# Bottom-up AdS/QCD through a few examples

Y. Kim (KIAS)

# Contents

- Hard-wall model.
- Examples: vector-vector correlator, vector-meson mass, chiral condensate, Deconfinement temperature in AdS/QCD.
- Dense matter
- Summary

# AdS/CFT Dictionary

- 4D CFT (QCD) --- 5D AdS
- 4D generating functional --- 5D (classical) effective action
- Operator --- 5D bulk field
- [Operator] --- 5D mass
- Current conservation --- gauge symmetry
- Large Q --- small z
- Confinement --- Compactified z
- Resonances --- Kaluza-Klein states

# Bottom-up AdS/QCD model

( look at QCD first !!! )

Let's start from 2-flavor QCD at low energy and attempts to guess its 5D holographic dual, AdS/CFT dictionaries.

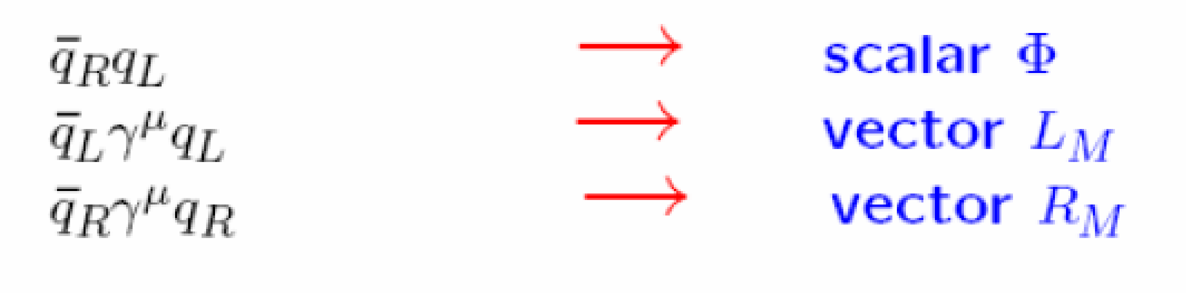
$$\text{4D generating functional} : Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i \int \phi_0(x)\mathcal{O}\} ,$$

$$\text{5D (classical) effective action} : \Gamma_5[\phi(x,z) = \phi_0(x)] \,;\, \phi_0(x) = \phi(x, z=0).$$

