Chemical and kinetic freeze-outs in heavy ion collisions

Exploration for QCD phase diagram
HaPhy & HIM

May 26th 2017
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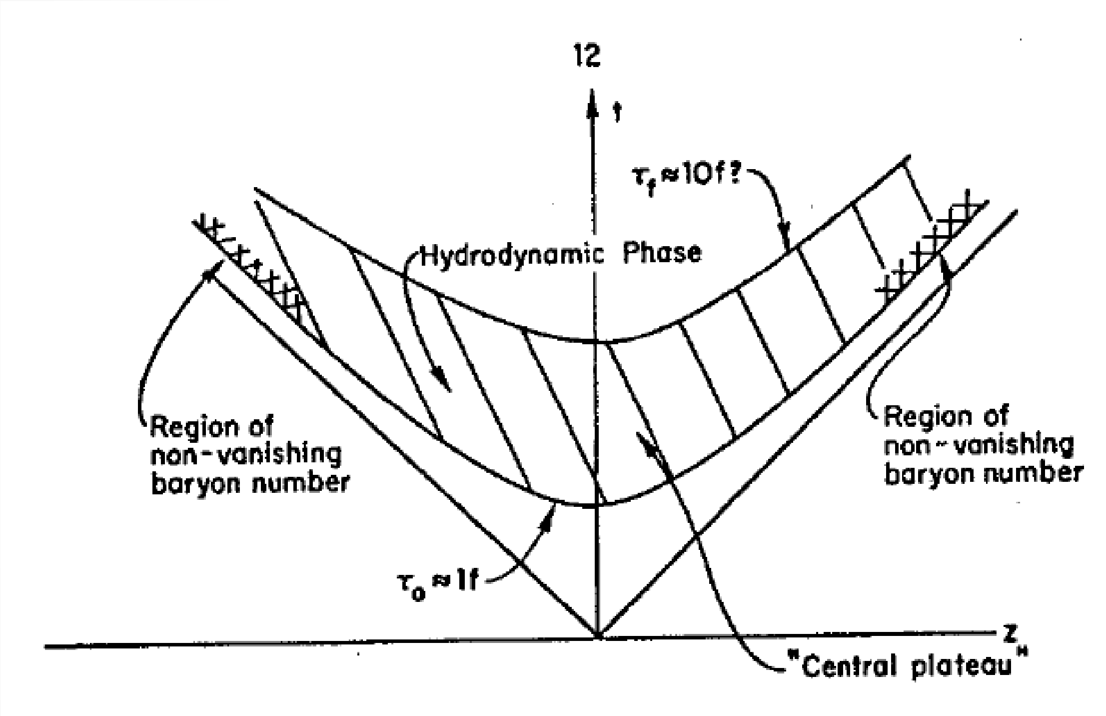
Outline

- Introduction
- Hadron production models
- Chemical freeze-out in heavy ion collisions
- Hadronic interactions
- Kinetic freeze-out in heavy ion collisions
- Conclusion

Introduction

– Time evolutions after heavy ion collisions



$$= \sqrt{t^2 - z^2}$$

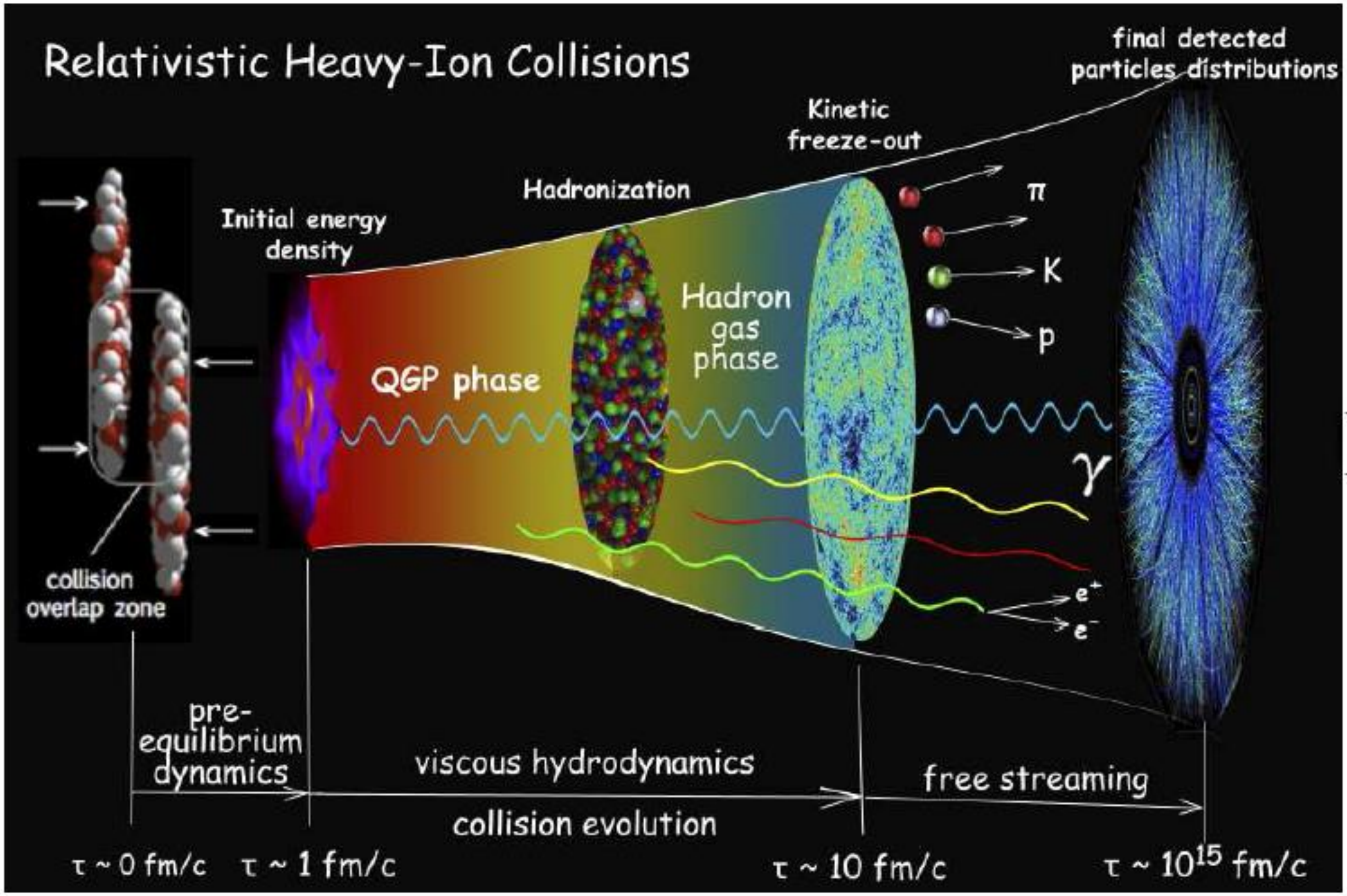
J. D. Bjorken,
Phys. Rev. D **27**, 140 (1983)

