Inha University, Republic of Korea

In-medium Nucleons, Nuclear Matter and Finite Nuclei (Chiral Soliton Approach)

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Motivation

To construct a model which describes at same footing

the single nucleon properties

- **in free space considering it as a structure-full system**
- **in nuclear medium (possible structure changes)**

as well as the properties of the whole nucleonic systems

- **infinite nuclear matter properties (EOS, volume and symmetry energy properties)**
- **matter under extreme conditions (e.g. neutron stars)**
- **few/many (ordinary/exotic) nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)**
- **nucleon knock-out reactions (lepton-nucleus scattering experiments)**
- **possible changes in in-medium NN interactions**
- **etc**

Strategy

How to construct a theoretical framework?

- the best way is to start from QCD and to arrive some an effective framework (it is not completely understood yet)
- therefore, as much as possible main peculiarities of QCD must be taken into account in arriving an effective theory or in constructing a phenomenological approach which describe the hadrons and their interactions
- at low energies main peculiarities (which obvious in a single hadron sector) are
  - chiral symmetry and its spontaneous breaking
  - quark confinement (the mechanism is not understood yet)
- in addition one should take into account the structure changes of in-medium nucleons in constructing the nuclear many body systems

Content

- Topological models (describe structure-full hadrons)
- Medium modifications (interactions with surrounding environment)
- Nucleon in nuclear matter (structure changes due to surrounding environment)
- Nuclear matter (takes into account structure changes of the constituents)
- Neutron stars (extrapolations to high density regions)
- Consistency (difference) with (from) other approaches
- Nucleon in finite nuclei (non-spherical deformations)
- Properties of finite nuclei (example: mirror nuclei)
- Summary and Outlook

Topological Models

Topological models

Why topological models?

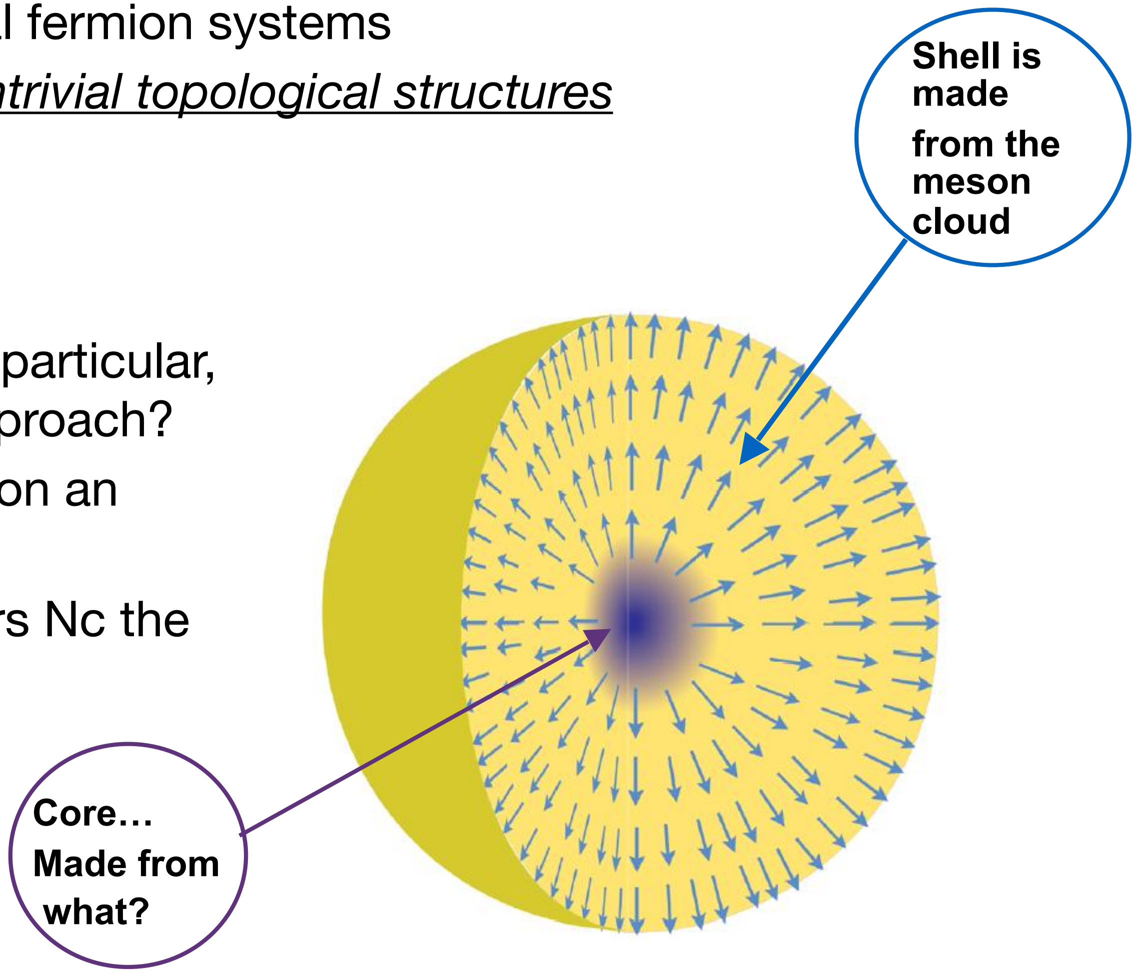
At fundamental level we may have

- fermions -> then bosons are trivial fermion systems
- bosons -> then fermions are *nontrivial topological structures*

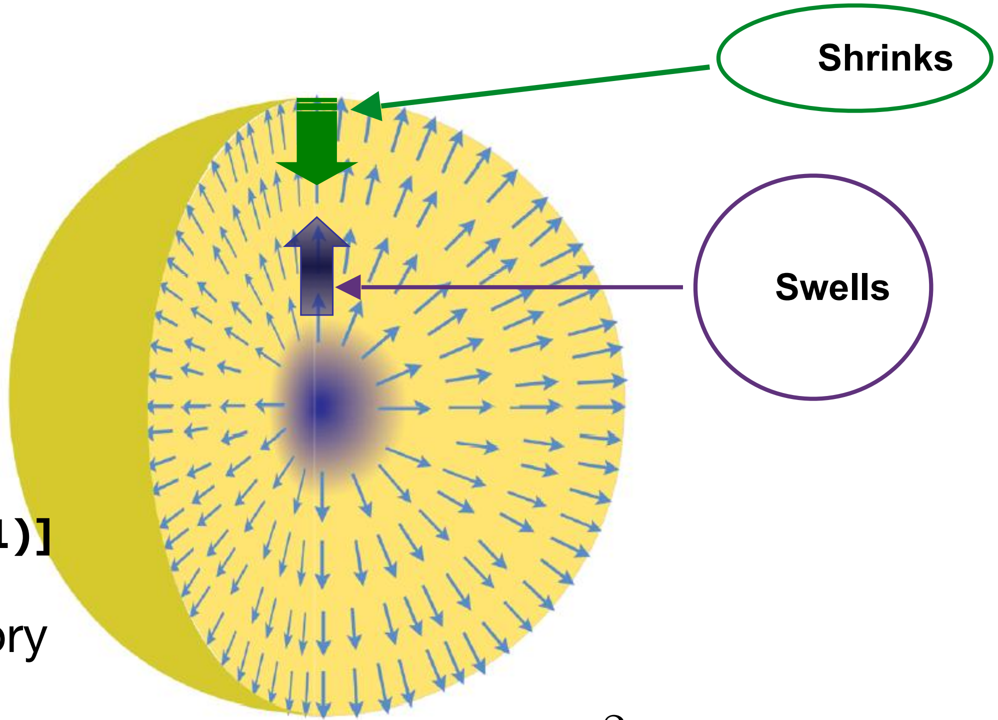
Structure

From what is made a nucleon and, in particular, its core in a starting boson picture approach?

- The structure treatment depends on an energy scale
- At the limit of large number colours Nc the core still has the mesonic content



Stabilisation mechanism



- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian

Prototype: **Skyrme model**

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2$$

Shrinking term ← → Swelling term

- *Hedgehog* solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

Topological models

The free space Lagrangian (which was widely in use)
[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2} \mathrm{Tr}\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16} \mathrm{Tr}\left(U + U^\dagger - 2\right)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) ***A***

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta} Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+ \partial_\alpha U$$

$$A = \int d^3 r B^0$$

- Nucleon is quantized state of the classical soliton-skyrmion which rotates in the ordinary and an internal spaces

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|S = T, s, t> = (-1)^{t+T}\sqrt{2T+1} D^{S=T}_{-t,s}(A)$$

Medium Modifications

Medium modifications

What happens in the nuclear medium?

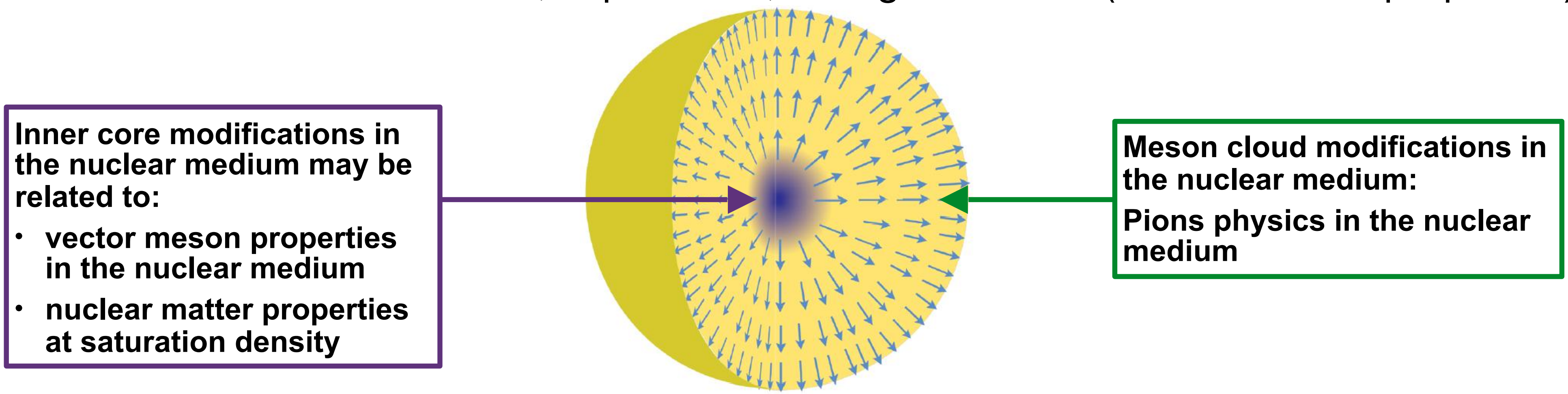
The possible medium effects

- Deformations (swelling or shrinking, multipole deformations) of nucleons
- Characteristic changes in: effective mass, charge distributions, all possible form factors
- NN interactions may change
- etc.

One should be able to describe all those phenomena

Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)



Medium modifications

"Outer shell" modifications

- In free space three types of pions can be treated separately: isospin breaking

$$(\partial^\mu \partial_\mu + m_\pi^2)\vec{\pi}^{(\pm,0)} = 0$$

- In nuclear matter: three types of polarization operators

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\text{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Optic potential approach: parameters from the pion-nucleon scattering (including the isospin dependents)

	π-atom	$T_\pi = 50$ MeV
b_0 $[m_\pi^{-1}]$	- 0.03	- 0.04
b_1 $[m_\pi^{-1}]$	- 0.09	- 0.09
c_0 $[m_\pi^{-3}]$	0.23	0.25
c_1 $[m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

"Outer shell" modifications in the Lagrangian [U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16}\alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \mathrm{Tr}\left(2 - U - U^\dagger\right)$$

- Due to the non-locality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters the following parts of the kinetic term are modified in different forms:
  - Temporal part
  - Space part

$$\hat{\Pi}^0 = 2\omega U_{\mathrm{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla}\cdot\chi_p(\rho, b_0, c_0)\vec{\nabla}$$

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
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g'	0.47	0.47

Medium modifications

“Inner core” modifications
[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_\tau}\mathrm{Tr}\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

may be related to

- Vector meson properties in nuclear matter
- Nuclear matter properties

$$\zeta_{\tau,s} = \zeta_{\tau,s}(\rho, \delta\rho, \mathrm{parameters})$$

Medium modifications

Final Lagrangian
[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}$$

- Isoscalar part

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_m$$

- Isovector part

$$\mathcal{L}^*_{\rm asym} = \mathcal{L}^*_{\delta m} + \mathcal{L}^*_{\delta\rho}$$

- **Nuclear matter stabilization**
- **Asymmetric matter properties**

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\alpha_\tau {\rm Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s {\rm Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}{\rm Tr}\left[U^\dagger\partial_0 U, U^\dagger \partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}{\rm Tr}\left[U^\dagger\partial_i U, U^\dagger \partial_j U\right]^2$$

