Aspects of iso-spin asymmetry in QCD

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Outline

I. **Background – phenomenology within iso-spin asymmetry**

II. QCD approaches – QCD sum rules
- Symmetry energy
- Nucleon and hyperons

III. QCD approaches – dense QCD in cold limit
- HDL resummation
- 2-color superconductivity

IV. Future prospects

Nuclear symmetry energy

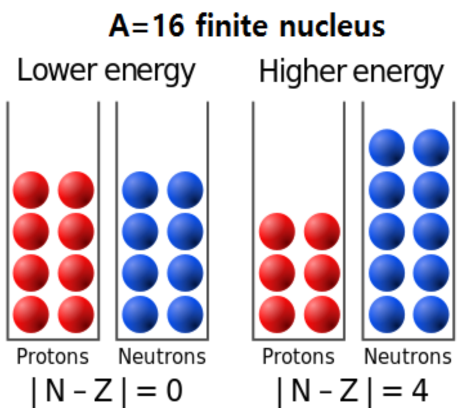
- For finite nucleus

From equation of state

Bethe-Weizsäcker formula for liquid-drop model

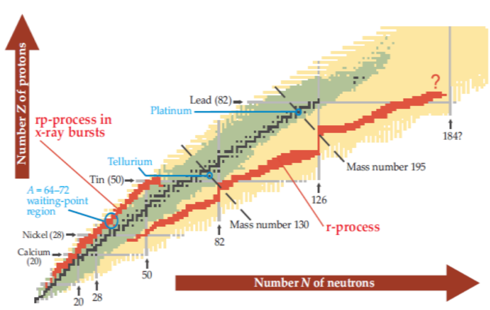
$$M_{\mathrm{nucl}} = Nm_n + Zm_p - E_B/c^2,$$
$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - \boxed{a_A} I^2 A + \delta(A, Z)$$
$$I = (N - Z)/A$$

Asymmetric term a_A (=32 MeV) accounts for shifted energy of nuclear matter per nucleon

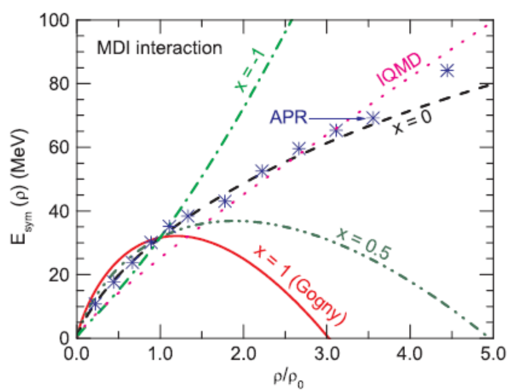


(Figure from Wikipedia)

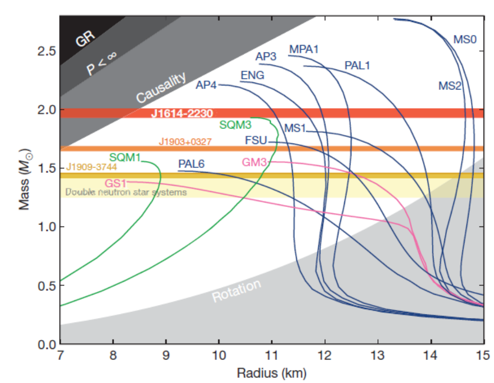
- From rare iso-tope to neutron star core



(Physics Today November 2008)



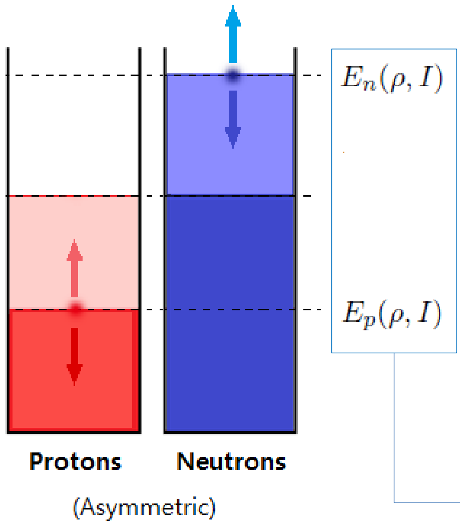
(PRL 102 (2009) 062502 Z. Xiao et al.)



(Nature 467 (2010) 1081 P. B. Demorest et al.)

Nuclear symmetry energy

- For continuous (infinite) matter



Similarly, from equation of state

$$\frac{E(\rho_N, I)}{A} \equiv \bar{E}(\rho_N, I) = E_0(\rho_N) + E_{\rm sym}(\rho_N) I^2 + O(I^4) + \cdots$$

$$E_{sym}(\rho_N) = \frac{1}{2!}\frac{\partial^2}{\partial I^2}\bar{E}(\rho_N, I) \qquad I = (\rho_n - \rho_p)/\rho_N$$

If one assume linear density dependent potential, the symmetry energy can be easily read off from potential

$$E_{\rm sym} = \frac{1}{4I}\left(E_n(\rho, I) - E_p(\rho, I)\right) \longrightarrow$$ Linearly dependent on (ρ, I)

- Quasi-nucleon self-energies

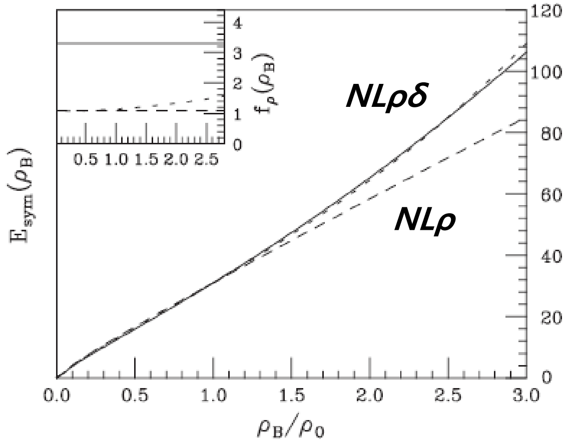
  - In continuous matter, nuclear potential can be understood as self-energy of quasi-nucleon
  - Energy dispersion relation can be written in terms of self-energies (RMFT)

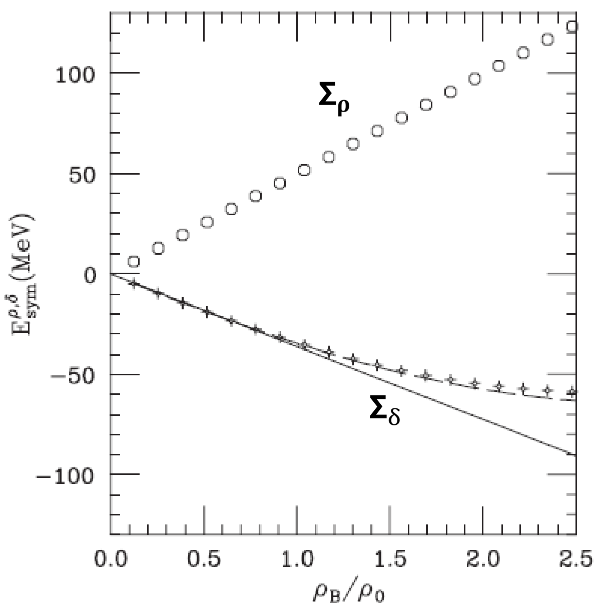
$$G(q) = -i \int d^4x e^{iqx} \langle \Psi_0 | \mathrm{T}[\psi(x)\bar{\psi}(0)] | \Psi_0 \rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$ (near quasi-pole)

  - Self-energies can be calculated in QCD sum rules

Nuclear symmetry energy – stiff or soft?

- Nuclear phenomenology (RMFT) (Phys. Rept. 410 (2005) 335 V. Baran et al.)





NLρδ model (Iso-spin dependent interaction)

$$\mathcal{L}=\bar{\psi}[i\gamma_\mu\partial^\mu-(M_N-g_\sigma\phi-g_\delta\vec{\tau}\cdot\vec{\delta})-g_\omega\gamma_\mu\omega^\mu$$
$$-g_\rho\gamma^\mu\vec{\tau}\cdot\vec{b}_\mu]\psi+\tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_\sigma^2\phi^2)-U(\phi)$$
$$+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\tfrac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu+\tfrac{1}{2}(\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta}-m_\delta^2\vec{\delta}^2)$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\tfrac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}.$$

$$E_{\rm sym}=\frac{1}{6}\frac{k_F^2}{E_F^*}+\frac{1}{2}\left[f_\rho-f_\delta\left(\frac{m^*}{E_F^*}\right)^2\right]\rho_B$$
$$\simeq\frac{1}{6}\frac{k_F^2}{E_F^*}+\Sigma_\rho^0+\frac{m^*}{E_F^*}\Sigma_\delta$$

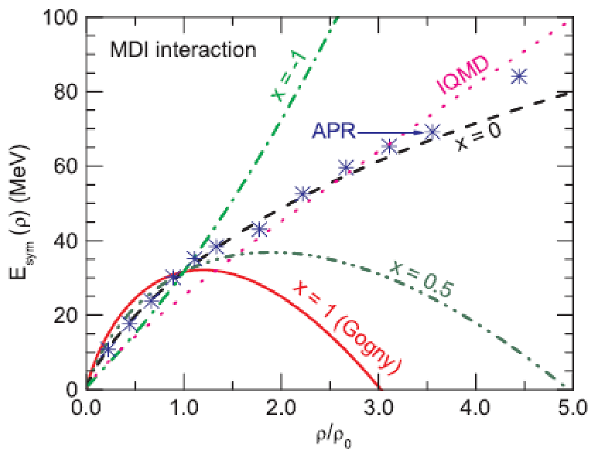
Iso-vector scalar channel (attraction) becomes weaker at dense matter ⇒ stiffly increases

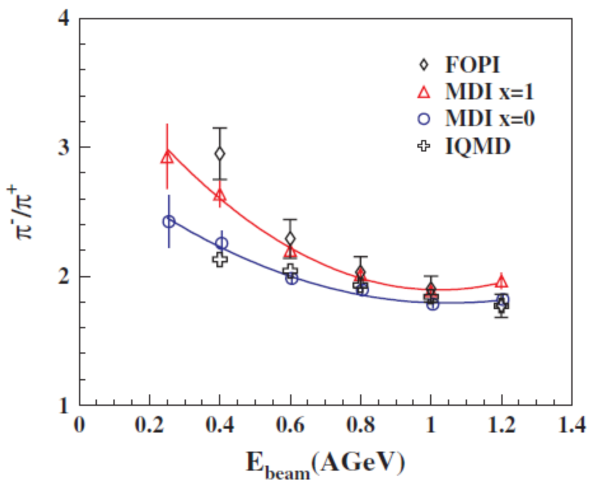
If symmetry energy is stiff, quasi-neutron Fermi sea will become unstable at dense regime

- causes *nn → pΔ- type* scattering
- matter becomes iso-spin symmetric
- subsequent changed hadron yield such as π^-/π^+ from final state will become smaller

Nuclear symmetry energy – stiff or soft?

- Nuclear phenomenology (MDI) (PRL 102 (2009) 062502 Z. Xiao et al.)





Momentum dependent mean-field potential

$$U(\rho, \delta, \mathbf{p}, \tau) = A_u(x)\frac{\rho_{\tau'}}{\rho_0} + A_l(x)\frac{\rho_\tau}{\rho_0} + B\left(\frac{\rho}{\rho_0}\right)^\sigma(1 - x\delta^2) - 8x\tau\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'} + \sum_{t=\tau,\tau'}\frac{2C_{\tau,t}}{\rho_0}\int d^3\mathbf{p}'\frac{f_t(\mathbf{r},\mathbf{p}')}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}$$

x parameter determines density behavior of symmetry energy

***x*=-1** corresponds to **stiff** symmetry energy
***x*=1** corresponds to **soft** symmetry energy

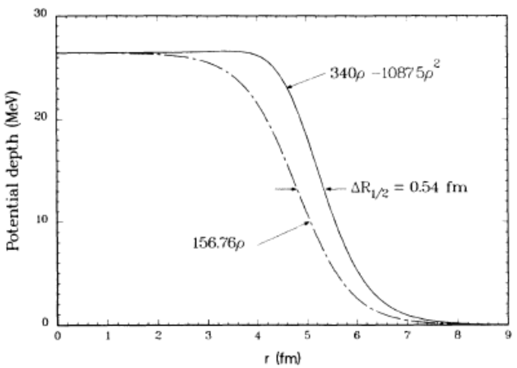
Soft symmetry energy is required to keep high iso-spin density represented as π^-/π^+ ratio in **FOPI** data

Hyperons and neutron star

- Hyperons (S≠0) in medium (**In vacuum**, M_N~940 MeV, M_Λ~1115 MeV, M_Σ~1190 MeV)

Λ is bounded (V ~ -30 MeV)

$$-U(r) = 56.67 f(r) - 30.21 f^2(r)$$



(PRC38 (1988) 2700 D. J. Millener et. al.)

Σ potential is repulsive (V ~ +100 MeV)

$$U(r) = (V_0 + iW_0) f(r) + V_{\rm spin}(r, \vec{l}\cdot\vec{\sigma}) + V_{\rm Coulomb}(r)$$

	Σ-nucleus pot.	
	$U_\Sigma^{R\,a}$	$U_\Sigma^{S\,a}$
V_0 (MeV)	+150	-10
W_0 (MeV)	-15	-10
V_{SO} (MeV)	0	0
c (fm)	3.3[c]	3.3[c]
z (fm)	0.67	0.67

$$f(r) = (1 + \exp[(r-c)/z])^{-1}$$

$$c = 1.1 \times (A-1)^{1/3}$$

At normal density

(PRL89 (2002) 072301 H. Noumi et al.)

- If hyperon energy becomes lower than nucleon energy? ($\rho > \rho_0$, I=1)
  - New degree of freedom (hyperon) can appear in the nuclear matter
    → matter becomes softer → maximum neutron star mass will be bounded near 1.5$M_\odot$
  - 2$M_\odot$ neutron star has been observed (Nature 467 (2010) 1081 P. B. Demorest et al.)
    → should we exclude hyperons in neutron star? How such a stiff EOS could be constructed?
    → related with density behavior of hyperons and symmetry energy
  - Hyperon self-energies can be compared with nucleon self-energies in sum rules context

Outline

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QCD Sum Rules: Overview

- Correlator for baryon current

$$\Pi(q) \equiv i \int d^4x \, e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$
$$= \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u}$$

Correlation of the quantum number contained in **η**
q stands for external momentum
u stands for medium velocity → (1,**0**) in rest frame

- Energy dispersion relation and OPE (in **QCD degree of freedom**)

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials},$$

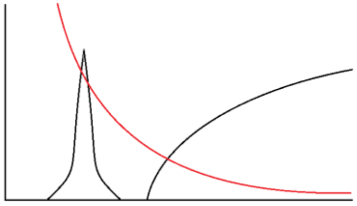
- Phenomenological Ansatz (in **hadronic degree of freedom**)

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

Equating both sides, hadronic quantum number can be expressed in QCD degree of freedom

- Weighting - Borel transformation

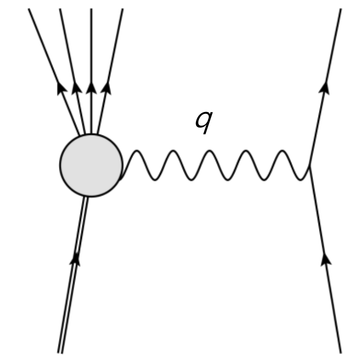
$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n\to\infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|)$$