$$\text{AdS/CFT correspondence} : Z_4 = \Gamma_5.$$

# ☆ 5D field contents

Operator → 5D bulk field



$\bar{q}_R q_L \longrightarrow$ scalar $\Phi$

$\bar{q}_L \gamma^\mu q_L \longrightarrow$ vector $L_M$

$\bar{q}_R \gamma^\mu q_R \longrightarrow$ vector $R_M$

[Operator] → 5D mass

$$(\Delta - p)(\Delta + p - 4) = m_5^2 \qquad m_\phi^2 = -3$$

# ☆ 5D Symmetry

Current conservation → gauge symmetry

$SU(2)_L X SU(2)_R$ gauge symmetry in $AdS_5$

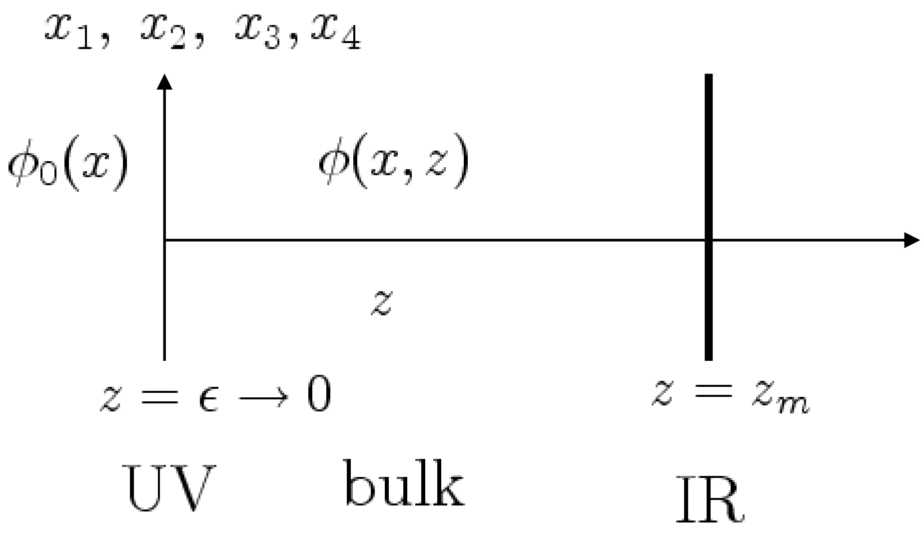
☆ Background: $AdS_5$

$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

# ☆ Confinement

Polchinski & Strassler '00

Confinement → IR cutoff in 5th direction



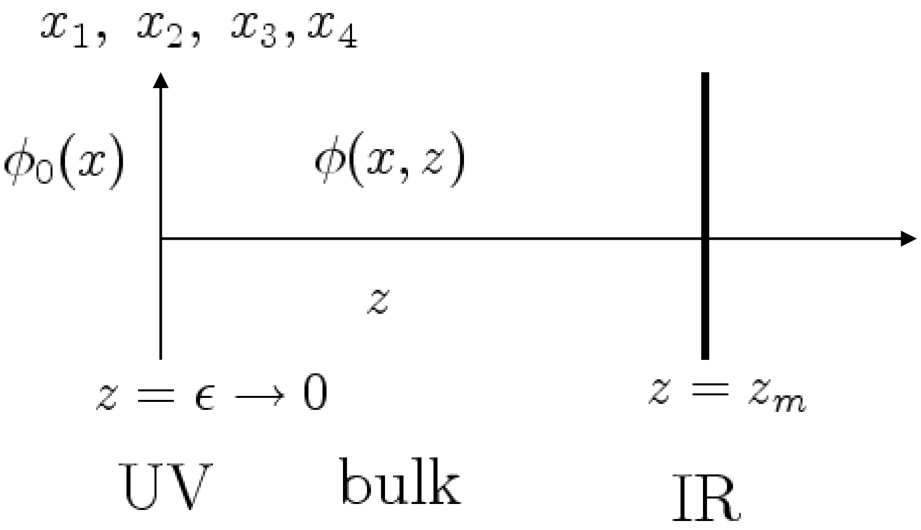
# Hard wall model

$$S_I = \int d^4x dz \sqrt{g} \mathcal{L}_5 \,,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[ -\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right] \,,$$

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)



$$V_M \sim L_M + R_M\,, \quad A_M \sim L_M - R_M\,,$$

$$\Phi = Se^{iP/v(z)}, \;\; v(z) \equiv \langle S \rangle\,,$$

$$\Phi \leftrightarrow X, \quad v(z) \leftrightarrow X_0\,.$$

# Example: vector–vector correlator

$$Z_4[\phi_0(x)] = \int D[\Phi]\, \exp\left[iS_4 + i\int \phi_0(x)\,\mathcal{O}\right]$$

$$\langle \mathcal{O}\,\mathcal{O} \rangle \sim \frac{\delta Z_4[\phi_0]}{\delta\phi_0\,\delta\phi_0}$$

$$\begin{tikzcd}
\underset{\sim}{\mathcal{O}} : \overset{\underline{4D}}{\bar{q}\gamma^\mu t^a q} \arrow[r, leftrightarrow, "\text{AdS/CFT}"'] & \overset{\underline{5D}}{V^{\mu a}(x,z)} \arrow[d, "\text{Sol. of E.O.M}", "z \to 0"'] & \\
& V_0^{\mu a} \arrow[r, hook] & \underset{\sim}{\phi_0}
\end{tikzcd}$$

$$Z_4[\phi_0(x)] \xleftrightarrow{\text{AdS/CFT}} \Gamma_5[\phi_0(x)]$$

$$\Downarrow$$

$$\underline{\underline{\Gamma_5[V_0]}} = Z_4[V_0]$$

$$S_5 = -\frac{1}{4g_5^2}\overset{\text{tr}}{\checkmark}\int d^5x\,\sqrt{g}\;V_{MN}V^{MN}$$