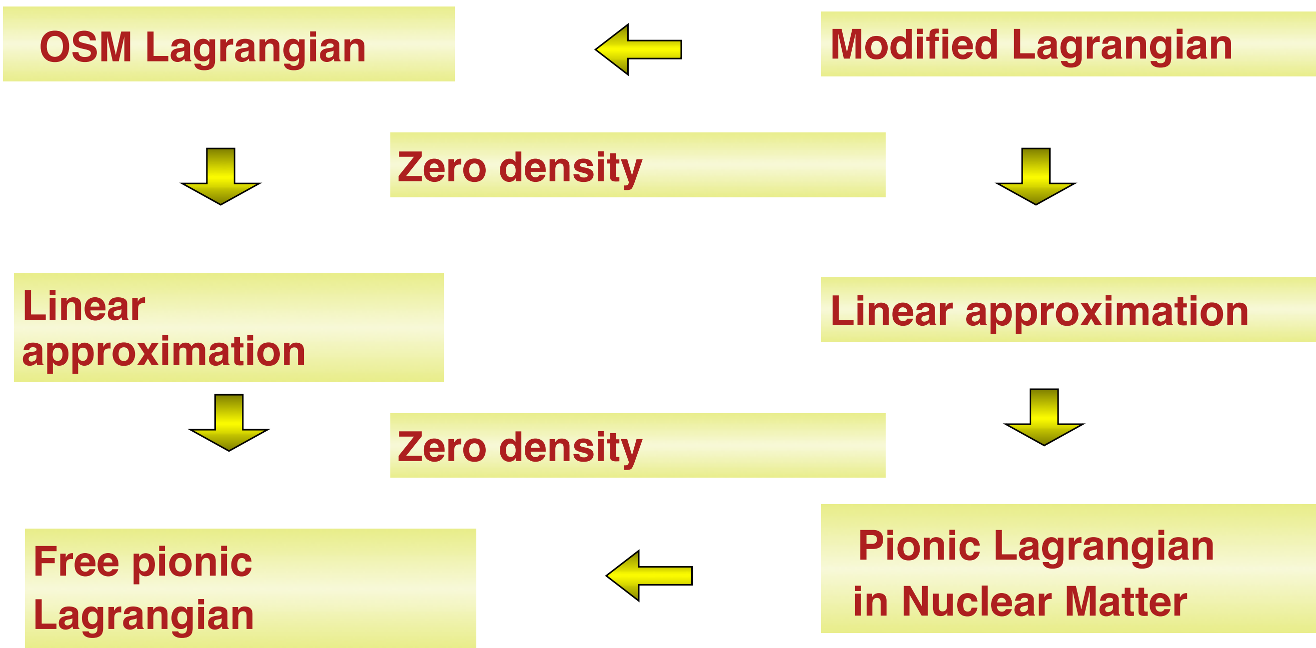
$$\mathcal{L}^*_m = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m {\rm Tr}\left(2 - U - U^+\right)$$

$$\mathcal{L}^*_{\delta m} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m^2_{\pi^\pm} - m^2_{\pi^0}){\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_a U^\dagger\right)$$

$$\mathcal{L}^*_{\delta\rho} = -\frac{F_\pi^2}{16} m_\pi \alpha_e\, \varepsilon_{ab3}{\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_b \partial_0 U^\dagger\right)$$

Medium modifications

- Modifications of the mesonic sector modifies the baryonic sector
- Lagrangian satisfies some limiting conditions



Medium modifications

Reparametrization
[UY, PRC88 (2013)]

- Five density dependent parameters

- Rearrangment (technical simplification to describe nuclear matter)

Shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau, \quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

$$+C_1\frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$+C_2\frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2\gamma_s}$$

$$+C_3\frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0\gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n-\rho_p}{\rho_0} = \frac{C_4\frac{\rho}{\rho_0}}{1+C_5\frac{\rho}{\rho_0}}\frac{\rho_n-\rho_p}{\rho_0}$$

Nucleon in Nuclear Matter

Nucleon in nuclear matter

Structure studies 1: Energy momentum tensor

- It allows to address the questions like:
  - How are the total angular momentum and angular momentum of the nucleon shared among its constituents?
  - How are the strong forces experienced by its constituents distributed inside the nucleon?
- EMT form factors studied in lattice QCD, ChPT and in different models (chiral quark soliton model, Skyrme model, etc.)
- We made further step studying EMT form factors in nuclear matter

Nucleon in nuclear matter

Structure studies 1: Energy momentum tensor

- Definition

$$\langle p'|\hat{T}_{\mu\nu}(0)|p\rangle = \bar{u}(p',\, s')\left[M_2(t)\,\frac{P_\mu P_\nu}{M_N} + J(t)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N}\right]u(p,\, s)\,,$$

- Three form factors give an information about energy distribution, angular momentum distribution and about the stabilization of strong forces inside the nucleon

$$T^*_{00}(r) = \frac{F^{*2}_{\pi,s}}{8}\left(\frac{2\sin^2 F}{r^2} + F'^2\right) + \frac{\sin^2 F}{2\,e^{*2}\,r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right) + \frac{m^{*2}_\pi F^{*2}_{\pi,s}}{4}\,(1-\cos F)\,,$$

$$T^*_{0k}(\boldsymbol{r},\boldsymbol{s}) = \frac{\epsilon^{klm} r^l s^m}{(\mathrm{s}\times\mathrm{r})^2}\,\rho^*_J(r)\,,$$

$$T^*_{ij}(\boldsymbol{r}) = s^*(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p^*(r)\,\delta_{ij}$$

$$M^*_2(t) - \frac{t}{5M^{*2}_N}\,d^*_1(t) = \frac{1}{M^*_N}\int \mathrm{d}^3r\; T^*_{00}(r)\; j_0(r\sqrt{-t})\,,$$

$$d^*_1(t) = \frac{15M^*_N}{2}\int \mathrm{d}^3r\; p^*(r)\;\frac{j_0(r\sqrt{-t})}{t}\,,$$

$$J^*(t) = 3\int \mathrm{d}^3r\;\rho^*_J(r)\;\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}\,,$$

$$M^*_2(0) = \frac{1}{M^*_N}\int \mathrm{d}^3r\; T^*_{00}(r) = 1\,, \quad J^*(0) = \int \mathrm{d}^3r\;\rho^*_J(r) = \frac{1}{2}\,.$$

Structure studies1: Energy momentum tensor related quantities

[H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

Different quantities related to the nucleon EMT densities and their form factors: $T_{00}^{*}(0)$ denotes the energy in the center of the nucleon; $\langle r_{00}^2\rangle^*$ and $\langle r_J^2\rangle^*$ depict the mean square radii for the energy and angular momentum densities, respectively; $p^*(0)$ represents the pressure in the center of the nucleon, whereas r_0^* designates the position where the pressure changes its sign; d_1^* is the value of the $d_1^*(t)$ form factor at the zero momentum transfer.

ρ/ρ_0	$T_{00}^{*}(0)$ [GeV fm^{-3}]	$\langle r_{00}^2\rangle^*$ [fm^2]	$\langle r_J^2\rangle^*$ [fm^2]	$p^*(0)$ [GeV fm^{-3}]	r_0^* [fm]	d_1^*
0	1.45	0.68	1.09	0.26	0.71	−3.54
0.5	0.96	0.83	1.23	0.18	0.82	−4.30
1.0	0.71	0.95	1.35	0.13	0.90	−4.85

Structure studies 1: Pressure distribution inside the nucleon in free space and in symmetric matter [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

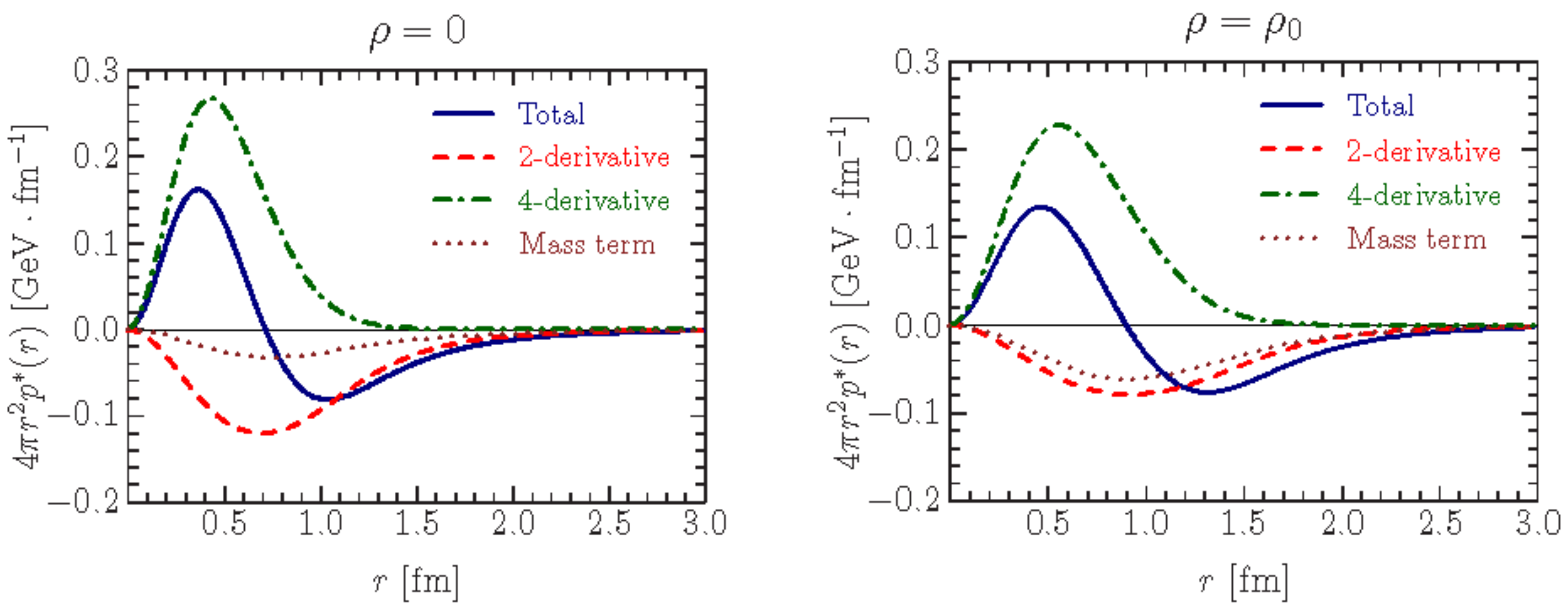


FIG. 3: (Color online) The decomposition of the pressure densities $4\pi r^2 p^*(r)$ as functions of r, in free space ($\rho = 0$) and at $\rho = \rho_0$, in the left and right panels, respectively. The solid curves denote the total pressure densities, the dashed ones represent the contributions of the 2-derivative (kinetic) term, the long-dashed ones are those of the 4-derivative (stabilizing) term, and the dotted ones stand for those of the pion mass term.

Nucleon in nuclear matter

Stability and applicability [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

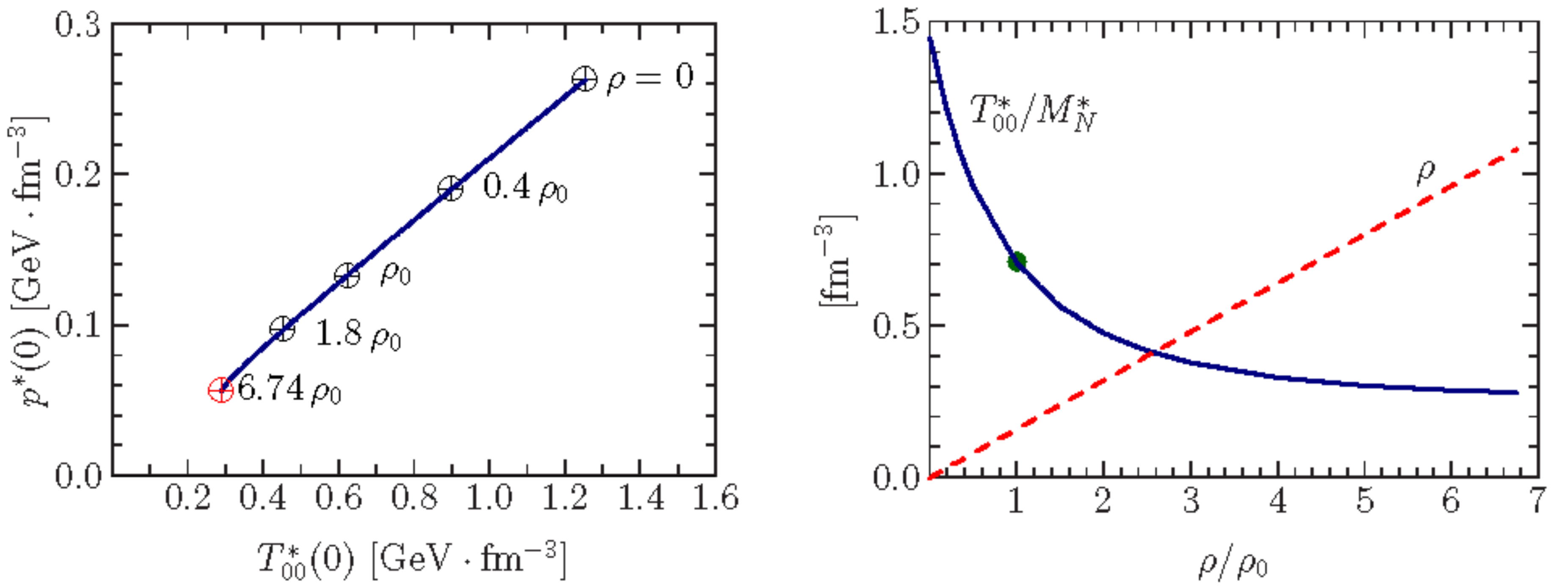


FIG. 5: (Color online) In the left panel, the correlated change of $p^*(0)$ and $T^*_{00}(0)$ drawn with ρ varied. In the right panel, the T^*_{00}/M^*_N and ρ depicted as a function of ρ/ρ_0. The maximal density is given as about $6.74\,\rho_0$, above which the Skyrmion does not exist anymore. The filled circle on the solid curve represents the value of T^*_{00}/M^*_N at normal nuclear matter density.