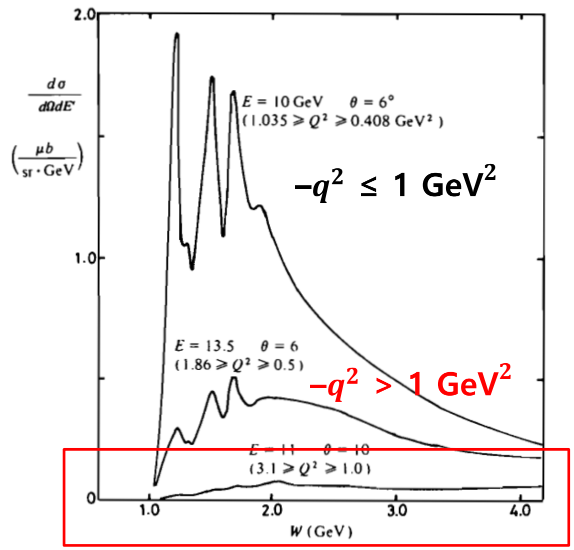
Interpolating fields – parton in hadron

- **Proton** is not a point-like particle

Inelastic scattering: **ep → e + hadrons**



Functions on invariant mass W

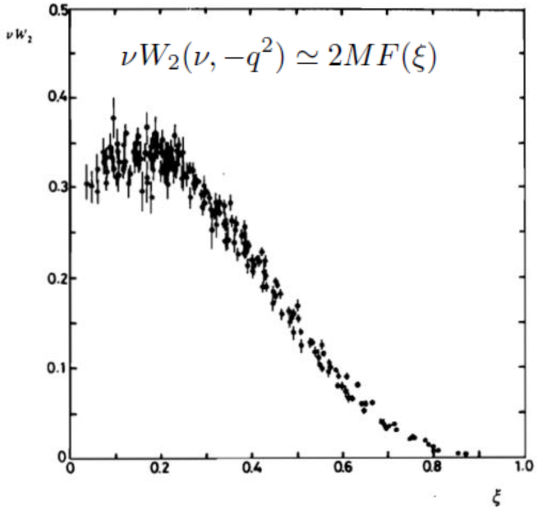


$$\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M \left[2W_1(\nu, -q^2)\tan^2\frac{\theta}{2} + W_2(\nu, -q^2)\right] \Big/ 2M \qquad \nu = k'_0 - k_0$$

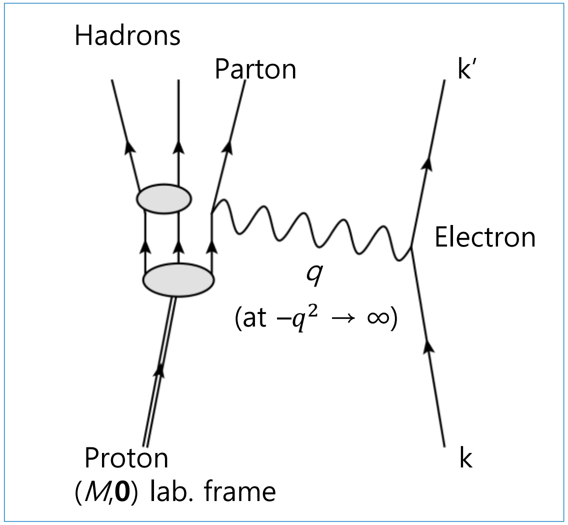
In Bjorken limit (large-momentum transferred region), there are no resonances
→ the scattering can be approximated by point-like free particles (partons)

Parton in hadron

- Bjorken scaling (at $-q^2 \to \infty$, $-q^2/(2M(k'_0 - k_0)) \to \xi$ (fixed) limit)



Behaves as well defined function of ξ

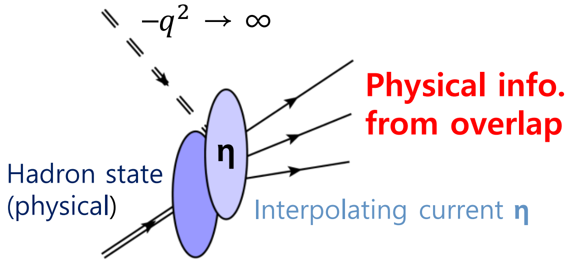


Point-like particle description leads $\nu W_2(\nu, -q^2)$ on function of fixed $\xi = -q^2/(2M(k'_0 - k_0))$
→ confirmed by experimental observation

OPE (at $-q^2 \to \infty$, $-q^2/(2M(k'_0 - k_0)) = \xi$) reproduces Bjorken scaling
→ quantum number of hadron can be interpolated with explicit QCD current

Interpolating current for baryons

- To obtain physical information



a. Quasi-particle state will be extracted from the overlap
b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
c. Our object: **N**, **P**, **Λ**, and **Σ family**

- Example: constructing **proton** current

Required quantum number: $\boldsymbol{I = 1/2,\ J^P = (1/2)^+}$

Simplest structure: **[$I = 0, J = 0$ di-quark structure] X [single quark with $I = 1/2,\ J = 1/2$]**

Di-quark structure: $\epsilon_{abc}(u_a^T C\gamma_5 d_b)$, $\epsilon_{abc}(u_a^T C d_b)$ ***u*** and ***d*** flavor in antisymmetric combination

Positive parity matching: $\eta_1 = \epsilon_{abc}(u_a^T C d_b)\gamma_5 u_c$, $\eta_2 = \epsilon_{abc}(u_a^T C\gamma_5 d_b) u_c$

Ioffe's choice: $\eta = 2(\eta_1 - \eta_2) = \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu d_c$

→ chiral symmetry breaking term appears in leading order

- Ioffe's choice for hyperon (**Λ** and **Σ+**) current

$$\eta_\Lambda \sim \epsilon_{abc}\left[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c - (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c\right]$$
$$\eta_{\Sigma+} \sim \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu s_c$$

Required quantum number
$I = 0,\ J^P = (1/2)^+$
$I = 1,\ J^P = (1/2)^+$

Ioffe's choice for Σ

- Σ_0 interpolating field in general combination

$$\eta_{\Sigma^0} = \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c + [d_a^T C s_b]\gamma_5 u_c + t\left([u_a^T C\gamma_5 s_b]d_c + [d_a^T C\gamma_5 s_b]u_c\right)\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c.$$

Requirement: (1) spin-0 di-quark structure, (2) total *I=0* combination

Using Fierz arrangement,

(a) $$\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{L,c} + [d_{R,a}^T C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$$

(b) $$\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c = 4\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{R,c} + [d_{R,a}^T C s_{R,b}]u_{R,c} - (L \leftrightarrow R)\right)$$

Quark propagation in perturbative regime (separation scale ~ 1 GeV)

$$\langle T[q_\beta^a(x)\bar{q}_\alpha^b(0)]\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{m_q - \not{p}} \simeq \underline{\frac{i}{2\pi^2}\delta_{ab}\frac{1}{(x^2)^2}[\not{x}]_{\alpha\beta}} - \underline{\frac{m_q}{4\pi^2}\frac{1}{x^2}\delta_{ab}\delta_{\alpha\beta}}$$

Light quark has chiral symmetry → propagation from each helicity state to itself
The symmetry is broken for strange quark ($m_s \neq 0$) → mixed propagation between helicity states

Correlator of each basis can be expressed in diagrammatical way

Ioffe's choice for Σ

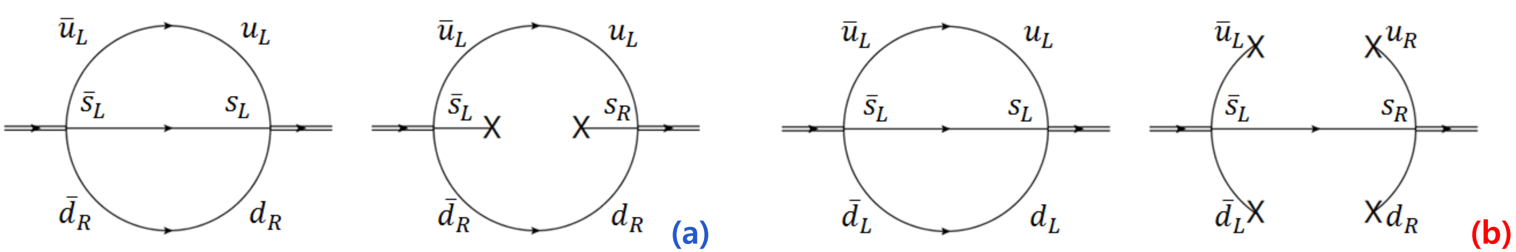
- Σ_0 interpolating field (continued)

$$\eta_{\Sigma^0} = \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c + [d_a^T C s_b]\gamma_5 u_c + t\left([u_a^T C \gamma_5 s_b] d_c + [d_a^T C \gamma_5 s_b] u_c\right)\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C \gamma_\mu d_b]\gamma_5\gamma^\mu s_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C \sigma_{\mu\nu} d_b]\gamma_5 \sigma^{\mu\nu} s_c.$$

(a) $\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}] d_{L,c} + [d_{R,a}^T C s_{R,b}] u_{L,c} - (L \leftrightarrow R)\right)$

(b) $\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c = 4\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}] d_{R,c} + [d_{R,a}^T C s_{R,b}] u_{R,c} - (L \leftrightarrow R)\right)$

Lowest mass dimensional quark condensate



Correlator of basis (a): strange quark condensate (dim-3)
Correlator of basis (b): four-quark condensate (dim-6)

Lack of clear information for four-quark condensate → choice of basis (a) can be better

Ioffe's choice for **proton**

- **Proton** case

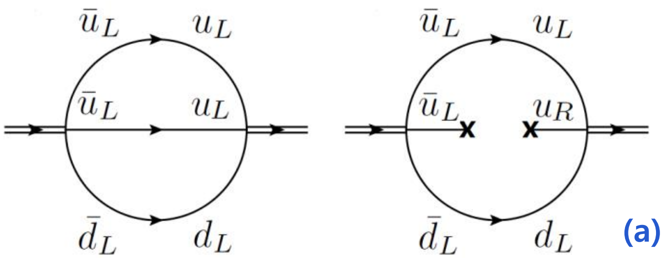
$$\eta_{p(t)} = 2\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 u_c + t[u_a^T C\gamma_5 d_b]u_c\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c$$

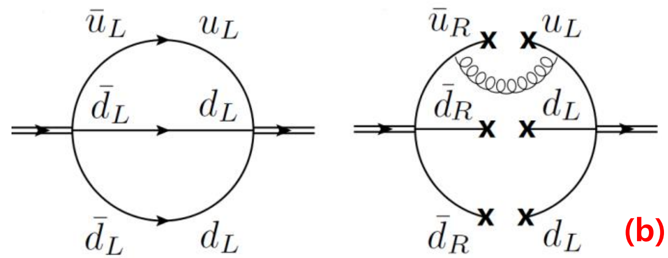
Requirement: (1) spin-0 di-quark structure, (2) total *I=0* combination

After Fierz arrangement,

(a) $\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{L,c} - [u_{L,a}^T C d_{L,b}]u_{R,c}\right)$

(b) $\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{R,c} - [u_{L,a}^T C d_{L,b}]u_{L,c}\right)$





Correlator of basis **(a)**: chiral condensate (dim-3)
Correlator of basis **(b)**: six-quark condensate (dim-9) with perturbative gluon attachment

Same reason for six-quark condensate → choice of basis **(a)** would be better (Ioffe's choice)

Generalized Interpolating field for Λ

- Special case: Λ

Possible I=0 combination with spin-0 di-quark structure

$$\{\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c,\ \epsilon_{abc}[u_a^T C\gamma_5 d_b] s_c,\ \underline{\epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right)},\ \underline{\epsilon_{abc}\left([u_a^T C\gamma_5 s_b] d_c - [d_a^T C\gamma_5 s_b] u_c\right)}\}$$

3rd and 4th basis can be expressed as

$$\epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right) = \frac{1}{2}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + [u_a^T C\gamma_5 d_b] s_c - [u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

$$\epsilon_{abc}\left([u_a^T C\gamma_5 s_b] d_c - [d_a^T C\gamma_5 s_b] u_c\right) = \frac{1}{2}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + [u_a^T C\gamma_5 d_b] s_c + [u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

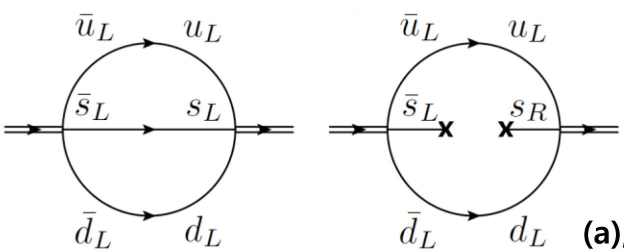
→ basis set can be reduced to 3-independent bases set

Representation in helicity bases

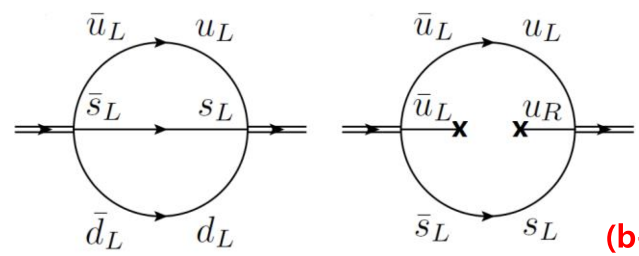
(a) $$\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}] s_{R,c} - [u_{R,a}^T C d_{R,b}] s_{L,c} - (L \leftrightarrow R)\right)$$

(b) $$\epsilon_{abc}[u_a^T C\gamma_5 d_b] s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}] s_{R,c} + [u_{R,a}^T C d_{R,b}] s_{L,c} - (L \leftrightarrow R)\right)$$

(c) $$\epsilon_{abc}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}] d_{L,c} - [d_{R,a}^T C s_{R,b}] u_{L,c} - (L \leftrightarrow R)\right)$$



(a), (b), (c)



(b-c)

Generalized Interpolating field for Λ

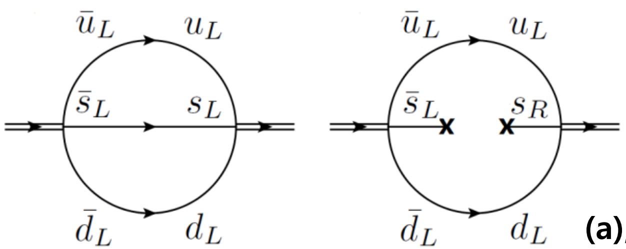
- Special case: **Λ** (continued)

Set of basis and **lowest mass dimensional quark condensate**

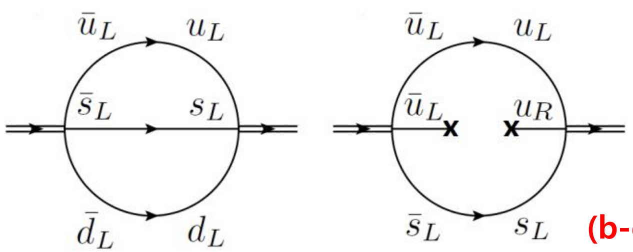
(a) $$\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c = \epsilon_{abc}\left([u^T_{R,a} C d_{R,b}]s_{R,c} - [u^T_{R,a} C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$$

(b) $$\epsilon_{abc}[u_a^T C \gamma_5 d_b] s_c = \epsilon_{abc}\left([u^T_{R,a} C d_{R,b}]s_{R,c} + [u^T_{R,a} C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$$

(c) $$\epsilon_{abc}[u_a^T C \gamma_5 \gamma_\mu d_b]\gamma^\mu s_c = 2\epsilon_{abc}\left([u^T_{R,a} C s_{R,b}]d_{L,c} - [d^T_{R,a} C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$$



(a), (b), (c)



(b-c)

Correlator of basis **(a)**, **(b)**, **(c)**: strange quark condensate (dim-3)
Cross correlator of basis **(b-c)**: chiral condensate (dim-3)
→ the chiral condensate term gives additional strong attraction to scalar self-energy

General form of **Λ** interpolating field

$$\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

Where *A* determines overall normalization and coupling strength to physical **Λ** state

QCD Sum Rules: dispersion relation

- Simplest case: Nucleon in vacuum

$$\Pi(q) \equiv i \int d^4x e^{iqx} \langle 0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | 0 \rangle = \Pi_s(q^2) + \Pi_q(q^2)\not{q}.$$

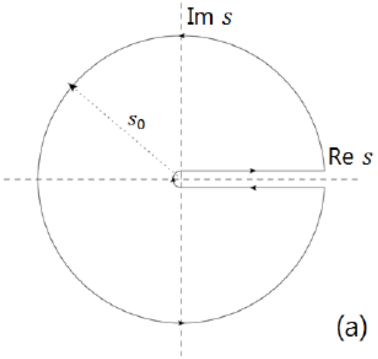
- Using Cauchy relation

$$\Pi_i(q^2) = \frac{1}{2\pi i} \int_0^\infty ds \frac{\Delta\Pi_i(s)}{s - q^2} + \text{polynomials}$$

$$\Delta\Pi_i(q^2) \equiv \lim_{\epsilon \to 0^+} [\Pi_i(q^2 + i\epsilon) - \Pi_i(q^2 - i\epsilon)]$$

$$= (2\pi)^4 i \sum_\alpha \Big(\delta^4(q - p_\alpha)\langle 0|\eta(0)|\alpha\rangle\langle\alpha|\bar{\eta}(0)|0\rangle - \delta^4(q + p_\alpha)\langle 0|\bar{\eta}(0)|\alpha\rangle\langle\alpha|\eta(0)|0\rangle \Big)$$

