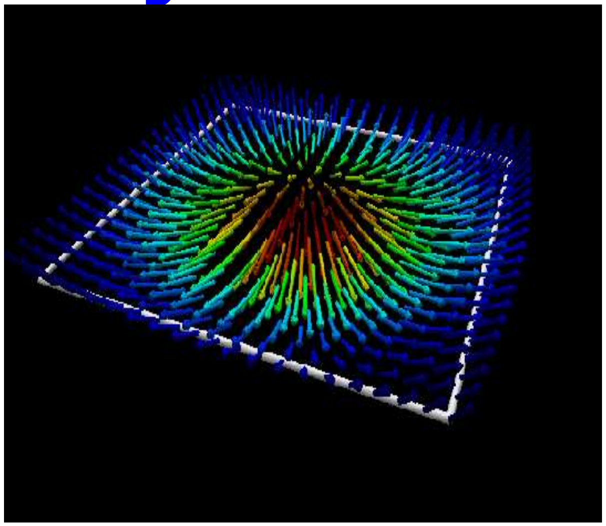
A View on AdS/QCD: Baryon story



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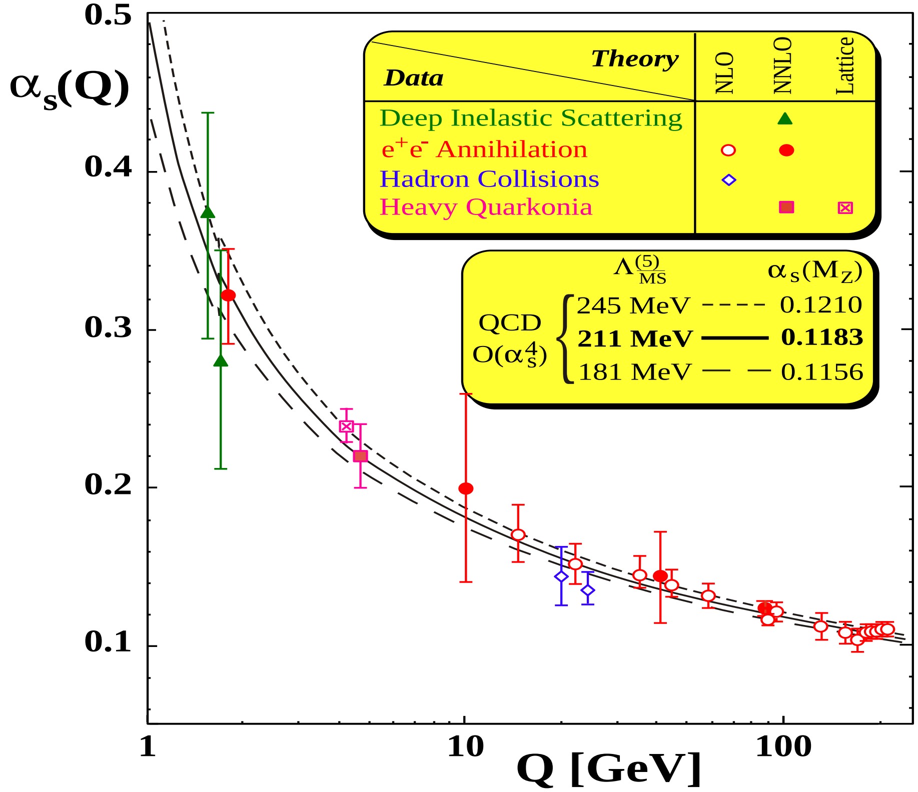
I. Introduction

- QCD is believed to be the theory of strong interaction.

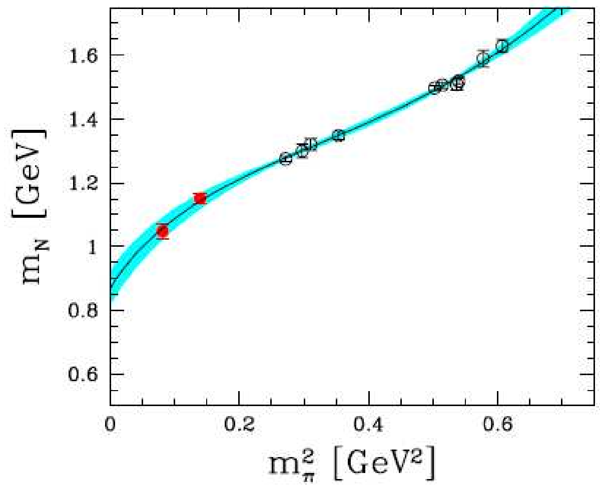
$$\mathcal{L} = -\frac{1}{4} {F^a_{\mu\nu}}^2 + \bar{q}_i \left(i \not{D} - m \right) q_i + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \,. \tag{1}$$

- Evidences?