$M = (\mu, z)$

$$= -\frac{1}{4g_5^2}\int d^5x\,\frac{1}{z^5}\,\overset{g^{zz}g^{\mu\mu}}{\overset{\swarrow}{z^4}}\left(-2V_{z\mu}V_{z\mu} + V_{\mu\nu}V^{\mu\nu}\right)$$

gauge choice:

$\underline{V_z = 0}$

$$= -\frac{1}{4g_5^2}\int d^5x\,\frac{1}{z}\left[-2(\partial_z V_\mu)^2 + (V_{\mu\nu})^2\right]$$

$\nearrow$ E.o.M

$$= -\frac{1}{2g_5^2}\int d^5x\, V_\mu \Big[ z\,\partial_z\big(\tfrac{1}{z}\partial_z\big)\eta_{\mu\nu} - \partial_4^2\,\eta_{\mu\nu}$$

$$+\partial_\mu\partial_\nu \Big] V_\nu \frac{1}{z}$$

$$+\frac{1}{2g_5^2}\int d^4x \int dz\, \partial_z\left(\tfrac{1}{z} V_\mu \partial_z V_\mu\right)$$

$$\rightsquigarrow S_4 \,,\ S_{\text{boundary}}$$

$$\text{E.o.M} : z\,\partial_z\left(\tfrac{1}{z}\partial_z V_\mu\right) - \left(\partial_4^2 V_\mu - \partial_\mu(\partial\cdot V)\right) = 0$$

$$\Big\Downarrow \text{F.T. in 4D}$$

$$z\,\partial_z\left(z^{-1}\partial_z V_\mu\right) + \underbrace{\left(q^2\delta_\mu^{\ \nu} - q_\mu q^\nu\right)V_\nu}_{\Rightarrow \underline{V_\nu^\perp \cdot q^2}_{\text{transverse}}} = 0$$

$$S_4 = -\frac{1}{2g_5^2}\int d^4x \left(\tfrac{1}{z} V_\mu \partial_z V^\mu\right)_{z=\varepsilon}$$

$$V_\mu(q,z) = V(q,z)\, V_0^\mu(q) \,, \quad V(q,\epsilon)=1$$

↳ source term of the vector current $\sim \bar{q}\gamma_\mu t^a q$

$$S_4 = -\frac{1}{2g_5^2}\int d^4x\, V_0^\mu(q)\, \frac{1}{z}\partial_z V(q,\epsilon)\, V_{0,\mu}(q)$$

## V-V correlator

$$\int e^{iq\cdot x}\langle J_\mu(x) J_\nu(0)\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_V(Q^2)$$

$$Q^2 = -q^2$$

i) $S_b$, $S_4$

$$\Pi_V(Q^2) = -\frac{1}{2g_5^2}\ln Q^2$$

$$\therefore\ g_5^2 = \frac{12\pi^2}{N_c}$$

ii) OPE

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2}\ln Q^2$$

# Example: 4D vector meson mass

$$V(x,z)=\sum f_v(z)\tilde{V}(x)$$

$$[\partial_z^2-\frac{1}{z}\partial_z+q^2]f_v(z)=0,\ q^2=m_n^2$$

$$m_n\simeq(n-\frac{1}{4})\frac{\pi}{z_m}$$

$$m_1=m_\rho\ ,\quad \frac{1}{z_m}\simeq 320\ \mathrm{MeV}.$$

# Example: Chiral condensate

Klebanov and Witten ‘99

$$\phi(z, \boldsymbol{x}) \to z^{d-\Delta}[\phi_0(\boldsymbol{x}) + O(z^2)] + z^{\Delta}[A(\boldsymbol{x}) + O(z^2)],$$

$$A(\boldsymbol{x}) = \frac{1}{2\Delta - d}\langle\mathcal{O}(\boldsymbol{x})\rangle.$$

$$\phi(z) = c_1 z + c_2 z^3,\ z \to 0;\ c_1 \sim m_q,\quad c_2 \sim \langle\bar{q}q\rangle.$$

