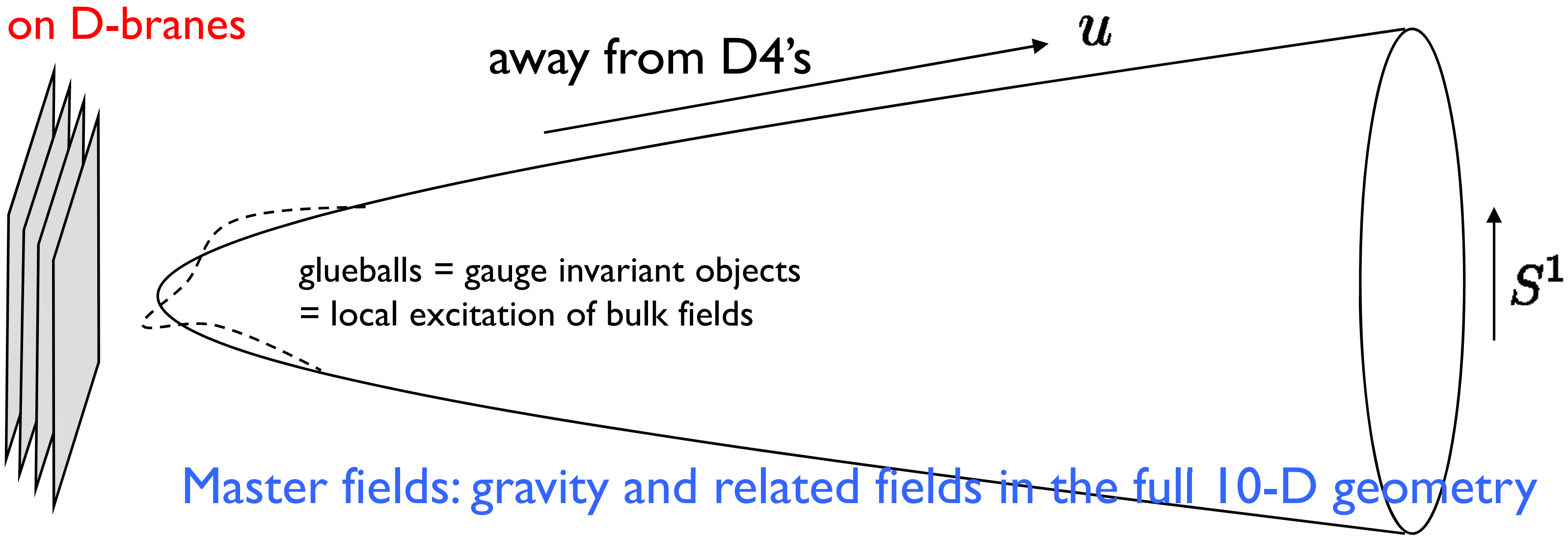
the question:
what are the master fields for large n QCD (without SUSY)
and the classical theory governing them?

D-branes can be used to construct QCD-like theory
at the low energy limit, so why not repeat what was done for
maximally SUSY YM with stack of D3 branes.

→ Witten's Theory with D4 on a thermal circle introduced earlier !

Holographic QCD is a theory of the large n Master Field

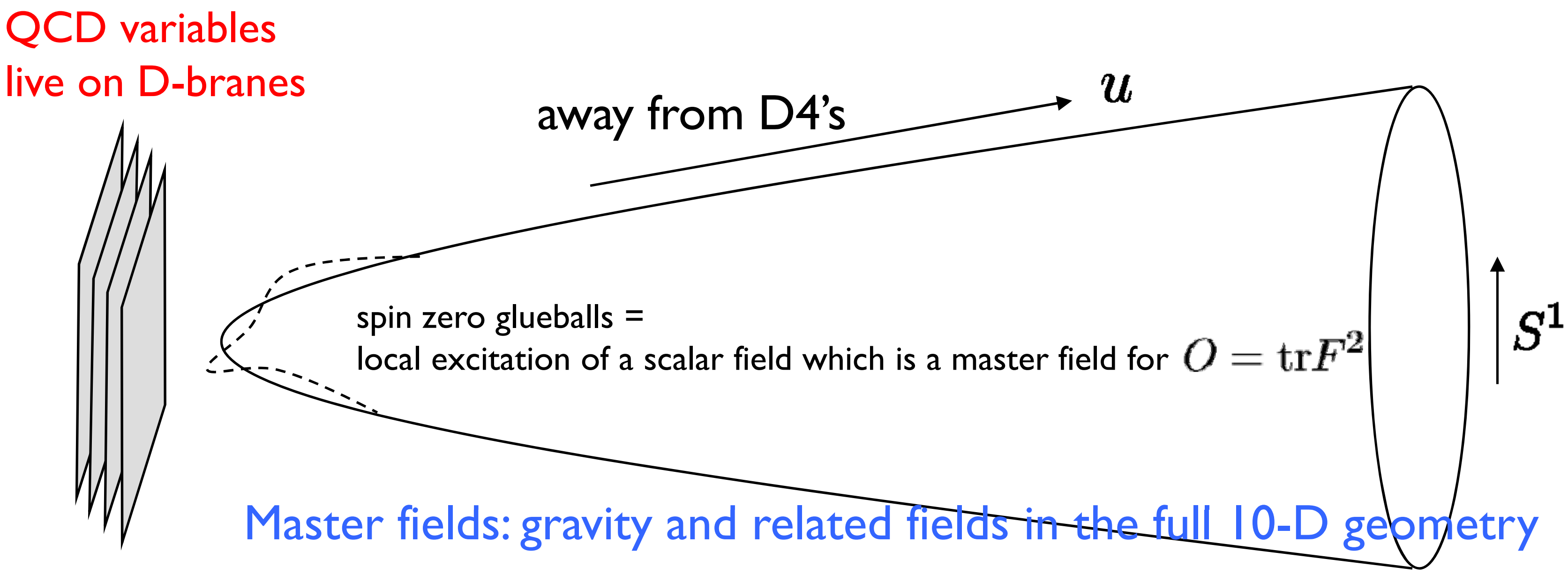
QCD variables
live on D-branes



Master fields: gravity and related fields in the full 10-D geometry

$$\times R^{3+1} \times S^4$$

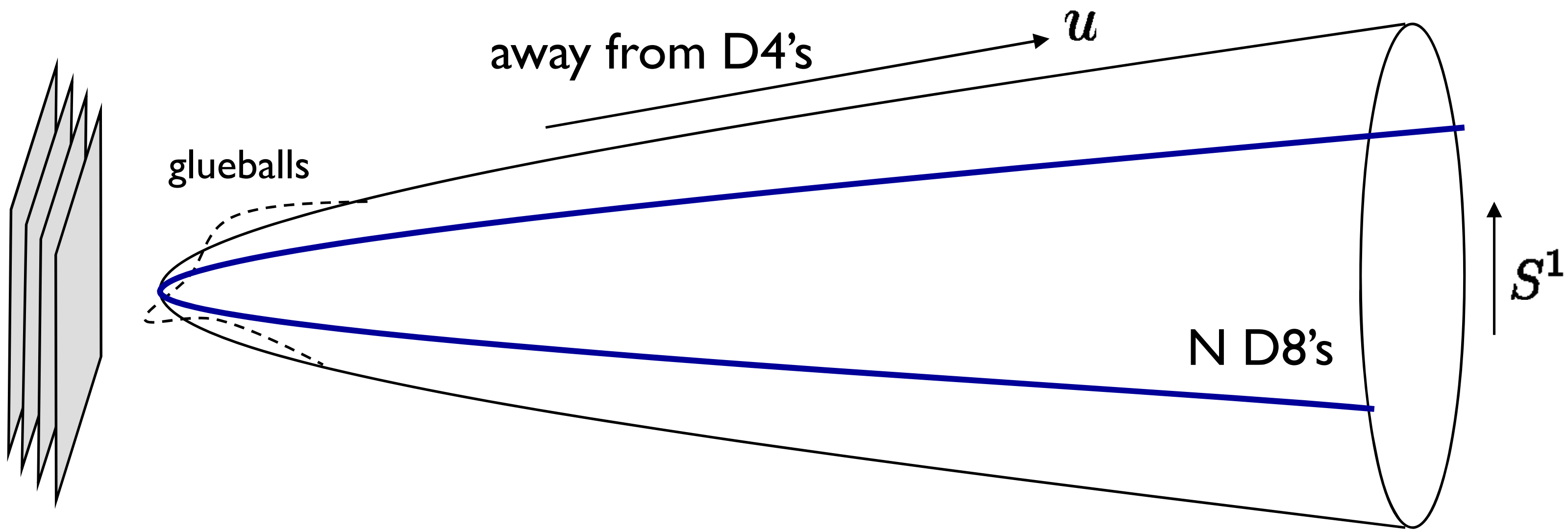
Holographic QCD is a theory of the large n Master Field