- Solving QCD is hard, since it's strongly coupled and has no expansion parameter.

- Lattice (ICHEP06)



Stat. error $\lesssim 3\%$

QCDSF

- Recent discovery of AdS/CFT correspondence provides a new scheme to solve strongly coupled gauge theories.

  $\longrightarrow$ Holographic QCD or AdS/QCD, 5D gravity dual to QCD.

- There are several models on holographic QCD, top-down or bottom-up, as good as other models ($< 30\% \sim 1/N_c$).

- But, what's more important is its model-independent features, insensitive to $1/N_c$ corrections, in contrast with other models:

  – Low energy parameters are related to each other and we have new sum rules, $\cdots$

$$g_A \sim \mu_{\rm an}, \quad \mu_{\rm an}^p + \mu_{\rm an}^n = 0 \; (\approx 0.12 \mu_N),$$
$$d_p + d_n = 0 \;\; (\approx 0.026 - 0.021 \bar{\theta}\, e \cdot {\rm fm}, \; {\rm Shintani\ et\ al\ 07}) \,, \; \cdots$$

  – Baryons are instantonic solitons in 5D and have specific couplings with vectors, thus have interesting features in form factors

II. Baryons as AdS Instantons

- If you look at vector mesons,

$$\rho(770), \quad \rho^{(1)}(1450), \ \rho^{(2)}(1700), \cdots , \tag{2}$$

$$\omega(782), \quad \omega^{(1)}(1420), \ \omega^{(2)}(1650), \cdots . \tag{3}$$

- They might be obtained by KK reduction from a 5D vector field,

$$A_\mu(x,z) = \sum_{n=0}^{\infty} f_n(z) A_\mu^{(n)}(x), \quad D_z f_n(z) = -m_n^2 f_n(z) \,. \tag{4}$$

- If you think of them as a single 5D field, the couplings of each vector mesons are constrained to relate to each other: holographic QCD.

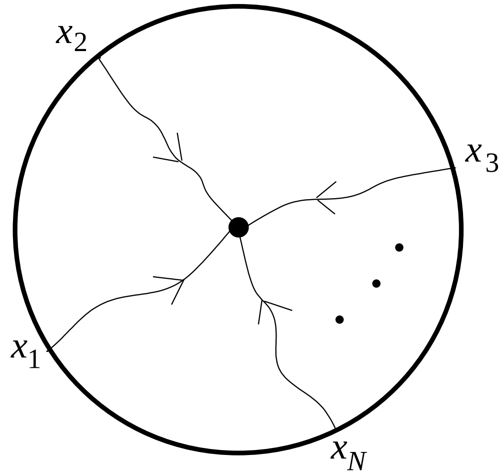
- One of the consequences of holographic QCD is vector meson dominance, where whole tower of vectors contribute. $\rightarrow$ New VMD.

- If this new VMD is verified experimentally, it will indicate strongly that QCD has a hidden symmetry, which is best described in a five dimensional spacetime with a warped factor.
- The VMD will be prominent in the EW form factors.

- $A_\mu \sim A_\mu + \partial_\mu \Lambda$, since the vector mesons coupling to (conserved) vector currents, $A_\mu J^\mu$.
- Mesons are described by a 5D $U(N_F)$ gauge theory (HLS by Bando et al), endowed with a CS term

$$S_{CS} = \frac{N_c}{24\pi^2} \int \omega_5(A), \quad \omega_5 = \mathrm{Tr}\left(AF^2 - \frac{1}{2}A^3 F + \frac{1}{10}A^5\right) \quad (5)$$

- What are baryons in AdS/QCD? It must be solitons.
- In $AdS_5 \times S^5$, D5 brane wrapping S^5 is the baryon vertex (Witten).