This imaginary part contains all possible hadronic resonance α



Vacuum q^2 integration contour

- Emphasizing ground state – Borel sum rules

$$\mathcal{B}[\Pi_i(q^2)] = \frac{1}{2\pi i} \int_0^\infty ds\, e^{-s/M^2} \Delta\Pi_i(s)$$

$$\equiv \lim_{\substack{-q^2, n \to \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q^2} \right)^n \Pi_i(q^2) \equiv \hat{\Pi}_i(M^2)$$

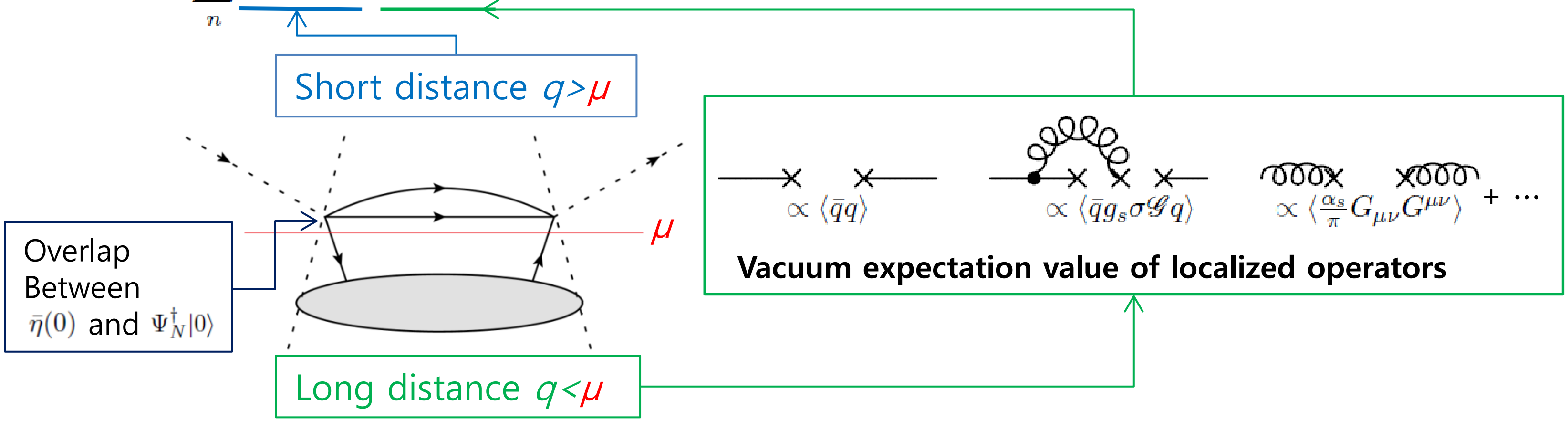
The continuum will be suppressed by setting M ~ hadronic mass scale

  - The moment (Borel mass $-q^2/n = M^2$) is a fictitious value (non-physical) In principle, physical values such as mass should not depends on M
  - As OPE is truncated, actually it depends → the value can be read off at plateau of Borel curve

QCD SR: operator product expansion

- Operator product expansion (Example: 2-quark condensate diagram)

$$\Pi_i(q^2) = \sum_n C_n^i(q^2,\mu)\langle \hat{O}_n(\mu)\rangle_0$$



- Separation scale is set to be hadronic scale (≤ 1 GeV)
  - **Wilson coefficient** contains perturbative contribution above separation scale – short-ranged partonic propagation in hadron
  - **Condensate** contains non-perturbative contribution below separation scale – long ranged correlation in low energy part of hadron
  - Quark confinement inside hadron is low energy QCD phenomenon
  - Genuine properties of hadron are reflected in **the condensates**

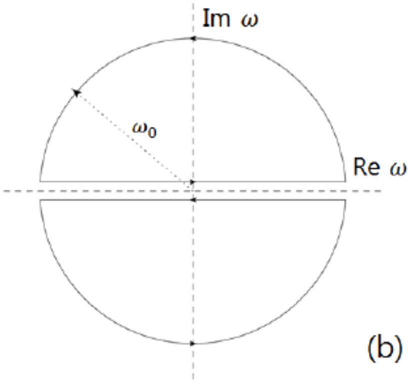
QCD Sum Rules: in-medium case

- Energy dispersion relation with fixed 3-momentum

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials},$$

$$\Delta\Pi_i(\omega, |\vec{q}|) \equiv \lim_{\epsilon\to 0^+} [\Pi_i(\omega + i\epsilon, |\vec{q}|) - \Pi_i(\omega - i\epsilon, |\vec{q}|)]$$

  - In-medium hadronic excitation is certainly not symmetric as in the vacuum case
  - Medium reference frame occurs – energy sum rules for quasi-particle state is proper choice



In-medium energy integration contour

- Energy Borel sum rules

$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] = \frac{1}{2\pi i}\int_{-\omega_0}^{\omega_0} d\omega\ W(\omega)\Delta\Pi_i(\omega, |\vec{q}|)$$

$$\equiv \lim_{\substack{-q_0^2, n\to\infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|) \equiv \hat{\Pi}_i(M^2, |\vec{q}|),$$

$$W(\omega) = (\omega - \bar{E}_q)e^{-\omega^2/M^2}$$

Anti-state is suppressed, only quasi-particle part is emphasized

QCD Sum Rules: spectral ansatz

- According to relativistic mean field theory

$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

Quasi-hadron propagator in **RMFT**

Each invariant can be assumed according to phenomenological Ansatz
Via Borel transformation, self-energies can be obtained in terms of invariants

$$\Pi_s(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{M_N^*}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

$$\Pi_q(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{1}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

$$\Pi_u(q_0, |\vec{q}|) = +\lambda_N^{*2} \frac{\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

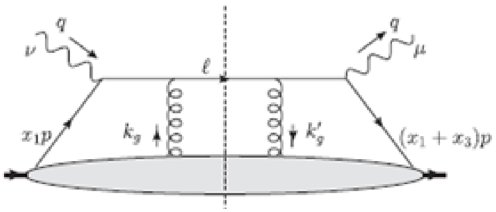
Borel transf. $\Rightarrow$

$$\bar{\mathcal{B}}[\Pi_s(q_0^2, |\vec{q}|)] = \lambda_N^{*2} M_h^* e^{-(E_q^2 - \vec{q}^2)/M^2}$$

$$\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)] = \lambda_N^{*2} e^{-(E_q^2 - \vec{q}^2)/M^2}$$

$$\bar{\mathcal{B}}[\Pi_u(q_0^2, |\vec{q}|)] = \lambda_N^{*2} \Sigma_v^h e^{-(E_q^2 - \vec{q}^2)/M^2}$$

- Condensates and in-medium properties



DIS diagram

I. In-medium properties are included in low energy scale (long-ranged)
II. PCAC (Gellman-Oakes-Renner relation), Chiral perturbation theory, Lattice QCD, DIS experiment can be used to obtain in-medium condensates

In-medium condensates

- Simplest guess: linear Fermi gas approximation

$$\langle \hat{O} \rangle_{\rho,I} = \langle \hat{O} \rangle_{\text{vac}} + \langle n|\hat{O}|n\rangle \rho_n + \langle p|\hat{O}|p\rangle \rho_p$$
$$= \langle \hat{O} \rangle_{\text{vac}} + \tfrac{1}{2}(\langle n|\hat{O}|n\rangle + \langle p|\hat{O}|p\rangle)\rho$$
$$+ \tfrac{1}{2}(\langle n|\hat{O}|n\rangle - \langle p|\hat{O}|p\rangle) I\rho.$$

[Vacuum condensate] +
[nucleon expectation value] **x** [density]

Iso-spin symmetric and asymmetric part

- Example: chiral condensates

Iso-spin symmetric part

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q}\rho$$

Nucleon-pion sigma term

$$\sigma_N = \frac{1}{3}\sum_{a=1}^{3}\left(\langle \tilde{N}|[\mathcal{Q}_A^a,[\mathcal{Q}_A^a,H_{QCD}]]|\tilde{N}\rangle - \langle 0|[\mathcal{Q}_A^a,[\mathcal{Q}_A^a,H_{QCD}]]|0\rangle\right)$$
$$= 2m_q \int d^3x \left(\langle \tilde{N}|\bar{q}q|\tilde{N}\rangle - \langle 0|\bar{q}q|0\rangle\right) \equiv 2m_q \langle N|\bar{q}q|N\rangle$$

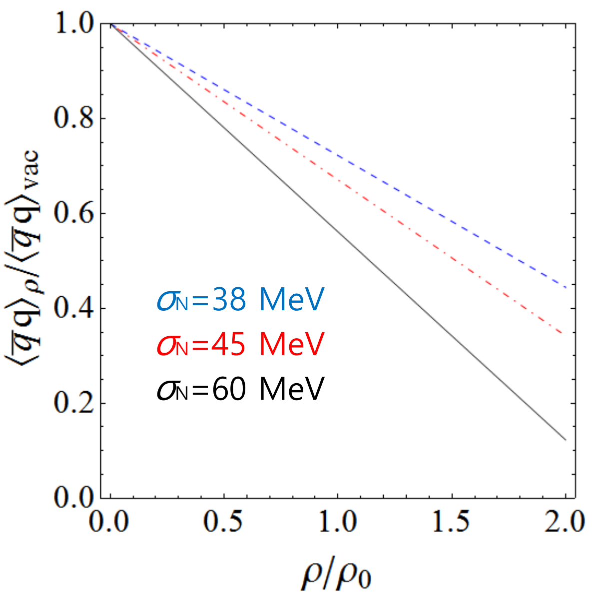
where, $H_{QCD} = \int d^3x (2m_q \bar{q}q + m_s \bar{s}s + \cdots)$

With Hellman-Feynman theorem

$$2m_q \langle \psi|\bar{q}q|\psi\rangle = m_q \frac{d}{dm_q}\langle \psi|H_{QCD}|\psi\rangle$$

and linear density approximation $\mathcal{E} \sim M_N \rho$

Sigma term determines dropping rate



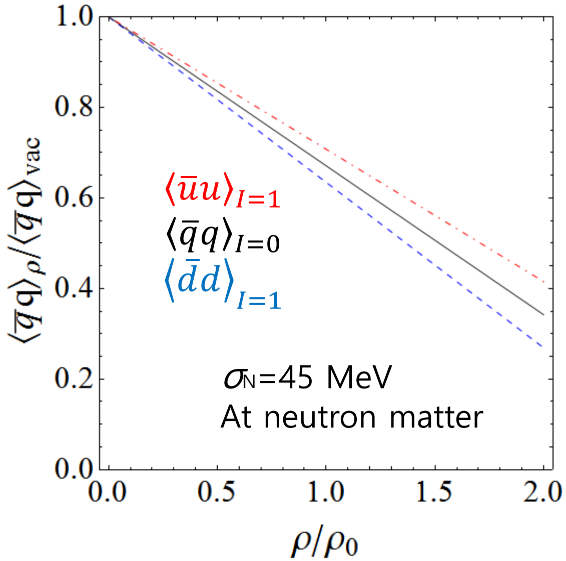
In-medium condensates

- Asymmetric part

From trace anomaly and heavy quark expansion

$$T^{\mu}_{\ \mu} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum_{h=c,t,b} m_h \bar{h}h + \cdots$$

$$= \left(-\frac{9\alpha_s}{8\pi}\right) G^2 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + O(\mu^2/4m_h^2)$$



Low-lying baryon octet mass relation

$$\begin{aligned} m_p &= A + m_u B_u + m_d B_d + m_s B_s \\ m_n &= A + m_u B_d + m_d B_u + m_s B_s \\ m_{\Sigma^+} &= A + m_u B_u + m_d B_s + m_s B_d \\ m_{\Sigma^-} &= A + m_u B_s + m_d B_u + m_s B_d \\ m_{\Xi^0} &= A + m_u B_d + m_d B_s + m_s B_u \\ m_{\Xi^-} &= A + m_u B_s + m_d B_d + m_s B_u \end{aligned}$$

$$\Rightarrow \quad \frac{1}{2}\left(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle\right) = \frac{1}{2}\left(\frac{(m_{\Xi^0} + m_{\Xi^-}) - (m_{\Sigma^+} + m_{\Sigma^-})}{2m_s - (m_u + m_d)}\right)$$

where $A \equiv \langle (\bar{\beta}/4\alpha_s) G^2 \rangle_p, \ \ B_u \equiv \langle \bar{u}u \rangle_p, \ \ B_d \equiv \langle \bar{d}d \rangle_p$

- Strange contents

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_{\rm vac} + \langle \bar{s}s \rangle_N \rho$$

$$= (0.8)\langle \bar{q}q \rangle_{\rm vac} + y\frac{\sigma_N}{2m_q}\rho$$

$$y = \langle \bar{s}s \rangle_N / \langle \bar{q}q \rangle_N$$

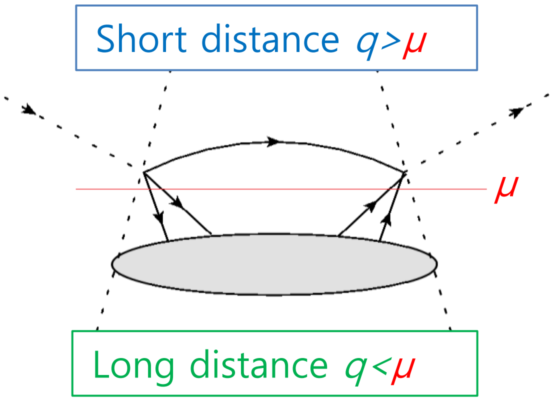
Ratio **0.8** is determined from vacuum sum rule for hyperon

y can be determined from direct lattice QCD
→ recent lattice results says ***y*** should be small
***y*~0.05** (PRD87 074503, PRD91 051502, PRD94 054503)

We confined ***y* → 0.1**

4-quark condensates – baryon sum rules

- In 3-quark constituted baryon sum rules



- There is no loop in leading order diagram
  → no suppression factor comes from loop diagram
- Can be numerically important in sum rules
- Indeed, 4-quark condensates give non-negligible contribution to baryon sum rules
- For twist-4 condensates, DIS data can be used (within linear density approximation)