$$\times R^{3+1} \times S^4$$

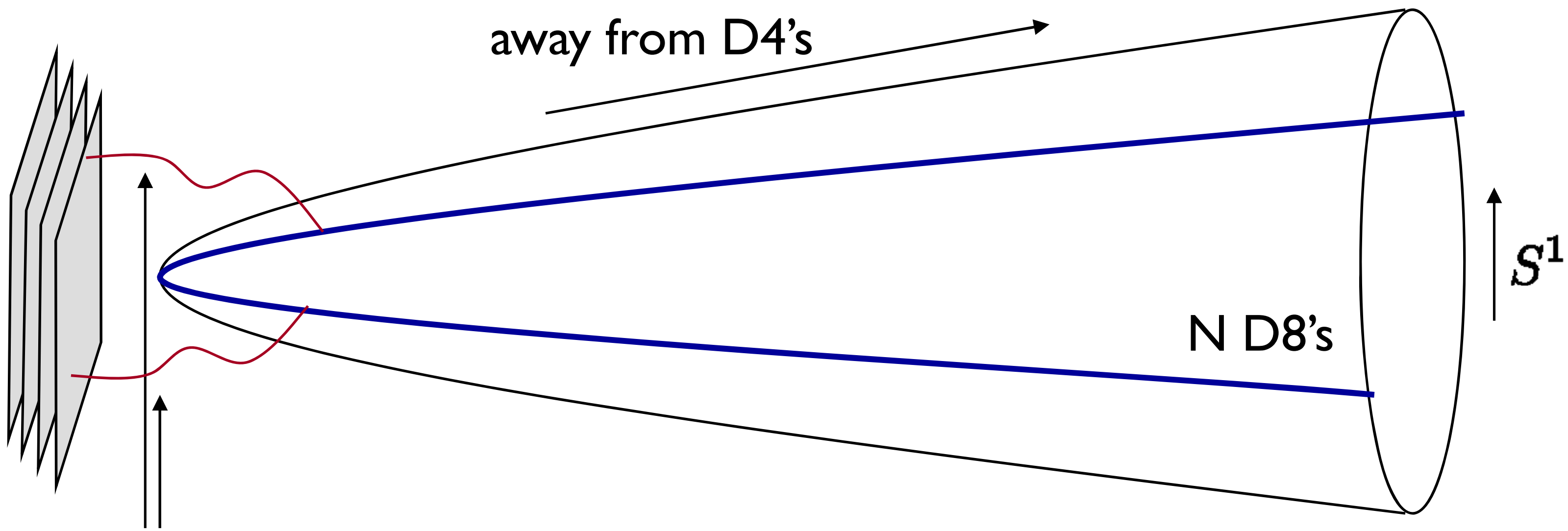
Sakai-Sugimoto

T. Sakai and S. Sugimoto 2004



$$\times R^{3+1} \times S^4$$

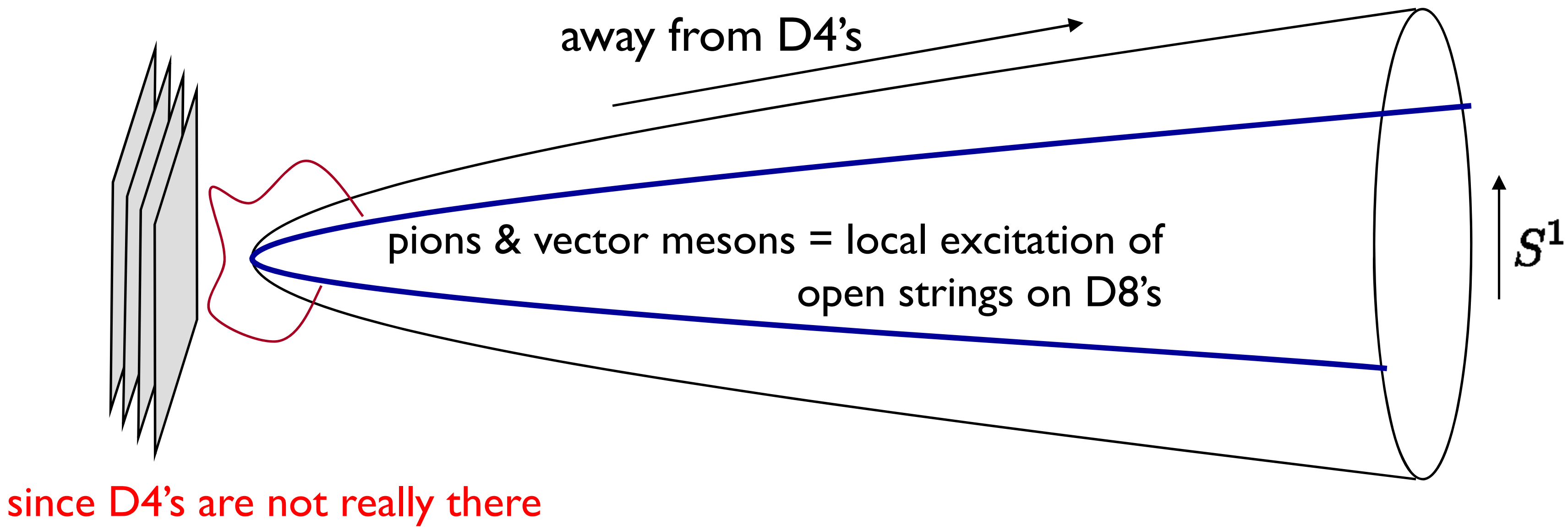
Sakai-Sugimoto



massless fermions in the bi-fundamental representation under SU(n)xU(N)_L or SU(n)xU(N)_R