$\phi = m_q z + C z^3$ 4D: $m_q \bar{q} q$

$\hookrightarrow \Phi(x\, z)$

$\downarrow^{z \to 0}$

$\phi_0 \sim m_q$

$$S_5 \sim \frac{1}{2} \int \sqrt{g}\, \partial_M \phi\, \partial^M \phi$$

$$= \frac{1}{2} \int \frac{1}{z^5}\, \partial_z \phi\, \partial_z \phi\, g^{zz} \rightsquigarrow -z^2$$

$$\sim \frac{1}{2} \int \underbrace{\phi\, \partial_z \left( \frac{1}{z^3} \partial_z \phi \right)}_{\swarrow \atop \text{E.O.M}} - \int \underbrace{\partial_z \left( \frac{1}{z^3} \phi \partial_z \phi \right)}_{\downarrow \atop \text{Boundary term}}$$

$\swarrow$

$S_4$, $S_b$

$$S_b \sim \frac{1}{z^3} \phi\, \partial_z \phi \Big|_{z \to \epsilon} \xrightarrow{\epsilon \to 0}$$

$$\sim \frac{1}{2} \frac{m_q^2}{\epsilon^2} + 3 m_q \cdot C \Rightarrow S_4$$

F-H theorem,

$$\frac{\delta \mathcal{L}_4}{\delta m_q} \sim \frac{m_q}{\epsilon^2} + 3C \Big|_{m_q \to 0}$$

$$\therefore C \sim \langle \bar{q} q \rangle$$

# Example: Deconfinement tempreature:

## Hawking-Page in a cut-off AdS$_5$

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998),
C. P. Herzog, Phys. Rev. Lett.98, 091601 (2007)

$$I = -\frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left( R + \frac{12}{L^2} \right) .$$

$$\kappa \sim 1/N_c, \quad F_\pi^2 \sim N_c$$

So, gravitational action:~$N_c^2$
Meson action:~$N_c$

1. thermal AdS:

$$ds^2 = L^2 \left( \frac{dt^2 + d\vec{x}^2 + dz^2}{z^2} \right)$$

$\beta'$ : the periodicity in the timedirection, (undetermined)

2. AdS black hole: $f(z) = 1 - \frac{z^4}{z_h^4}$

$$ds^2 = \frac{L^2}{z^2} \left( f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right) \qquad 0 \leq t < \pi z_h$$

Transition between two backgrounds ⟷ (De)confinement transition

$$R = -\frac{20}{L^2} \qquad I = \frac{4}{L^2\kappa^2}\int d^5x\sqrt{g}$$

1. Cut-off thermal AdS:

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^{z_0} dz\, z^{-5}$$

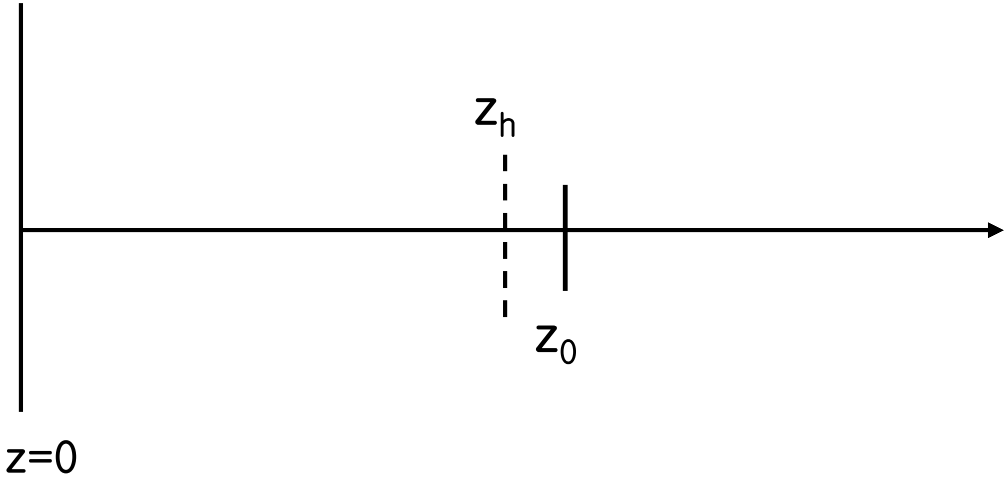
2. Cut-off AdS black hole:

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{\min(z_0,z_h)} dz\, z^{-5}$$

$$\beta' = \pi z_h \sqrt{f(\epsilon)}$$

$$\begin{aligned} \Delta V &= \lim_{\epsilon \to 0} \left( V_2(\epsilon) - V_1(\epsilon) \right) \\ &= \begin{cases} \frac{L^3 \pi z_h}{\kappa^2} \frac{1}{2 z_h^4} & z_0 < z_h \\ \frac{L^3 \pi z_h}{\kappa^2} \left( \frac{1}{z_0^4} - \frac{1}{2 z_h^4} \right) & z_0 > z_h \end{cases} \end{aligned}$$

$$T_c = 2^{1/4} / (\pi z_0)$$



# Hawking-Page at finite density

YK, B.-H. Lee, S. Nam, C. Park, S.-J. Sin, hep-ph/07062525, to appear in PRD.

$$\begin{aligned} \mu_q \bar{\psi}\psi \quad &\longleftrightarrow \quad V_0(z) = \mu_q + \cdots \\ \mu_I \bar{\psi}\tau^3\psi \quad &\longleftrightarrow \quad \tilde{V}_0(z) = \mu_I \ \mathrm{diag}(1,-1) + \cdots \end{aligned}$$

For example,

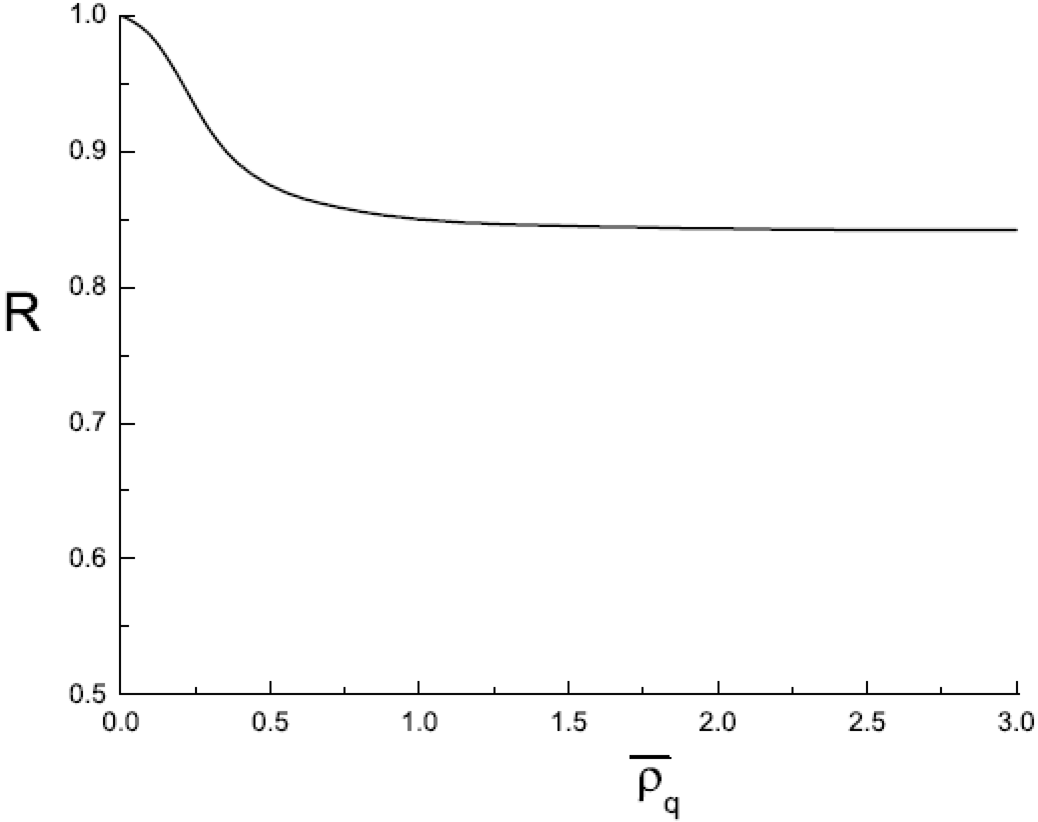
$$\partial_z \left[\frac{1}{z}\partial_z V_\tau(z)\right] = 0\,, \qquad V_\tau = c_1 + c_2 z^2\,.$$