- Twist-4 ops. in baryon OPE

Operator type	$\gamma-\gamma$	$\gamma_5\gamma-\gamma_5\gamma$	$\sigma-\sigma$
t^A-t^A	$\langle\bar{q}_1\gamma_5\gamma t^A q_1\bar{q}_2\gamma_5\gamma t^A q_2\rangle_{p,\text{s.t.}}\equiv T^1_{q_1q_2}$	$\langle\bar{q}_1\gamma t^A q_1\bar{q}_2\gamma t^A q_2\rangle_{p,\text{s.t.}}\equiv T^2_{q_1q_2}$	$\langle\bar{q}_1\sigma t^A q_1\bar{q}_2\sigma t^A q_2\rangle_{p,\text{s.t.}}\equiv T^5_{q_1q_2}$
$I-I$	$\langle\bar{q}_1\gamma_5\gamma q_1\bar{q}_2\gamma_5\gamma q_2\rangle_{p,\text{s.t.}}\equiv T^3_{q_1q_2}$	$\langle\bar{q}_1\gamma q_1\bar{q}_2\gamma q_2\rangle_{p,\text{s.t.}}\equiv T^4_{q_1q_2}$	$\langle\bar{q}_1\sigma q_1\bar{q}_2\sigma q_2\rangle_{p,\text{s.t.}}\equiv T^6_{q_1q_2}$

$$\langle p|\bar{q}_1\Gamma_i^\alpha q_1\bar{q}_2\Gamma_i^\beta q_2|p\rangle_{\text{s.t.}}=\left(u^\alpha u^\beta-\frac{1}{4}g^{\alpha\beta}\right)\frac{1}{4\pi\alpha_s}\frac{M_n}{2}T^i_{q_1q_2}$$

q_1 and q_2 stand for light quark flavor

Matrix elements can be obtained from DIS experiment data

Borel transformed OPE

- Nucleon OPE (**neutron**)

$$\overline{\mathcal{W}}_M[\Pi_{n,s}(q_0^2,|\vec{q}|)] = \lambda_n^{*2} M_n^* e^{-(E_{n,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{n,s}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n,s}^o(q_0^2,|\vec{q}|)]$$
$$= -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle \bar{u}u\rangle_{\rho,I},$$

$$\overline{\mathcal{W}}_M[\Pi_{n,q}(q_0^2,|\vec{q}|)] = \lambda_n^{*2} e^{-(E_{n,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{n,q}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n,q}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{1}{32\pi^2} M^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}$$
$$- \left[\frac{1}{9\pi^2} M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right] \langle u^\dagger i D_0 u\rangle_{\rho,I} L^{-\frac{4}{9}} - \left[\frac{4}{9\pi^2} M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right] \langle d^\dagger i D_0 d\rangle_{\rho,I} L^{-\frac{4}{9}}$$
$$- \frac{1}{2}\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{tr.}} - \frac{1}{2}\langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{tr.}} + \frac{3}{2}\langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{tr.}} + \frac{5}{2}\langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{tr.}}$$
$$- \frac{1}{2}\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} - \frac{1}{2}\langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}}$$
$$+ \bar{E}_{p,q}\left[\frac{1}{6\pi^2} M^2 \left[\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}\right] E_0 L^{-\frac{4}{9}}\right],$$

$$\overline{\mathcal{W}}_M[\Pi_{n,u}(q_0^2,|\vec{q}|)] = \lambda_n^{*2} \Sigma_v^n e^{-(E_{n,q}^2-\vec{q}^2)/M^2} \mathcal{B}[\Pi_{n.u}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n.u}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{1}{12\pi^2}(M^2)^2 \left[7\langle d^\dagger d\rangle_{\rho,I} + \langle u^\dagger u\rangle_{\rho,I}\right] E_1 L^{-\frac{4}{9}}$$
$$+ \bar{E}_{p,q}\left[\frac{4}{9\pi^2} M^2 \langle u^\dagger i D_0 u\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} + \frac{16}{9\pi^2} M^2 \langle d^\dagger i D_0 d\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}\right.$$
$$\left. + 2\left[\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{s.t.}} - \langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} + \langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{s.t.}} - \langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}}\right]\right].$$

Borel transformed OPE

- $\boldsymbol{\Sigma}^+$ hyperon OPEs

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,s}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2} M_{\Sigma^+}^{*} e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{\Sigma^+,s}^{e}(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\mathcal{B}[\Pi_{\Sigma^+,s}^{o}(q_0^2,|\vec{q}|)]$$

$$= \frac{m_s}{16\pi^4}(M^2)^3 E_2 L^{-\frac{8}{9}} - \frac{1}{4\pi^2}(M^2)^2 E_1 \langle \bar{s}s\rangle_{\rho,I} + m_s\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\rm tr.} - m_s\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\rm tr.}$$

$$+ \bar{E}_{\Sigma^+,q}\left[\frac{m_s}{2\pi^2}M^2\langle u^\dagger u\rangle_{\rho,I}E_0 L^{-\frac{8}{9}} - \frac{4}{3}\langle \bar{s}s\rangle_{\rm vac}\langle u^\dagger u\rangle_{\rho,I}\right],$$

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,q}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2} e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{\Sigma^+,q}^{e}(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\mathcal{B}[\Pi_{\Sigma^+,q}^{o}(q_0^2,|\vec{q}|)]$$

$$= \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} - +\frac{1}{32\pi^2}M^2\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}$$

$$- \left[\frac{1}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right]\langle s^\dagger iD_0 s\rangle_{\rho,I}L^{-\frac{4}{9}} - \left[\frac{4}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right]\langle u^\dagger iD_0 u\rangle_{\rho,I}L^{-\frac{4}{9}}$$

$$- \frac{1}{2}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\rm tr.} - \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\rm tr.} + \frac{3}{2}\langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\rm tr.} + \frac{5}{2}\langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\rm tr.}$$

$$- \frac{1}{2}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\rm s.t.} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\rm s.t.} - \frac{1}{2}\langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\rm s.t.} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\rm s.t.}$$

$$+ \bar{E}_{\Sigma^+,q}\left[\frac{1}{6\pi^2}M^2\langle u^\dagger u\rangle_{\rho,I}E_0 L^{-\frac{4}{9}}\right],$$

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,u}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2}\Sigma_v^{\Sigma^+} e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2}\mathcal{B}[\Pi_{\Sigma^+,u}^{e}(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\mathcal{B}[\Pi_{\Sigma^+,u}^{o}(q_0^2,|\vec{q}|)]$$

$$= \frac{7}{12\pi^2}(M^2)^2\langle u^\dagger u\rangle_{\rho,I}E_1 L^{-\frac{4}{9}}$$

$$+ \bar{E}_{\Sigma^+,q}\left[\frac{4}{9\pi^2}M^2\langle s^\dagger iD_0 s\rangle_{\rho,I}E_0L^{-\frac{4}{9}} + \frac{16}{9\pi^2}M^2\langle u^\dagger iD_0 u\rangle_{\rho,I}E_0 L^{-\frac{4}{9}}\right.$$

$$\left. + 2\left[\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\rm s.t.} - \langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\rm s.t.} + \langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\rm s.t.} - \langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\rm s.t.}\right]\right].$$

Borel transformed OPE

- **Λ** hyperon OPE with **Generalized interpolating field**

$$
\begin{aligned}
\overline{\mathcal{W}}_M[\Pi_{\Lambda,s}(q_0^2,|\vec{q}|)] = \lambda_\Lambda^{*2} M_\Lambda^* e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} &= \bar{\mathcal{B}}[\Pi_{\Lambda,s}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\bar{\mathcal{B}}[\Pi_{\Lambda,s}^o(q_0^2,|\vec{q}|)] \\
&= -\frac{(1-\tilde{a}^2+2\tilde{b}^2)}{64\pi^4} m_s (M^2)^3 E_2 L^{-\frac{8}{9}} \\
&\quad + \frac{(1-\tilde{a}^2+2\tilde{b}^2)}{16\pi^2}(M^2)^2 \langle \bar{s}s\rangle_{\rho,I} E_1 - \frac{\tilde{a}\tilde{b}}{4\pi^2}(M^2)^2\langle\bar{q}q\rangle_{\rho,I} E_1 \\
&\quad - \frac{(1-\tilde{a}^2-2\tilde{b}^2)}{128\pi^2} m_s M^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} E_0 L^{-\frac{8}{9}} \\
&\quad + \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{4} m_s \langle \bar{u}u\bar{d}d\rangle_{\mathrm{tr.}} + \frac{(1+\tilde{a}^2-4\tilde{b}^2)}{4} m_s \langle \bar{u}\gamma_5 u \bar{d}\gamma_5 d\rangle_{\mathrm{tr.}} \\
&\quad - \frac{(1-\tilde{a}^2-2\tilde{b}^2)}{4} m_s \langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\mathrm{tr.}} - \frac{(1-\tilde{a}^2+2\tilde{b}^2)}{4} m_s \langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\mathrm{tr.}} - \frac{(1+\tilde{a}^2)}{8} m_s \langle \bar{u}\sigma u \bar{d}\sigma d\rangle_{\mathrm{tr.}} \\
&\quad - \bar{E}_{\Lambda,q}\left[\frac{(1-\tilde{a}^2+2\tilde{b}^2)}{8\pi^2} m_s M^2 \langle q^\dagger q\rangle_{\rho,I} E_0 L^{-\frac{8}{9}} + \frac{2\tilde{a}\tilde{b}}{3}\langle q^\dagger q\rangle_{\rho,I}\langle \bar{q}q\rangle_{\mathrm{vac}} - \frac{(1-\tilde{a}^2+4\tilde{b}^2)}{3}\langle q^\dagger q\rangle_{\rho,I}\langle\bar{s}s\rangle_{\mathrm{vac}} \right]
\end{aligned}
$$

$$
\begin{aligned}
\overline{\mathcal{W}}_M[\Pi_{\Lambda,u}(q_0^2,|\vec{q}|)] = \lambda_\Lambda^{*2}\Sigma_v^\Lambda e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} &= \bar{\mathcal{B}}[\Pi_{\Lambda,u}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\bar{\mathcal{B}}[\Pi_{\Lambda,u}^o(q_0^2,|\vec{q}|)] \\
&= \frac{(1+\tilde{a}^2+14\tilde{b}^2)}{48\pi^2}(M^2)^2\langle q^\dagger q\rangle_{\rho,I} E_1 L^{-\frac{4}{9}} - \frac{2\tilde{a}\tilde{b}}{3} m_s \langle q^\dagger q\rangle_{\rho,I}\langle\bar{s}s\rangle_{\mathrm{vac}} \\
&\quad + \bar{E}_{\Lambda,q}\Big[\tilde{b}^2\langle\bar{u}\gamma u\bar{d}\gamma d\rangle_{\mathrm{s.t.}} - \tilde{b}^2\langle\bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\mathrm{s.t.}} + \tilde{b}^2\langle\bar{u}\sigma u\bar{d}\sigma d\rangle_{\mathrm{s.t.}} \\
&\qquad + \frac{(1+\tilde{a}^2-2\tilde{b}^2)}{2}\langle\bar{u}\gamma u\bar{s}\gamma s\rangle_{\mathrm{s.t.}} + (\tilde{a}+\tilde{b}^2)\langle\bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\mathrm{s.t.}} - \tilde{a}\tilde{b}\langle\bar{u}\sigma u\bar{s}\sigma s\rangle_{\mathrm{s.t.}} \Big].
\end{aligned}
$$

Borel transformed OPE

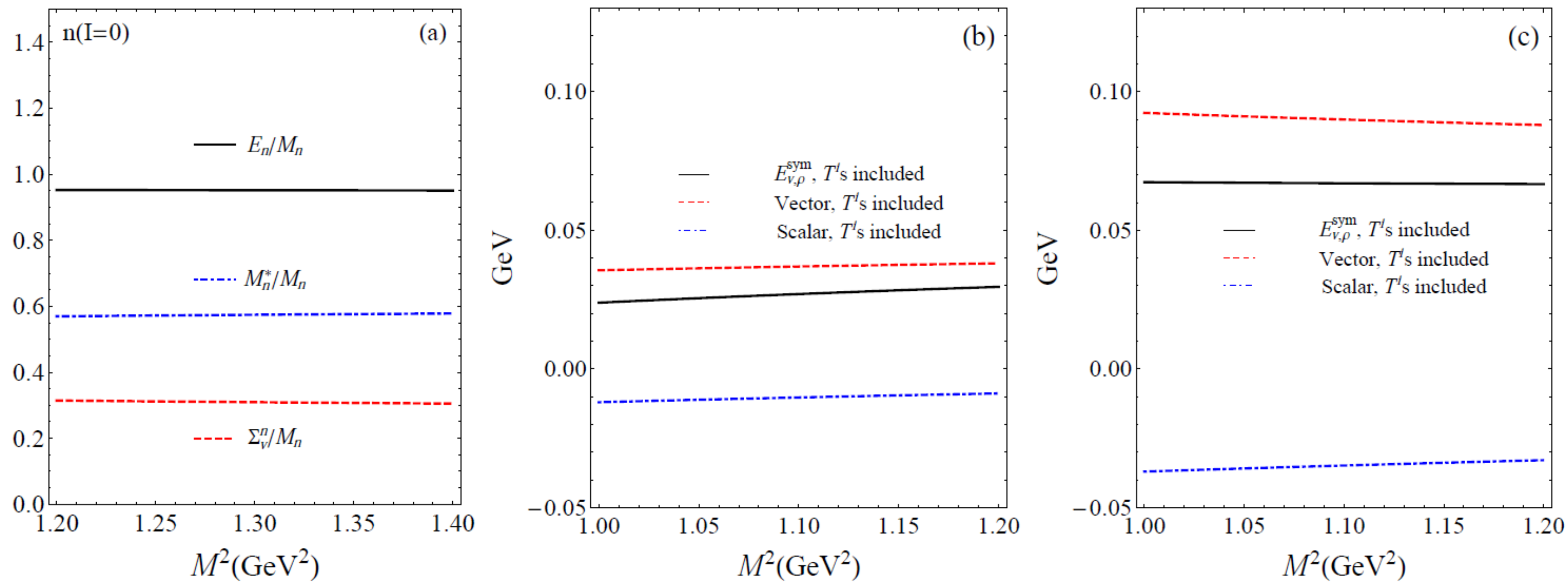
- **Λ** hyperon OPE with **Generalized interpolating field** (continued)

$$
\begin{aligned}
\overline{\mathcal{W}}_M[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)] &= \lambda_\Lambda^{*2} e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{\Lambda,q}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\mathcal{B}[\Pi_{\Lambda,q}^o(q_0^2,|\vec{q}|)] \\
&= \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{256\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{256\pi^2} M^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\
&\quad - \frac{\tilde{a}\tilde{b}}{4\pi^2} m_s M^2 \langle \bar{q}q\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} + \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{32\pi^2} m_s M^2 \langle \bar{s}s\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\
&\quad - \frac{(1-\tilde{a}^2+2\tilde{b}^2)}{4}\langle \bar{u}u\bar{d}d\rangle_{\rm tr.} - \frac{(1-\tilde{a}^2-2\tilde{b}^2)}{4}\langle \bar{u}\gamma_5 u\bar{d}\gamma_5 d\rangle_{\rm tr.} + \frac{(1+\tilde{a}^2-\tilde{b}^2)}{4}\langle \bar{u}\gamma u\bar{d}\gamma d\rangle_{\rm tr.} \\
&\quad + \frac{(1+\tilde{a}^2+\tilde{b}^2)}{4}\langle \bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\rm tr.} + \frac{(1-\tilde{a}^2)}{8}\langle \bar{u}\sigma u\bar{d}\sigma d\rangle_{\rm tr.} + \tilde{a}\tilde{b}\langle \bar{q}q\bar{s}s\rangle_{\rm tr.} + \tilde{b}\langle \bar{q}\gamma_5 q\bar{s}\gamma_5 s\rangle_{\rm tr.} \\
&\quad + \frac{(1+\tilde{a}^2-10\tilde{b}^2)}{8}\langle \bar{q}\gamma q\bar{s}\gamma s\rangle_{\rm tr.} + \frac{(\tilde{a}-3\tilde{b}^2)}{4}\langle \bar{q}\gamma_5\gamma q\bar{s}\gamma_5\gamma s\rangle_{\rm tr.} - \frac{\tilde{a}\tilde{b}}{4}\langle \bar{q}\sigma q\bar{s}\sigma s\rangle_{\rm tr.} \\
&\quad - \frac{\tilde{b}^2}{4}\langle \bar{u}\gamma u\bar{d}\gamma d\rangle_{\rm s.t.} + \frac{\tilde{b}^2}{4}\langle \bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\rm s.t.} - \frac{\tilde{b}^2}{4}\langle \bar{u}\sigma u\bar{d}\sigma d\rangle_{\rm s.t.} \\
&\quad - \frac{(1+\tilde{a}^2-2\tilde{b}^2)}{8}\langle \bar{q}\gamma q\bar{s}\gamma s\rangle_{\rm s.t.} + \frac{(\tilde{a}+\tilde{b}^2)}{4}\frac{1}{q^2}\langle \bar{q}\gamma_5\gamma q\bar{s}\gamma_5\gamma s\rangle_{\rm s.t.} + \frac{\tilde{a}\tilde{b}}{4}\langle \bar{q}\sigma q\bar{s}\sigma\gamma s\rangle_{\rm s.t.} \\
&\quad + \bar{E}_{\Lambda,q}\left[\frac{(1+\tilde{a}^2+2\tilde{b}^2)}{24\pi^2} M^2 \langle q^\dagger q\rangle_{\rho,I} E_0\right],
\end{aligned}
$$

Sum rule result I - nucleons

- **Neutron** sum rules and symmetry energy (at $\rho=\rho_0$)

Neutron sum rules (***I=0***) | Sym. energy without twist-4 ops. | Sym. energy with twist-4 ops.