i. Collision ii. Pre-equilibrium state and Quark-gluon plasma

iii. Hydrodynamic expansion iv. Chemical freeze-out

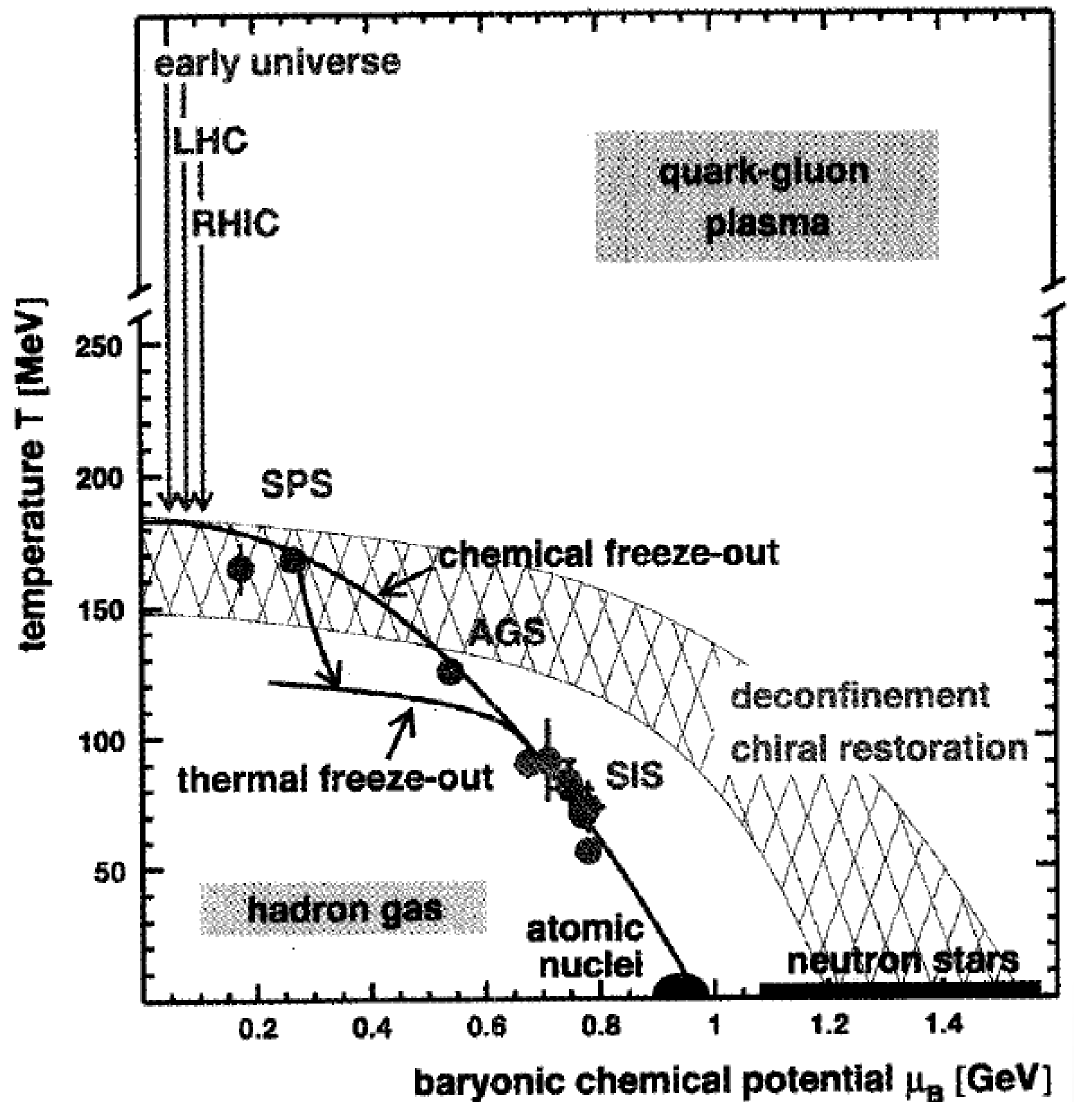
v. Kinetic freeze-out

– Relativistic heavy ion collisions



U. W. Heinz, J. Phys. Conf. Ser. **455**, 012044 (2013)

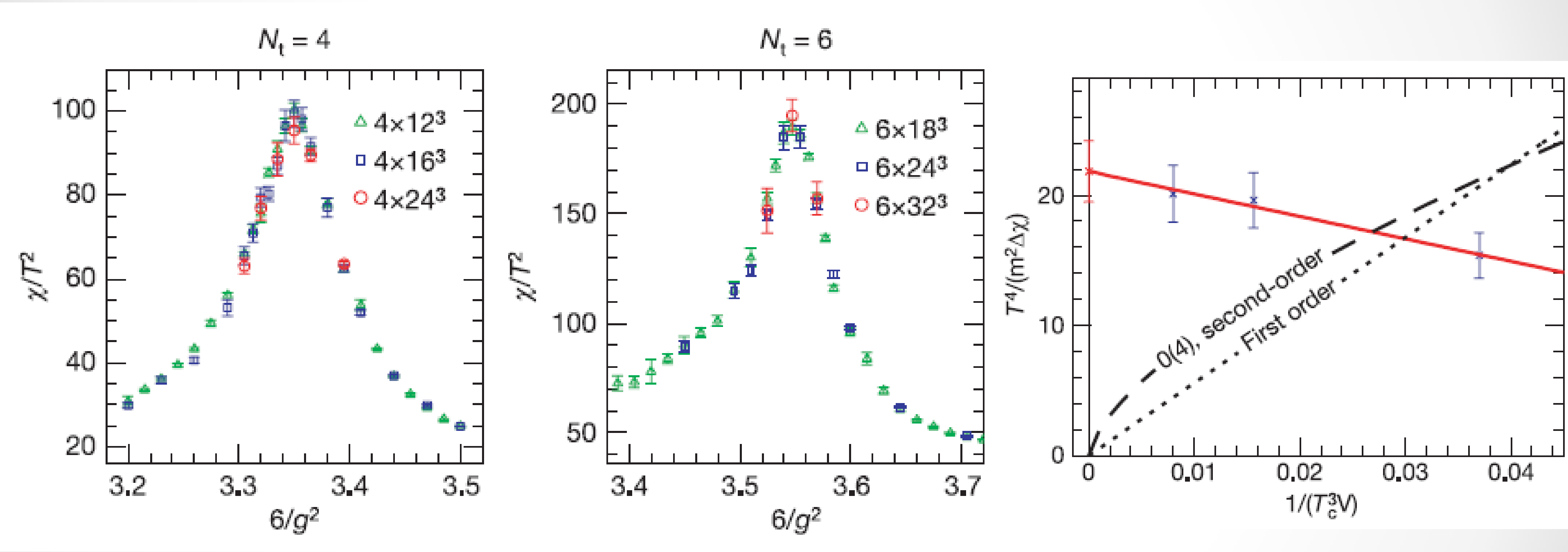
– The QCD Phase diagram



P. Braun-Munzinger and J. Stachel, Nucl. Phys. A **690**, 119c (2001)

– The QCD phase transition

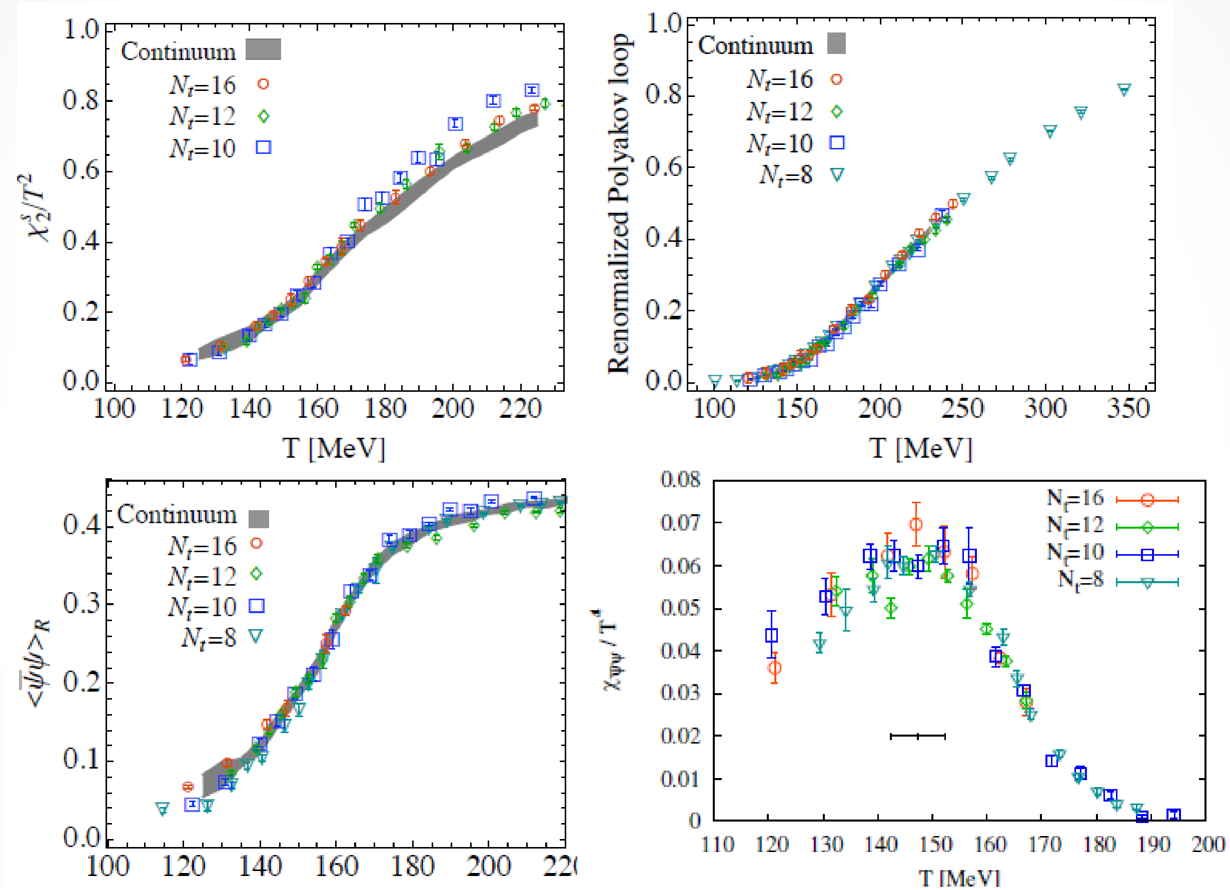
1) An analytic crossover in QCD



the chiral susceptibilities $\chi(N_s, N_t) = \partial^2/(\partial m_{ud}^2)(T/V)\log Z,$

Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, and K. K. Szabo, Nature, **443**, 675 (2006)

2) The various observables lead to different transition temperature, between 150 and 170 MeV



	$\chi_{\bar{\psi}\psi}/T^4$	$\Delta_{l,s}$	$\langle\bar{\psi}\psi\rangle_R$	χ_2^s/T^2	ϵ/T^4	$(\epsilon-3p)/T^4$
this work	147(2)(3)	157(3)(3)	155(3)(3)	165(5)(3)	157(4)(3)	154(4)(3)

S Borsanyi, Z. Fodor, C. Hoelbling, S.D. Katz,, S. Krieg, C. Ratti, and K. K. Szabo, JHEP, **09**, 073 (2010)

Hadron production models

– Statistical Hadronization model

P. Braun-Munzinger, J. Stachel, J. P. Wessels, N. Xu, Phys. Lett. **B344**, 43 (1995)

1) The particle production yield In a chemically and thermally equilibrated system of non-interacting hadrons and resonances,