$$c_2 = 12\pi^2 \rho_q / N_c$$

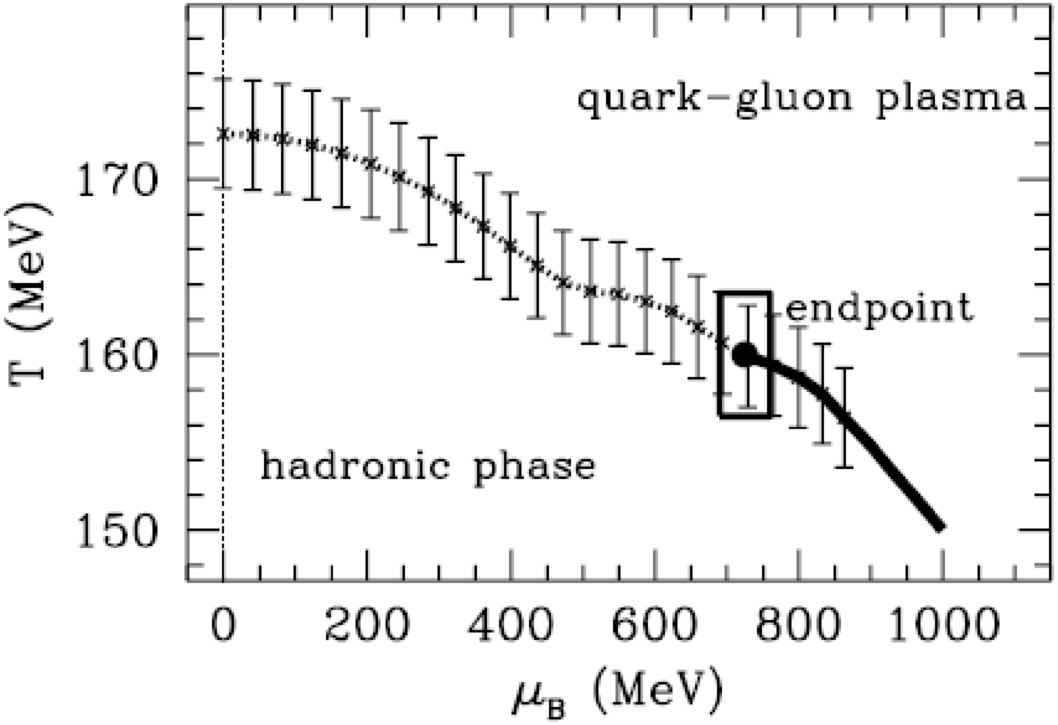
$$\begin{aligned} \rho_q &= \frac{\partial F}{\partial \mu_q} \\ &= \frac{1}{g_5^2} c_2\,, \end{aligned} \qquad F = \frac{1}{2g_5^2}\frac{1}{z}V_0\partial_z V_0 = \frac{1}{g_5^2}\mu_q c_2$$

The final result for $z_h < z_{IR}$ is

$$\Delta V = \frac{L^3 \pi z_h}{\kappa^2} \left[ \frac{1}{z_{IR}^4} - \frac{1}{2 z_h^4} - \frac{L^4 N_f c_2^2}{48 N_c} \left( z_{IR}^2 - z_h^2 \right) \right]$$



$$\mathrm{R} \equiv \frac{T_c}{T_0}, \quad \bar{\rho}_q \equiv \rho_q z_{IR}^3$$



Fodor and Katz ‘01

# Nuclear matter in AdS/QCD

YK, C.-H. Lee, H-U. Yee, hep-ph/07072637

Hard wall model with baryons:

D. K. Hong, T. Inami and H.-U. Yee, Phys. Lett. **B 646**, 165(2007).

$$
\begin{aligned}
S_{\text{kin}} &= \int dz \int dx^4 \sqrt{G_5}\left[i\bar{N}_1\Gamma^M D_M N_1 + i\bar{N}_2\Gamma^M D_M N_2 - \frac{5}{2}\bar{N}_1 N_1 + \frac{5}{2}\bar{N}_2 N_2\right] \\
S_m &= \int dz \int dx^4 \sqrt{G_5}\left[-g\bar{N}_1 X N_2 - g\bar{N}_2 X^\dagger N_1\right] \quad ,
\end{aligned}
$$

$$
f_{1L,R}(p,z)\,\psi_{1L,R}(p) = \int \mathrm{d}^4x\, N_{1L,R}(x,z)e^{ip\cdot x} \,,
$$

$$
\begin{pmatrix} \partial_z - \frac{\Delta}{z} & -\frac{1}{2}g\sigma z^2 \\ -\frac{1}{2}g\sigma z^2 & \partial_z - \frac{4-\Delta}{z} \end{pmatrix}
\begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} = -|p| \begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} \,,
$$

$$
\begin{pmatrix} \partial_z - \frac{4-\Delta}{z} & \frac{1}{2}g\sigma z^2 \\ \frac{1}{2}g\sigma z^2 & \partial_z - \frac{\Delta}{z} \end{pmatrix}
\begin{pmatrix} f_{1R} \\ f_{2R} \end{pmatrix} = |p| \begin{pmatrix} f_{1L} \\ f_{2L} \end{pmatrix} \,,
$$

# Mean field approach:

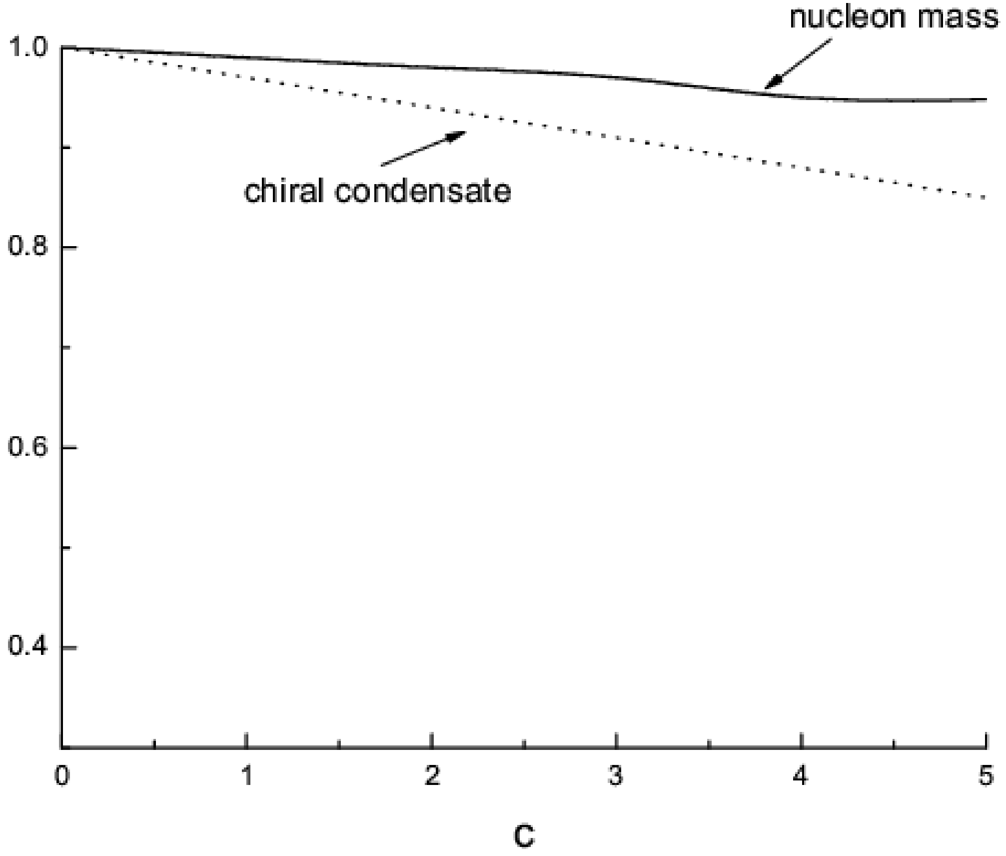
$$\langle \bar{N}(x,z)\gamma^0 N(x,z)\rangle = \sum f(z)^2 \langle \psi(x)^\dagger \psi(x)\rangle \, ; \rho_B = \langle \psi(x)^\dagger \psi(x)\rangle$$