$$\times R^{3+1} \times S^4$$

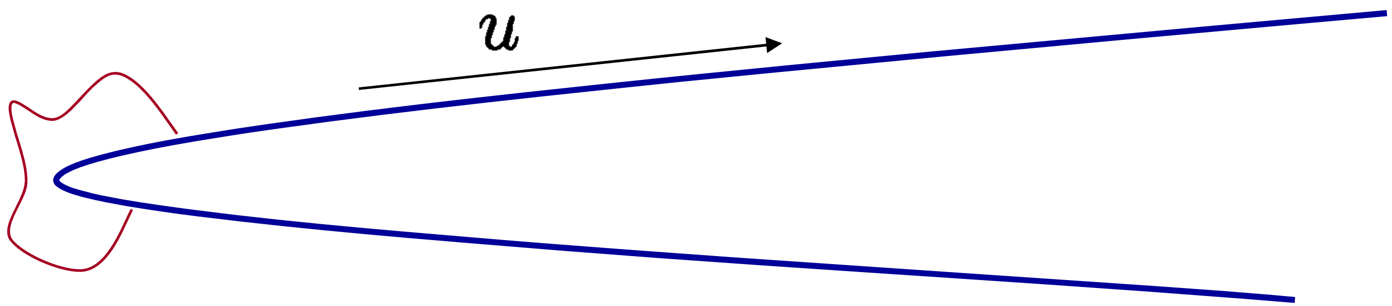
Sakai-Sugimoto



$$\times R^{3+1} \times S^4$$

Sakai-Sugimoto

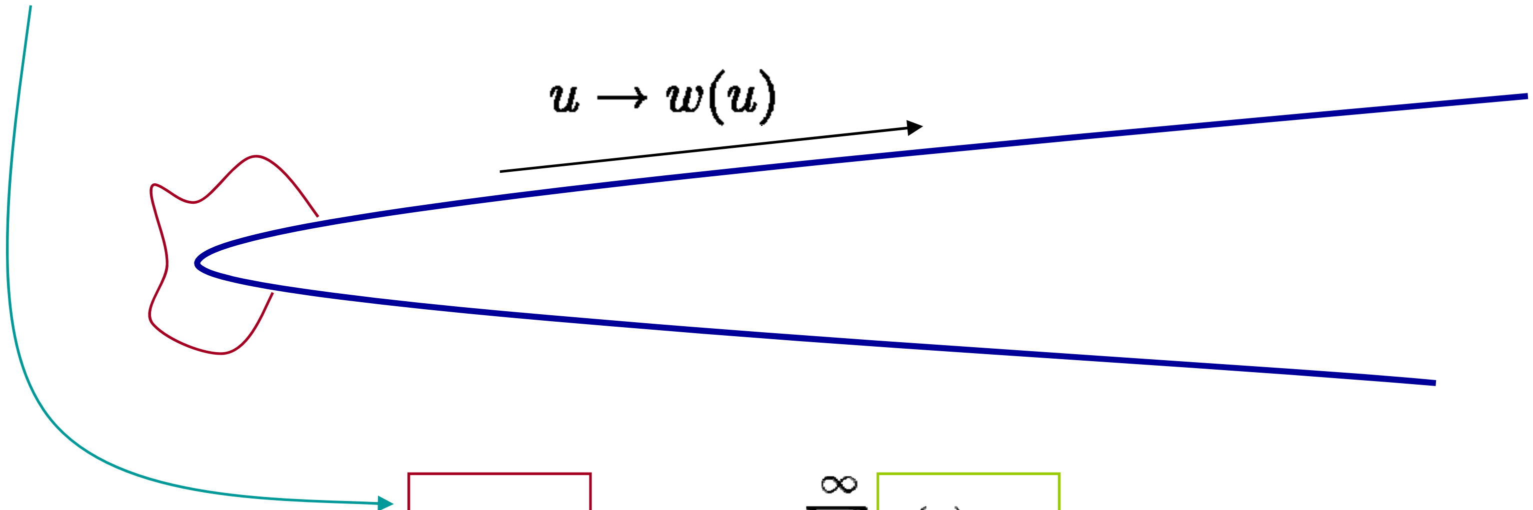
Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's



Only 5D vectors along noncompact direction remain massless
due to compactification on S^4 and also due to broken SUSY.

Sakai-Sugimoto

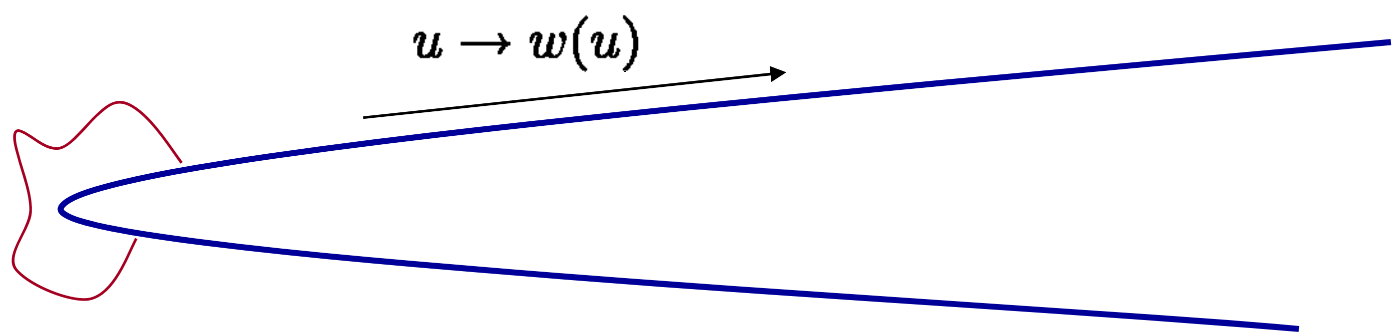
Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's



$$
\begin{aligned}
A_5(x^\mu, w) &= \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w) \\
A_\mu(x^\mu, w) &= \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)
\end{aligned}
$$

Sakai-Sugimoto

Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's



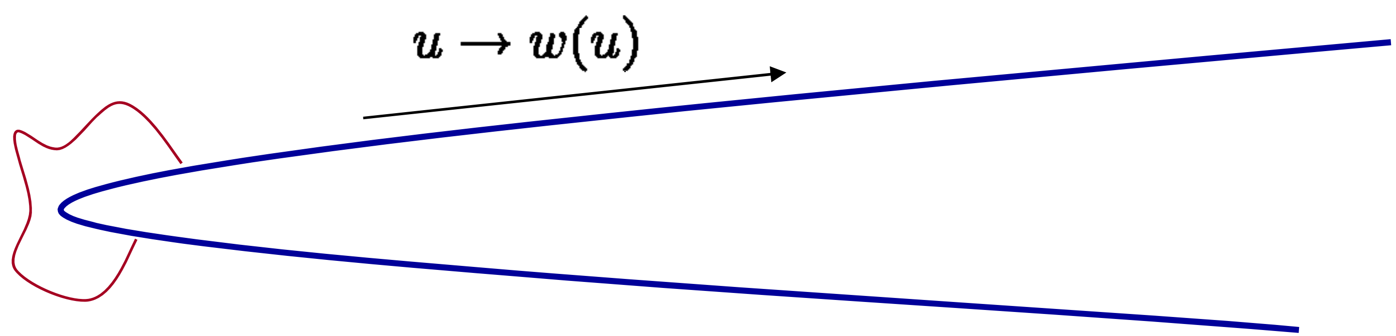
$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

eaten up → massive vector mesons

Sakai-Sugimoto

Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's



$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

eaten up → massive vector mesons

Sakai-Sugimoto

Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's

$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

eaten up → massive vector mesons

$\psi_n(w) = \psi_n(-w)$ → vector meson

$\psi_n(w) = -\psi_n(-w)$ → pseudo-vector meson

Sakai-Sugimoto

Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's

$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w\psi_0(w) + \sum_{n=1}^{\infty}\phi^{(n)}(x^\mu)\partial_w\psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

$$e^{2i\pi(x^\mu)/f_\pi} = U(x^\mu) \equiv P e^{i\int A_5(x^\mu, w)dw}$$

Sakai-Sugimoto

$$e^{2i\pi(x^\mu)/f_\pi} = U(x^\mu) \equiv P e^{i\int A_5(x^\mu,w)dw}$$

$$\int dx^{3+1}\left\{\frac{f_\pi^2}{4}\mathrm{tr}\left(U^{-1}\partial U\right)^2 + \frac{1}{32e_{Sk}^2}\mathrm{tr}\left[U^{-1}\partial U, U^{-1}\partial U\right]^2\right\} + \cdots$$

$$f_\pi^2 = \frac{(g_{YM}^2 n_c)n_c}{54\pi^2}M_{KK}^2 \qquad \frac{1}{e_{Sk}^2} = \frac{61(g_{YM}^2 n_c)n_c}{54\pi^2}$$

chiral Lagrangian for QCD
coupled to all massive
(pseudo-) vector mesons

Sakai-Sugimoto

$$e^{2i\pi(x^\mu)/f_\pi} = U(x^\mu) \equiv Pe^{i\int A_5(x^\mu,w)dw}$$

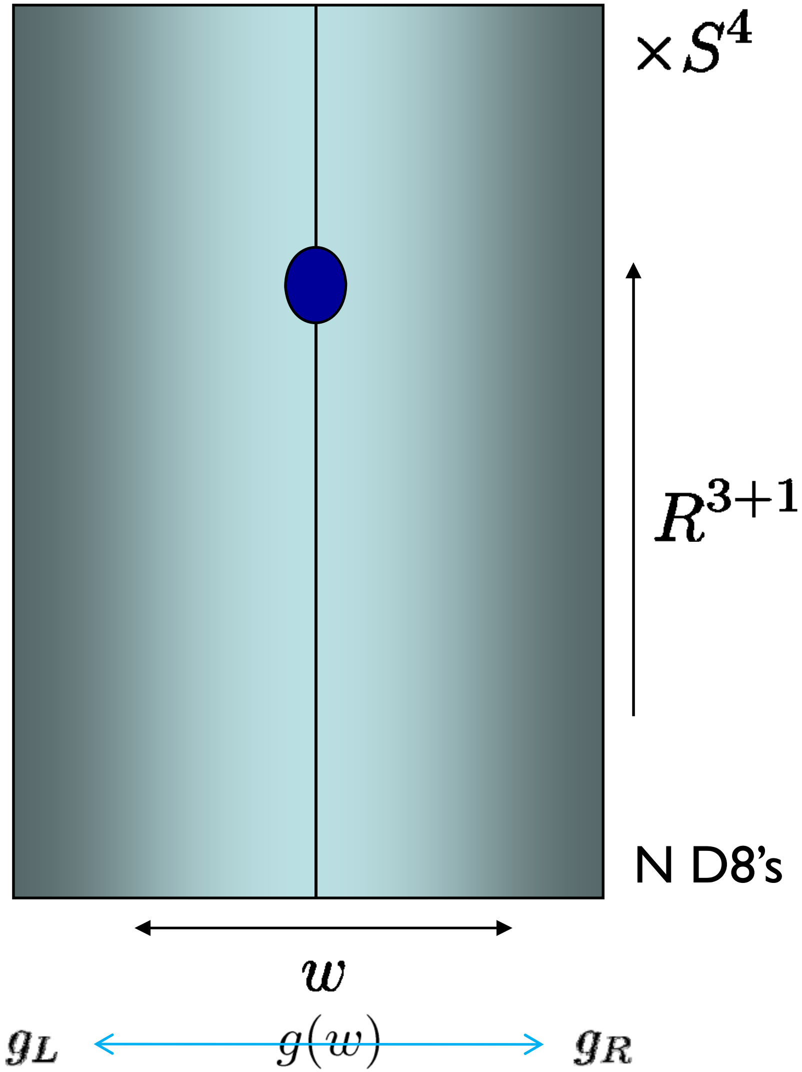
how is the axial symmetry realized ?
answer: as a large gauge transformation !