- Topological currents in a 5D gauge theory

$$\sqrt{g} j_5^{\mu} = \frac{1}{8\pi^2} \epsilon^{\mu\nu\lambda\rho\sigma} \mathrm{Tr}\, F_{\nu\lambda} F_{\rho\sigma} \tag{6}$$

- The topological charge is nothing but the $4D$ instanton number,

$$\int \mathrm{d}^3x\, \mathrm{d}z \sqrt{g} j_5^0 = \frac{1}{8\pi^2} \int \mathrm{Tr} F\tilde{F} = \frac{i}{3} \int \mathrm{d}^3x\, \mathrm{Tr}\left(h^{-1}\mathrm{d}h\right)^3 . \tag{7}$$

- The instanton number is the Skyrme number, if we interpret the wilson line as a pion field (Atiyah-Manton),

$$\Sigma(x) = P\mathrm{exp}\left(i \int \mathrm{d}z A_z(x, z) \right) . \tag{8}$$

- At low energy the baryons are described as bulk spinors.
- In the SS model the DBI action tends to shrink the baryons but the Coulomb repulsion stabilizes them and the radius of baryons (Rho+Yee+Yi+DKH, Hata+Sakai+Sugimoto+Yamato)

$$\rho_{baryon} \sim \frac{9.6}{M_{KK}\sqrt{g_{YM}^2 N_c}} \tag{9}$$

- The effective Lagrangian becomes in conformal coordinates

$$\int d^4x dw \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w) \bar{\mathcal{B}}\mathcal{B} + g_5(w) \frac{\rho_{baryon}^2}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right]$$
$$- \int d^4x dw \frac{1}{4e^2(w)} \,\mathrm{tr}\, F_{mn} F^{mn} \,, \tag{10}$$

- The spinor sources YM fields

$$\nabla^2 A_m^a = 2g_5(0)\rho^2_{baryon}\bar{\eta}^a_{mn}\partial_n\delta^{(4)}(x) \, , \tag{11}$$

whose solution goes as

$$A_m^a = -\frac{g_5(0)\rho^2_{baryon}}{2\pi^2}\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} \tag{12}$$

to compare with the 't Hooft ansatz

$$A_m^a = -\bar{\eta}^a_{mn}\partial_n \log\left(1+\frac{\rho^2}{r^2+w^2}\right) \simeq -\rho^2\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2} \, , \tag{13}$$

- Including the quantum fluctuations to match the long-range instanton tale (Adkins+Nappi+Witten),

$$g_5(0) = \frac{2\pi^2}{3} \tag{14}$$

- The Lagrangian is unique up to operators with two derivatives in the large N_c and large $\lambda = g_s^2 N_c$ and valid for $E < M_{KK}$.
- Though the coefficient of the Pauli term might be model dependent, the fact that it contains only the nonabelian part of the flavor symmetry is model-independent!
  $\longrightarrow$ The $U(1)$ coupling does not have the Pauli term.
- One immediate consequence of this is that the Pauli form factor

$$F_2^p(q^2) = -F_2^n(q^2)\,. \tag{15}$$

- Especially for instance $\mu_{\rm an}^p + \mu_{\rm an}^n = 0$, which is very close to the experimental value, $(\mu_{\rm an}^p + \mu_{\rm an}^n)_{\rm exp} = 1.79\mu_N - 1.91\mu_N = -0.12\mu_N$

III. Phenomenology: Static properties of baryons

- Once you are given the holographic action, you can get various couplings of baryons after KK reduction.

- Vector couplings of baryons,

$$
\begin{aligned}
g_{\rm min}^{(n)} &= \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(n)}(w) \, , \\
g_{\rm mag}^{(n)} &= 2C \int_w dw \left(\frac{g_5(w) U(w)}{g_5(0) U_{KK} M_{KK}} \right) |f_L(w)|^2 \, \partial_w \psi_{(n)}(w) \, .
\end{aligned}
\tag{16}
$$

- For SS model in the large N_c,

$$
C = \frac{6}{\pi^2} \frac{\lambda N_c}{108\pi^3} (\rho_{baryon} M_{KK})^2 \simeq 0.18 N_c \, . \tag{17}
$$

For bottom-up, C can be fixed by the anomalous magnetic moment.