Nucleon in nuclear matter

Structure studies 2: Transverse EM charge densities

- Definition of EM ff's $\langle N(p', S') | J_\mu^{EM}(0) | N(p, S) \rangle$

$$= \bar{u}_N(p', S') \left[\gamma_\mu F_1^*(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2m_N} F_2^*(q^2) \right] u_N(p, S)$$

- These Pauli and Dirac ff's can be expressed by Sachs ff's

$$G_E(Q^2) = F_1(Q^2) + \frac{Q^2}{4M_N^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$

- They give an information about transverse charge distributions inside the nucleon

$$\rho_0^*(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) \frac{G_E^*(Q^2) + \tau G_M^*(Q^2)}{1 + \tau}$$

$$\rho_T^*(\boldsymbol{b}) = \rho_0^*(b) - \sin(\phi_b - \phi_S) \times \int_0^\infty \frac{Q^2\,dQ}{4\pi m_N} J_1(bQ) \frac{-G_E^*(Q^2) + G_M^*(Q^2)}{1 + \tau}.$$

$$\boldsymbol{b} = b(\cos\phi_b \hat{\boldsymbol{e}}_x + \sin\phi_b \hat{\boldsymbol{e}}_y)$$

Structure studies 2: Transverse EM charge densities inside an unpolarized nucleon [UY, H.C.Kim, Phys.Lett. B726 (2013)]

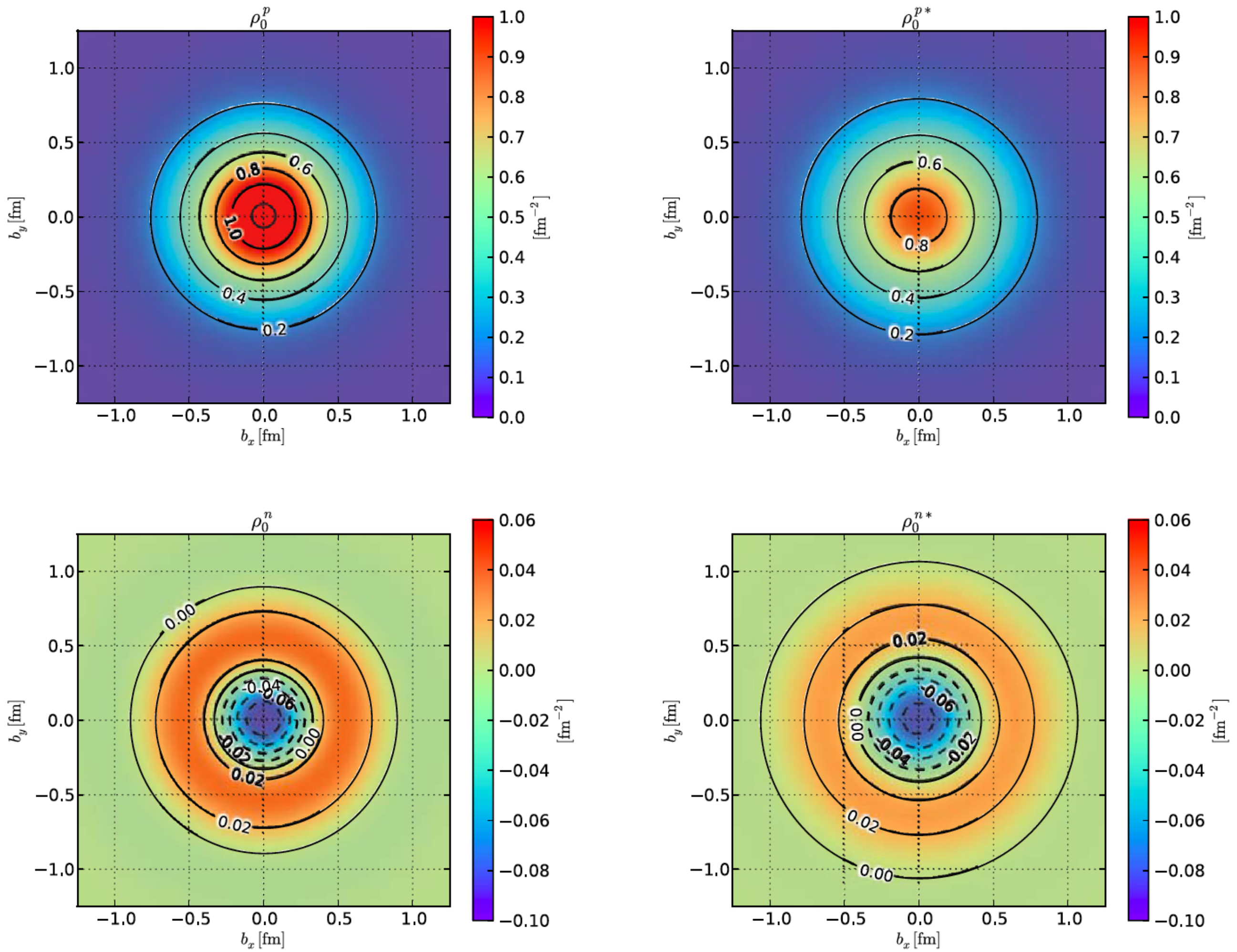


Fig. 3. Quark transverse charge densities inside an unpolarized proton (upper panels) and a neutron (lower panels) in free space (left panels) and at nuclear matter density $\rho_0 = 0.5m_\pi^3$ (right panels).

Nucleon in nuclear matter

Structure studies 2: Transverse EM charge densities inside the polarized nucleon [UY, H.C.Kim, Phys.Lett. B726 (2013)]

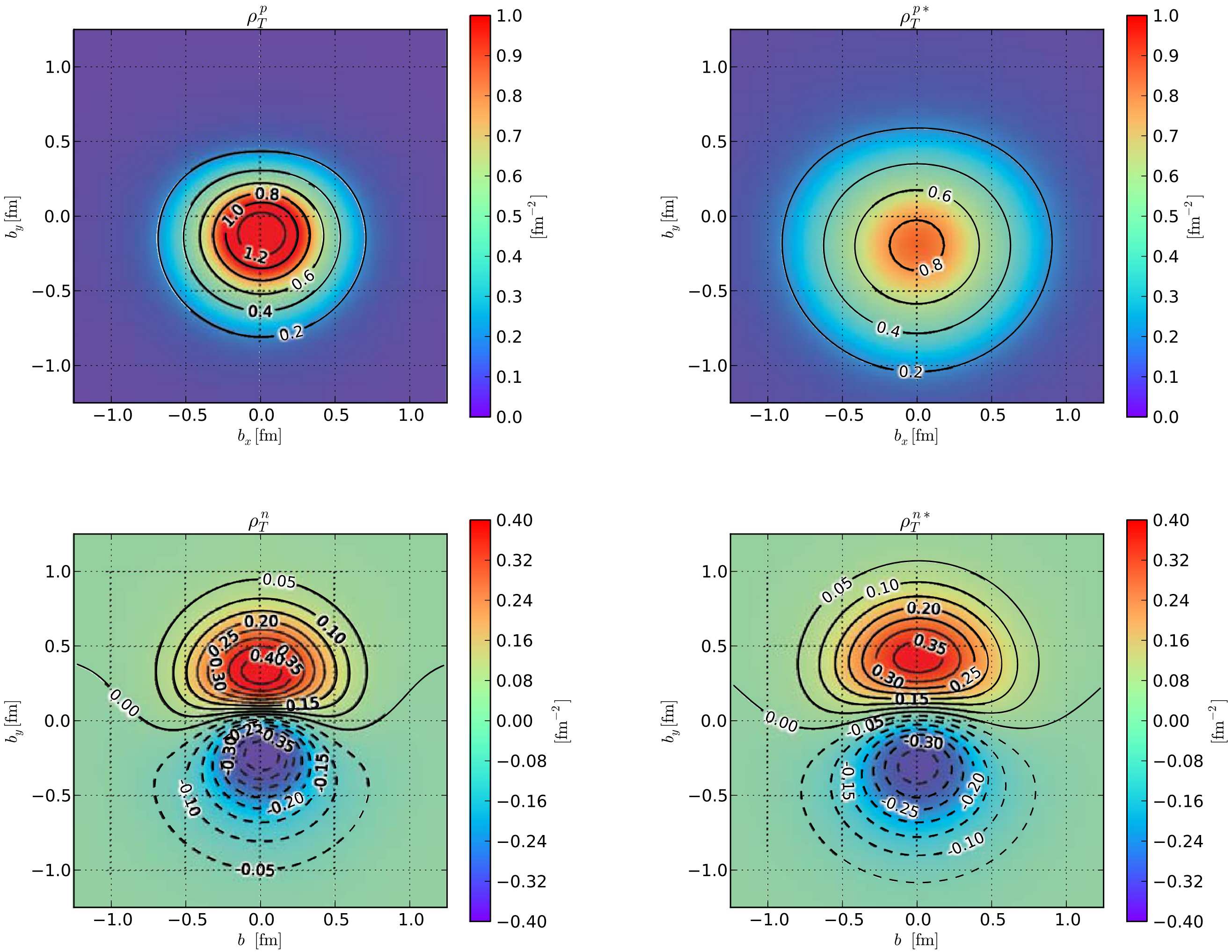


Fig. 4. Transverse charge densities of the proton (upper panels) and neutron (lower panels) in free space (left panels) and in nuclear matter with the density $\rho_0 = 0.5 m_\pi^3$ (right panels).

Nucleon in nuclear matter

Masses [UY, PRC88 (2013)]

- Isoscalar effective mass

$$m^*_{N,s} = M^*_S + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2}\left(a^{*2} + \frac{\Lambda^{*2}_{\text{env}}}{\Lambda^{*2}}\right)$$

- Isovector effective mass (relevant to: universe evolution in Early stage; stability of drip line nuclei; mirror nuclei; transport in heavy-ion collisions; asymmetric nuclear matter)

$$\Delta m^*_{np} = a^* + \frac{\Lambda^*_{\text{env}}}{\Lambda^*}$$

- Effective masses of the nucleons

$$m^*_{n,p} = m^*_{N,s} - \Delta m^*_{np} T_3$$

Nuclear Matter

Nuclear matter

From the Bethe-Weizsacker formula

$$\varepsilon\,(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \text{[illegible]}$$

The binding-energy-formula terms in the framework of present model can be obtained considering

We are ready to reproduce (Volume term, Asymmetry term)

- Volume term
  - Symmetric infinite nuclear matter
- Asymmetry term
  - Isospin asymmetric environment
- Surface and Coulomb terms
  - Nucleons in a finite volume
- Finite nuclei properties
  - Local density approximation

The volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda,\delta) = \varepsilon_V(\lambda) + \varepsilon_S \delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda,\delta)$$

  - λ is normalised nuclear matter density
  - δ is asymmetry parameter
  - ε_S is symmetry energy

- In our model
  - Symmetric matter
  - Asymmetric matter

$$\varepsilon_V(\lambda) = m^*_{N,s}(\lambda,0) - m_N^{\text{free}}$$

$$\varepsilon_A(\lambda,\delta) = \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda)$$
$$= m^*_{N,s}(\lambda,\delta) - m^*_{N,s}(\lambda,0) + m^*_{N,V}(\lambda,\delta)\delta$$

Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left. \frac{\partial \varepsilon_V(\lambda)}{\partial \lambda} \right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left. \frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2} \right|_{\rho=\rho_0} \quad Q = 27\lambda^3 \left. \frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3} \right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \cdots$$

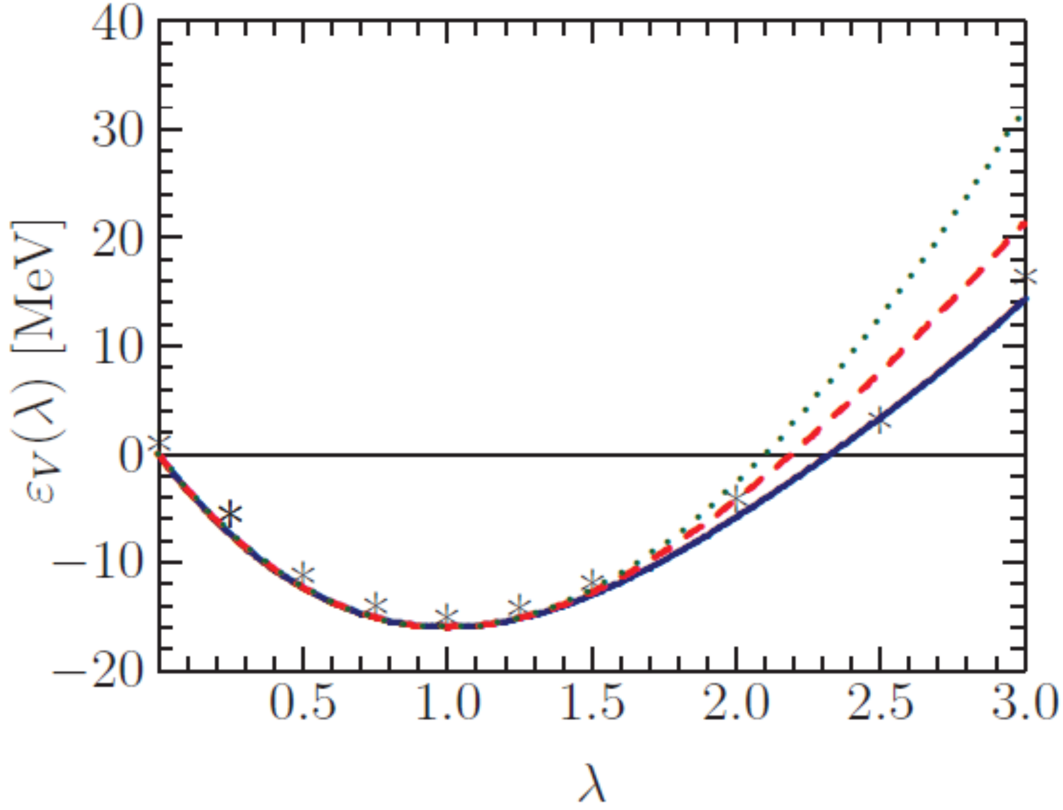
Symmetric matter

Volume energy
[UY, PRC88 (2013)]

- Set I – solid
- Set II – dashed
- Set III – dotted

For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.