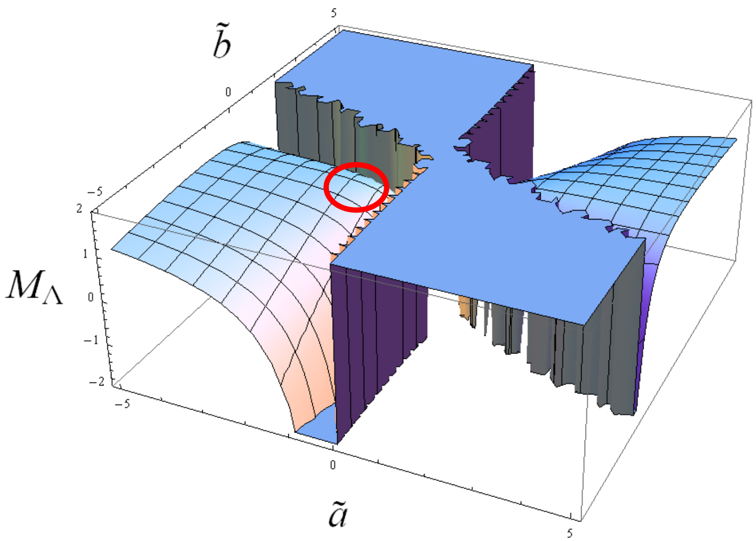


1. The quasi-**neutron** energy is slightly reduced → represents bounding at $\rho=\rho_0$
2. Large cancelation mechanism in both of the quasi-neutron and the symmetry energy
3. Twist-4 matrix elements enhance the strength of cancelation mechanism
   → simple linear gas approximation would not be good choice

Determination of $\{\tilde{a},\tilde{b}\}$ in Λ sum rules

- 3D plot with $\tilde{a}$ and $\tilde{b}$

Vacuum sum rules with $\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$



1. Self-energies will be obtained by taking ratio
   $$M_\Lambda = \bar{\mathcal{B}}[\Pi_{\Lambda,s}(q_0^2,|\vec{q}|)]/\bar{\mathcal{B}}[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)]$$
   $$\Sigma_v^\Lambda = \bar{\mathcal{B}}[\Pi_{\Lambda,v}(q_0^2,|\vec{q}|)]/\bar{\mathcal{B}}[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)]$$
2. Overall normalization *A* becomes meaningless in practical calculation
   → free parameter reduces to $\tilde{a}$ and $\tilde{b}$
3. Plane $\{\tilde{a},\tilde{b}\}$ has stable/unstable region

- **Ioffe's choice** corresponds to $\{\tilde{a},\tilde{b}\} = \{-1,-1/2\}$ and $A_{(-1,-1/2)} = \sqrt{8/3}$

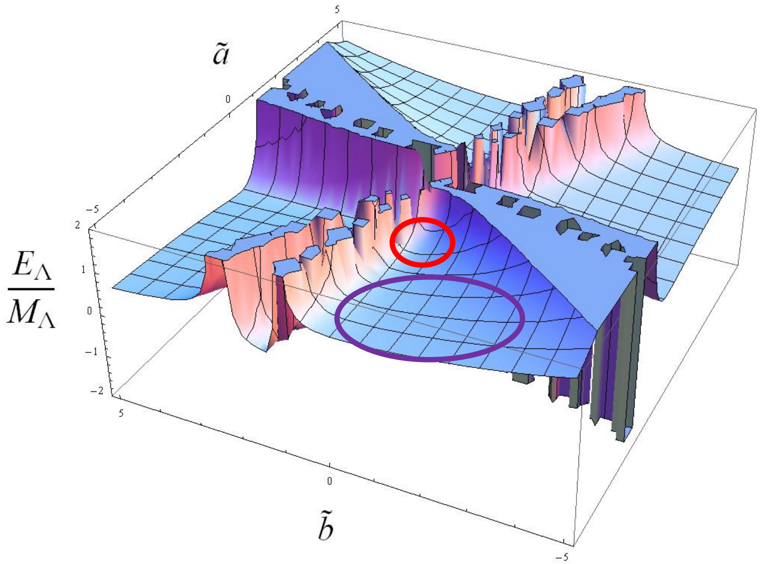
$$\eta_{\Lambda(-1,-1/2)} \Rightarrow \sqrt{\frac{2}{3}}\epsilon_{abc}\left([u_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu d_c - [d_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu u_c\right)$$

- This linear combination is located on boundary of stable region
  → mass can be drastically changed via even small variation of $\{\tilde{a},\tilde{b}\}$

Determination of $\{\tilde{a},\tilde{b}\}$ in Λ sum rules

- 3D plot with $\tilde{a}$ and $\tilde{b}$

In-medium sum rules with $\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$



1. The OPE **does not contain** the derivative expansion and $\bar{s}\gamma s$ dependent Ops.
2. **Ioffe's choice** is located on unstable point and the quasi-Λ energy/M_Λ ~ 1.5
3. To control the repulsive tendency of the quasi-Λ energy, one can try derivative expansion

$$\bar{s}\gamma_\mu s\bar{q}q = (\bar{s}\gamma_\mu s\bar{q}q) + x^\nu\,(\bar{s}\gamma_\mu D_\nu s\bar{q}q)$$

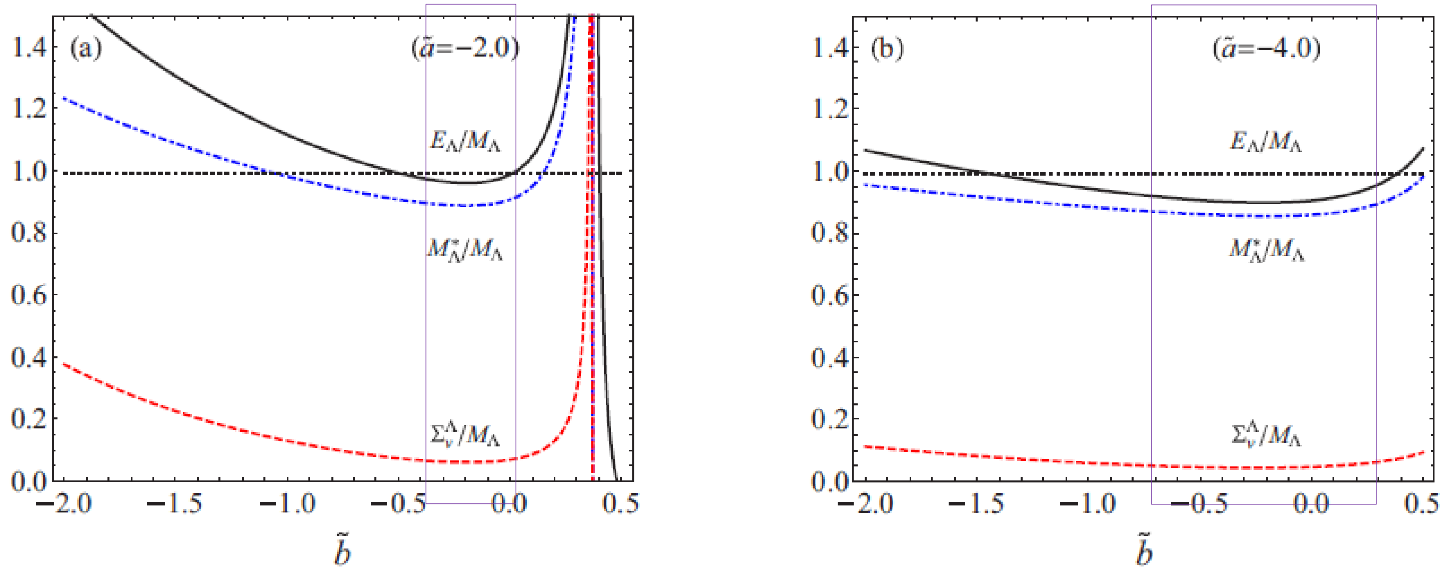
$$\langle\bar{s}\gamma_\mu D_\nu s\bar{q}q\rangle = \underline{\frac{1}{4}g_{\mu\nu}m_s\langle\bar{s}s\bar{q}q\rangle} + \frac{4}{3}\left(\langle\bar{s}\gamma_0 D_0 s\bar{q}q\rangle - \frac{1}{4}m_s\langle\bar{s}s\bar{q}q\rangle\right)\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right)$$

→ **Trace part can reduce** the quasi-Λ energy but contains large uncertainty
→ It is worthwhile to **try new linear combination** with $\tilde{a}$ and $\tilde{b}$

Stable $\{\tilde{a},\tilde{b}\}$ for Λ sum rules

- Confining stable points on $\{\tilde{a},\ \tilde{b}\}$ plain

Cross section with fixed $\tilde{a} = -2.0$ and $\tilde{a} = -4$



As $|\tilde{a}|$ becomes large, sum rules become stable and weakly depend on $\tilde{b}$

- 9 stable points

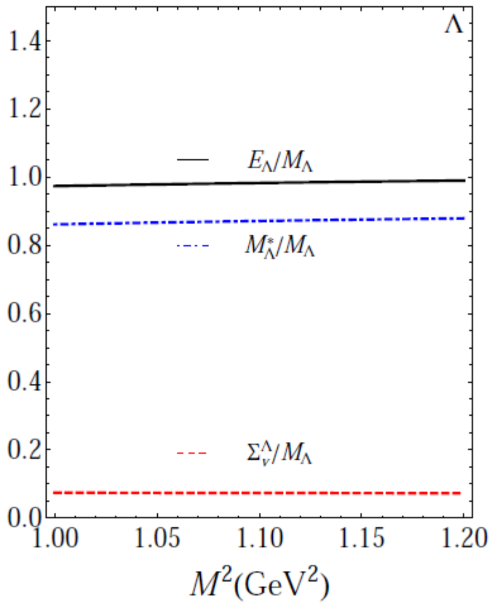
Sum rules have been obtained by averaging results on following 9 points:

$\{\tilde{a},\tilde{b}\}$ = {(-1.80,-0.10), (-1.80,-0.15), (-1.80,-0.30), (-2.00,-0.10), (-2.00,-0.20), (-2.00,-0.30),(-2.20,-0.10), (-2.20,-0.30), (-2.20,-0.50)}

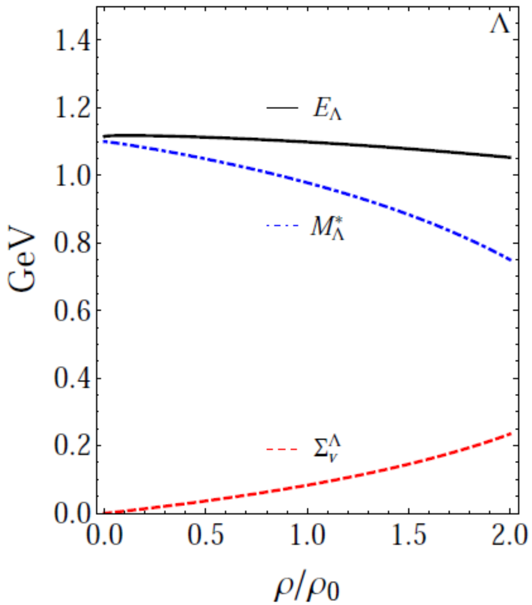
Sum rule result II – Λ hyperon

- Λ sum rules with new interpolating field

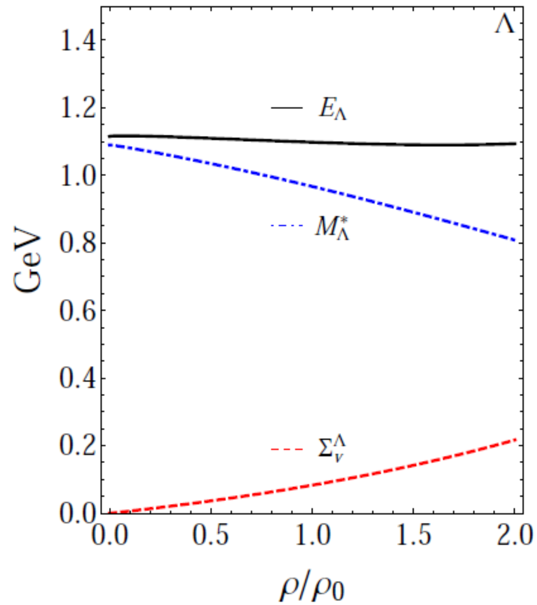
Λ sum rules



Constant negative pole



Density dependent negative pole



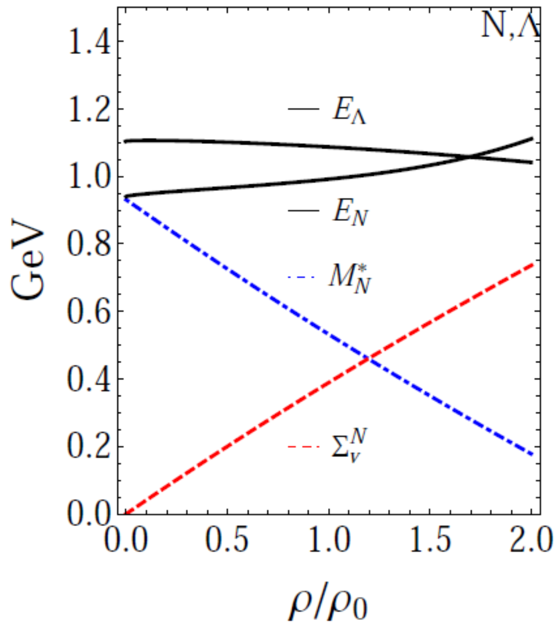
1. The quasi-Λ energy is slightly reduced → represents bounding at normal nuclear density
2. Weak attraction and weak repulsion → **scalar**: $V_{s\Lambda}$ / V_{sN} ~ 0.31 **vector**: $V_{v\Lambda}$ / V_{vN} ~ 0.26
   → naïve quark counting for determination of N-H force strength may not be good
3. Constant negative anti-Λ pole case (2nd graph) and density dependent case (3rd graph)

$$\bar{E}_q = \Sigma_v(\bar{E}_q) - \sqrt{\vec{q}^2 + M^*(\bar{E}_q)^2}$$ (anti-Λ pole)

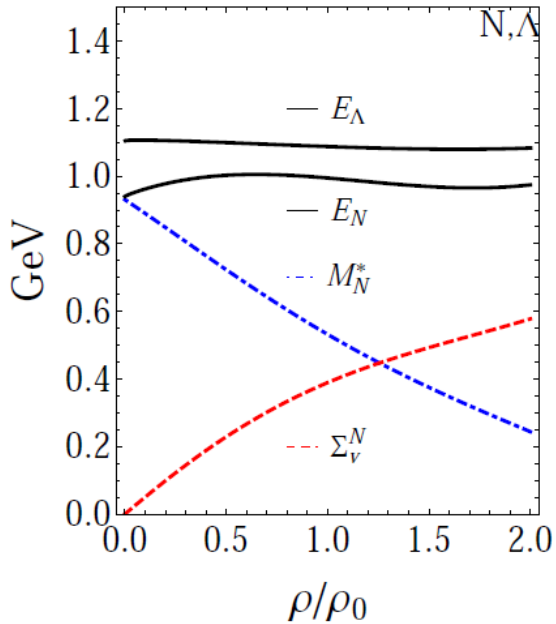
Sum rule result III – density behavior

- Comparison of density behavior (neutron matter)

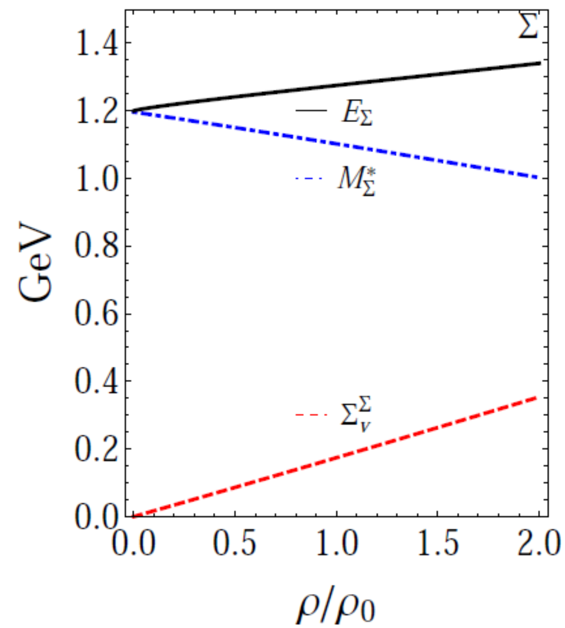
Constant negative pole



Density dependent negative pole



Density dependent negative pole (Σ_+)



1. Constant negative pole case: the quasi energy of **Λ** and **neutron** crosses at $\rho/\rho_0 = 1.8$
2. Density dependent case: never crosses
3. In **Σ_+** sum rules, there is only small difference between constant- and density dependent-case
4. **Within new interpolating field for Λ, the early onset of the hyperon in the dense nuclear matter is unlikely**

Outline

I. Motivations – phenomenology with iso-spin asymmetry

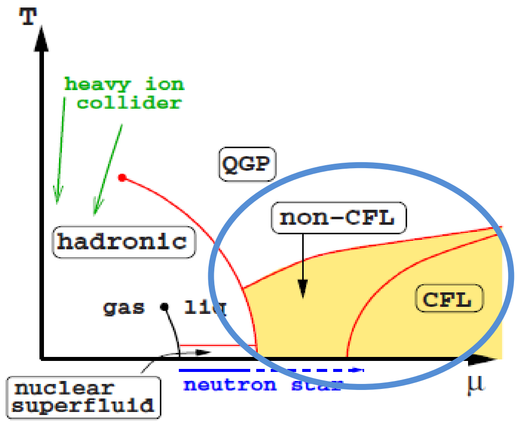
II. QCD approaches – QCD sum rules
- Symmetry energy
- Nucleon and hyperons

III. QCD approaches – dense QCD in cold limit
- **HDL resummation**
- **2-color superconductivity**

IV. Future prospects

At extremely high density?