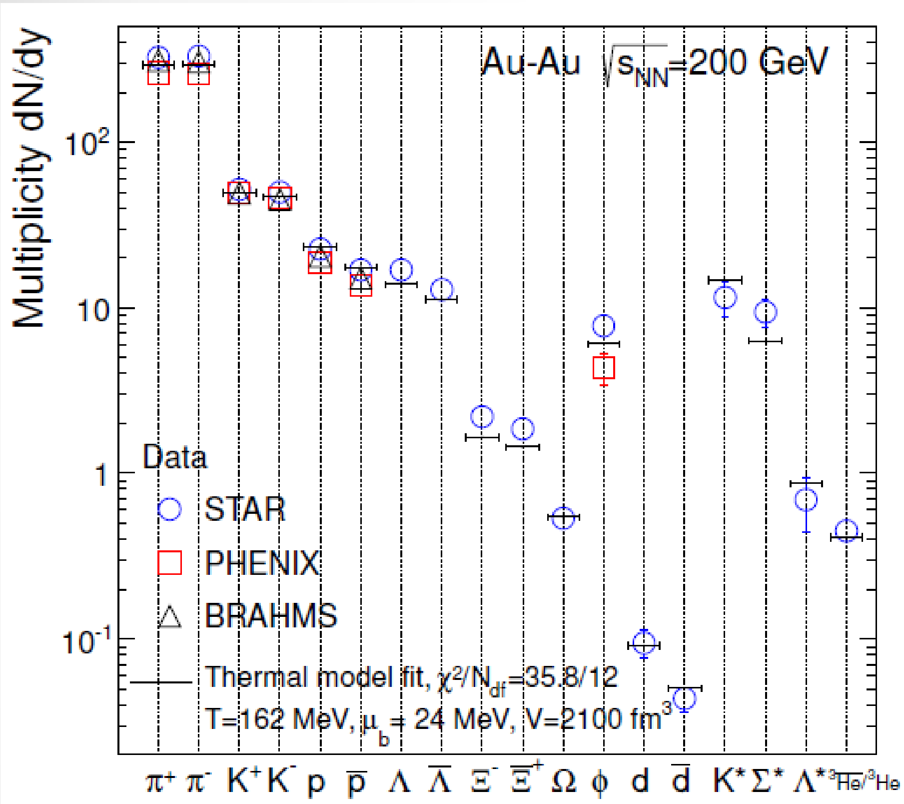
$$N_i = V_H \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\gamma_i^{-1} e^{E_i/T_H} \pm 1} \qquad E_i = \sqrt{m_i^2 + p_i^2}$$

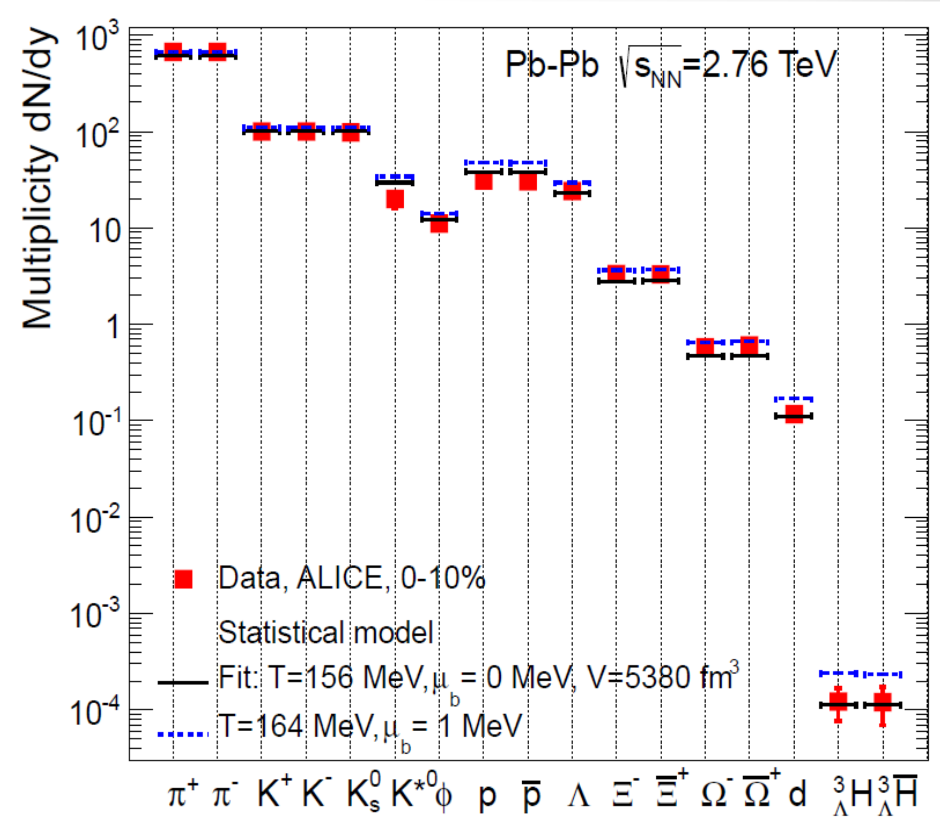
the fugacity for incomplete strange and charm quarks equilibrium

$$\gamma = \gamma_c^{n_c + n_{\bar{c}}} e^{[\mu_B n_B + \mu_s n_s]}$$

2) Two parameters, the hadronization temperature and the chemical potential determined from the experimental data

– Recent statistical model analysis





A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nucl. Phys. A **904-905**, 535c (2013)
J. Stachel, A. Andronic, P. Braun-Munzinger, and K. Redlich, J. Phys. Conf. Ser. **509**, 012019 (2014)

– Coalescence model

1) Yields of hadrons

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)
R. J. Freis. B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g\int\left[\prod_{i=1}^{n}\frac{1}{g_i}\frac{p_i\cdot d\sigma_i}{(2\pi)^3}\frac{d^3p_i}{E_i}f(x_i,p_i)\right]f^W(x_1,\cdots,x_n:p_1,\cdots,p_n)$$

with the Wigner function, the coalescence probability function

$$f^W(x_1,\cdots,x_n:p_1,\cdots,p_n)$$
$$=\int\prod_{i=1}^{n}dy_i e^{p_i y_i}\psi^*\left(x_1+\frac{y_1}{2},\cdots,x_n+\frac{y_n}{2}\right)\psi\left(x_1-\frac{y_1}{2},\cdots,x_n-\frac{y_n}{2}\right)$$

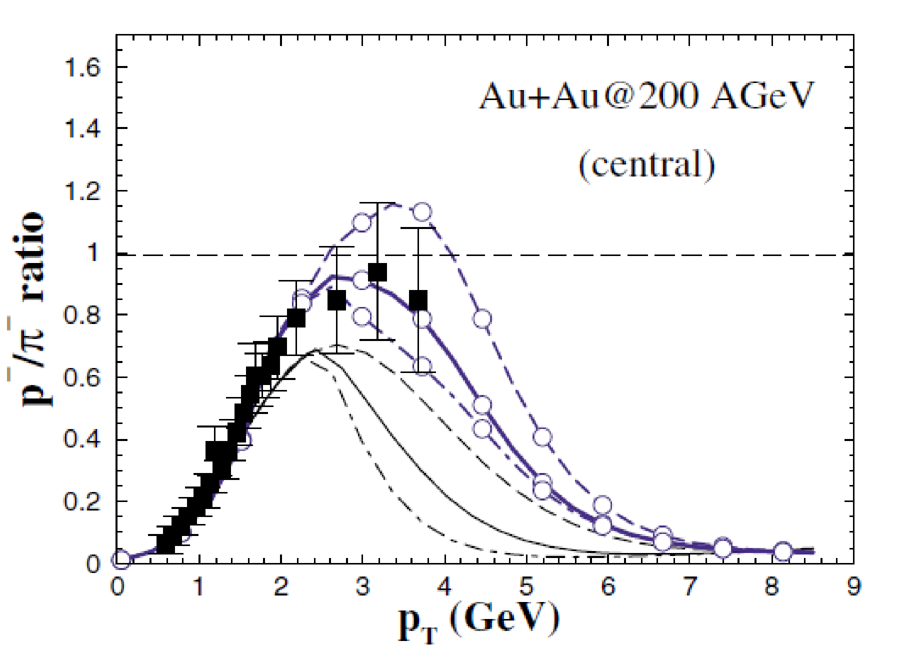
i. A Lorentz-invariant phase space integration of a space-like hypersurface constraints the number of particles in the system

$$\int p_i\cdot d\sigma_i\frac{d^3p_i}{(2\pi)^3E_i}f(x_i,p_i)=N_i$$

– Quark coalescence or quark recombination in heavy ion collisions

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. Lett. **90**, 202302 (2003)
R. J. Freis. B. Muller, C. Nonaka, and S. Bass, Phys. Rev. Lett. **90**, 202303 (2003)

1) The puzzle in antiproton /pion ratio



i. A competition between two particle production mechanisms exists
: A fragmentation dominates at large transverse momenta and a coalescence prevails at lower transverse momenta

2) The transverse momentum spectra

$$\frac{dN_M}{d^2\mathbf{p}_T} = g_M \frac{6\pi}{\tau \Delta y R_\perp^2 \Delta_p^3} \int d^2\mathbf{p}_{1T} d^2\mathbf{p}_{2T} \left.\frac{dN_q}{d^2\mathbf{p}_{1T}}\right|_{|y_1|\leq \Delta y/2} \left.\frac{dN_{\bar{q}}}{d^2\mathbf{p}_{2T}}\right|_{|y_2|\leq \Delta y/2}$$

$$\times \delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{1T} - \mathbf{p}_{2T}) \Theta\left(\Delta_p^2 - \frac{1}{4}(\mathbf{p}_{1T} - \mathbf{p}_{2T}) - \frac{1}{4}[(m_{1T} - m_{2T})^2 - (m_1 - m_2)^2]\right).$$

$$f_M(x_1, x_2; p_1, p_2) = \frac{9\pi}{2(\Delta_x \Delta_p)^3} \Theta\left(\Delta_x^2 - (x_1 - x_2)^2\right)$$

$$\times \Theta\left(\Delta_p^2 - \frac{1}{4}(p_1 - p_2)^2 + \frac{1}{4}(m_1 - m_2)^2\right).$$