$$U \rightarrow L^\dagger U R$$

$$\updownarrow$$

$$(Pe^{i\int_L^R A}) \rightarrow L^\dagger (Pe^{i\int_L^R A}) R$$

Sakai-Sugimoto

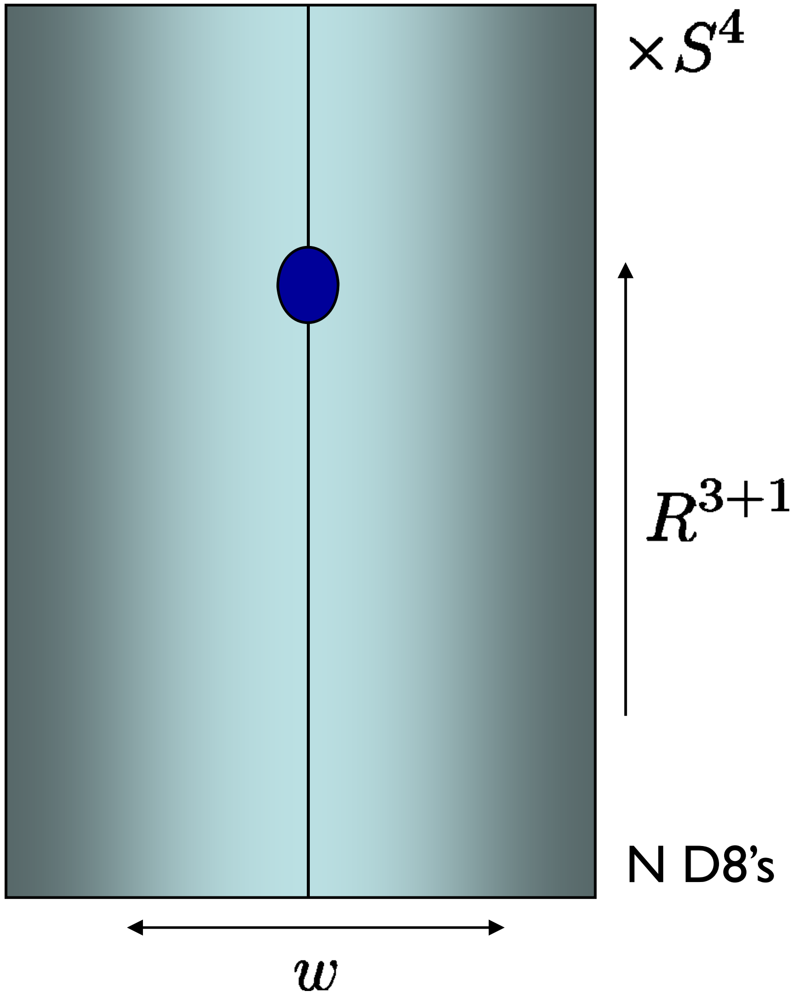


$$(Pe^{i\int_L^R A}) \to g_L^\dagger (Pe^{i\int_L^R A}) g_R$$

$$U \to g_L^\dagger U g_R$$

$$(L = g_L,\ R = g_R)$$

Baryons



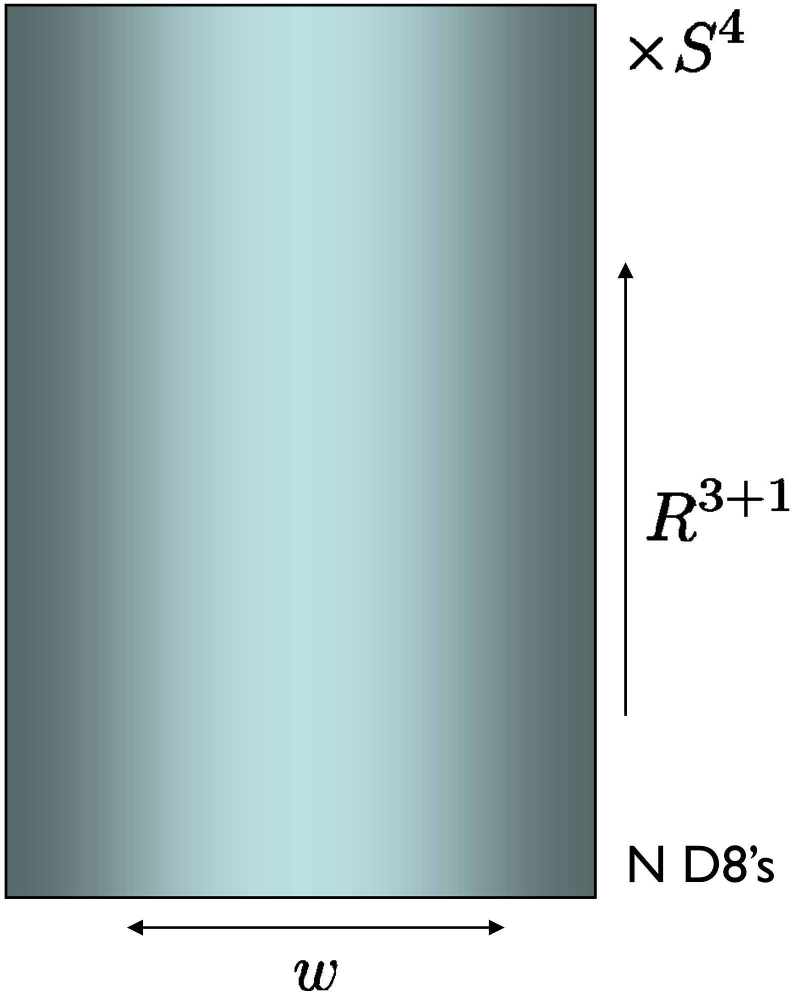
The horizontal wilson line
of an instanton soliton carries
the Skyrmion winding number

$$U = Pe^{i\int_L^R A_5(x^\mu, w)dw}$$

Because an instanton mediates
jump between two topological
vacua with the 3rd homotopy
winding numbers
which differ by unit.

Atiyah and Manton 1988

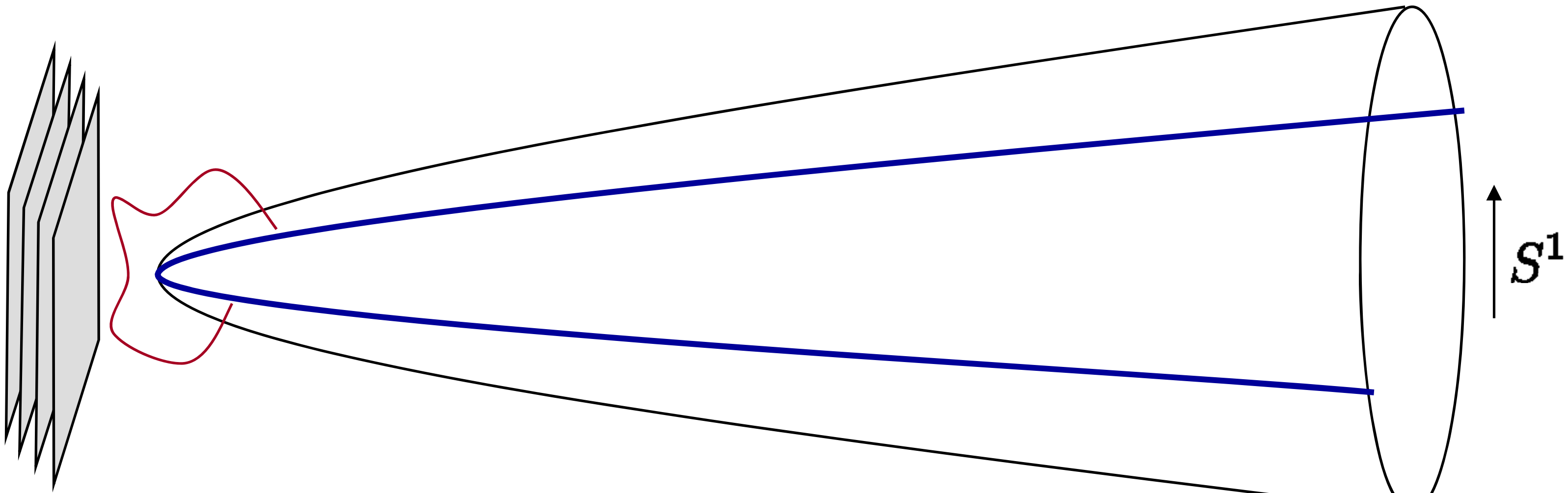
Baryons



The horizontal wilson line
of an instanton soliton carries
the Skyrmion winding number