- The axial coupling for the SS model with $\lambda N_c = 50$

$$g_A \approx 1.30 - 1.31, \quad g_A^{\rm exp} = 1.2670 \pm 0.0035 \tag{18}$$

- **The ρNN and ωNN coupling constants:**

1. In the large λ limit

$$|g_{\omega^{(k)}NN}| \simeq N_c \times |g_{\rho^{(k)}NN}| \tag{19}$$

2. For $\lambda N_c = 50$ in the SS model the couplings get corrections from the subleading Pauli term

$$g_{\rho NN} \approx 3.6, \quad g_{\omega NN} \approx 12.6 \tag{20}$$

Thus the relation (19) is modified to

$$\mathcal{R} \equiv \frac{g_{\omega NN}}{3 g_{\rho NN}} \approx 1.2 \tag{21}$$

$$g_{\rho NN}^{\rm emp} \approx 4.2 - 6.5, \;\; \mathcal{R} \approx 1.1 - 1.5. \tag{22}$$

- Magnetic moments:

$$\frac{\mu^{an}_{proton}}{e_{EM}} = \frac{0.18 N_c}{M_{KK}}, \qquad \frac{\mu^{an}_{neutron}}{e_{EM}} = -\frac{0.18 N_c}{M_{KK}}. \tag{23}$$

- With the shift $N_c \to N_c + 2$ and $m_B \simeq M_{KK}$

$$\mu_p = 1 + 1.08 \left(\frac{N_c + 2}{3} \right) \simeq 2.8, \; \mu_n = -1.08 \left(\frac{N_c + 2}{3} \right) \simeq -1.8$$

The experimental values, $\mu_p = 2.79 \mu_N$ and $\mu_n = -1.91 \mu_N$.

- Form factors:

$$\langle p'|\, J^{\mu}(x)\,|p\rangle = e^{iqx}\,\bar{u}(p')\,\mathcal{O}^{\mu}(p,p')\,u(p), \quad q = p' - p \tag{24}$$

$$\mathcal{O}^{\mu} = \gamma^{\mu}\left[\frac{1}{2}F_1^S(q^2) + F_1^a(q^2)\tau^a\right] + \frac{\gamma^{\mu\nu}}{2m_B}q_{\nu}\left[F_2^S(q^2) + F_2^a(q^2)\tau^a\right], \tag{25}$$

- Vector meson dominance as a direct consequence of AdS/CFT

$$F_{1\mathrm{min}}(q^2) \;=\; \int_w |f_L(w)|^2\; A(q,z(w))\,, \tag{26}$$

$$F_{1\mathrm{mag}}(q^2) \;\simeq\; 2\times 0.18 N_c \int_w |f_L(w)|^2\,\partial_w A(q,z(w))\,.$$

where $f_{L,R}(z)$ are the left(right)-handed normalizable modes, corresponding to the nucleon state and A is dual to the external current as

$$A_{\mu}(x,z) = \int_q A_{\mu}(q)A(q,z)\,e^{iqx}\,. \tag{27}$$

- The Pauli form factor is given as

$$F_2^3(q^2) \simeq 0.18 N_c \times \frac{4m_B}{M_{KK}} \int_w f_L^*(w) f_R(w)\, A(q,w) \quad (28)$$

$$F_2^S(q^2) = 0 \quad (29)$$

- If we expand the non-normalizable mode in terms of the normalizable modes $\phi_n(z)$

$$A(q,z) = \sum_n \frac{f_n \phi_n(z)}{q^2 + m_n^2}\,, \quad f_n = \xi_n m_{2n+1}^2\,, \quad (30)$$

we get

$$F_1(p^2) = \sum_k \frac{g_{v^{(k)}} g_V^{(k)}}{p^2 + m_{2k+1}^2}, \quad F_2^3(p^2) = \sum_k \frac{g_2^{(k)} \xi_k m_{2k+1}^2}{p^2 + m_{2k+1}^2} \quad (31)$$

- Sachs form factors

$$G_M^p(q^2) = F_{1\mathrm{min}}(q^2) + \frac{1}{2}F_{1\mathrm{mag}}(q^2) + \frac{1}{2}F_2^3(q^2) \tag{32}$$

$$G_E^p(q^2) = F_{1\mathrm{min}}(q^2) + \frac{1}{2}F_{1\mathrm{mag}}(q^2) - \frac{q^2}{4m_B^2}\frac{1}{2}F_2^3(q^2) \tag{33}$$

$$G_M^n(q^2) = -\frac{1}{2}F_{1\mathrm{mag}}(q^2) - \frac{1}{2}F_2^3(q^2) \tag{34}$$

$$G_E^n(q^2) = -\frac{1}{2}F_{1\mathrm{mag}}(q^2) + \frac{q^2}{4m_B^2}\frac{1}{2}F_2^3(q^2)\,. \tag{35}$$