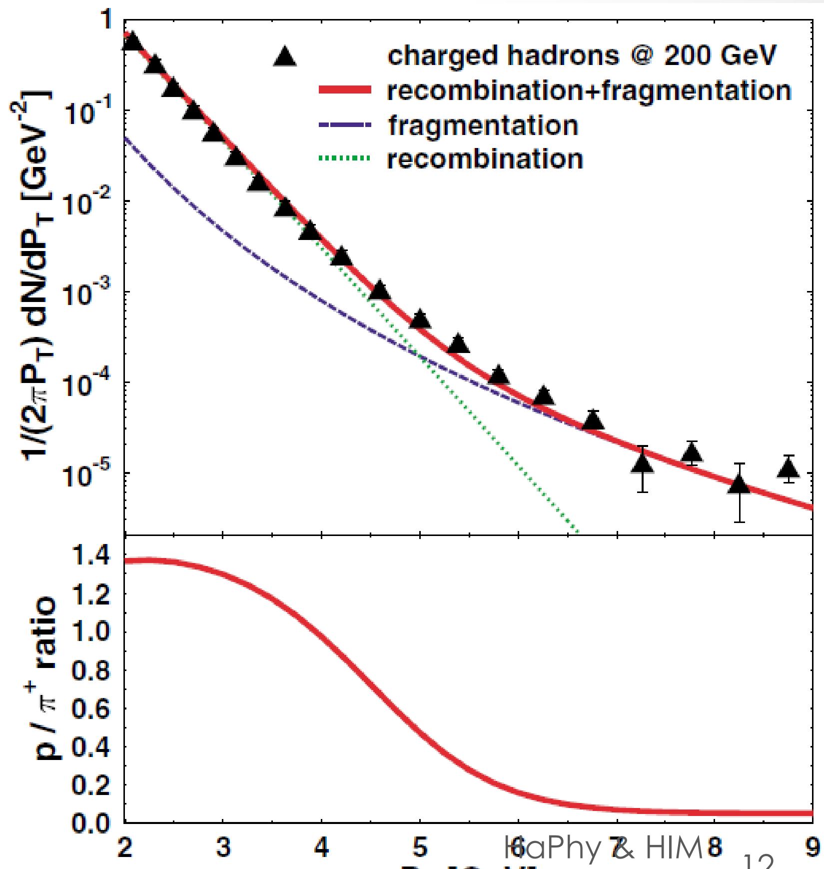
and

$$E\frac{dN_M}{d^3P} = C_M \int_\Sigma \frac{d^3 R P \cdot u(R)}{(2\pi)^3} \int \frac{d^3 q}{(2\pi)^3}$$

$$\times w_a\left(R; \frac{\mathbf{P}}{2} - \mathbf{q}\right) \Phi_M^W(\mathbf{q}) w_b\left(R; \frac{\mathbf{P}}{2} + \mathbf{q}\right)$$

$$\Phi_M^W(\mathbf{q}) = \int d^3 r \Phi_M^W(\mathbf{r}, \mathbf{q})$$

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\cdot\mathbf{r}'} \varphi_M\left(\mathbf{r} + \frac{\mathbf{r}'}{2}\right) \varphi_M^*\left(\mathbf{r} - \frac{\mathbf{r}'}{2}\right)$$



3) Quark number scaling of the elliptic flow

Denes Molnar and Sergei A. Voloshin, Phys. Rev. Lett **91**, 092301 (2003)

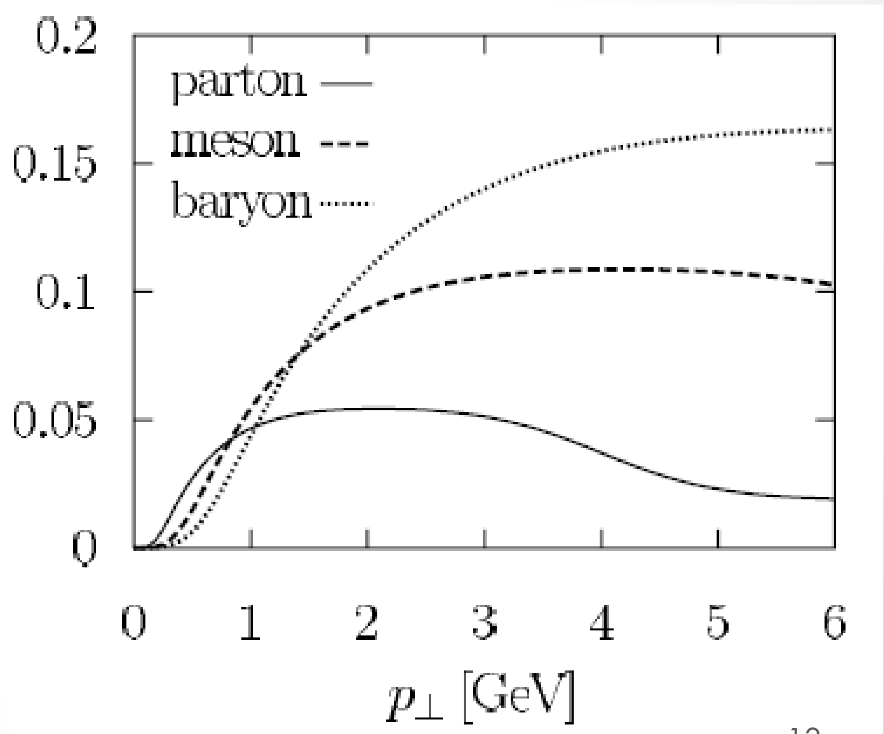
$$v_2(p_T) = \frac{\int d\varphi \cos 2\varphi \frac{d^2N}{dp_T^2}}{\int d\varphi \frac{d^2N}{dp_T^2}}, \quad \frac{dN_q}{p_T dp_T d\varphi} = \frac{1}{2\pi}\frac{dN_q}{p_T dp_T}\left[1 + 2v_{2,q}(p_T)\cos(2\varphi)\right]$$

i. Coalescence model predicts by assuming that partons have elliptical anisotropy

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1 + 2v_{2,q}^2(p_T/2)}$$

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3) + 3v_{2,q}^3(p_T/3)}{1 + 6v_{2,q}^2(p_T/3)}$$

$$v_{2,h}(p_T) \approx n v_{2,q}\left(\frac{1}{n}p_T\right)$$



– Yields in the coalescence model

S. Cho *et al*. [ExHIC Collaboration], Phys. Rev. Lett. **106**, 212001 (2011)
S. Cho *et al*. [ExHIC Collaboration], Phys. Rev. C **84**, 064910 (2011)

1) Yields at mid-rapidity $\sigma_i = \frac{1}{\sqrt{\mu_i \omega}}$

$$N_h^{Coal} \cong g \prod_{j=1}^{n} \frac{N_i}{g_i} \prod_{i=1}^{n-1} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \frac{(2l_i)!!}{(2l_i+1)!!} \left[\frac{2\mu_i T\sigma_i^2}{(1+2\mu_i T\sigma_i^2)} \right]^{l_i} \qquad \frac{1}{\mu_i} = \frac{1}{m_{i+1}} + \frac{1}{\sum_j^i m_j}$$

2) The internal structure of hadrons is taken into consideration

s-wave $\frac{N_i}{g_i} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \sim 0.168$

p-wave $\frac{N_i}{g_i} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \frac{2}{3} \left[\frac{2\mu_i T\sigma_i^2}{(1+2\mu_i T\sigma_i^2)} \right] \sim 0.040$

d-wave $\frac{N_i}{g_i} \frac{(4\pi\sigma_i^2)^{3/2}}{V(1+2\mu_i T\sigma_i^2)} \frac{8}{15} \left[\frac{2\mu_i T\sigma_i^2}{(1+2\mu_i T\sigma_i^2)} \right]^2 \sim 0.011$

: Yields of multi-quark hadrons are suppressed

Chemical freeze-out in heavy ion collisions

– Hadron production at chemical freeze-out

1) Start from the hadronization temperature and volume in the statistical hadronization model

$$T_H^{RHIC} = 162 \text{ MeV}, \quad V_H^{RHIC} = 2100 \text{ fm}^3$$
$$T_H^{LHC} = 156 \text{ MeV}, \quad V_H^{LHC} = 5380 \text{ fm}^3$$

2) Satisfy the entropy conservation during the expansion of the system, $s_H V_H = s_C V_C$ at both RHIC and LHC using the Lattice results for entropy at different temperatures

S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP **1011**, 077 (2010)

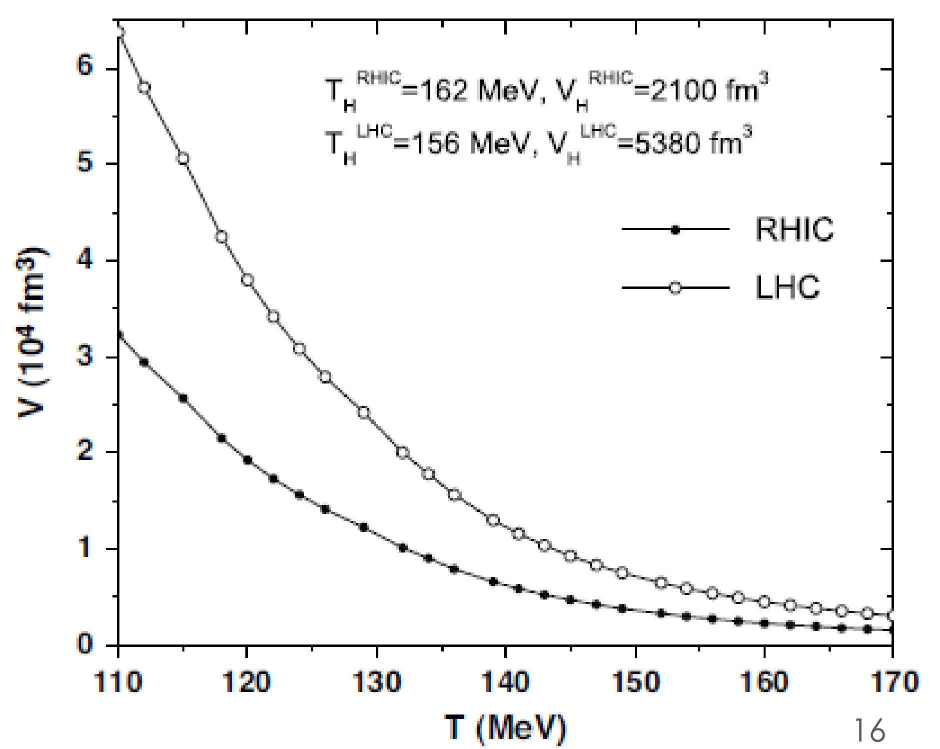
3) Require the size of rho and omega mesons produced at the critical temperature by coalescence of thermal quarks in QGP to be equal at both RHIC and LHC