Baryon ~ Skyrmion ~
instanton soliton on
noncompact part of D8's

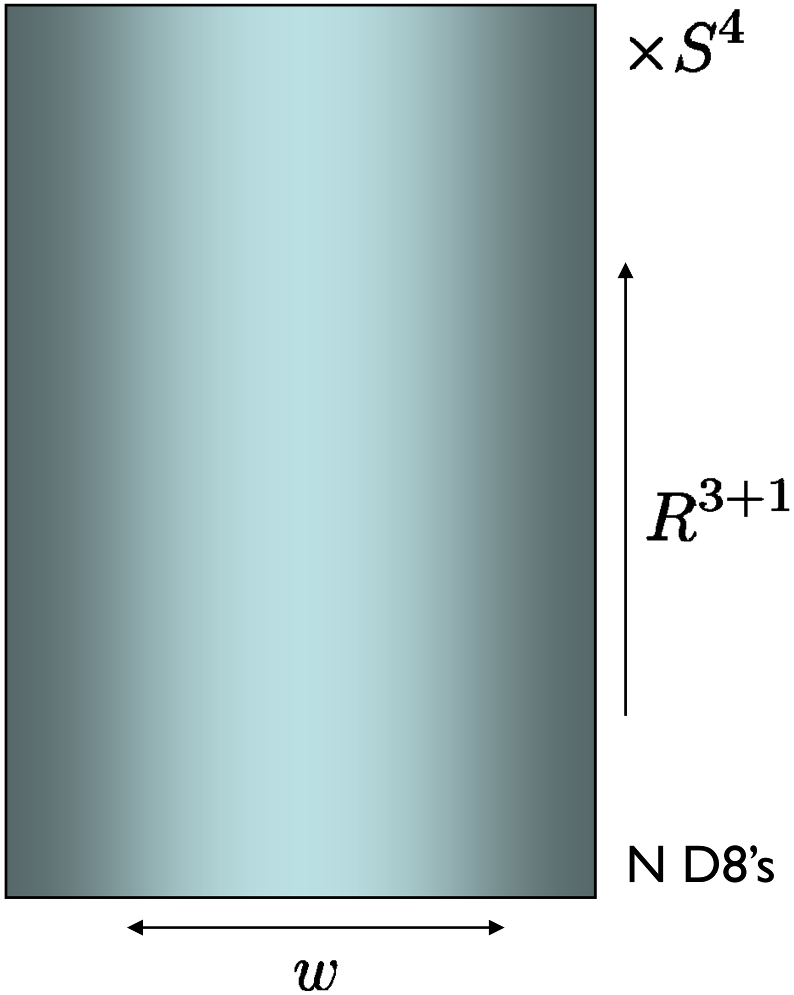
Baryons



$$\times R^{3+1} \times \boxed{S^4}$$

wrap a single D4 on it

Baryons

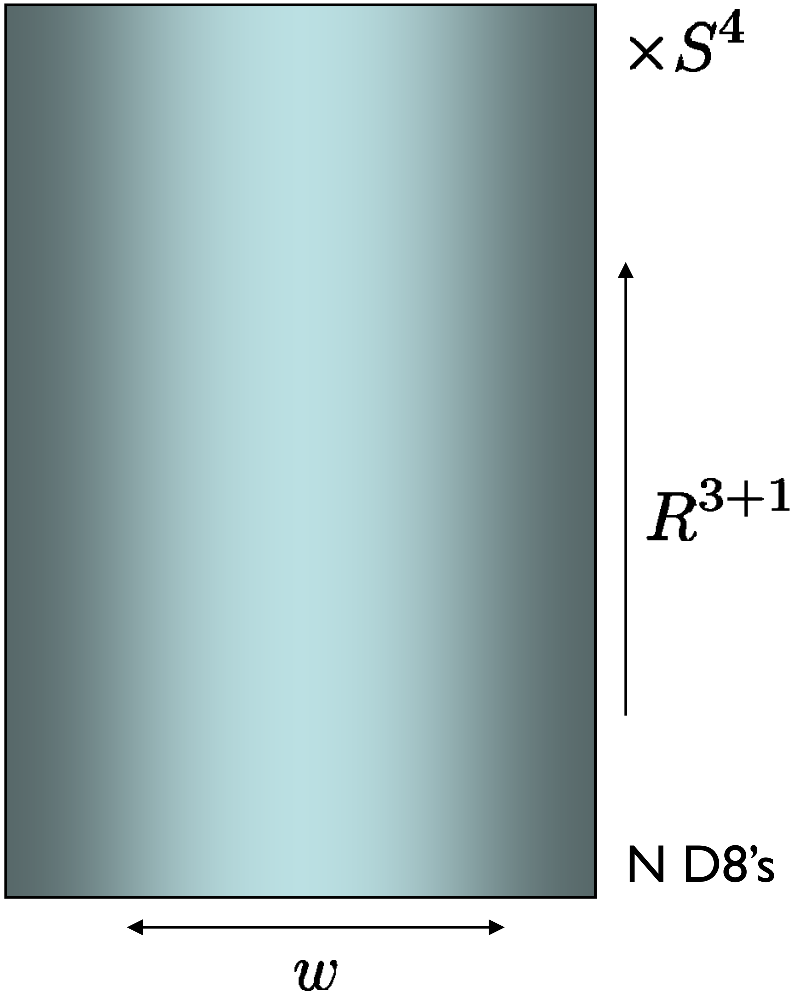


The horizontal wilson line of an instanton soliton carries the Skyrmion winding number

D4 on S^4 looks like an instanton soliton to D8's

Baryon ~ D4 on S^4

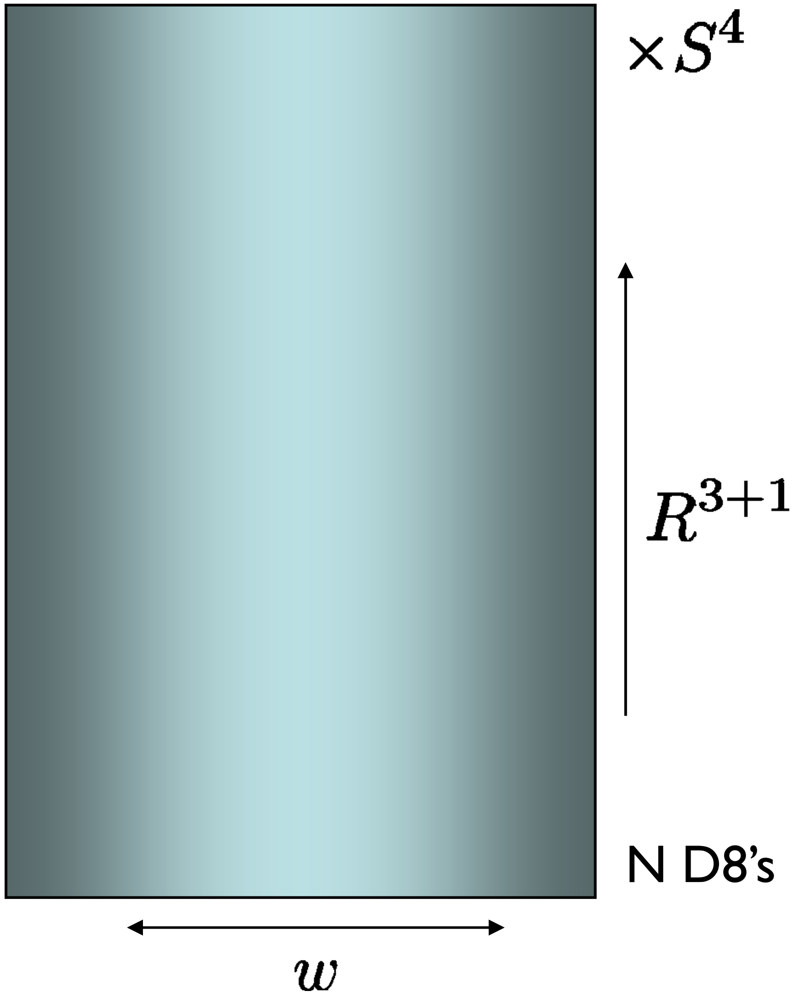
Baryons



Tadpole cancellation on S^4
Demands n fundamental strings
Attached to the D4 whose other
Ends can only attach to D8's