1. Chiral condensate $X_0 = \langle X \rangle$

$$[\partial_z^2 - \frac{3}{z}\partial_z + \frac{3}{z^2}]X_0 = \frac{1}{4}\frac{g}{z^2}(f_{2R}^2 - f_{1R}^2)\rho_s \quad \text{where } \rho_s \equiv \langle \bar{\psi}(x)\psi(x)\rangle .$$

$$X_0(z) = \frac{1}{2}m_q z + \frac{1}{2}\sigma z^3 \, ,$$

2. In-medium nucleon mass (iteratively)



$$\rho_B = c\rho_0$$

We may have to consider back-reaction of matters on the background.

# More phenominological approach

$$X_0(z) = (\frac{1}{2} m_q z + \frac{1}{2} \sigma_0 z^3) \mathbf{1} \,,$$

$$\sigma(\rho_B) \approx \sigma(\rho_B = 0) \left( 1 - 0.37 \frac{\rho_B}{\rho_0} \right) .$$

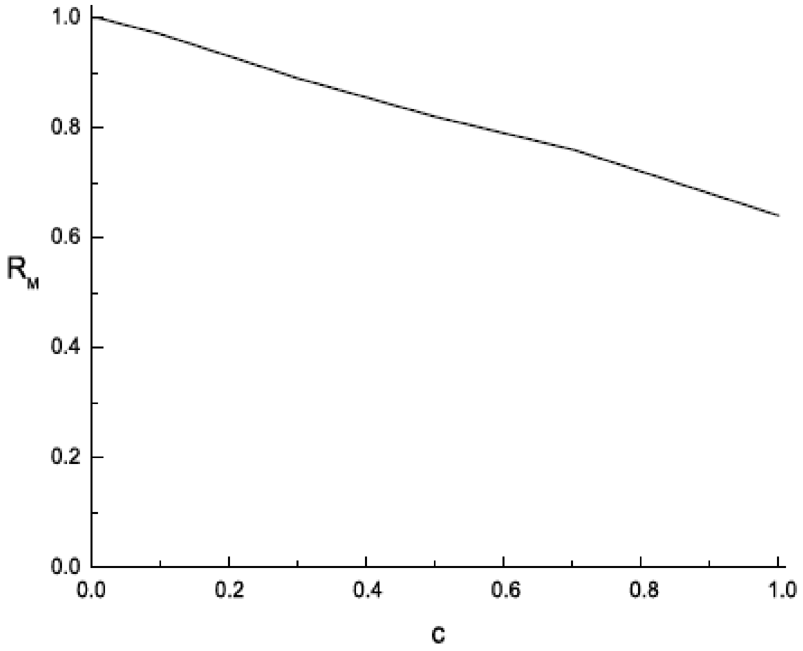


Figure 3: In-medium nucleon mass in the more phenomenological approach. Here $R_M \equiv M_N(\rho_B)/M_N(\rho_B = 0)$.

# Summary

- Although there is no robust proof, bottom-up AdS/QCD approaches are describing QCD relatively well.
- It is important to see what we can do, what we cannot do and what we should not do with the approach.