- QCD phase transition



$E \sim \mu$	Hard scale (separation point between slow-fast modes)
$E \sim g\mu$	Soft scale for screening and damping
$E \sim \mu \exp(-1/g)$	Gap scale for color superconductivity
$E \sim \mu \exp(-1/g^2)$	Non-Fermi liquid effect
	Fermi surface $p_F = \mu$ > Λ_{QCD}

In $q \geq \mu > \Lambda_{QCD}$ region, **QCD** can be directly applicable

Static quantities can be obtained from partition function for dense QCD

$$\mathcal{Z}_\Omega = \mathrm{Tr}\exp\left[-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})\right] = \int \mathcal{D}(\bar{\psi}, \psi, A, \eta)\exp\left[-\int_0^\beta d\tau \int d^3x \mathcal{L}_E(\bar{\psi}, \psi, A, \eta)\right]$$

Dense QCD Lagrangian (Euclidean)

$$\mathcal{L}_E = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{\eta}^a(\partial^2\delta_{ab} + g f_{abc}\partial_\mu A^c_\mu)\eta^b + \sum_f^{n_f}\left[\psi_f^\dagger \partial_\tau \psi_f + \bar{\psi}_f(-i\gamma^i\partial_i + m_f)\psi_f - \mu_f \psi_f^\dagger \psi_f - g\bar{\psi}_f \not{A}\psi_f\right]$$

Full QCD and effective approach within scale hierarchy

Effective approach for cold dense matter

- Lagrangian near Fermi surface (analogous with NRQCD)

Dirac equation in dense medium

$$p_0\psi \;=\; (\vec{\alpha}\cdot\vec{p}-\mu)\,\psi \qquad\qquad p_0 = E_\pm \equiv -\mu \pm |\vec{p}\,|$$

Subtracting out dense-Fermi momentum and projecting energy eigenstate

$$\psi(x) = {\sum_{\vec{v}}}' e^{-i\mu v\cdot x}\left[\psi_+(x)+\psi_-(x)\right] \qquad \psi_\pm \;=\; P_\pm\psi \;\equiv\; \frac{1}{2}\left(1\pm\frac{\vec{\alpha}\cdot\vec{p}}{|\vec{p}|}\right)\,\psi$$

Matter part can be written as

$$\begin{aligned}\mathcal{L}_{\text{quark}} &= \bar{\psi}(x)\big(\mathrm{i}\not{D}+\mu\gamma^0\big)\psi(x)\\ &= \sum_{\vec{v}_F}\big[\bar{\psi}_+(\vec{v}_F,x)\mathrm{i}\gamma^0 V\cdot D\psi_+(\vec{v}_F,x)+\bar{\psi}_-(\vec{v}_F,x)\gamma^0(2\mu+\mathrm{i}\bar{V}\cdot D)\psi_-(\vec{v}_F,x)\\ &\quad+\bar{\psi}_-(\vec{v}_F,x)\mathrm{i}\gamma_\perp^\mu D_\mu\psi_+(\vec{v}_F,x)+\bar{\psi}_+(\vec{v}_F,x)\mathrm{i}\gamma_\perp^\mu D_\mu\psi_-(\vec{v}_F,x)\big]\end{aligned}$$

By using equation of motion, fast mode can be integrated out

$$\psi_- = -\frac{i}{2\mu+i\tilde{V}\cdot D}\gamma_0\,\not{D}_\perp\,\psi_+ \qquad \boxed{\mathcal{L}_D = {\sum_{\vec{v}}}'\left[\psi^\dagger iV\cdot D\psi-\psi^\dagger\frac{1}{2\mu+i\tilde{V}\cdot D}\not{D}_\perp^2\psi\right]}$$

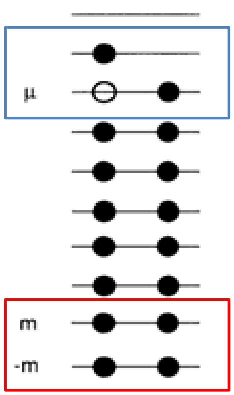
Where $\not{D}_\perp = \gamma_\perp^\mu D_\mu$ with $\gamma_\perp^\mu = \gamma^\mu - \gamma_\parallel^\mu$ and $\gamma_\parallel^\mu = (\gamma^0, \vec{v}_f\vec{v}_f\cdot\vec{\gamma})$, $V^\mu = (1,\vec{v}_f)$, $\tilde{V}^\mu = (1,-\vec{v}_f)$

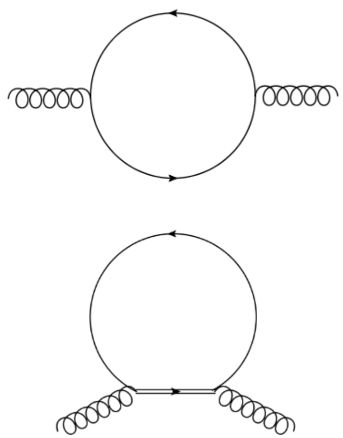
Matter loops in HDET

- Two matter loops

$$\mathcal{L}_D = {\sum_{\vec{v}}}' \left[\psi^\dagger i V \cdot D \psi - \psi^\dagger \frac{1}{2\mu + i\tilde{V}\cdot D} \not{D}_\perp^2 \psi \right]$$

For **soft** gluon propagation matter loop correction can be obtained as





$$\Pi_{\mu\nu}(Q)|_{\text{q-h}} = m^2 \int \frac{d\Omega}{4\pi} \hat{K}_\mu \hat{K}_\nu \left(-1 + \frac{i\omega}{Q\cdot\hat{K}} \right)$$

Quark-hole loop (slow mode loop)

$$\Pi_{\mu\nu}(Q)|_{\text{q-a}} = m^2 \int \frac{d\Omega}{4\pi} \left(\delta_{\mu 4}\delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \right)$$

Quark-antiquark loop (slow-fast mode loop)

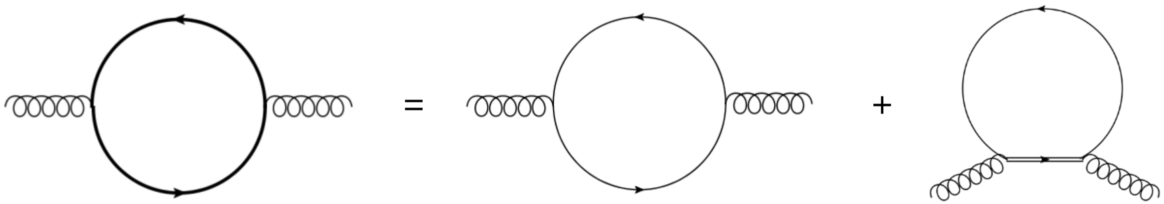
Double line denotes fast mode (propagating in Dirac sea)

Each quark-hole and quark-antiquark loop correction is not transverse

Sum of these diagrams constitutes transverse gauge-invariant matter loop known as **hard scale loop correction**

Hard dense loop

- Hard dense loop from full QCD (for soft external momenta)



Bold Fermion line denotes thermal quark propagator in full QCD

$$S_F(P) = \frac{m_f - \not{P}}{-(i\omega_n + \mu_f)^2 + \vec{p}^2 + m_f^2}$$

Loop integration is dominated by hard scale $\boldsymbol{\mu}$

$$\Pi^{ab}_{\mu\nu}(Q) = g^2\delta^{ab}\int \frac{d^4K}{(2\pi)^4}\mathrm{Tr}\left[\gamma_\mu S_F(K)\gamma_\nu S_F(K-Q)\right]$$
$$= m^2\delta^{ab}\int \frac{d\Omega}{4\pi}\left(\delta_{\mu 4}\delta_{\nu 4} + \hat{K}_\mu \hat{K}_\nu \frac{i\omega}{Q\cdot\hat{K}}\right),$$

$$T\sum_n \int \frac{d^3k}{(2\pi)^3} S_F(Q-K)S_F(K)$$
$$\sim \int d\Omega \int \frac{dk k^2}{(2\pi)^3}\tilde{n}_\pm(k)\frac{1}{E_1E_2}\frac{k}{k^2}\frac{k}{(k-q)^2} \sim \mu^2$$

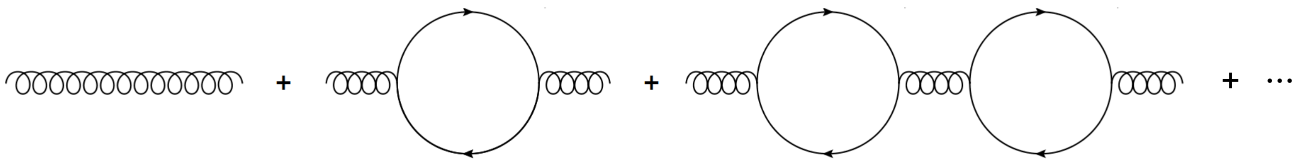
$$m^2 = \frac{1}{3}g^2T^2\left(C_A + \frac{1}{2}n_f\right) + \frac{1}{2}g^2\sum_f \frac{\mu_f^2}{\pi^2}$$

Relevant in cold limit

PRD53 (1996) 5866 C. Manuel
PRD48 (1993) 1390 J. P. Blaizot and J. Y. Ollitrault

Hard dense loop

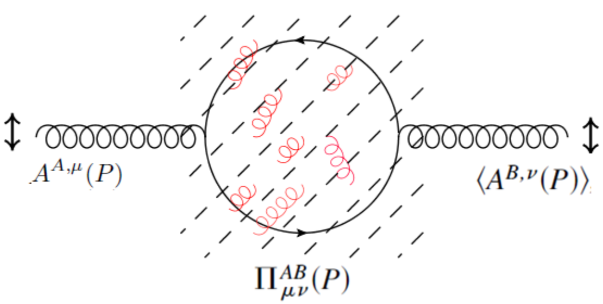
- Gluon propagator is saturated at soft scale (*Q~gμ*)



$$\sim \frac{1}{(g\mu)^2} \qquad \sim \frac{1}{(g\mu)^2} g^2\mu^2 \frac{1}{(g\mu)^2} \qquad \sim \frac{1}{(g\mu)^2} g^2\mu^2 \frac{1}{(g\mu)^2} g^2\mu^2 \frac{1}{(g\mu)^2}$$

In cold dense matter, soft gauge interaction should be **resummed**
Resummation of matter loop generates iso-spin dependence of interaction

- Gluon polarization as a kernel for linear response



$$\begin{aligned} J^{A,\mathrm{ind}}_{\mu}(P) &= J^{A,\mathrm{tot}}_{\mu}(P) - \mathbf{J}^{A}_{\mu}(P) \\ &= i[(D^{-1})^{AB}_{\mu\nu}(P) - (\mathcal{D}^{-1})^{AB}_{\mu\nu}(P)]\langle A^{B,\nu}(P)\rangle \\ &\equiv \Pi^{AB}_{\mu\nu}(P)\langle A^{B,\nu}(P)\rangle, \\ \langle A^{A}_{\mu}(P)\rangle &= -i\mathcal{D}^{AB}_{\mu\nu}(P)\mathbf{J}^{\nu,B}(P) \end{aligned}$$

Soft gluon contains collective iso-spin information of the matter

Polarization

- Projection along polarization

Euclidean propagator

$$*D_{\mu\nu} = \frac{1}{Q^2+\delta\Pi^L} P^L_{\mu\nu} + \frac{1}{Q^2+\delta\Pi^T} P^T_{\mu\nu} + \frac{1}{f_e}\frac{Q_\mu Q_\nu}{Q^2}$$

$$P^T_{ij} = \delta_{ij} - \hat{q}_i\hat{q}_j, P^T_{44} = P^T_{4i} = 0$$

$$P^L_{\mu\nu} = \delta_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} - P^T_{\mu\nu}$$

Longitudinal and transverse part

In ω->0 limit

$$\delta\Pi^L = \sum_f \left(\frac{1}{2}g^2\frac{\mu_f^2}{\pi^2}\right)\frac{Q^2}{q^2}\left(1-\left(\frac{i\omega}{q}\right)Q_0\left(\frac{i\omega}{q}\right)\right) \quad \Rightarrow \quad m^2 = \frac{1}{2}g^2\sum_f \frac{\mu_f^2}{\pi^2}$$

$$\delta\Pi^T = \frac{1}{2}\sum_f \left(\frac{1}{2}g^2\frac{\mu_f^2}{\pi^2}\right)\left(\frac{i\omega}{q}\right)\left[\left(1-\left(\frac{i\omega}{q}\right)^2\right)Q_0\left(\frac{i\omega}{q}\right)+\left(\frac{i\omega}{q}\right)\right] \quad \Rightarrow \quad 0$$

$$Q_0(x) = \tfrac{1}{2}\ln\left[(x+1)/(x-1)\right]$$

- Debye mass and effective description

Effective Lagrangian for static limit

$$\mathcal{L} = -\frac{1}{4}F^2 \to \frac{1}{2}A_\mu(-Q^2 g^{\mu\nu} + m^2 P_L^{\mu\nu} + O(\omega/q)P_T^{\mu\nu} + \cdots)A_\nu$$

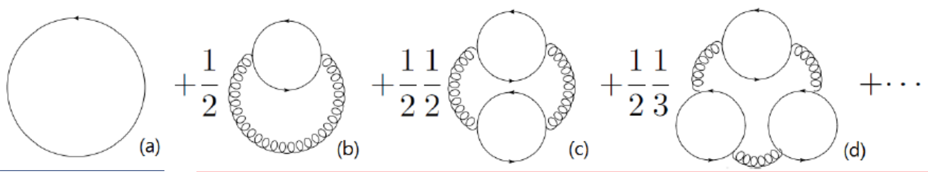
Iso-spin dependence mainly comes from **Debye mass** of hard (dense) matter loop

HDL resummed thermodynamic potential

- Free energy from partition function

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \hat{\vec{N}} \rangle = -\frac{1}{\beta} \ln \mathcal{Z}_\Omega \qquad Z_\Omega \sim \exp\left[\sum \text{connected diagrams}\right]$$

- Relevant ring diagrams in HDL resummation



At $T{\sim}Q{\sim}g\mu$, gluon is saturated
In other region, the integrand decays fastly

$$\ln \mathcal{Z}_{\Omega_{q,0}} \simeq \beta V \left(\frac{N_c}{12} \sum_{f=u,d} \frac{\mu_f^4}{\pi^2} \right)$$

Ideal quark gas

$$\begin{aligned}\ln \mathcal{Z}_{\Omega_{g,\mathrm{HDL}}} &= -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \Pi_{\mu\nu}(Q) D_F^{\nu\mu}(Q)\right] \\ &= -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \left(\ln\left[1 + \delta\Pi^L(Q)\frac{1}{Q^2}\right] + 2\ln\left[1 + \delta\Pi^T(Q)\frac{1}{Q^2}\right] \right)\end{aligned}$$

$$\mathcal{L} \equiv -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + m^2 \left(1 - \frac{i\omega}{2q} \ln \frac{i\omega + q}{i\omega - q}\right) \frac{1}{q^2}\right],$$

$$\mathcal{T} \equiv -\frac{(N_c^2-1)}{2} \beta V \int \frac{d^4Q}{(2\pi)^4} \ln\left[1 + \frac{m^2}{2}\left(\frac{i\omega}{q}\right)\left[\left(1 - \left(\frac{i\omega}{q}\right)^2\right) Q_0\left(\frac{i\omega}{q}\right) + \left(\frac{i\omega}{q}\right)\right] \frac{1}{Q^2}\right]$$