Baryon = D4 on S^4
with n fundamental
string hairs attached

Baryons



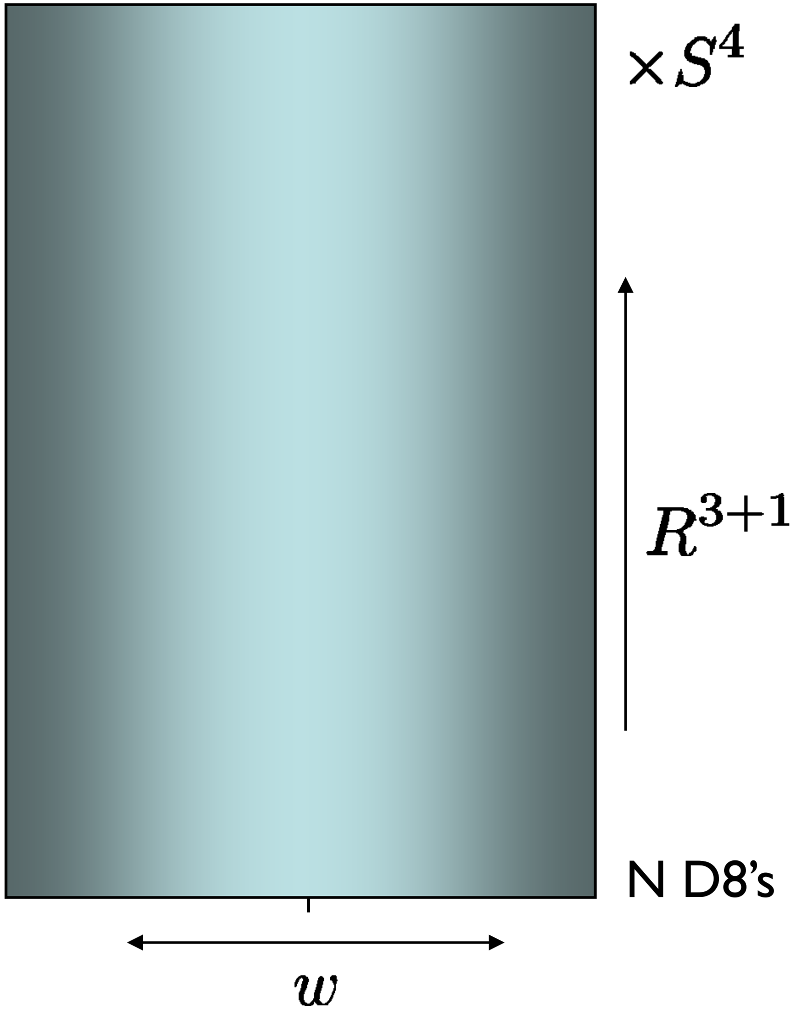
Baryon = D4 on S^4

with fundamental string hairs

Baryon = instanton soliton

with n unit of additional electric charges

Baryons: Topology, Charge, and Energetics

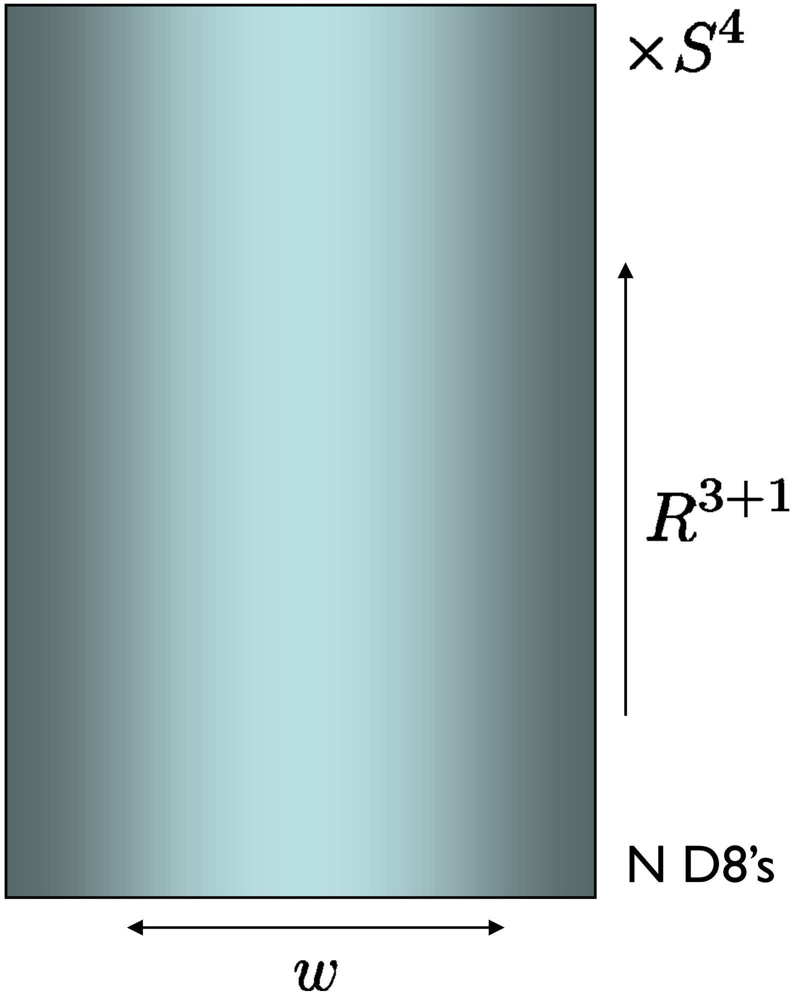


Baryon = instanton soliton
with n unit of additional
electric charges

We need actual solutions with such characteristics to know how they interact with mesons:

Look for Yang-Mills soliton with unit Pontryagin number and n electric charge, which actually solves the equation of motion.

Baryons



the action to extremize:

$$\int dx^{3+1} dw \frac{1}{4e(w)^2} \,\mathrm{tr}\, F^{mn} F_{mn}$$

$$+ \frac{n}{24\pi^2} \boxed{\int \omega_5(A)}$$

Chern-Simons terms

$$\sim \mathbf{3\,tr}\, \boldsymbol{A \wedge F \wedge F} + \cdots$$

$$\frac{1}{e(w)^2} = \frac{(g_{YM}^2 n)n}{108\pi^3} \boxed{M_{KK}} \frac{u(w)}{u_0}$$

~ 0.94 GeV
to fit mesons

Baryons: Topology, Charge, and Energetics

$$G_{9-1} = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{3-1} + f(u)d\theta^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

induced metric on D8's:

$$\begin{aligned} h_{8+1} &= \left(\frac{u}{R}\right)^{3/2} \eta_{3+1} + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right) \\ &= \frac{u(w)^{3/2}}{R^{3/2}} \left(\eta_{3+1} + dw^2\right) + R^{3/2} u(w)^{1/2} d\Omega_4^2 \end{aligned}$$

dilaton:

$$e^{\Phi} = g_s \times \left(\frac{u(w)}{R}\right)^{3/4}$$

Baryons: Topology, Charge, and Energetics

$$\mu_8 \int dx^{8+1} \frac{(2\pi\alpha')^2}{4e^{\Phi}} \sqrt{-h}\, \mathrm{tr}\, F_{MN} F_{KL} h^{MK} h^{NL} + \frac{\mu_8 (2\pi\alpha')^3}{6} \int dC_3^{\mathrm{RR}} \wedge \omega_5(A)$$

integrate over S^4
employ the conformal coordinate
recall quantized RR-flux through S^4

$$\int dx^{3+1} dw \frac{1}{4e(w)^2}\, \mathrm{tr}\, F^{mn} F_{mn} + \frac{n}{24\pi^2} \int \omega_5(A)$$