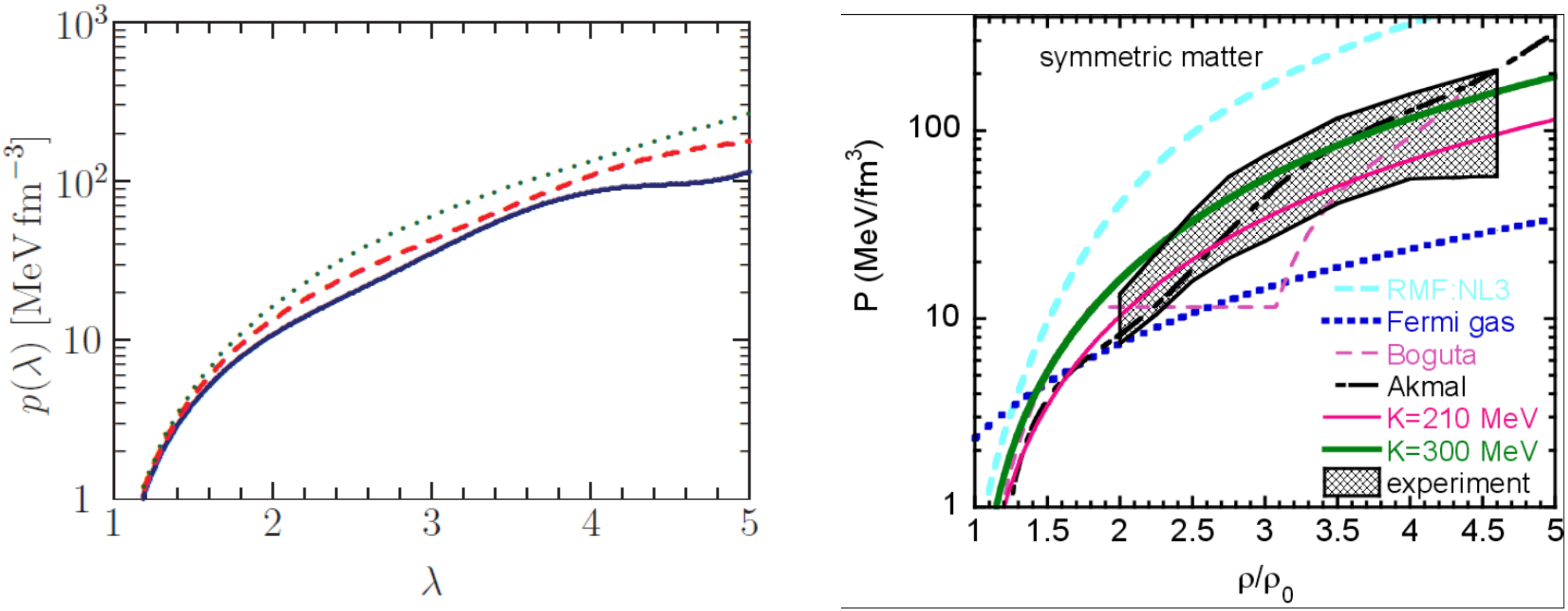
(From Arigonna 2 body interactions + 3 body interactions)



Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

Symmetric matter

Pressure
[UY, PRC88 (2013)]



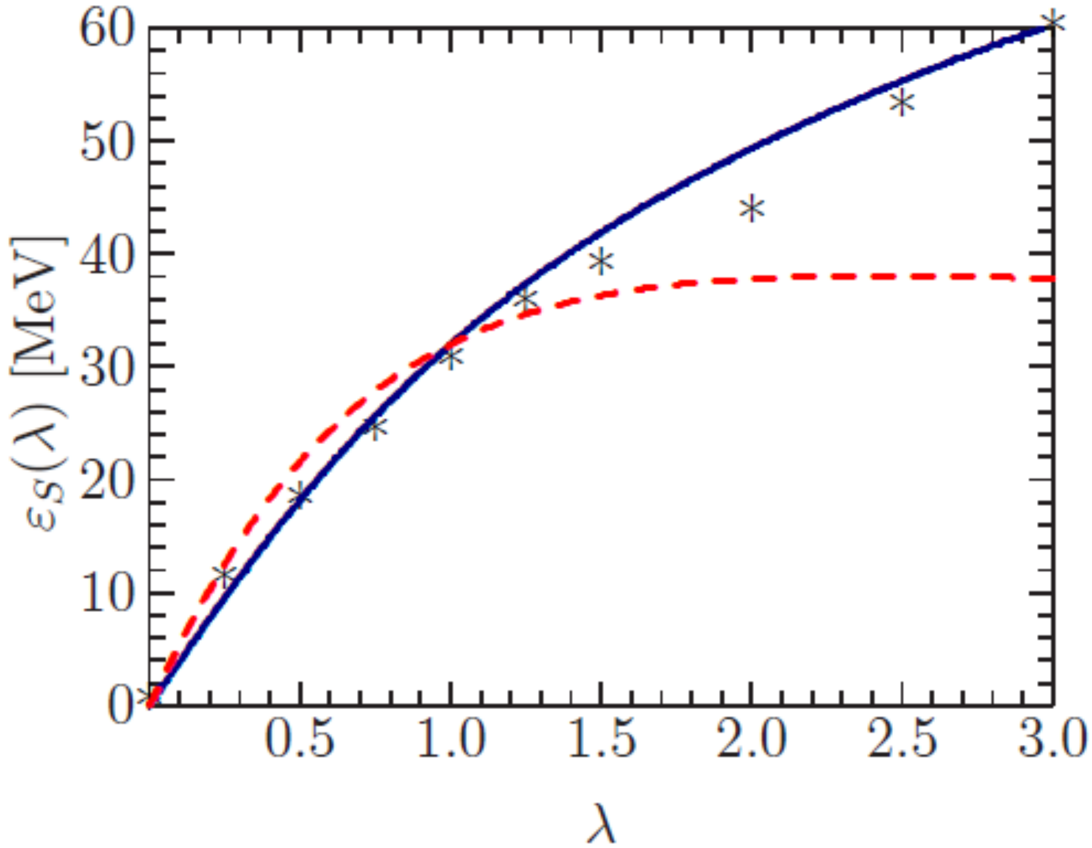
For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

Symmetry energy

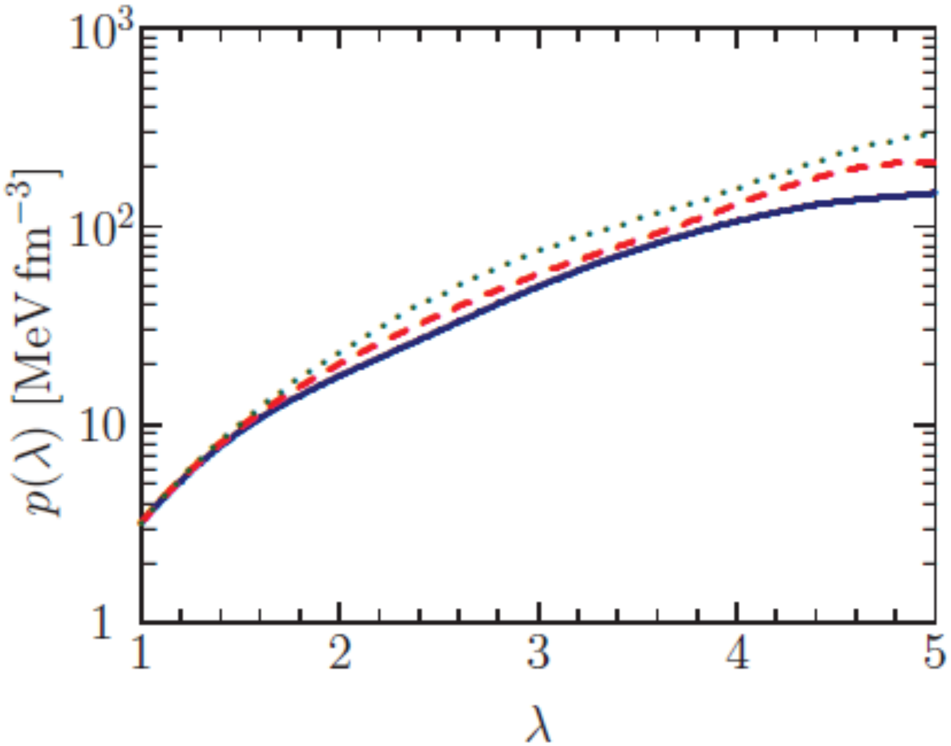
- Solid $L_s = 70\,\mathrm{MeV}$
- Dashed $L_s = 40\,\mathrm{MeV}$

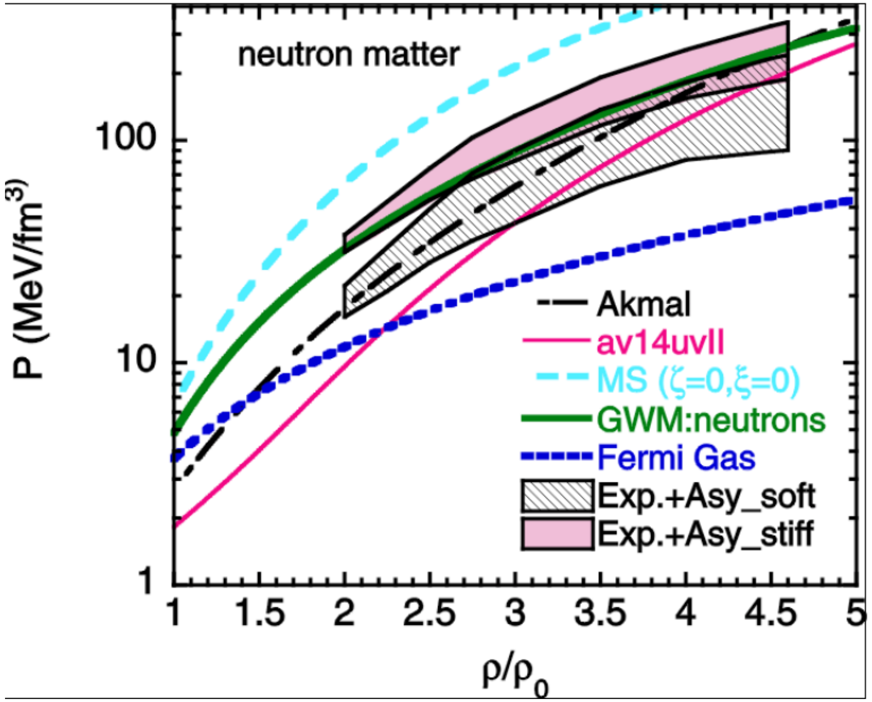
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.