HDL resummed thermodynamic potential

- After regularization

$$\mathcal{L} = (N_c^2-1)\beta V \frac{1}{(2\pi)}\frac{d\Omega_3}{(2\pi)^3}\frac{(m^2)^2}{4}\left[\left(1-\ln\frac{m^2}{\pi\mu_4^2}\right)\alpha-\beta+\frac{1}{\epsilon}\alpha\right]$$

Longitudinal mode is important

$$\Rightarrow \beta V\left[\alpha_s^2\frac{2}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln 2-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)-\ln\alpha_s\right)\alpha-\beta\right]\right]_{\text{finite}}$$

$$\alpha = 0.321336$$
$$\beta = -0.176945$$

$$\mathcal{T} = (N_c^2-1)\beta V \frac{1}{(2\pi)}\frac{d\Omega_3}{(2\pi)^3}\frac{(m^2)^2}{8}\left[\left(1-\ln\frac{m^2}{2\pi\mu_4^2}\right)\frac{1}{2}\bar{\alpha}-\frac{1}{2}\bar{\beta}+\frac{1}{2\epsilon}\bar{\alpha}\right]$$

$$\Rightarrow \beta V\left[\alpha_s^2\frac{1}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)-\ln\alpha_s\right)\frac{1}{2}\bar{\alpha}-\frac{1}{2}\bar{\beta}\right]\right]_{\text{finite}}$$

$$\bar{\alpha} = 0.142727$$
$$\bar{\beta} = -0.200869$$

- Total logarithm

$$\ln\mathcal{Z}_\Omega = \beta V\left(\frac{1}{4}\sum_{f=u,d}\frac{\mu_f^4}{\pi^2}\left[1-4\left(\frac{\alpha_s}{\pi}\right)+\left(\frac{8}{3}-\frac{4}{9}\pi^2\right)\left(\frac{\alpha_s}{\pi}\right)^2\right]\right.$$
$$\left.+\alpha_s^2\frac{2}{\pi}\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\right)^2\left[\left(1-\ln\alpha_s-\ln\left(\sum_{f=u,d}\frac{\mu_f^2}{\pi^2}\frac{1}{\mu_4^2}\right)\right)\Lambda_1-\Lambda_2-\alpha\ln 2\right]\right)$$

→ Quark resummation (optionally considered)

$$\Lambda_1 \equiv \alpha+\tfrac{1}{2}\bar{\alpha}$$
$$\Lambda_2 \equiv \beta+\tfrac{1}{2}\bar{\beta}$$

HDL resummed symmetry energy

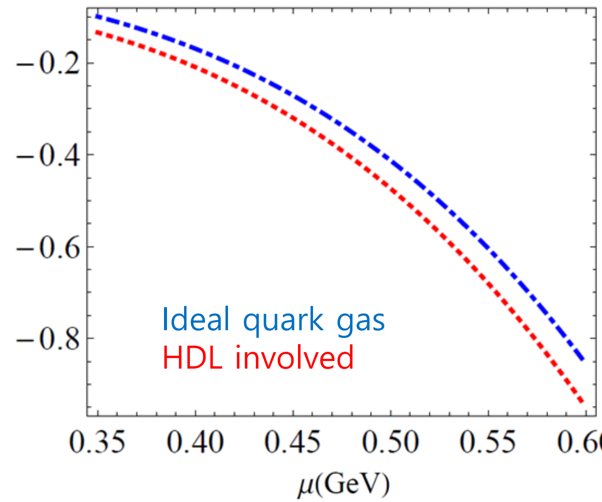
- Thermodynamic quantities can be obtained from $\Omega(\mu)$

$$\Omega(\mu) = \langle \hat{H} \rangle - \vec{\mu} \cdot \langle \hat{\vec{N}} \rangle = -\frac{1}{\beta V} \ln \mathcal{Z}_\Omega,$$

$$\rho_i(\mu) = \frac{\langle \hat{N}_i \rangle}{V} = \frac{1}{\beta V} \frac{\partial}{\partial \mu_i} \ln \mathcal{Z}_\Omega,$$

$$\epsilon(\mu) = \frac{\langle \hat{H} \rangle}{V} = -\frac{1}{V} \left(\frac{\partial}{\partial \beta} - \frac{1}{\beta} \vec{\mu} \cdot \frac{\partial}{\partial \vec{\mu}} \right) \ln \mathcal{Z}_\Omega,$$

$$I_B = \frac{\rho_3}{\rho_B} = 3 \frac{\rho_u - \rho_d}{\rho_u + \rho_d} \qquad \mu^u_d = \mu \left(1 \pm \frac{1}{3} I_B \right)^{\frac{1}{3}}$$



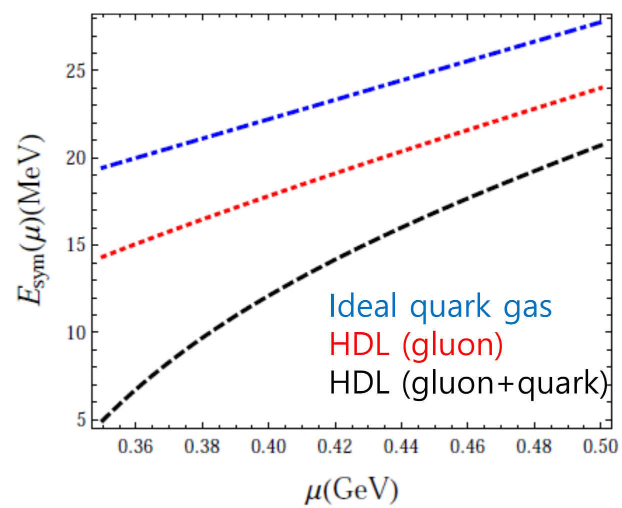
- **Quark matter symmetry energy**

$$\frac{\epsilon(\mu, I_B)}{\rho(\mu, I_B)} = \frac{E(\mu, I_B)}{N_B} = \bar{E}(\mu, I_B) = \bar{E}(\mu) + \bar{E}_{sym}(\mu) I_B^2 + \cdots$$

$$\bar{E}_{sym}(\mu) = \frac{1}{2} \frac{\partial^2}{\partial I_B^2} \bar{E}(\mu, I_B)$$

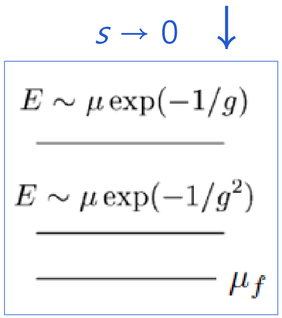
$$= \widetilde{E}^{q,0}_{sym}(\mu) + \underline{\widetilde{E}^{g,HDL}_{sym}(\mu)} \quad < 0$$

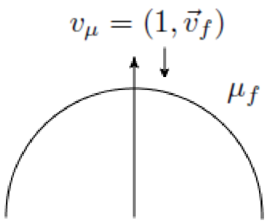
With gauge interaction, the symmetry energy becomes **even smaller**

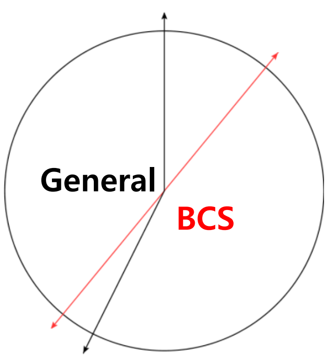


At extremely low temperature

- **At T~0 limit,** quark is mainly confined near Fermi sea







If one scales longitudinal momentum to near Fermi surface

$$\int d^4p \to \mu_f^2 \int d\Omega \int dl^2 s^2 \quad \text{where} \quad l = (l_0, (\vec{l} \cdot \vec{v}_f)\vec{v}_f)$$

Free fermion part should be invariant under scaling

$$\int d^2 l s^2 \psi^\dagger_{\vec{v}_f} s(l_0 - l_\parallel)\psi_{\vec{v}_f} \quad \to \quad \psi \sim s^{-\frac{3}{2}}$$

Four-quark interaction

General scattering

$$\int \Pi_i^4 \left(dk_\perp^2 dl^2\right)_i \left[\psi^\dagger(k_3)\psi(k_1)V(k)\psi^\dagger(k_4)\psi(k_2)\right] \delta(k_1 + k_2 - (k_3 + k_4))$$

scales as s^2 : irrelevant in $s \to 0$ scaling

Interaction **between opposite velocity (BCS type)**

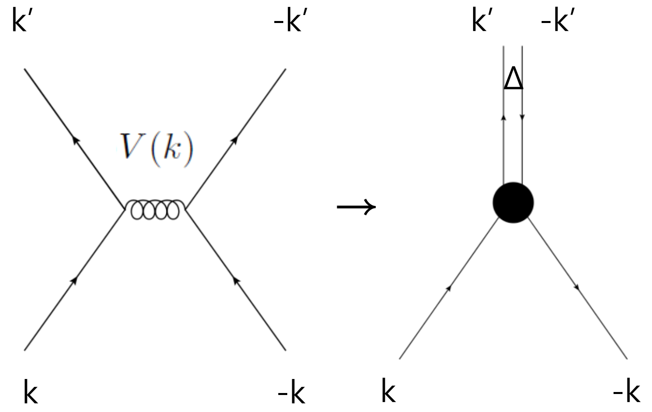
$$\int \Pi_i^4 \left(dk_\perp^2 dl^2\right)_i \left[\psi^\dagger(k_3)\psi(k_1)V(k)\psi^\dagger(-k_3)\psi(-k_1)\right] \delta(l_1 + l_2 - (l_3 + l_4))$$

scales as s^0: **marginal** in $s \to 0$ scaling

In QCD, there is no relevant interaction which scales as s^{-n}
→ **BCS** type interaction becomes most important at scaling

Color BCS paired states

- **4 quark interaction in QCD** (N_c=3)



Anti-triplet channel is attractive (V<0)

$$\tau^a_{ij}\tau^a_{kl} = \frac{1}{6}(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}) - \frac{1}{3}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})$$

→ BCS condensation in low energy limit

To take entire Fermi surface, spin-0 condensate is favored → in same helicity (asymmetric in spin)

For asymmetric wave function as for fermion, flavor should be in asymmetric configuration

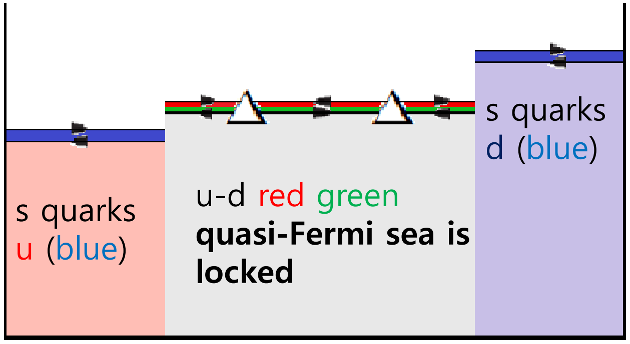
In non negligible M_s^2/μ , **2SC** state is favored

$$\langle \psi^\alpha_a C\gamma_5 \psi^\beta_b \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}$$

$$\langle \psi^T_{L,\alpha i} C\psi_{L,\beta j}\rangle = -\langle \psi^T_{R,\alpha i} C\psi_{R,\beta j}\rangle = \frac{\Delta}{2}\epsilon_{\alpha\beta 3}\epsilon_{ij3}$$

$$\mathcal{L}_\Delta = -\frac{\Delta}{2}\psi^T_L C\epsilon\psi_L\epsilon - (L \to R) + \text{h.c.}$$

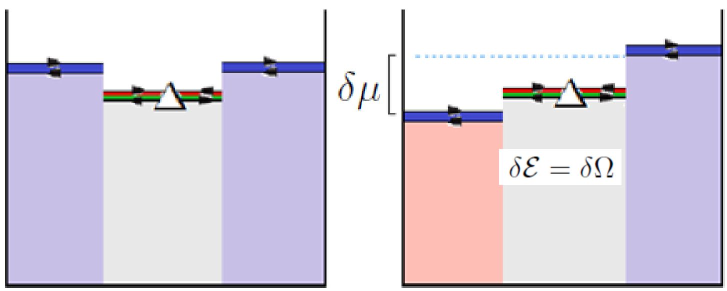
- Modification of Fermi-sea



In 2SC phase, u-d red-green states are trapped in gap and only s quarks and u-d blue quarks can be liberated

Asymmetrization in 2SC phase

- **Only** **Blue state (1/3)** can affect iso-spin asymmetry



BCS phase remains in $\delta\mu < (1/\sqrt{2})\Delta \sim \Lambda$

(PRL9 (1962) 266 A. M. Clogston)

Only u-d blue states can be asymmetric

The other 4-gapped quasi-states are **locked**
$(I_{\tilde{B}} = I_B/3)$

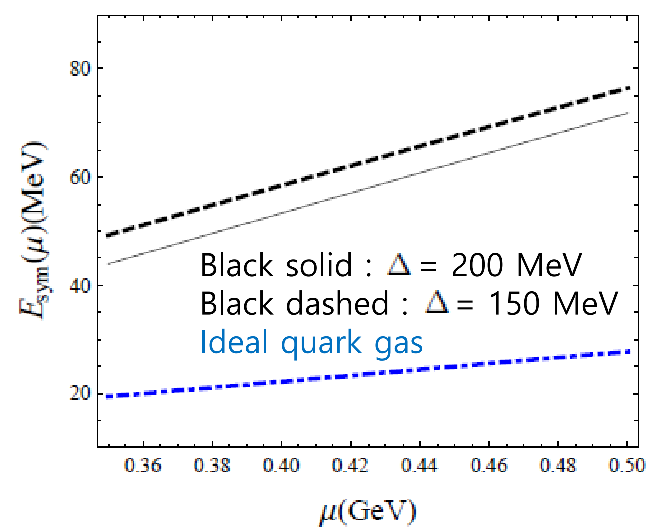
- **Symmetry energy in 2SC** (only non-perturbative gap is considered)

$$\Omega_\Delta(\mu) \simeq -\frac{1}{12}\sum^{N_c}\sum_{f=u,d}\frac{\mu_f^4}{\pi^2} - \sum_i^{2SC}\frac{\mu_i^2\Delta^2}{4\pi^2},$$

$$\rho_i(\mu) = \frac{1}{3}\frac{\mu_i^3}{\pi^2}, \quad \rho_{\Delta i}(\mu) = \frac{1}{3}\frac{\mu_i^3}{\pi^2} + \frac{\mu_i\Delta^2}{2\pi^2},$$

$$\begin{aligned}\epsilon_\Delta(\mu) &= \epsilon_{\text{unpaired}}(\mu) + \epsilon_{\text{paired}}(\mu) \\ &= \frac{1}{4}\sum_i^{\text{unpaired}}\frac{\mu_i^4}{\pi^2} + \frac{1}{4}\sum_i^{2SC}\left[\frac{\mu_i^4}{\pi^2} + \frac{\mu_i^2\Delta^2}{\pi^2}\right]\end{aligned}$$

$$\frac{\epsilon(\mu, I_{\tilde{B}})}{\rho_{\tilde{B}}(\mu, I_{\tilde{B}})} = \bar{E}(\mu, I_{\tilde{B}}) \quad E_{sym}^{2SC}(\mu) = \frac{1}{2!}\frac{\partial^2}{\partial I_{\tilde{B}}^2}\bar{E}(\mu, I_{\tilde{B}}),$$



Quasi-quark states in **2SC phase**

- **2SC** description in linear combination of Gellman matrices

Gapped (A=0,1,2,3) and un-gapped (A=4,5) quasi-state

$$\psi_{+,\alpha i} = \sum_{A=0}^{5} \frac{(\tilde{\lambda}_A)_{\alpha i}}{\sqrt{2}} \psi_+^A \qquad \chi = \begin{pmatrix} \psi_+ \\ C\psi_-^* \end{pmatrix}$$

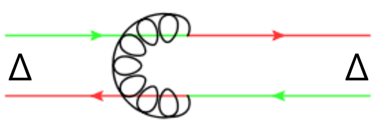
+ and – represents direction of Fermi velocity

$$\tilde{\lambda}_0 = \frac{1}{\sqrt{3}}\lambda_8 + \frac{2}{3}I;\ \tilde{\lambda}_A = \lambda_A\ (A = 1,2,3);\ \tilde{\lambda}_4 = \frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5);\ \tilde{\lambda}_5 = \frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7),$$

These Hermitian representations $(\tilde{\lambda}_A)_{\alpha i}$ are color(α)-flavor(i) matrix

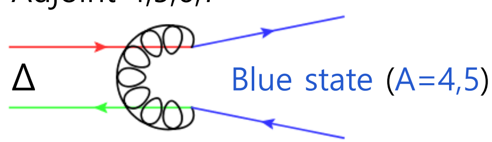
Color interaction can mediate transition of quasi-quark state

Adjoint 1,2,3



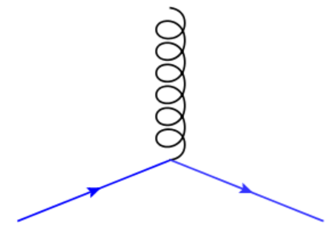
Gluon in adjoint 1,2,3 is trapped in gapped states

Adjoint 4,5,6,7



Gluon in adjoint 4,5,6,7 can dissolve gapped state into liberated state (Requires large momentum transf.)