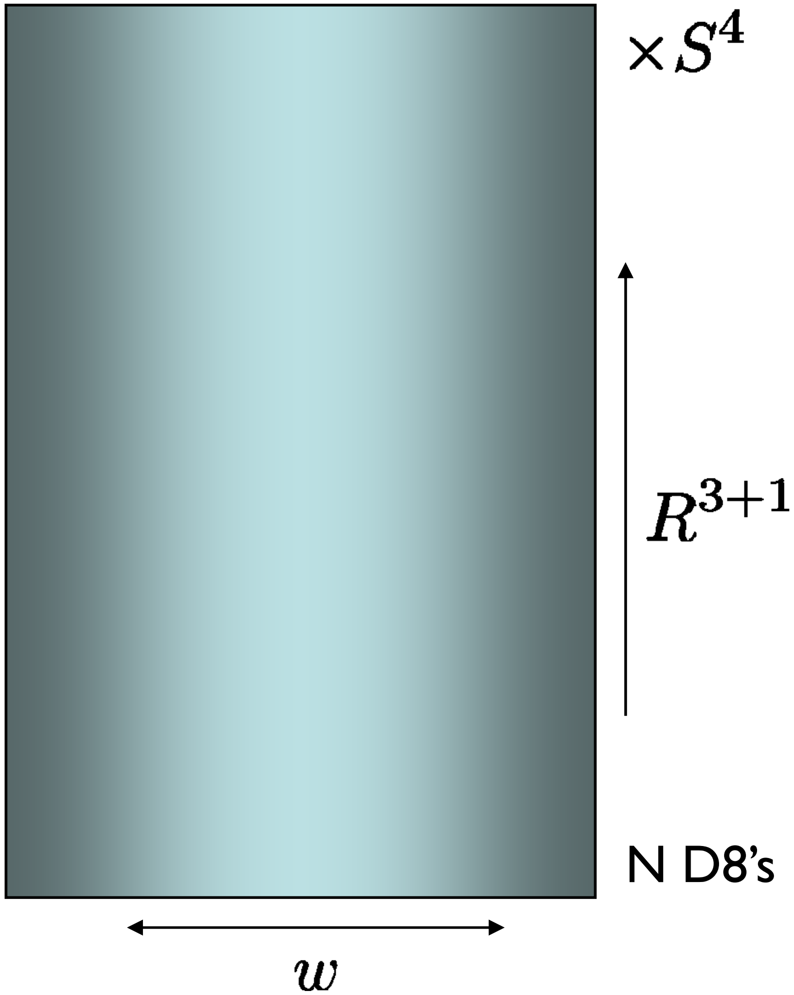
Baryons

$$\frac{1}{e(w)^2} = \mu_8 (2\pi\alpha')^2 e^{-\Phi} V_{S^4} \left(\frac{u(w)}{R}\right)^{3/4}$$

$$\frac{1}{e(w)^2} = \frac{(g_{YM}^2 n)n}{108\pi^3} M_{KK} \frac{u(w)}{u_0}$$

$$\frac{u(w)}{u_0} \simeq 1 + \frac{1}{3} M_{KK}^2 w^2 + \cdots$$

Baryons



the action to extremize:

$$\int dx^{3+1} dw \frac{1}{4e(w)^2} \, \mathrm{tr}\, F^{mn} F_{mn} + \frac{n}{24\pi^2} \boxed{\int \omega_5(A)}$$

Chern-Simons terms

$$\sim \mathbf{3\, tr}\, \boldsymbol{A \wedge F \wedge F} + \cdots$$

$$\frac{1}{e(w)^2} = \frac{(g_{YM}^2 n)n}{108\pi^3} \boxed{M_{KK}} \frac{u(w)}{u_0}$$

~ 0.94 GeV

Baryons

Using the ordinary instanton as trial configurations

$$A^a_m = \bar{\eta}^a_{mk} \partial_k \log\left(1 + \rho^2/(\vec{x}^2 + w^2)\right)$$

which is an instanton soliton for

$$\int dx^{3+1} dw \frac{1}{4e_0^2} \, \mathrm{tr}\, F^{mn} F_{mn}$$

Baryons

Using the ordinary instanton as trial configurations

$$A_m^a = \bar{\eta}_{mk}^a \partial_k \log\left(1 + \rho^2/(\vec{x}^2 + w^2)\right)$$

Energy can be estimate for small size limit as

$$E(\rho) = \frac{(g_{YM}^2 n)n}{27\pi} M_{KK} \times \left(1 + \frac{1}{6} M_{KK}^2 \rho^2 + \cdots\right) + \frac{e(0)^2 n^2}{20\pi^2 \rho^2} - \cdots$$

Extra F^2 energy due to increasing 1/e(w)^2

Coulomb energy due to electric charge proportional to the instanton density =

Extra energy from corrections due to the Chern-Simons term in the action

Baryons

Hong, Rho, Yee, Yi 2007
Hata, Sakai, Sugimoto, Yamato 2007

Using the ordinary instanton as trial configurations

$$A^a_m = \bar{\eta}^a_{mk}\partial_k \log\left(1 + \rho^2/(\vec{x}^2 + w^2)\right)$$

Energy can be estimated for small size limit as

$$E(\rho) = \frac{(g^2_{YM}n)n}{27\pi} M_{KK} \times \left(1 + \frac{1}{6}M^2_{KK}\rho^2 + \cdots\right) + \frac{e(0)^2 n^2}{20\pi^2\rho^2} - \cdots$$

Minimization gives a definite size to the 5D baryon = instanton + hair

$$\rho_{\rm baryon} \simeq \frac{(2\cdot 3^7\cdot \pi^2/5)^{1/4}}{M_{KK}\sqrt{g^2_{YM}n}} \simeq \frac{9.6}{M_{KK}\sqrt{g^2_{YM}n}}$$

Effective Field Theory of Isospin ½ Baryons in 5D

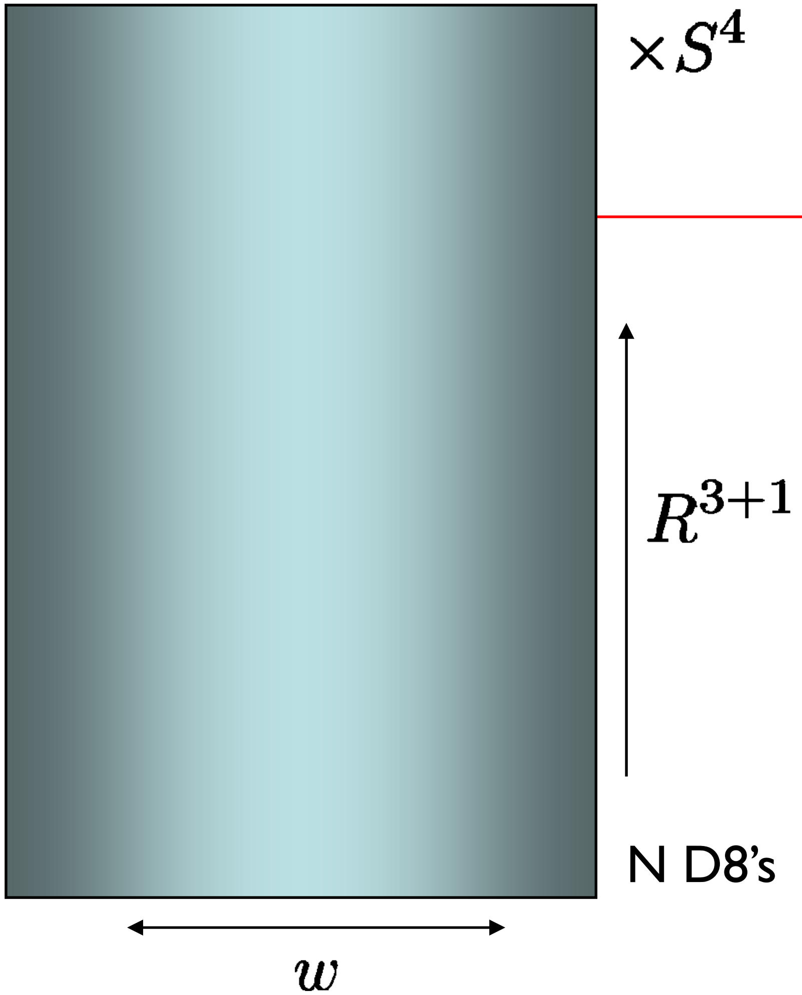
with small soliton size, we may introduce an effective (Dirac) field for the (isospin ½) baryon and try to incorporate the property of the latter into an effective action.

$$n \gg 1, \;\; g_{YM}^2 n \gg 1$$

Compton Length of Baryons << Soliton Size << Compton Lengths of Mesons

→ The baryon can be treated pointlike for interaction with mesons, yet, the classical properties of the soliton can still be reliable.