4) Force the yields of rho & omega mesons produced at T_C

$$= N_\rho^{coal} = \frac{3\cdot 3}{(2\cdot 3)^2} N_u N_u \frac{(4\pi/\omega_l)^{3/2}}{V_C(1+2T_C/\omega_l)} \Big(\frac{M_u+M_u}{M_u^2}\Big)^{3/2}$$

$$N_\rho^{stat} = V_H \frac{3\cdot 3}{2\pi^2} \int_0^\infty \frac{p^2 dp}{e^{\sqrt{m_\rho^2+p^2}/T_H} - 1}$$

in the coalescence model
to be equal to those at T_H
in the statistical hadronization model



S. Cho *et al.* [ExHIC Collaboration], Prog. Part. Nucl. Phys. **95** 279 (2017)

5) Parameter determinations

Table 3.1

Statistical and coalescence model parameters for Scenario 1 and 2 at RHIC (200 GeV), LHC (2.76 TeV) and LHC (5.02 TeV), and those given in Refs. [14,15]. Quark masses are taken to be $m_q = 350$ MeV, $m_s = 500$ MeV, $m_c = 1500$ MeV and $m_b = 4700$ MeV. In Refs. [14,15], light quark masses were taken to be $m_q = 300$ MeV.

	RHIC		LHC (2.76 TeV)		LHC (5.02 TeV)		RHIC	LHC (5 TeV)
	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Sc. 1	Sc. 2	Refs [14,15]	
T_H (MeV)	162		156				175	
V_H (fm^3)	2100		5380				1908	5152
μ_B (MeV)	24		0				20	0
μ_s (MeV)	10		0				10	0
γ_c	22		39		50		6.40	15.8
γ_b	4.0×10^7		8.6×10^8		1.4×10^9		2.2×10^6	3.3×10^7
T_C (MeV)	162	166	156	166	156	166	175	
V_C (fm^3)	2100	1791	5380	3533	5380	3533	1000	2700
ω (MeV)	590	608	564	609	564	609	550	
ω_s (MeV)	431	462	426	502	426	502	519	
ω_c (MeV)	222	244	219	278	220	279	385	
ω_b (MeV)	183	202	181	232	182	234	338	
$N_u = N_d$	320	302	700	593	700	593	245	662
$N_s = N_{\bar{s}}$	183	176	386	347	386	347	150	405
$N_c = N_{\bar{c}}$	4.1		11		14		3	20
$N_b = N_{\bar{b}}$	0.03		0.44		0.71		0.02	0.8
T_F (MeV)	119		115				125	
V_F (fm^3)	20355		50646				11322	30569

Hadronic interactions

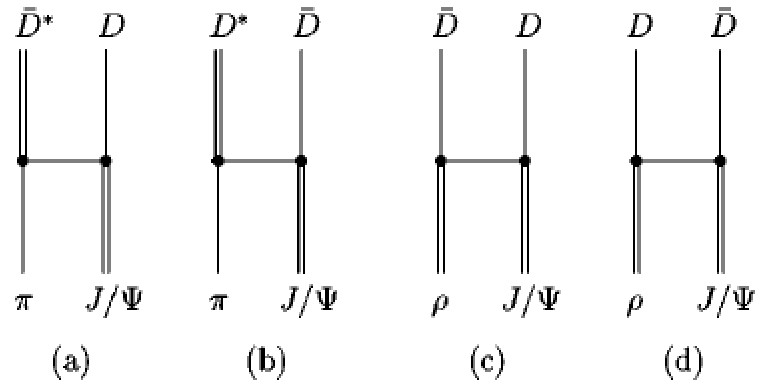
– A meson exchange model with an effective Lagrangian

S. G. Matinyan and B. Muller, Phys. Rev. C **58**, 2994 (1998)

K. L. Haglin, Phys. Rev. C **61**, 031902(R) (2000)

Z. Lin and C. M. Ko, Phys. Rev. C **62**, 034903 (2000)

Y. Oh, T. Song, and S. –H. Lee, Phys. Rev. C **63**, 034901 (2000)



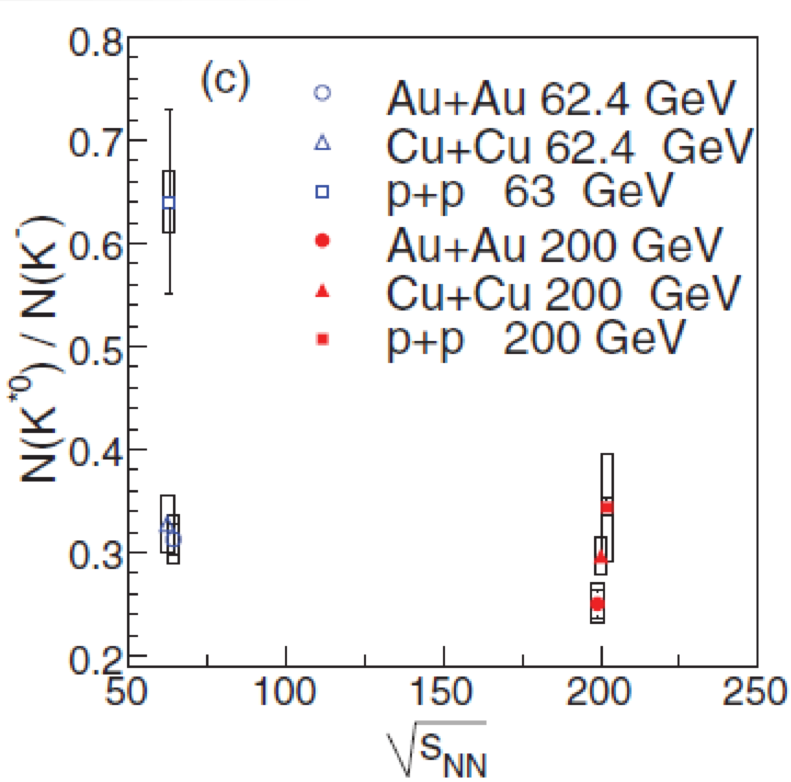
$$\mathcal{L}_0 = \mathrm{Tr}(\partial_\mu P^\dagger \partial^\mu P) - \frac{1}{2}\mathrm{Tr}(F^\dagger_{\mu\nu}F^{\mu\nu}),$$

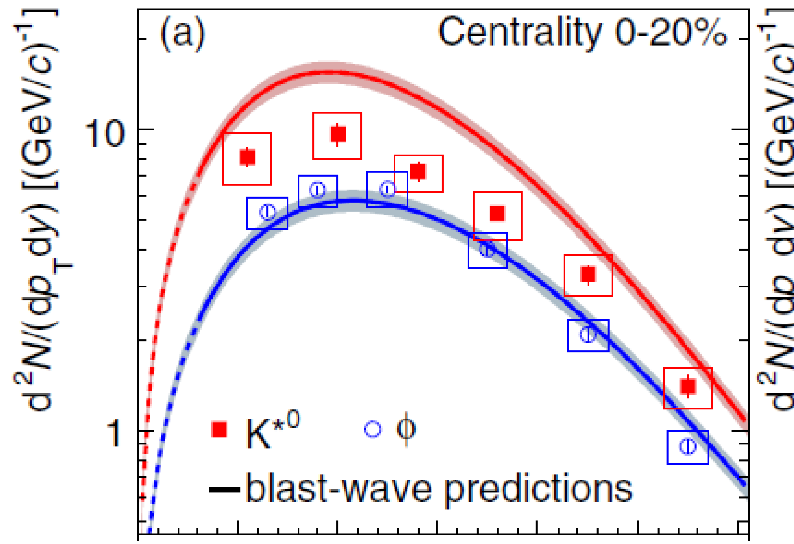
$$\begin{aligned}
\mathcal{L}_{\pi K K^*} &= i g_{\pi K K^*} K^{*\mu} \vec{\tau} \cdot (\bar{K}\partial_\mu \vec{\pi} - \partial_\mu \bar{K}\vec{\pi}) + \text{H.c.}, \\
\mathcal{L}_{\rho K K} &= i g_{\rho K K}(K\vec{\tau}\partial_\mu \bar{K} - \partial_\mu K \vec{\tau}\bar{K}) \cdot \vec{\rho}^{\mu}, \\
\mathcal{L}_{\rho K^* K^*} &= i g_{\rho K^* K^*} \, [(\partial_\mu K^{*\nu}\vec{\tau}\bar{K}^*_\nu - K^{*\nu}\vec{\tau}\partial_\mu \bar{K}^*_\nu) \cdot \vec{\rho}^{\mu} \\
&+ (K^{*\nu}\vec{\tau} \cdot \partial_\mu \vec{\rho}_\nu - \partial_\mu K^{*\nu}\vec{\tau} \cdot \vec{\rho}_\nu)\bar{K}^{*\mu} \\
&+ K^{*\mu}(\vec{\tau} \cdot \vec{\rho}^{\nu}\partial_\mu \bar{K}^*_\nu - \vec{\tau} \cdot \partial_\mu \vec{\rho}^{\nu}\bar{K}^*_\nu)],
\end{aligned}$$