(From arigonna 2 body interactions + 3 body interactions)



Pressure in neutron matter
[UY, PRC88 (2013)]





For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

Asymmetric matter

Low density behaviour of symmetry energy

For comparison:
Trippa-Colo-Vigezzi
[PRC 77, 061304 (2008)];
From analysis of GDR
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\text{fm}^{-3}) < 24.9\ \text{MeV}$$

Consequently one can predict in this model:

$$K_\tau = K_s - 6L_s$$

$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_S$$

$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\text{fm}^{-3})$ [MeV]
32	40	−181	−301	−257	25.15
32	50	−160	−310	−254	24.15
32	60	−126	−306	−239	23.22
32	70	−80	−290	−211	22.37
32	80	−21	−261	−172	21.57
32	90	50	−220	−119	20.82
32	100	134	−166	−55	20.13

Neutron Stars

Neutron star properties

- **TOV equations**

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1-\frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1+\frac{P(r)}{\mathcal{E}(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

- **Energy-pressure relation**

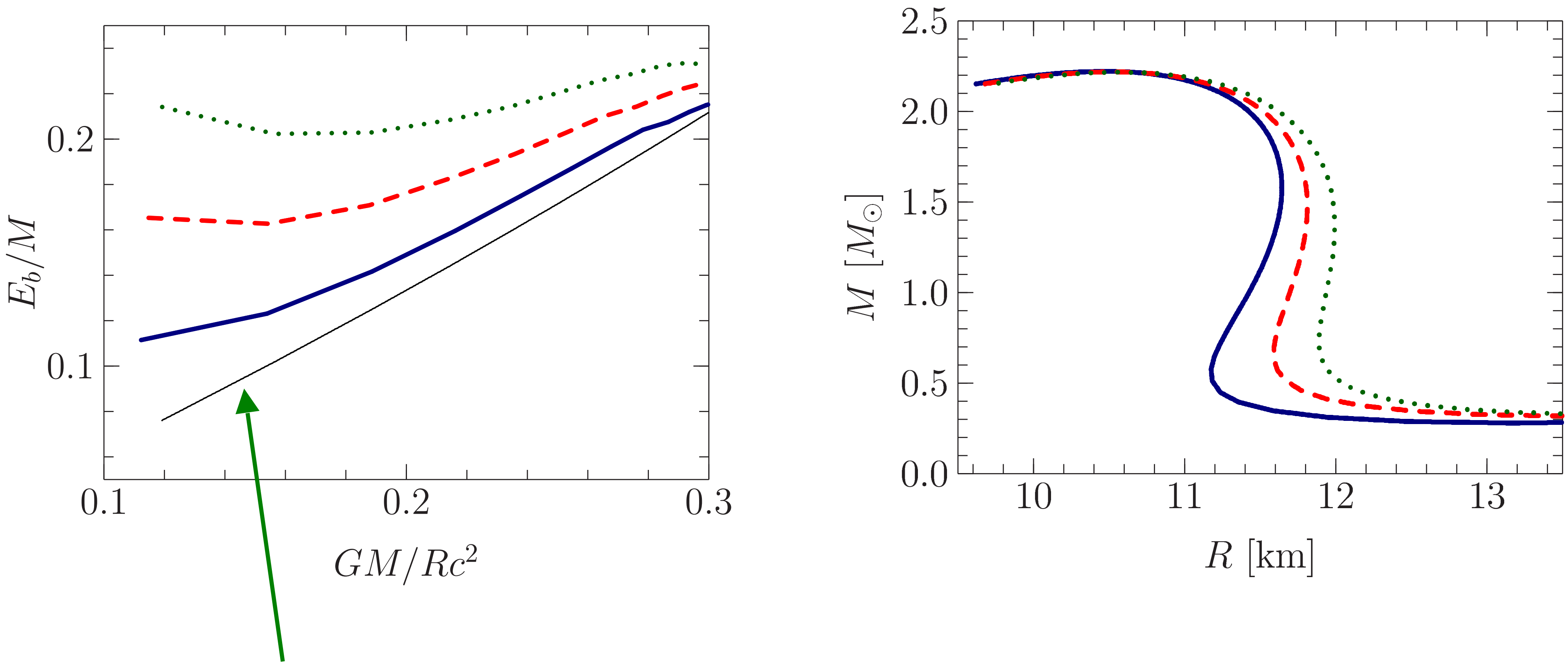
$$P = P(\mathcal{E})$$

$$P(\lambda) = \rho_0 \lambda^2 \frac{\partial \varepsilon(\lambda,1)}{\partial \lambda}\,,$$
$$\mathcal{E}(\lambda) = [\varepsilon(\lambda,1)+m_N]\lambda\rho_0 .$$

- **Neutron star's mass**

$$\mathcal{M}(r) = 4\pi \int_0^r \mathrm{d}r\, r^2 \mathcal{E}(r)\,.$$

Neutron star properties [UY, PLB749 (2015)]



From Ref. [J.M. Lattimer & M. Prakash, Astrophys. J. 550 (2001)].

Neutron star properties

[UY, PLB749 (2015)]

TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters): n_c is central number density, ρ_c is central energy-mass density, R is radius of the neutron star, $M_{\max}$ is possible maximal mass, A is number of baryons in the star, E_b is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass $M_{\max}$ and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

Set	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	$M_{\max}$ [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	M [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

Consistency (difference) with (from) other approaches

Consistency (difference) with (from) other approaches

One can find density functionals from the reparametrization scheme
[UY, PRC88 (2013)]

- Five density dependent parameters

Shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau, \quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

- Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \frac{\rho}{\rho_0}}{1 + C_5 \frac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Consistency (difference) with (from) other approaches

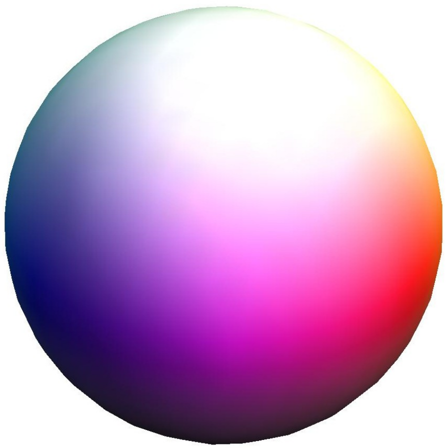
Low energy constants in nuclear at normal nuclear matter density ρ_0

	Present model	ChPT [1]	QCD sum rules [2]
$F^*_{\pi,t} / F_\pi$	0.37	0.74	0.79
$F^*_{\pi,s} / F_\pi$	0.72	< 0	0.78

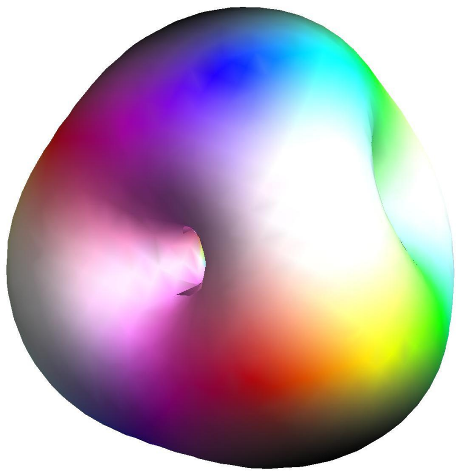
[1] U. Meissner, J. Oller, A. Wirzba, Annals Phys. 297 (2002) 27.
[2] H. Kim, M. Oka, NPA720 (2003) 368.

Consistency (difference) with (from) other approaches

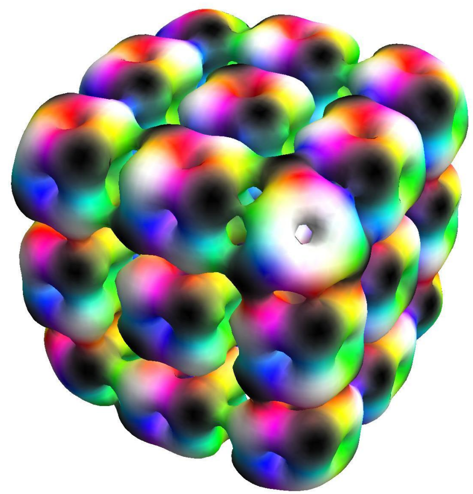
Surface of constant baryon density skyrmions **[Feist, D.T.J.** ***et al.*** **Phys.Rev. D87 (2013)]**



$B = 1$



$B = 3$



$B = 104$

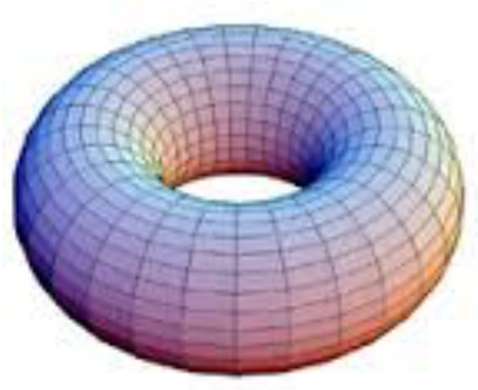
$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2} \mathrm{Tr}\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2$$

Changes from a nucleus to a nucleus (“calibration”)

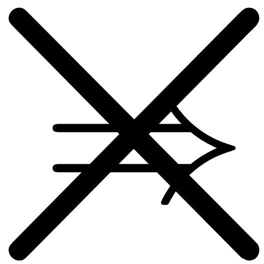
Consistency (difference) with (from) other approaches

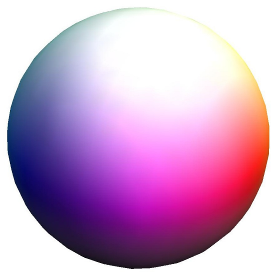
?

Topological non-triviality

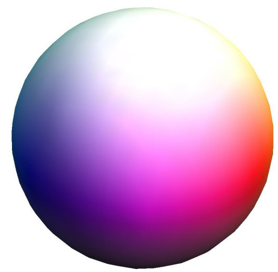


$B = 2$

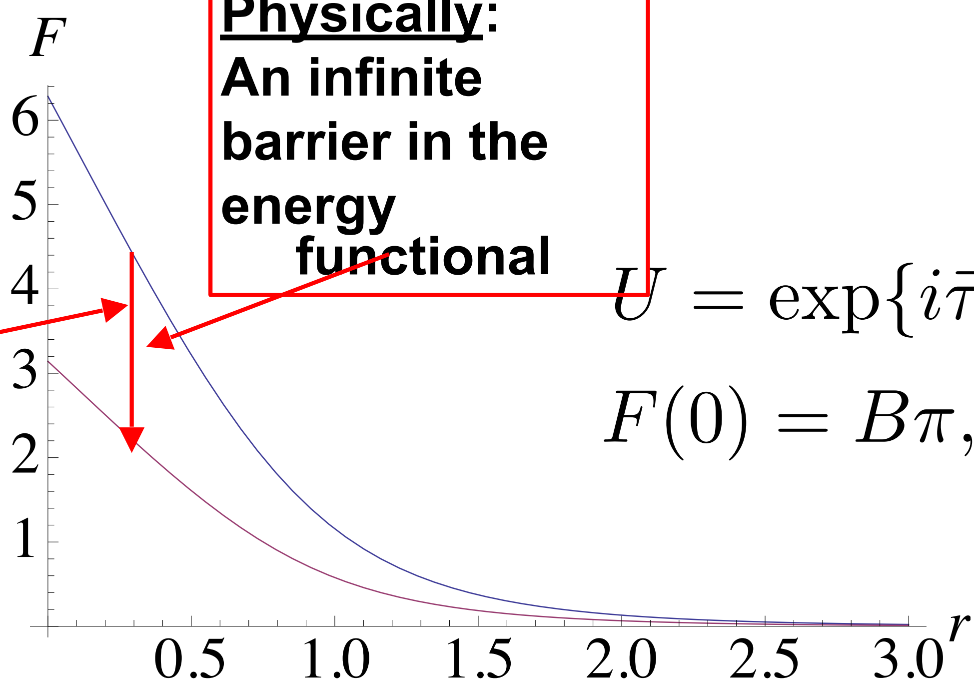




$B = 1$



$B = 1$



<u>Physically</u>:
An infinite barrier in the energy functional

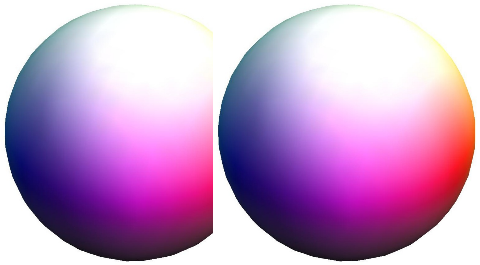
<u>Mathematically</u>:
There is no smooth transition between topologically distinct mappings

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

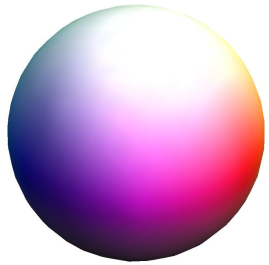
$$F(0) = B\pi, \qquad F(\infty) = 0$$

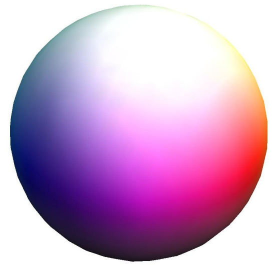
Consistency (difference) with (from) other approaches

Physically consistent picture (ansatz product)



$$\xRightarrow{\text{Well defined}}$$





$B = 2$ $\qquad$ $B = 1$ $\qquad$ $B = 1$

Overlapping at small distances

Well separated at large distances

One can reproduce SS potential and project it into NN potential.

$$U_{\text{system}} = U(\vec{r}_1)U(\vec{r}_2)$$

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

$$F(0) = \pi, \qquad F(\infty) = 0$$

Consistency (difference) with (from) other approaches

Other approaches

- Classical crystalline structures
  - Cubic structure
    [M. Kutschera *et al*. Phys. Rev. Lett. **53** (1984)]
    [I. R. Klebanov, Nucl. Phys. B **262** (1985)]