Effective Field Theory of Isospin ½ Baryons in 5D



excitation quanta of Dirac spinor $\mathcal{B}^a_\alpha(x^\mu, w)$ in some representation under U(N)

Effective Field Theory of Isospin ½ Baryons in 5D

Hong, Rho, Yee, Yi 2007

The result turns out to be exceedingly simple:

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_B(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B}\right]$$

$$g_5(0) = \frac{2\pi^2}{3}$$

Effective Field Theory of Isospin ½ Baryons in 5D

two main issues

Necessity & Uniqueness

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\text{baryon}}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B} \right]$$

$$g_5(0) = \frac{2\pi^2}{3}$$

Couplings: Position Dependence & Reliability

Effective Field Theory of Baryons in 5D

two main issues

Uniqueness & Necessity of the Direct Coupling to the SU(N) Field Strength

Necessity: A coupling to the magnetic field strength must be present to reproduce the self-dual long-range tail of the instanton soliton.
The same observation was used by Adkins-Nappi-Witten to extract pion-nucleon coupling from the Skyrmion picture of baryons.

Effective Field Theory of Baryons in 5D

two main issues

Uniqueness & Necessity of the Direct Coupling to the SU(N) Field Strength

Necessity: A coupling to the magnetic field strength must be present to reproduce the self-dual long-range tail of the instanton soliton.
The same observation was used by Adkins-Nappi-Witten to extract pion-nucleon coupling from the Skyrmion picture of baryons.

Uniqueness: Power counting shows that the necessary term is dimension-6, i.e. one higher power than the minimal coupling. The present term is the only gauge-invariant dimension-6 term, involving a baryon current and a single gauge field. Nontrivial things to show is that this term actually does the job necessary. (See the paper, sorry)

Effective Field Theory of Baryons in 5D

two main issues

Couplings: Position Dependence & Reliability

Two separate questions here:

1)whether the classical value of the couplings is reliable?

the answer is yes, within the usual AdS/CFT with large n and large ‘tHooft limit, which of course does not guarantee the reliability for the realistic regime. the basic tenet of AdS/CFT in that limit is that bulk side is treated at tree-level.

Effective Field Theory of Baryons in 5D

two main issues

Couplings: Position Dependence & Reliability

Two separate questions here:

1)whether the classical value of the couplings is reliable?

the answer is yes, within the usual AdS/CFT with large n and large ‘tHooft limit, which of course does not guarantee the reliability for the realistic regime. the basic tenet of AdS/CFT in that limit is that bulk side is treated at tree-level.

2)to what extent the specific coordinate-dependence can be trusted?

even at tree-level the value of couplings away from origin is difficult to estimate precisely. its coordinate-dependence must be taken with a grain of salk. therefore whet we must do is to consider physical quantities which are dictated by the value at origin.

Effective Field Theory of Isospin ½ Baryons in 5D

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\rm baryon}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F^{SU(N)}_{mn}\mathcal{B} \right]$$

$$g_5(0) = \frac{2\pi^2}{3}$$

this effective action with **only one nontrivial term** is capable of reproducing all the interaction between Nucleons and the entire tower of pions and (pseudo-)vector mesons, including some subleading corrections in 1/n expansion. Furthermore, this effective action dictates all electromagnetic interaction, with a vector-dominance.

Effective Field Theory of Isospin ½ Baryons in 5D

How does this 5D effective action for Nucleons
arises and in what regime of validity in the parameter space ?

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_B(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\text{baryon}}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B} \right]$$

$$g_5(0) = \frac{2\pi^2}{3}$$

How does the 4D effective action arise from this and
what kind of interactions does it encode ?

How well do the structures and numbers of the resulting 4D effective action
compare against the nature (or at least against the quenched QCD) ?

Effective Field Theory of Nucleons in 4D

KK reduction along the fifth direction

$$\mathcal{B}(x,w) = \begin{pmatrix} B_+(x)f_+(w) \\ B_-(x)f_-(w) \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\pm\partial_w f_\pm(w) + m_{\mathcal{B}}(w) f_\pm(w) = M_B f_\mp(w)$$

take the smallest eigenvalue M_B ⟶ 4D nucleon mass

$$B(x) \equiv \begin{pmatrix} B_+(x) \\ B_-(x) \end{pmatrix}$$ ⟶ 4D Dirac field for nucleons

Effective Field Theory of Nucleons in 4D

$$\int dx^{3+1}\int dw\left[-i\bar{\mathcal{B}}\gamma^{m}(\partial_m - iA_m^{U(N)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\text{baryon}}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B}\right]$$

$$\mathcal{B}(x,w) = \begin{pmatrix} B_+(x)f_+(w) \\ B_-(x)f_-(w) \end{pmatrix}$$

$$B(x) \equiv \begin{pmatrix} B_+(x) \\ B_-(x) \end{pmatrix}$$

A_5

A_μ

$F_{5\mu}$

$F_{\mu\nu}$

$$\int dx^{3+1}\left[-i\bar{B}\gamma^\mu\partial_\mu B - iM_B\bar{B}B\right] + \cdots$$

Effective Field Theory of Nucleons in 4D

Pions and (pseudo-)vector mesons are contained in U(N) gauge field on N D8's

$$A_5(x^\mu, w) = \phi_0(x^\mu)\partial_w \psi_0(w) + \sum_{n=1}^{\infty} \phi^{(n)}(x^\mu)\partial_w \psi_n(w)$$

$$A_\mu(x^\mu, w) = \sum_{n=1}^{\infty} a_\mu^{(n)}(x^\mu)\psi_n(w)$$

eaten up → massive vector mesons

$\psi_n(w) = \psi_n(-w)$ → vector meson

$\psi_n(w) = -\psi_n(-w)$ → pseudo-vector meson

Effective Field Theory of Nucleons in 4D

$$\int dx^{3+1} \int dw \left[-i\bar{\mathcal{B}}\gamma^m(\partial_m - iA_m^{U(N)})\mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g_5(w)\rho^2_{\mathrm{baryon}}}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(N)}\mathcal{B}\right]$$

$$\mathcal{B}(x,w) = \begin{pmatrix} B_+(x)f_+(w) \\ B_-(x)f_-(w) \end{pmatrix}$$

$$B(x) \equiv \begin{pmatrix} B_+(x) \\ B_-(x) \end{pmatrix}$$

A_5

A_μ

$F_{5\mu}$

$F_{\mu\nu}$

$$\int dx^{3+1} \ [-i\bar{B}\gamma^\mu\partial_\mu B - iM_B\bar{B}B] + \cdots$$

meson-nucleon-nucleon or

$$\sim \int dw \ f_+(w)^*\psi_n(w)f_\pm(w)$$

meson-meson-nucleon-nucleon

$$\sim \int dw \ f_+(w)^*\psi_n(w)\psi_m(w)f_\pm(w)$$

Effective Field Theory of Nucleons in 4D

$$\int dx^{3+1}\ \left[-i\bar{B}\gamma^\mu\partial_\mu B - iM_B\bar{B}B\right] + \cdots$$

meson-nucleon-nucleon or

$$\sim \int dw\ f_+(w)^*\psi_n(w)f_\pm(w)$$

meson-meson-nucleon-nucleon

$$\sim \int dw\ f_+(w)^*\psi_n(w)\psi_m(w)f_\pm(w)$$

Cubic Terms:

$$g_A\ \bar{B}\partial_\mu\pi\gamma^\mu\gamma^5 B$$

$$g_V^{(k)}\ \bar{B}a_\mu^{(2k+1)}\gamma^\mu B$$

$$g_A^{(k)}\ \bar{B}a_\mu^{(2k)}\gamma^\mu\gamma^5 B$$

$$\int dw\ \frac{g_5\rho^2}{e^2}f_+(w)^*\psi_n(w)f_\pm(w)$$

$$\int dw\ f_+(w)^*\psi_n(w)f_\pm(w)$$

↔

Vector Dominance

and ?

- take the effective action at face-value and use it for nuclear phenomenology
  →
  seemingly works much better than rigorously justified and
  in particular improves the existing framework quite a bit.
  the improvements largely come from the systematic and
  unambiguous incorporation of the first few vector mesons.

- try to better the formulation by incorporating things which should have been, such as adding the 3^{rd} flavor, generating pion mass, going beyond the quenched approximation, going beyond large n, etc, etc.

- apply to astrophysical regimes.

- take it as a model building paradigm and do things to the model that string theorists does not approve. (bottom-up, say)