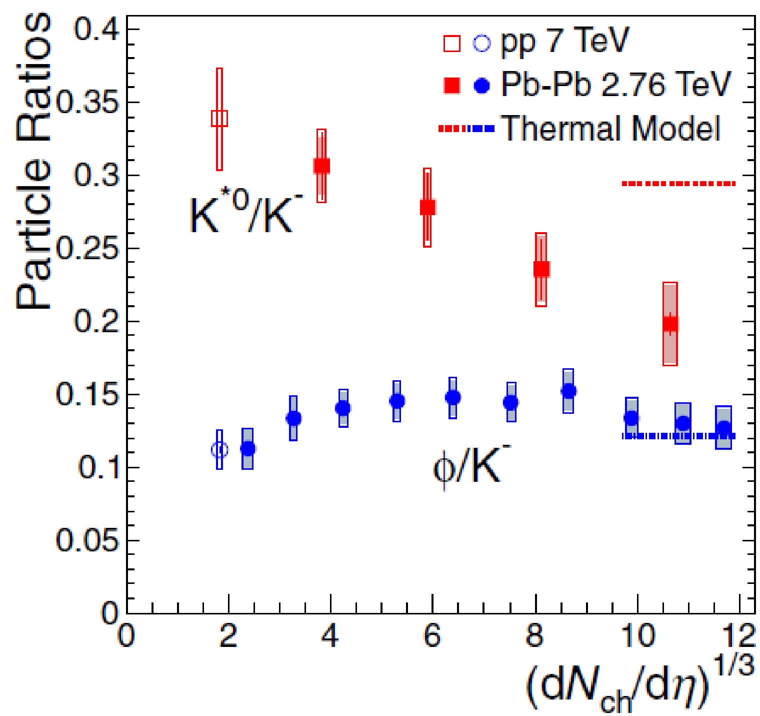
$$V = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}$$

– K* mesons in heavy ion collisions



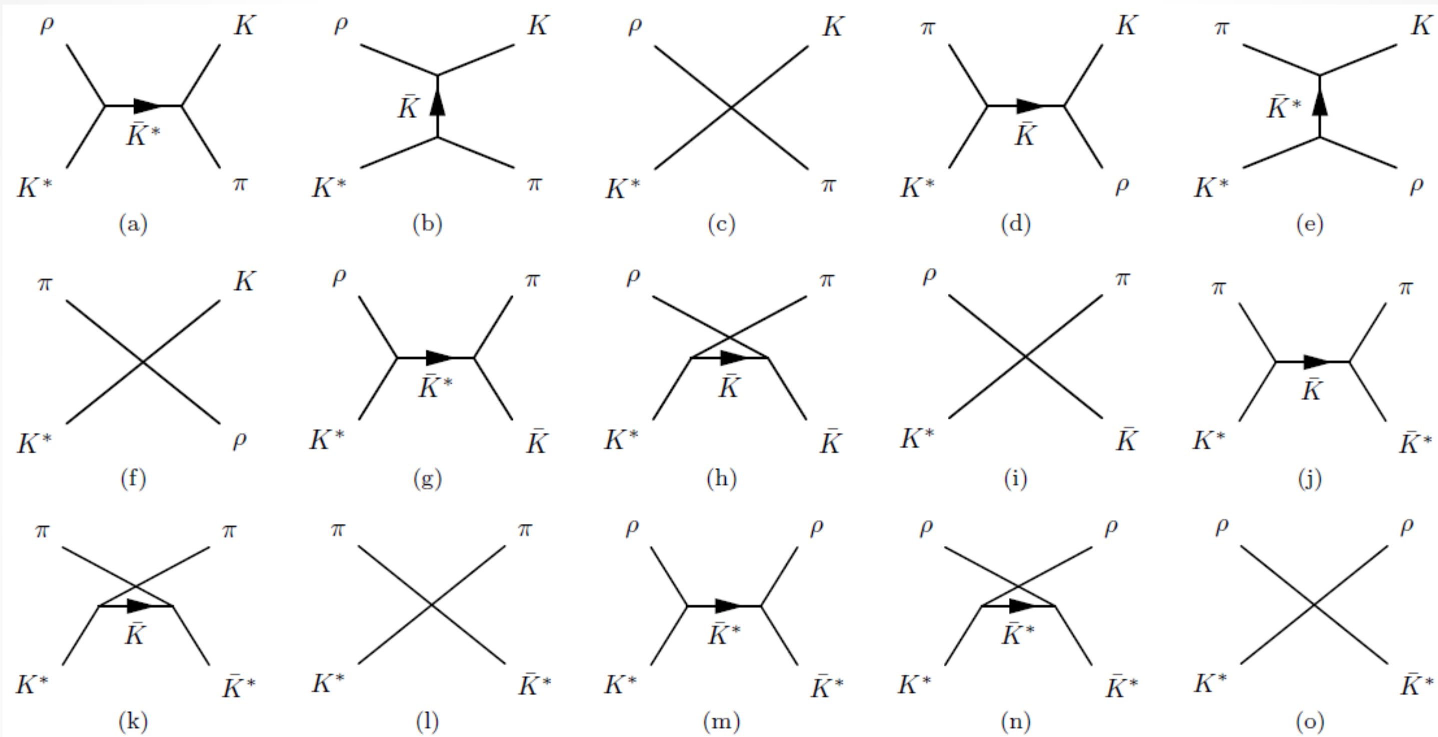




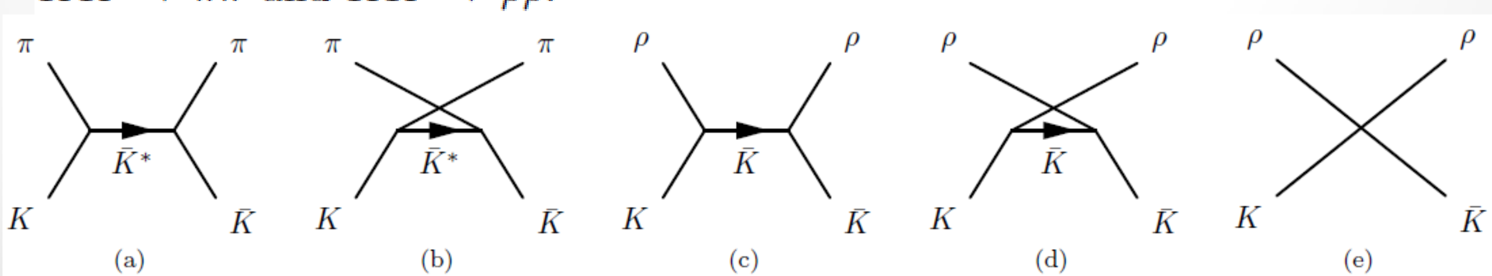
M. M. Aggarwal et al, [STAR Collaboration], Phys. Rev. C **84**, 034909 (2011)
B. Abelev et al. [ALICE Collaboration], Phys. Rev. C **91**, 024609 (2015)

– K* meson and kaon interactions

$K^*\pi \to \rho K,\ K^*\rho \to \pi K, K^*\bar{K} \to \rho\pi,\ K^*\bar{K}^* \to \pi\pi,$ and $K^*\bar{K}^* \to \rho\rho.$



$K\bar{K} \to \pi\pi$ and $K\bar{K} \to \rho\rho.$



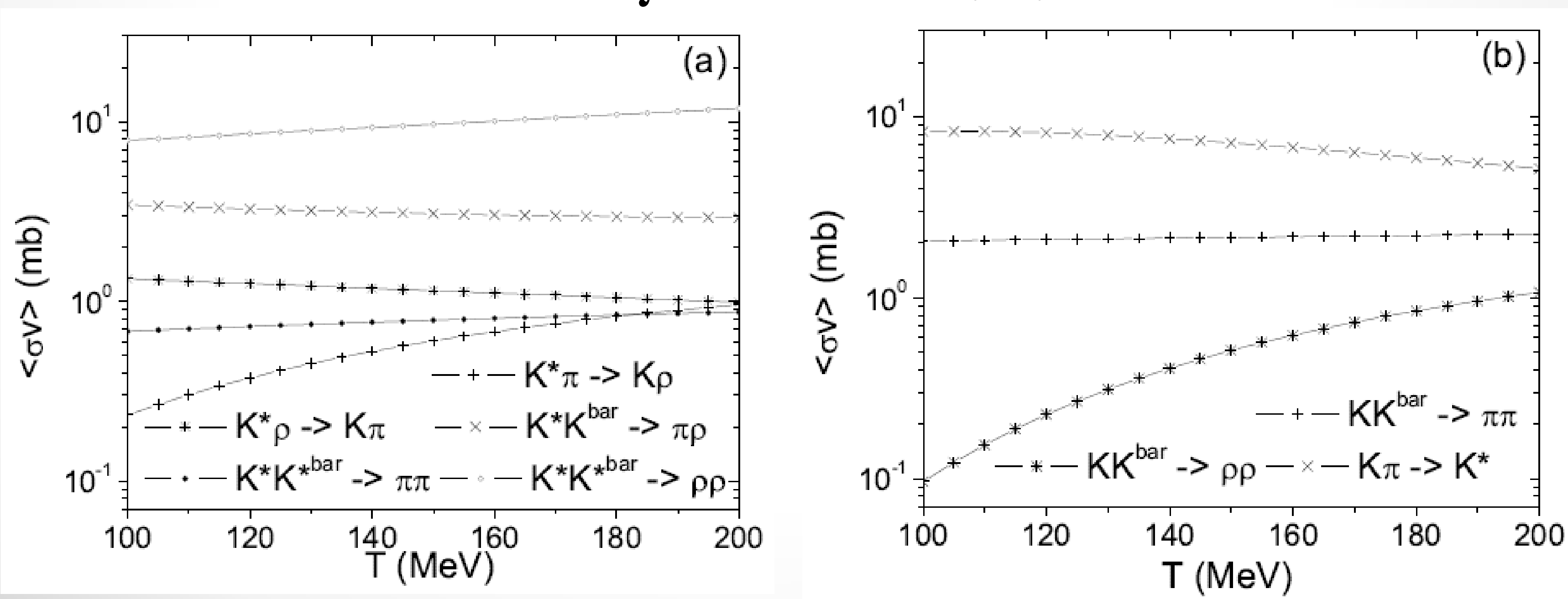
1) K* meson production from kaons and pions & K* meson decay to kaons and pions

$$\sigma_{K\pi\to K^*} = \frac{g_{K^*}}{g_K g_\pi}\frac{4\pi}{p_{cm}^2}\frac{s\Gamma^2_{K^*\to K\pi}}{(m_{K^*}-\sqrt{s})^2 + s\Gamma^2_{K^*\to K\pi}}, \quad \Gamma_{K^*\to K\pi}(\sqrt{s}) = \frac{g^2_{\pi K^* K}}{2\pi s}p^3_{cm}(\sqrt{s}),$$