  - Phase structure analysis using FCC crystal
    [H.-J. Lee *et al.* Nucl. Phys. A **723** (2003)]

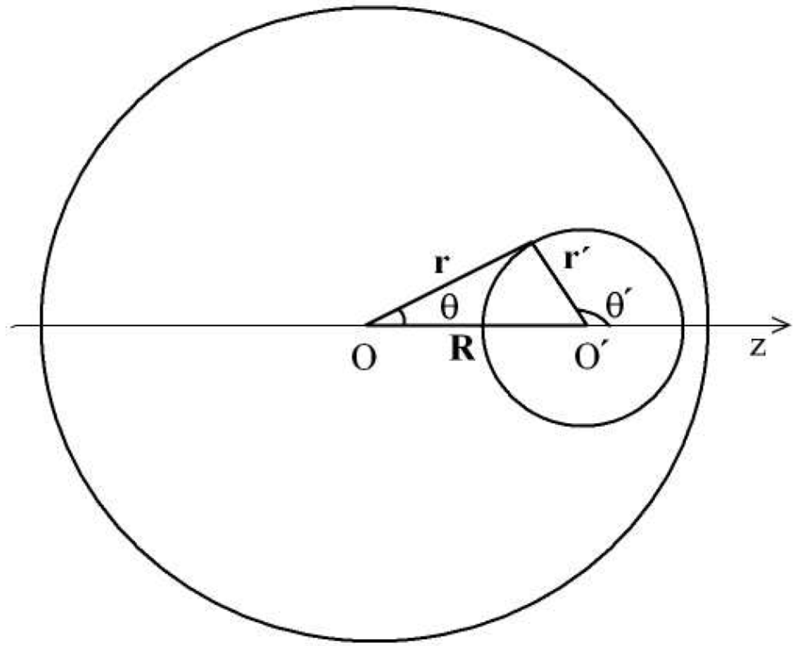
- Skyrmions in hypersphere
  - System properties from the single skyrmion in hypersphere
    [N. S. Manton and P. J. Ruback, Phys. Lett. B **181** (1986)]

Nucleon in Finite Nuclei

Nucleon in finite nuclei

The nucleon in a nucleus will include

- Local density approach for environment
- R dependence of a results
- Deformations
  - In particular, axially symmetric case allows the deformations in polar direction
- Polar deformations can be represented
  - in the isotopic vector and
  - in the profile function



$$\boldsymbol{N}(\boldsymbol{r}-\boldsymbol{R}) = \begin{pmatrix} \sin\Theta(\boldsymbol{r}-\boldsymbol{R})\cos\varphi \\ \sin\Theta(\boldsymbol{r}-\boldsymbol{R})\sin\varphi \\ \cos\Theta(\boldsymbol{r}-\boldsymbol{R}) \end{pmatrix}$$

$$P = P(|\boldsymbol{r}-\boldsymbol{R}|,\theta), \qquad \Theta = \Theta(|\boldsymbol{r}-\boldsymbol{R}|,\theta)$$

$$U(\boldsymbol{r}-\boldsymbol{R}) = \exp\left[i\boldsymbol{\tau}\cdot\boldsymbol{N}(\boldsymbol{r}-\boldsymbol{R})P(\boldsymbol{r}-\boldsymbol{R})\right]$$

The Equations of Motion

- The coupled partial differential equations (not an easy problem)

$$f\,(F_{\tilde{r}\tilde{r}},\, F_{\theta\theta},\, F_{\tilde{r}},\, F_{\theta},\, \Theta_{\theta},\, F,\, \Theta) = 0,$$

$$g(\Theta_{\theta\theta},\, \Theta_{\theta},\, F_{\tilde{r}},\, F_{\theta},\, \Theta,\, F) = 0,$$

- A numerical variational method can be applied

$$P(r,\theta) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+m_\pi r)\big(1+u(\theta)\big)\right\} e^{-f(r)r}$$

$$\Theta(r,\theta) = \theta + \zeta(r,\theta),$$

$$F(r) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+m_\pi r)\right\} e^{-f(r)r}, \qquad u(\theta) = \sum_{n=1}^{\infty} \gamma_n \cos^n\theta$$

$$f(r) = \beta_0 + \beta_1 e^{\beta_2 r^2}.$$

$$\zeta(r,\theta) = r e^{-\delta_0^2 r^2} \sum_{n=1}^{\infty} \delta_n \sin 2n\theta,$$

$$\lim_{r\to 0} F(r) = \pi - Cr,$$

$$\lim_{r\to\infty} F(r) = D\,(1+m_\pi r)\,\frac{e^{-m_\pi r}}{r^2},$$

Accuracy of the variational method

- In spherically symmetric approximation (e.g. nucleon in the centre of the spherical nucleus) one can explicitly solve Equations of Motion and compare with results of variational method
- Skyrme term is not modified in nuclear matter (table below)

Element		r_0 [fm]	$10\beta_0$ [m_π]	β_1 [m_π]	β_2 [m_π^2]	m_p^* [MeV]	Δm_{np}^* [MeV]	$\Delta m_{np}^{*(EM)}$ [MeV]	μ_p^* [n.m.]	μ_n^* [n.m.]	$\langle r^2\rangle_{E,S}^{*1/2}$ [fm]	$\langle r^2\rangle_{E,V}^{*1/2}$ [fm]
	i)	–	–	–	–	938.268	1.291	−0.686	1.963	−1.236	0.481	0.739
free space	ii)	0.954	0.075	1.311	−0.009	938.809	1.313	−0.687	1.966	−1.241	0.481	0.739
	i)	–	–	–	–	593.285	1.668	−0.526	2.355	−1.276	0.656	0.850
^{14}N	ii)	1.393	0.076	0.920	0.226	598.505	1.655	−0.536	2.230	−1.209	0.648	0.810
	i)	–	–	–	–	585.487	1.697	−0.517	2.393	−1.297	0.667	0.863
^{16}O	ii)	1.426	0.076	0.907	0.219	590.175	1.685	−0.527	2.341	−1.232	0.660	0.825
	i)	–	–	–	–	558.088	1.804	−0.480	2.584	−1.422	0.722	0.942
^{38}K	ii)	1.493	0.076	0.841	0.153	559.957	1.802	−0.485	2.550	−1.377	0.718	0.910
	i)	–	–	–	–	557.621	1.804	−0.478	2.569	−1.428	0.724	0.947
^{40}Ca	ii)	1.489	0.076	0.839	0.149	559.378	1.802	−0.483	2.557	−1.383	0.720	0.914

Nucleon in finite nuclei

The Hamiltonian of the model

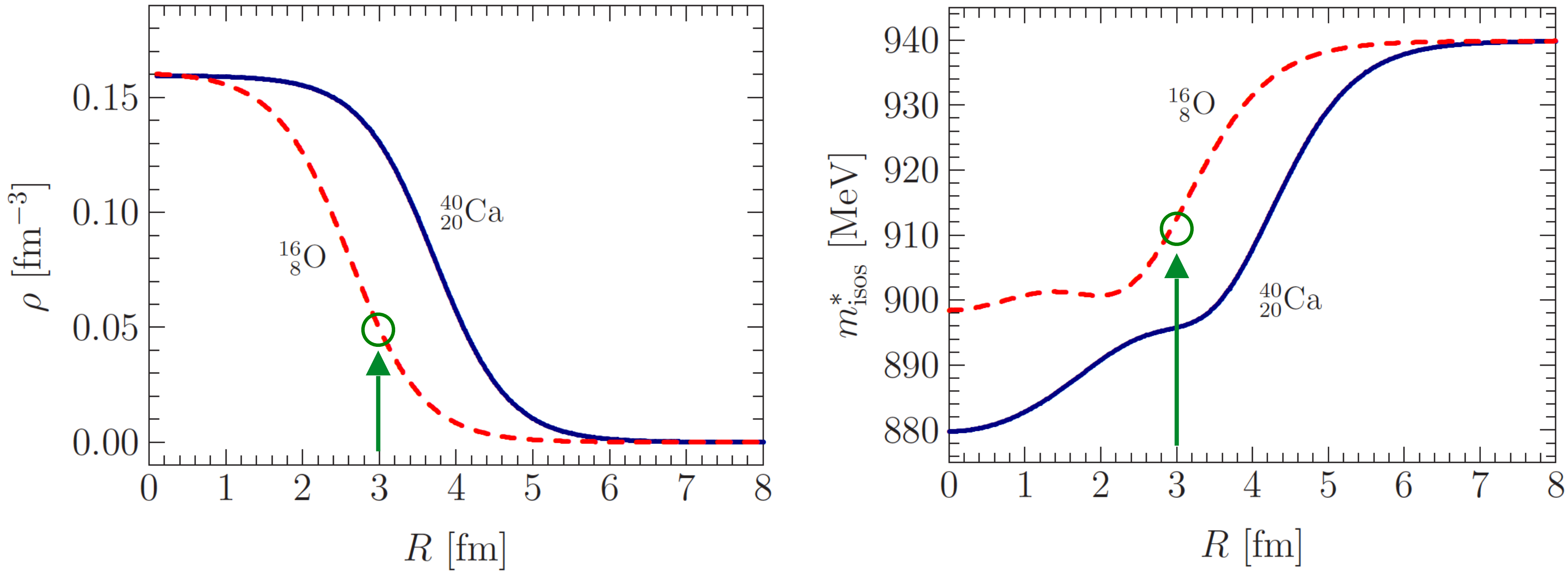
- Has the form as in the case of symmetric top

$$\hat{H} = M^*_{\mathrm{NP}} + \mathcal{M}_-^2 \Lambda_{\mathrm{mes}} + \frac{\Lambda^{*2}_{\mathrm{env}}}{2\Lambda^*_{\omega\Omega,33}} + \frac{(\hat{T}_1^2 + \hat{T}_2^2)\Lambda^*_{\Omega\Omega,12} + (\hat{J}_1^2 + \hat{J}_2^2)\Lambda^*_{\omega\omega,12}}{2(\Lambda^*_{\omega\omega,12}\Lambda^*_{\Omega\Omega,12} - \Lambda^{*2}_{\omega\Omega,12})}$$
$$+ \frac{(\hat{T}_1\hat{J}_1 + \hat{T}_2\hat{J}_2)\Lambda^*_{\omega\Omega,12}}{\Lambda^*_{\omega\omega,12}\Lambda^*_{\Omega\Omega,12} - \Lambda^{*2}_{\omega\Omega,12}} + \frac{\hat{T}_3^2}{2\Lambda^*_{\omega\Omega,33}} - \boxed{\left(a^* + \frac{\Lambda^*_{\mathrm{env}}}{\Lambda^*_{\omega\Omega,33}}\right)}\hat{T}_3 \,.$$

Neutron-proton mass difference in finite nuclei

Nucleon in finite nuclei

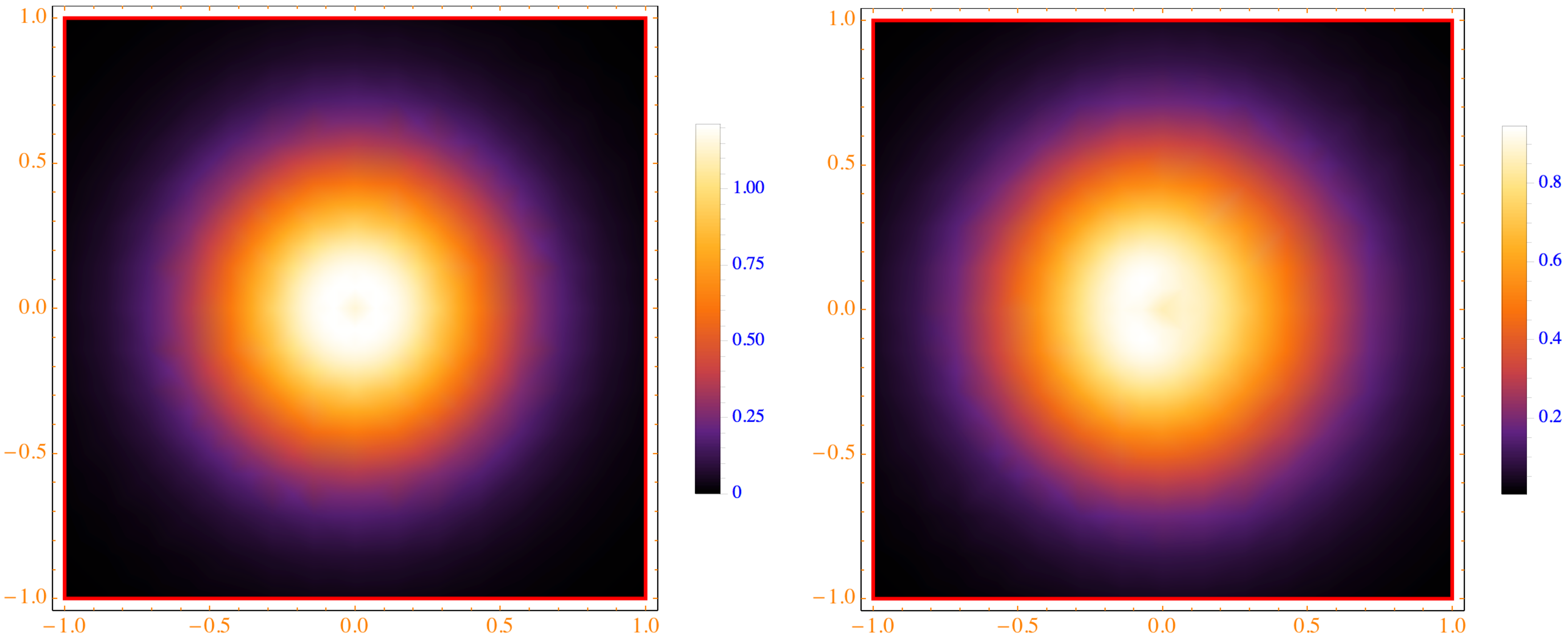
The densities of nuclei (left) and the isoscalar mass in nuclei (right)