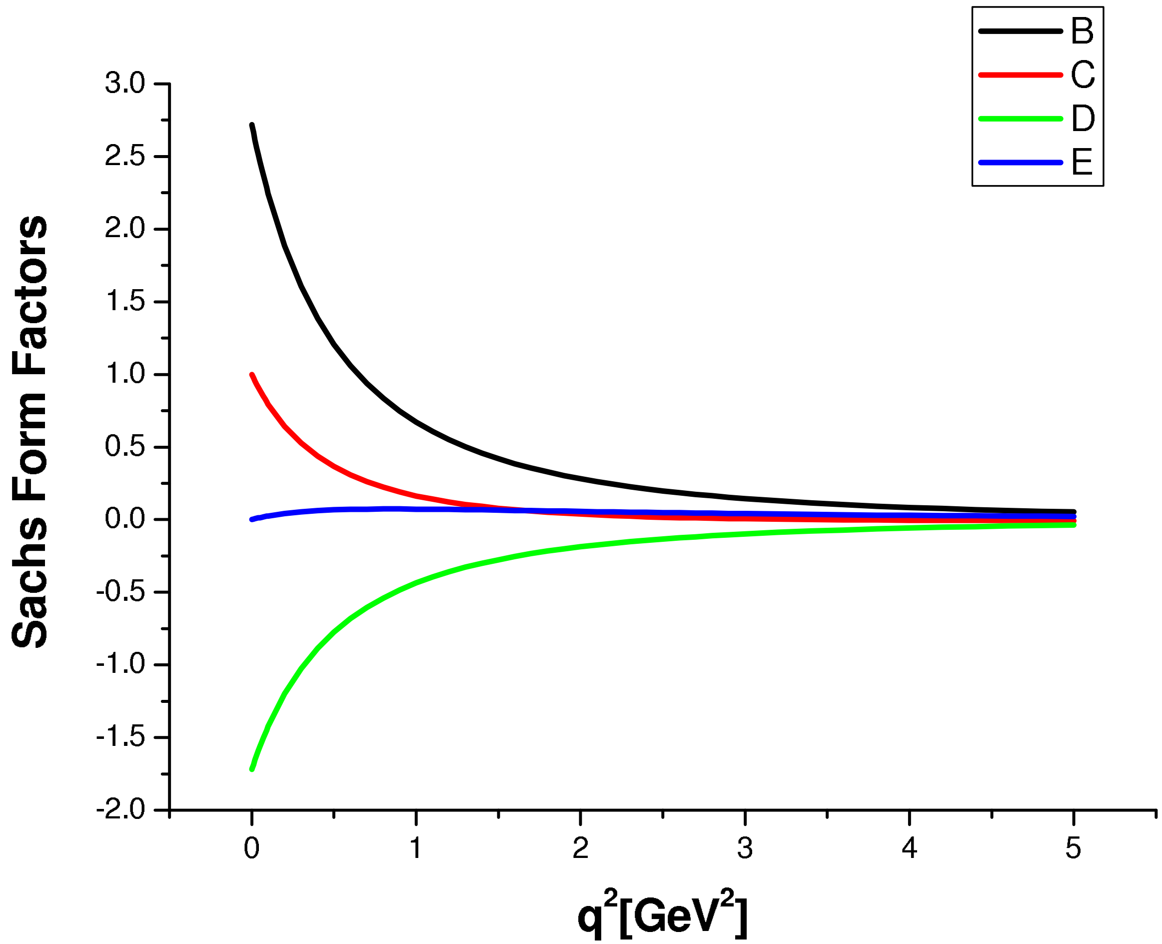
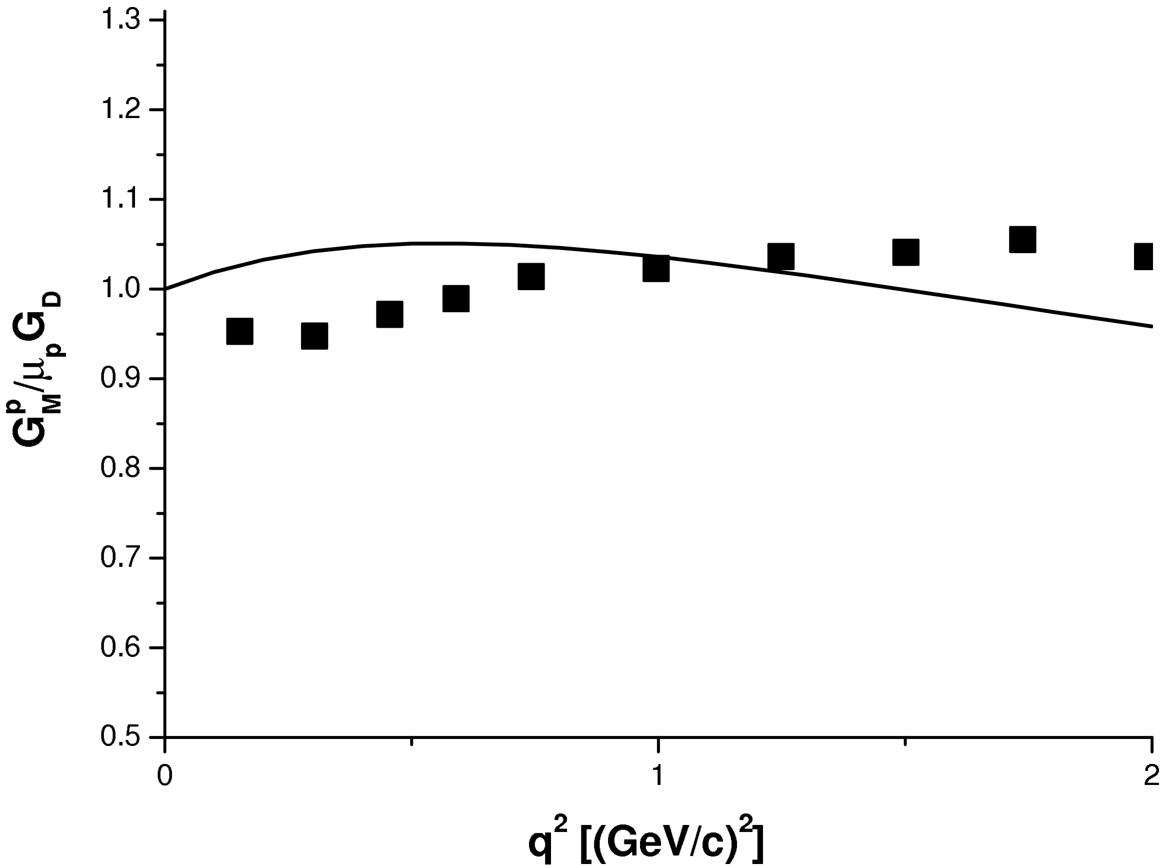