Adjoint 4,5,6,7,8



Blue state (A=4,5)

The transition can be determined from factor $\kappa_{AaB} = \frac{1}{2}\mathrm{Tr}[\tilde{\lambda}_A \tau_a \tilde{\lambda}_B]$ in $\psi_{f+}^{\dagger}(iV \cdot D)\psi_{f+}$

Gluon polarization from HDET (2SC)

- Gluon rest masses from HDET Lagrangian

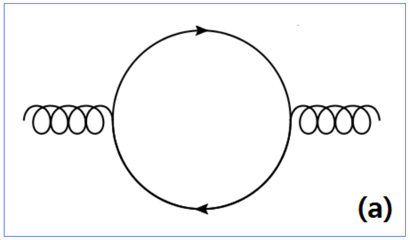
High density effective Lagrangian in Nambu-Gorkov form

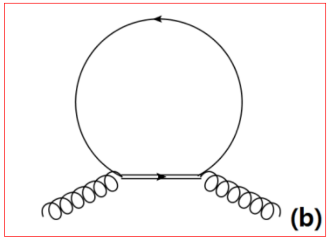
$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_{\vec{v}_f}^{\text{half}}\sum_{A,B=0}^{5}\Bigg[\chi^{A\dagger}\begin{pmatrix} iV\cdot\partial\delta_{AB} & \Delta_{AB} \\ \Delta_{AB} & i\bar{V}\cdot\partial\delta_{AB}\end{pmatrix}\chi^B + igA^a_\mu\chi^{A\dagger}\begin{pmatrix} iV^\mu\kappa_{AaB} & 0 \\ 0 & -i\bar{V}^\mu\kappa^*_{AaB}\end{pmatrix}\chi^B$$ **(a)**

$$+ g^2A^c_\mu A^d_\nu\chi^{A\dagger}\begin{pmatrix} \frac{1}{2\mu_f+i\bar{V}\cdot D}\xi^{cd}_{AB} & 0 \\ 0 & \frac{1}{2\mu_f+iV\cdot D^*}\xi^{cd*}_{AB}\end{pmatrix}P^{\mu\nu}\chi^B\Bigg] + (L\to R),$$ **(b)**

Where $P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}(V^\mu\bar{V}^\nu + V^\nu\bar{V}^\mu)$, $\kappa_{AaB} = \frac{1}{2}\text{Tr}[\tilde{\lambda}_A\tau_a\tilde{\lambda}_B]$, $\xi^{cd}_{AB} = \frac{1}{2}\text{Tr}[\tilde{\lambda}_A\tau_c\tau_d\tilde{\lambda}_B]$

$$\Delta_{AB} = \frac{\Delta}{2}\text{Tr}[\epsilon\sigma^T_A\epsilon\sigma_B] \quad (A,B=0,1,2,3,\ \text{otherwise}\ \Delta_{AB}=0)$$
$$= \Delta_A\delta_{AB} \quad \text{with} \quad \Delta_A = (-\Delta,\Delta,\Delta,\Delta,0,0)$$





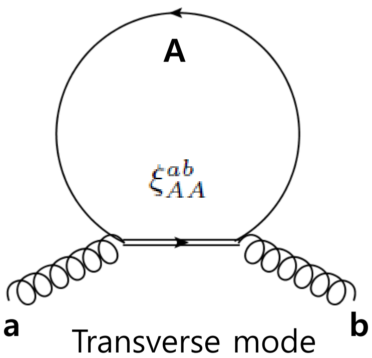
(a) Shares common iso-spin symmetric Fermi-sea

Only **(b)** has explicit iso-spin dependence

From gluon rest masses, effect of gauge interaction can be anticipated

Gluon rest masses in 2SC phase

- Relevant diagram for iso-spin asymmetry



	$a,b=1,2,3$	$a,b=4,5,6,7$	$a,b=8$
Paired (A=0,1,2,3)	$\xi^{ab}_{AA}=\frac{1}{2}\delta^{ab}$	$\xi^{ab}_{AA}=\frac{1}{4}\delta^{ab}$	$\xi^{ab}_{AA}=\frac{1}{6}\delta^{ab}$
Unpaired (A=4,5)	$\xi^{ab}_{AA}=0$	$\xi^{ab}_{AA}=\frac{1}{4}\delta^{ab}$	$\xi^{ab}_{AA}=\frac{1}{3}\delta^{ab}$

	$a,b=1,2,3$	$a,b=4,5,6,7$	$a,b=8$
$\Pi^{ab}_{00}(0)$	0	$\frac{1}{2}m^2\delta^{ab}$	$m^2\delta^{ab}$
$-\Pi^{ab}_{ij}(0)$	0	$\frac{1}{12}g^2\sum_f^{4,5}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$	$\frac{1}{9}g^2\sum_f^{4,5}(\mu_f^2/\pi^2)\delta_{ij}\delta^{ab}$

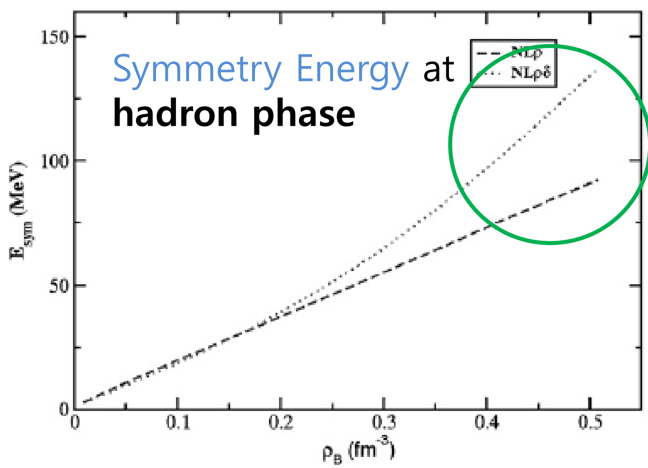
$m^2=(g^2\mu^2/\pi^2)$

- Loop factor ξ^{ab}_{AA} and rest masses
  - Tadpole can be obtained by calculating $\xi^{cd}_{AB}=\frac{1}{2}\mathrm{Tr}[\tilde{\lambda}_A\tau_c\tau_d\tilde{\lambda}_B]$,
  - Total counter term should be $\delta\Pi^{ab}_{ij}(0)=-\frac{1}{3}m^2\delta_{ij}\delta^{ab}$ (PRD62 (2000) 034007 D. H. Rischke)
  - Unbroken gluons in SU(2) (for gapped state) do not have rest masses
  - Only Meissner mass of broken gluon (adjoint 4,5,6,7,8) has iso-spin dependence
  - The portion is very small: minimal contribution to static quantities
  - For super-soft gluons, transverse mode can become important

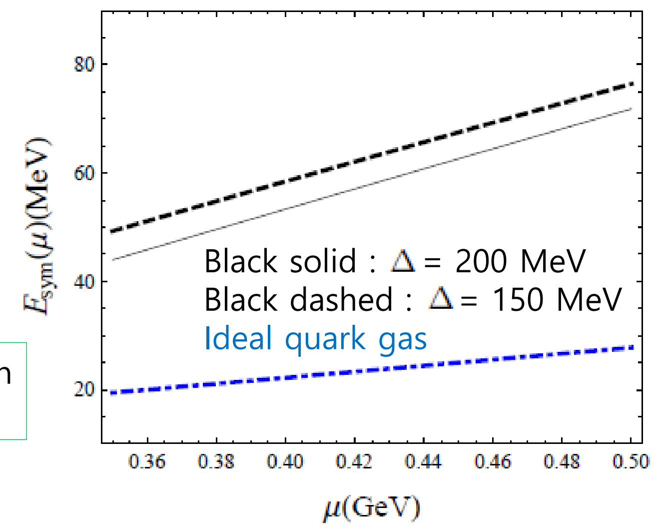
Anticipation for H-Q mixed phase

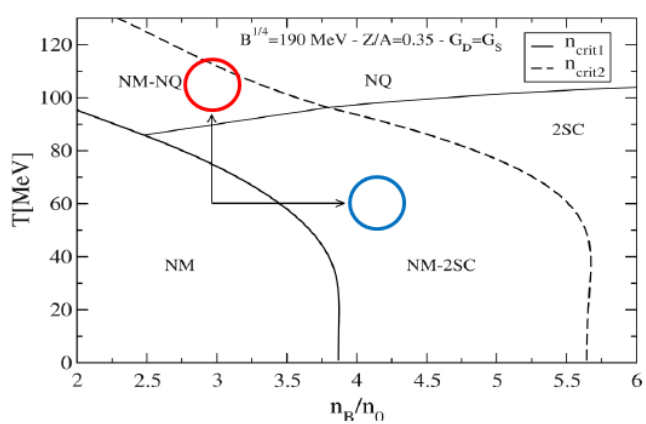
- Iso-spin distillation and π^-/π^+ ratio (in agreement with PRD81 (2010) 094024)

(Phys.Rept. 410 (2005) 335 V. Baran et al.)



Pushing neutron
→ Evaporation





Large symmetry energy leads iso-spin evaporation

$E_{\rm sym}^{\rm nuclear}(\mu) \gg E_{\rm sym}^{\rm quark}(\mu)$ (*NLρδ* model and this calculation)
→ Iso-spin distillation can occur at mixed phase

At **2SC phase** the distillation will be **reduced**

Eventually, π^-/π^+ ratio will be **reduced**

Can be an indirect evidence for **2SC** phase

Outline

I. Motivations – phenomenology with iso-spin asymmetry

II. QCD approaches – QCD sum rules

- Symmetry energy
- Nucleon and hyperons

III. QCD approaches – dense QCD

- HDL resummation
- 2-color superconductivity

IV. Future prospects

Naïve future plan – diquark

- Diquark paring pattern

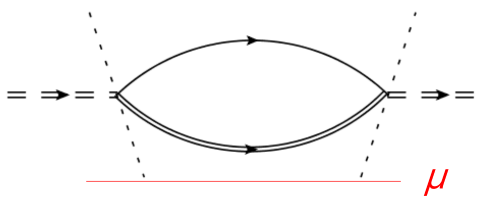
I. In Λ structure, scalar (*I=0*) light diquark structure should be emphasized

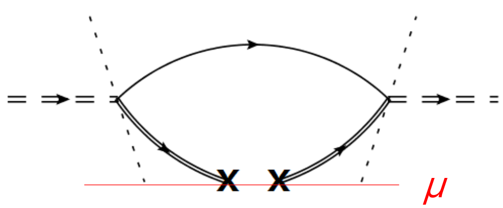
$$\eta_\Lambda = \epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + (\tilde{a} \leq -2)[u_a^T C\gamma_5 d_b]s_c + (\tilde{b} \sim -1/8)[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

II. 2SC BCS paring at cold dense matter

$$\langle \psi^T_{L,\alpha i} C\psi_{L,\beta j}\rangle = -\langle \psi^T_{R,\alpha i} C\psi_{R,\beta j}\rangle = \frac{\Delta}{2}\epsilon_{\alpha\beta 3}\epsilon_{ij3}$$

- The diquark structure corresponds to the light 4q ops.

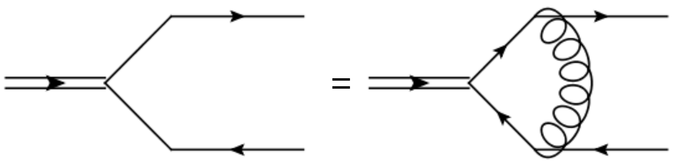




- Just above QCD scale ($q > \mu \sim \Lambda_{QCD}$), diquark may be weakly bounded state in perturbative interaction
- But below the scale ($q < \mu \sim \Lambda_{QCD}$), diquark contribution may mainly overlapped with four-quark condensates
- $\langle \bar{q}_{1a'}\Gamma q_{1a}\bar{q}_{2b'}\Gamma q_{2b}\rangle \simeq \langle \bar{q}_{1a'}\tilde{\Gamma}\bar{q}_{2b'}q_{1a}\tilde{\Gamma}q_{2b}\rangle$
- Scalar condensates correspond to `good' diquark (s=0) correlation
- Twist-4 condensates correspond to `bad' diquark (s=1) correlation

Naïve future plan – similarities

- Dense matter and heavy quark system



Singlet state can be obtained by solving Bethe-Salpeter equation

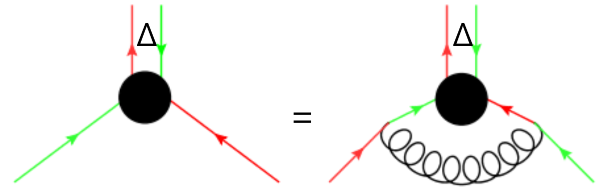
$$\Gamma_\mu(p_1,-p_2)=iC_{\text{color}}\int\frac{d^4k}{(2\pi)^4}g^2V(k)\gamma^\nu\Delta(p_1+k)$$
$$\times\Gamma_\mu(p_1+k,-p_2+k)\Delta(-p_2+k)\gamma_\nu$$

In non-relativistic and heavy mass limit

$$\Gamma_\mu(q/2+p,-q/2+p)$$
$$=-\left(\varepsilon-\frac{\mathbf{p}^2}{m}\right)\sqrt{\frac{M_\Phi}{N_c}}\psi(\mathbf{p})\frac{1+\gamma_0}{2}\gamma_i\delta_{\mu i}\frac{1-\gamma_0}{2}$$

Coulombic bound state

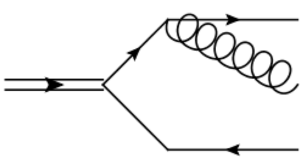
$$\left(\varepsilon-\frac{\mathbf{p}^2}{m}\right)\psi(\mathbf{p})=-g^2C_{\text{color}}\int\frac{d^3k}{(2\pi)^3}V(\mathbf{k})\psi(\mathbf{p}+\mathbf{k})$$



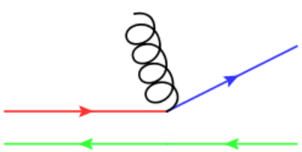
Large gap size can be obtained by solving gap equation with one gluon exchange

$$\Delta(k)=ig^2\int\frac{d^4q}{(2\pi)^4}\left(\gamma_\mu\frac{\lambda^a}{2}\right)^T S_{21}(q)\left(\gamma_\nu\frac{\lambda^a}{2}\right)D_{\mu\nu}(q-k)$$

$$\Delta/\mu=(b/g^5)\exp\left(-3\pi^2/\sqrt{2}g\right)\qquad b=256\pi^4$$



adj. 4,5,6,7



Singlet → Octet   Gap → Ungapped

External gluon attachment can dissolve the bound state

For color BCS state, Meissner mass screens dissociation of gapped state → requires large momentum transf.

Naïve future plan – effective Lagrangian

- Dense matter and heavy quark system
  - Wilsonian approach can be tried to describe good `diquark' quantum number
  - As a first step, one can try separation and matching scheme of heavy quark effective Lagrangian QCD → NRQCD → pNRQCD
- In the matching scheme, RG analysis will play important role
  - For now, diquark structure is just the candidate based on phenomenology
  - The important, relevant operators could be constrained via RG analysis
  - The relation between 4-quark condensate in the classical sum rules and the constituent quark models could be clarified in terms of the relevant degree of freedom in the effective Lagrangian for intermediate density region
  - New constraints for the nuclear matter and the consequence can be checked in the near future planned experiments such as FAIR, NICA, and RAON
- Thanks for attention