On the right panel R is a distance between the geometrical centres of nucleus and nucleon

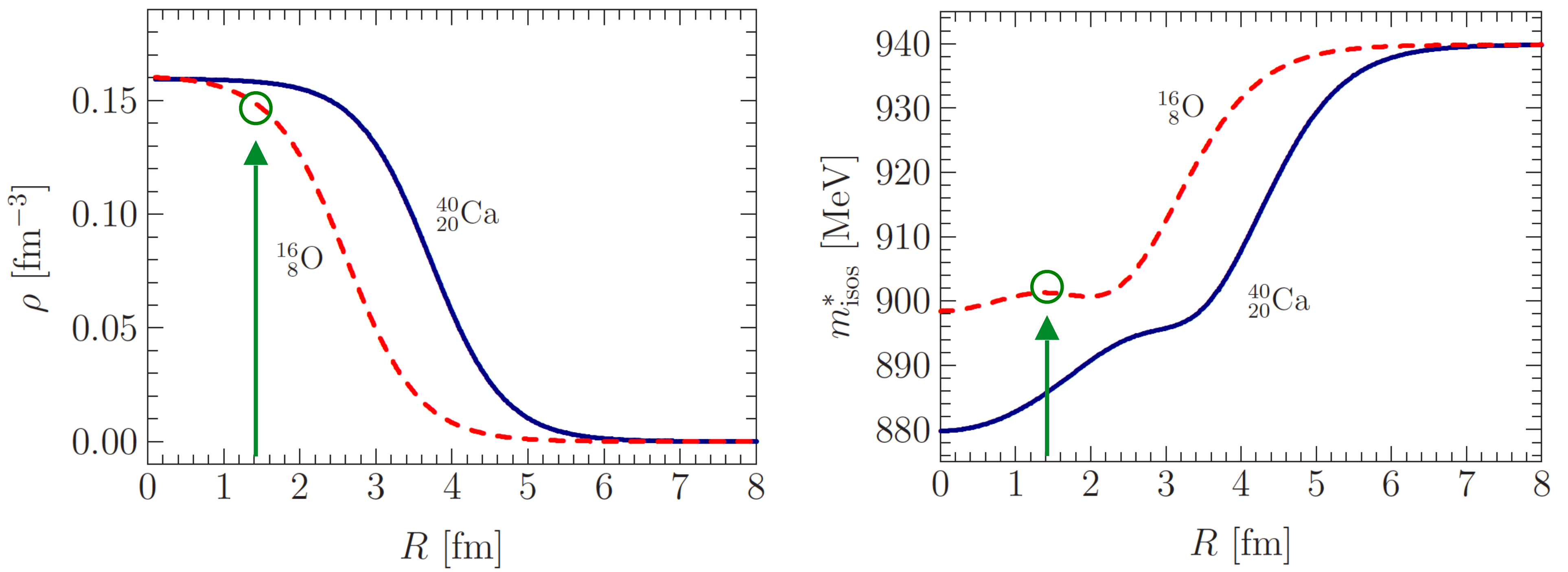
Nucleon in finite nuclei

Baryon charge distribution inside the nucleon under the consideration



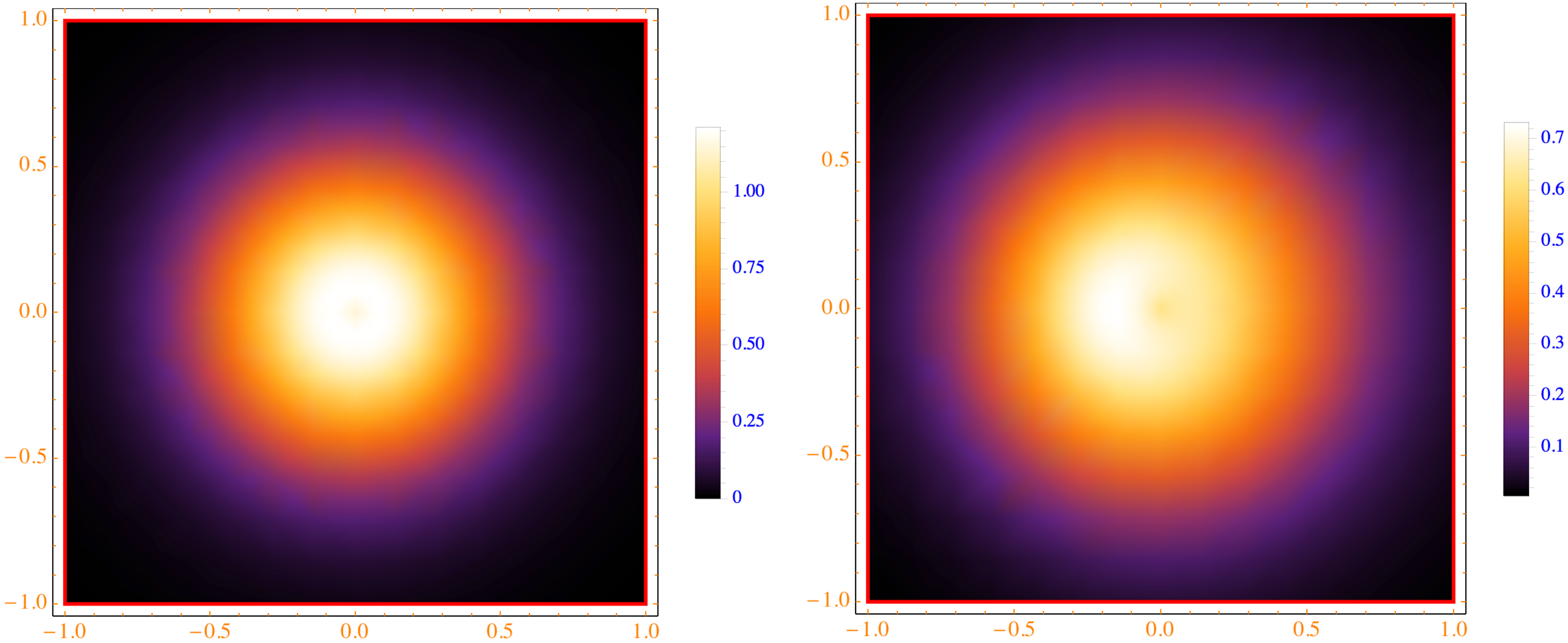
In free space (left) and in O16 (right), R = 3fm

The densities of nuclei (left) and the isoscalar mass in nuclei (right)



On the right panel R is a distance between the geometrical centres of nucleus and nucleon

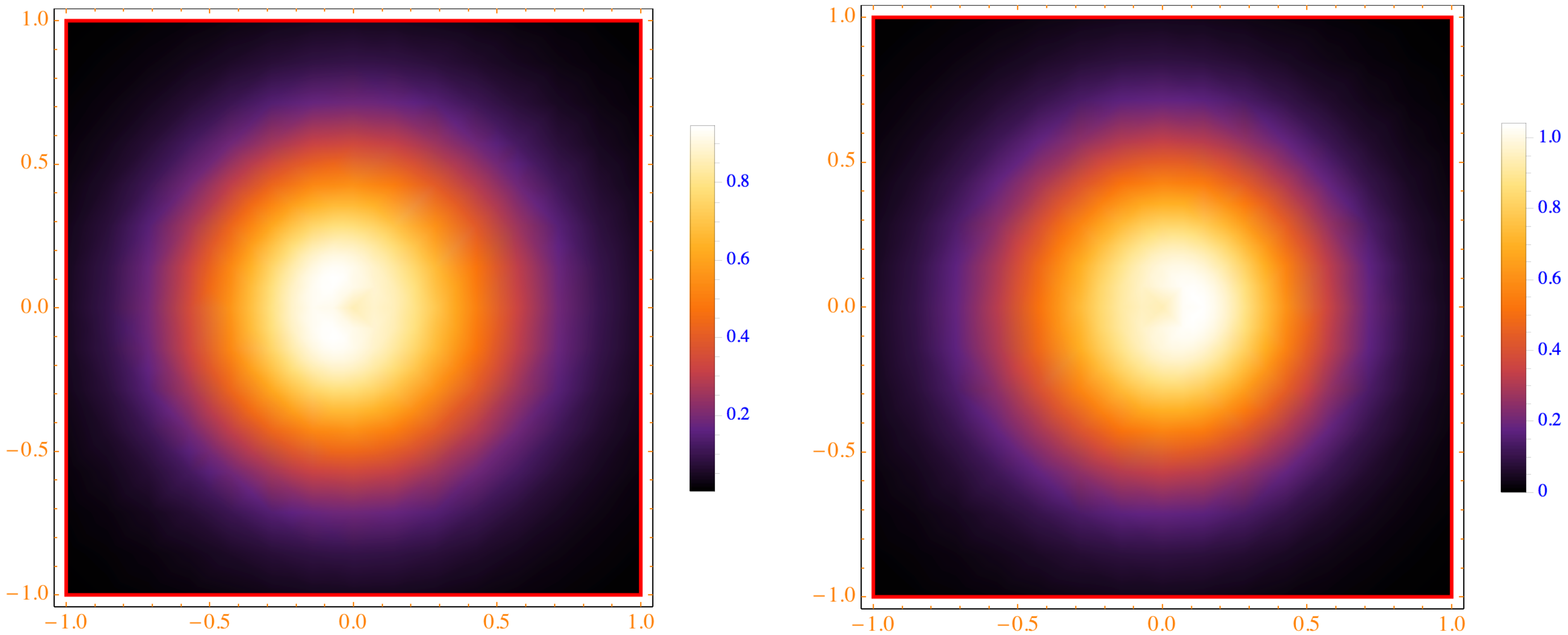
Baryon charge distribution inside the nucleon under the consideration



In free space (left) and in O16 (right), R = 1.5fm

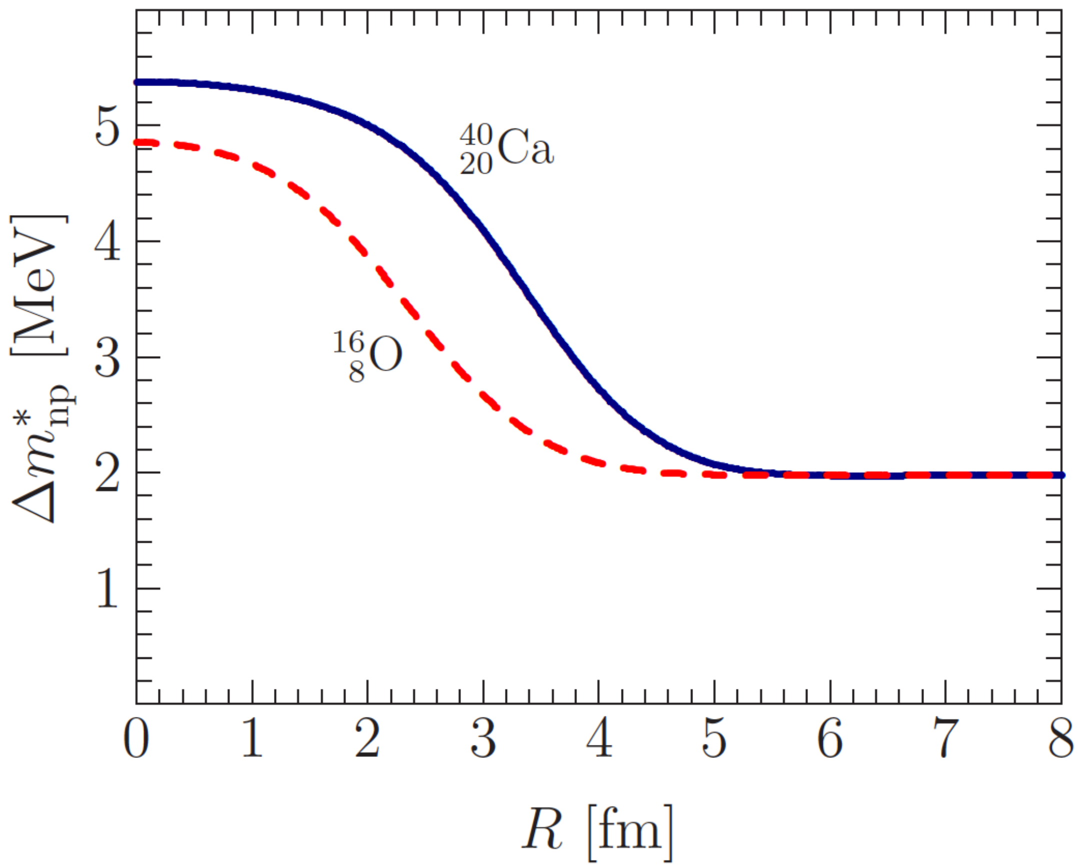
Nucleon in finite nuclei

Baryon charge distribution inside the nucleon under the consideration



In O16 (left), R = 3fm and in Ca40 (right), R = 4.5fm

The neutron-proton mass difference in finite nuclei



R is a distance between the geometrical centres of nucleus and nucleon

Properties of Finite Nuclei

Mirror nuclei

The Nolen-Schiffer anomaly (NSA)