Figure 1: The Sachs form factors: B=G_M^p, C=G_E^p, D=G_M^n, and E=G_E^n

- To see how well our form factors fit the experimental data (R. C. Walker *et al.* (1994)), we plot the ratio with the dipole form factor, $G_D = 1/(1+q^2/0.71)^2$: (Rho+Yee+Yi+DKH, To appear)



IV. Conclusion and Outlook

- Baryons are realized as instanton solitions in holographic QCD, which uniquely determines its chiral Lagrangian up to the Pauli term.
- New VMD is a key feature of holgraphic QCD: Form factors, $\cdots$.
- As a model to QCD, holographic QCD is as good as other models, $\sim 1/N_c$: Mass spectrum, magnetic moments of baryons, g_A, various couplings with vector mesons, $\cdots$.
- But it has model-independent predictions, insensitive to $1/N_c$ corr.
  1. various sum rules due to the instanton nature of baryons:
  $$F_2^p(q^2) + F_2^n(q^2) = 0, \quad d_n + d_p = 0. \tag{36}$$
  2. Low energy parameters of hadrons are unified into a few parameters in 5D: $g_A \sim \mu_{\rm an}, \quad g_{\omega NN} \approx N_c\, g_{\rho NN}, \cdots$.
- Extension to finite density and temperature is under progress.