2) Thermally averaged cross sections for K* mesons and kaons

P. Koch, B. Muller, and J. Rafelski, Phys. Rept., **142**, 167 (1986)

$$\left\langle \sigma_{ih\to jk} v_{ih} \right\rangle = \frac{\int d^3p_i d^3p_h f_i(p_i) f_j(p_j) \sigma_{ih\to jk} v_{ih}}{\int d^3p_i d^3p_h f_i(p_i) f_j(p_j)}$$

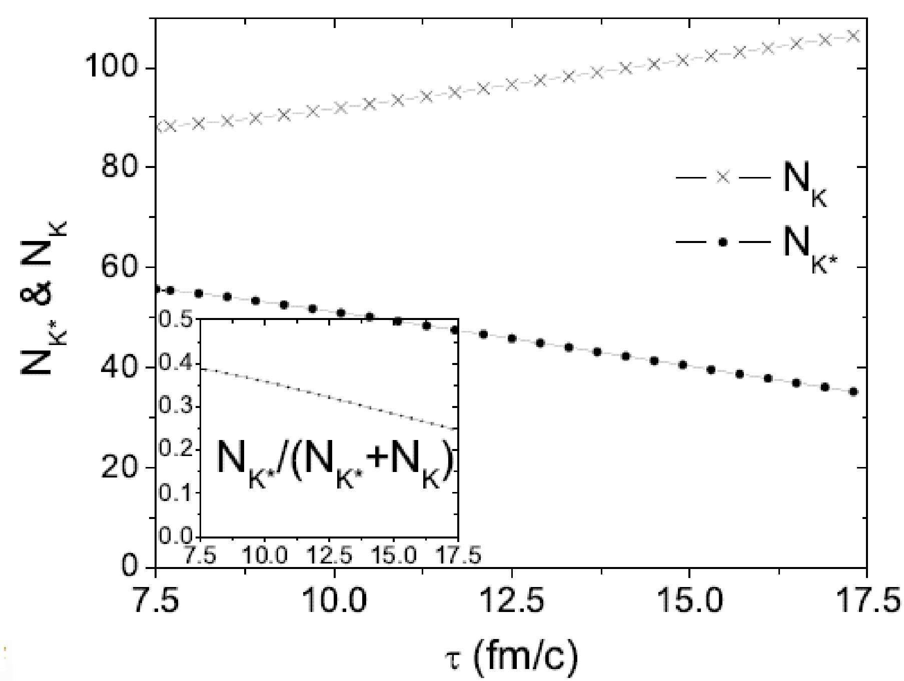


– Time evolution of the K* and K meson abundances

: Rate equations for K* & K meson abundances

$$
\begin{aligned}
\frac{dN_{K^*}(\tau)}{d\tau} = {} & \langle\sigma_{K\rho\to K^*\pi}v_{K\rho}\rangle n_\rho(\tau)N_K(\tau) - \langle\sigma_{K^*\pi\to K\rho}v_{K^*\pi}\rangle n_\pi(\tau)N_{K^*}(\tau) + \langle\sigma_{K\pi\to K^*\rho}v_{K\pi}\rangle n_\pi(\tau)N_K(\tau) \\
& - \langle\sigma_{K^*\rho\to K\pi}v_{K^*\rho}\rangle n_\rho(\tau)N_{K^*}(\tau) + \langle\sigma_{\rho\pi\to K^*\bar{K}}v_{\rho\pi}\rangle n_\pi(\tau)N_\rho(\tau) - \langle\sigma_{K^*\bar{K}\to\rho\pi}v_{K^*\bar{K}}\rangle n_K(\tau)N_{K^*}(\tau) \\
& + \langle\sigma_{\pi\pi\to K^*\bar{K}^*}v_{\pi\pi}\rangle n_\pi(\tau)N_\pi(\tau) - \langle\sigma_{K^*\bar{K}^*\to\pi\pi}v_{K^*\bar{K}^*}\rangle n_{\bar{K}^*}(\tau)N_{K^*}(\tau) + \langle\sigma_{\rho\rho\to K^*\bar{K}^*}v_{\rho\rho}\rangle n_\rho(\tau)N_\rho(\tau) \\
& - \langle\sigma_{K^*\bar{K}^*\to\rho\rho}v_{K^*\bar{K}^*}\rangle n_{\bar{K}^*}(\tau)N_{K^*}(\tau) + \langle\sigma_{\pi K\to K^*}v_{\pi K}\rangle n_\pi(\tau)N_K(\tau) - \langle\Gamma_{K^*}\rangle N_{K^*}(\tau),
\end{aligned}
$$

$$
\begin{aligned}
\frac{dN_K(\tau)}{d\tau} = {} & \langle\sigma_{\pi\pi\to K\bar{K}}v_{\pi\pi}\rangle n_\pi(\tau)N_\pi(\tau) \\
& - \langle\sigma_{K\bar{K}\to\pi\pi}v_{K\bar{K}}\rangle n_{\bar{K}}(\tau)N_K(\tau) \\
& + \langle\sigma_{\rho\rho\to K\bar{K}}v_{\rho\rho}\rangle n_\rho(\tau)N_\rho(\tau) \\
& - \langle\sigma_{K\bar{K}\to\rho\rho}v_{K\bar{K}}\rangle n_{\bar{K}}(\tau)N_K(\tau) \\
& + \langle\sigma_{K^*\pi\to K\rho}v_{K^*\pi}\rangle n_\pi(\tau)N_{K^*}(\tau) \\
& - \langle\sigma_{K\rho\to K^*\pi}v_{K\rho}\rangle n_\rho(\tau)N_K(\tau) \\
& + \langle\sigma_{K^*\rho\to K\pi}v_{K^*\rho}\rangle n_\rho(\tau)N_{K^*}(\tau) \\
& - \langle\sigma_{K\pi\to K^*\rho}v_{K\pi}\rangle n_\pi(\tau)N_K(\tau) \\
& + \langle\sigma_{\rho\pi\to K^*\bar{K}}v_{\rho\pi}\rangle n_\pi(\tau)N_\rho(\tau) \\
& - \langle\sigma_{K^*\bar{K}\to\rho\pi}v_{K^*\bar{K}}\rangle n_{\bar{K}}(\tau)N_{K^*}(\tau) \\
& + \langle\Gamma_{K^*}\rangle N_{K^*}(\tau) - \langle\sigma_{\pi K\to K^*}v_{\pi K}\rangle n_\pi(\tau)N_K(\tau).
\end{aligned}
$$



Kinetic freeze-out in heavy ion collisions

– The abundance ratio of K* mesons to kaons in heavy ion collisions

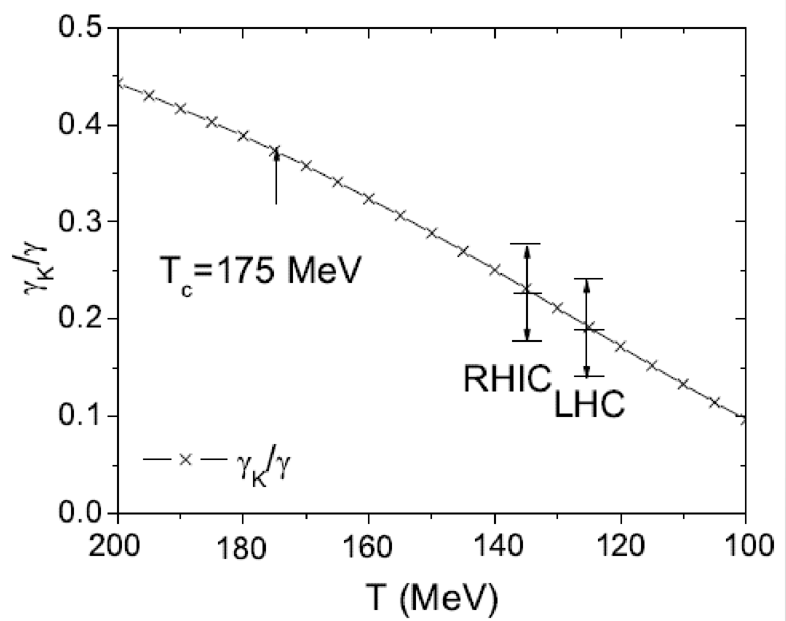
1) Simplified rate equations

$$\frac{dN_{K^*}(\tau)}{d\tau} = \gamma_K N_K(\tau) - \gamma_{K^*} N_{K^*}(\tau),$$
$$\frac{dN_K(\tau)}{d\tau} = -\gamma_K N_K(\tau) + \gamma_{K^*} N_{K^*}(\tau),$$