- The mass difference of mirror nuclei

$$\Delta M \equiv {}^{A}_{Z+1}\mathbf{M}_{N} - {}^{A}_{Z}\mathbf{M}_{N+1} = \Delta E_{\rm EM} - \Delta m^{*}_{\rm np}$$

- EM part was calculated with high accuracy (within 1% error) in very detailed form (e.g., the exchange term, the center-of-mass motion, finite-size effects of the proton and neutron charges, magnetic interactions, vacuum effects, the dynamical effect of the neutron-proton mass difference, and short-range two-body correlations, etc.)
- If neutron-proton mass difference is not changed in nuclear matter then the above formula cannot be satisfied.

$$\overline{\Delta}_{\rm NSA} = \Delta m_{\rm np} - \left(\Delta \overline{m}^{*(1)}_{\rm np} + \Delta \overline{m}^{*(2)}_{\rm np} \right)$$

Mirror nuclei

The Nolen-Schiffer anomaly (NSA)

- Is defined as ("bar" means averaging over the R)

$$\overline{\Delta}_{\rm NSA} = \Delta m_{\rm np} - \left(\Delta \overline{m}^{*(1)}_{\rm np} + \Delta \overline{m}^{*(2)}_{\rm np} \right)$$

- where

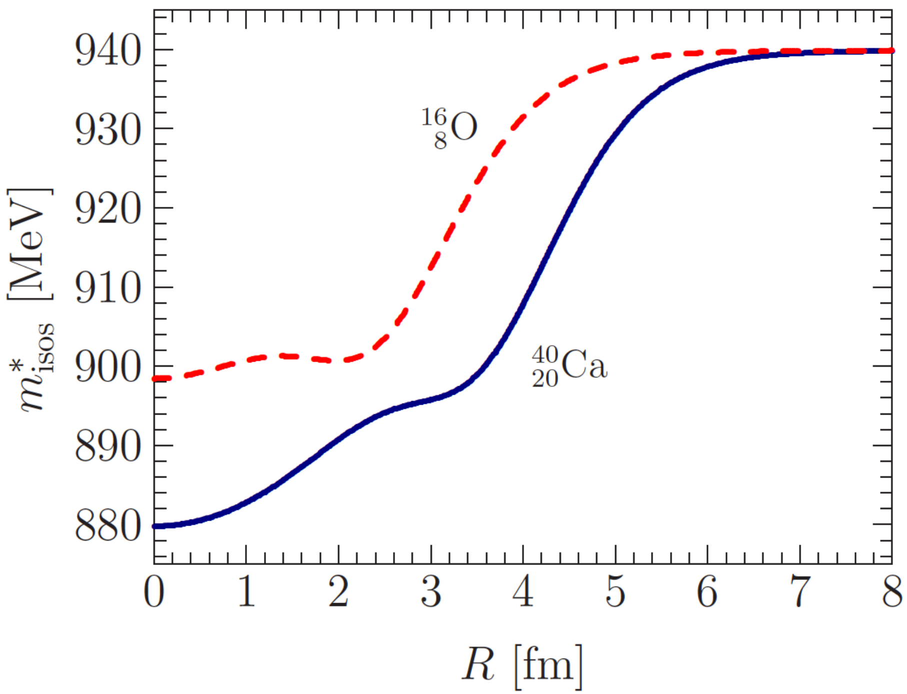
$$\Delta \overline{m}^{*}_{\rm np} \approx \int \left(\Delta \psi^{(2)}_{\rm np} m^{*}_{\rm p} + \left(\psi^{(p)}\right)^2 \Delta m^{*}_{\rm np} \right) {\rm d}^3 R$$
$$\equiv \Delta \overline{m}^{*(1)}_{\rm np} + \Delta \overline{m}^{*(2)}_{\rm np} \, ,$$

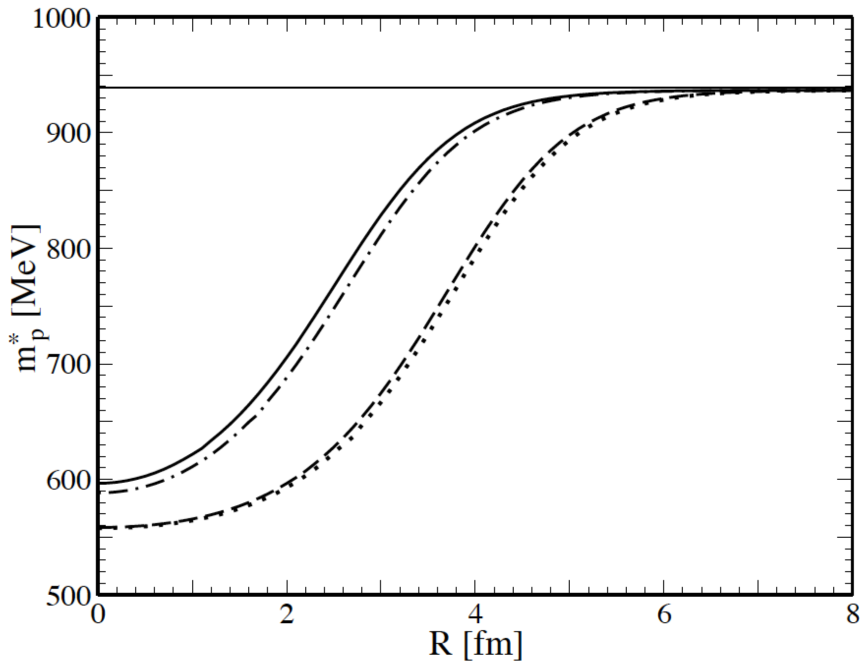
Nuclei	$\overline{m}^*_{\rm p}$		Present approach						$\overline{\Delta}_{\rm NSA}$	$\overline{\Delta}_{\rm NSA}$
			$\alpha_{\rm ren}=0$			$\alpha_{\rm ren}=0.95$				
	$\alpha_{\rm ren}=0$	$\alpha_{\rm ren}=0.95$	$\Delta\overline{m}^{*(1)}_{\rm np}$	$\Delta\overline{m}^{*(2)}_{\rm np}$	$\overline{\Delta}_{\rm NSA}$	$\Delta\overline{m}^{*(1)}_{\rm np}$	$\Delta\overline{m}^{*(2)}_{\rm np}$	$\overline{\Delta}_{\rm NSA}$	ref. [16]	ref. [17]
^{15}O-^{15}N	767.45	928.30	-4.27	1.56	4.02	-0.21	1.33	0.20	–	0.16 ± 0.04
^{17}F-^{17}O	812.35	930.54	-5.53	1.52	5.33	-0.28	1.32	0.27	0.31	0.31 ± 0.04
^{39}Ca-^{39}K	724.78	926.16	-8.11	1.67	7.75	-0.41	1.33	0.37	–	0.22 ± 0.08
^{41}Sc-^{41}Ca	771.71	928.51	-9.74	1.62	9.44	-0.49	1.33	0.47	0.62	0.59 ± 0.08

U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)]

Mirror nuclei

The nucleon mass in nuclei

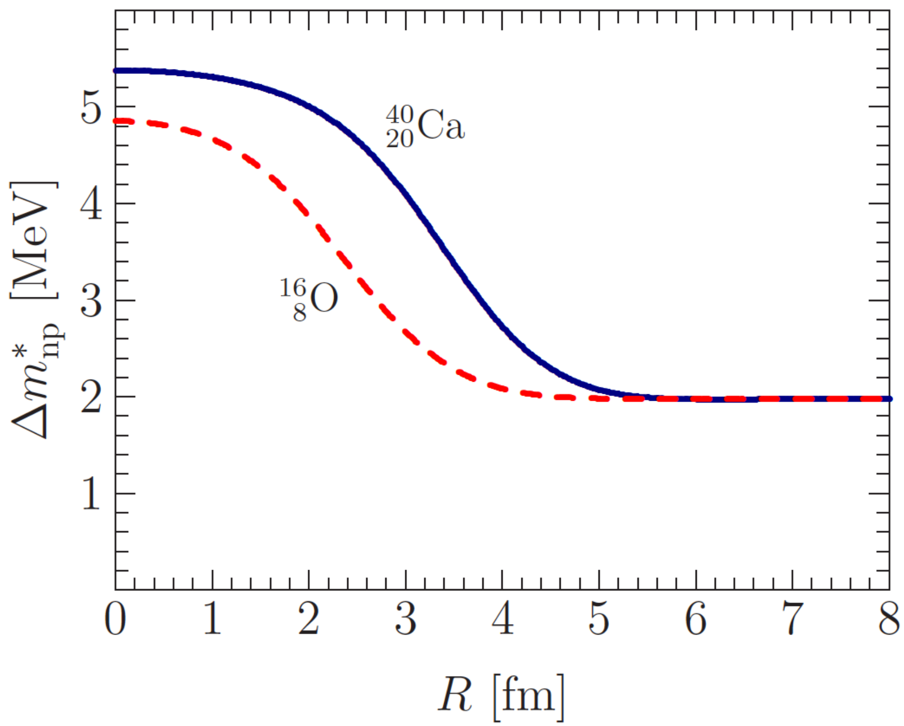


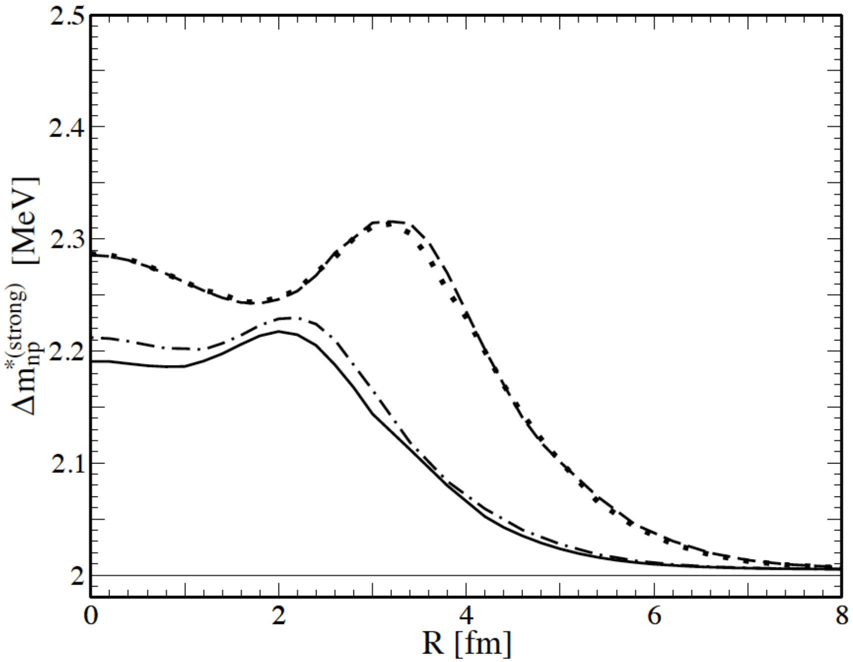


Present work (left) and U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)] (right)

Mirror nuclei

The neutron-proton mass difference in finite nuclei





Present work (left) and U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)] (right)

Summary and outlook

Summary and Outlook

The present model describes at same footing

the single nucleon properties

- **in free space considering it as a structure-full system**
- **in nuclear matter (EM and EMT form factors)**

as well as the properties of the whole nucleonic systems

- **infinite nuclear matter properties (volume and symmetry energy properties)**
- **matter under extreme conditions (e.g. neutron stars)**
- **few/many nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)**
- **nucleon knock-out reactions (lepton-nucleus scattering)**
- **possible changes in in-medium NN interactions**
- **etc**

Summary and Outlook

Applicability and extensions of the approach so far

- **Nucleon tomography in free space/nuclear medium**
  - **[H.Ch. Kim, P. Schweitzer, UY, PLB718 (2012)]**
  - **[H.Ch. Kim, UY, PLB726 (2013)]**
  - **[J.H.Jung, UY, H.Ch.Kim, Jour. Phys. G41 (2014)]**
  - **[J.H.Jung, UY, H.Ch.Kim, P. Schweitzer. PRD89 (2014)]**
- **Nucleon properties in asymmetric nuclear matter**
  - **[UY, Prog. Theor. Exp. Phys. 2014 (2014)]**
- **Isospin symmetric/asymmetric nuclear matter**
  - **[UY, PRC88 (2013)]**
- **Neutron stars**
  - **[UY, PLB749 (2015)]**
- **Vector mesons in nuclear matter**
  - **[J.H.Jung, UY, H.Ch.Kim, PLB 723 (2013)]**

Thank you very much for your attention!