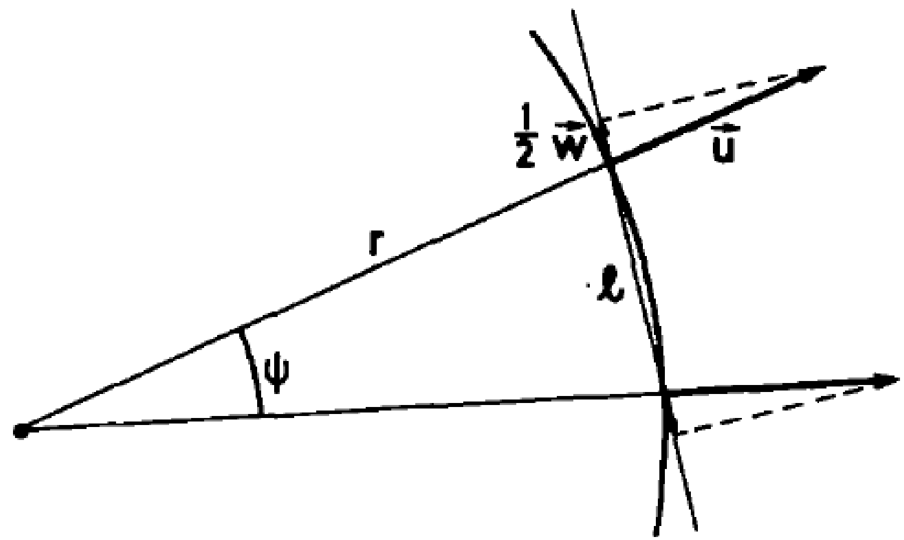
$$\gamma_{K^*} = \langle \sigma_{K^*\rho\to K\pi} v_{K^*\rho}\rangle n_\rho + \langle \sigma_{K^*\pi\to K\rho} v_{K^*\pi}\rangle n_\pi + \langle \Gamma_{K^*}\rangle,$$
$$\gamma_K = \langle \sigma_{K\pi\to K^*\rho} v_{K\pi}\rangle n_\pi + \langle \sigma_{K\rho\to K^*\pi} v_{K\rho}\rangle n_\rho + \langle \sigma_{K\pi\to K^*} v_{K\pi}\rangle n_\pi.$$

2) K* and K meson abundances

$$N_{K^*}(\tau) = \frac{\gamma_K}{\gamma}N^0 + \left(N^0_{K^*} - \frac{\gamma_K}{\gamma}N^0\right)e^{-\gamma(\tau-\tau_h)},$$
$$N_K(\tau) = \frac{\gamma_{K^*}}{\gamma}N^0 + \left(N^0_K - \frac{\gamma_{K^*}}{\gamma}N^0\right)e^{-\gamma(\tau-\tau_h)},$$
$$R(\tau) = \frac{N_{K^*}(\tau)}{N_{K^*}(\tau)+N_K(\tau)} = \frac{N_{K^*}(\tau)}{N^0} = \frac{\gamma_K}{\gamma} + \left(\frac{N^0_{K^*}}{N^0} - \frac{\gamma_K}{\gamma}\right)e^{-\gamma(\tau-\tau_h)}.$$



– Geometrical concept of the freeze-out



$$w(r,t) = \frac{l(r,t)}{r}\, u(r,t)$$

$$w(r,t_b) = v(r,t_b).$$

J. P. Bondorf, S. I. A. Garpman, J. Zimanyi, Nucl. Phys. A **296**, 320 (1978)

The freeze-out criterion
: the time for a macroscopic flow element is equal to the microscopic interaction time which is a function of local density, mean speed, and cross sections

The scattering rate and expansion rate

$$\tau_{exp} = \frac{1}{\partial \cdot u} = \tau^{i}_{scatt} = \frac{1}{\sum_j \langle \sigma_{ij} v_{ij} \rangle n_j}$$

– The kinetic freeze-out condition

S. Cho, T. Song, and S-H. Lee, arXiv:15011.08019

1) Rate equations for the abundances of K* and K mesons

$$\frac{dN_{K^*}(\tau)}{d\tau} = \frac{1}{\tau^{K}_{scatt}} N_K(\tau) - \frac{1}{\tau^{K^*}_{scatt}} N_{K^*}(\tau),$$

$$\frac{dN_K(\tau)}{d\tau} = \frac{1}{\tau^{K^*}_{scatt}} N_{K^*}(\tau) - \frac{1}{\tau^{K}_{scatt}} N_K(\tau),$$

with $1/\tau^{K^*}_{scatt} = \sum_i \langle \sigma_{K^*i} v_{K^*i} \rangle n_i, \; 1/\tau^{K}_{scatt} = \sum_j \langle \sigma_{Kj} v_{Kj} \rangle n_j.$

3) The yield ratio between K* mesons and kaons

$$R(\tau) = R_0 + \left(\frac{N^0_{K^*}}{N^0} - \frac{\tau_{scatt}}{\tau^{K}_{scatt}} \right) e^{-\frac{\tau - \tau_h}{\tau_{scatt}}}.$$

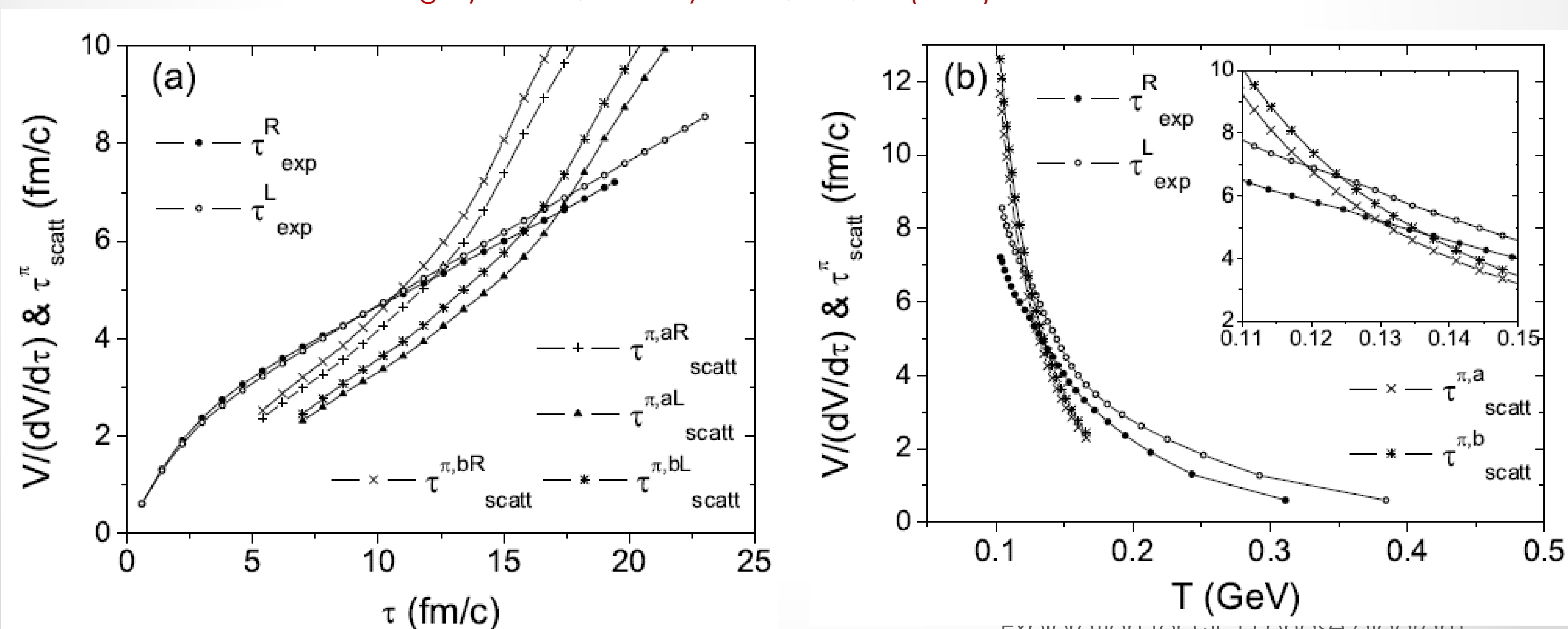
with $R_0 = \frac{\tau_{scatt}}{\tau^{K}_{scatt}} = \frac{\tau^{K^*}_{scatt}}{\tau^{K}_{scatt} + \tau^{K^*}_{scatt}}$ and $\tau_{scatt} = \frac{\tau^{K}_{scatt}\tau^{K^*}_{scatt}}{\tau^{K}_{scatt} + \tau^{K^*}_{scatt}}$

– The freeze-out condition of the pion

1) The scattering time for pions

$$\frac{1}{\tau^{\pi,a}_{scatt}} \approx \langle v \rangle n \sigma \qquad \frac{1}{\tau^{\pi,b}_{scatt}} = 59.5\, fm^{-1} \left(\frac{T}{GeV} \right)^{3.45}$$

C. M. Hung and Edward V. Shuryak, Phys. Rev. C **57**, 1891 (1998)
Ulrich Heinz and Gregory Cestin, Eur. Phys. J. ST, **155**, 75 (2008)



Conclusion

– Chemical and kinetic freeze-outs in heavy ion collisions

1) Heavy ion collision experiments provide various ways to investigate the phase diagrams of QCD
2) The study on the hadron production is helpful in identifying chemical and kinetic freeze-out conditions in heavy ion collisions
3) The final yield ratio between K* mesons and kaons may reflect the condition at the last stage of the hadronic effects on K* and K mesons, or the kinetic freeze-out temperature
4) The smaller ratio of K*/K measured at the LHC energy may indicate a lower kinetic freeze-out temperature compared to that at RHIC